Cosmic Gravitational Microwave Background

Carlos Tamarit, Technische Universität München

JCAP 03 (2021) 054, arXiv:2011.04731 [hep-ph]

in collaboration with...

Andreas Ringwald
DESY

Jan Schütte-Engel
University of Illinois-Urbana

The aim

Present state-of the art predictions for the spectrum of **gravitational waves** sourced by **thermal fluctuations**

Discuss current **bounds** and **prospects** for **experimental detection**

The novelty

We **generalized** theoretical **predictions** for the SM to **arbitrary models**

Study of **experimental prospects** of detection from **GW/EMW** conversion in **axion experiments** with **HET/SPD** dectectors and in proposal with **Gaußian beam**

The plan

Gravitational waves from the thermal plasma

Features and predictions for thermal spectra

Current bounds

Future prospects

Gravity waves from the thermal plasma

Gravitational wave basics

Excitations of the metric field sourced by anisotropies in the stress-energy momentum tensor. In local Minkowski frame

$$ds^2 \supset -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j\,,$$
$$\partial^i h_{ij} = 0$$

$$\Box\left(h_{ij} - \frac{1}{2}\delta_{ij}h\right) = \frac{2}{M_P^2} T^{\rm TT}_{ij},$$
$${T^{\rm TT}}^i_{\ i} = 0,$$
$$\partial^i T^{\rm TT}_{ij} = 0$$

Sources of GWs in the early universe

Most studied sources of stochastic backgrounds of **gravitational waves**:

- inflation
- preheating
- first-order phase transitions

In the **Big Bang plasma,** transverse traceless stress-energy generated by:

- **Viscosity** effects in the plasma at large length scales
- Plasma **absorption-emission** effects at small scales

A **guaranteed** background of GWs relying on well **known physics**

Why is this background of interest?

Despite Planck suppresion of rate, **emission accumulates** since Big Bang

Peak of emission at

$$\nu_{\text{peak}} \sim T$$

redshifts at approx same rate as frequencies of previously emitted waves,

$$\nu(t) = \frac{a(t_i)}{a(t)}\,\nu(t_i) = \frac{1}{T_i}\left(\frac{g_{\star s}(T_i)}{g_{\star s}(T)}\right)^{1/3}\nu(t_i)T$$

energy at peak adds up constructively

Could a measurement of this background inform us of the temperature reached by the Hot Big Bang?

Peak in microwave regime: Cosmic Gravitational Microwave Background (CGMB)

Ingredients to compute the CGMB spectrum

The **CGMB spectrum** can be **computed** for an arbitrary theory **knowing**:

Dynkin indices of particle representations R

$$\mathrm{Tr} T^a_{n,R} T^b_{n,R} = T_{n,R} \delta^{ab} \quad a, b = 1, \ldots, N_n$$

Index labelling gauge group

Number of generators in group n

Gauge and Yukawa couplings and their running

Three **functions counting the degrees of freedom** in the plasma, which can be derived from thermal effective potential

$$g_{*\rho}(T), g_{*s}(T), g_{*c}(T)$$

Ingredients to compute thermal spectrum

Energy density

$$\rho = \frac{\partial U}{\partial V} \equiv \frac{\pi^2}{30} g_{*\rho}(T) T^4$$

Entropy density

$$s = \frac{\partial S}{\partial V} = \frac{2\pi^2}{45} g_{*s}(T) T^3$$

Heat capacity

$$c = \frac{1}{V} \frac{\partial U}{\partial T}\bigg|_V = T \frac{\partial s}{\partial T}\bigg|_V = \frac{2\pi^2}{15} g_{*c}(T) T^3$$

Relation to **effective potential** at finite T, $\Delta V(T)$

$$g_{*\rho} = \frac{30}{\pi^2 T^4} \left(\Delta V(T) - T \frac{\partial \Delta V(T)}{\partial T} \right),$$

$$g_{*s} = -\frac{45}{2\pi^2 T^3} \frac{\partial \Delta V(T)}{\partial T},$$

$$g_{*c} = -\frac{15}{2\pi^2 T^2} \frac{\partial^2 \Delta V(T)}{\partial T^2},$$

Rate of gravitational wave production

$$\rho_{\rm gw} = \frac{M_P^2}{4}\langle \dot h_{ij}(t,\mathbf{x})\,\dot h_{ij}(t,\mathbf{x})\rangle$$

Production rate neglecting backreaction related to 2-point correlator of stress-energy tensor [Laine, Ghiglieri]

$$(\partial_t + 4H)\rho_{\rm CGMB} = \frac{1}{2M_P^2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int d^4x\, e^{ikx}\langle T_{ij}^{\rm TT}(x)T_{ij}^{\rm TT}(0)\rangle \equiv \frac{4T^4}{M_p^2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\,\hat\eta\left(T,\frac{k}{T}\right)$$

Small k (large length scales): source from viscosity effects dominated by hypercharge interactions of right-handed leptons

$$\hat\eta(T,\hat k) = \frac{\eta_{\rm shear}}{T^3 g_1(T)^4\log(5/\hat m_1)} \qquad \hat k \equiv \frac{k}{T}$$

$$\hat m_n^2(T) = \frac{m_n^2(T)}{T^2} = g_n^2(T)\left(\frac{1}{3}T_{n,\rm Ad} + \frac{1}{6}\sum_{\hat\imath} T_{n,\hat\imath} + \frac{1}{6}\sum_{\hat\alpha} T_{n,\hat\alpha}\right)$$

Thermal Debye mass

Dynkin indices of gauge fields, real scalars, Weyl fermions

Rate of gravitational wave production

Large k (short length scales): Source from thermal excitations in the plasma. Generalized results of [Laine & Ghiglieri] to arbitrary theories

$\hat{\eta}_{\mathrm{HTL}}(T,\hat{k})$

$$+\sum_{n=1}^{N_g} g_n(T)^2 N_n \left(\frac{1}{2} T_{n,\mathrm{Ad}}\,\eta_{gg}(\hat{k}) + \sum_{\hat{\imath}} T_{n,\hat{\imath}}\,\eta_{sg}(\hat{k}) + \frac{1}{2}\sum_{\hat{\alpha}} T_{n,\hat{\alpha}}\,\eta_{fg}(\hat{k})\right) + \frac{1}{4}\sum_{i\alpha\beta} |y^i_{\alpha\beta}(T)|^2\,\eta_{sf}(\hat{k}),$$

Leading log contribution

$$\hat{\eta}_{\mathrm{HTL}}(T,\hat{k}) = \frac{\hat{k}}{16\pi(e^{\hat{k}}-1)} \sum_n N_n \hat{m}_n^2(T) \log\left(1 + 4\frac{\hat{k}^2}{\hat{m}_n^2(T)}\right).$$

