Double scaling limits for field theory defects

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Based on 2202.03472⊕2206.09935 w/ Jorge Russo

- Studying defects in QFT is interesting for a number of reasons

    - Explore all operators in a QFT: extended operators may be sensible to finer details (e.g. topology of space, global properties of gauge group…)

    - If topological, they give rise to generalized symmetries

    - May serve as a diagnose the phases of the theory

    - Describe impurities coupled to the system

    - …

- However QFT (with/without defects) is hard…
- One strategy which has proved very successful is to look for small parameters on which one can expand. Celebrated examples include
    - the semiclassical approximation
    - large N
    - large spin
- In the recent past one new item added to the list
    - large charge sectors
- Inspired by this: can we access new information about defects in QFT???

Contents

- Motivation
- Local operators in N=2 SCFT’s in 4d at large charge and a double scaling limit
- Taking it over to Wilson lines in the k-symm product
- Defects in Wilson-Fisher
- Final comments

Correlation functions in N=2 and large charge

- N=2 theories are interesting playgrounds to tinker with QFT: they have SUSY enough so as to constrain dynamics to accessible limits but not too much so as to “trivialize”

- Some of them have holographic duals

- In particular, one can exploit SUSY to compute observables exactly

LOCALIZATION

This includes correlators, defect operators and even the partition function itself (meaningful for 4d N=2)

- The 4d superconformal algebra contains

$$\{\overline{Q}^a_{\dot\alpha}, \overline{S}^b_{\dot\beta}\} = \epsilon_{\dot\alpha\dot\beta}\epsilon^{ab}\left(\Delta - \frac{R}{2}\right) + \epsilon^{ab}M_{\dot\alpha\dot\beta} + \epsilon_{\dot\alpha\dot\beta}J^{ab}$$

- Hence an interesting shortening condition is

cf. Raffaele's talk!!

$$[\overline{Q}^a_{\dot\alpha}, O] = 0 \rightsquigarrow \Delta_O = \frac{R_O}{2},\ j_L = s = 0,\ (\text{and } \mathrm{j_R} = 0) \longrightarrow$$ **Chiral Primary Operator (CPO)**

- CPO's have a non-singular OPE (not to violate the BPS bound). As a consequence, they form a ring: the chiral ring

- Their 2-point functions are

$$\langle O_I(0)\overline{O}_{\overline{J}}(x)\rangle = \frac{g_{I\overline{J}}(\tau^i, \overline{\tau}^i)}{|x|^{2\Delta_I}}\,\delta_{\Delta_I,\Delta_{\overline{J}}}$$

Endowes the Coulomb branch of a very interesting geometry…but that's another story. See Papadodimas; Baggio, Niarchos & Papadodimas

- The 2-point functions can be mapped to the sphere

$$\langle A(x)\overline{B}(0)\rangle = \frac{C_{AB}}{|x|^{2\Delta_A}}\,\delta_{\Delta_A\Delta_B} \;\rightsquigarrow\; \langle |x|^{2\Delta_A} A(x)\overline{B}(0)\rangle = C_{AB}\delta_{\Delta_A\Delta_B}$$

- To extract C, we can take the large x limit

$$\lim_{|x|\to\infty} |x|^{2\Delta_A} A(x) = 4^{\Delta_A} \lim_{|x|\to\infty} \left(1+\frac{|x|^2}{4}\right)^{\Delta_A} A(x)$$

- Since

$$ds^2_{\mathbb{R}^4} = \left(1+\frac{|x|^2}{4}\right)^4 ds^2_{\mathbb{S}^4}$$

- …it follows that $4^{\Delta_A}\langle A(N)\overline{B}(S)\rangle_{\mathbb{S}^4} = C_{AB}$

Computed through a matrix integral thanks to localization

- There is one subtlety, though: due to the conformal anomaly there can be mixing

$$O_\Delta^{\mathbb{R}^4} \to O_\Delta^{\mathbb{S}^4} + \frac{\alpha_1}{R^2}\, O_{\Delta-2}^{\mathbb{S}^4} + \frac{\alpha_2}{R^4}\, O_{\Delta-4}^{\mathbb{S}^4} + \cdots$$

- Remove this mixing by Gram-Schmidt orthogonalization!

