HQL2021 - The XV Int. Conf. on Heavy Quarks and Leptons 13-17 Sep. 2021

SM Theory of muon g-2

UNIVERSITY OF LIVERPOOL

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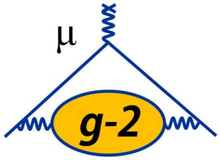
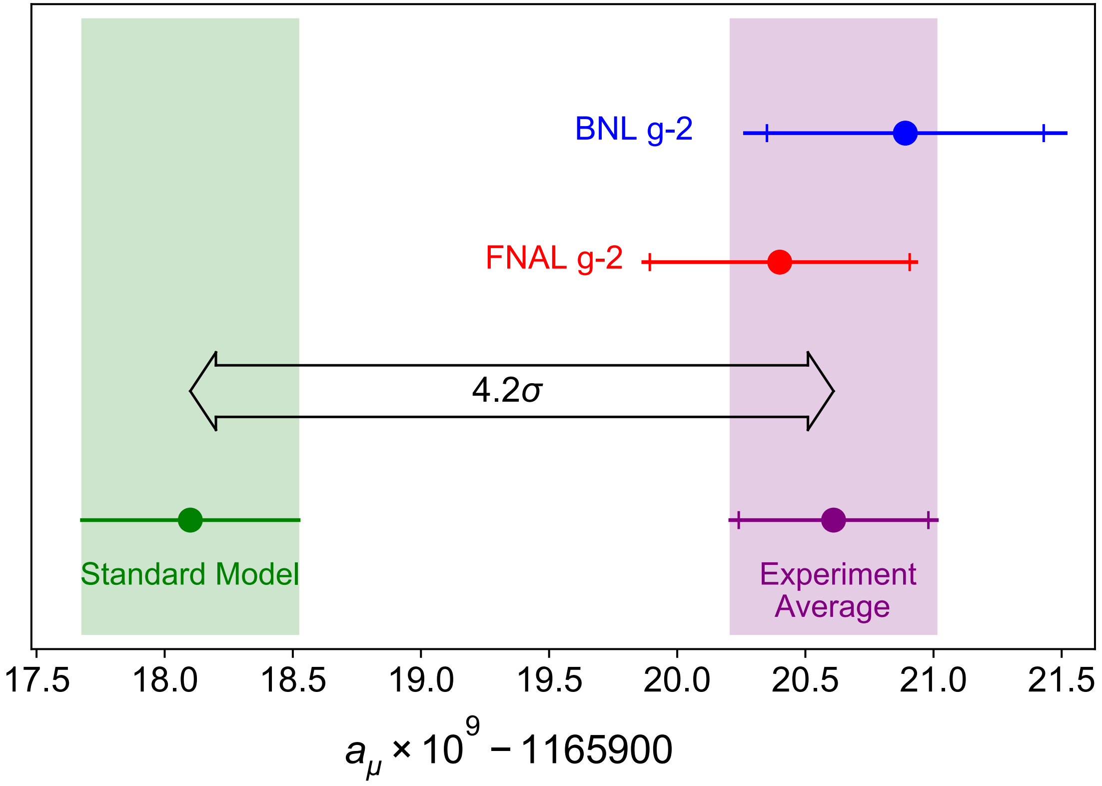


- Introduction
- Overview of $\mathbf{a_\mu^{SM}}$ from the Muon g-2 Theory Initiative
- Data-driven HVP evaluation: basic ingredients, main features & results
- Outlook & conclusions

SM theory vs. Experiment

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

[Phys. Rev. Lett. 126 (2021) 14, 141801]



- Unblinding of Run 1 analyses:
  25 February ’21
- FNAL confirms BNL
- Release of result:
  7 April ’21
- PRL already has > 280 cites
  (mostly from BSM)
- Run 1 is only 6% of total
  expected statistics

➤ But what about the Standard Model prediction?

Muon g-2 Theory Initiative est. 2017

https://muon-gm2-theory.illinois.edu

``… map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental result.''

- Organised 7 international workshops in 2017-2021
- **White Paper** posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

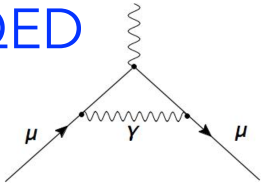
  ``**The anomalous magnetic moment of the muon in the Standard Model**''

  [T. Aoyama et al, arXiv:2006.04822, Phys. Rept. 887 (2020) 1-166] ➤ please follow citation recommendations

Group photo from the Seattle workshop in September 2019

SM **WP20** prediction from the TI White Paper **(0.37 ppm)**

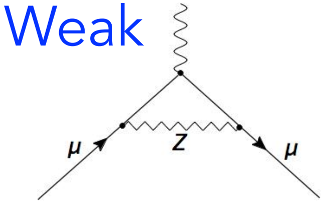
QED



+…

$116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak



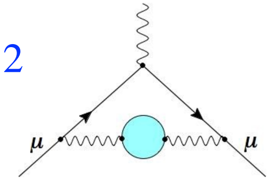
+…

$153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

α^2

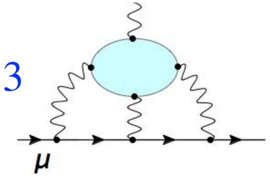


+…

$6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

…Light-by-Light (HLbL)

α^3



+…

$92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

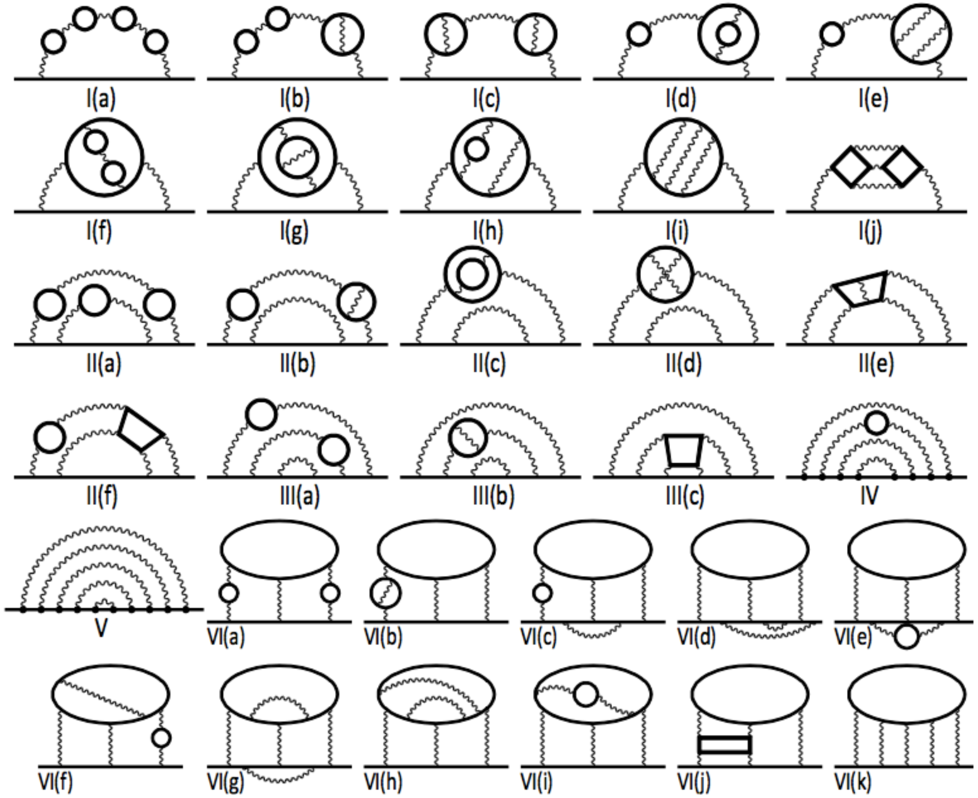
- Uncertainty dominated by hadronic contributions, now **δ HVP > δ HLbL**

a_μ^{QED} & a_μ^{weak} : a triumph for perturbative QFT

QED: Kinoshita et al. + many tests

- g-2 @ 1, 2, 3, 4 & **5 loops**
- Subset of 12672 5-loop diagrams:
- code-generating code, including
- renormalisation
- multi-dim. numerical integrations

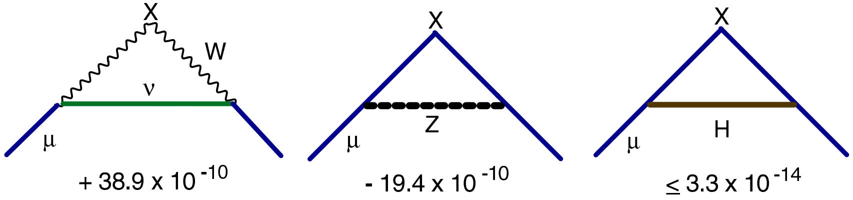
$$a_\mu^{QED} = 116\,584\,718.9\,(1) \times 10^{-11}$$ ✓



Weak: (several groups agree)

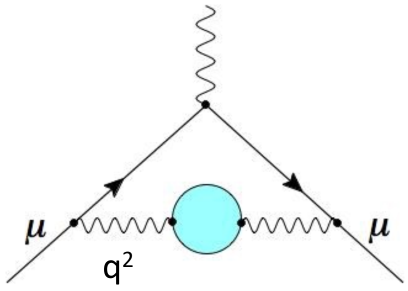
- done to 2-loop order, 1650 diagrams
- the first full 2-loop weak calculation

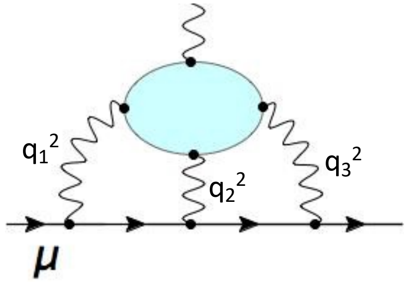
$$a_\mu^{weak} = \quad 153.6\,(1.0) \times 10^{-11}$$ ✓



SM weak 1-loop diagrams

a_μ^{hadronic} : non-perturbative, the limiting factor of the SM prediction





- **Q:** What's in the hadronic (**Vacuum Polarisation** & **Light-by-Light scattering**) blobs?

