Jet Formation with Spectral Clustering

arXiv:2104.01972

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Jet physics

Spectral clustering theory

Aim

Relaxation

Embedding space

Appearance

Results

Conclusion

Backup slides

Timing

Parameters

Shape variables

Outline

1 *Jet physics*

2 *Spectral clustering theory*
- Aim
- Relaxation

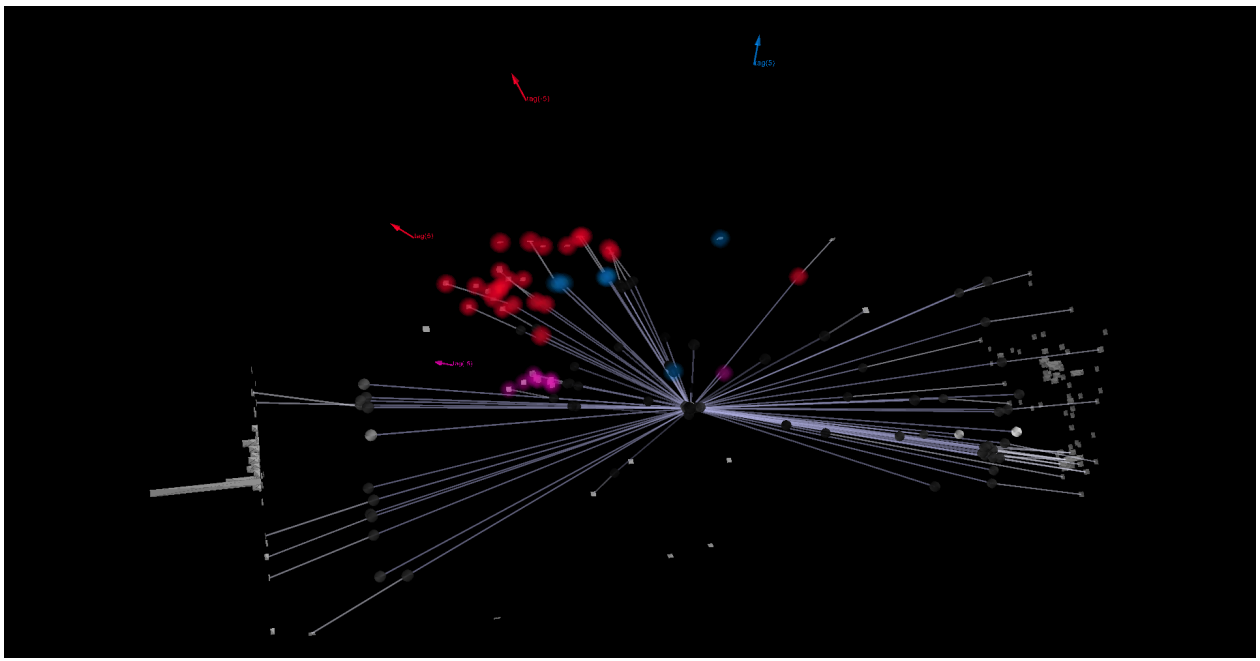
3 *Embedding space*
- Appearance

4 *Results*
- Conclusion

5 *Backup slides*
- Timing
- Parameters
- Shape variables

Jet Formation

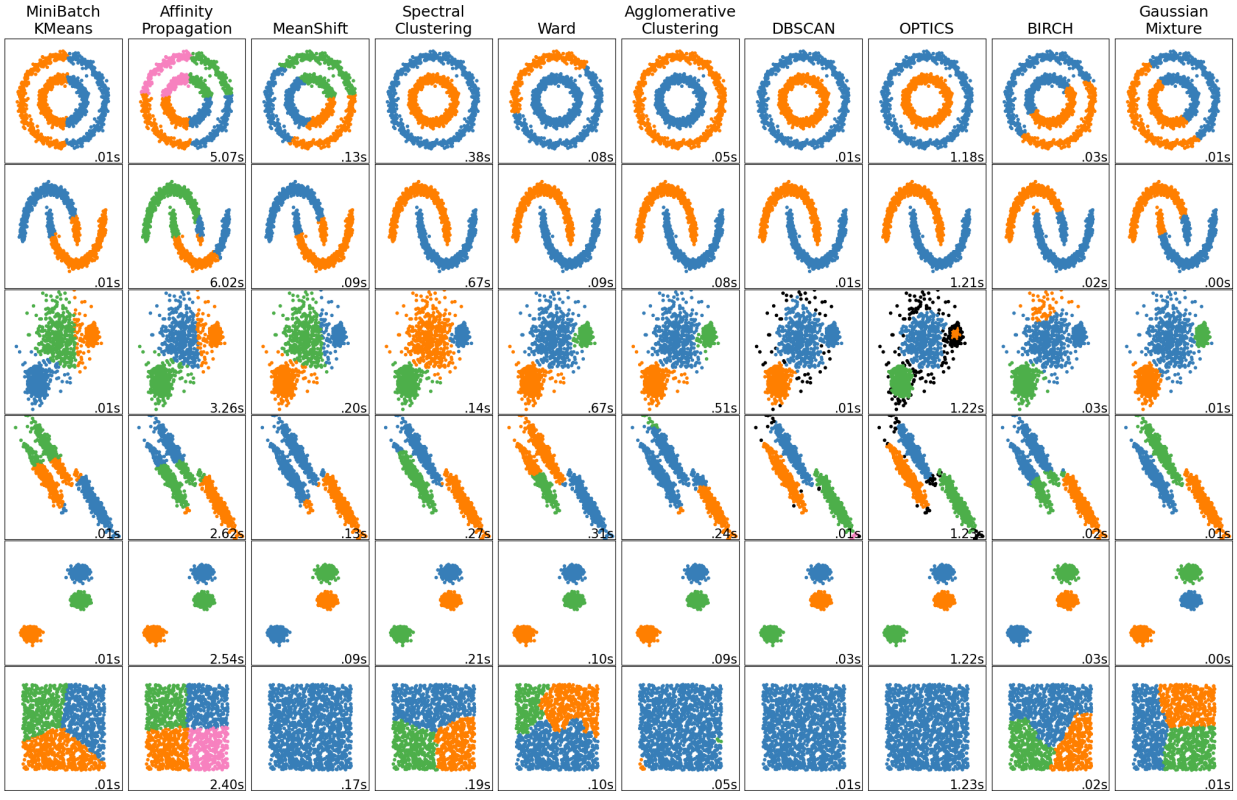
Jet formation involves grouping decay products to estimate the momentum and identity of the particle that decayed



This activity has been compared to reading tea leaves...
`www.quantumdiaries.org/2011/04/22/when-youre-a-jet-youre-a-jet-all-the-way/`

