Status of lattice QCD

Heavy quark CKM physics

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Heavy Quarks & Leptons 2021, U of Warwick

Outline

FLAG 2020, Eur. Phys. J. C (2020) 80:113

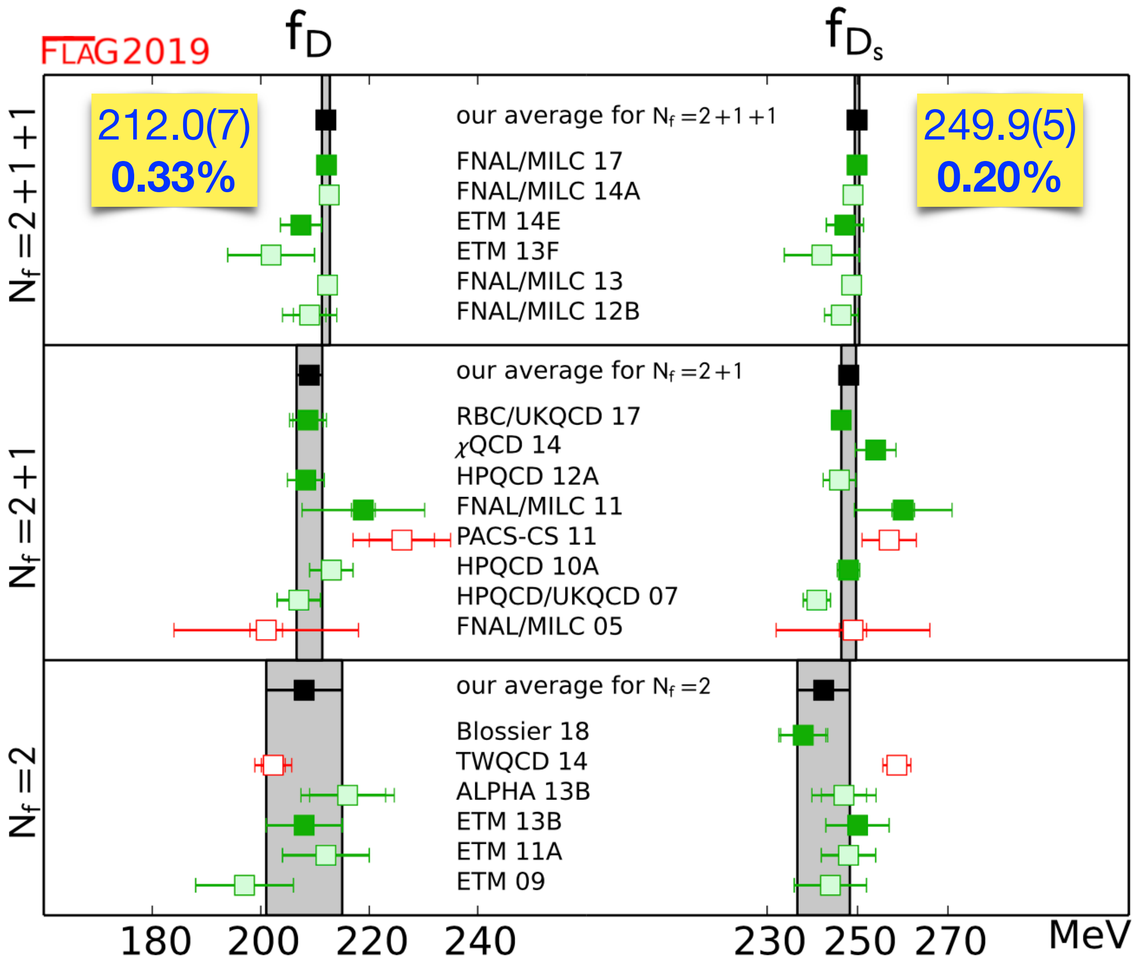
Current status I'll treat FLAG 2020 as baseline and focus on (select) new results

- Decay constants
- Semileptonic decays $(B \to D^*,\ B_s \to D_s^*,\ \Lambda_b \to \Lambda_c^*,\ D \to K)$

Ongoing & next steps

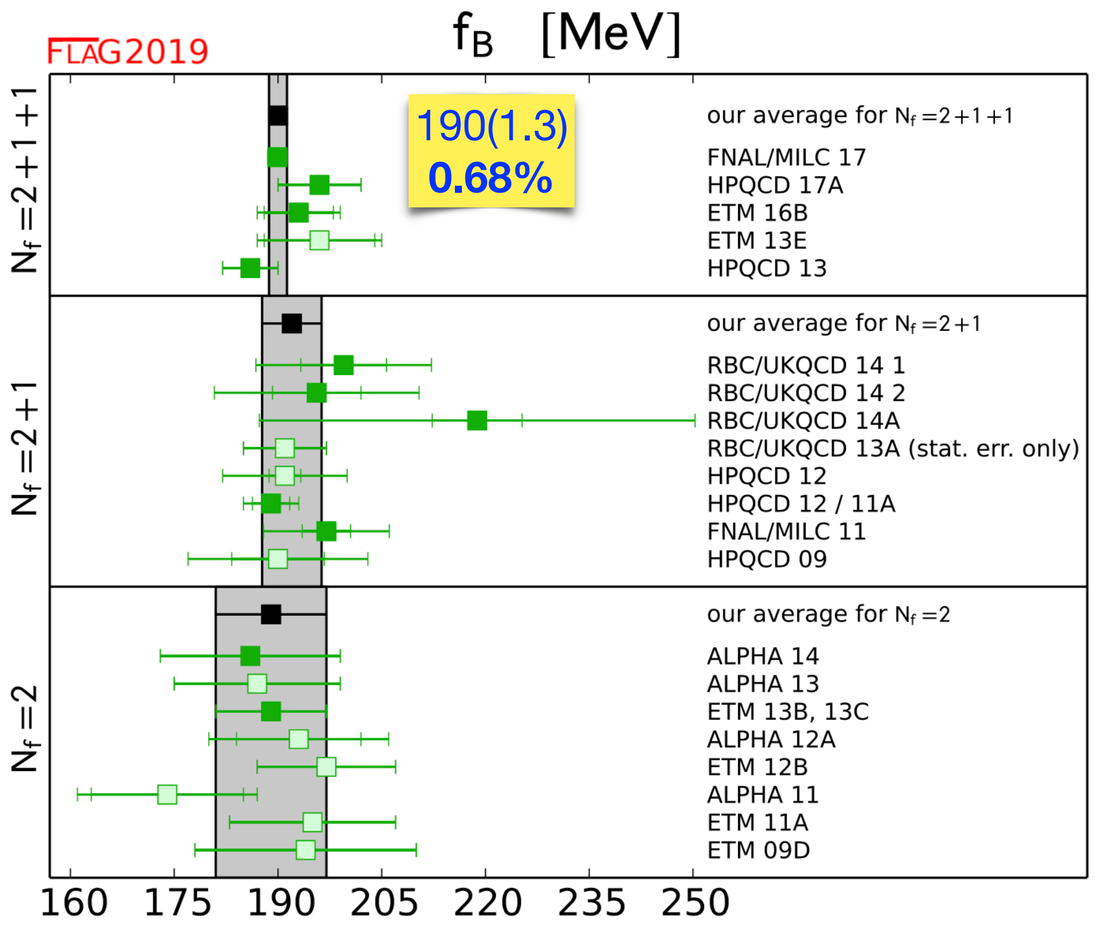
- Improved treatment of heavy quarks in heavy-light decays
- Inclusive decays on the lattice

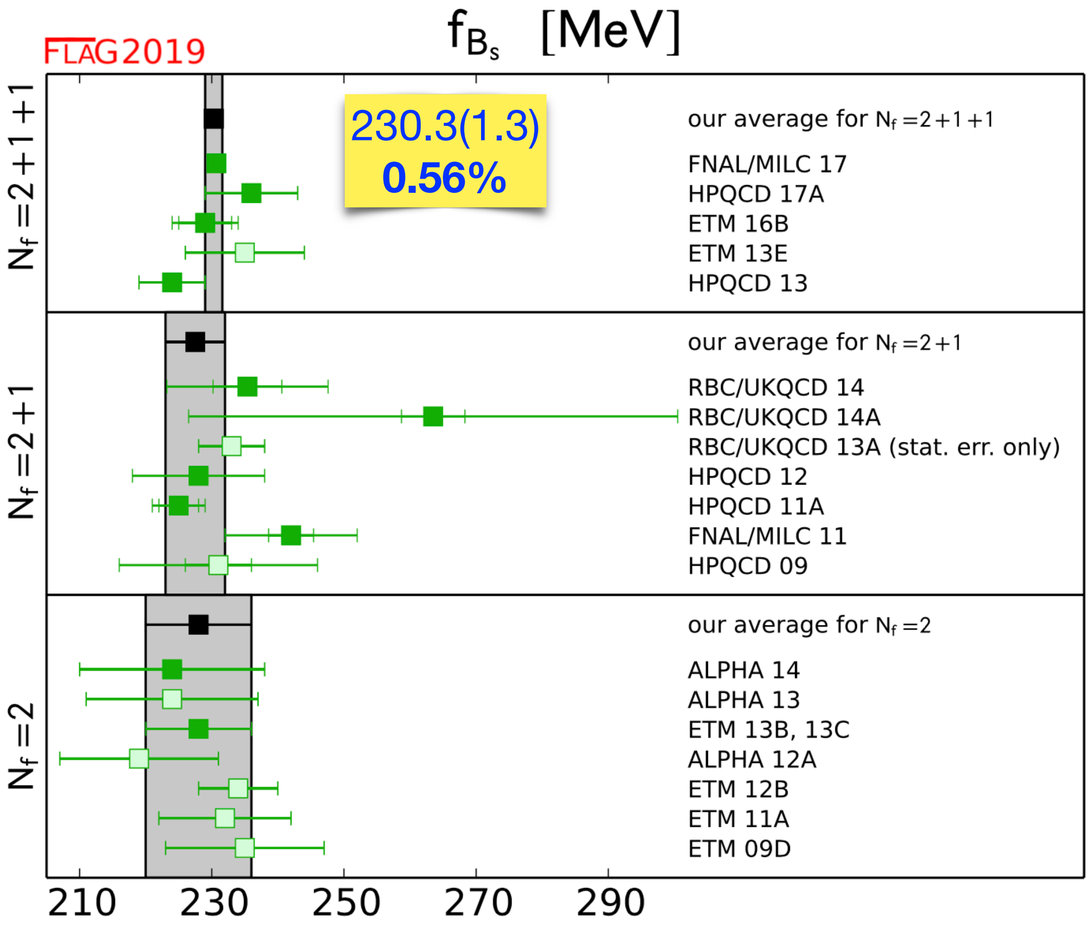
Summary



FLAG 2020, Eur. Phys. J. C (2020) 80:113

- Sub-percent precision from LQCD
- 1-2% expected at Belle II (50/ab) and BES III (20/fb) 1002.5012, 0809.1869
- For CKM elements, LQCD improvement not pressing issue. Would require:
  - Isospin-breaking effects
  - QCD + QED
  - Being done for kaons and pions (Martin Gorbahn's talk)





