Stability abalysis of a non-unitary CFT

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7 July 2022, Les Diablerets

Based on [2203.08843] and WIP with Ohad Mamroud and Adar Sharon

Motivation

- (1) How much information is needed to guess the large-charge behaviour?
- What if we know the dimension of ϕ^n at fixed n in the ϵ expansion? Does the analyticity in n help, if it exists?
- What if we know the dimension of ϕ^n in the double-scaling parameter ϵn fixed? Here the analyticity in ϵn exists and helps for sure.
- (2) What's the relation between the three regimes, (a) n fixed, ϵ small, (b) double-scaling limit , and (c) ϵ fixed, n large?
- (3) Can we for example prove that the concavity in the small charge region means the imaginary part of the operator dimension using the above?
- I will answer (1) and partially (2) today. I would like any suggestions about (3).

$O(2)$ Wilson-Fisher theory

- This is already day 4 but let's still review the large-charge expansion using the simplest example.
- We consider the $D = 3$, $O(2)$ Wilson-Fisher fixed point, and try to compute the operator dimension of ϕ^Q for $Q \gg 1$.
- The UV Lagrangian is given by

$$L = |\partial\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4.$$

On the IR fixed-point, we tune m^2 and $\lambda = O(1)$.

Key ingredient 1: State-operator correspondence

- Now that we consider a CFT, we can use the state-operator correspondence to compute the dimension of ϕ^Q.
- ϕ^Q is the lowest operator at charge Q – This corresponds to the ground state energy at charge Q on $S^2 \times \mathbb{R}$, with radius 1.
- It is very important that this is a ground state of some sector of the theory. Ideally we would want to compute ϕ^n for the real ϕ^4 theory, but we don't know how this is defined.
- We don't even know how to do $\phi^n \bar{\phi}^m$. They all seem to have an unstable periodic orbit in phase space though. Periodic orbit theory and quantum scarring?

Grand canonical ensemble

- Anyway, in order to compute the ground state energy at charge Q, we can use the grand-canonical ensemble,

$$Z[\beta, \mu] = \mathrm{Tr}_{\mathcal{H}_{S^2}} \left[e^{-\beta(H-\mu Q)} \right]$$

$$\langle Q \rangle = \frac{1}{\beta}\frac{\partial}{\partial \mu} \log Z, \quad \langle E \rangle = -\frac{\partial}{\partial \beta} \log Z + \mu \langle Q \rangle$$

- In the low temp limit ($\beta \to \infty$), we have

$$F \equiv -\frac{1}{\beta} \log Z \approx -\frac{\partial}{\partial \beta} \log Z = \langle E \rangle - \mu \langle Q \rangle$$

- So we just need to evaluate the grand-canonical free energy and then do the Legendre transform.

Key ingredient 2: Saddle point analysis

- We will represent the grand partition function using the path integral, and then use the saddle-point approximation.
- According to the path-integral formulation,

$$Z(\beta,\mu) \equiv \mathrm{Tr}\left[e^{-\beta(H-\mu\hat{Q})}\right] = \int D\phi\, e^{-(S[\phi]-\mu Q)}$$

- Since $\rho \equiv i(\bar{\phi}\partial_0\phi - \phi\partial_0\bar{\phi})$, if we redefine $\Phi \equiv e^{-i\mu t}\phi$, the Lagrangian becomes

$$L - \mu Q = |\partial\Phi|^2 + (m^2-\mu^2)|\Phi|^2 + g|\Phi|^4$$

- The relevant saddle point is the vacuum configuration. Quite simply, $\Phi = (\text{const.})$, or $\phi = A\, e^{i\mu t}$.
- Did you notice that I skipped the logic a little? If not, fine. If yes, it's not too relevant so let's go on.

Summary so far

- All the complications aside, what we have to do is clear now.
- Find the lowest saddle point configuration of the original Lagrangian

$$L = |\partial\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4,$$

  which has the dependence $\phi = A\, e^{i\mu t}$, on $S^2 \times \mathbb{R}$.
- Compute the energy of the configuration, $E(\mu)$, in terms of chemical potential, μ, and perform the Legendre transform to get $E(Q)$ in terms of charge.
- At large Q, the action evaluated at the saddle point becomes large. The saddle-point analysis is therefore controlled.