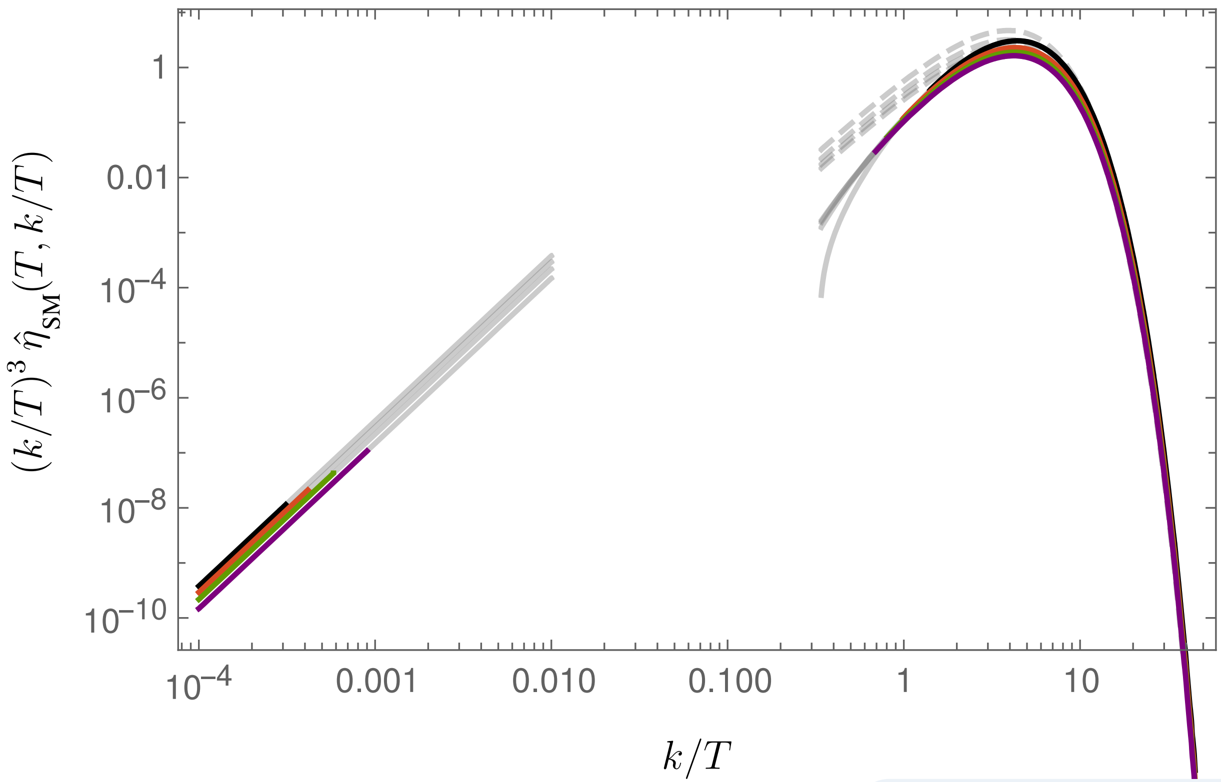
Gauge and Yukawa couplings

Dynkin indices of representations

Thermal loop functions computed by [Laine & Ghiglieri]

State of the art calculation of $\hat{\eta}$ in the SM

[Laine & Ghiglieri]



Production from **shear viscosity** effects

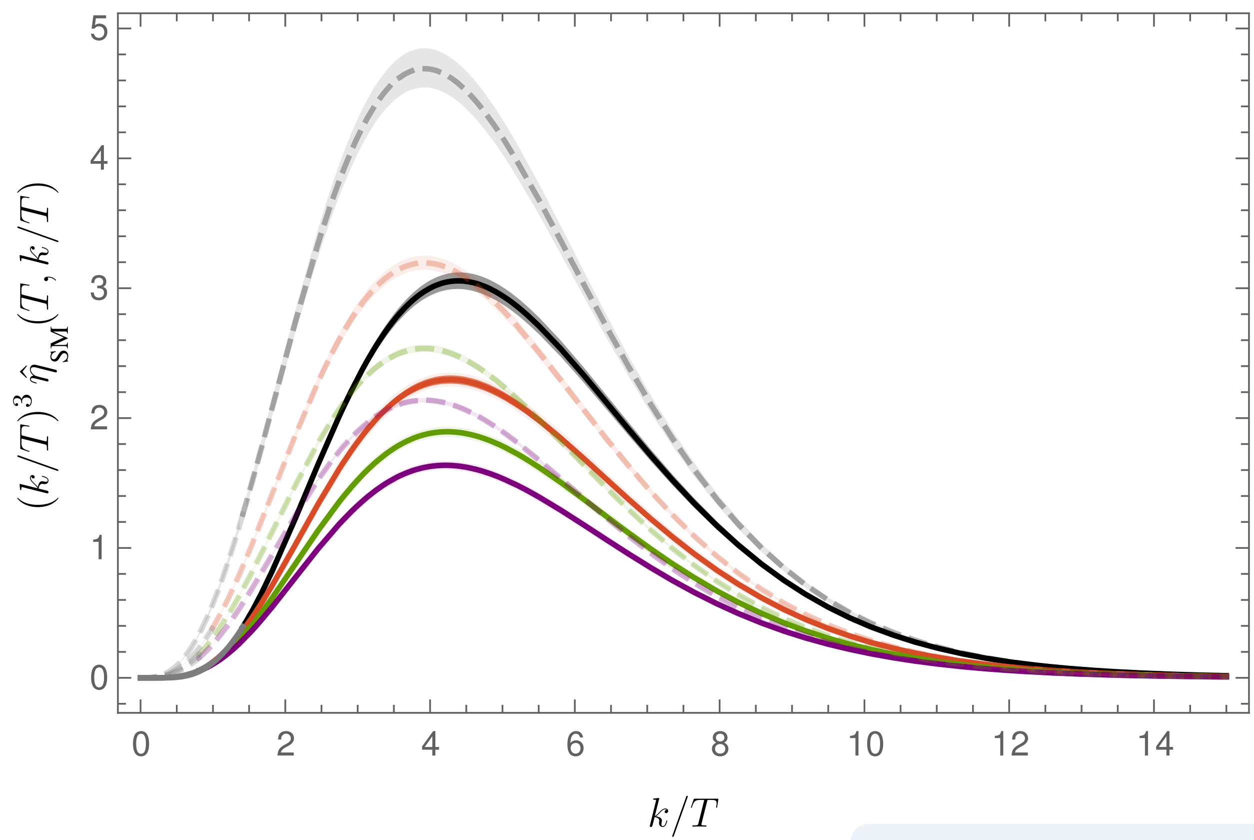
Dominant production from **particle excitations** in the plasma

T= 10^3 GeV , 10^8 GeV, 10^{13} GeV, Mp

Subleading explicit T dependence

State of the art calculation of $\hat{\eta}$ in the SM

Production from **particle excitations** in the plasma [Laine & Ghiglieri]



Leading log vs full-leading order

Small theoretical error!