FLAG 2020, Eur. Phys. J. C (2020) 80:113

- Similar for leptonic B decays: LQCD precision should suffice for CKM elements through Belle II
- Future improvements require isospin-breaking effects and QCD+QED

New results for semileptonic decays …

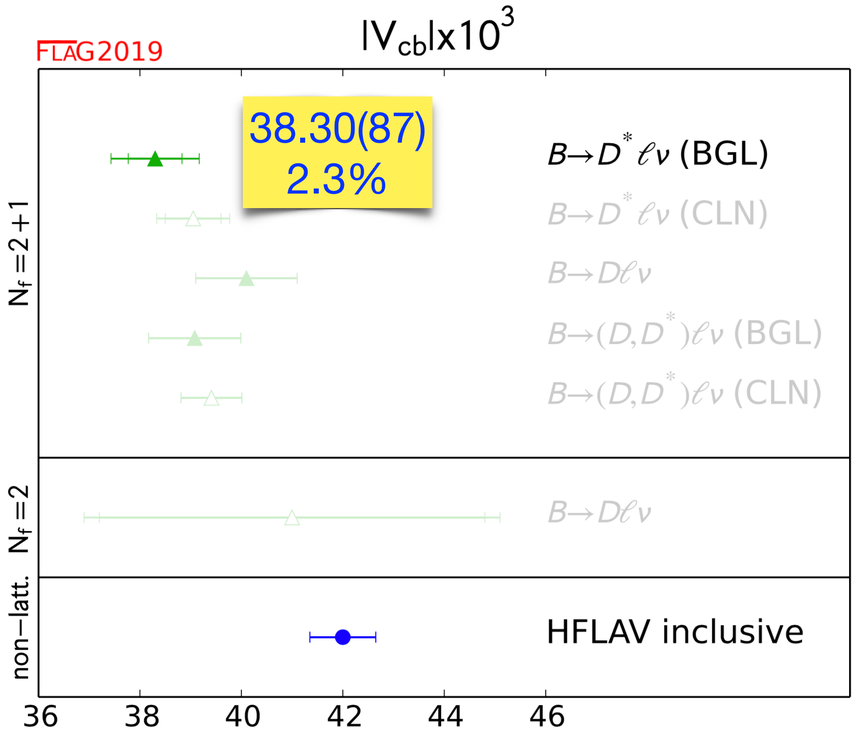
$B \to D^*$ as of FLAG 2020

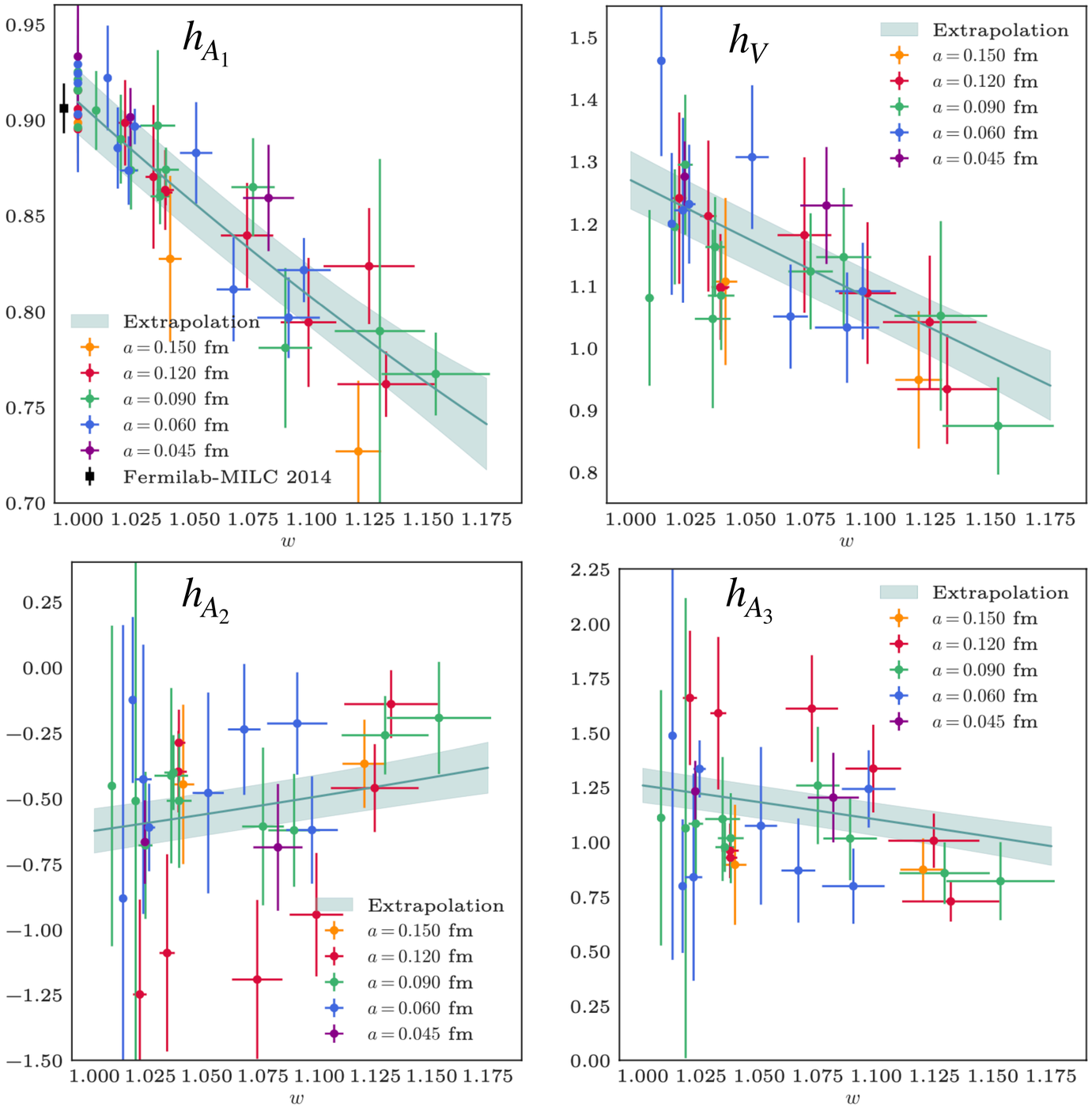
FLAG, Eur. Phys. J. C (2020) 80:113

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$w = 1$ form factor / ratio		
FNAL/MILC 14	[629]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.906(4)(12)	1.4%
HPQCD 17B	[635]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.895(10)(24)	2.3%

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3}(M_B - M_{D^*})^2 \chi(w)\sqrt{w^2-1}\,(\eta_{\rm EW}\,|\,V_{cb}\,|)^2\,|\,\mathcal{F}(w)\,|^2$$

- Historically focus on zero recoil, $w = 1$
- $\eta_{\rm EW}$ universal short-distance corrections to G_F
- $\chi(w)$ a kinematic function