Plot Twist: Use of effective action

- We can go on and actually compute the classical action and the 1-loop corrections and so on, if it were a weakly-coupled theory.
- In fact, one could do this in the ϵ-expansion, where $\lambda = O(\epsilon)$. We will get back to this later on.
- However, in the absence of such a weak-coupling parameter, we can just use the effective field theory around the saddle-point!
- This is basically the idea of the large charge expansion.

- You know what to do. We just need to write down the action invariant under conformal and the $O(2)$ symmetry.
- This can be done by integrating out the massive radial mode a and writing down the conformal sigma model.
- At leading order the mass of a is $|\partial\chi|$, so it can come in the denominator.
- At leading order in μ ($\chi = \mu t$), the effective Lagrangian becomes

$$L = c_{3/2}|\partial\chi|^3$$

- The whole effective action is the following,

$$L = c_{3/2}|\partial\chi|^3 + c_{1/2}\mathtt{Ric}_3|\partial\chi| + O(Q^{-1/2})$$

- The ground state energy, realised at $\chi = \mu t$, is the dimension of the operator ϕ^Q.
- The first term is $O(\mu^3)$, and from Noether theorem we have $\mu = O(\sqrt{Q})$, so it's $O(Q^{3/2})$.
- The second is $Q^{1/2}$. It's only scale-invariant but not Weyl-invariant, but one can complete it. The completion is $O(Q^{-1})$ though.
- No terms is available at order $O(1)$. This is important later. Have you ever followed the proof of this? If not I can do it on the board.

Aside: Difference between this EFT and that EFT

- We have been treating the $O(2)$ model and wrote the EFT for the original action at large charge, using the field χ.
- People have considered writing down the EFT at large chemical potential for the action with chemical potential term added.
- This is less advantageous, having less symmetries.
- In particular, the conformal (in particular the Lorentz) symmetry is explicitly broken, so the EFT is

$$L = (\partial_0 \chi_{\text{fluc}})^2 + c_s^2 (\partial_i \chi_{\text{fluc}})^2 + \circ\circ\circ$$

  Underlying conformal invariance $T^{\mu}_{\mu} = 0$ dictates $c_s^2 = 1/2$.
- Using our EFT, this can be easily derived by setting $\chi = \mu t + \chi_{\text{fluc}}$.
- This becomes more important when we realise the inhomogeneous ground states, but not for today.

- Now we have the operator dimension of ϕ^Q from th effective action,

$$L = c_{3/2}|\partial\chi|^3 + c_{1/2}\mathtt{Ric}_3|\partial\chi| + O(Q^{-1/2})$$

- Up to $O(Q^0)$ term, the result is

$$\Delta[Q] = \tilde{c}_{3/2}Q^{3/2} + \tilde{c}_{1/2}\mathtt{Ric}_3 Q^{1/2} - 0.094 + O(Q^{-1/2}).$$

- The 1-loop correction is $O(1)$. Since there are no effective operator there, the prediction gives a precise prediction as a number, not as an undetermined coefficient in the EFT.
- It would be important to check this from other perturbative methods. Done at large-N by de la Fuente.

Aside: Numerical stability of the fit

- Anton computed the operator dimension of monopole operators at large-N and fitted numerically.
- He really fitted with

$$\Delta[Q] = \tilde{c}_{3/2} Q^{3/2} + \tilde{c}_{1/2} \mathtt{Ric}_3 Q^{1/2} + c_0 + O(Q^{-1/2}).$$

- We could also fit with

$$Q^2 \Delta(Q) - \left(\frac{Q^2}{2} + \frac{Q}{4} + \frac{3}{16}\right) \Delta(Q-1) - \left(\frac{Q^2}{2} - \frac{Q}{4} + \frac{3}{16}\right) \Delta(Q+1)$$
$$= \frac{3}{8} c_0 + O(Q^{-1/2}).$$

Remark: Weak-coupling expansion

- This result is weird though. Why is the operator dimension of ϕ^Q not proportional to Q? Using Feynman diagrams don't we get $\Delta[Q] = Q + \circ \circ \circ$?
- At $D = 4$, the Wilson-Fisher fixed point is a free theory. If we do the same analysis in $D = 4$, we get $\Delta[Q] \propto Q^{4/3}$ and not $\Delta[Q] = Q$.
- In general, we get $\Delta[Q] \propto Q^{D/(D-1)}$ for general D as this only comes from the dimensional analysis.
- (This use of EFT is nothing but a sophisticated version of the dimensional analysis in the first place.)