Spectral Clustering

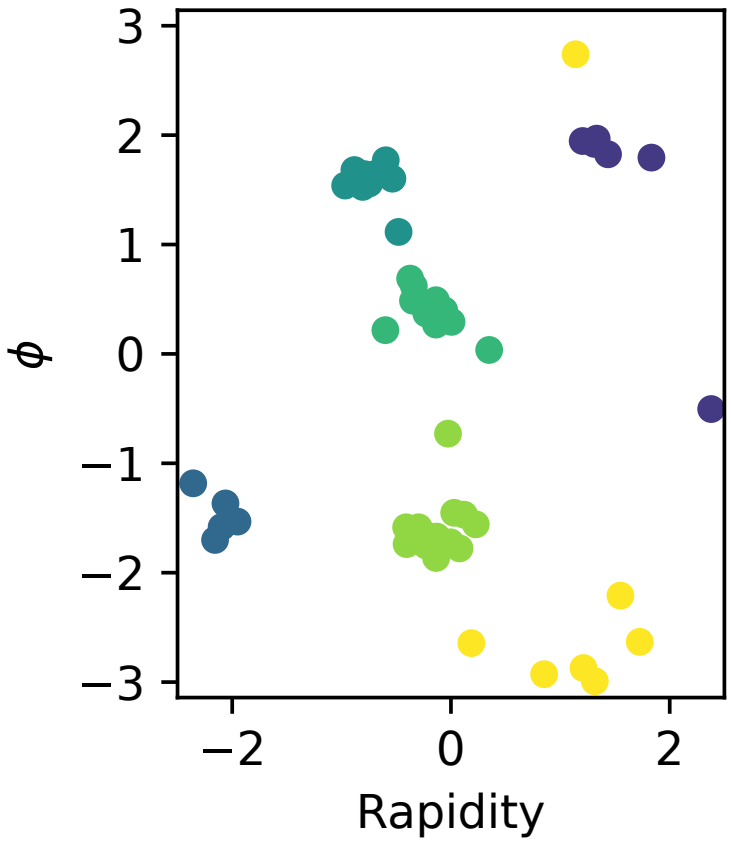
Spectral clustering is a machine learning technique for picking out clusters. It doesn't use a neural net



https://scikit-learn.org/stable/modules/clustering.html

Spectral Clustering

Inputs

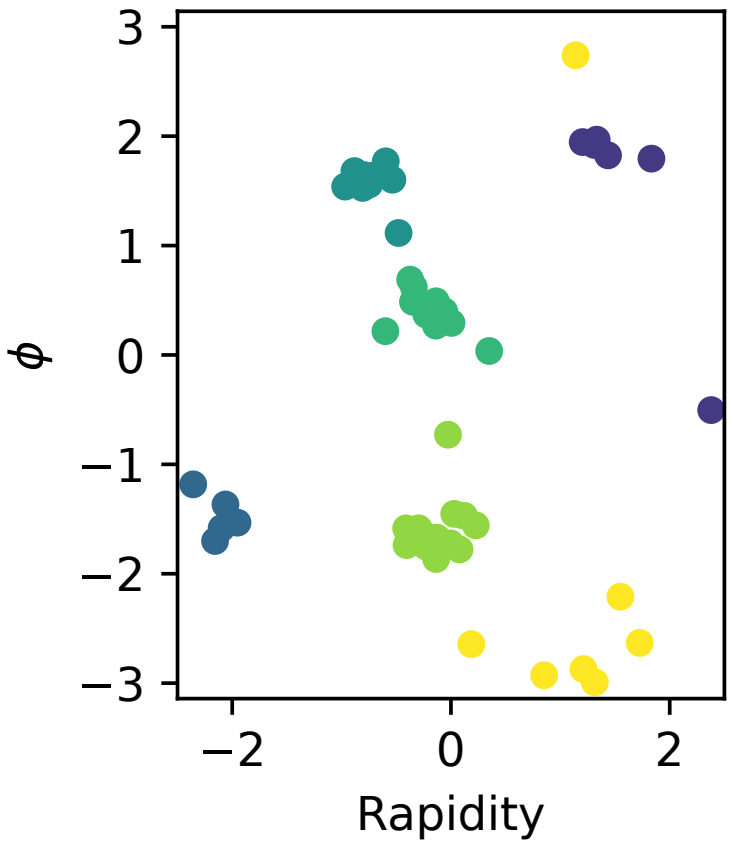


Here is a image representing hits on the unrolled barrel. Each point is a detected track, its colour roughly indicates which shower it was generated by. It contains;

- 4 *b*-showers from a light Higgs cascade.
- Background showers from initial state radiation.

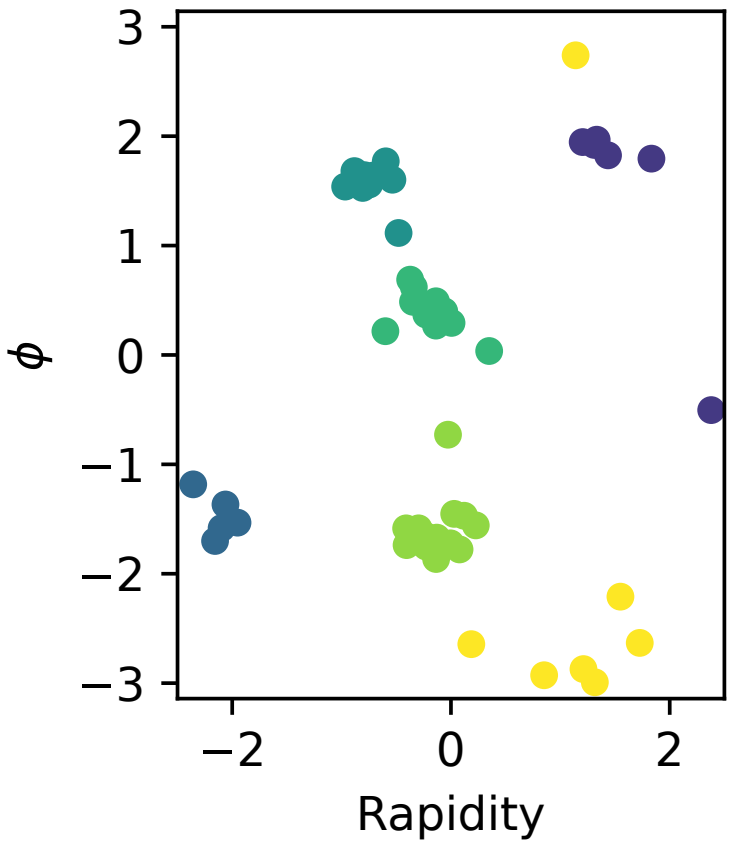
Spectral Clustering

Inputs



How likely are two particles to belong in the same jet?
Distance; $d_{i,j}$

$$= \sqrt{\delta\phi_{i,j}^2 + \delta y_{i,j}^2}$$



Spectral Clustering

Inputs

How likely are two particles to belong in the same jet?
Distance; $d_{i,j}$

$$= \sqrt{\delta\phi_{i,j}^2 + \delta y_{i,j}^2}$$

Similarity; $a_{i,j}$

$$= \exp\left(\frac{-d_{i,j}^2}{\sigma_v}\right)$$

Spectral Clustering

Cost function

Ideally, particles connected by high similarity would be in the same jet. Particles separated into different jets would only have low similarity between them.

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Cost function;

$$\text{NCut} = \sum_k \frac{\sum_{i \in G_k, j \in \bar{G}_k} a_{i,j}}{|G_k|}$$

Where

The numerator is the similarity crossing the boundary of G_k.

$|G_k| = \sum_{i \in k} \sum_j a_{i,j}$ seeks to balance cluster size.

Spectral Clustering

Indicator vectors

Let's create some indicator vectors

$$f(k)_i = \begin{cases} \frac{1}{\sqrt{|G_k|}} & \text{if } i \in G_k \\ 0 & \text{otherwise} \end{cases}$$

If these could be found it would define the jets.

Spectral Clustering

Indicator vectors

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A Laplacian helps with that; let $D_{i,j} = \delta_{i,j} \left(\sum_k a_{i,k} \right)$

$$L_{i,j} = D_{i,j} - a_{i,j}$$

If these could be found it would define the jets.