10^3 GeV , 10^8 GeV, 10^{13} GeV, Mp

Integrating the rate until today

To get current spectrum one has to **integrate over time** and **account for expansion** of the universe, using:

Relation between **temperature** and **time** from entropy conservation and Friedmann's equations

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{\pi}{\sqrt{90}} \left[g_{*\rho}(T)\right]^{1/2} \frac{g_{*s}(T)}{g_{*c}(T)} \frac{T^3}{M_P}$$

Redshifting of frequencies from entropy conservation

$$\hat{k} = \frac{2\pi f}{T_0} \left(\frac{g_{*s}(T)}{g_{*s}(\mathrm{fin})}\right)^{1/3}$$

Integrating the rate until today

$$\Omega_{\rm CGMB}(f) = \frac{1}{\rho_{\rm tot}}\frac{\rho_{\rm CGMB}}{d\log k}$$

$$\simeq \frac{1440\sqrt{10}}{2\pi^2 M_P}\,\Omega_\gamma\,[g_{*s}(\text{fin})]^{1/3}\,\frac{f^3}{T_0^3}\times\int_{T_{\rm ewco}}^{T_{\rm max}} \mathrm{d}T\,\frac{g_{*c}(T)}{[g_{*s}(T)]^{4/3}\,[g_{*\rho}(T)]^{1/2}}\,\hat{\eta}\left(T, 2\pi\left[\frac{g_{*s}(T)}{g_{*s}(\text{fin})}\right]^{1/3}\frac{f}{T_0}\right).$$

Integral dominated by high T in which all g functions tend to a common limit

$$\Omega_{\rm CGMB}(f) \approx 4.03\times 10^{-12}\left[\frac{T_{\rm max}}{M_P}\right]\left[\frac{g_{*s}(T_{\rm max})}{106.75}\right]^{-5/6}\left[\frac{f}{\rm GHz}\right]^3\hat{\eta}\left(T_{\rm max}, 2\pi\left[\frac{g_{*s}(T_{\rm max})}{g_{*s}(\text{fin})}\right]^{1/3}\frac{f}{T_0}\right)$$

Linear effect with $T_{\rm max}$: A true thermometer!

[3% accuracy for SM]

As usual we introduce the **dimensionless strain**

$$h_c(f) = 1.26\times 10^{-18}\left[\frac{\rm Hz}{f}\right]\sqrt{h^2\,\Omega_{\rm GW}^{(0)}(f)}$$

What if T_{max} goes above M_P?

Cannot ignore backraction of produced gravitons. Expect gravitons to **reach thermal equilibrium** and **decouple at $T = M_P$**

Spectrum given by the equilibrium distribution redshifted from $T = M_P$ to today

$$\Omega_{\text{Eq.CGMB}}(f) = \frac{16\pi^2}{3M_P^2 H_0^2} \frac{f^4}{e^{2\pi f/T_{\text{grav}}} - 1}, \qquad T_{\text{grav}} = \left(\frac{g_{*s}(\text{fin})}{g_{*s}(M_P)}\right)^{1/3} T_0.$$

Features and predictions

Main features

Peak frequencies

$$f_{\rm peak}^{\Omega_{\rm CGMB}}(T_{\rm max}) \approx 79.8\,{\rm GHz}\,\left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{1/3},$$

$$f_{\rm peak}^{h_c^{\rm CGMB}}(T_{\rm max}) \approx 40.5\,{\rm GHz}\,\left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{1/3},$$

Measuring peak value and position can lead to estimate of $T_{\rm max}$ and $g_{*s}(T_{\rm max})$

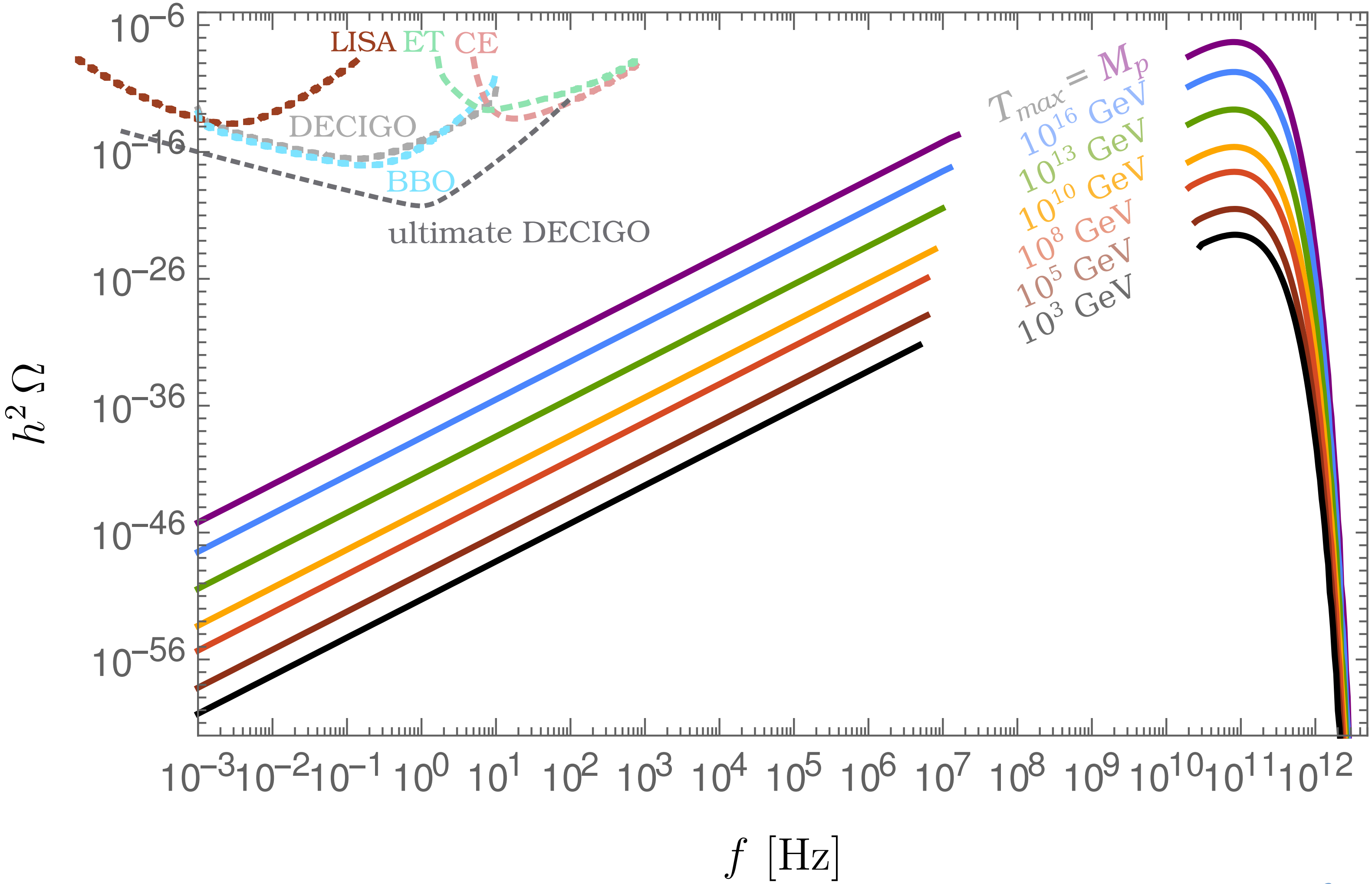
For **weakly coupled theories** [up to 30% deviations in MSSM]

