Gegenbauer Goldstones

and the small Higgs vev

Gauthier Durieux
(CERN)

[2110.06941]
with Matthew McCullough and Ennio Salvioni

Joint Israeli Particle Seminar, 17 Nov 2021

The Higgs is a mystery.

Why does it look lighter than the SM cutoff?

Why do its couplings look SM-like?

Small mass

Global spontaneous symmetry breaking leads to massless scalars, Nambu-Goldstone bosons (NGBs).

Small explicit symmetry breakings lead to small masses.
NGBs become pNGBs.

e.g. pions much lighter than Λ_{QCD}:

SSB: $SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_{L+R}$
ESB: quark masses/charges

Composite Higgs

Another strong-sector confinement triggers a global SSB at a scale f set by dimensional transmutation.

Small explicit breakings give NGB a potential, including a mass and a EWSB vev.

The Higgs is realized as a pNGB.

SM-like couplings

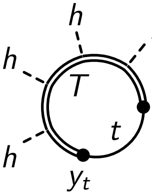
Higgs coupling modifications are controlled by v^2/f^2,
and naturally of order 1 in vanilla pNGB Higgs models.

Fine-tuning $v^2/f^2 \ll 1$ is then required,
but is that *generic* or *specific* to vanilla models?

Vanilla composite Higgs

Minimal SO(5) → SO(4) spontaneous breaking

Minimal explicit breaking from top and gauge sectors

$V(h) \sim$ 

$y_t + \cdots \sim \kappa \, \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2 \frac{h}{f} + \delta \, \sin^4 \frac{h}{f} \right)$

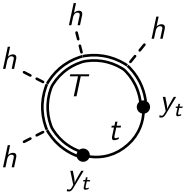
$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \qquad \text{vs.} \qquad \frac{c_{hVV}}{c_{hVV}^{\rm SM}} = \sqrt{1 - \frac{v^2}{f^2}} = 1 \pm 6\%$$

$$\rightarrow m_h^2 = \frac{y_t^2 N_c}{16\pi^2} M_T^2 \; 4\kappa \left(1 - \frac{1}{2\delta} \right) \qquad \text{vs.} \qquad M_T \gtrsim 1.3\,\text{TeV}$$

Vanilla composite Higgs

Minimal SO(5) $\to$ SO(4) spontaneous breaking

Minimal explicit breaking from top and gauge sectors

$V(h) \sim$ 

$+ \cdots \sim \kappa \, \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2\frac{h}{f} + \delta\, \sin^4\frac{h}{f} \right)$

$\to \frac{v^2}{f^2} = \sin^2\frac{\langle h\rangle}{f} = \frac{1}{2\delta}$ $\quad$ $1/\delta \lesssim 0.2$ $\quad$ $\frac{g_{hVV}}{g_{hVV}^{SM}} = \sqrt{1-\frac{v^2}{f^2}} = 1 \pm 6\%$

$\to m_h^2 = \frac{y_t^2 N_c}{16\pi^2} M_T^2 \, 4\kappa \left(1 - \frac{1}{2\delta}\right)$ $\quad$ $\kappa \lesssim 0.1$ $\quad$ $M_T \gtrsim 1.3$ TeV

Percent tuning wrt $\kappa \sim 1, \delta \lesssim 1$ expectation

Natural $v/f \ll 1$ receipe

not addressing m_h^2 aka κ tuning!

New source of explicit breaking,

radiatively stable,

with structure at small field values.

pNGB potentials

Abelian example

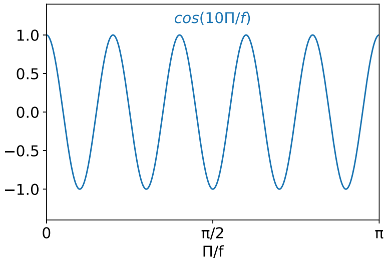
Spontaneously broken global U(1)

One Nambu-Goldstone boson Π
Non-linear field parameterisation: $\phi = e^{i\Pi/f}$