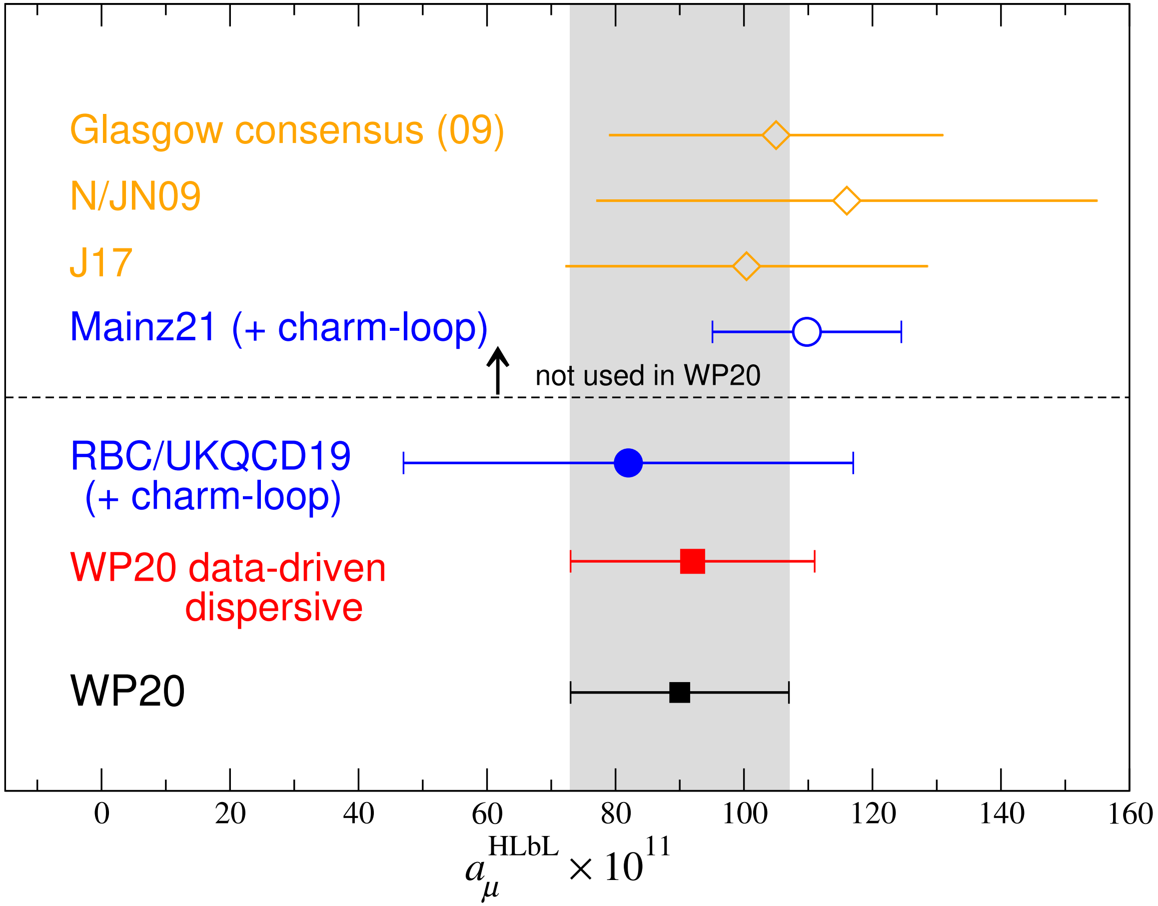
  A: Anything `**hadronic**' the virtual photons couple to, i.e. **quarks** with **gluons** & photons

  But: low q^2 photons dominate loop integral(s) ➠ cannot calculate blobs with perturbation theory

- **Two very different strategies:**

  1. use wealth of hadronic data, `**data-driven dispersive methods**':
     - data combination from many experiments, radiative corrections required
  2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `**lattice QCD**':
     - finite size, finite lattice spacing, artifacts from lattice actions, QCD + QED needed
     - numerical Monte Carlo methods require large computer resources

a_μ^{HLbL} : WP Status/Summary of Hadronic Light-by-Light contributions



hadronic models + pQCD

new lattice QCD + QED

lattice QCD + QED

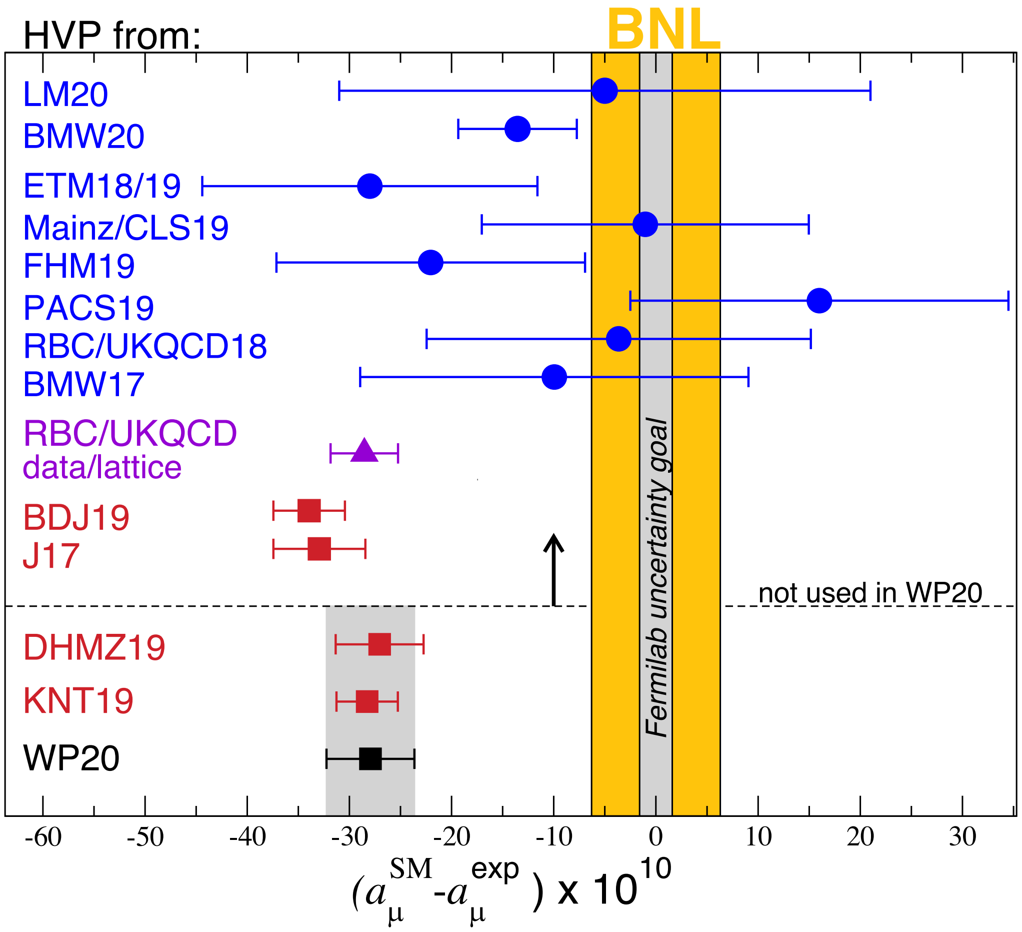
data-driven

TI White Paper 2020 value:

a_μ^{HLbL} **= 92 (18) × 10^{-11}** ✓

- **data-driven dispersive** & **lattice** results have confirmed the earlier model-based predictions
- **uncertainty much better under control** and at 0.15ppm already **sub-leading compared to HVP**
- lattice predictions now competitive, good prospects for combination and error reduction to ≤ 10%

a_μ^{HVP} : WP20 Status/Summary of Hadronic VP contributions



Lattice QCD + QED

- impressive progress, but…
- large spread between results
- tensions when looking at `Euclidean time window' comparisons
- large systematic uncertainties (e.g. from non-trivial extrapolation to continuum limit, finite size)

Dispersive/lattice hybrid

(`window' method)

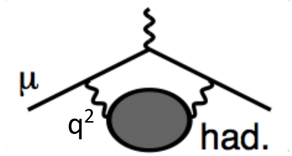
For WP20: Dispersive data-driven from **DHMZ** and **KNT**

TI White Paper 2020 value:

$$\mathbf{a_\mu^{HVP} = 6845\ (40) \times 10^{-11}}$$

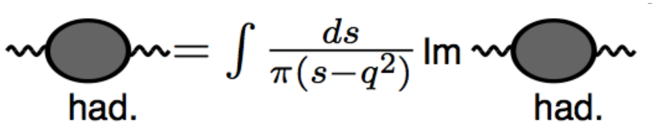
- **TI WP20** prediction uses dispersive data-driven evaluations with **minimal model dependence**
- a_μ^{HVP} **value and error** obtained **by merging** procedure ➠ accounts for tensions in input data and differences in data treatment & combination (going beyond usual χ^2_{min} inflation)

a_μ^{HVP} : Basic principles of dispersive method



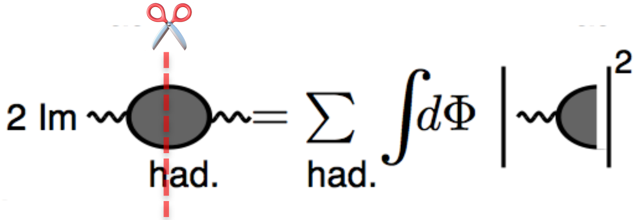
One-loop diagram with hadronic blob =

integral over q^2 of virtual photon, 1 HVP insertion



Causality ➠ analyticity ➠ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➠ Optical Theorem:

imaginary part (`cut diagram') =

sum over |cut diagram|^2, i.e.

∝ sum over all total hadronic cross sections

$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

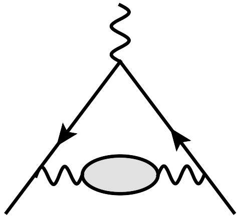
$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

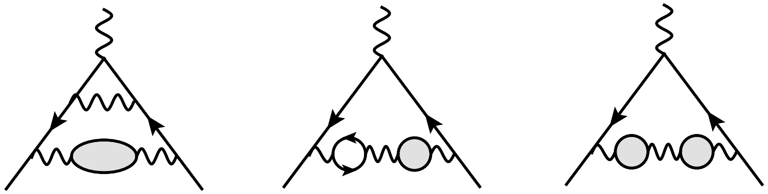
- Total hadronic cross section **σ_{had}** from > 100 **data** sets for **e^+e^- ➞ hadrons** in > 35 final states
- Uncertainty of a_μ^{HVP} prediction from statistical & systematic uncertainties of input data
- Pert. QCD used only at large s, **no modelling** of $\sigma_{\mathrm{had}}(s)$ **required**, direct data integration

a_μ^{HVP} : Higher orders & QED power counting; WP20 values in 10^{-11}

- All hadronic blobs also contain photons, i.e. real + virtual corrections in $\sigma_{had}(s)$



- LO: 6931(40)