Weak-coupling expansion

- Of course as you all know this is not weird at all.
- Imagine computing this using Feynman diagrams. Let's say we have the weak-coupling parameter g. g is the strength of two-body interaction.
- There are Q particles here, and the $O(g^{k-1})$ correction to $\Delta[Q] = Q$ comes from k body interactions between them.
- The way to pair k particles up inside a total of Q particles is $\binom{Q}{k} \sim Q^k$. So in total the correction will go as $O(g^{k-1}Q^k)$.
- This means that the actual expansion parameter is $O(gQ)$. Feynman diagram computation can only capture the regime $gQ \ll 1$, while our EFT captured the regime $gQ \gg 1$.
- Indeed, the Feynman diagram computation shows

$$\Delta[Q] = Q\left[\left(\frac{D}{2} - 1\right) + \frac{\epsilon}{10}(Q-1) + O\left((\epsilon Q)^2\right) + O\left((\epsilon Q)^3\right) + \circ \circ \circ\right]$$

Weak-coupling expansion

- The idea of combining large charge with the ϵ-expannsion is that We can explicitly compute and show this if we go back to our grand canonical ensemble.
- We were expanding our original action around the saddle $\phi = A\, e^{i\mu t}$. We had no weak coupling parameter, so we had no way of knowing what A was.
- However, if we have the weak coupling parameter, ϵ, we can compute A and compute the energy of the configuration.

Weak-coupling expansion

- Let us quickly look for the lowest configuration $\phi = Ae^{i\mu t}$ in the ϵ-expansion.
- The Lagrangian was

$$L = |\partial\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

and

$$\lambda/(4\pi)^2 = \epsilon/5 + O(\epsilon^2) \quad m = 1 - \epsilon/2$$

m is the conformal coupling on $S^{D-1} \times \mathbb{R}$.

- We use the EOM and the charge fixing constraint to determine A and μ in terms of Q,

$$\partial^2\phi = m^2\phi + \frac{\lambda}{2}|\phi|^2\phi, \quad Q = i(\phi\partial_0\bar{\phi} - \bar{\phi}\partial_0\phi)\frac{2\pi^{D/2}}{\Gamma(D/2)}$$

- We can solve for A and μ by solving a cubic equation, so there is an analytic solution.

Aside: ϵ-expansion on the torus

- You all know how to proceed from now on, so let's skip the derivation.
- Let me comment one thing though. The crossover of the solution at $Q \sim 1/\epsilon$ is due to the competition between conformal coupling and the interaction in

$$L = |\partial\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

- (Equivalently, this means that the double-scaling limit is taken so that the mass of the dilaton is fixed.)
- On the torus, $m = 0$ from the beginning and one would have never found out the crossover. But this would be enough to fix the leading order coefficient in the EFT. This number is the same on the sphere or on the torus.
- In that case $c_{3/2}$ and $c_{1/2}$ are related as there is only one free parameter in the EFT, and this might also help. (Also the computation of the universal term...? But we start from 4D so how should I regularise...?)

Final result of the Weak-coupling expansion

- The final result in the free limit ($\epsilon Q \ll 1$) is

$$\Delta = Q\left[\left(\frac{D}{2} - 1\right) + \frac{\epsilon}{10}(Q-1) - \frac{\epsilon^2}{50}\left(Q^2 - 4Q + O(1)\right) + \circ\circ\circ\right]$$

and the result in the interacting limit ($\epsilon Q \ll 1$) is

$$\Delta = \left(\frac{15}{8\epsilon} + \alpha + O(\epsilon)\right)\left(\frac{2\epsilon Q}{5}\right)^{\frac{4-\epsilon}{3-\epsilon}} + \left(\frac{5}{4\epsilon} + \beta + O(\epsilon)\right)\left(\frac{2\epsilon Q}{5}\right)^{\frac{2-\epsilon}{3-\epsilon}} + \circ\circ\circ$$

- We recover the EFT result when $\epsilon Q \gg 1$.

Comment on the general structure

- The general structure can be packaged into a double scaling limit with $x \equiv \epsilon Q$ fixed,

$$\Delta = \frac{1}{\epsilon} F_0(x) + F_1(x) + \epsilon F_2(x) + \circ \circ \circ$$

- Even though it was at weak coupling, by taking the number of particles, Q, large, we were able to probe the strongly coupled region. This is similar to the ordinary 't Hooft limit.
- This type of phenomena is ubiquitous. For example, it is also similar to the Horowitz-Polchinski solution of self gravitating strings. Even when the string coupling is small, having a large number of fundamental strings changes the physics qualitatively.