$$\Omega_{\rm CGMB}(f^{\Omega}_{\rm peak}(T_{\rm max})) \approx \left(\frac{g_{*s,{\rm SM}}(T_{\rm max})}{g_{*s}(T_{\rm max})}\right)^{11/6} \Omega_{\rm CGMB,SM}(f^{\Omega}_{\rm peak,SM}(T_{\rm max})),$$

$$h_c^{\rm CGMB}(f^{h_c}_{\rm peak}(T_{\rm max})) \approx \left(\frac{g_{*s,{\rm SM}}(T_{\rm max})}{g_{*s}(T_{\rm max})}\right)^{7/12} h_c^{\rm CGMB,SM}(f^{h_c}_{\rm peak,SM}(T_{\rm max})),$$

SM leads to **higher power** in gravitational waves

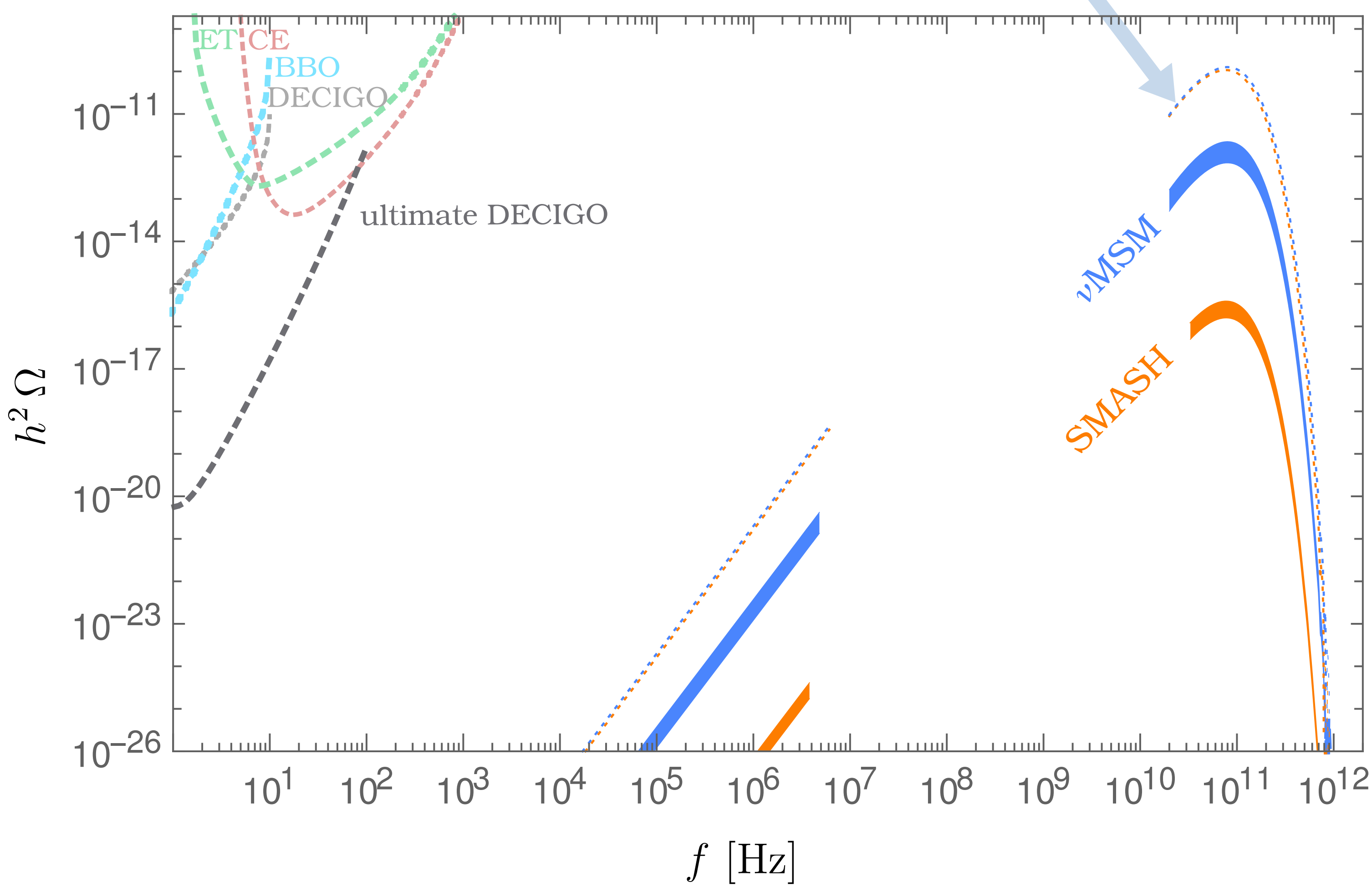
Spectrum in the SM



[0.1% accuracy for SM]