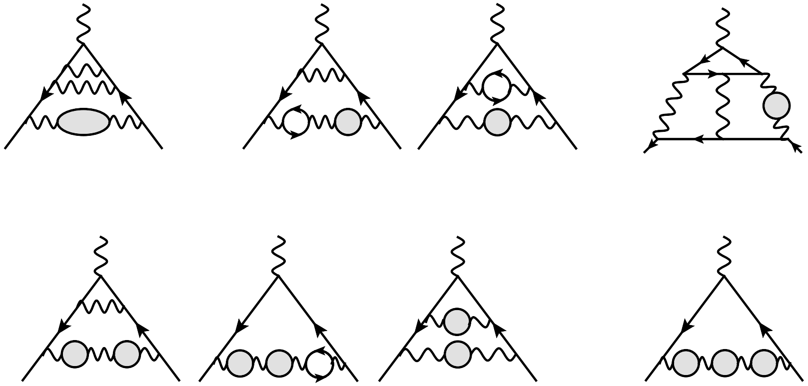


- NLO: - 98.3(7)

  from three classes of graphs:

  - 207.7(7) + 105.9(4) + 3.4(1) [KNT19]

  (photonic, extra e-loop, 2 h-loops)



- NNLO: 12.4(1) [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]

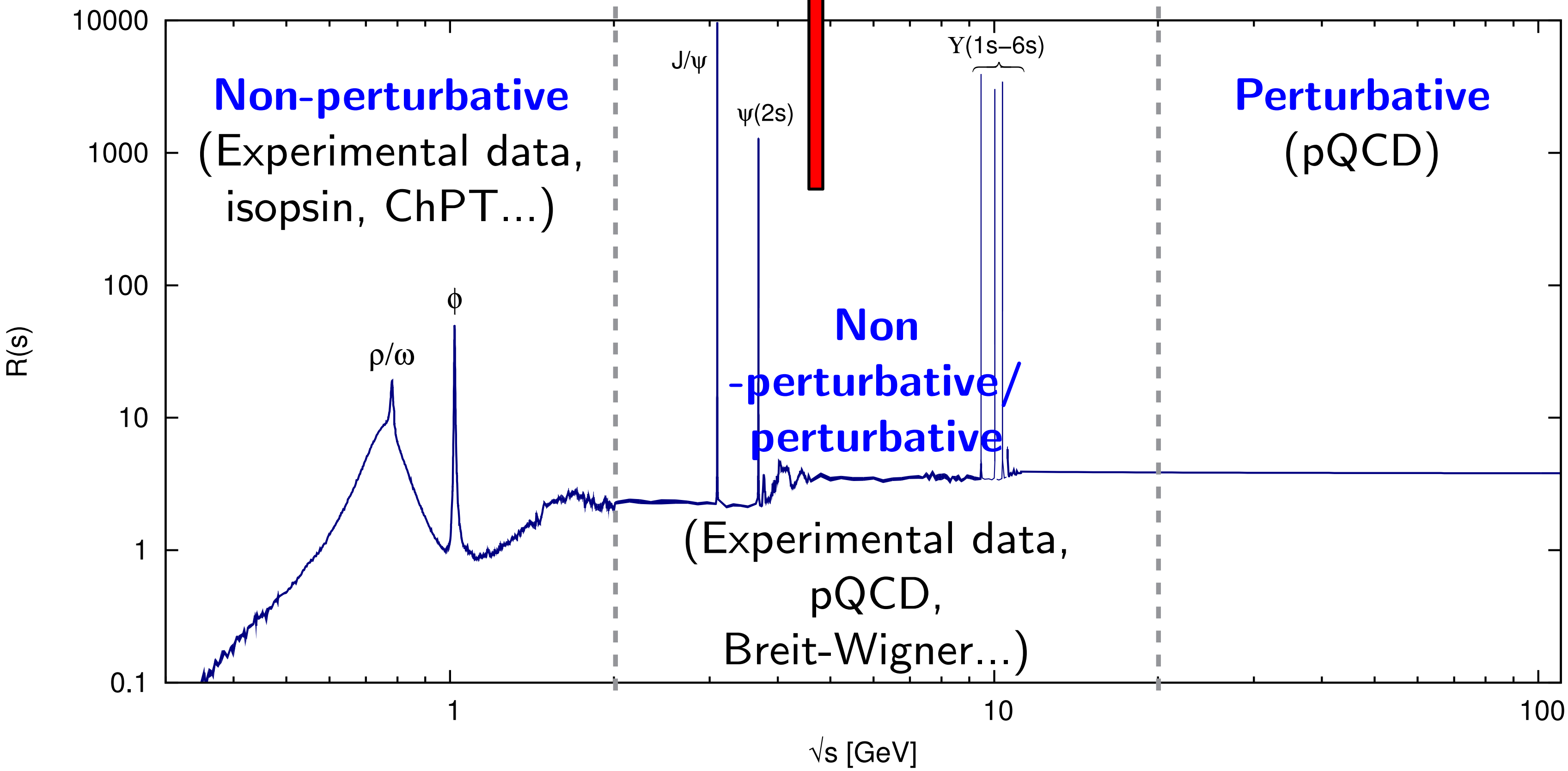
  from five classes of graphs:

  8.0 - 4.1 + 9.1 - 0.6 + 0.005

➥ good convergence, iterations of hadronic blobs very small

HVP disp.: cross section (in terms of R-ratio) input

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\,R(s)K(s),\ \text{where}\ R(s)=\frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



Non-perturbative
(Experimental data, isopsin, ChPT...)

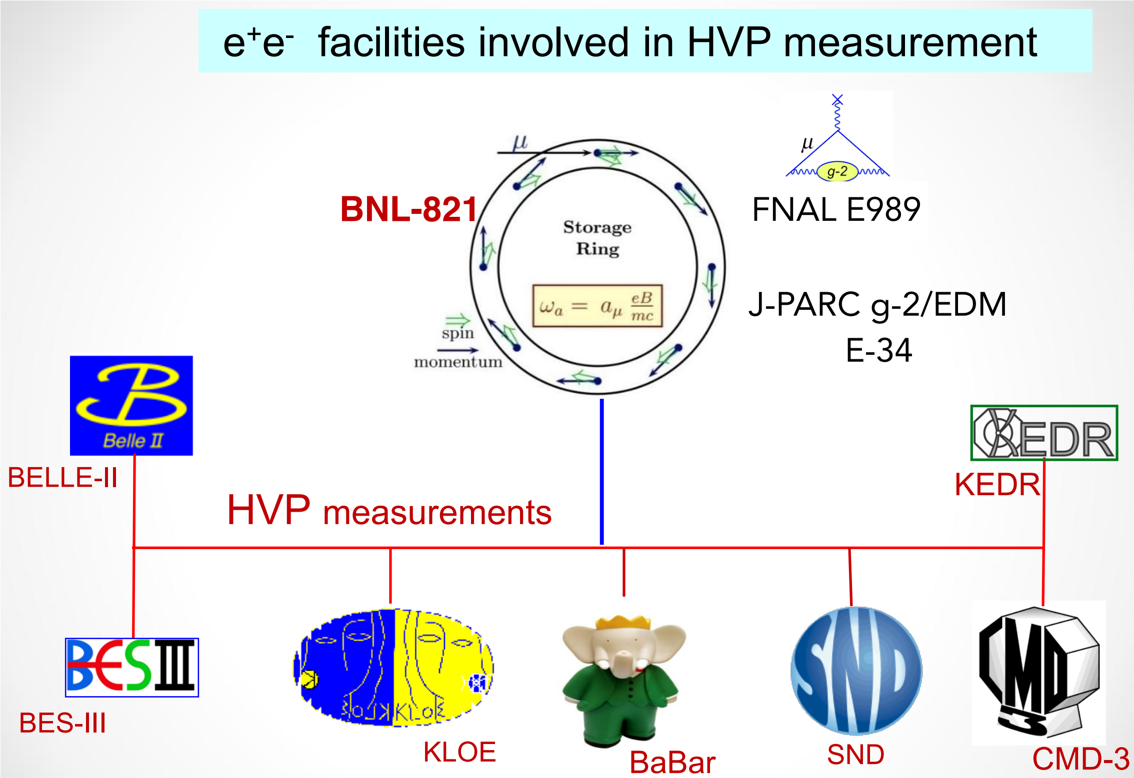
Non -perturbative/ perturbative
(Experimental data, pQCD, Breit-Wigner...)

Perturbative
(pQCD)

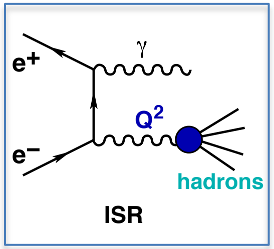
Must build full hadronic cross section/R-ratio...

a_μ^{HVP} : Recent (of 25+ years) experiments providing input $\sigma_{had}(s)$ data

S. Serednyakov (for SND) @ HVP KEK workshop



e^+e^- facilities involved in HVP measurement



- Different methods: \`**Direct Scan**’ (tunable e^+e^- beams) &
  \`**Radiative Return**’ (Initial State Radiation scan at fixed cm energy) ↗
- Over last decades detailed studies of **radiative corrections** & **Monte Carlo Generators** for $\sigma_{had}(s)$
  - ➤ **RadioMonteCarLow** Working Group report: Eur. Phys. J. C66 (2010) 585-686
  - ➤ full NLO radiative corrections in ISR MC *Phokhara*: Campanario et al, PRD 100(2019)7,076004

HVP dispersive: cross section compilation

How to get the most precise σ^0_{had}? Use of $e^+e^- \rightarrow$ hadrons (+γ) data:

- **Low energies:** sum ~35 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, …,
  [now very limited use iso-spin relations for missing channels]

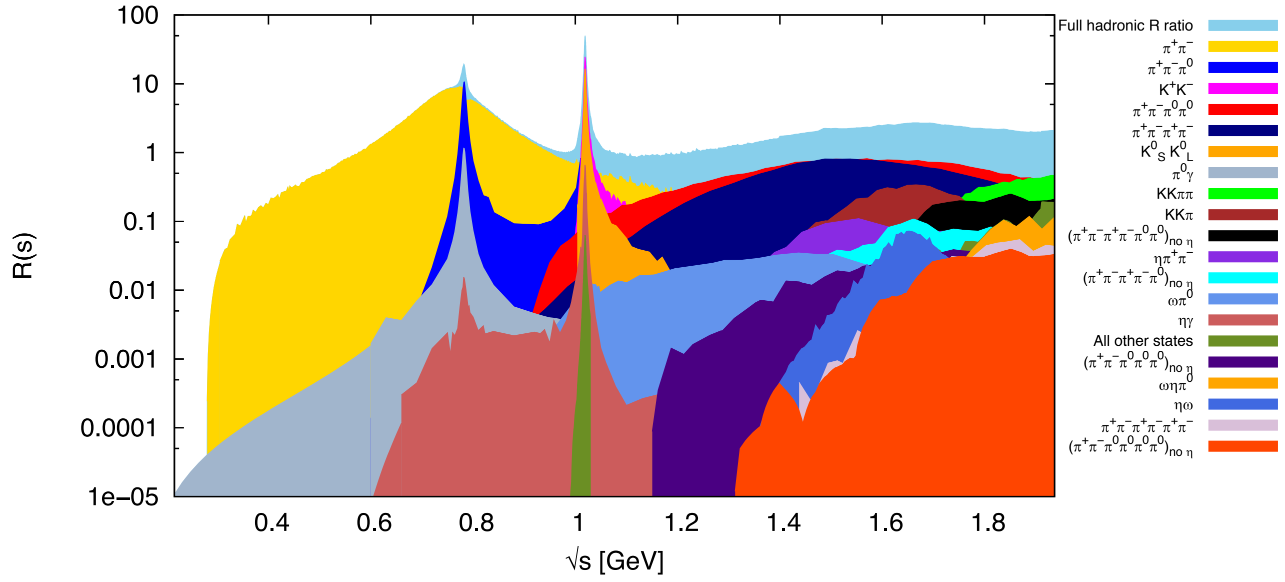
- **Above ~1.8 GeV:** use of inclusive data or pQCD (away from flavour thresholds),
  supplemented by narrow resonances (J/Ψ, Υ)