Spectral Clustering

Indicators

$$
\begin{aligned}
f(k)' L f(k) &= \sum_{i,j} f(k)_i L_{i,j} f(k)_j \\
&= \sum_{i,j} f(k)_i \left(\delta_{i,j} \sum_p a_{i,p} - a_{i,j} \right) f(k)_j \\
&= \sum_i \left(f(k)_i^2 \sum_p a_{i,p} - \sum_j f(k)_i f(k)_j a_{i,j} \right) \\
&= \sum_{i,j} a_{i,j} \left(f(k)_i^2 - f(k)_i f(k)_j \right) \\
&= \frac{1}{2} \sum_{i,j} a_{i,j} \left(f(k)_i - f(k)_j \right)^2
\end{aligned}
$$

Spectral Clustering

Indicator vectors

Use the definition of indicator vectors again;

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This gives something familiar looking...

$$f(k)' L f(k) = \sum_{i \in G_k, j \notin G_k} \frac{a_{i,j}}{|G_k|}$$

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So if F is a matrix with rows of $f(n)$;

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Spectral Clustering

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So if F is a matrix with rows of $f(n)$;

$$\text{NCut} = F' L F$$

Finally, let $T = D^{\frac{1}{2}} F$. Then $T'T = I$, so

$$\text{NCut} = \frac{T' D^{-\frac{1}{2}} L D^{-\frac{1}{2}} T}{T'T}$$

Spectral Clustering

Embedding space

To minimise this cost function, take eigenvectors of

$L = D^{-\frac{1}{2}}(D - A)D^{-\frac{1}{2}} \rightarrow L\mathbf{v_n} = \lambda_n \mathbf{v_n}$

Spectral Clustering

Embedding space

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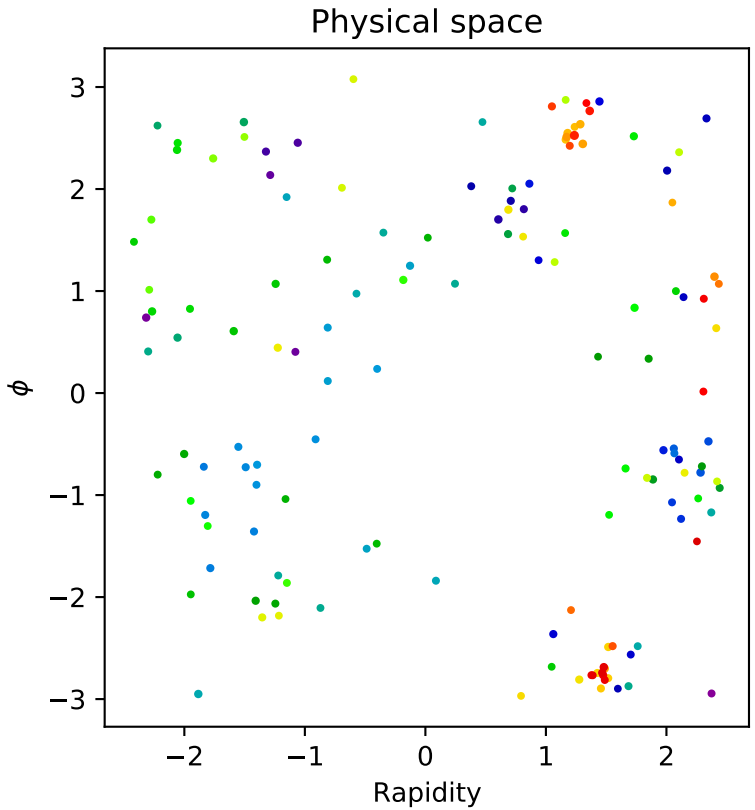
$$\mathbf{v_1}, \mathbf{v_2} = \begin{bmatrix} \mathbf{v_1}_1 \\ \mathbf{v_1}_2 \\ \vdots \\ \mathbf{v_1}_N \end{bmatrix}, \begin{bmatrix} \mathbf{v_2}_1 \\ \mathbf{v_2}_2 \\ \vdots \\ \mathbf{v_2}_N \end{bmatrix}$$

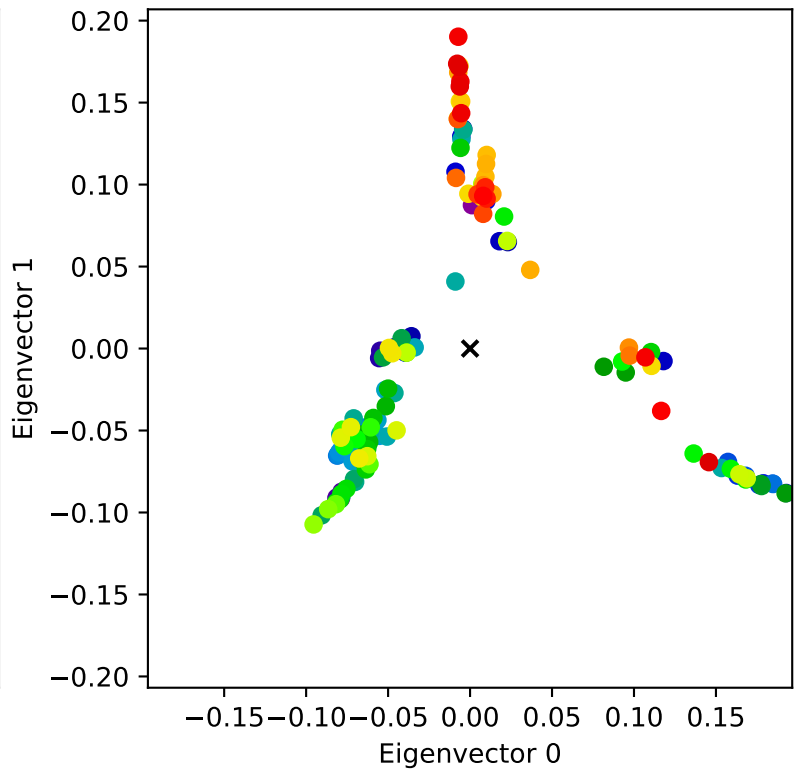
Spectral Clustering

Embedding space

The eigenvectors of L create a multidimensional embedding space.

