Gravitomagnetic resonance and gravitational waves

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Outline

★ Description of the GW field

★ The role of Fermi Coordinates

★ GWs in Fermi coordinates: gravitoelectric and gravitomagnetic forces

★ Gravitomagnetic Resonance

GWs as solutions of Einstein field equations

Looking for solutions of Einstein equations in vacuum in weak field approximation

$$\Box \bar{h}_{\mu\nu} = 0. \quad \Rightarrow \quad \bar{h}_{\mu\nu} = -\left(h^{+} e^{+}_{\mu\nu} + h^{\times} e^{\times}_{\mu\nu}\right)$$

GW propagating along the x axis

$$h^{+} = A^{+} \sin\left(\omega t - kx\right), \quad h^{\times} = A^{\times} \cos\left(\omega t - kx\right)$$

$$e^{+}_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad e^{\times}_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Polarization tensors

TT Gauge

GWs as solutions of Einstein field equations

Line element: $$ds^2 = -c^2dt^2 + dx^2 + (1 - h^+)dy^2 + (1 + h^+)dz^2 - 2h_\times dydz$$

Geodesics: $$x^\alpha(\tau) = \delta_0^\alpha \tau + \text{constants}$$

An inertial mass initially at rest does not change its coordinates in the field of the GW in TT gauge

But the physical distance changes with time

$$\ell = \Delta y \left(1 + \frac{h_+}{2}\right)$$

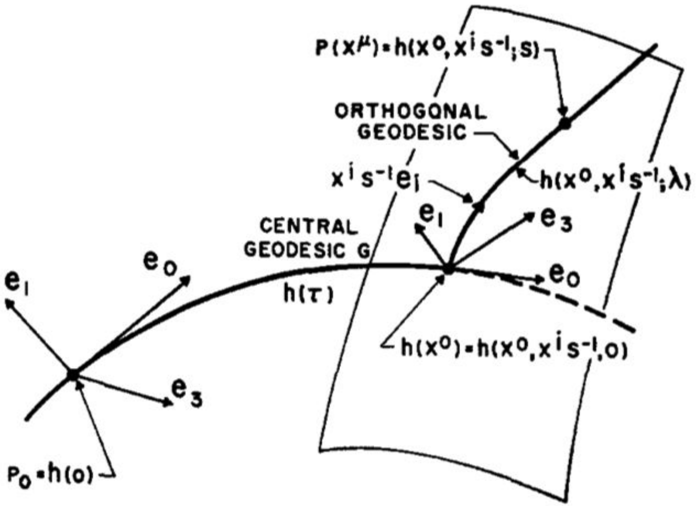
$$R_{ij0l}(x) = \frac{1}{2}\left(\partial_0\partial_j h_{il} - \partial_0\partial_i h_{jl}\right).$$

Geodesics deviation due to Riemann!

Fermi coordinates

In the tangent space along the observer's world-line we consider an orthonormal tetrad. The time coordinate is the proper time, while the space coordinates are defined using space-like geodesics.

These coordinates are determined by the observer's world-line and by the background space-time where he is moving



from Misner, Manasse (1963)

Inertial effects

$$ds^2 = -\left[\left(1+\frac{\mathbf{a}\cdot\mathbf{X}}{c^2}\right)^2 - \frac{1}{c^2}\left(\mathbf{\Omega}\wedge\mathbf{X}\right)^2 + R_{0i0j}X^iX^j\right]c^2dT^2 + \left[\frac{1}{c}\left(\mathbf{\Omega}\wedge\mathbf{X}\right)_i - \frac{4}{3}R_{0jik}X^jX^k\right]cdTdX^i +$$
$$+\left(\delta_{ij} - \frac{1}{3}R_{ikjl}X^kX^l\right)dX^idX^j.$$

curvature effects

Gravitoelectromagnetic formalism

$$ds^2 = -\left(1 - 2\frac{\Phi}{c^2}\right) c^2 dT^2 - \frac{4}{c}(\mathbf{A} \cdot d\mathbf{X}) dt + \delta_{ij} dX^i dX^j .$$

Gravitoelectromagnetic potentials $(\Phi, \mathbf{A})$ **Inertial and curvature effects**

$$\Phi(T, \mathbf{X}) = \Phi^I(\mathbf{X}) + \Phi^C(T, \mathbf{X}), \quad , \quad \mathbf{A}(T, \mathbf{X}) = \mathbf{A}^I(\mathbf{X}) + \mathbf{A}^C(T, \mathbf{X}),$$

$$\Phi^I(\mathbf{X}) = -\mathbf{a} \cdot \mathbf{X} - \frac{1}{2}\frac{(\mathbf{a} \cdot \mathbf{X})^2}{c^2} + \frac{1}{2}\left[|\mathbf{\Omega}|^2 |\mathbf{X}|^2 - (\mathbf{\Omega} \cdot \mathbf{X})^2\right] \qquad \Phi^C(T, \mathbf{X}) = -\frac{1}{2} R_{0i0j}(T) X^i X^j$$

$$A_i^I(\mathbf{X}) = -\left(\frac{\mathbf{\Omega} c}{2} \wedge \mathbf{X}\right)_i \qquad A_i^C(T, \mathbf{X}) = \frac{1}{3} R_{0jik}(T) X^j X^k$$