- Challenge of **data combination** (locally in √s, with **error inflation if tensions**):
  - many experiments, different energy ranges and bins,
  - statistical + systematic errors from many different sources,
  - use of correlations; must avoid inconsistencies, bias

  [Significant differences between DHMZ and KNT in use of correlated errors:
  - KNT allow non-local correlations to influence mean values,
  - DHMZ restrict this but retain correlations for errors and also betw. channels]

- σ^0_{had} means the `bare' cross section, i.e. excluding `running coupling' (VP) & ISR effects,
  but including Final State (γ) Radiation; data are subject to Radiative Corrections

a_μ^{HVP} : Landscape of $\sigma_{had}(s)$ data & most important $\pi^+\pi^-$ channel



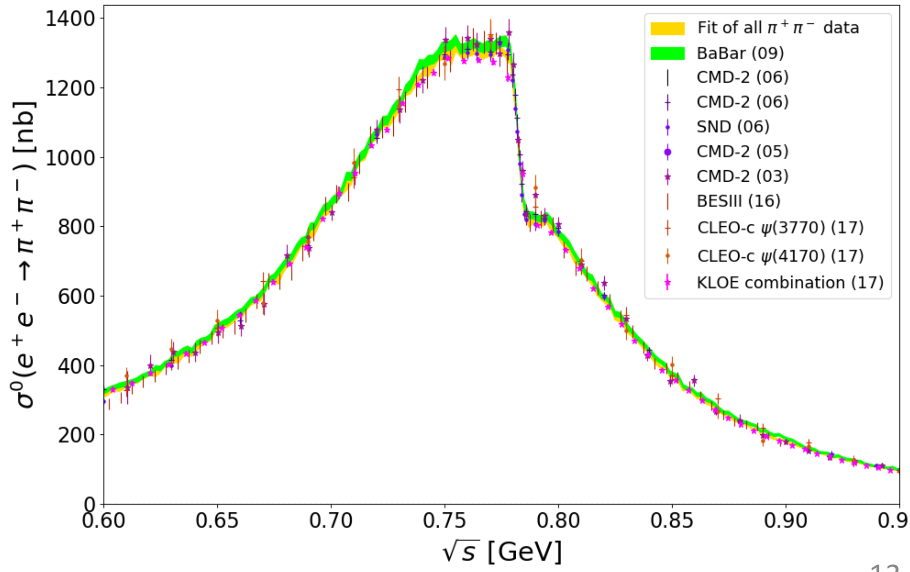
[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of 2π

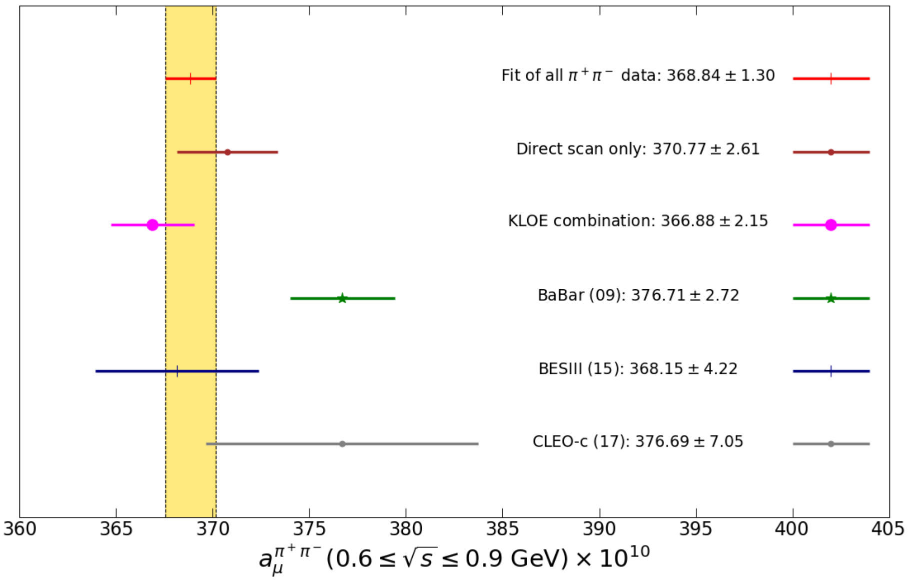
$\pi^+\pi^-$:

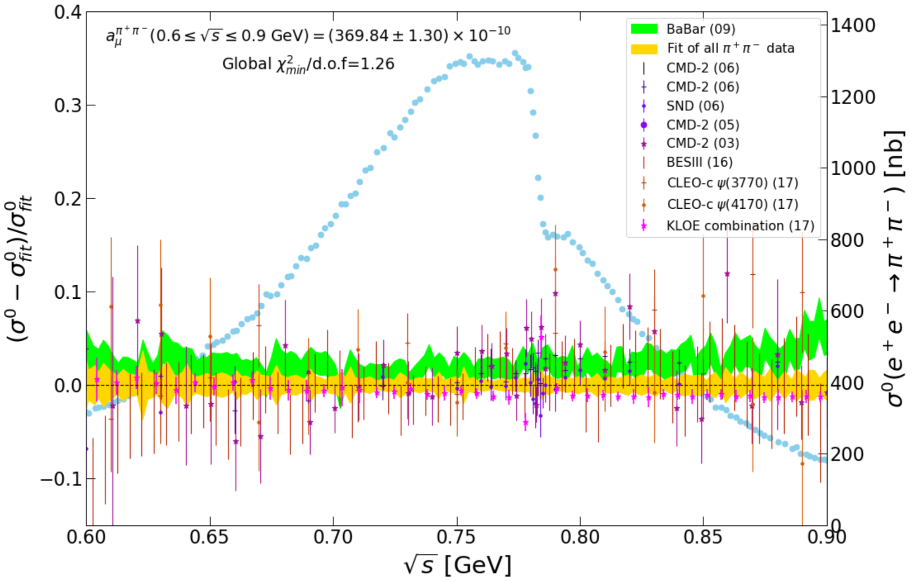
- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from 6 independent experiments with different systematics and different radiative corrections
- Data sets from Radiative Return dominate
- Some tension in data accounted for by local χ^2_{min} inflation and via WP merging procedure

[KNT19, PRD101, 014029]



HVP: $\pi^+\pi^-$ channel [KNT19, Phys. Rev. D 101(2020)1, 014029]





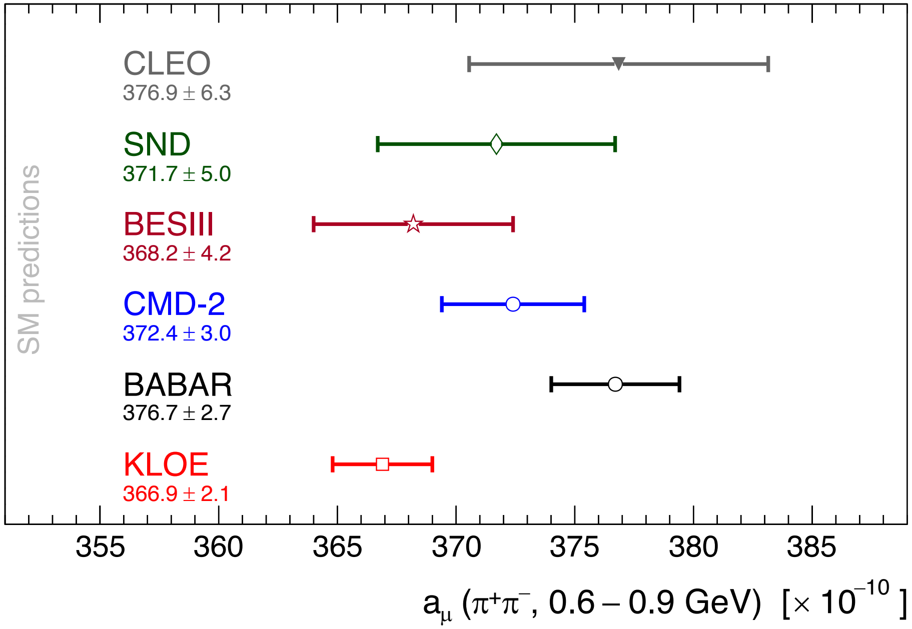
- **Tension** between different sets, especially between the most precise 4 sets from BaBar and KLOE
- Inflation of error with **local χ^2_{min}** accounts for tensions, leading to a **~14% error inflation**
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods, covered by WP merging

HVP: $\pi^+\pi^-$ channel [DHMZ, Eur. Phys. J. C 80(2020)3, 241]

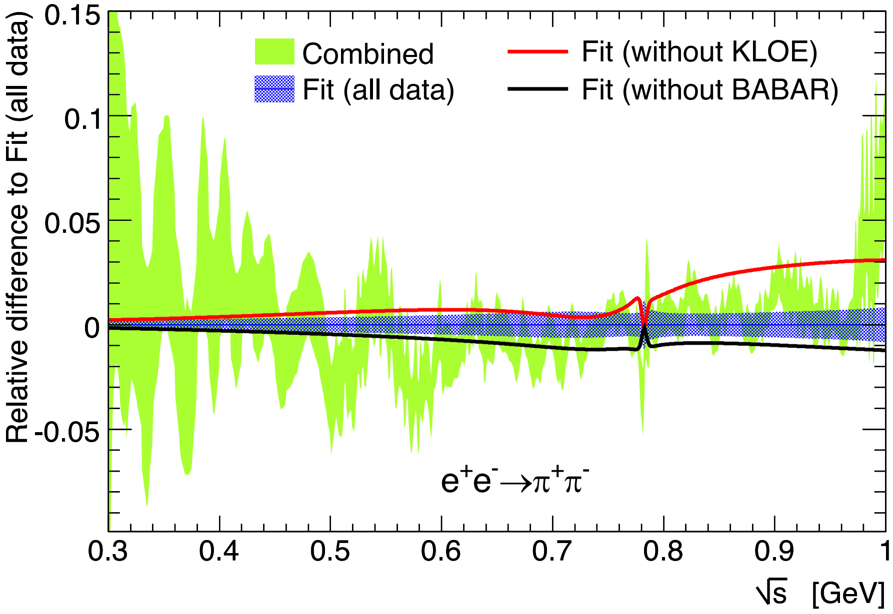
- In addition they employ a fit, based on **analyticity + unitarity + crossing symmetery,** similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraints/lower errors at low energies