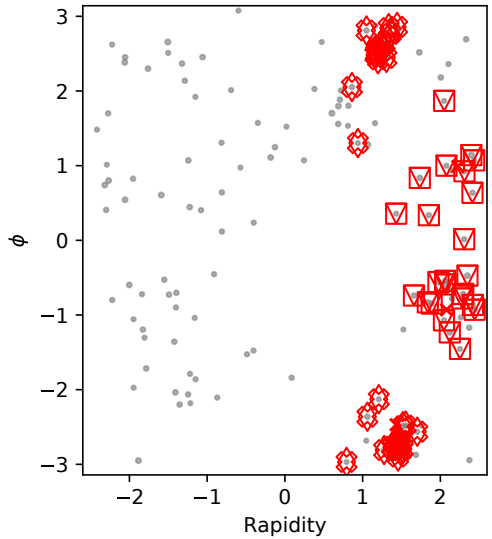
$$Lv_n = \lambda_n v_n$$





Spectral Clustering

Jet shape



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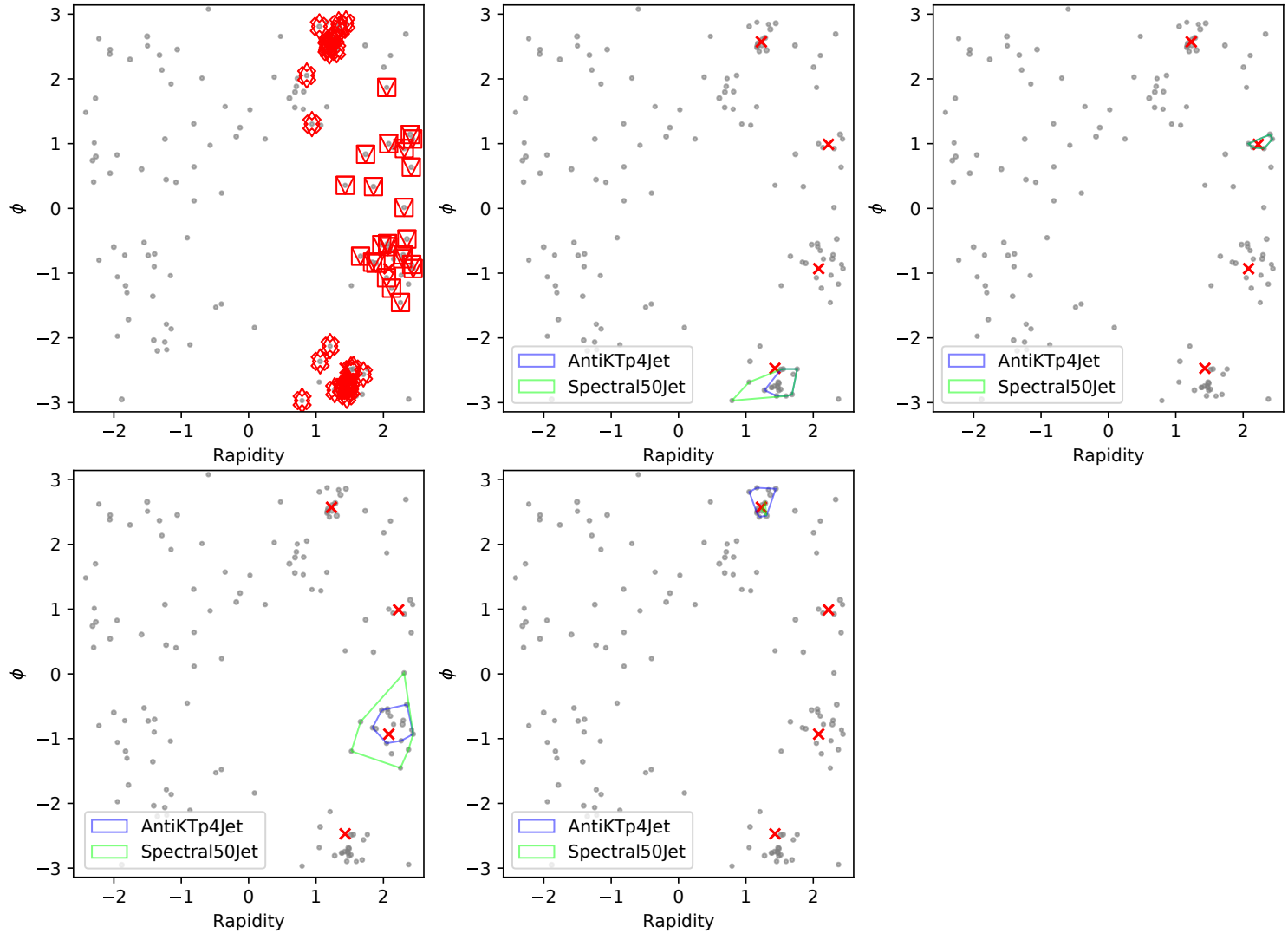

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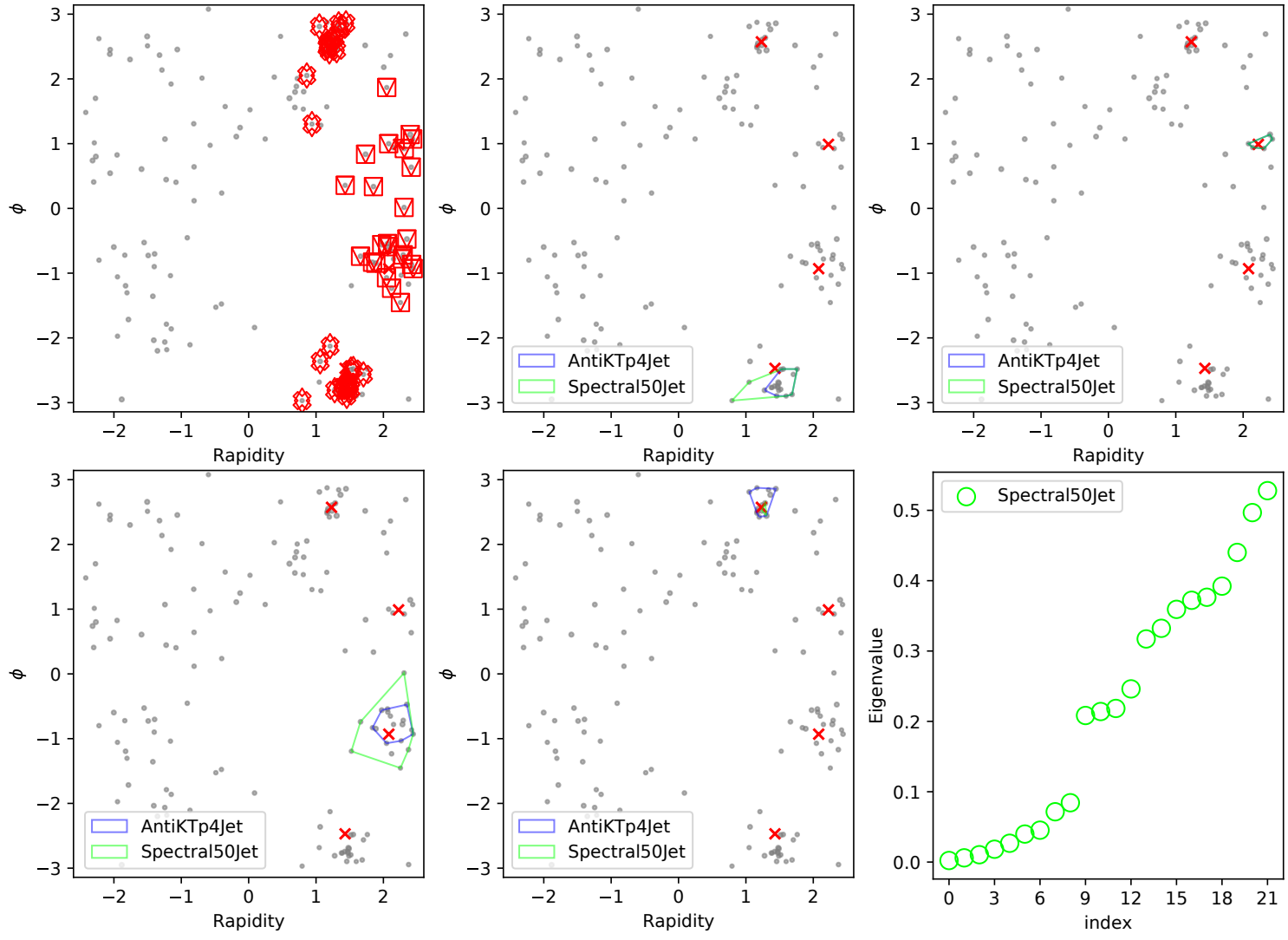
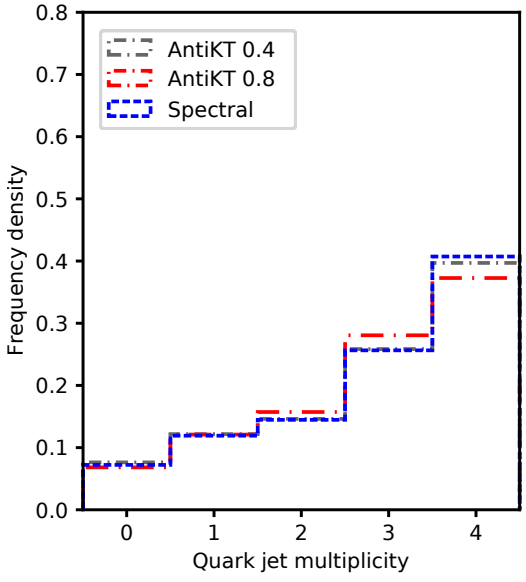
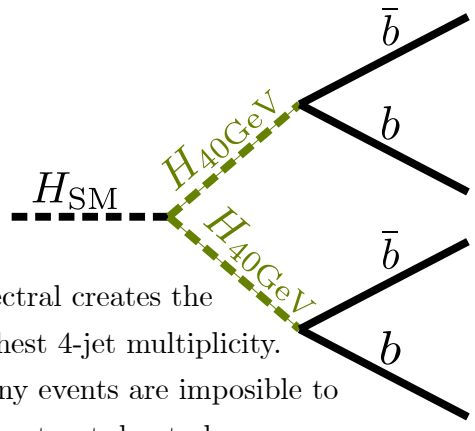