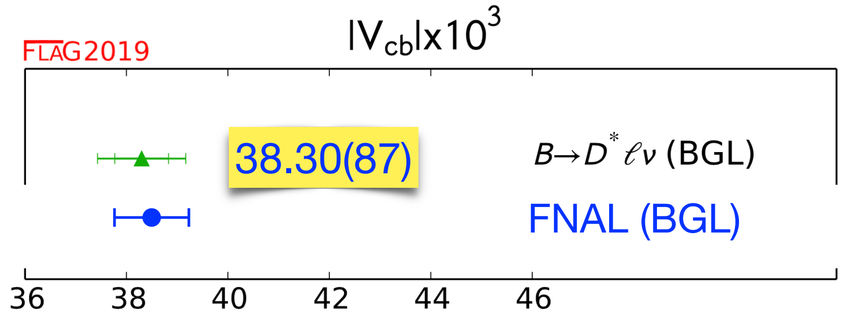


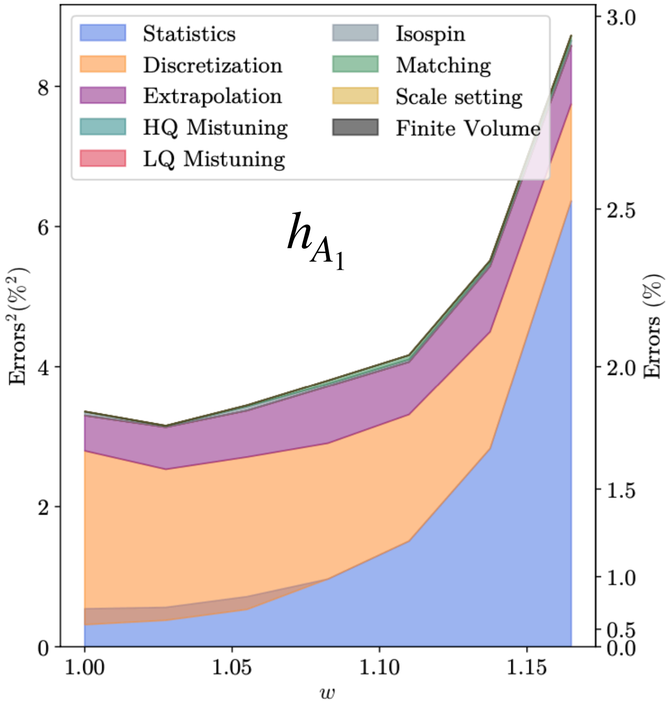
New: $B \to D^*$ at nonzero recoil

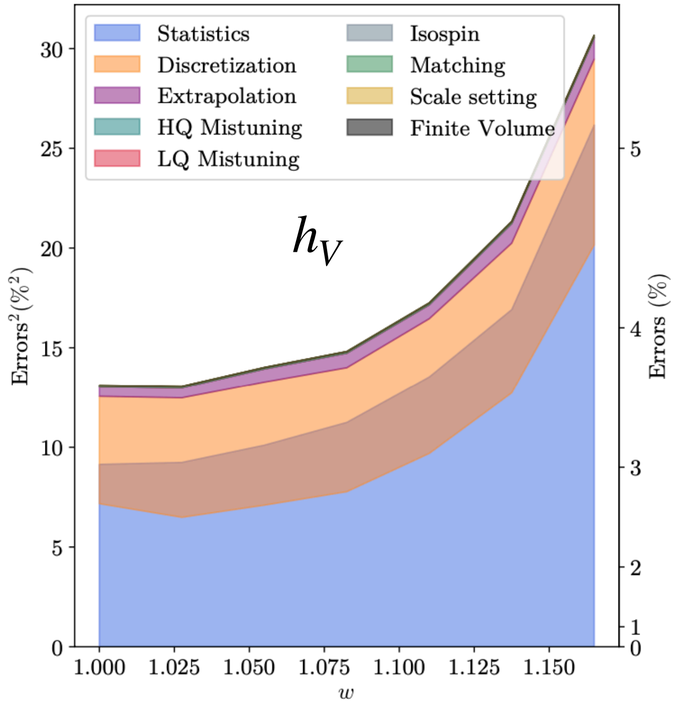
FNAL/MILC, 2105.14019

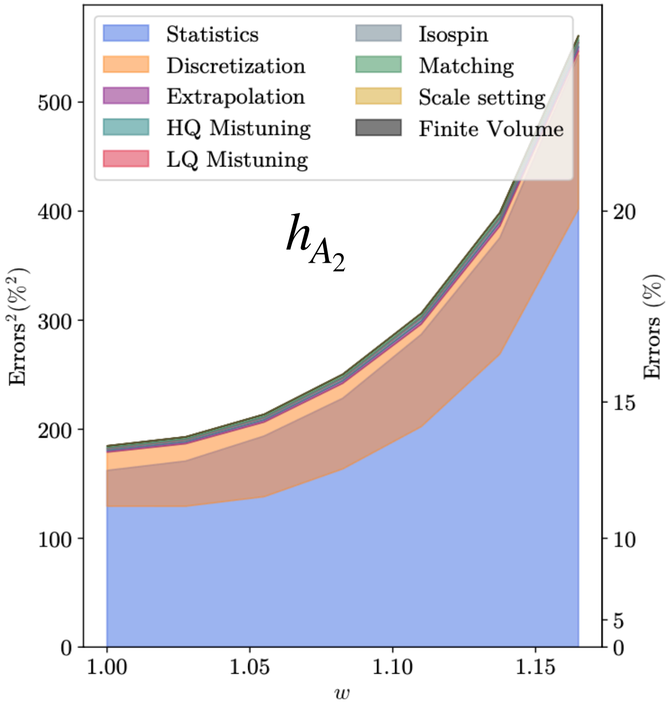
- First unquenched LQCD calculation at nonzero recoil, 2+1 flavors of sea quarks
- Pion masses: 180-560 MeV
- 5 lattice spacings: 0.15-0.045 fm
- 'Fermilab' b, c quarks (effective theory)
- 'asqtad' staggered light quarks

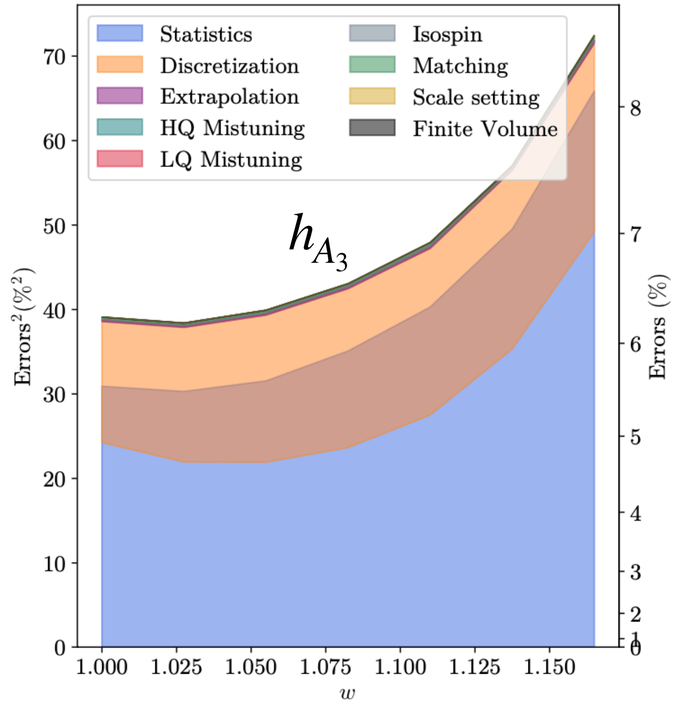
$$10^3 \, |V_{cb}| = 38.40 \pm 0.66_{\text{th}} \pm 0.34_{\text{exp}}$$











Exciting first result. Just as exciting, future improvements straightforward:

- Next generation MILC 2+1+1 lattices: better statistics, reduced sea quark/ gluon discretization effects
- HISQ (Highly Improved Staggered Quarks) vs Fermilab b, c: should reduce discretization effects

Dominant FNAL form factor uncertainties:

- Statistics (blue)
- Discretization (orange)
- Where they overlap (brown)

All HISQ calculation (including b, c) addresses both dominant errors. FNAL and HPQCD both pursuing this.

$B_s \to D_s^*$ as of FLAG 2020

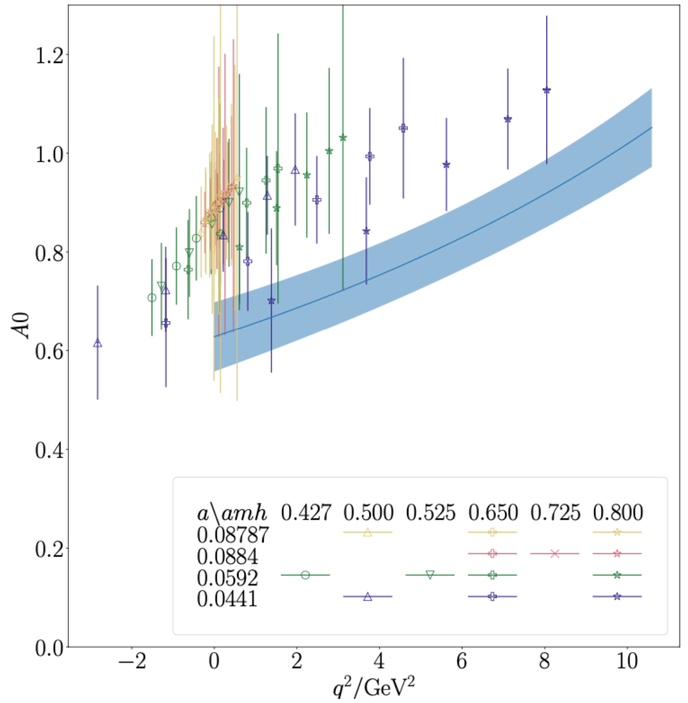
FLAG, Eur. Phys. J. C (2020) 80:113

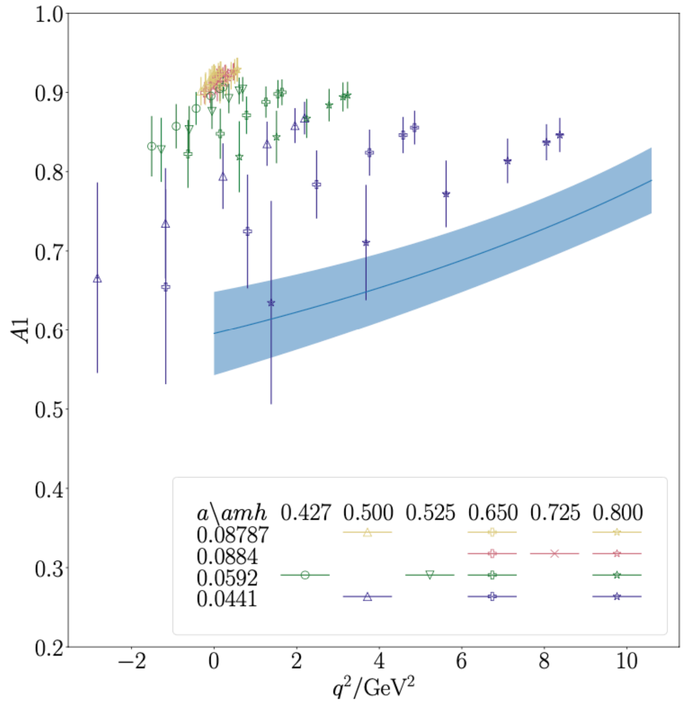
Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$w = 1$ form factor / ratio		
HPQCD 17B	[635]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B_s \to D_s^*}(1)$	0.883(12)(28)	3.4%

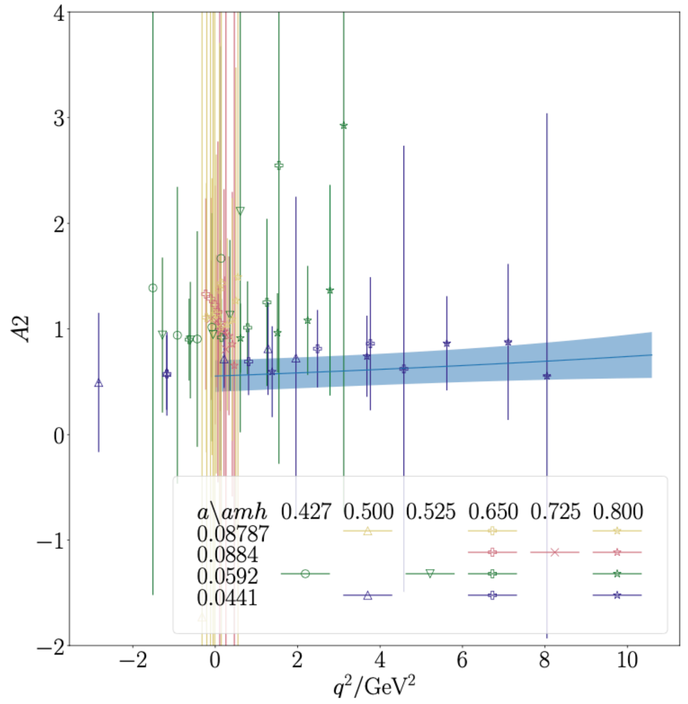
$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3}(M_{B_s} - M_{D_s*})^2 \chi(w) \sqrt{w^2 - 1}\, (\eta_{\mathrm{EW}} |\, V_{cb}\,|)^2\, |\, \mathcal{F}(w)\,|^2$$

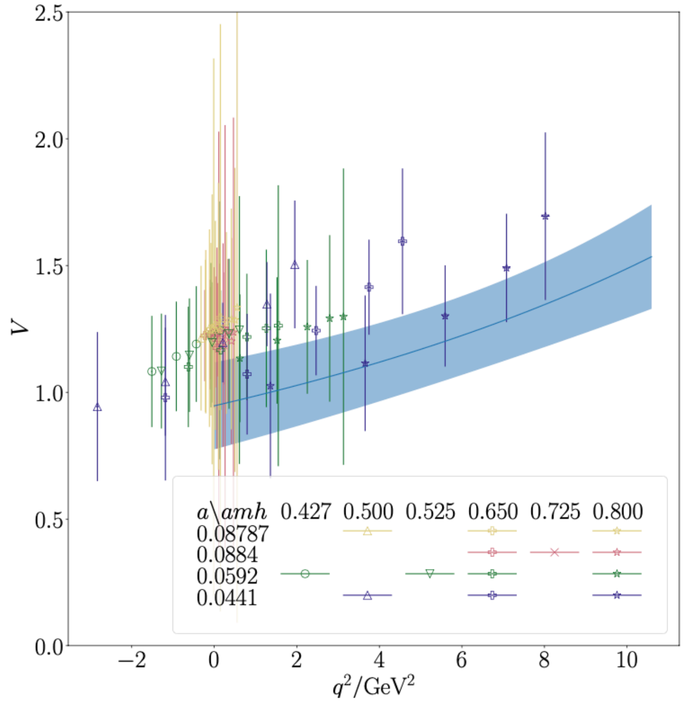
- One result, at zero recoil, $w = 1$
- Combination with LHCb results not yet done