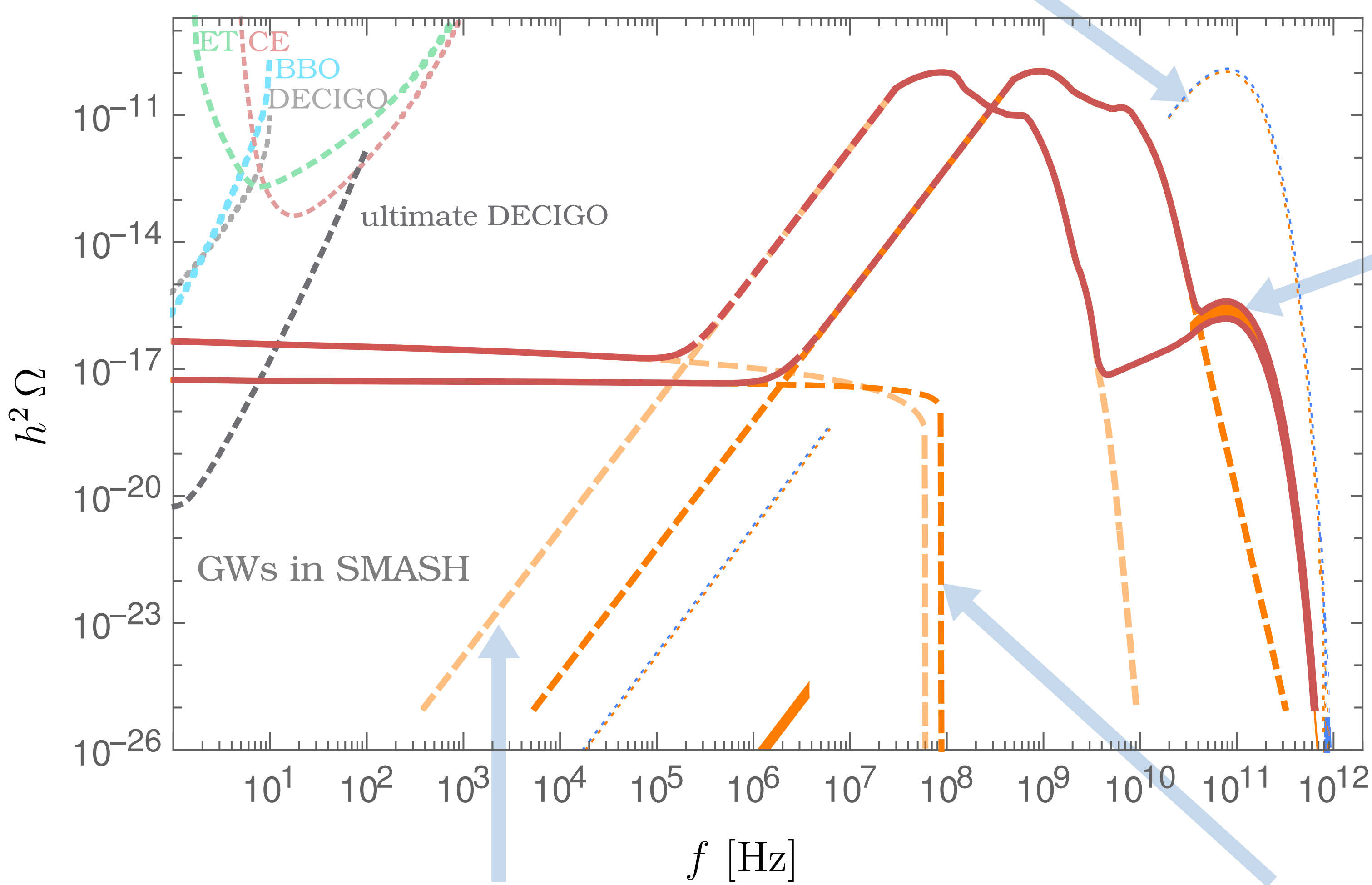
Spectrum in SMASH/νMSM

Single-field inflation upper bound to CGMB



Is the CGMB still visible in complete spectra?

Single-field inflation upper bound to SM CGMB



CGMB

[Ringwald, Tamarit] to appear

Gws from preheating

GWs from inflation

Current experimental bounds

How to detect high-freq gravitational waves?

Direct searches

Interferometers [Nishizawa et al, Akutsu et al, Chou et al]

Cavity experiments measuring **rotation of polarization** [Cruise & Ingley]

Experiments measuring **resonant spin precession** of electrons [Ito et al]

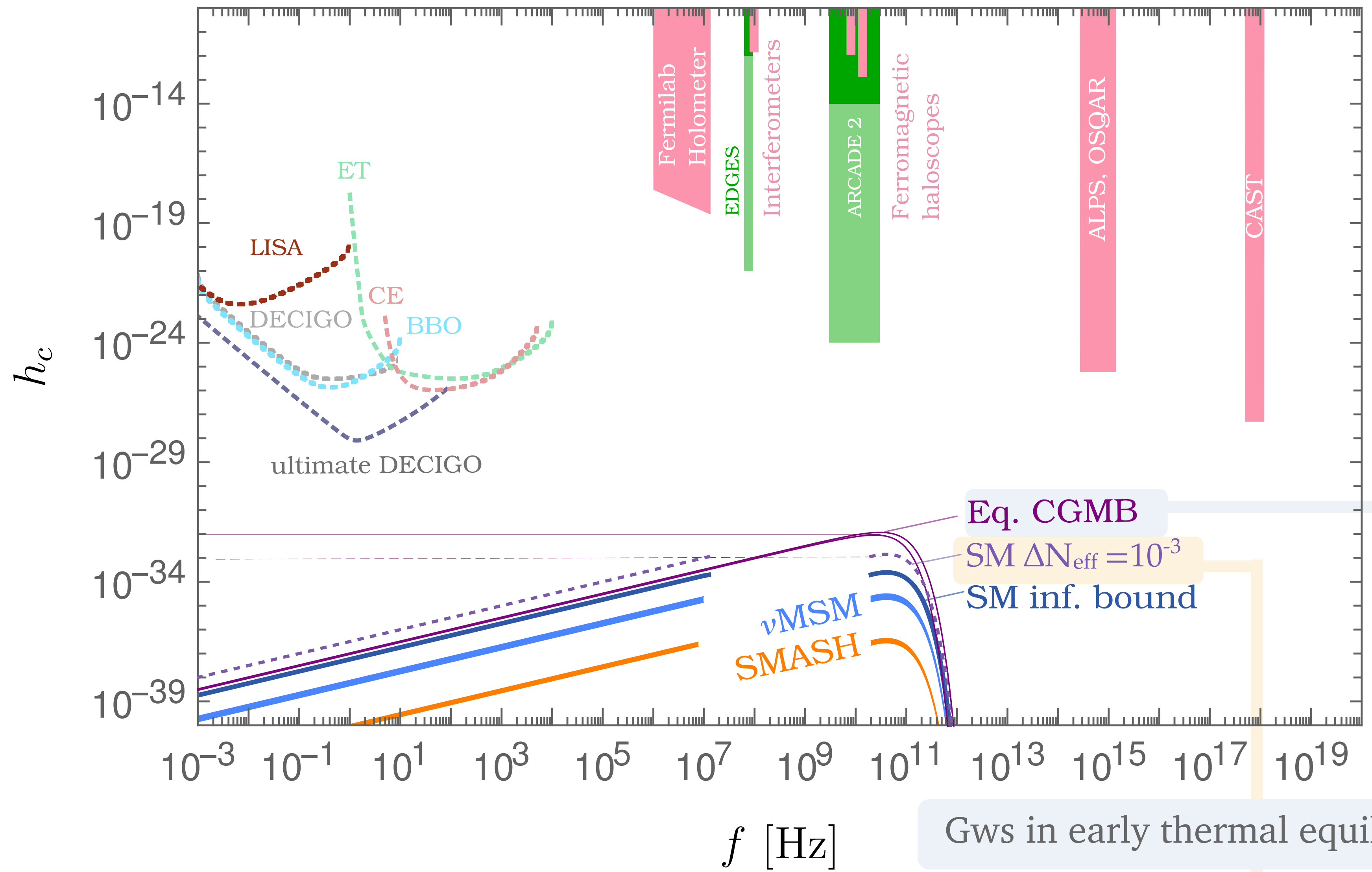
Inverse Gertsenshtein effect: Partial conversion of gravitational waves into electromagnetic radiation