- For 2**π**, based on difference between result for $a_\mu^{\pi\pi}$ w/out KLOE and BaBar, sizeable additional systematic error is applied and mean value adjusted

arXiv:1908.00921 Figure 5:



arXiv:1908.00921 Figure 6:



HVP: White Paper comparison & merging procedure

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, HHKS 2019]

HVP: White Paper comparison & merging procedure

Conservative merging procedure developed during 2019 Seattle TI workshop:

- Accounts for the different results obtained by different groups based on the same or similar experimental input
- Includes correlations and their different treatment as much as possible
- Allows to give one recommended (merged) result, which is conservative w.r.t. the underlying (and possibly underestimated) uncertainties
- Note: Merging leads to a bigger error estimate compared to individual evaluations

➠ $a_\mu^{\mathrm{HVP,\,LO}} = 693.1\,(4.0) \times 10^{-10}$ is the result used in the WP `SM2020’ value

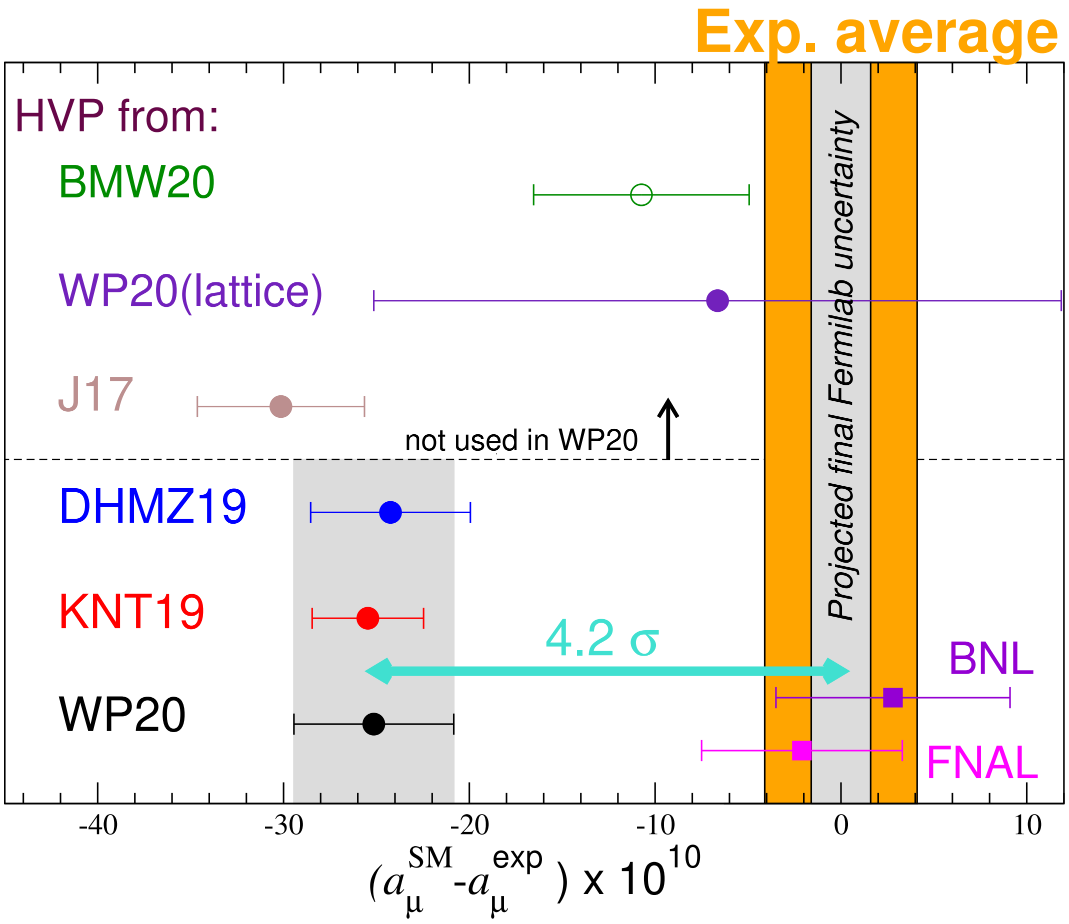
- This result does not include lattice, but is compatible with published lattice results apart from the BMW prediction:

  $a_\mu^{\mathrm{HVP,\,LO}}$ (BMW) = $707.5\,(5.5) \times 10^{-10}$ [Nature]

Efforts are ongoing in the community to check their result, with a topical online workshop from the g-2 Theory Initiative in November 2020 shedding first light.

Muon g-2 SM prediction from the TI WP vs FNAL+BNL

Experiment vs. theory **with** the FNAL g-2 Run-1 result announced 7th April:



SM prediction:

$$a_\mu^{\mathrm{SM}} = 116\,591\,810(43) \times 10^{-11}$$

FNAL E989 (2021):

$$a_\mu^{\mathrm{E989}} = 116\,592\,040(54) \times 10^{-11}$$

Combined with BNL E821 (2004):

$$a_\mu^{\mathrm{exp}} = 116\,592\,061(41) \times 10^{-11}$$

$$a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}} = 251(59) \times 10^{-10} \quad (4.2\,\sigma)$$

Outlook & conclusions

- The still **unresolved muon g-2 discrepancy** has triggered new experiments and a lot of theory activities, including and helped by the Muon g-2 Theory Initiative

- **Much progress** has been made for **HLbL** which previously was seen as the bottleneck.
  New data driven dispersive approaches & lattice have confirmed earlier model estimates and now allow a **reliable error estimate**, and **more work is in progress**

- For **HVP dispersive**, the **TI published a conservative & robust consensus**.
  Soon **new hadronic data for 2π** will come from **BaBar, CMD-3, BESIII and Belle-II**

- Longer term: direct HVP measurement planned with e-μ scattering: **MUonE** at CERN

- **Lattice** has started to deliver impressive results with **high precision**.
  Further work needed and ongoing to scrutinize, check & improve different approaches

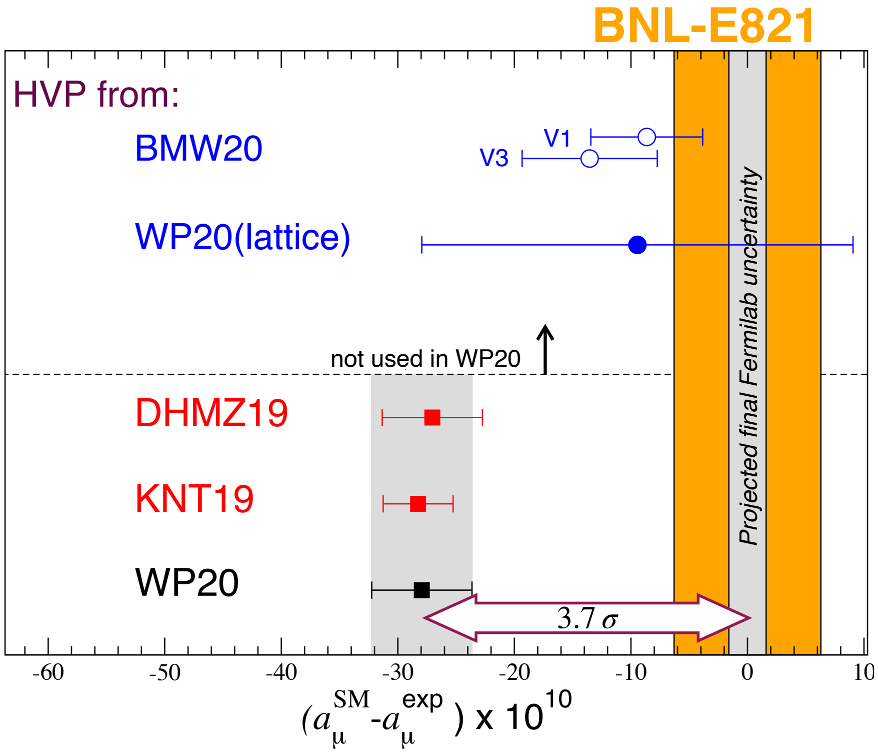
- The **Muon g-2 Theory Initiative** will continue to facilitate this work and to publish agreed & conservative SM predictions for g-2 prior to new experimental results

- With the **WP20 SM** prediction and the first g-2 result from **FNAL**,

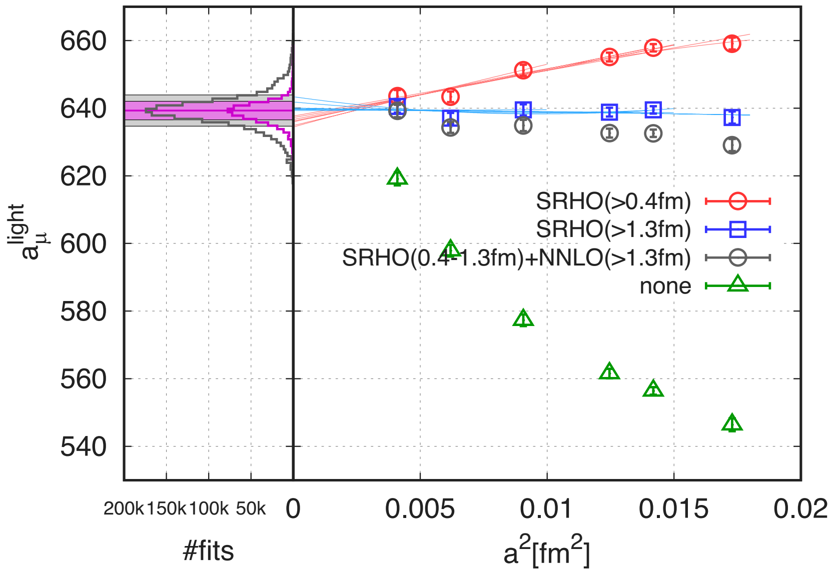
  the discrepancy stands at 4.2σ and is more intriguing than ever.