Gerchkovitz, Gomis, Ishtiaque, Komargodski & Pufu, 1602.05971

- Let us look to the correlators of the simplest operators $\mathcal{O}_n = (\mathrm{Tr}\phi^2)^n$ in SU(N) SQCD

$$\begin{aligned} \frac{G_{2n}^{\mathrm{QCD}}}{G_{2n}^{\mathcal{N}=4}} &= 1 - \frac{9\, n\, (N^2+2n-1)\, \zeta(3)}{4\pi^2\, (\mathrm{Im}\tau)^2} \\ &+ \frac{5\, n\, (2N^2-1)\, (3N^4+(15n-3)N^2+(20n^2-15n+4))\, \zeta(5)}{4\pi^3\, N\, (N^2+3)\, (\mathrm{Im}\tau)^3} + \cdots . \end{aligned}$$

$$G_{2n}^{\mathcal{N}=4,\mathfrak{g}} = \frac{n!\, 2^{2n}}{(\mathrm{Im}\tau)^{2n}} \alpha\, (1+\alpha)_{n-1}\,, \qquad \alpha = \frac{1}{2}\mathrm{dim}(\mathfrak{g})\,.$$

- The polynomial in n multiplying each order in the coupling is just the appropriate so as to define the double scaling limit (at FIXED N!)

$$n \to \infty\,, \qquad g \to 0\,, \qquad \lambda \equiv g^2 n = \text{fixed}\,,$$

(Gauge instantons truly supressed!)

Bourget, R-G & Russo, 1803.00580

- Going beyond this tower by explicit computation is hard. The next simplest case is SU(3): there is only one more CPO. Explicitly computing the correlators shows that the limit continues to exist

Beccaria, 1809.06280
Beccaria, 1810.10483

- It turns out that the existence of the limit is rooted in the structure of the correlators: the GS can be recasted as a matrix model

  - Very sketchy: for SU(2) there is only one CPO, whose sphere correlators are derivatives of Z wrt. the coupling. The flat space correlators are ratios of subdeterminants of the matrix of derivatives

  - It turns out that each such subdeterminant can be written as a matrix integral: convert the computation of correlators into a matrix model!

$$\det \mathcal{M}_{(n)} = \frac{1}{n!}\int_0^\infty \prod_{j=0}^{n-1} dx_j e^{-4\pi \operatorname{Im}\tau x_j} x_j^{\frac{1}{2}} Z_{\text{1-Loop}}\left(\sqrt{x_j}\right) \prod_{j<k}(x_j - x_k)^2 \ .$$

  - The 't Hooft limit of this matrix model is well defined: it is our double scaling limit (strictly speaking, the latter is the weak 't Hooft coupling regime)

Grassi, Komargodski & Tizziano, 1908.10306
Beccaria, Galvagno & Hasan, 2001.06645

(note that in any case, gauge instantons are safely supressed in this regime)

Wilson loops in the k-fold symmetrized product

- Consider now circular Wilson loops in the k-fold symmetrized representation. The exact formula is

$$\langle W_k \rangle = \frac{1}{N}\,\frac{1}{Z_N}\int d^N a \prod_{i<j}(a_i - a_j)^2\, Z_{1-\mathrm{loop}}\, Z_{\mathrm{inst}}\, e^{-\frac{8\pi^2}{g^2}\sum_{i=1}^{N} a_i^2}\, W_k\,,$$

- For N=4 both the instanton and 1-loop contributions are 1

- The insertion is the character of the k-fold symm rep (of U(N)/SU(N)). This is easy to compute: the generating function is by definition the PE of the fundamental. Then

$$W_k = (-1)^{N-1} \sum_{i=1}^{N} \frac{e^{2\pi(N-1)\, a_i + 2k\pi a_i}}{\prod_{j\neq i} e^{2\pi a_j} - e^{2\pi a_i}} \,. \qquad \longleftrightarrow \qquad W_k = \sum_{1\leq i_1 \leq i_2 \cdots \leq i_k \leq N} e^{2\pi a_{i_1} + 2\pi a_{i_2} + \cdots + 2\pi a_{i_k}} \,.$$

- Hence in the end

$$\langle W_k \rangle = \frac{(-1)^{N-1}}{Z_{U(N)}} \int d^N a \prod_{k<l} (a_k - a_l)^2 \, Z_{1-\mathrm{loop}} \, Z_{\mathrm{inst}} \, e^{-\frac{8\pi^2}{g^2} \sum_{m=1}^{N} a_m^2} \frac{e^{2\pi(N-1)\, a_N + 2k\pi a_N}}{\prod_{j\neq N} e^{2\pi a_j} - e^{2\pi a_N}} \,.$$