Gravitoelectromagnetic formalism

Gravitoelectromagnetic fields

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{2}{c}\frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \boldsymbol{\nabla} \wedge \mathbf{A}$$

The motion of free test masses *relative* to a reference mass, at rest at origin of the Fermi frame is determined by the geodesics which can be written in the form of a Lorentz-like force equation

$$m\frac{\mathrm{d}^2\mathbf{X}}{\mathrm{d}T^2} = -m\mathbf{E} - 2m\frac{\mathbf{V}}{c} \times \mathbf{B}.$$

q_E=-m and q_B=-2m are the gravitoelectric and gravitomagnetic charge

E.g. Motion in non inertial frames if We consider only the inertial contributions

$$\frac{\mathrm{d}^2\mathbf{X}}{\mathrm{d}T^2} = -\mathbf{a}\left(1 + \frac{\mathbf{a}\cdot\mathbf{X}}{c^2}\right) - \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{X}) - 2\boldsymbol{\Omega} \wedge \mathbf{V}$$

GWs in the Gravitoelectromagnetic formalism

$$E_X = 0, \quad E_Y = -\frac{\omega^2}{2}\left[A^+ \sin(\omega T) Y + A^\times \cos(\omega T) Z\right], \quad E_Z = -\frac{\omega^2}{2}\left[A^\times \cos(\omega T) Y - A^+ \sin(\omega T) Z\right].$$

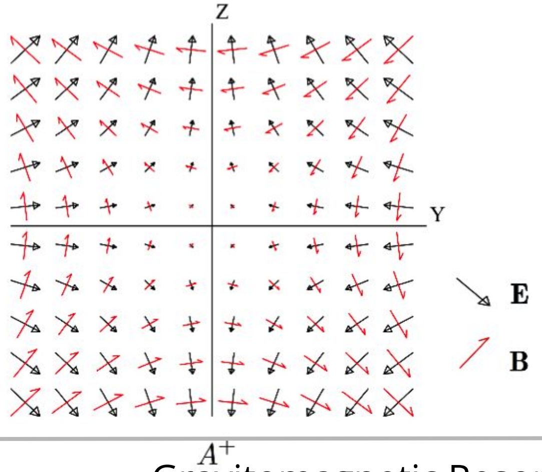
$$B_X = 0, \quad B_Y = -\frac{\omega^2}{2}\left[-A^\times \cos(\omega T) Y + A^+ \sin(\omega T) Z\right], \quad B_Z = -\frac{\omega^2}{2}\left[A^+ \sin(\omega T) Y + A^\times \cos(\omega T) Z\right]$$

Gravitoelectromagnetic fields associated to a plane GW wave (tidal fields)

The fields are transverse to the propagation direction and orthogonal to each other

$$\mathbf{E} \cdot \mathbf{B} = 0$$

$$|\mathbf{E}|^2 - |\mathbf{B}|^2 = 0.$$



GWs in the Gravitoelectromagnetic formalism

$$m\frac{\mathrm{d}^2\mathbf{X}}{\mathrm{d}T^2} = -m\mathbf{E} - 2m\frac{\mathbf{V}}{c}\times\mathbf{B}$$

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T} = \frac{1}{c}\mathbf{B}\times\mathbf{S}$$

Gravitoelectric effects (interferometers)

Gravitomagnetic effects (new!)

Let the location of the mass before the passage of the wave be $\mathbf{X}_0 = (0, L, 0)$

The distance between the reference mass at the origin and the test mass changes with time:

$$Y(T) = L\left[1 - \frac{A^+}{2}\sin(\omega T)\right]$$

Gravitomagnetic Resonance

In a rotating Fermi frame with a given rotation rate Ω, the total gravitomagnetic field is

$$\mathbf{B}' = \mathbf{B}^I + \mathbf{B}^C, \text{ where } \mathbf{B}^I = -\mathbf{\Omega} c$$

If **Ω** is along the propagation direction of the Gw the spin evolution equation is

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T} = \frac{1}{c}\left[\mathbf{B}^C(T) + \mathbf{B}^I\right] \times \mathbf{S} \quad \Longrightarrow \quad \left(\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T}\right)_{\mathrm{rot}} = \left[\Delta\omega \mathbf{u}_{X'} + \frac{1}{c}\mathbf{B}^C\right] \times \mathbf{S} = \frac{1}{c}\mathbf{B}_{eff} \times \mathbf{S}$$

$$\omega - \frac{1}{c}B^I = \omega - \Omega = \Delta\omega$$

In the rotating frame spin precesses around the B_{eff} field.