Results

Light Higgs cascade

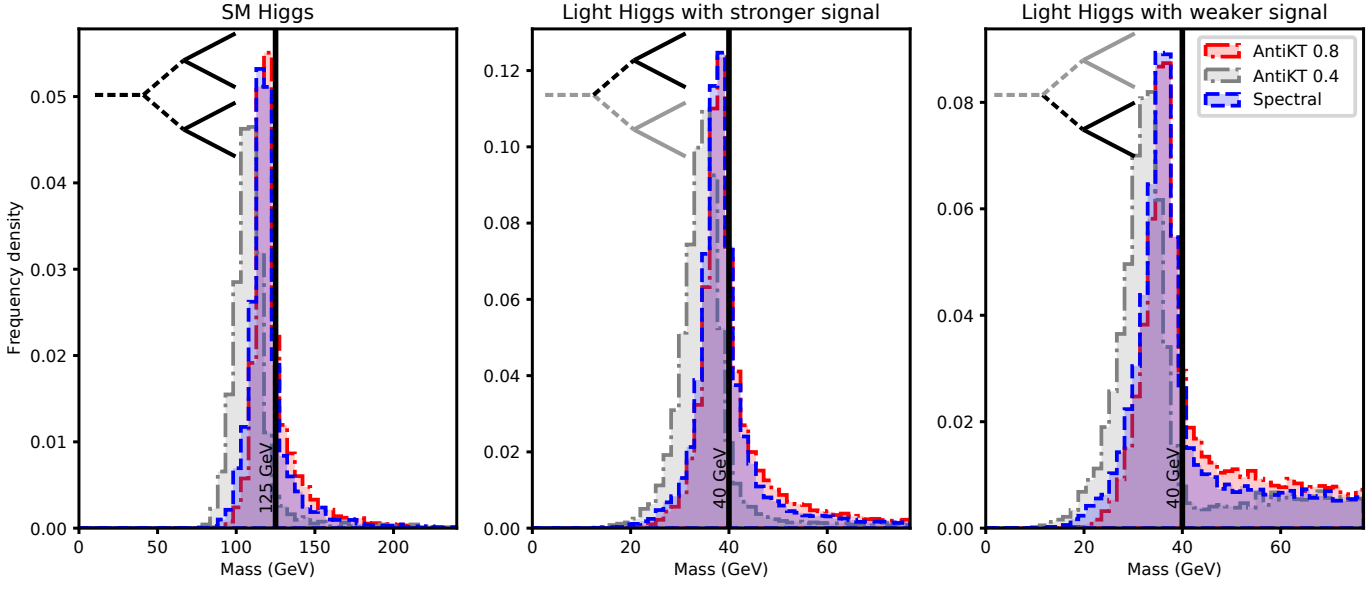




Spectral and anti-kt radius 0.8 both give good narrow mass peaks with the correct location.

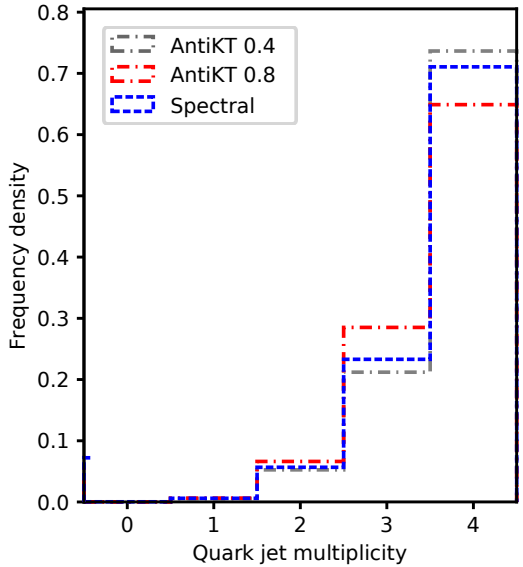
Only spectral is producing good mass peaks and multiplicity

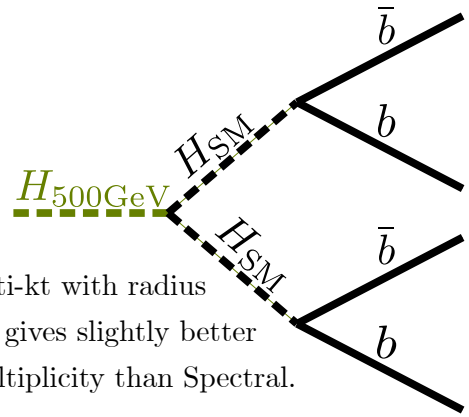
Spectral creates the highest 4-jet multiplicity. Many events are imposible to reconstruct due to low energy.