Can use **axion experiments** featuring photon-axion conversion! [Cruise, Ellji et al]

Indirect searches

Gertsenshtein effect can modify the **CMB** spectrum! [Domcke & Cely]

Current direct and indirect bounds



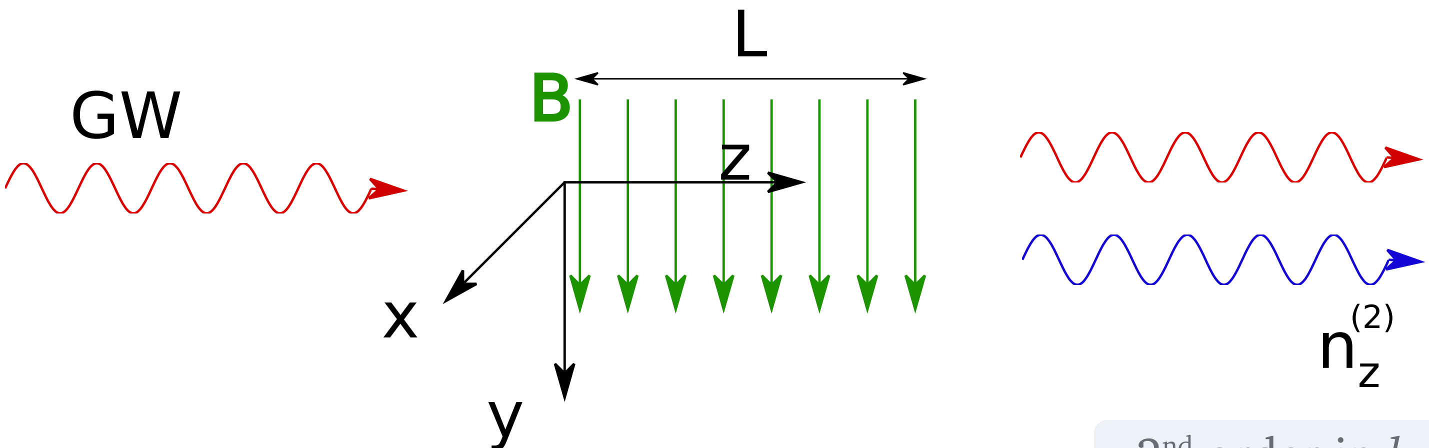
Gws in early thermal equilibrium

Future dark radiation limit

Future experimental prospects

Magnetic GW-EMW conversion in vacuum

Gertsenshtein effect



2^{nd} order in h_c

Average power at terminal position of magnetic field (assuming coherence)

$$f\,\frac{\mathrm{d}P^{(2)}_{\mathrm{EMW}}}{\mathrm{d}f} \simeq \pi^2\, f^2\, h_c^2(f)\, B^2\, L^2\, A = 4.20\times 10^{-23}\,\mathrm{W}\left[\frac{f}{40\,\mathrm{GHz}}\right]^2 \left[\frac{h_c(f)}{10^{-21}}\right]^2 \left[\frac{B}{\mathrm{T}}\right]^2 \left[\frac{L}{\mathrm{m}}\right]^2 \left[\frac{A}{\mathrm{m}^2}\right].$$

If B field surrounded by tube, **coherence** safe from waveguide effect for

$$f \gg f_c \equiv \frac{c_{11}}{\pi^2}\frac{L}{d^2} \simeq 5.5\times 10^7\,\mathrm{Hz}\left[\frac{L}{\mathrm{m}}\right]\left[\frac{\mathrm{m}}{d}\right]^2$$

Heterodyne/single photon detectors

Sensitivity of Heterodyne radio wave receiver

$$\left[h_c^{\mathrm{CGMB}}\right]^{\mathrm{HET}}_{\mathrm{sens}} \simeq 9.65 \times 10^{-21} \left[\frac{\mathrm{S/N}}{2}\right]^{1/2} \left[\frac{\Delta t}{\pi \times 10^7\ \mathrm{s}}\right]^{-1/4} \left[\frac{f}{40\ \mathrm{GHz}}\right]^{-3/4} \left[\frac{\Delta f}{f}\right]^{-1/4} \times$$
$$\times K_{\mathrm{rec}}^{1/2} \left[\frac{T_{\mathrm{sys}}}{4\,\mathrm{K}}\right]^{1/2} \left[\frac{B}{\mathrm{T}}\right]^{-1} \left[\frac{L}{\mathrm{m}}\right]^{-1} \left[\frac{A}{\mathrm{m}^2}\right]^{-1/2} .$$

Factor order 1

System noise temperature

Sensitivity of single photon detectors

$$\left[h_c^{\mathrm{CGMB}}\right]^{\mathrm{SPD}}_{\mathrm{sens}} \simeq 7.71 \times 10^{-24} \left[\frac{\mathrm{S/N}}{2}\right]^{1/2} \left[\frac{\Delta t}{\pi \times 10^7\ \mathrm{s}}\right]^{-1/4} \left(\frac{\Delta \omega}{10^{-4}\,\mathrm{eV}}\right)^{-1/2} \times$$
$$\times \epsilon^{-1/2} \left[\frac{\Gamma_{\mathrm{D}}}{10^{-3}\,\mathrm{Hz}}\right]^{1/4} \left[\frac{B}{\mathrm{T}}\right]^{-1} \left[\frac{L}{\mathrm{m}}\right]^{-1} \left[\frac{A}{\mathrm{m}^2}\right]^{-1/2} .$$

Photon detection efficiency

Dark detection rate

Heterodyne/single photon detectors

BLA$^{1/2}$ sensitivty shared by **light-shining-through wall** experiments with optical cavities, **helioscopes** searching for magnetic conversion of axions