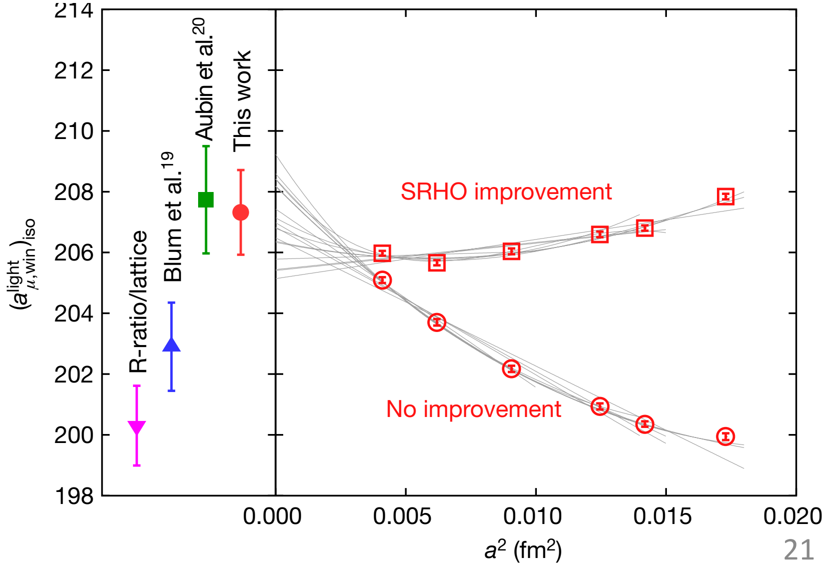
Extras/discussion

Lattice HVP: Tension betw. BMW & data-driven. Systematics



BMW20 [Borsanyi et al, arXiv:2002.12347, 2021 Nature]





BMW20: large systematics from **continuum limit,** large taste-breaking corrections (`SRHO')

- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central `window' compared to other lattice and data-driven results (**3.7σ** tension)

Lattice HVP: Cross checks, window method (I)

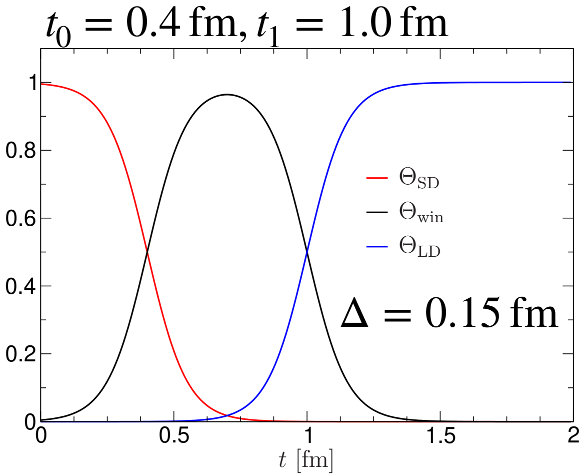
$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)}$$

- Use windows in Euclidean time to consider the different time regions separately.

Short Distance (SD) $t : 0 \to t_0$

Intermediate (W) $t : t_0 \to t_1$

Long Distance (LD) $t : t_1 \to \infty$



- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

Lattice HVP: Cross checks, window method (II)

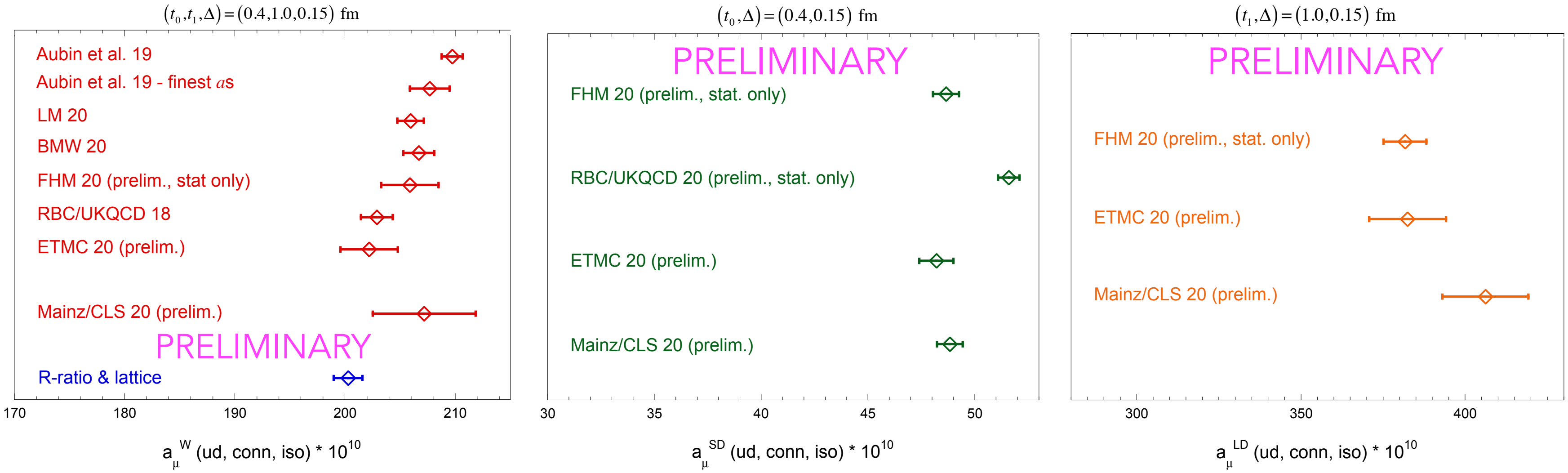
H. Wittig @ Lattice HVP workshop

$$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$$
$$\Delta = 0.15\,\mathrm{fm}$$

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

“Window” quantities

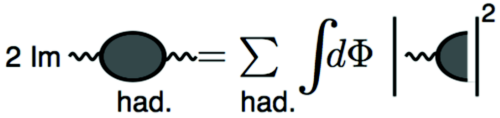
(Plots from Davide Giusti)



- Straightforward reference quantities
- Can be applied to individual contributions (light, strange, charm, disconnected,…)
- **Large discrepancies** between different results, also with data-driven: **BMW vs KNT: 3.7σ**
- Individual results **must sum up**, and different groups & discretisations **must agree** (universality)

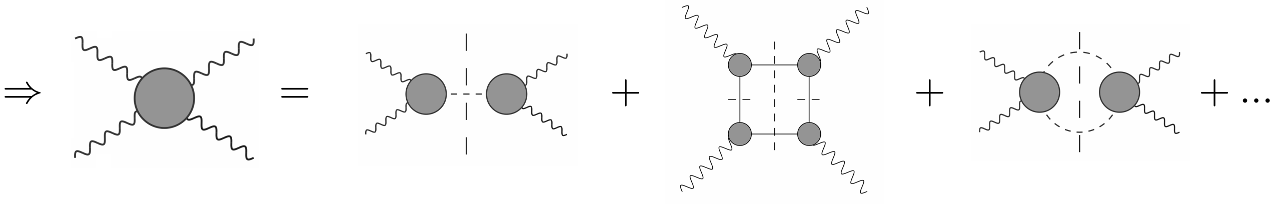
$a_\mu^{\rm HLbL}$: Hadronic Light-by-Light: Dispersive approach

For HVP ⇒



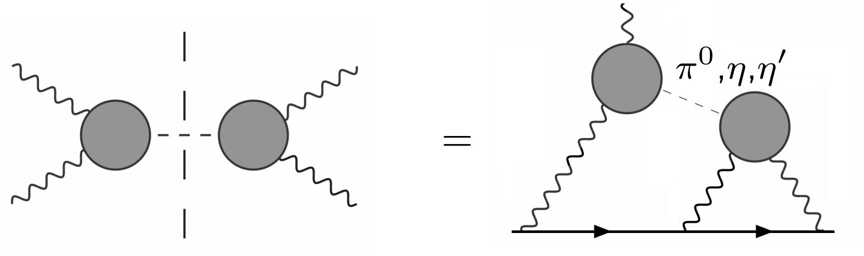
⇒ $\mathrm{Im}\Pi_{\rm had}(s) = \left(\frac{s}{4\pi\alpha}\right)\sigma_{\rm had}(s)$

For HLbL ⇒ $\Pi_{\mu\nu\lambda\sigma} = \Pi^{\rm pole}_{\mu\nu\lambda\sigma} + \Pi^{\rm box}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$



⇒ Dominated by pole (pseudoscalar exchange) contributions

$\Pi^{\rm pole}_{\mu\nu\lambda\sigma}$ =



⇒ Sum all possible diagrams to get $a_\mu^{\rm HLbL}$

- See also review by Danilkin+Redmer+Vanderhaeghen using dispersive techniques estimates $(8.7 \pm 1.3) \times 10^{-10}$ [Prog. Part. Nucl. Phys. 107 (2019) 20]
- With new results & progress, L-by-L can now be reliably predicted! ✓

Rad. Corrs.: Final State γ Radiation

- Real + virtual , <u>must be included</u> in $\boldsymbol{\sigma^0_{\mathrm{had}}}$ as part of the hadronic dynamics,
- but some events with real radiation cut-off by experimental analyses
  (no problem if γ missed and event counted, but possible problem of mis-identifies)
- Experiments (or compilations) account for this and add FSR back;
  - based on MC and **scalar QED** for pions (detailed studies, checked to work well)
  - contributes to systematic uncertainties
  - intrinsic part of Radiative Return analyses of dominant recent data sets
- Notes:
  - at low energies and at resonances, hard radiation is limited by phase space
  - different compilations apply **additional uncertainty** to cover possible problems of the **FSR** (& **VP**/undressing) treatment, e.g.

➤ KNT: $\boldsymbol{\delta a_\mu^{\mathrm{had,\,FSR}} = 7.0 \times 10^{-11}}$, and also $\boldsymbol{\delta a_\mu^{\mathrm{had,\,VP}} = 2.1 \times 10^{-11}}$

a_μ^{HVP} : Hadronic tau decay data

- Historically, hadronic tau decay data, e.g. $\tau^- \to \pi^0\pi^-\nu_\tau$, were used to improve precision of e^+e^- based evaluations

- However, with the increased precision of the e^+e^- data there is now limited merit in this (DHMZ have dropped it), and

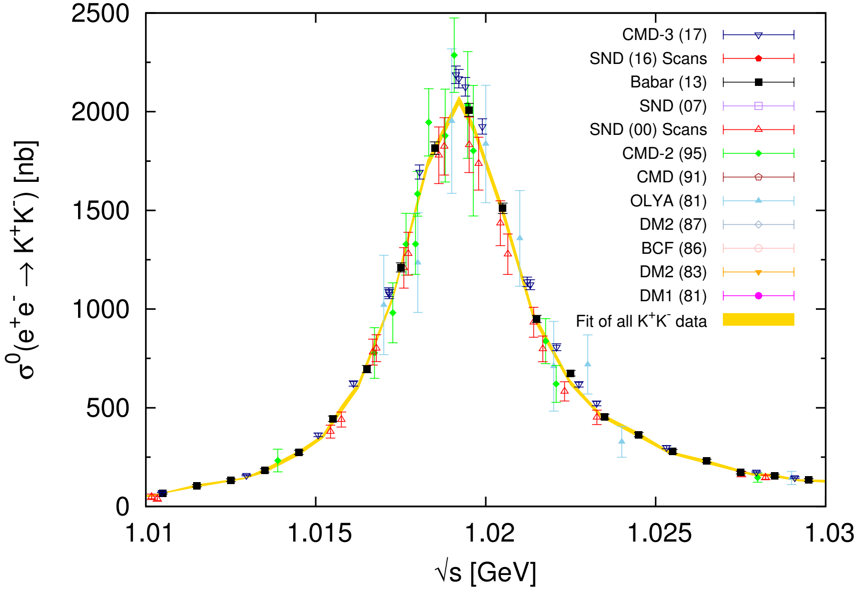
- The required iso-spin breaking corrections re-introduce a model-dependence and connected systematic uncertainty (there is, e.g., no ρ–ω mixing in τ decays)

- Quote from the WP, where this approach is discussed in detail:

"Concluding this part, it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Sec. 3.4.2, may provide a solution to this problem."