Results

Heavy Higgs cascade

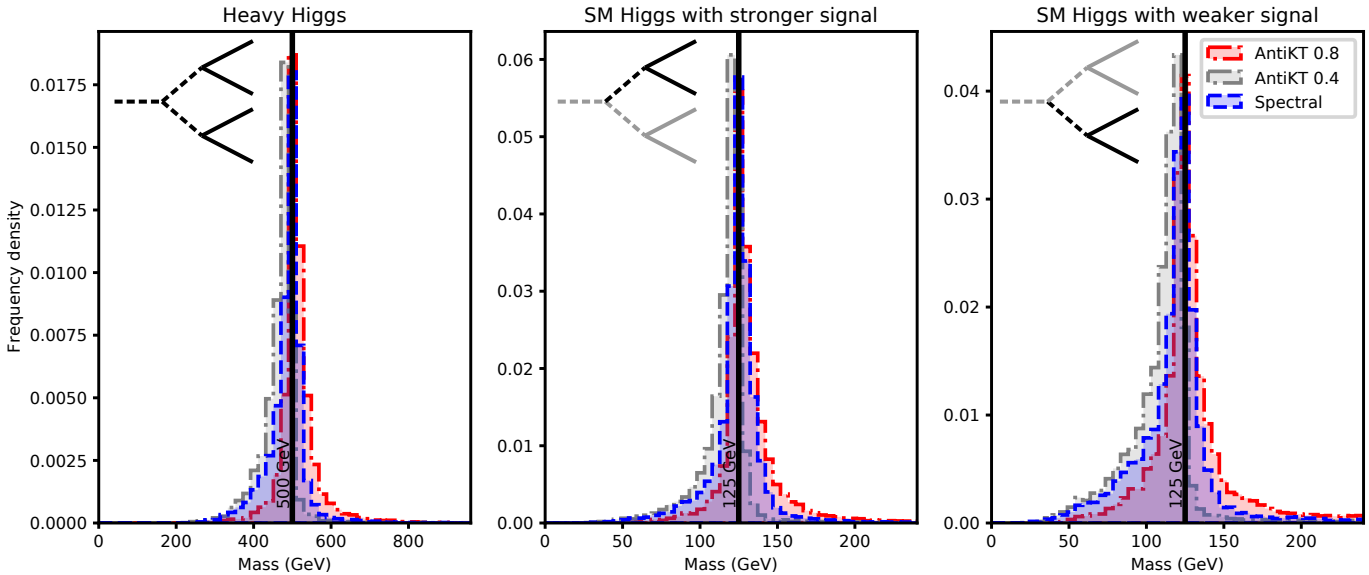




Spectral and anti-kt radius 0.8 both give good narrow mass peaks with the correct location.

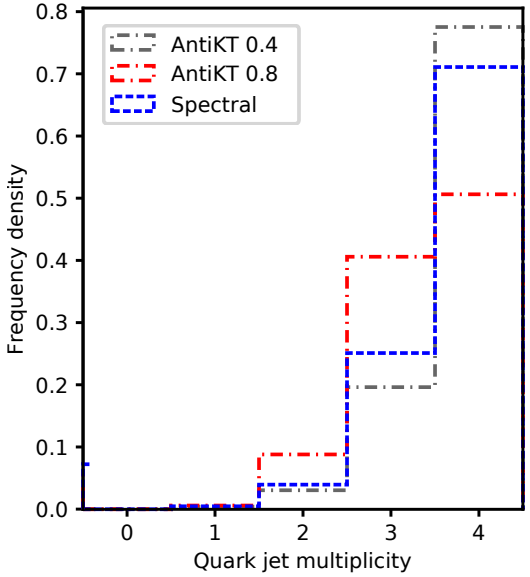
Anti-kt with radius 0.4 gives slightly better multiplicity than Spectral.

Spectral is producing good mass peaks and multiplicity

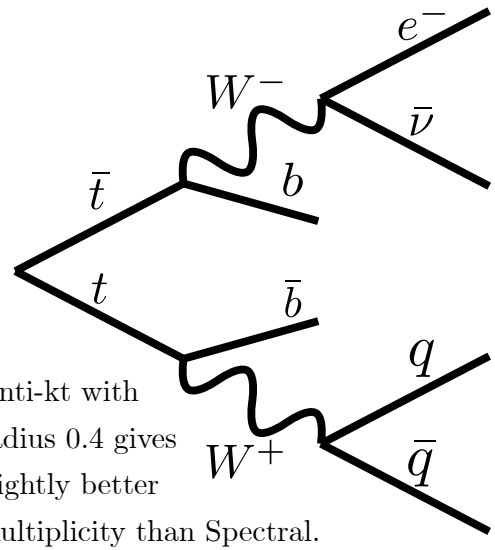


Results

Semileptonic top decay

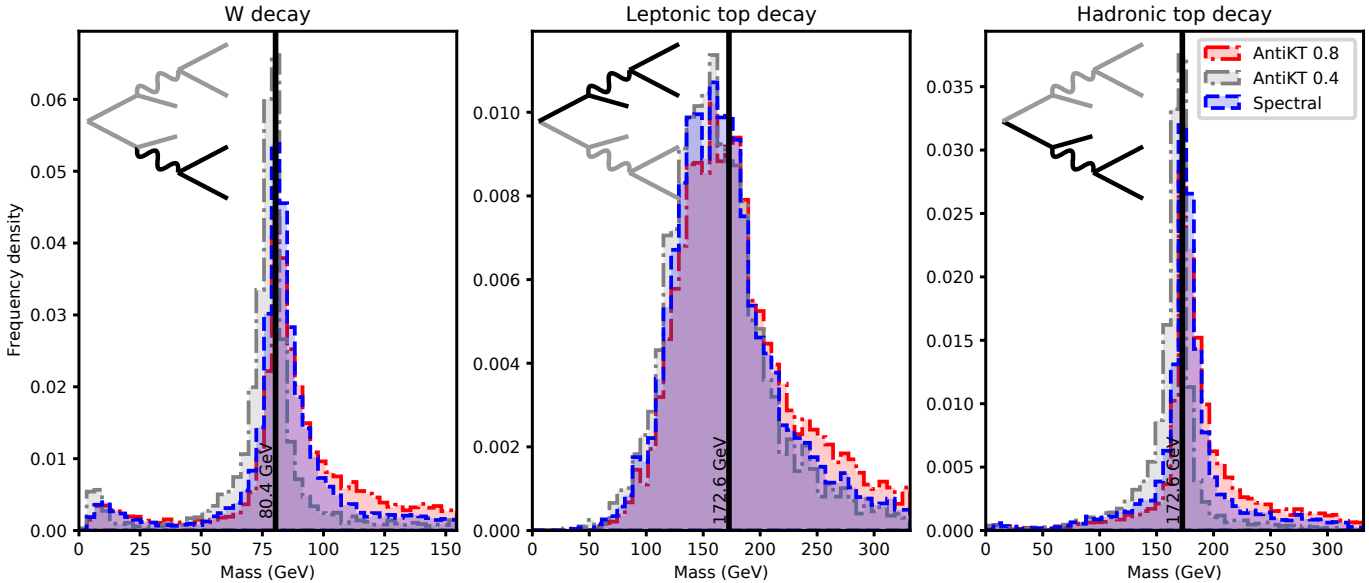


Anti-kt with radius 0.4 gives slightly better multiplicity than Spectral.



Anti-kt radius 0.4 produces the best mass peaks. Spectral is an improvement on anti-kt radius 0.8.

Anti-kt 0.4 is producing good mass peaks and multiplicity, spectral is partially mimicking this.