- For SU(N) impose

$$\textstyle\sum_{i=1}^{N} a_i = 0$$

- Introduce now $\kappa = g^2 k$
- Then (we specify to U(N) N=4)

$$\langle W_k \rangle = \frac{(-1)^{N-1}}{Z_{U(N)}} e^{\frac{k\kappa}{8}(1+\frac{N-1}{k})^2} \int d^N a \prod_{k<l}(a_k - a_l)^2 \, e^{-k\frac{8\pi^2}{\kappa}\sum_{m=1}^{N-1} a_m^2} \left(\frac{e^{-k\frac{8\pi^2}{\kappa}\left(a_N - a_N^\star\right)^2}}{\prod_{j\neq N} e^{2\pi a_j} - e^{2\pi a_N}} \right).$$

$$a_N^\star \equiv \frac{\kappa}{8\pi}(1 + \frac{N-1}{k})$$

- This suggests the limit FOR FIXED N

$$g \to 0\,, \quad k \to \infty\,, \qquad g^2 k = \kappa = \text{fixed}\,.$$

c.f. as well
Beccaria, Giombi & Tseytlin
Cuomo, Komargodski, Mezei & Raviv-Moshe

- Note that in this limit gauge instantons are completely supressed (just like in the large charge limit)

- The N-th eigenvalue gets stabilized at a much larger scale than the rest… so the integral breaks in two pieces

$$\langle W_k \rangle = \frac{(-1)^{N-1}}{Z_{U(N)}}\, e^{\frac{k\kappa}{8}(1+\frac{N-1}{k})^2} \int d^N a \prod_{k<l}(a_k - a_l)^2\, e^{-k\frac{8\pi^2}{\kappa}\sum_{m=1}^{N-1} a_m^2} \left(\frac{e^{-k\frac{8\pi^2}{\kappa}\left(a_N - a_N^\star\right)^2}}{\prod_{j\neq N} e^{2\pi \cancel{a_j}} - e^{2\pi a_N}} \right).$$

$$\sim 0$$

$$a_N^\star \equiv \frac{\kappa}{8\pi}\left(1 + \cancel{\frac{N-1}{k}}\right)$$

$$\langle W_k \rangle = \frac{Z_{U(N-1)}}{Z_{U(N)}}\, e^{\frac{k\kappa}{8}(1+\frac{N-1}{k})^2} \int da_N \left(\frac{a_N^2}{e^{2\pi a_N} - 1} \right)^{N-1} e^{-k\frac{8\pi^2}{\kappa}(a_N - \frac{\kappa}{8\pi})^2}$$

- Doing the last integral (saddle) and putting all factors

$$\langle W_k \rangle = \frac{1}{N!}\left(\frac{k\,\kappa}{4}\right)^{N-1} e^{\frac{k\kappa}{8}(1+\frac{N-1}{k})^2} \left(e^{\frac{\kappa}{4}} - 1\right)^{1-N}$$

- Note that

We could take N large provided it is much smaller than k

$$\langle W_k\rangle = \frac{(-1)^{N-1}}{Z_{U(N)}}\, e^{\frac{k\kappa}{8}\,(1+\frac{N-1}{k})^2} \int d^N a \prod_{k<l}(a_k-a_l)^2\, e^{-k\frac{8\pi^2}{\kappa}\sum_{m=1}^{N-1} a_m^2} \left(\frac{e^{-k\frac{8\pi^2}{\kappa}\left(a_N-a_N^\star\right)^2}}{\prod_{j\neq N} e^{2\pi a_j}-e^{2\pi a_N}}\right).$$

$$a_N^\star \equiv \frac{\kappa}{8\pi}(1+\frac{N-1}{k})$$

- Our result becomes then

$$\langle W\rangle = \frac{e^{-S}}{\sqrt{2\pi N}}\,, \qquad S = -\frac{k\kappa}{8} - N\log\left(\frac{k\,\kappa}{4N}\right) - N + N\,\log\left(1-e^{-\frac{\kappa}{4}}\right)$$

$$S = -\frac{k\kappa}{8} - N\log\left(\frac{k\,\kappa}{4N}\right) - N$$

- Lets compare with the holographic/matrix model@large N result

Drukker & Fiol

$$S_{\rm DF} = -2N\left[\frac{k\sqrt{\lambda}}{4N}\sqrt{1+\frac{k^2\lambda}{16N^2}} + \text{arcsinh}\left(\frac{k\sqrt{\lambda}}{4N}\right)\right] \longrightarrow S_{DF} \sim -\frac{k\kappa}{8} - N\log\left(\frac{k\,\kappa}{4N}\right) - N$$

- What about SU(N)? Do

$$\sum_{i=1}^{N} a_i^2 = \sum_{i=1}^{N} \hat{a}_i^2 + \frac{1}{N}(\sum_{i=1}^{N} a_i)^2\,, \qquad \hat{a}_i = a_i - \frac{1}{N}\sum_{i=1}^{N} a_i$$

- Then

$$\langle W_k \rangle = \frac{(-1)^{N-1}}{Z_N} \int d^N a \prod_{k<l} (\hat{a}_k - \hat{a}_l)^2 \, e^{-\frac{8\pi^2}{g^2}\sum_{m=1}^{N} \hat{a}_m^2} \, e^{-\frac{8\pi^2 N}{g^2}x^2 - 2\pi k x} \frac{e^{2\pi(N-1)\,\hat{a}_N + 2k\pi\hat{a}_N}}{\prod_{j\neq N} e^{2\pi\hat{a}_j} - e^{2\pi\hat{a}_N}}\,, \qquad x = \tfrac{1}{N}\sum_{i=1}^{N} a_i$$