LHCb, Phys. Rev. D 101, 072004 (2020)









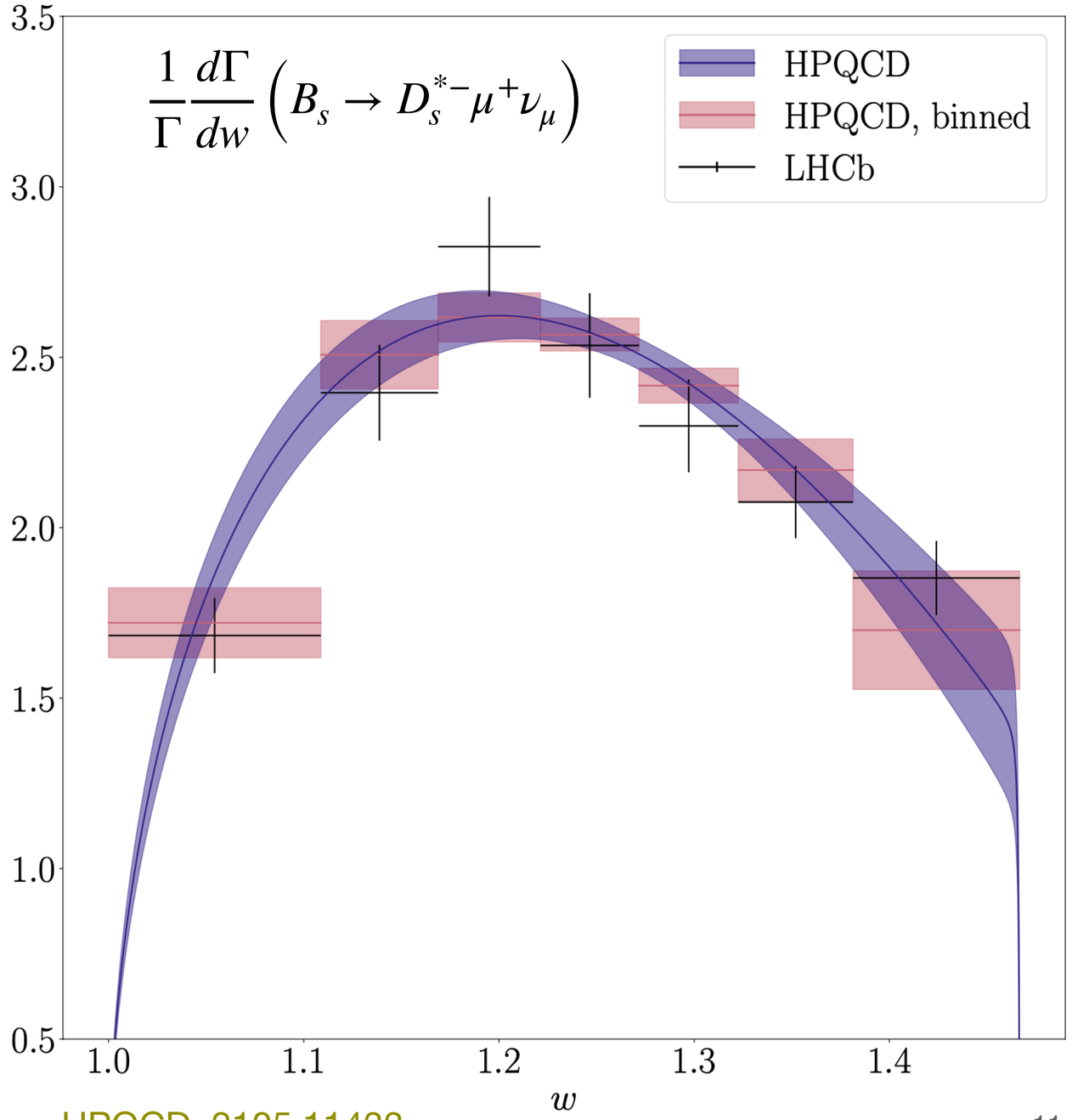
New: $B_s \to D_s^*$ at nonzero recoil

Davies & Harrison (HPQCD), 2105.11433

- First LQCD calculation at nonzero recoil
- Includes physical pion mass
- 3 lattice spacings: 0.09, 0.06, 0.045 fm
- All HISQ quarks
- HISQ b, c quarks - *'heavy HISQ'*
  - Proxy for b is heavy quark h

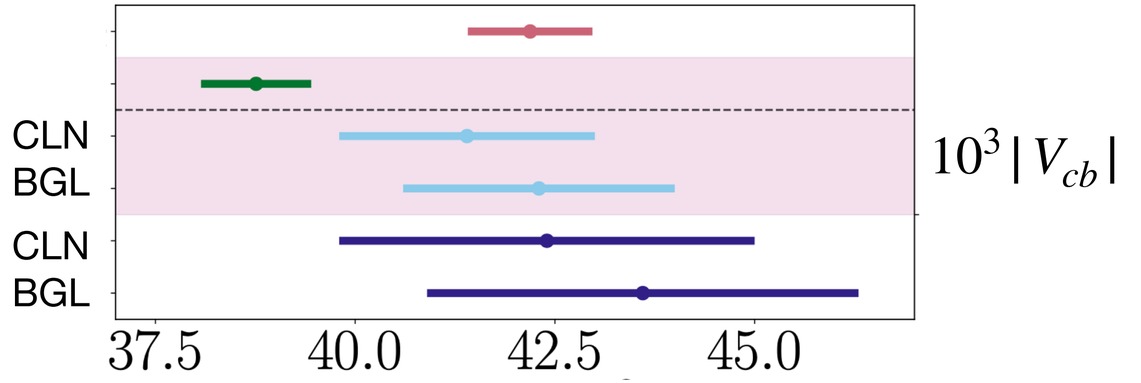
    $$m_c \leq m_h \lesssim m_b$$

  - Reach in m_h limited by $\mathcal{O}(am_h)^2$ errors, smaller a means larger m_h
  - Simulate range of m_h, extrapolate to m_b
  - Fully relativistic action removes effective theory (NRQCD) matching error
  - Renormalizaton of lattice matrix elements done nonperturbatively (or absolutely normalised via PCVC/PCAC relations)



HPQCD, 2105.11433

- (Left) shape of $d\Gamma/dw$ agrees well with LHCb
- (Below) joint fit to LHCb data and HPQCD's $B_s \to D_s^*$ form factors yields $|V_{cb}|$ (dark blue)



- Comparison with:
  - Inclusive $b \to c$ (red)
  - $B \to D^{(*)}$ at zero recoil (green)
  - LHCb $B_s \to D_s^{(*)}$ and zero recoil LQCD (blue)
- LHCb results given as BGL and CLN parameters, instead of binned $d\Gamma/dq^2$ (seems a bad-ish idea, maybe I'm missing something) PRD 101, 072004 (2020)

% errors

Source	$\Gamma/\lvert\eta_{\rm EW}V_{cb}\rvert^2(1+\delta_{\rm EM})$ $\ell=\mu$	$\ell=\tau$	$R(D_s^*)$
$M_{\eta_h}\to M_{\eta_b}$	3.2	2.65	1.4
$am_c\to 0$	2.8	2.55	0.32
$am_h\to 0$	4.97	4.95	0.653
$\delta^{\rm val}_{m_c}$	0.41	0.40	0.05
$\delta^{\rm sea}_{m_c}$	1.43	1.56	0.21
$\delta^{\rm val}_{m_s}$	0.05	0.06	0.01
$\delta^{\rm sea}_{m_s}$	0.72	0.77	0.09
$\delta^{\rm sea}_{m_l}$	1.16	1.2	0.08
w_0/a, w_0	0.47	0.48	0.27
Statistics			
Set 1	1.68	1.33	0.86
Set 2	3.09	2.44	1.29
Set 3	6.11	5.46	2.0
Set 4	1.14	0.90	0.52
Other Priors	2.5	2.4	1.1
Total	10.2	9.32	3.23

- Dominant errors
  - Extrapolation $m_h \to m_b$
  - Discretization effects
  - Statistics on finest ensembles
- **Not independent**: increased stats on finest lattice (Set 3) would reduce error on largest m_h (helping extrapolation $m_h \to m_b$) and smallest a (helping extrapolation $a \to 0$)
- MILC 0.03 fm lattice would allow simulation at $m_h = m_b$
- Improvement straightforward (common theme in heavy HISQ semileptonic form factors)
  - $B_c \to J/\Psi$ HPQCD, PRD 102, 094518 (2020)
  - $B_c \to B_{(s,d)}$ HPQCD, PRD 102, 014513 (2020)

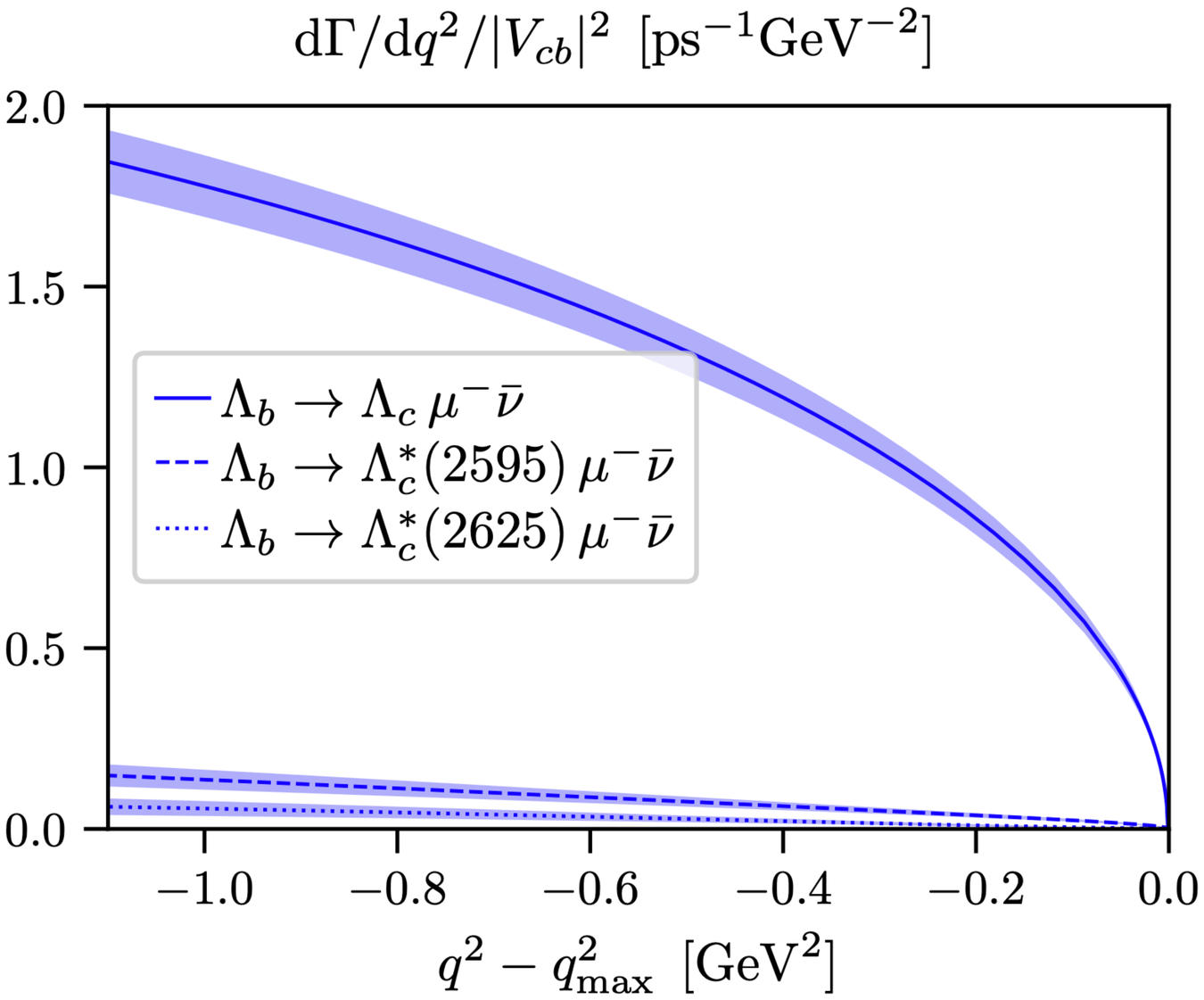
New: $\Lambda_b \to \Lambda_c^*(2595, 2625)$ Meinel & Rendon, PRD 103, 094516 (2021)