What does this mean?

- This means that we can sort of guess the strong-coupling result given weak-coupling, in the double-scaling limit.
- We could analytically continue Δ_0 at small ϵn to a larger ϵn. Then we would recover the EFT result, in principle.
- At small ϵn, the result presumably recover the Feynman diagram computation at small ϵ and fixed n, so these two regions are also connected.
- Given the resummation of $\epsilon Q^{4/3} log(\epsilon Q)$, we could say that this is also connected smoothly to large Q, fixed ϵ region. We would really need to include the worldline instanton corrections though.
- Are you all happy now?

Remaining question

- I'm not still happy. We know (but I will expalin again later) that the operator dimension becomes complex at large ϵQ, of the $O(2)$ WF theory in $D = 4 + \epsilon$.
- And we say that it's real for small ϵQ.
- This is not against the analyticity in term of ϵQ. There simply is a branch cut in the ϵn plane.
- (This is still weird as there seems to be a phase transition in a finite volume system.)
- If this is really connected to finite Q, then the result remains real.
- But it's against the actual computation. No wonder, the potential is unbounded so there must be an imaginary part, albeit nonperturbatively small.

Non-unitary CFT above four dimensions

- Let us now talk about the Wilson-Fisher theory in $4+\epsilon$ dimensions.
- Let us remember the result in $D=4-\epsilon$, in the free limit ($\epsilon Q \ll 1$).
- The final perturtive result in this limit

$$\Delta = Q\left[\left(\frac{D}{2}-1\right)+\frac{\epsilon}{10}(Q-1)-\frac{\epsilon^2}{50}\left(Q^2-4Q+O(1)\right)+\circ\circ\circ\right]$$

presumably matches the Feynman diagram computation to all orders.

Nonpertubative corrections – Free limit

- To get the result of the $O(2)$ model in $4+\epsilon$ dimensions, we simply replace ϵ with $-\epsilon$, so we have

$$\Delta = Q\left[\left(\frac{D}{2}-1\right)-\frac{\epsilon}{10}(Q-1)-\frac{\epsilon^2}{50}\left(Q^2-4Q+O(1)\right)+\circ\circ\circ\right].$$

  This matches Feynman diagram computation, but this is not the end of the story.

- Since the model is a complex CFT, the operator dimension gets corrected and obtain nonperturbative imaginary parts from instanton corrections to the sphere partition function (See Giombi et al).
- Indeed, the operator dimension of ϕ gets an imaginary part of $O(e^{-1/\epsilon})$, so the result above is not enough.
- The imaginary part is very intuitive, since the potential of the model is the inverse ϕ^4 potential, and experiences the tunnelling.

Aside – Interacting limit

- In the interacting limit $\epsilon Q \gg 1$, the imaginary part is no longer nonperturbative. Replace ϵ with $-\epsilon$ in the final expression, and we get

$$\Delta = -\frac{15}{8\epsilon}\left(\frac{-2\epsilon Q}{5}\right)^{\frac{4+\epsilon}{3+\epsilon}} + \circ\circ\circ = -e^{\pm\frac{2\pi i}{3}}\left(\frac{2\epsilon Q}{5}\right)^{\frac{4+\epsilon}{3+\epsilon}}$$

- The transition is because the saddle point solution cannot have a real solution anymore when $\epsilon Q > O(1)$.

$$A^2 = \frac{2(\mu^2 - m^2)}{\lambda}, \quad Q = 2A^2\mu\frac{2\pi^{D/2}}{\Gamma(D/2)}$$

$\lambda < 0$ here.

- As I said, there is a computation in the ϵ-expansion and the large-N expansion that there is indeed an imaginary part to the operator dimension. This comes from the instanton contribution from the sphere partition function.
- In our large charge formalism, it is more intuitive where such a nonperturbative correction comes from.
- In order to see this, let's go back to the original formalism using the grandcanonical partition function. It was

$$Z(\beta,\theta) \equiv \mathrm{Tr}\left[e^{-\beta H-\mu\hat{Q}}\right] = \int D\phi\, e^{-(S[\phi]+\mu Q)}$$

- We first separate the fields into two parts, $\phi = \frac{r}{\sqrt{2}}\, e^{i\chi}$.