- The a’s sum zero: relax this by introducing a delta

$$\langle W_k \rangle = \frac{(-1)^{N-1}}{Z_N} \int d^N \hat{a} \prod_{k<l} (\hat{a}_k - \hat{a}_l)^2 \, e^{-\frac{8\pi^2}{g^2}\sum_{m=1}^{N} \hat{a}_m^2} \frac{e^{2\pi(N-1)\,\hat{a}_N + 2k\pi\hat{a}_N}}{\prod_{j\neq N} e^{2\pi\hat{a}_j} - e^{2\pi\hat{a}_N}} \, \delta(\sum_{i=1}^{N} \hat{a}_i) \left(\int dx \, e^{-\frac{8\pi^2 N}{g^2}x^2 - 2\pi k x} \right)$$

- One recognizes the SU(N) result

$$\langle W_k \rangle_{U(N)} = \left(\frac{Z_{SU(N)}}{Z_{U(N)}} \int dx \, e^{-\frac{8\pi^2 N}{g^2} x^2 - 2\pi k x} \right) \langle W_k \rangle_{SU(N)} \longrightarrow \langle W_k \rangle_{U(N)} = e^{\frac{g^2 k^2}{8N}} \langle W_k \rangle_{SU(N)}$$

- The prefactor is a loop for the U(1) part

$$\frac{Z_{SU(N)}}{Z_{U(N)}} \int dx \, e^{-\frac{8\pi^2 N}{g^2} x^2 - 2\pi k x} = \frac{\int da \, e^{-\frac{8\pi^2}{g^2} a^2 - 2\pi \frac{k}{N} a}}{\int da \, e^{-\frac{8\pi^2}{g^2} a^2}}$$

- If k>N this is a (leading) contribution: this observable is sensible to U vs SU!!!

- One can also compute correlators of loops with CPO's. This has info about the OPE

- The first few such correlators are

$$\mathcal{O}_1 = \mathrm{Tr}\phi - \frac{\langle \mathrm{Tr}\phi \rangle}{\langle \mathbb{1} \rangle}\, \mathbb{1}\,, \qquad \mathcal{O}_2 = \mathrm{Tr}\phi^2 - \frac{\langle \mathrm{Tr}\phi^2 \rangle}{\langle \mathbb{1} \rangle}\, \mathbb{1} \qquad \cdots\,.$$

- But $\frac{\langle \mathrm{Tr}\phi^n \rangle}{\langle 1 \rangle} \sim g^n \sim \kappa^{\frac{n}{2}}\, k^{-n}$, so in this limit only the ``leading term" contributes. To compute it

$$\begin{aligned} \langle \mathrm{Tr}\phi^{n_1} \cdots \mathrm{Tr}\phi^{n_m}\, W_k \rangle \;&=\; \frac{1}{Z_{U(N)}} \int d^N a \prod_{k<l} (a_k - a_l)^2\, Z_{1-\mathrm{loop}}\, Z_{\mathrm{inst}}\, e^{-\frac{8\pi^2}{g^2} \sum_{m=1}^N a_m^2} \\ &\times\; \frac{e^{2\pi(N-1)\, a_N + 2k\pi a_N}}{\prod_{j\neq N} \left(e^{2\pi a_N} - e^{2\pi a_j}\right)} \Big(\sum_{i=1}^N a_i^{n_1}\Big) \cdots \Big(\sum_{i=1}^N a_i^{n_m}\Big)\,. \end{aligned}$$

- Just as before in the large k limit the integral can be done via saddle point giving

$$\langle \mathrm{Tr}\phi^{n_1} \cdots \mathrm{Tr}\phi^{n_m} \, W_k \rangle = \frac{Z_{U(N-1)}}{Z_{U(N)}} \, e^{\frac{k\kappa}{8}\,(1+\frac{N-1}{k})} \int da_N \left(\frac{a_N^2}{e^{2\pi a_N}-1} \right)^{N-1} a_N^{n_1+\cdots+n_m} \, e^{-k\frac{8\pi^2}{\kappa}(a_N-\frac{\kappa}{8\pi})^2} \, .$$

- Hence

$$\langle \mathrm{Tr}\phi^{n_1} \cdots \mathrm{Tr}\phi^{n_m} \, W_k \rangle = \left(\frac{\kappa}{8\pi} \right)^{n_1+\cdots+n_m} \langle W_k \rangle \, .$$