- Continues previous work by Meinel et al. on heavy baryon semileptonic decays

$\Lambda_b \to p\ell\nu$, via $b \to u$
$\Lambda_b \to \Lambda_c\ell\nu$, via $b \to c$
} Detmold, Lehner & Meinel, PRD 92, 034503 (2015)

$\Lambda_c \to n\ell\nu$, via $c \to d$ Meinel, PRD 97, 034511 (2018)

- Proceeds via $b \to c$, complements $B_{(s)} \to D_{(s)}^{(*)}$
- $\Lambda_c^*(2595, 2625)$ are lightest $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ charm baryons
- First LQCD calculation of form factors (full set of vector, axial and tensor)
- Lattice spacings: 0.11, 0.08 fm
- Pion masses: 300, 340, 430 MeV
- Kinematics in vicinity of zero recoil



- CDF measured integrated decay rates

$$\Gamma(\Lambda_b \to \Lambda_c^*(2625)) \approx 1.7 \times \Gamma(\Lambda_b \to \Lambda_c^*(2595))$$

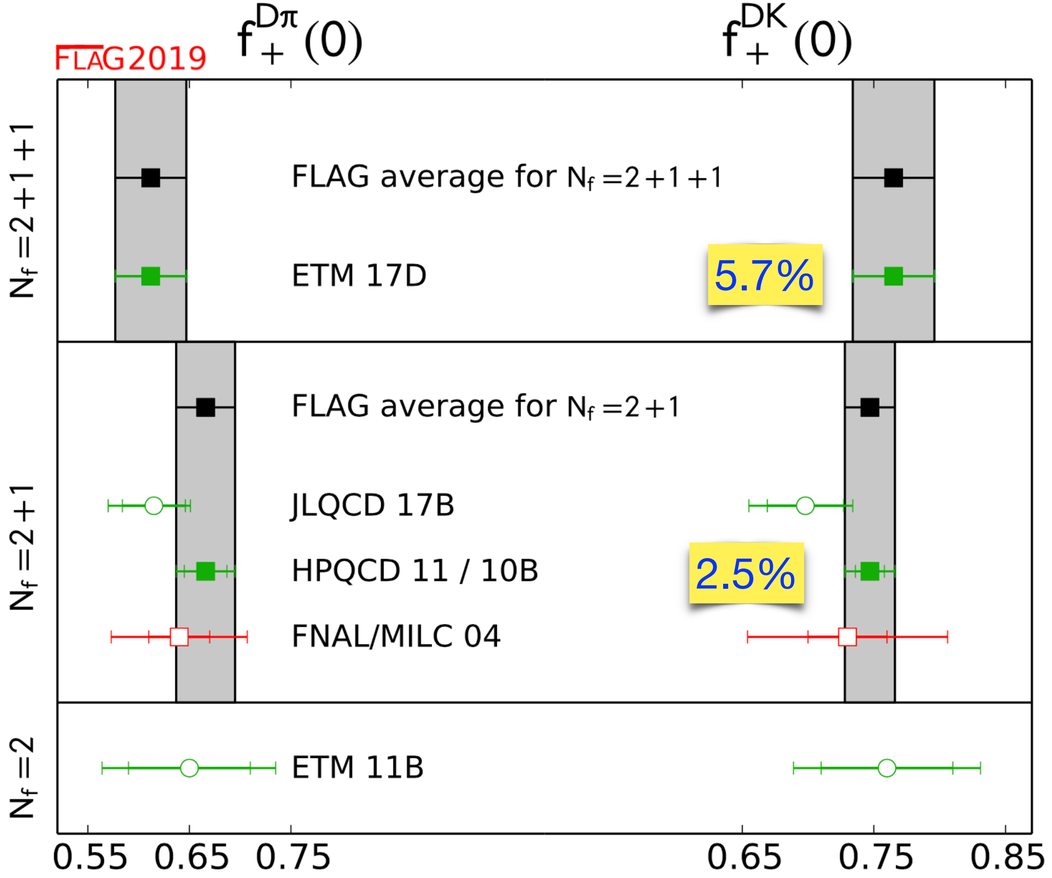
CDF, Phys. Rev. D 79 (2009) 032001

suggesting a crossing at lower q^2

  - Opposite LO HQET prediction
  - Consistent with expected importance of subleading corrections

- Offers checks of lepton flavor universality that complement R(D), R(D*), …
- Provides route to complementary determinations of $|V_{cb}|$

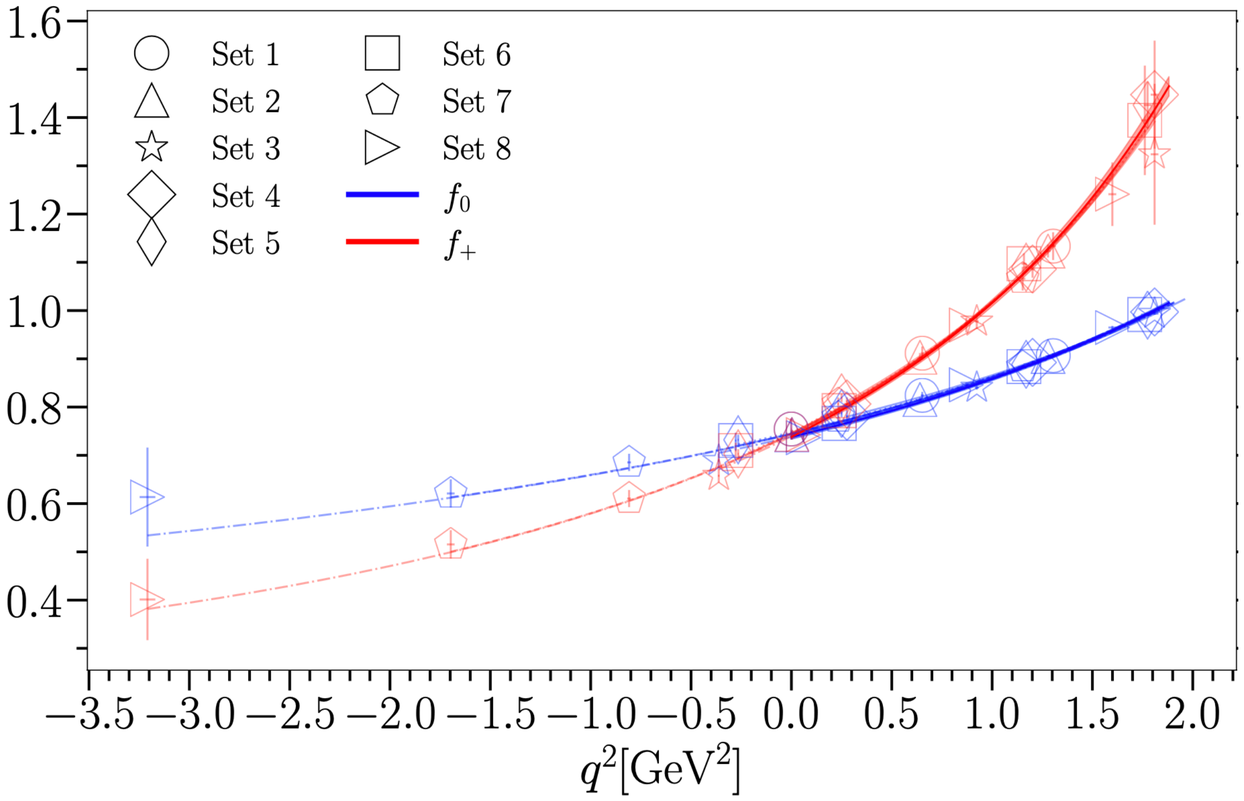
$D \to K$ as of FLAG 2020



HFLAV, EPJ C77 (2017) 895

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_K|^3 (\eta_{\rm EW} |V_{cs}|)^2 (1+\delta_{\rm EM}) |f_+(q)|^2$$

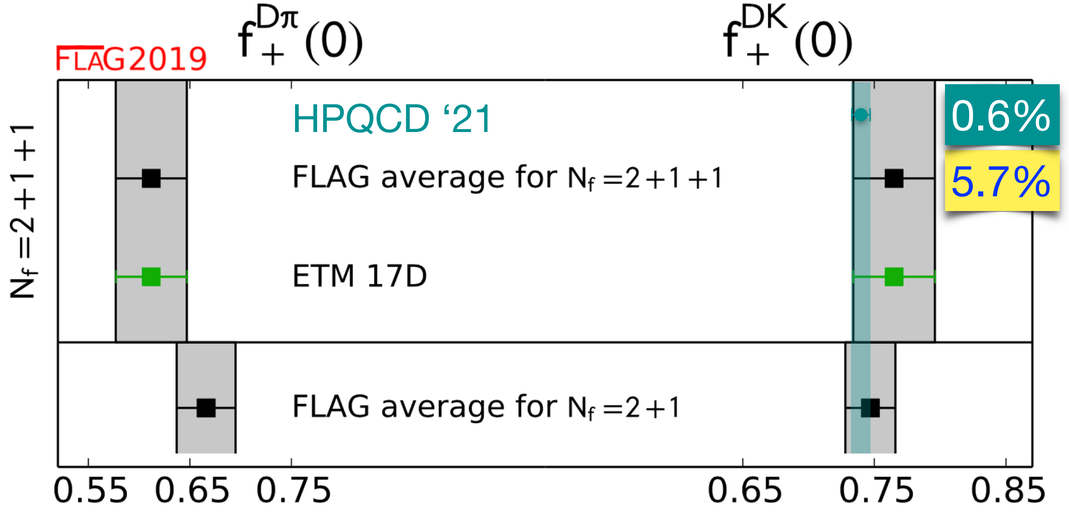
- Historically work at zero recoil, $q^2 = 0$
- $\eta_{\rm EW}$ universal short-distance corrections to G_F
- $\delta_{\rm EM}$ QED corrections. Depends on K^0 vs. $K^\pm$. Not yet calculated, historically ignored.