Small explicit breaking by a spurion K of charge $nQ_{\bar{\phi}}$:

$$V \sim K\,\phi^n \sim \cos\frac{n\Pi}{f}$$

Radiatively stable structure on $\frac{\Pi}{f} \sim \frac{1}{n}$ scale



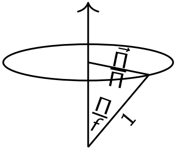
Non-Abelian case

Spontaneously broken global $\mathrm{SO}(N+1) \to \mathrm{SO}(N)$

Unspecified strong sector in the UV

N Nambu-Goldstone bosons $\vec{\Pi}$ in the low-energy EFT

Field parameterisation: $\phi = (\frac{\vec{\Pi}}{\Pi} \sin\frac{\Pi}{f},\ \cos\frac{\Pi}{f})$
with $\Pi \equiv |\vec{\Pi}|$



Potential

Explicit $\mathrm{SO}(N+1) \to \mathrm{SO}(N)$ breaking

$$V(\Pi) = \epsilon\, M^2 f^2\, G(\Pi/f)$$

with a dimensionless function G
and ϵ small

One-loop stability

Quadratic divergence (linear in ϵ) as diagnosis tool
from within the pNGB EFT

$$\delta V_{\text{1-loop}}^{\text{order }\epsilon} = \epsilon M^2 \frac{\Lambda^2}{32\pi^2} \left(G'' + (N-1) \cot \frac{\Pi}{f}\ G' \right)$$

One-loop stability

Quadratic divergence (linear in ϵ) as diagnosis tool
from within the pNGB EFT

$$\delta V_{\text{1-loop}}^{\text{order }\epsilon} = \epsilon M^2 \frac{\Lambda^2}{32\pi^2} \left(\underline{G'' + (N-1)\cot\frac{\Pi}{f}\, G'} \right)$$

Radiative stability at one-loop and linear ϵ order if $\propto G$

Gegenbauer polynomials $G(\frac{\Pi}{f}) = G_n^{(N-1)/2}(\cos\frac{\Pi}{f})$
satisfy exactly this differential eq!

All-loop stability at order ϵ

Explicit $\mathrm{SO}(N+1) \to \mathrm{SO}(N)$ breaking by an irrep spurion

$$K^{i_1 \dots i_n}\, \phi_{i_1} \cdots \phi_{i_n} \qquad \text{(symmetric traceless)}$$

No other invariant, linear in K, can be constructed,
so all-loop linear renormalization can only be multiplicative.

All-loop stability at order ϵ

Explicit $\mathrm{SO}(N+1) \to \mathrm{SO}(N)$ breaking by an irrep spurion

$$K^{i_1 \dots i_n}\, \phi_{i_1} \cdots \phi_{i_n} \qquad \text{(symmetric traceless)}$$

No other invariant, linear in K, can be constructed,
so all-loop linear renormalization can only be multiplicative.

And $\quad K^{i_1 \dots i_n}\, \phi_{i_1} \cdots \phi_{i_n} \;=\; G_n^{(N-1)/2}(\cos \frac{\Pi}{f})\;$!

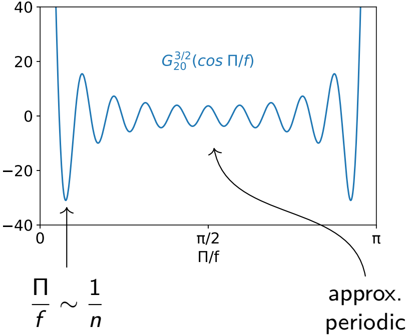
$$\phi = (\tfrac{\vec{\Pi}}{\Pi} \sin \tfrac{\Pi}{f},\ \cos \tfrac{\Pi}{f})$$

Gegenbauer polynomials

Eigenfunctions of linear renormalization
for $SO(N+1) \to SO(N)$ pNGB potentials.†

Generalisation of $\cos(n\Pi/f)$ for $N > 1$
of Legendre polynomials for $N > 2$
i.e. multipole expansion in *field* space

L.B. Gegenbauer
1849–1903



$\frac{\Pi}{f} \sim \frac{1}{n}$

for positive coefficient and n even

†Brézin, Zinn-Justin, Le Guillou '76: RG eigenfunctions in 2D

Gegenbauer Higgs

Pure Gegenbauer potential

minimal composite Higgs with $N = 4$

$$\Pi = h, \quad m_h = 125\,\text{GeV}, \quad v = 246\,\text{GeV}$$

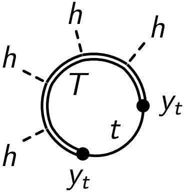
$\frac{v}{f} = \sin\frac{\langle h\rangle}{f} \approx \frac{5.1}{n}$ is naturally small for sizeable n