- So finally

$$\langle \mathcal{O}_\Delta \, W_k \rangle = \left(\frac{\kappa}{8\pi} \right)^{\Delta} \langle W_k \rangle \qquad \langle \mathcal{O}_\Delta(x) \, \mathcal{O}_\Delta(0) \rangle = \frac{C_\Delta}{|x|^{2\Delta}} \, , \qquad C_\Delta \equiv \frac{\Delta \, \lambda^\Delta}{(2\pi)^{2\Delta}}$$

$$\langle \hat{\mathcal{O}}_\Delta \, W_k \rangle = \frac{1}{\sqrt{\Delta}} \left(\frac{k\kappa}{16N} \right)^{\frac{\Delta}{2}} \langle W_k \rangle \, ,$$

Berenstein, Corrado, Fischler& Maldacena '98
Giombi, Ricci & Trancanelli '06

- A similar double scaling limit holds in general N=2 theories. For instance, for N=2*

c.f for SU(2) SQCD Cuomo, Komargodski, Mezei & Raviv-Moshe' 22

$$\langle W_k \rangle_{\mathcal{N}=2^*} = \frac{1}{Z_{\mathcal{N}=2^*}} \int d^N a \prod_{i<j}^{N-1} \frac{(a_i - a_j)^2 H(a_i - a_j)^2}{H(a_i - a_j + M) H(a_i - a_j - M)} e^{-\frac{8\pi^2}{g^2} \sum_{m=1}^{N-1} a_m^2}$$
$$e^{\frac{k^2 g^2}{8}\left(1+\frac{N-1}{k}\right)^2} \prod_{i=1}^{N-1} \frac{(a_i - a_N)^2 H(a_i - a_N)^2}{H(a_i - a_N + M) H(a_i - a_N - M)} \frac{e^{-\frac{8\pi^2}{g^2}(a_N - a_N^*)^2}}{\prod_{j \neq N}\left(e^{2\pi a_N} - e^{2\pi a_j}\right)},$$

- Introducing the same variables

$$\langle W_k \rangle_{\mathcal{N}=2^*} = \frac{e^{\frac{k\kappa}{8}\left(1+\frac{N-1}{k}\right)^2}}{Z_{\mathcal{N}=2^*}} \int d^{N-1} a \prod_{i<j}^{N-1} \frac{(a_i - a_j)^2 H(a_i - a_j)^2}{H(a_i - a_j + M) H(a_i - a_j - M)} e^{-\frac{8\pi^2 k}{\kappa} \sum_{m=1}^{N-1} a_m^2}$$
$$\int da_N \left(\frac{H(a_N)^2}{H(a_N + M) H(a_N - M)}\right)^{N-1} \left(\frac{a_N^2}{e^{2\pi a_N} - 1}\right)^{N-1} e^{-k\frac{8\pi^2}{\kappa}(a_N - a_N^*)^2},$$

- In the same double scaling limit at large k one then finds

$$\langle W_k\rangle_{\mathcal{N}=2^*} = \Big(\frac{H(a_N^*)^2\, H(M)^2}{H(a_N^*+M)H(a_N^*-M)}\Big)^{N-1} \langle W_k\rangle_{\mathcal{N}=4}$$

- It is interesting to look to the decompatification limit of large MR. N=2* undergoes a sequence of phase transitions…what about the large k loop? One finds

Russo & Zarembo

$$\begin{aligned}\log\langle W_k\rangle_{\mathcal{N}=2^*} \;&\approx\; \log\langle W_k\rangle_{\mathcal{N}=4} + 2(N-1)\log H(a_N^*) - \frac{1}{2}(N-1)R^2\Big[2M^2\log(MR)^2 \\ &-\; (M-a_N^*)^2\log(M-a_N^*)^2R^2 - (M+a_N^*)^2\log(M+a_N^*)^2R^2\Big] \\ &+\; (1-2\gamma)(N-1)(Ra_N^*)^2\,.\end{aligned}$$

- This suggests a potential non-analytic behavior at a* of order M. But this corresponds to

$$\kappa = 8\pi MR$$

- …so this happens “infinitely far away”: no phase transitions for this observable

- All in all, in the decompactification limit

$$\log\langle W_k\rangle_{\mathcal{N}=2^*} \;\longrightarrow\; \log\langle W_k\rangle_{\mathcal{N}=4} + (N-1)\left(2\log H(\frac{\kappa}{8\pi}) + \frac{\kappa^2}{32\pi^2}\Big[2-\gamma+\log\big(MR\big)\Big]\right)$$

Defects in WF

- Using the same strategy we can also study lines in the WF fixed point near d=4, 6d.

D.R-G.