New: $D \to K$ at nonzero recoil

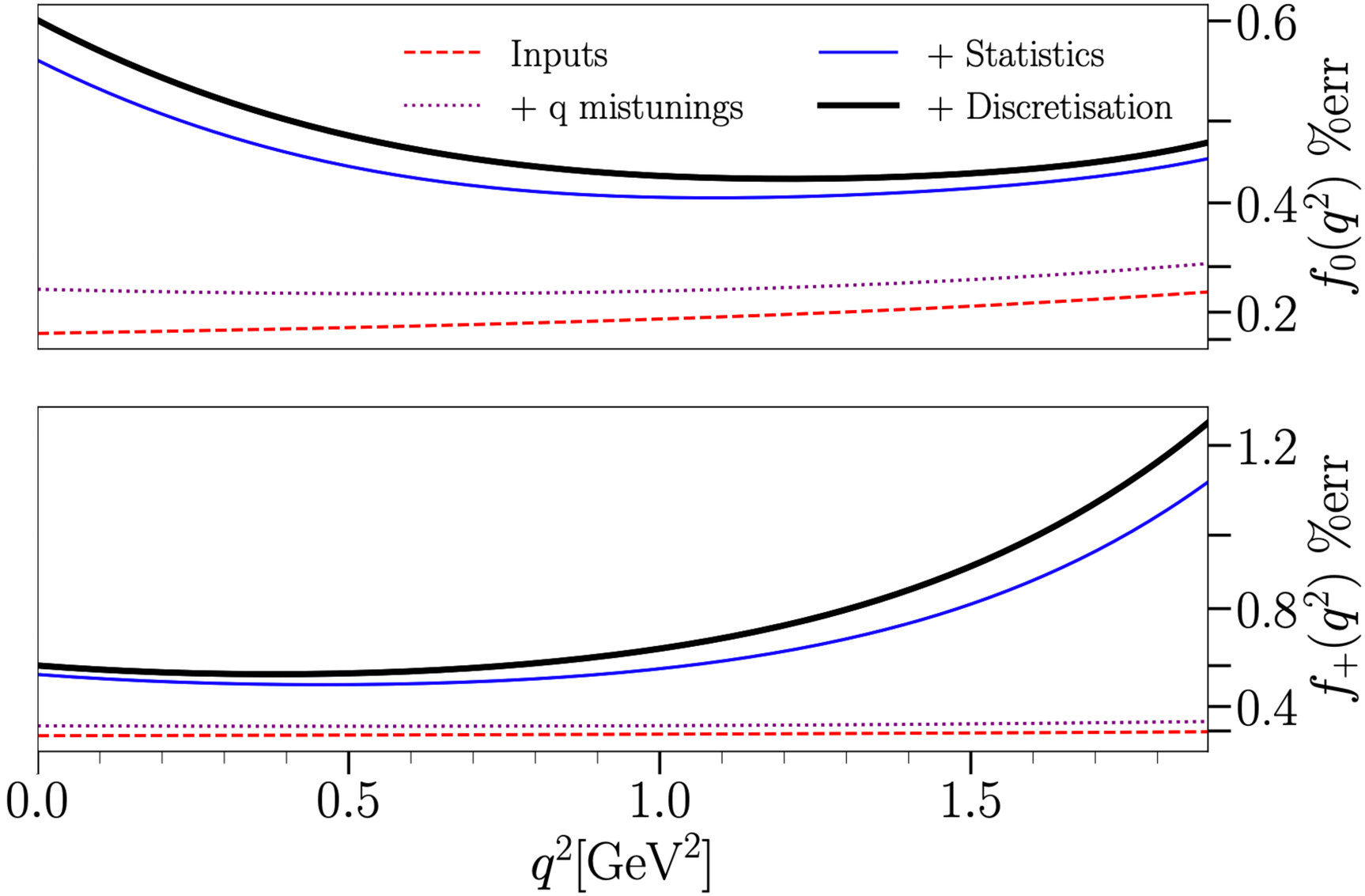
HPQCD, PRD 104, 034505 (2021)

- MILC lattices with 2+1+1 flavors of sea quarks
- All HISQ quarks
- Physical pion masses
- 5 lattice spacings: 0.15-0.045 fm
- Negligible discretisation effects visible in data



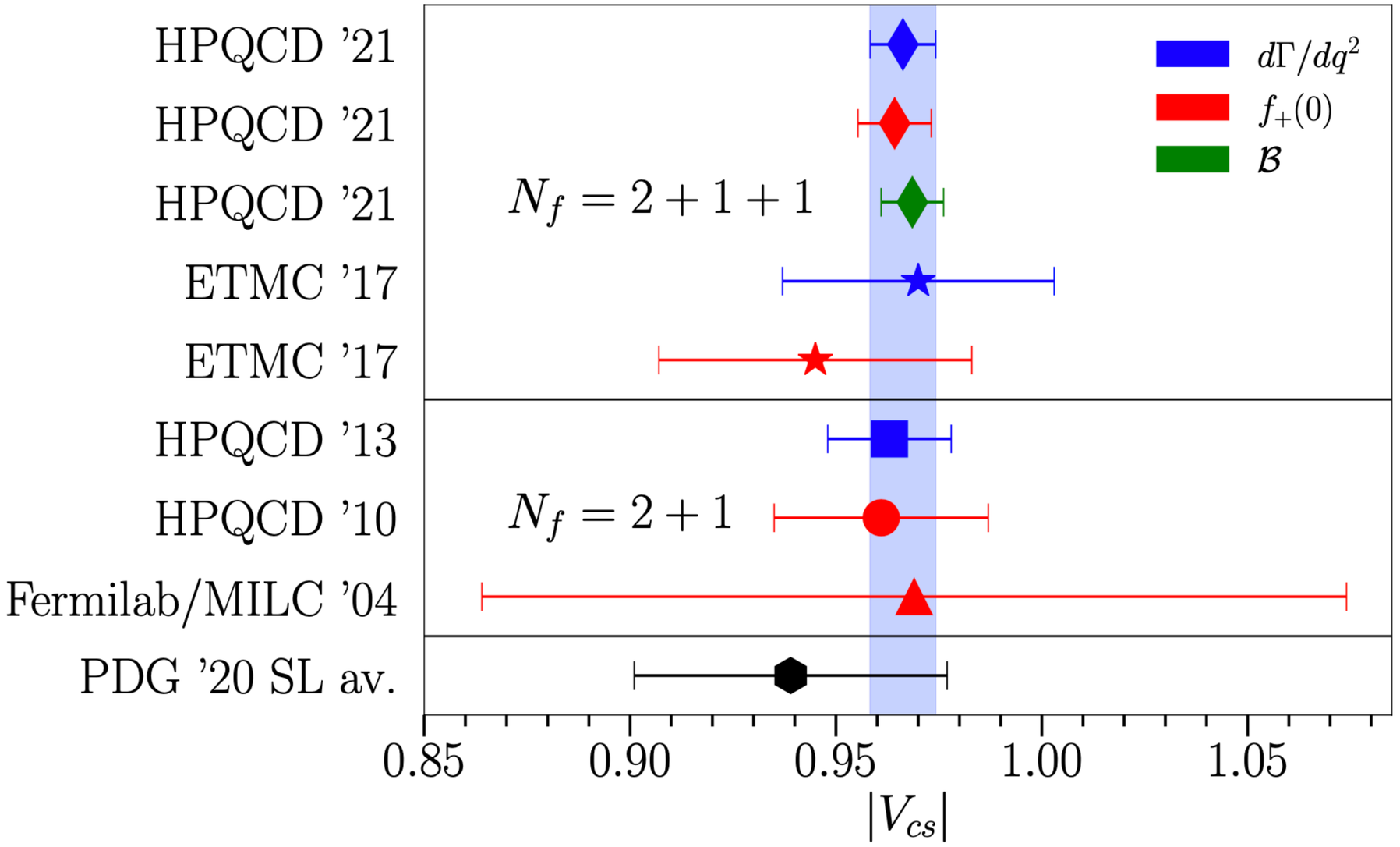
- Significant improvement over previous determinations

HPQCD, PRD 104, 034505 (2021)



- As with other ‘heavy HISQ’ results, dominant source of error is statistics
- Straightforward to improve form factors

$$|V_{cs}| = 0.9663(53)_{\text{latt}}(39)_{\text{exp}}(19)_{\eta_{\text{EW}}}(40)_{\text{EM}}$$ 0.8%



- LQCD error now commensurate with other errors
- EM corrections now important

HPQCD, PRD 104, 034505 (2021)

Ongoing & next steps …

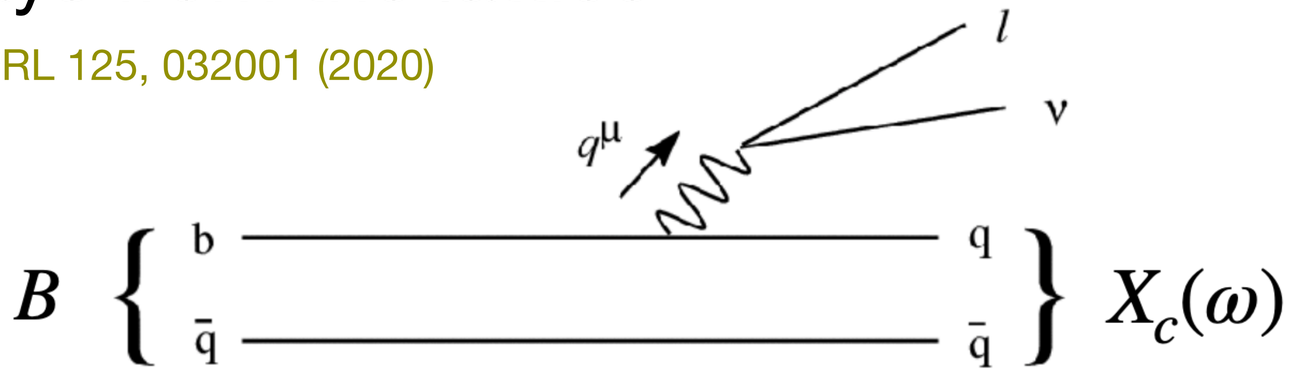
Improved treatment of b, c in heavy → light

- All HISQ $h \to u, d, s$
  - $B_s \to \eta_s$ test case PRD 103 (2021) 9, 094506
  - HPQCD and FNAL (underway): $B_s \to K$, $B \to \pi$
  - FNAL (underway): $D \to \pi$, $D_{(s)} \to K$
- All Domain Wall $h \to u, d, s$
  - RBC-UKQCD (underway): $D \to \pi$, $D_{(s)} \to K$
  - JLQCD (underway): $B \to \pi$ 1912.02409
- All Twisted Mass
  - ETMC (underway): $D \to \pi$, $D \to K$ 1911.02412