Nonpertubative corrections – Free limit

- We were talking about the grandcanonical partition function

$$Z(\beta,\theta) \equiv \mathrm{Tr}\left[e^{-\beta H-\mu\hat{Q}}\right] = \int D\phi\, e^{-(S[\phi]+\mu Q)}$$

- We are looking for the saddle of this path-integral. That is, the solution to the classical EOM. We take the low-temp limit, so only the lowest one matters. in particular, this is homogeneous in space.
- We separated the fields into two parts, $\phi = \frac{r}{\sqrt{2}}\, e^{i\chi}$.

- The EOM is

$$\partial^2\phi = m^2 + 2\lambda|\phi|^2\phi$$

This reduces to

$$\ddot{r} = r\dot{\chi}^2 - m^2 r - \lambda r^3 \quad r^2\dot{\chi} = Q/\alpha(D)$$

- One can eliminate χ and we get

$$\ddot{r} = -V'(r), \quad V(r) = \frac{\rho^2}{2r^2} + \frac{m^2}{2}r^2 - \frac{\lambda}{4}r^4,$$

where $\lambda/(4\pi^2) = \epsilon/5 > 0$ (as I changed the definition of λ by a minus sign).

- The lowest configuration of this QM system gives the log of the grand canonical partition function.

Nonpertubative corrections – Free limit

- In the Euclidean signature, we have the inverted potential and the EOM is

$$\ddot{r} = V'(r), \quad V(r) = \frac{\rho^2}{2r^2} + \frac{m^2}{2}r^2 - \frac{\lambda}{4}r^4$$

  actually, we can rescale $r \equiv R/\sqrt{|\lambda|}$ and have

$$V(r) = \frac{1}{|\lambda|}\left[\frac{(\lambda\rho)^2}{2R^2} + \frac{m^2}{2}R^2 - \frac{\lambda}{4}R^4\right]$$

- From here on $\gamma \equiv \lambda\rho$.
- (Memo: Write on the board how the potential looks like.)
- What we have been doing so far is to look for time independent solutions. Indeed a real local minima of $V(r)$ exists when $x \equiv \lambda Q < O(1)$. On the other side, there is not real saddle, meaning the operator dimension is really complex at the saddle point.
- But even when $\lambda Q < O(1)$, there exists a bounce solution, which is time dependent.

Nonpertubative corrections – Free limit

- It is now easy to compute the action of the bounce from the potential

$$V(r) = \frac{1}{|\lambda|}\left[\frac{(\lambda\rho)^2}{2R^2} + \frac{m^2}{2}R^2 - \frac{\lambda}{4}R^4\right] \equiv \frac{1}{|\lambda|}W(R)$$

- The action of the bounce is given by

$$S_b \equiv 2\alpha(D)\int_{r_0}^{r_1} dr\sqrt{2(V(r) - V(r_0))} = \frac{2\sqrt{2}\alpha(D)}{\lambda}s_b(\lambda\rho)$$

where

$$s_b(\gamma) \equiv \int_{R_0}^{R_1} dR\sqrt{(W(R) - W(R_0))}$$

Nonpertubative corrections – Free limit

- The computation can be in principle done analytically but let's do it numerically.
- We know that

$$s_b(0) = \frac{\sqrt{2}}{3}$$

  so we define $s_b(\gamma) = \frac{\sqrt{2}}{3} F(\gamma)$.
- Finally, we get

$$S_b = \frac{N+8}{3\epsilon} F(\epsilon Q)$$

  by plugging in the fixed point coupling.

Nonpertubative corrections – Free limit

- The final result for the imaginary part should then look like

$$\pm i f_b(\epsilon Q)\sqrt{\frac{2(N+8)}{\pi\epsilon}}\exp\left(-\frac{N+8}{3\epsilon}F(\epsilon Q)\right)+\cdots$$

- N as in $O(N)$ model, I'm shoing it as the logic is the same. Also i comes from the unstable direction around the saddle. This numerically looks like this.