In resonance condition (the rotation rate Ω is equal to the wave frequency ω) the passage of the wave completely flips the spin.

GRAVITOMAGNETIC RESONANCE

Gravitomagnetic Resonance

Probability transition for a 2-level system

$$P_{g\to e}(T) = \frac{(\omega^*)^2}{(\omega^*)^2 + \Delta\omega^2} \sin^2\left(\sqrt{(\omega^*)^2 + \Delta\omega^2}\frac{T}{2}\right)$$

GRAVITOMAGNETIC RESONANCE

When Δω =0 the probability of transition is equal to 1 independently of the strength of the gravito-magnetic field

$$T = \frac{2n+1}{(\omega^*)}\pi \qquad \frac{1}{c}B^C = \omega^*$$

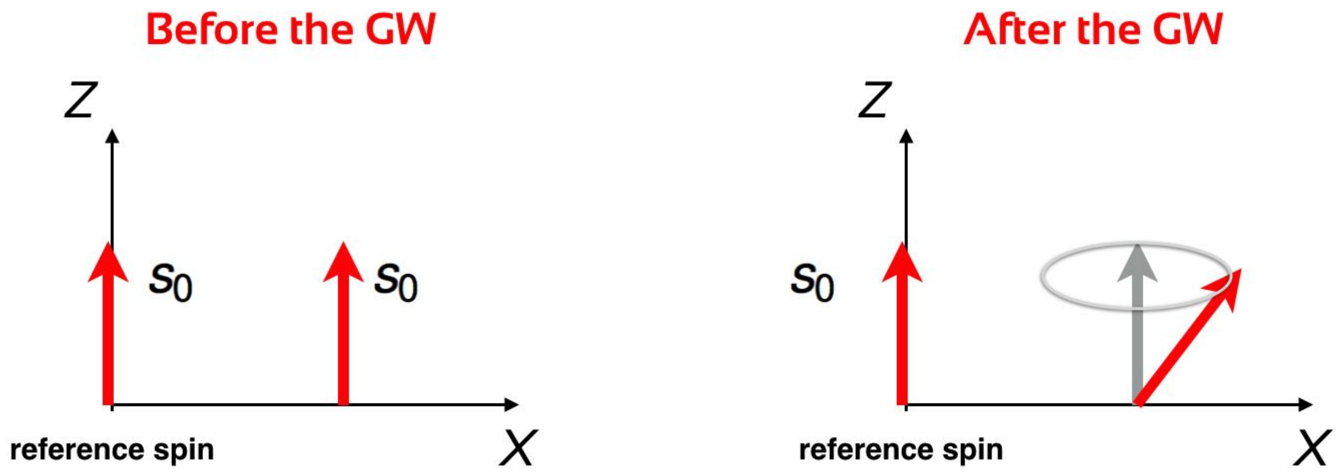
The time required is inversely proportional to the gravitomagnetic field of the wave

High Frequency GWs are ideal candidates

LARMOR THEOREM

Equivalence between a system of electric charges in a magnetic field, and the same system rotating with the Larmor frequency

Gravitomagnetic Resonance



$$\frac{\Delta S}{S} \rightarrow \frac{\Delta M}{M} \rightarrow \frac{\Delta B}{B} \simeq A$$

GW amplitude

→ **The passage of Gws can modify the magnetization of a sample (detection)**

Conclusions

- Describing GWs in Fermi coordinates shows their gravitomagnetic components
- Current detectors are not sensitive to the gravitomagnetic part of GWs
- Spinning particles could be used as a probe to measure gravitomagnetic effects due to GWs
- High Frequency GWs are ideal candidates

More... in these publications

- Ruggiero, Matteo Luca, and Antonello Ortolan. "Gravitomagnetic resonance in the field of a gravitational wave." *Physical Review D* 102.10 (2020): 101501.
- Ruggiero, Matteo Luca, and Antonello Ortolan. "Gravito-electromagnetic approach for the space-time of a plane gravitational wave." *Journal of Physics Communications* 4.5 (2020): 055013.
- Ruggiero, Matteo Luca. "Gravitational waves physics using Fermi coordinates: a new teaching perspective." *arXiv preprint arXiv:2101.06746* (2021) (American Journal of Physics 89, 639 (2021)
- Iorio, Lorenzo and Ruggiero, Matteo Luca. "Perturbations of the orbital elements due to the magnetic-like part of the field of a plane gravitational wave" *arXiv preprint arXiv:2106.14462* (2021) (*to appear in IJMPD*)