	B [T]	L [m]	d [m]	$\mathrm{n_{tubes}}$	$BLA^{1/2}$	f_c [Hz]	$[h_c^{\mathrm{CGMB}}]^{\mathrm{HET}}_{\mathrm{sens}}$	$[h_c^{\mathrm{CGMB}}]^{\mathrm{SPD}}_{\mathrm{sens}}$
ALPS IIc	5.3	211	0.05	1	49.6 Tm^2	4.6×10^{12}	–	–
BabyIAXO	2.5	10	0.7	2	21.9 Tm^2	1.1×10^{9}	4.41×10^{-22}	3.52×10^{-25}
MADMAX	4.83	6	1.25	1	32.1 Tm^2	1.9×10^{8}	3.01×10^{-22}	2.40×10^{-25}
IAXO	2.5	20	0.7	8	87.7 Tm^2	2.2×10^{9}	1.10×10^{-22}	8.79×10^{-26}

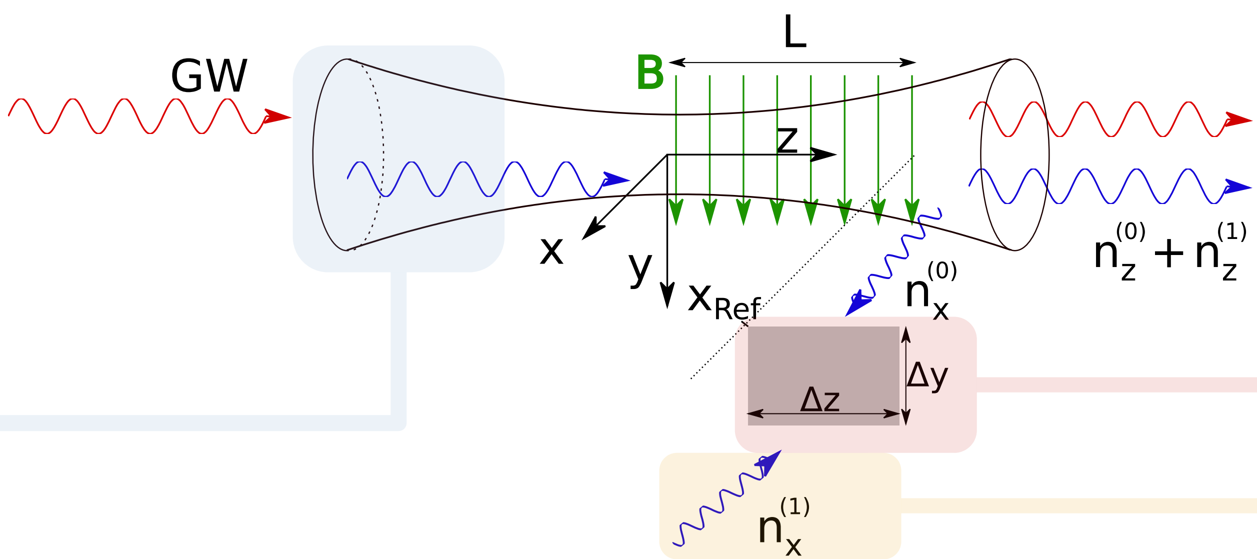
ALPS cannot access CGMB due to f_c above CGMB peak

Pretty hopeless with respect to:

CGMB with **early time equilibration** $h_c \sim 10^{-32}$
future **dark radiation** constraints $h_c \sim 10^{-33}$

GW-EMW conversion with Gaußian beam

[Li et Al] Gaußian beam proposal:



Powerful transverse **Gaußian beam**, polarized in x direction, tuned to GW freq.

Orthogonal photon flux, **first-order in h_c**!

Reflector

Pay the price of increased noise floor due to large EM fields

GW-EMW conversion with Gaußian beam

[Woods et al] argue that **main noise source** can be **dark count rate** of SPDs.

$$\left[h_c^{\rm CGMB}\right]^{\rm GB}_{\rm sens} \simeq 4.02\times 10^{-29}\,\eta^{-1}\left[\frac{\rm S/N}{2}\right]\left[\frac{\Delta t}{10^4\,s}\right]^{-1/2}\left[\frac{\frac{\Delta f_0}{f_0}}{10^{-6}}\right]^{-1}\times$$
$$\times\,\epsilon^{-1}\left[\frac{\Gamma_{\rm D}}{10^{-3}\,{\rm Hz}}\right]^{1/2}\left[\frac{E_0}{5\times 10^5\,{\rm V/m}}\right]^{-1}\left[\frac{B_y^{(0)}}{10\,{\rm T}}\right]^{-1}\left[\frac{L}{5\,{\rm m}}\right]^{-1}\left[\frac{\Delta y\Delta z}{0.01\,{\rm m}^2}\right]^{-1}\left[\frac{\mathcal{F}_x^{(1)}(x_{\rm Ref})}{10^{-5}}\right]^{-1}.$$

$0<\eta<1$ reflectivity of reflector

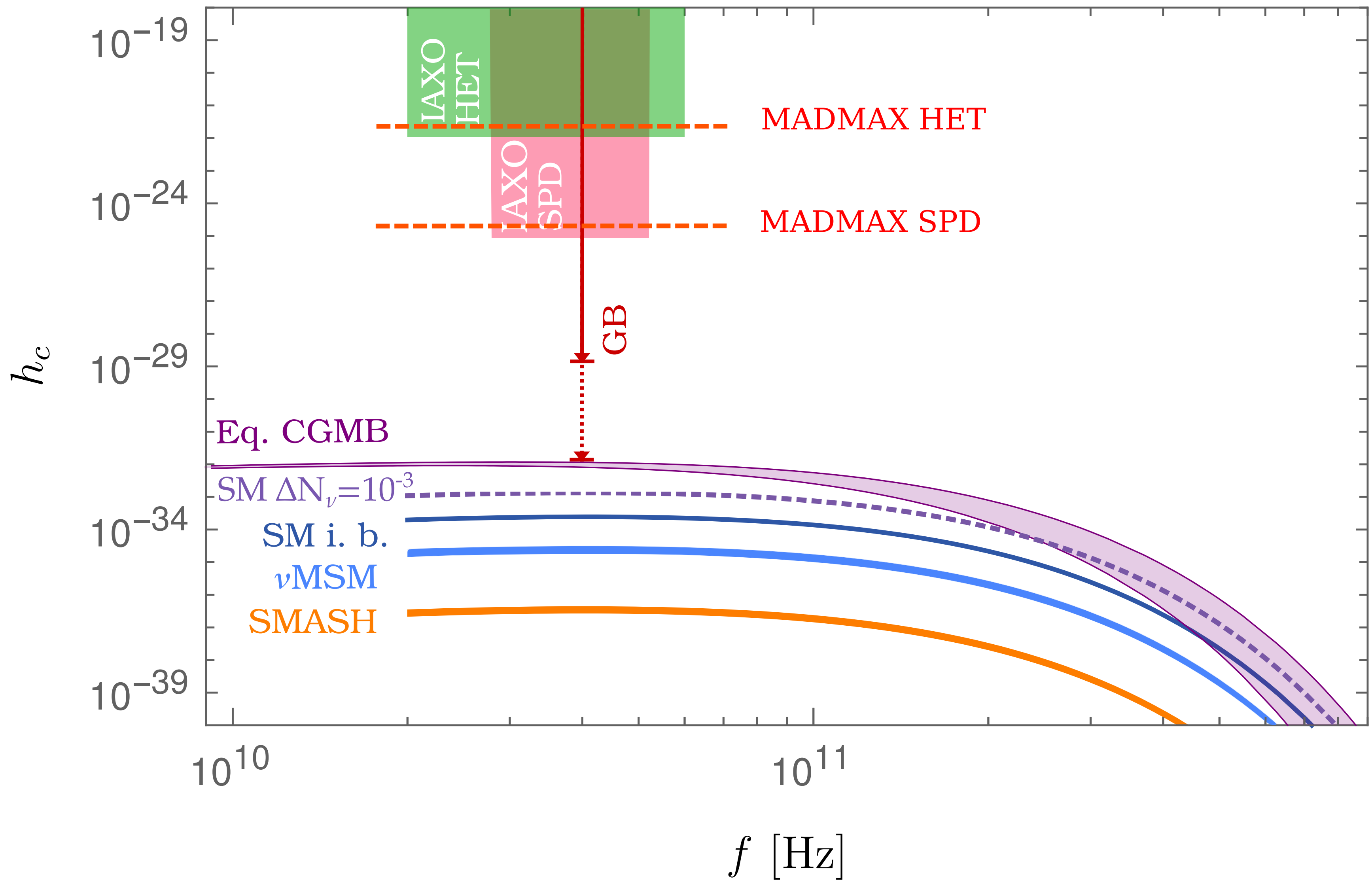
Quantity related to reflected flux reaching mirror