Inclusive decays from the lattice

Gambino & Hashimoto, PRL 125, 032001 (2020)

JLQCD, 2001.11678



- Then inclusive differential decay rate is

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3} l^{\mu\nu} W^{\mu\nu}$$

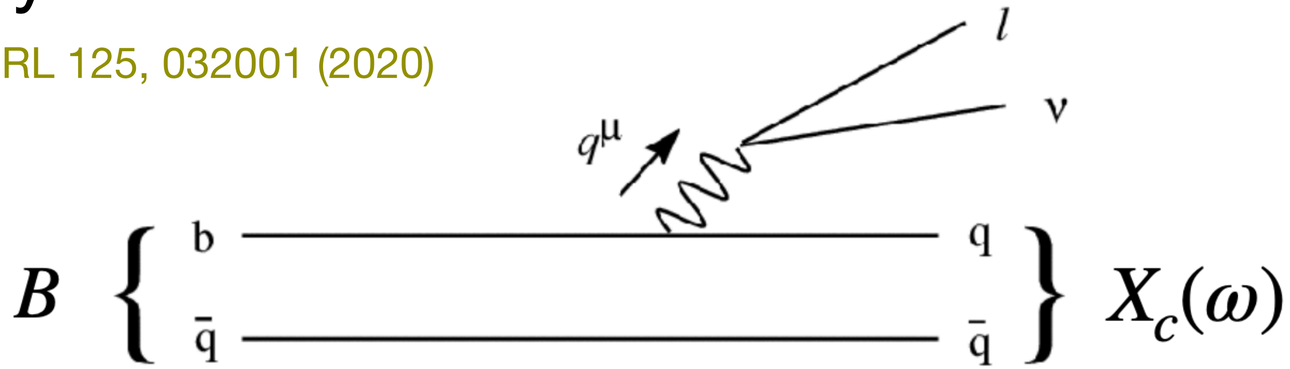
- With hadronic tensor

$$W^{\mu\nu} = \sum_X (2\pi)^2 \delta^{(4)}(p_B - p_X - q) \frac{1}{2M_B} \langle B(p_B) | J_\mu^\dagger(0) | X \rangle \langle X | J_\nu(0) | B(p_B) \rangle$$

Inclusive decays from the lattice

Gambino & Hashimoto, PRL 125, 032001 (2020)

JLQCD, 2001.11678



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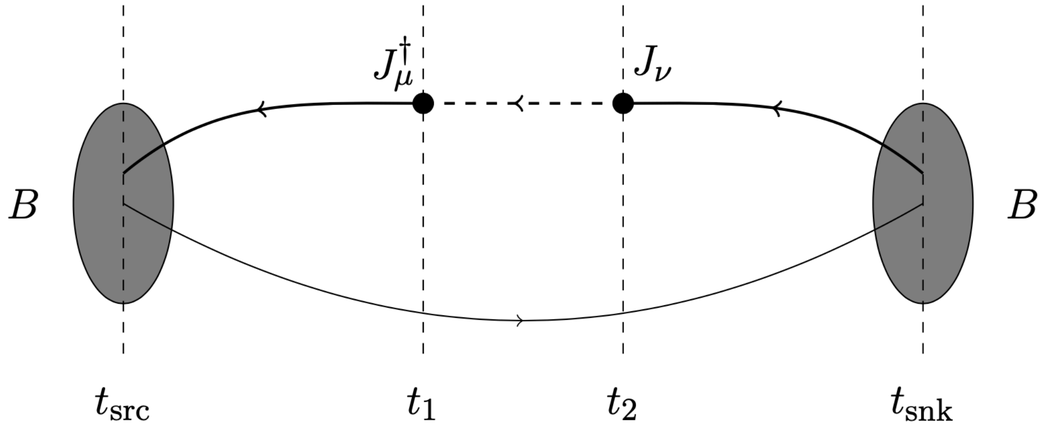
- With hadronic tensor

$$W^{\mu\nu} = \sum_X (2\pi)^2 \delta^{(4)}(p_B - p_X - q) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

$$\langle B(p_B)|J_\mu^\dagger(-\vec{q};t) \,\vdots\, J_\nu(\vec{q};0)|B(p_B)\rangle$$

- The sum over all states X is unfeasible, so use the optical theorem

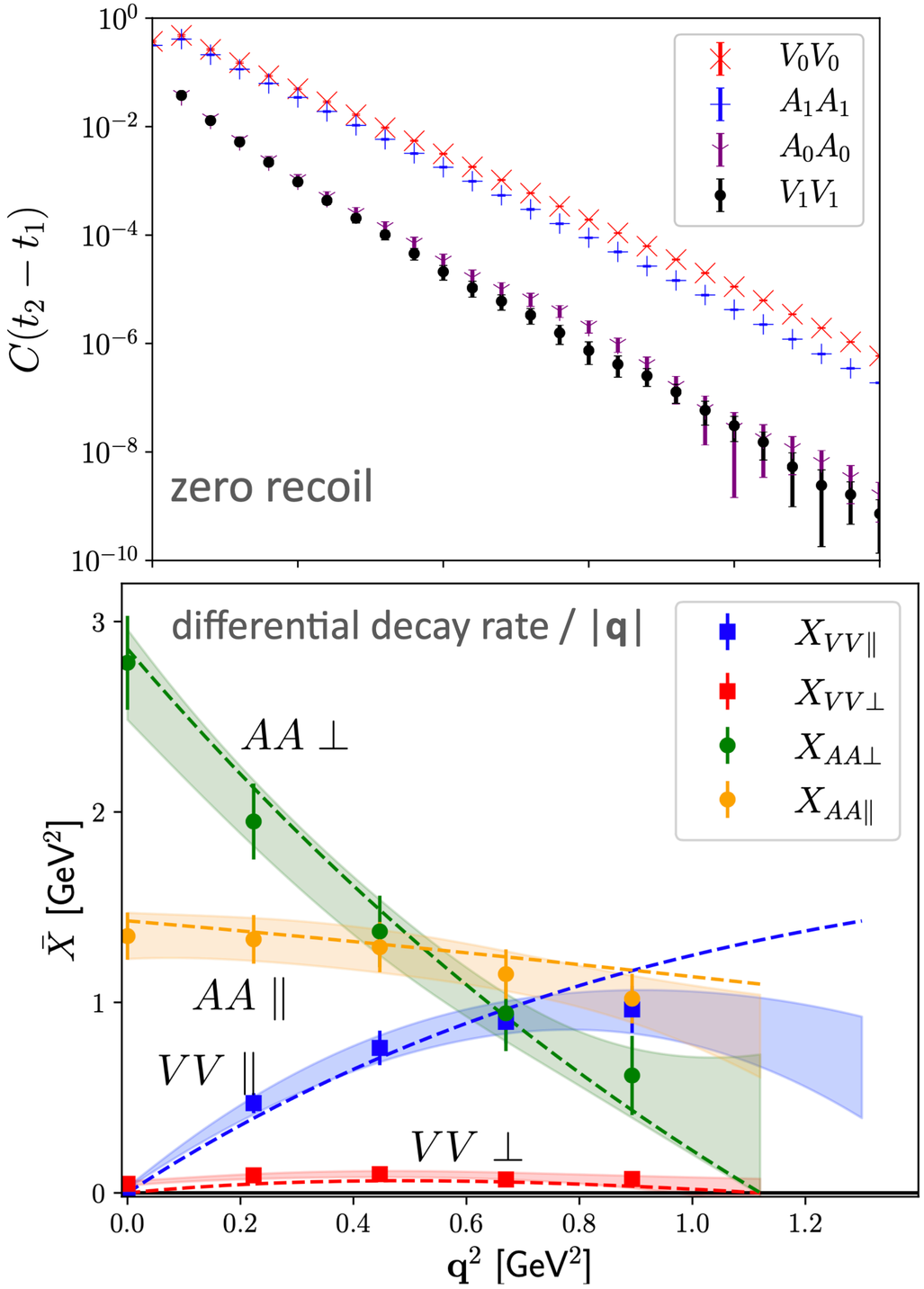
Construct 4pt correlation function



$$\langle B(p_B)\,|\,J_\mu^\dagger(-\vec{q};t_1)\;J_\nu(\vec{q};t_2)\,|\,B(p_B)\rangle$$

Feed back into differential decay rate, shown here for different currents with momenta $\|$ or $\perp$ to recoil momentum

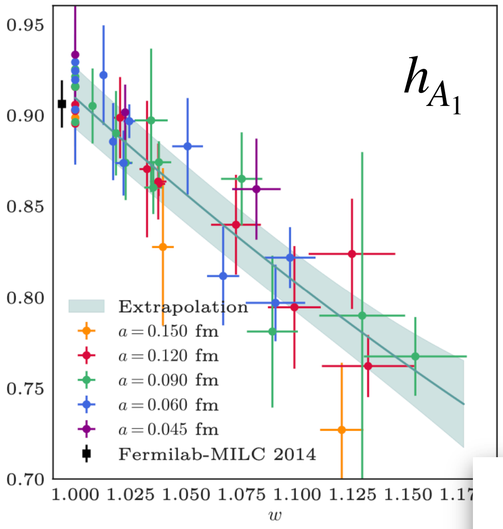
- Dashed lines are estimated sums over exclusive decays