→ small Higgs coupling modifications: $\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \sqrt{1 - \frac{v^2}{f^2}}$

→ opposite trilinear self-coupling: $\frac{c_{hhh}}{c_{hhh}^{\text{SM}}} = -\sqrt{1 - \frac{v^2}{f^2}}$

Leading top-sector contribution

The top sector provides sizeable explicit breaking.



Leading contribution to potential of the form

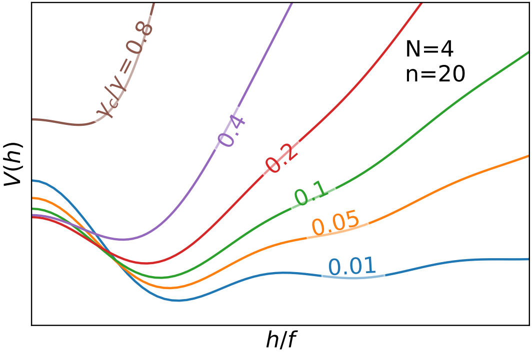
$$V_{\text{top}}(h) = \kappa \, \frac{N_c y_t^2}{16\pi^2} \, f^2 M_T^2 \, \left(+ \sin^2 \frac{h}{f} \right)$$

positive sign now

$$V_{\text{top}}^{\mathbf{5}_L+\mathbf{5}_R} \propto \frac{N_c}{8\pi^2}(y_L^2 - 2y_R^2) f^2 (M_1^2 - M_4^2) \sin^2 \frac{h}{f}$$

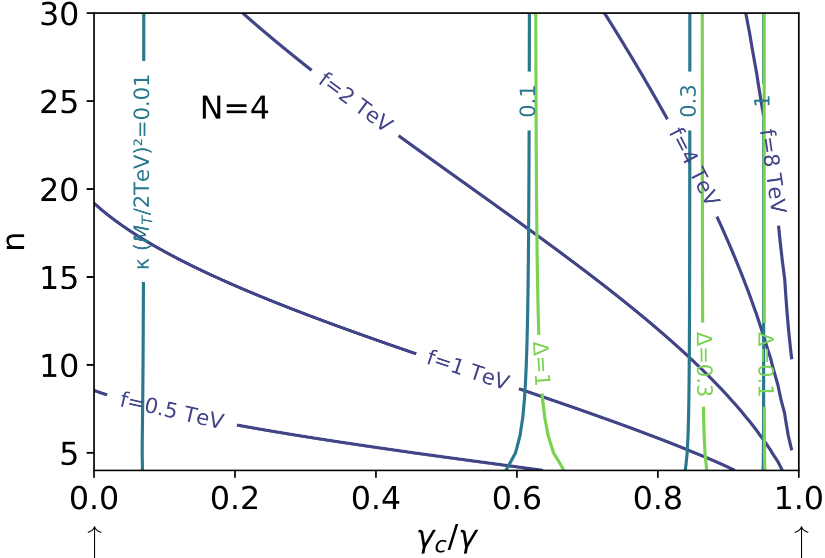
Top+Gegenbauer potential

$$V(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma\, G_n^{3/2}\left(\cos \frac{h}{f} \right) \right]$$