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Spectral clustering offers a deterministic crisp jet formation algorithm, without any use of learnt parameters or black box elements. It is remarkable for its ability to adapt to various data sets.

arXiv:2104.01972

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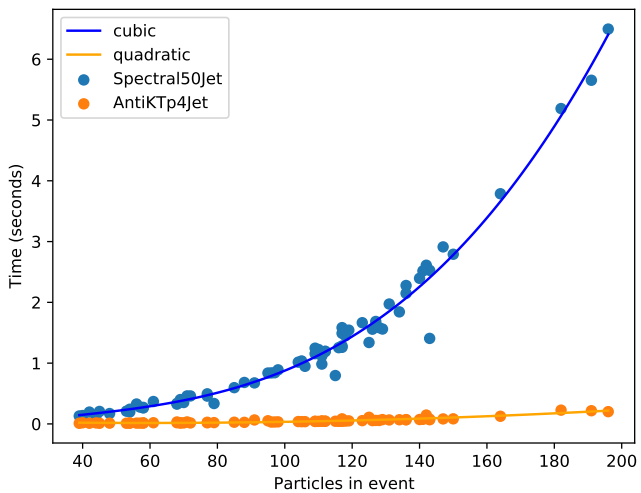
End

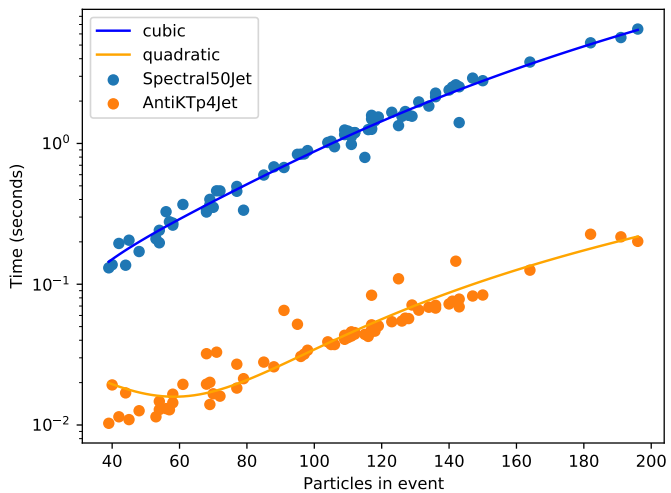
Thank you!

Spectral Clustering

Timing

The algorithm is currently $\mathcal{O}(N^3)$.





Spectral Clustering

Parameters

- σ_v and α are used to define the similarity; $a_{i,j} = e^{-d_{i,j}^{\alpha}/\sigma_v}$.

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Spectral Clustering

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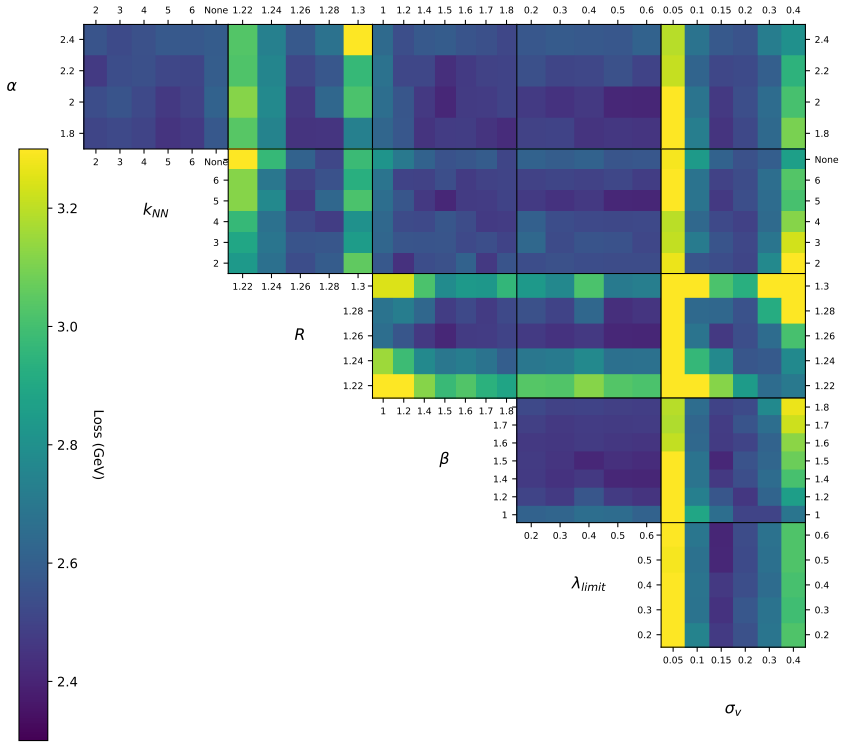
- σ_v and α are used to define the similarity; $a_{i,j} = e^{-d_{i,j}^{\alpha}/\sigma_v}$.
- k_{NN} removes some similarities to reduce noise.
- λ_{limit} and β make use of the information in the eigenvalues to prioritise the eigenvectors.
- When the mean distance in the embedding space rises over R, the clustering stops.

Spectral Clustering

Parameters

The parameters are not fine tuned. This can be seen if we plot the performance as the parameters change;

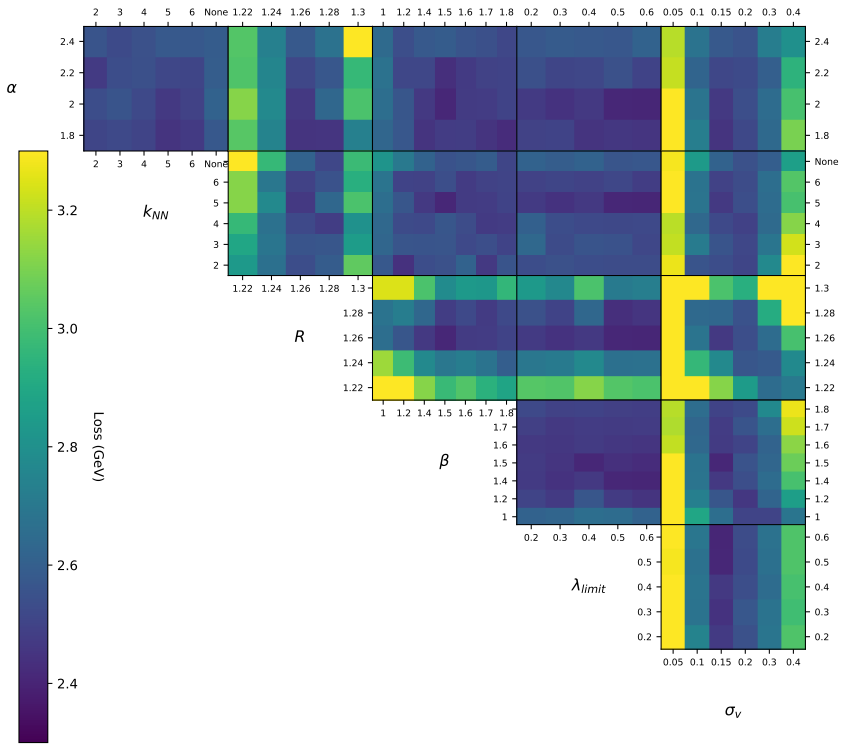


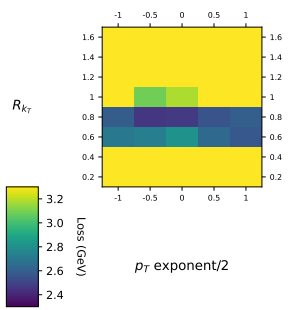
Dark blue areas, indicating low loss, show parameter combinations with good performance.