  - Also a similar double-scaling limit exists

    Arias-Tamargo, Russo & R-G,
    Watanabe,
    Badel, Cuomo, Monin & Rattazzi,
    Hellerman, Orlando, Reffert *et al.*

  - Can be interpreted as effective description of large spin impurities in magnets

    Cuomo, Komargodski, Mezei & Raviv-Moshe

- For instance, consider O(2N+1) near d=4

$$S = \int \frac{1}{2}|\partial\vec{\varphi}|^2 + \frac{g}{4}(\vec{\varphi}^2)^2\,,$$

- One may imagine the trivial line along one direction. It admits a deformation

$$\mathcal{D}(\vec{z}) = e^{-h\int d\tau\,\varphi^{2N+1}(\tau,\vec{z})} = e^{-h\int dx\,\varphi^{2N+1}\,\delta_T(\vec{x}-\vec{z})} \longrightarrow \langle\mathcal{D}(\vec{z})\rangle = \int e^{-\int \frac{1}{2}|\partial\vec{\varphi}|^2+\frac{g}{4}(\vec{\varphi}^2)^2+h\varphi^{2N+1}\,\delta_T(\vec{x}-\vec{z})}\,.$$

- Assume $g = \dfrac{\lambda}{n}\,, \quad h = \nu\, n$

- Then

$$\langle \mathcal{D}(\vec{z}) \rangle = \int e^{-nS_{\rm eff}}\,, \qquad S_{\rm eff} = \int \frac{1}{2}|\partial\vec{\phi}|^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2 + \nu\phi^{2N+1}\,\delta_T(\vec{x}-\vec{z})\,.$$

- So imagine taking large n with everything else fixed: use saddle point. The only relevant eq. can be easily solved

$$\partial^2\phi^{2N+1} - \lambda(\phi^{2N+1})^3 - \nu\,\delta_T(\vec{x}-\vec{z}) = 0\,.$$

- Finally

$$\begin{aligned} S_{\rm eff} \;&=\; \Big(-\frac{\nu^2}{2} + \frac{\lambda\nu^4}{128\pi^2\epsilon} + \frac{\lambda\nu^4}{128\pi^2}(3-\gamma_E+\log(4\pi))\Big)\, T \int \frac{d^{d-1}\vec{p}}{(2\pi)^{d-1}}\,\frac{1}{\vec{p}^2} \\ &\quad -\frac{\lambda\nu^4}{128\pi^2}\, T \int \frac{d^{d-1}\vec{p}}{(2\pi)^{d-1}}\,\frac{\log|p|^2}{\vec{p}^2}\,. \end{aligned}$$

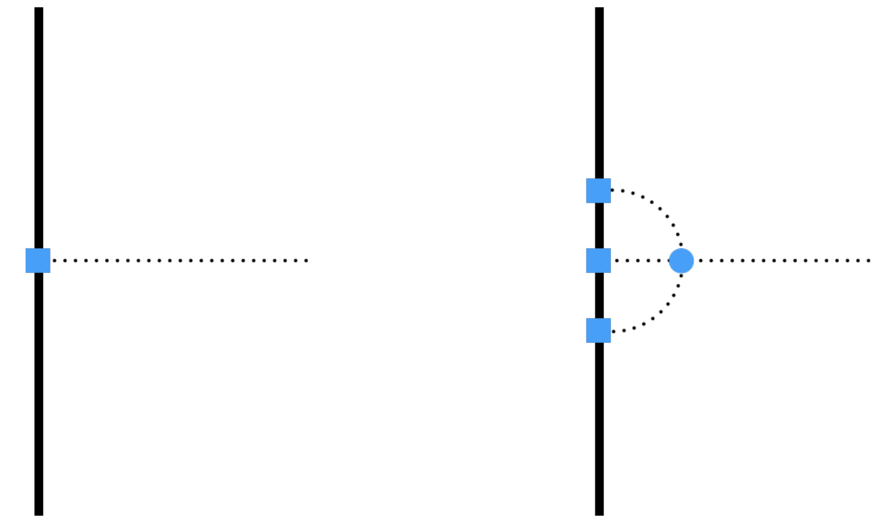
- Since $\frac{1}{\langle \mathcal{D} \rangle} \frac{d}{d\nu} \langle \mathcal{D} \rangle = -n \int \langle \phi^{2N+1} \rangle \, \delta_T(\vec{x} - \vec{z})$

- We can use this to define the renormalized coupling

$$\nu = \mu^{\frac{\epsilon}{2}} \left(\nu_R + \frac{\lambda \nu_R^3}{2(4\pi)^2 \epsilon} \right) ,$$

- We can compute the beta function, which shows a fixed point

$$\mu \frac{d\nu_R}{d\mu} = -\frac{\epsilon}{2} \nu_R + \frac{\lambda \nu_R^3}{(4\pi)^2} \, . \quad \longrightarrow \quad \nu_R^2 = \frac{8\pi^2}{\lambda} \epsilon \, , \quad \longleftrightarrow$$