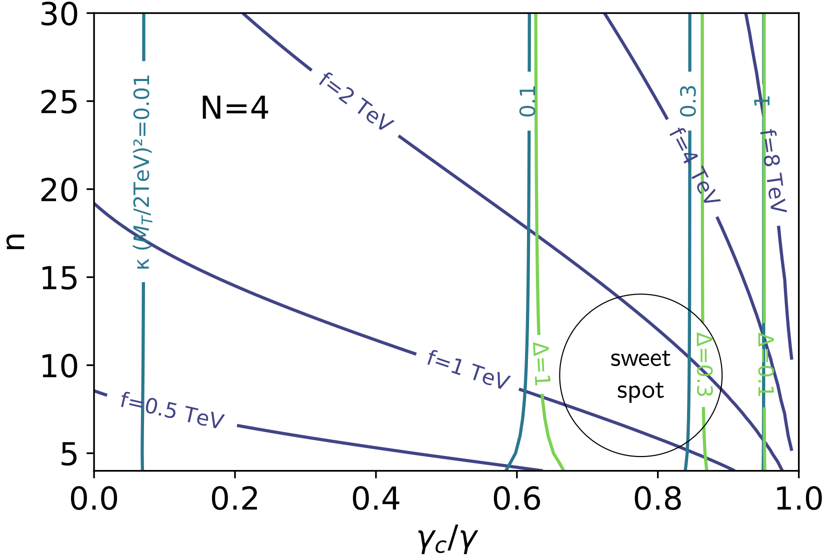
$$v/f \searrow 0 \quad \text{as} \quad \gamma \searrow \gamma_c$$

$$\frac{m_h^2}{\kappa \frac{N_c y_t^2}{16\pi^2} M_T^2} \searrow \quad \text{as} \quad \gamma \searrow \gamma_c \quad \text{relaxing} \quad \kappa \nearrow$$