Spectral Clustering

Parameters

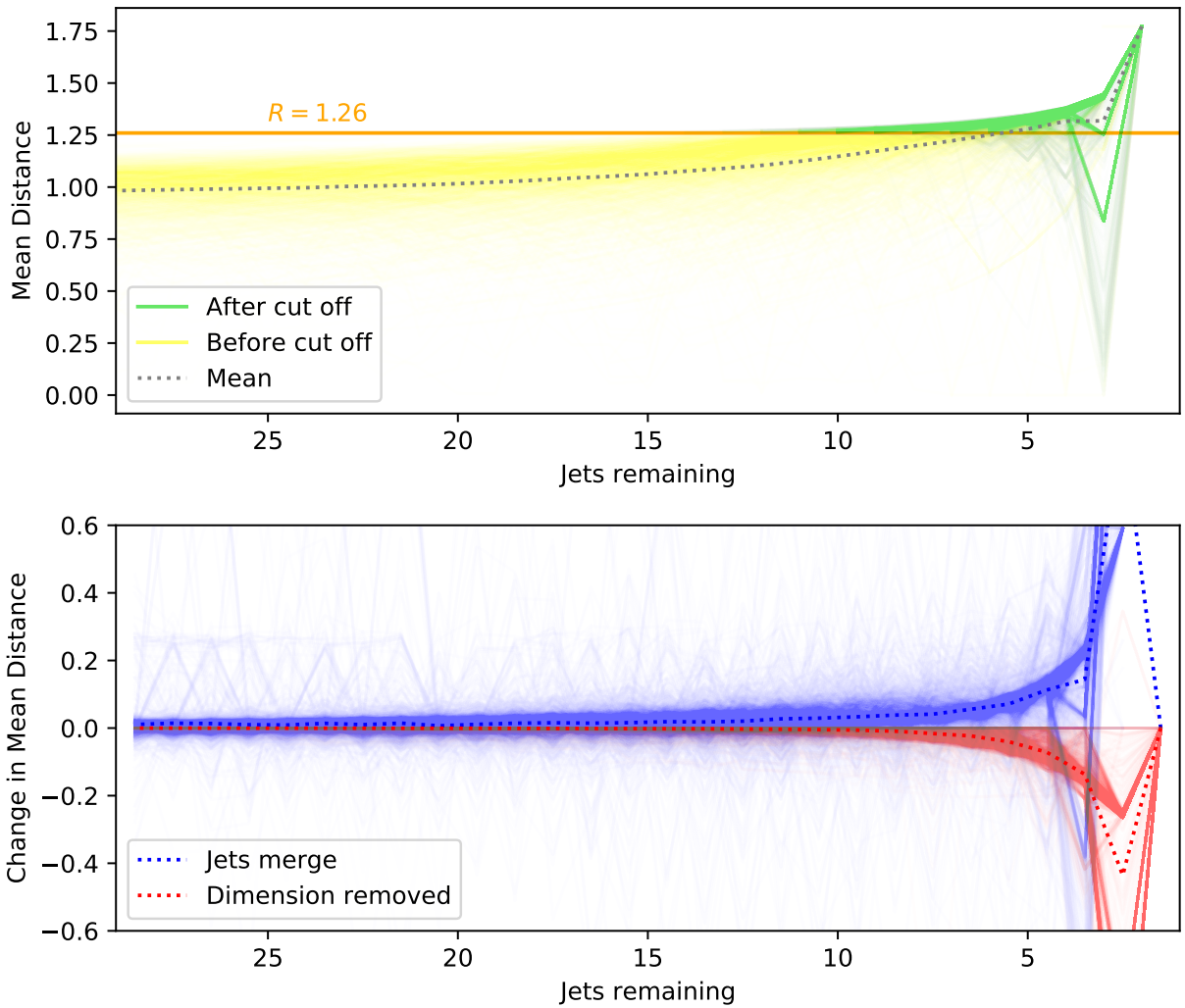
For contrast, on the right is the parameter space of the generalised k_T algorithm



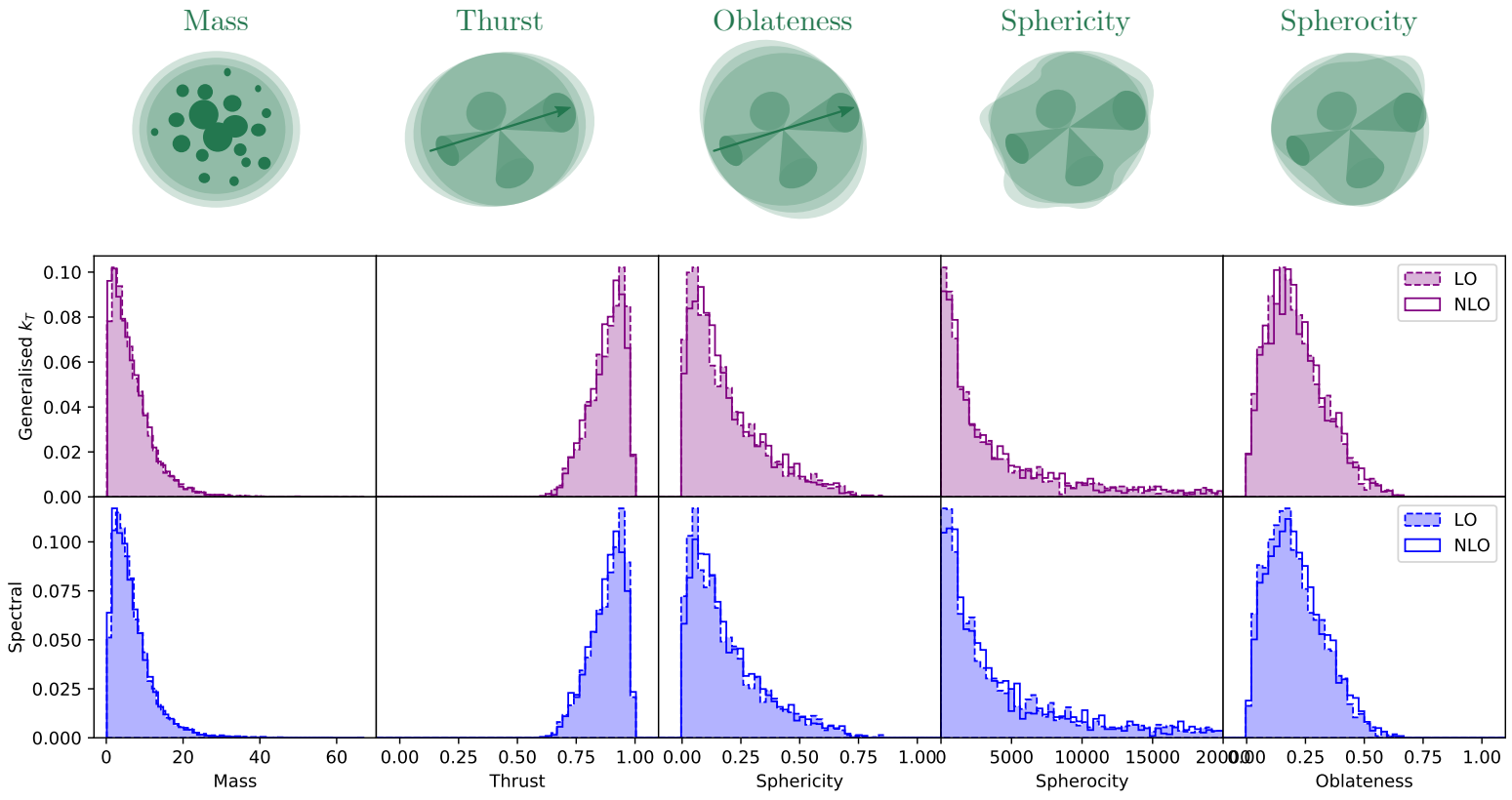


Spectral Clustering

Stopping condition



Spectral Clustering

Shape variables

Shape variables are dependant on the clustering algorithm