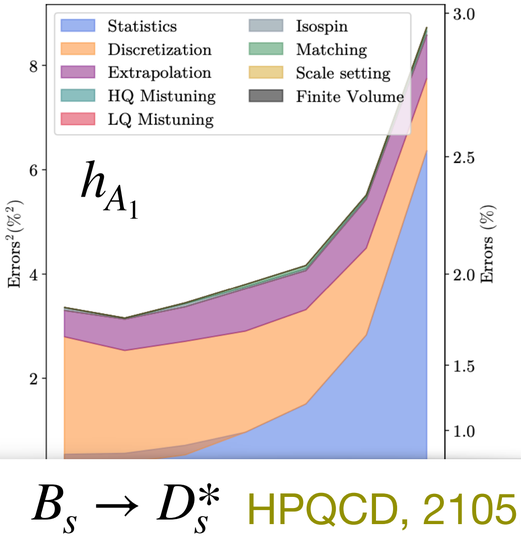


Summary

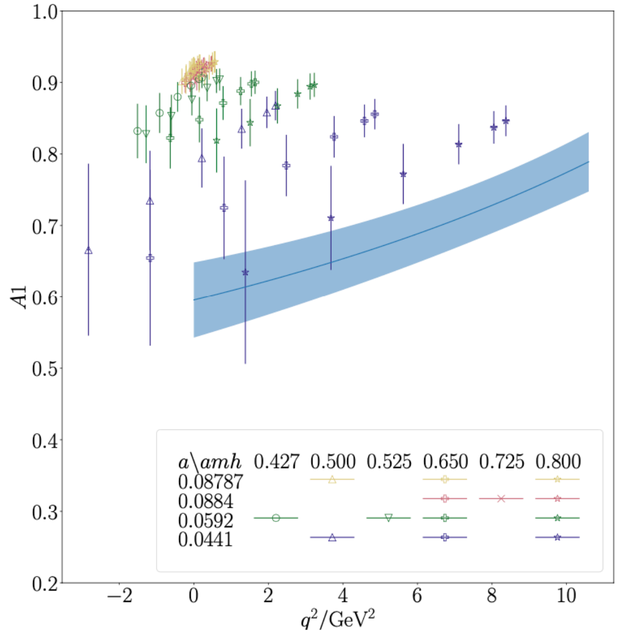
Current status

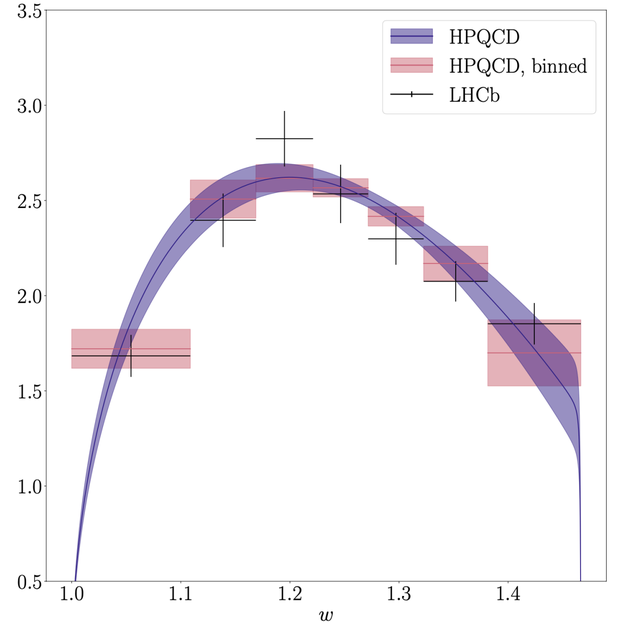
$B \to D^*$ FNAL/MILC, 2105.14019





$B_s \to D_s^*$ HPQCD, 2105.11433





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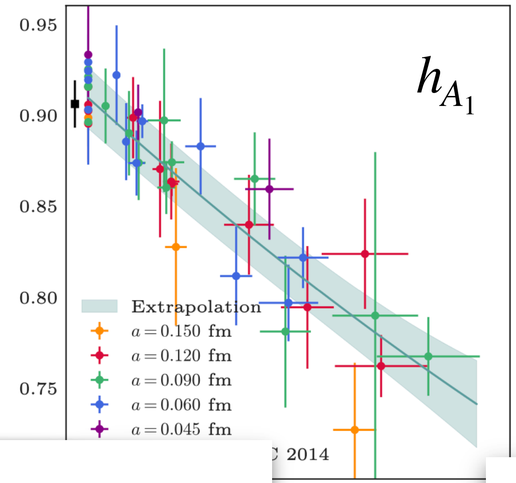
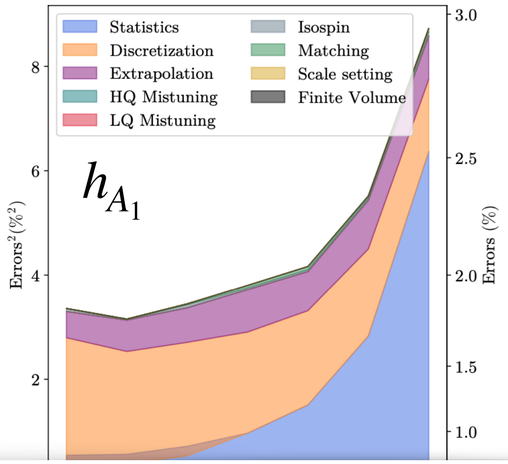
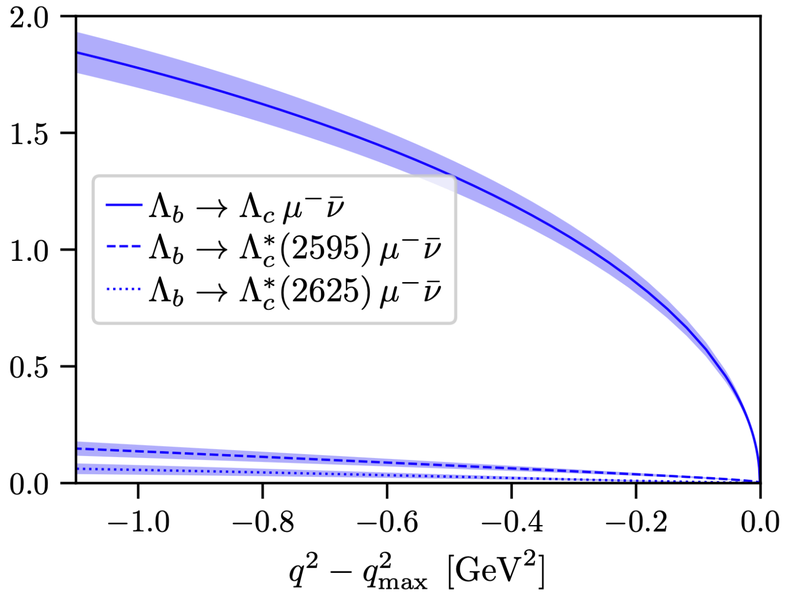
Current status

$B \to D^*$ FNAL/MILC, 2105.14019



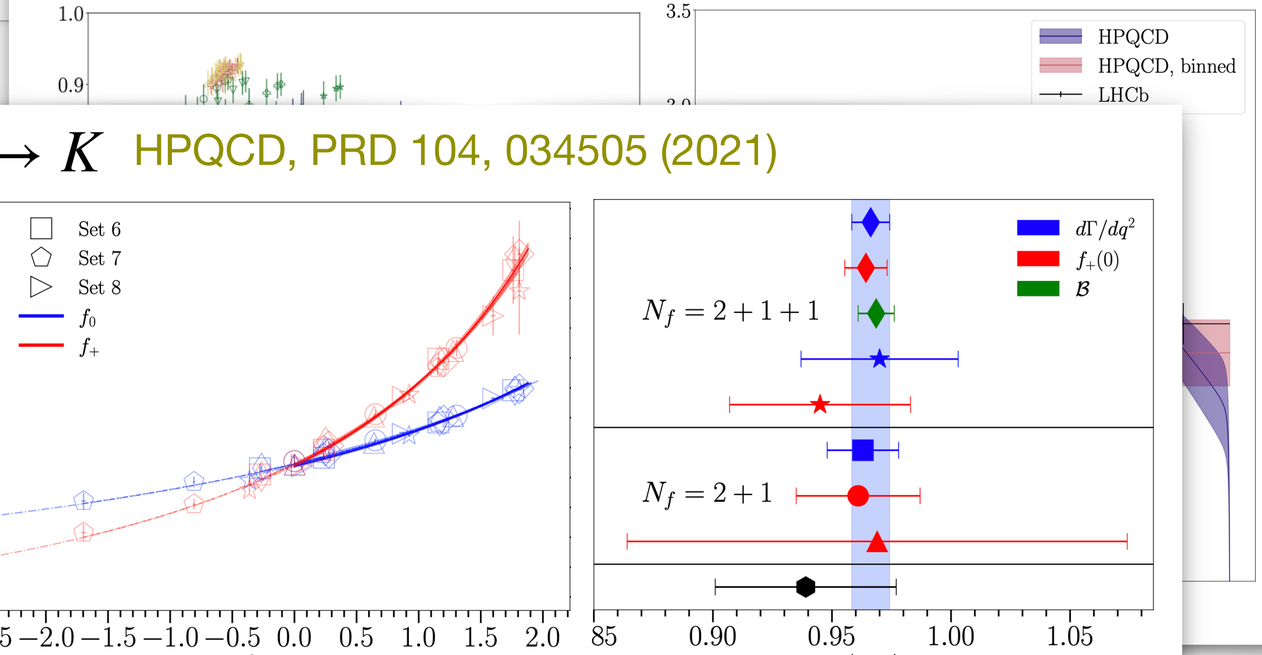


$\Lambda_b \to \Lambda_c^*(2595, 2625)$



Meinel & Rendon, PRD 103, 094516 (2021)

$B_s \to D_s^*$ HPQCD, 2105.11433

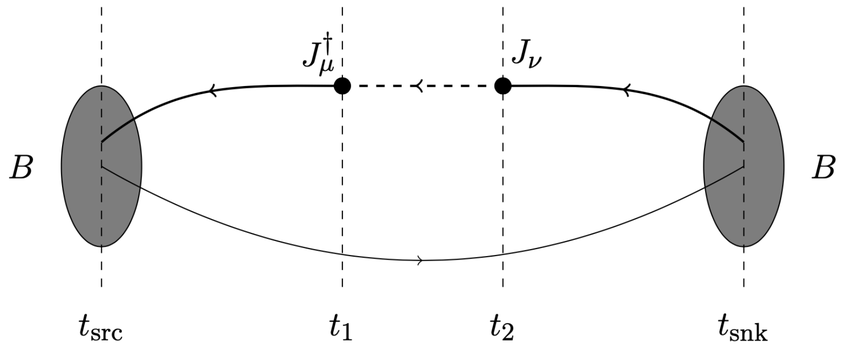
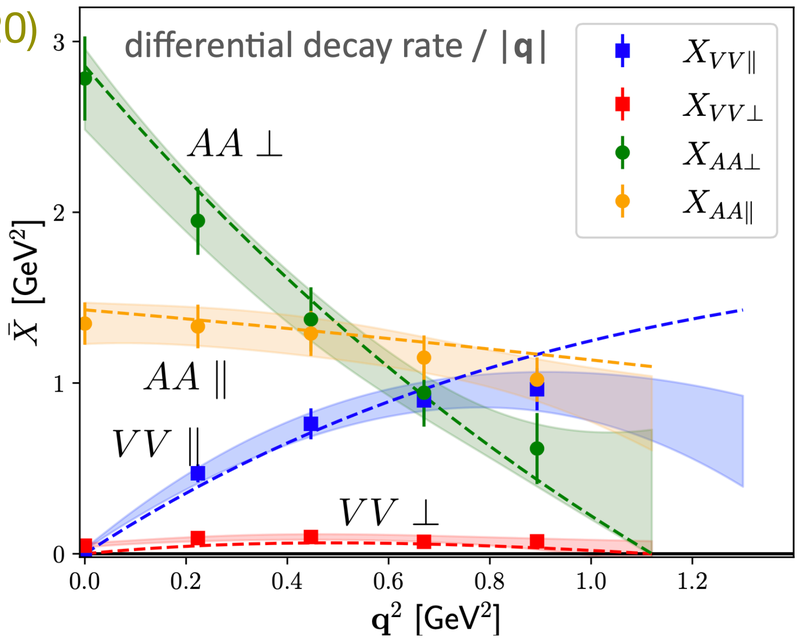


$D \to K$ HPQCD, PRD 104, 034505 (2021)

Summary

Ongoing & next steps

- HPQCD, FNAL, RBC-UKQCD, JLQCD, ETMC all working on fully relativistic heavy quarks
- Inclusive decays on the lattice

Gambino & Hashimoto, PRL 125, 032001 (2020)

JLQCD, 2001.11678