HVP: (subleading) KK channels [KNT18, PRD97, 114025]

K^+K^-



New data:

BaBar: [Phys. Rev. D 88 (2013), 032013.]
SND: [Phys. Rev. D 94 (2016), 112006.]
CMD-3: [arXiv:1710.02989.]

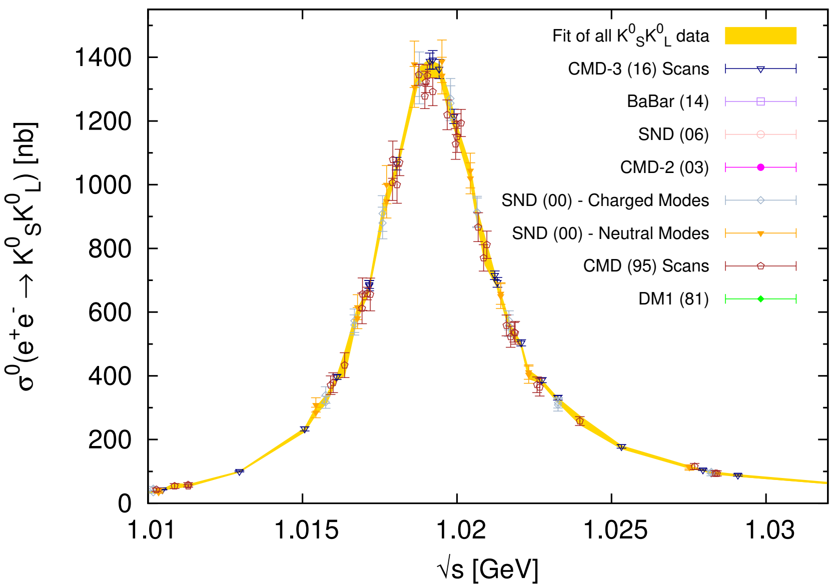
Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.] omitted as waiting reanalysis.

$$a_\mu^{K^+K^-} = 23.03 \pm 0.22_{\rm tot}$$

HLMNT11: $22.15 \pm 0.46_{\rm tot}$

Large increase in mean value

$K^0_S K^0_L$



New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.]
CMD-3: [Phys. Lett. B 760 (2016) 314.]

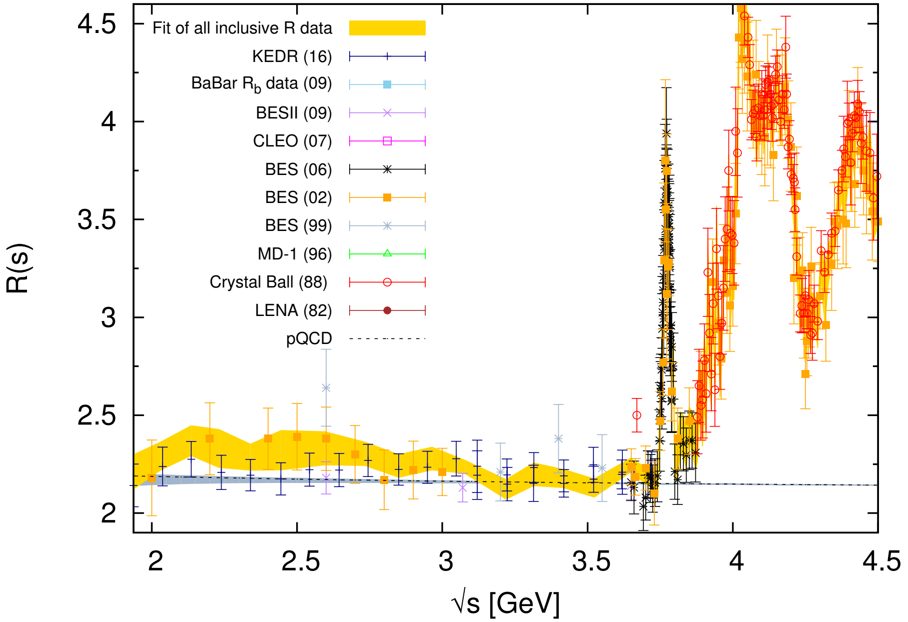
$$a_\mu^{K^0_S K^0_L} = 13.04 \pm 0.19_{\rm tot}$$

HLMNT11: $13.33 \pm 0.16_{\rm tot}$

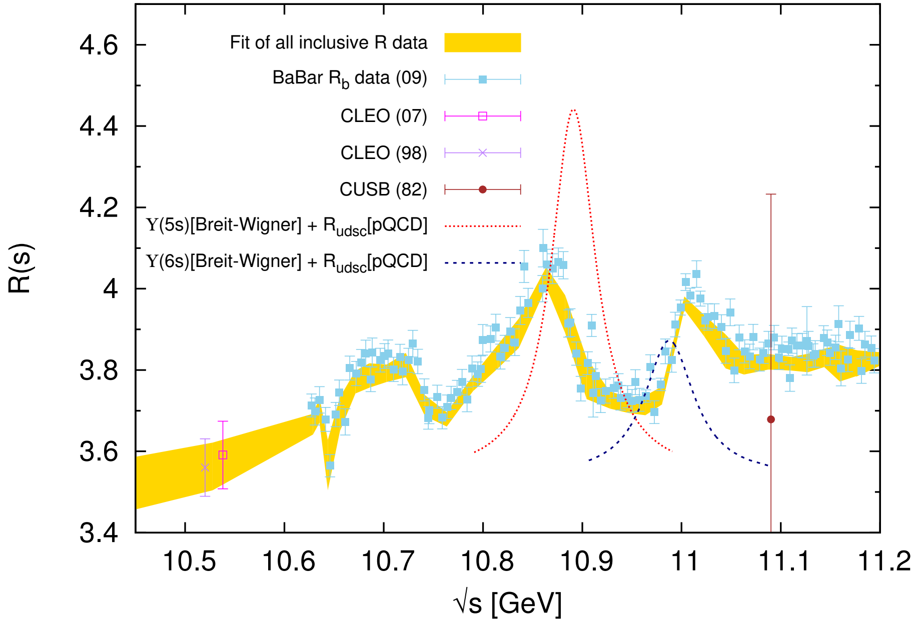
Large changes due to new precise measurements on ϕ

HVP: σ_{had} inclusive region [KNT18]