$10^3\times(h_c$ with early time equilibration)

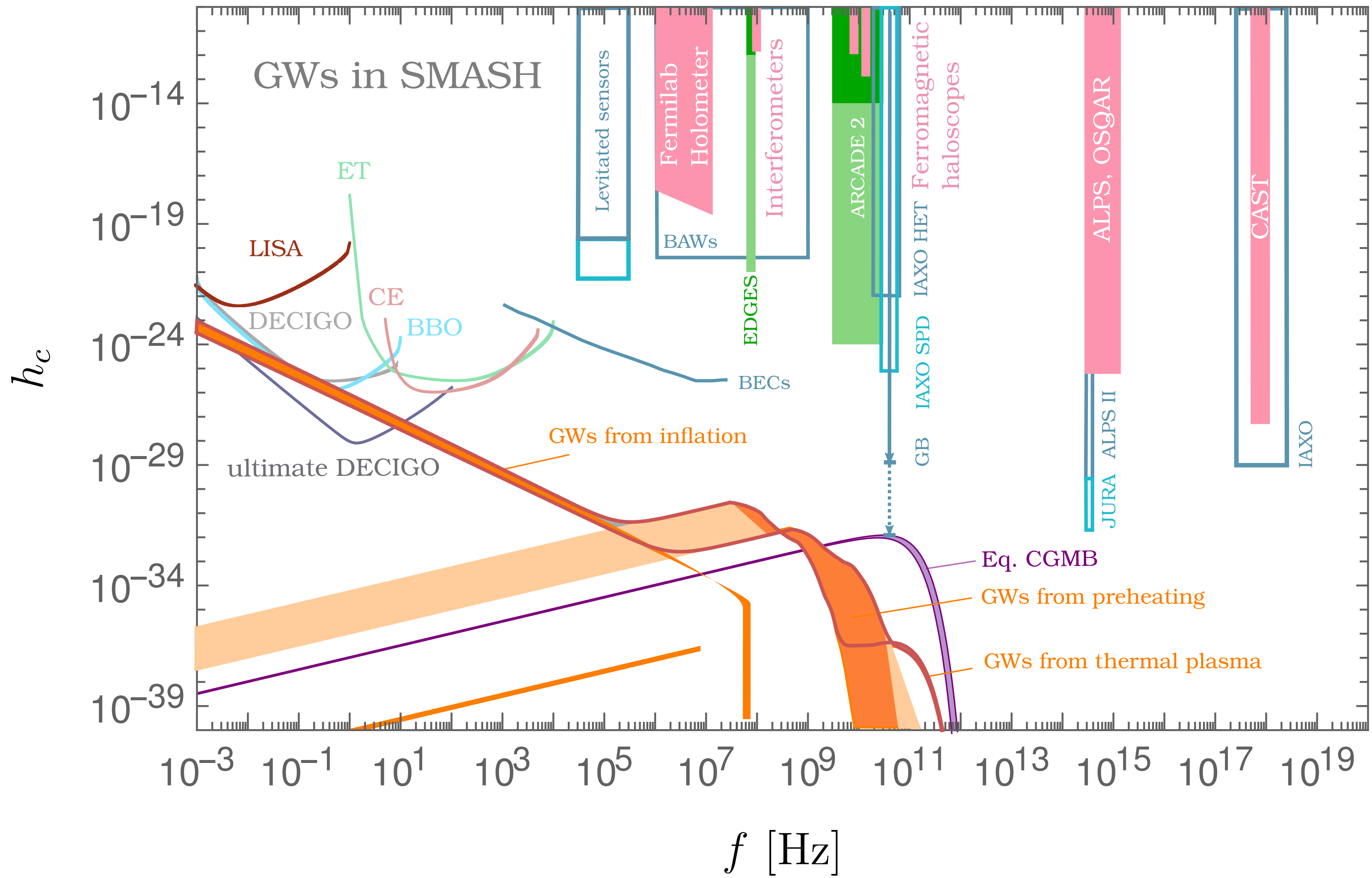
Possible 3 orders of mangitude increase from:

100× Higher power of gyrotron
100× stable running time
100× better dark count rate

GW-EMW conversion with Gaußian beam



The wider context



Conclusions

There is a **guaranteed background** of gravitational waves of thermal origin, **peaking in the microwave region**: the CGMB

The **peak value** and **frequency** are directly **related** to $T_{\rm max},\ g_\star(T_{\rm max})$

Current bounds are many **orders of magnitude above CGMB with early time equilibration** ($T_{\rm max}\sim M_P$) or reaching dark radiation limit $\Delta N_{\rm eff}\sim 10^{-3}$

Gertsenshtein effect of GW/EM wave conversion can be **exploited in axion experiments** using new HET/SPD detectors

Exploiting Gaußian beams with improved power and stability, with more efficient SPDs, could take us near **CGMB with early time equilibration**

Thank you!