$a)$ $b)$

Allais & Sachdev,
Cuomo, Komargodski & Mezei

- …where

$$\langle \phi^{2N+1} \rangle = -\nu_R \int \frac{d^{d-1}\vec{p}}{(2\pi)^{d-1}} \frac{e^{-i\vec{p}\cdot\vec{x}}}{|\vec{p}|^{2-\frac{\epsilon}{2}}} \sim \frac{1}{|\vec{x}_T|^{\frac{d-2}{2}}}$$

- One can also compute correlators of defect fields as well as correlators of defects themselves. For instance, for the latter

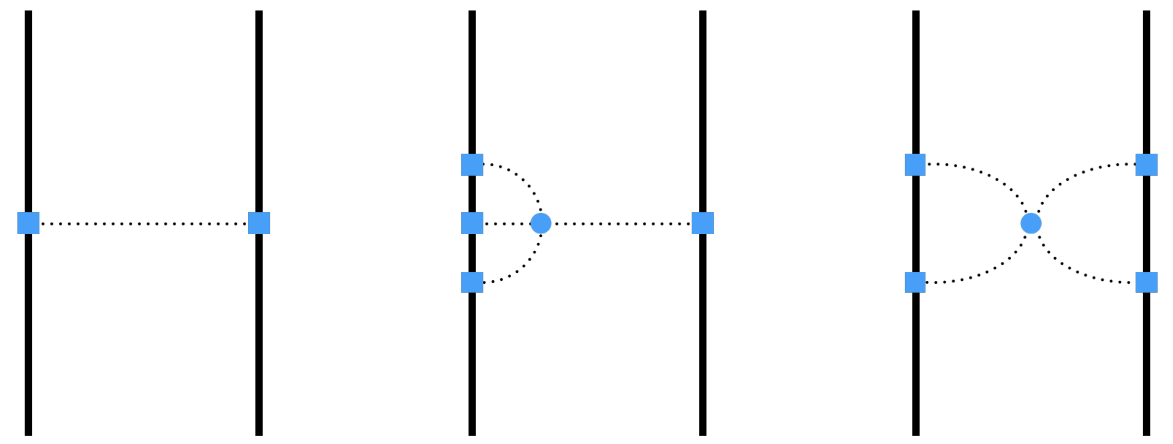
$$\langle \mathcal{D}(z_1)\mathcal{D}(z_2)\rangle = \int e^{-\int \frac{1}{2}|\partial\vec{\varphi}|^2+\frac{g}{4}(\vec{\varphi}^2)^2+h\varphi^{2N+1}\,\delta_T(\vec{x}-\vec{z}_1)+h\varphi^{2N+1}\,\delta_T(\vec{x}-\vec{z}_2)}\,.$$

- Assuming the same scaling

$$\langle \mathcal{D}(z_1)\mathcal{D}(z_2)\rangle = \int e^{-nS_{\text{eff}}}, \qquad S_{\text{eff}} = \int \frac{1}{2}|\partial\vec{\phi}|^2+\frac{\lambda}{4}(\vec{\phi}^2)^2+\nu\phi^{2N+1}\,\delta_T(\vec{x}-\vec{z}_1)+\nu\phi^{2N+1}\,\delta_T(\vec{x}-\vec{z}_2)\,.$$

- The saddle point eqs. are

$$\partial^2\phi^{2N+1} - \nu\,\delta_T(\vec{x}-\vec{z}_1) - \nu\,\delta_T(\vec{x}-\vec{z}_2) = 0\,. \longrightarrow \phi^{2N+1} = \rho_1(\vec{x})+\rho_2(\vec{x}), \qquad \rho_i(\vec{x}) = -\nu\int dy\, G(x-y)\delta_T(\vec{y}-\vec{z}_i)\,.$$



$a)$ $b)$ $c)$

- So finally

$$\langle \mathcal{D}(z_1)\mathcal{D}(z_2)\rangle = \langle \mathcal{D}(z_1)\rangle\langle \mathcal{D}(z_2)\rangle e^{-nS_{\mathrm{I}}} \longrightarrow S_{\mathrm{I}} = -\Big(\frac{\nu_R^2}{4\pi}+\frac{3\lambda\nu_R^4}{512\pi}-\frac{\lambda\nu_R^4}{64\pi^3}(3+\gamma_E+\log(4\pi))\Big)\frac{T}{|\vec{z}_1-\vec{z}_2|^{1+(-\epsilon+\frac{\lambda\nu_R^2}{8\pi^2})}}\,. \longrightarrow S_{\mathrm{I}} = -\Big(\frac{\nu_R^2}{4\pi}+\frac{3\lambda\nu_R^4}{512\pi}-\frac{\lambda\nu_R^4}{64\pi^3}(3+\gamma_E+\log(4\pi))\Big)\frac{T}{|\vec{z}_1-\vec{z}_2|}\,.$$

c.f. Soderberg (free case)