⇒ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

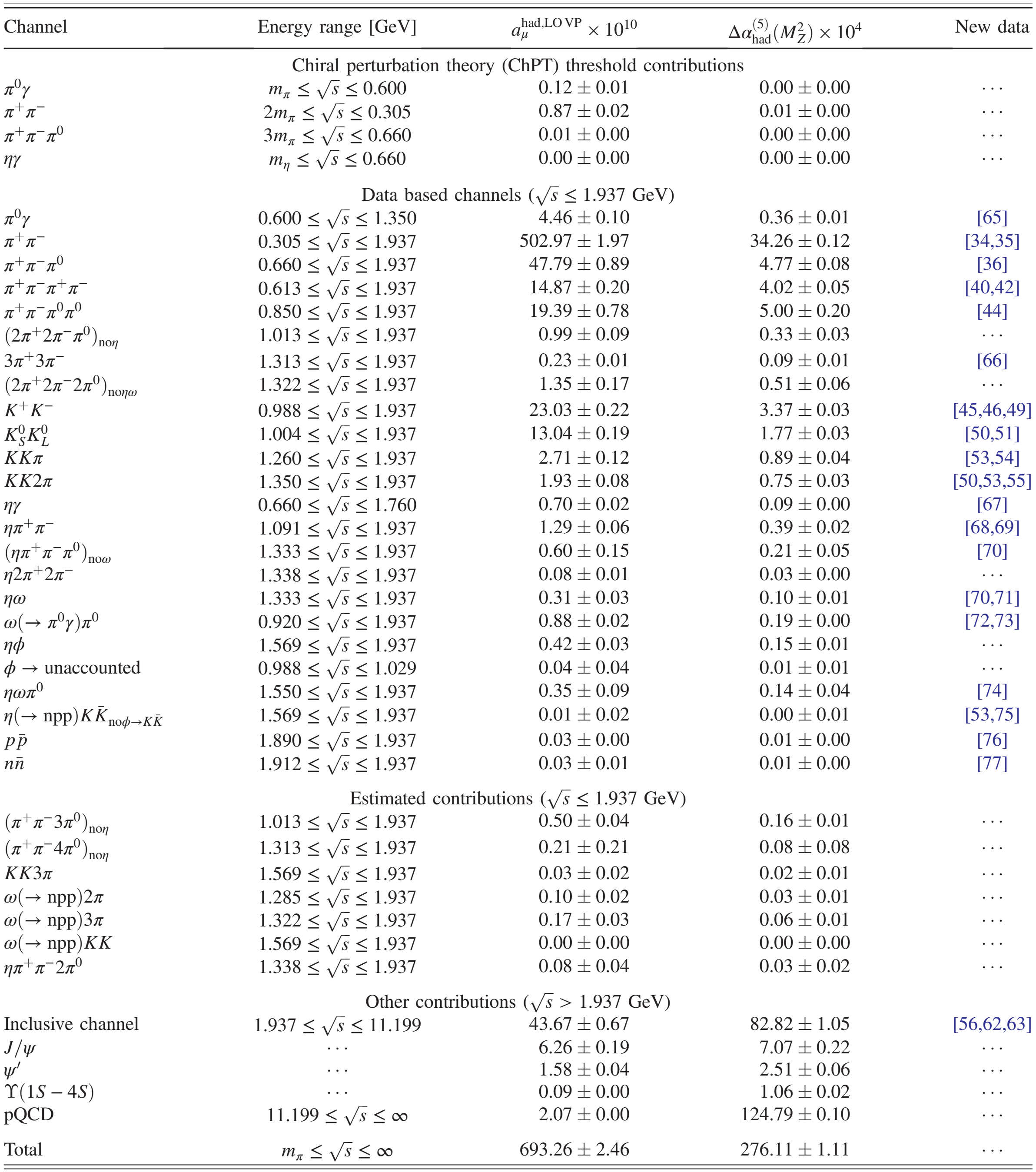
Table from KNT18,
PRD 97(2018)114025

Update: KNT19
LO+NLO HVP for
$a_{e,\mu,\tau}$ & hyperfine splitting of muonium

PRD101(2020)014029

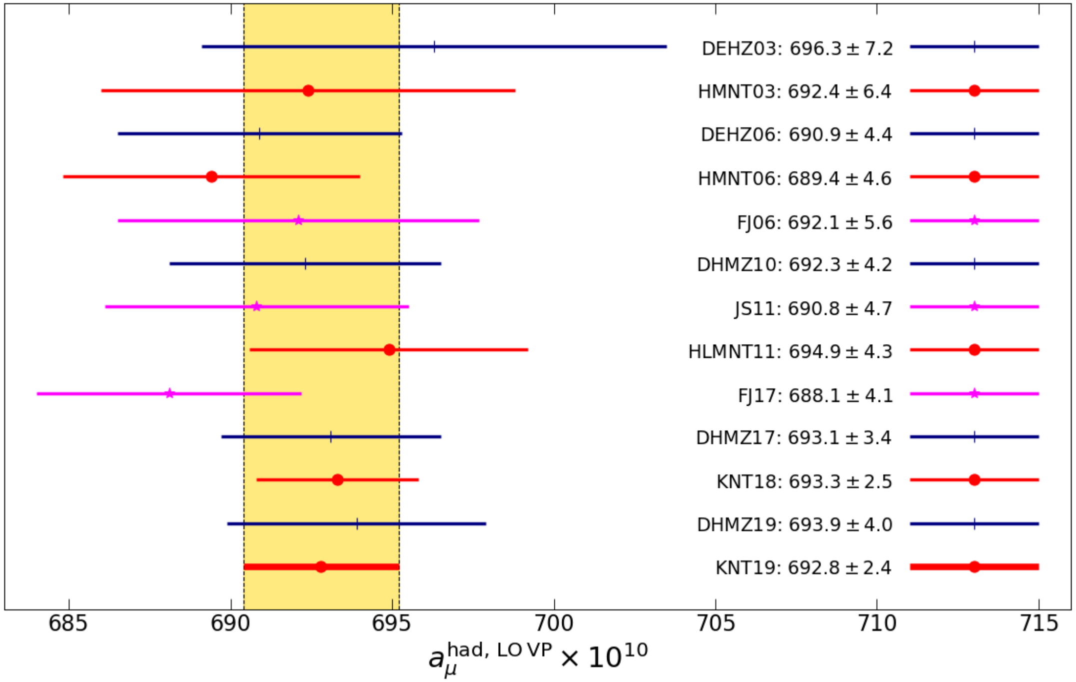
Breakdown of HVP contributions in ∼35 hadronic channels

From 2-11 GeV, use of inclusive data, pQCD only beyond 11 GeV



Channel	Energy range [GeV]	$a_\mu^{\rm had,LO\,VP} \times 10^{10}$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \times 10^4$	New data
		Chiral perturbation theory (ChPT) threshold contributions		
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	...
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	...
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	...
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	...
		Data based channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	...
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\rm no\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	...
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\rm no\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	...
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	...
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	...
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to {\rm npp})K\bar{K}_{{\rm no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
		Estimated contributions ($\sqrt{s} \le 1.937$ GeV)		
$(\pi^+\pi^-3\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	...
$(\pi^+\pi^-4\pi^0)_{\rm no\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	...
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	...
$\omega(\to {\rm npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	...
$\omega(\to {\rm npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	...
$\omega(\to {\rm npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	...
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	...
		Other contributions ($\sqrt{s} > 1.937$ GeV)		
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	...	6.26 ± 0.19	7.07 ± 0.22	...
ψ'	...	1.58 ± 0.04	2.51 ± 0.06	...
$\Upsilon(1S-4S)$	...	0.09 ± 0.00	1.06 ± 0.02	...
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	...
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	...

History plot of a_μ^{HVP} w. min. model dep. Pies.



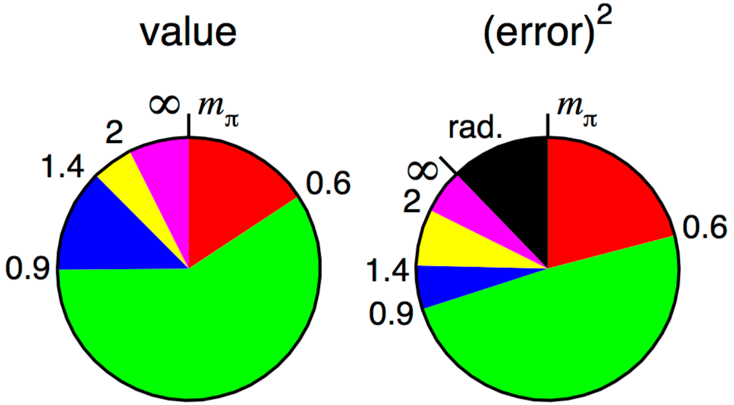
- Stability and consolidation over two decades thanks to more and better data input and improved compilation procedures

- Compare with `merged' DHMZ & KNT WP20 value:

$a_\mu^{\text{had, LO VP}}$(WP20) = 693.1(4.0)×$10^{-10}$

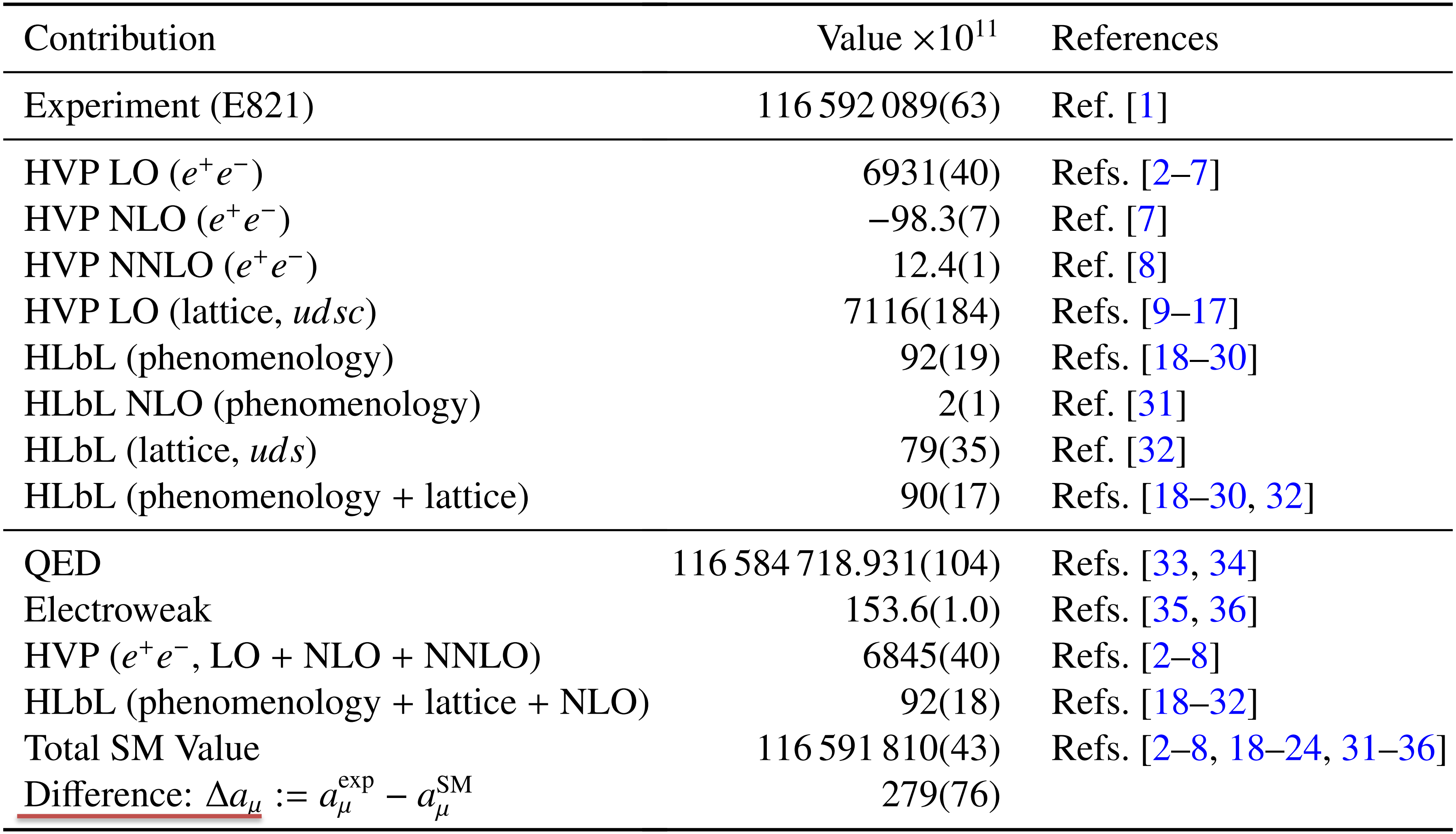
Pie diagrams [KNT]:

- error still dominated by two pion channel
- significant contribution to error from additional uncertainty from radiative corrections



a_μ (SM): White Paper https://doi.org/10.1016/j.physrep.2020.07.006

White Paper [T. Aoyama et al, arXiv:2006.04822], 132 authors, 82 institutions, 21 countries

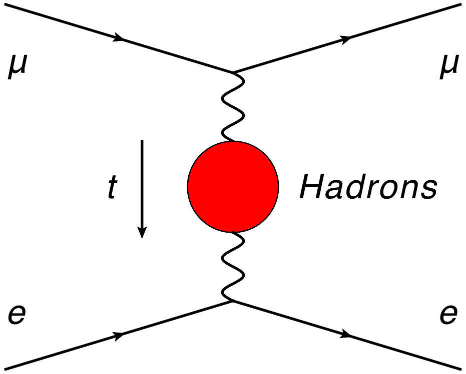


Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)	

w.r.t. BNL

HVP from electron-muon scattering in the space-like

M. Passera @ HVP KEK 2018 [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]



$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{\mathrm{had}}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- Jan 2020: SPSC recommends pilot run in 2021
- goal: run with full apparatus in 2023-2024