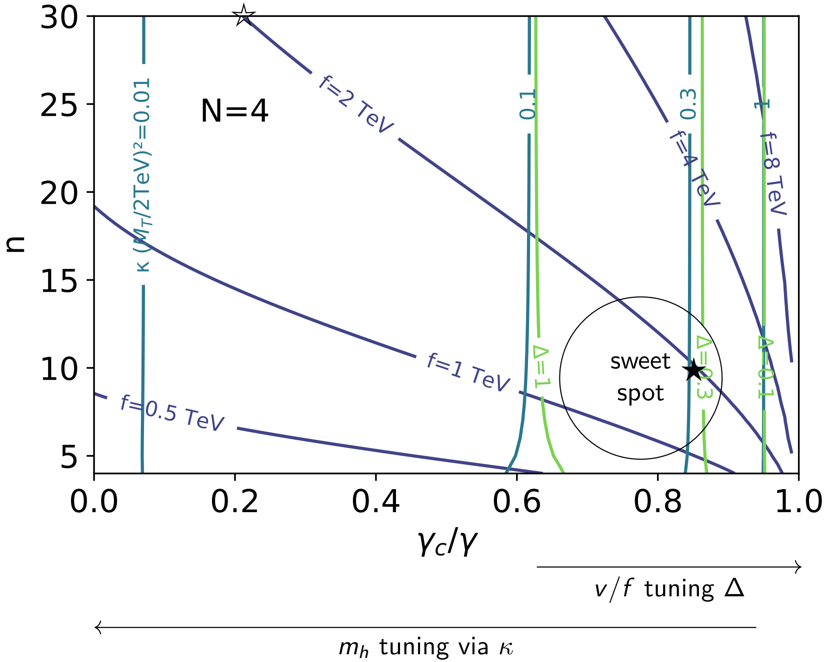
↑ pure Gegenbauer $v/f \approx 5.1/n$

↑ $v/f \to 0$

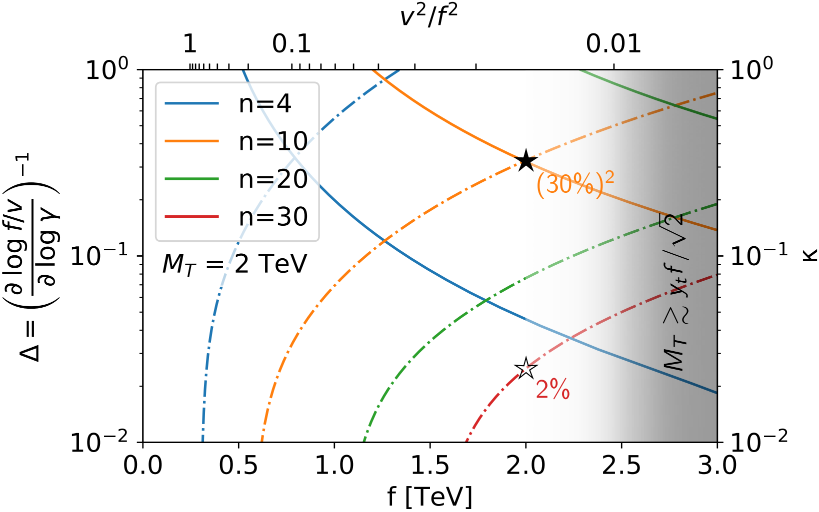


v/f tuning Δ

m_h tuning via κ


N=4
sweet spot
κ (M_T/2TeV)²=0.01
0.1
0.3
1
f=0.5 TeV
f=1 TeV
f=2 TeV
f=4 TeV
f=8 TeV
Δ=1
Δ=0.3
Δ=0.1
n
γ_c/γ
v/f tuning Δ
m_h tuning via κ

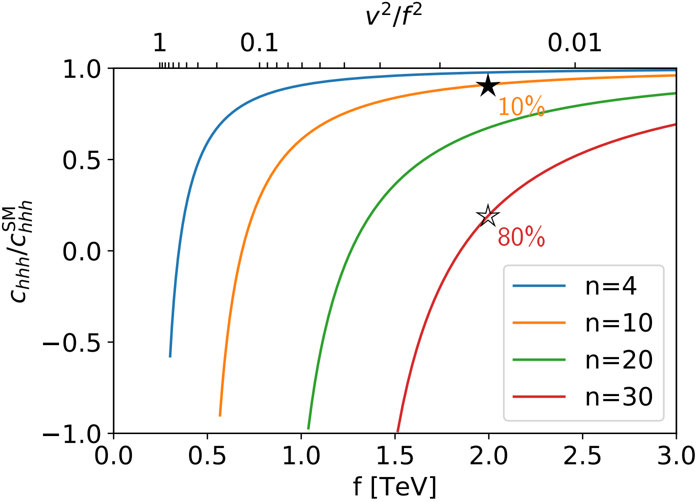
v/f and m_h tunings



$(m_t \sim M_T v/f \quad \text{for} \quad y_t f \gg M_T)$

$$\Delta \approx 30\% \left(\frac{f}{4v}\frac{5.1}{n}\right)^{-2.1} \qquad \kappa \approx 30\% \left(\frac{f}{4v}\frac{5.1}{n}\frac{2\,\text{TeV}}{M_T}\right)^{2}$$

Trilinear self-coupling



$$\frac{c_{hhh}}{c_{hhh}^{SM}} \approx 1 - 1.2\left(\frac{f}{v}\frac{5.1}{n+\lambda}\right)^{-2}$$ when close to 1

Benchmark phenomenology

$n = 10$ and $f \sim M_T \sim 2\,\text{TeV}$

$\rightarrow$ both v/f and m_h tunings $\sim 30\%$, so $\sim 10\%$ total

$\rightarrow$ top partners just escape HL-LHC searches

$\rightarrow$ Higgs coupling modifications $\frac{v^2}{f^2} \lesssim 1\%$ $\lesssim 2.6\%$ @HL-LHC, 2σ

$\rightarrow$ Higgs self-coupling modification $\lesssim 10\%$ $\lesssim 100\%$ @HL-LHC, 2σ

[ECFA '19]

Benchmark phenomenology

$n = 10$ and $f \sim M_T \sim 2\,\text{TeV}$

$\rightarrow$ both v/f and m_h tunings $\sim 30\%$, so $\sim 10\%$ total

$\rightarrow$ top part

Natural composite Higgs
will survive high luminosities!

$\rightarrow$ Hig $f^2 \lesssim 1\%$ $\lesssim 2.6\%$ @HL-LHC, 2σ

$\rightarrow$ Higgs self-coupling modification $\lesssim 10\%$ $\lesssim 100\%$ @HL-LHC, 2σ

[ECFA '19]

Gegenbauer Goldstones

and the small Higgs vev

Gegenbauer potentials are eigenfunctions of linear renorm.
for $SO(N+1) \rightarrow SO(N)$ pNGBs.

They naturally suppress v/f,
resulting in a SM-like pNGB Higgs couplings.

The trilinear Higgs self-coupling would be opposite to the SM,
in the absence of top-sector contributions to the potential.

Vanilla top-sector contributions preserve naturally small v/f,
but bring the trilinear close to the SM.

HL-LHC won't probe all natural pNGB Higgs models!