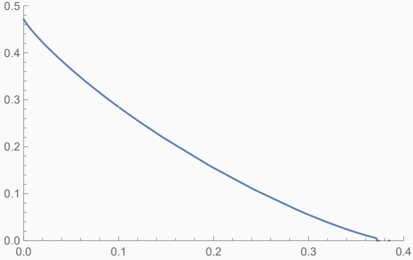

Figure 1: Plot for $F(\epsilon Q)$

Two interesting points

- We had

$$\operatorname{Im}\Delta(Q) = \pm f_b(\epsilon Q)\sqrt{\frac{2(N+8)}{\pi\epsilon}}\exp\left(-\frac{N+8}{3\epsilon}F(\epsilon Q)\right) + \cdots$$

- First of all, at $\epsilon Q = 0$, the non-perturbative correction is of order $O(\exp\left(-\frac{N+8}{3\epsilon}\right))$.
- This matches the non-perturbative correction computed using the ordinary ϵ-expansion, for the dimension of ϕ. This checks the consistency of our result.
- The regime ϵQ fixed is smoothly connected to ϵ small and Q fixed.

Two interesting points

- We had

$$\operatorname{Im}\Delta(Q) = \pm f_b(\epsilon Q)\sqrt{\frac{2(N+8)}{\pi\epsilon}}\exp\left(-\frac{N+8}{3\epsilon}F(\epsilon Q)\right) + \cdots$$

- Second, the point $\epsilon Q = x_0$ at which $F(x) = 0$ is also interesting.
- The action of the bounce becomes zero and the dilute-gas approximation is not valid there anymore.
- This is usually associated with some form of large-N [or Q] phase transitions (e.g., GWW transition).

Two interesting points

- Indeed, the "planar" limit of $\Delta(Q)$, which is $F_0(\epsilon Q)/\epsilon$ experiences the phase transition at $\epsilon Q = x_0$.
- This is presumably smoothed out by the growing instanton corrections near $\epsilon Q = x_0$. In particular, the imaginary part should smoothly connect to the very large imaginary part at large ϵQ.
- After some computation, we can also see that

$$\Delta(Q) = \pm f_b(\epsilon Q)\sqrt{\frac{2(N+8)}{\pi\epsilon}}\exp\left(-\frac{N+8}{3\epsilon}F(\epsilon Q)\right) + \cdots$$
$$F(\epsilon Q) \approx (\epsilon Q - x_0)^{5/4}/\epsilon$$

- One would be able to take a double-scaling limit which fixes $(\epsilon Q - x_0)^{5/4}/\epsilon$ to probe the region in more detail.
- These are just abstract comments but kind of hints towards a connection to matrix models. (Also remember the RMT paper by Komargodski et al.)

Final result with bounce corrections – Free limit

- The final result for the operator dimension in the free limit is

$$Q\left[\left(\frac{D}{2}-1\right)-\frac{\epsilon}{10}(Q-1)-\frac{\epsilon^2}{50}\left(Q^2-4Q+O(1)\right)+\circ\circ\circ\right]$$
$$\pm i f_b(\epsilon Q)\sqrt{\frac{2(N+8)}{\pi\epsilon}}\exp\left(-\frac{N+8}{3\epsilon}F(\epsilon Q)\right).$$

- As we aproach $\epsilon Q = x_0$, the non perturbative imaginary correction gets bigger and bigger.
- This connects nicely to the formula for $\epsilon Q \gg 1$,

$$\Delta = -e^{\pm\frac{2\pi i}{3}}\left(\frac{2\epsilon Q}{5}\right)^{\frac{4-\epsilon}{3-\epsilon}}+\circ\circ\circ$$

  The nonperturbative imaginary part comes from the proliferation of the instanton!

- This concludes what I wanted to say today.

If we have time

- (1) About the SUSY cubic model.
- (2) Mystery of the exponent $Q^{(4+\epsilon)/(3+\epsilon)}$

Future directions

- Resurgence structure of the double-scaling limit? How does this relate to the expansion with fixed ϵ and large Q, or fixed Q and small ϵ?
- Similar structure of double expansion in twisted superpotential of $\mathcal{N} = 2$ gauge theory. In this language, $\epsilon Q \gg 1$ is the electric description and $\epsilon Q \ll 1$ is the dyonic description. SW curve for this non-SUSY system!?
- What can we say about $D = 4 - \epsilon$ case? We expect no real bounce, but does this mean we have no non-perturbative corrections to the ϵ-expansion?
- We might have complex bounces. Would this contribute as a small nonperturbative correction to the ϵ-expansion?
- Any connection to matrix models, minimal strings, or 2D gravity?