- Near d=6 a similar story holds. Consider the proposed UV completion to the quartic theory above 4d

Fei, Giombi & Klebanov '14
Giombi, Huang, Klebanov, Pufu & Tarnopolski '19

$$S = \int \frac{1}{2}|\partial\vec{\varphi}|^2 + \frac{1}{2}\partial\eta^2 + \frac{g_1}{2}\eta\,|\vec{\varphi}|^2 + \frac{g_2}{6}\eta^3\,. \qquad g_{1\star} = \sqrt{\frac{6\,(4\pi)^3}{2N}\epsilon}\,\Big(1+\mathcal{O}(\frac{1}{N})\Big)\,, \qquad g_{2\star} = 6\sqrt{\frac{6\,(4\pi)^3}{2N}\epsilon}\,\Big(1+\mathcal{O}(\frac{1}{N})\Big)$$

- This is is the UV completion of the quartic theory. Upon Hubbar-Stronovich - zation this theory is described by

$$S_{\text{quartic}} = \int \partial\vec{\xi}^{\,2} + \sigma\,\vec{\xi}^{\,2}\,,$$

- For the HS field

$$\langle\sigma(x)\,\sigma(0)\rangle \sim \frac{1}{x^4}$$

- In the sextic theory one can consider surface defects deformed as

$$\mathcal{D} = e^{-h \int d^2x\, \eta}\,.$$

- This does not break the O(N) symmetry. A natural guess in the quartic theory

$$\mathcal{D}_{\text{quartic}} = e^{-\hat{h} \int d^2x \vec{\xi}^2}$$

- Assuming double scaling, we can use saddle point

$$S_{\text{eff}} = n \int \frac{1}{2}|\partial\vec{\phi}|^2 + \frac{1}{2}\partial\rho^2 + \frac{h_1}{2}\rho\,|\vec{\phi}|^2 + \frac{h_2}{6}\rho^3 + \nu\rho\,\delta_T(\vec{x}) \qquad g_i\sqrt{n} = h_i,\; h = \nu\sqrt{n}.$$

- Proceeding as in the 4d case, one can compute the defect beta function

$$\beta_\nu = -\frac{\epsilon}{2}\nu_R - \frac{h_2 \nu_R^2}{16\pi^2}\,.$$

- This has a fixed point at

$$\nu_R = -\frac{8\pi^2 \epsilon}{h_2}\,.$$

- One can also compute correlators of defects as well as defect fields…but we'll leave that for another day

Final comments

- Inspired by the large charge developments, we introduced a double scaling limit for defects
- In the WF fixed near 4/6d we considered lines/surfaces: a fixed point leading to a dCFT exists
- One can compute correlators of defect operators/defects themselves
- We introduced a novel double scaling limit for the k-fold symmetrized Wilson loop

Allows to compute exact observables for finite N in gauge theories (free of gauge instantons!)

- The loop distinguishes U vs SU…can this be seen holographically?

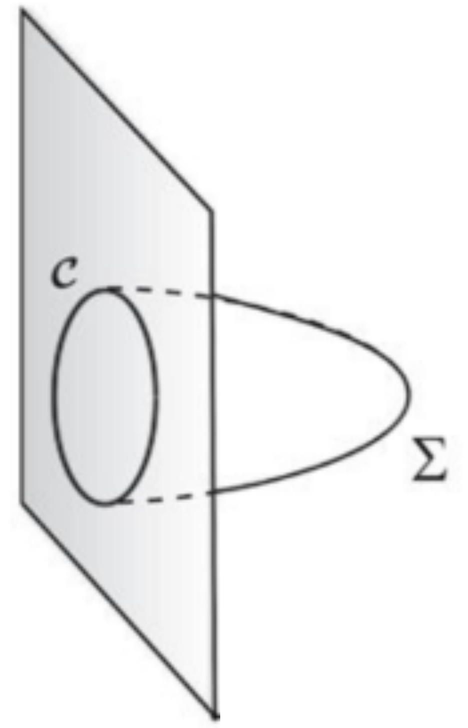
  - U vs SU encoded in a topological BF theory in AdS5…

$$S_{\rm top} = N \int C_2 \wedge dB_2$$

  - …the fluxed D3 dual to the loop would source the RR 2-form…gives rise to boundary term?

- …maybe one needs to do holography “the other way around”

cf. Shota’s talk!!



- The defect on the WF in this limit simplifies…can one study aspects of RG flows?
    - Perhaps toy models for interesting behaviors (fixed point annihilation?)
    - Can one study general aspects of RG flows such as entropy extremization?
    - …

Many thanks!!!