KKMC-hh Update: Recent IFI Results and Comments on ISR and PDFs

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Contents

1. We show a table of IFI results including the most complete calculations with the current C++ version of KKMC-hh and a semi-analytic implementation of a semi-soft approximation to it, KKhhFoam, which was described in my June talk. The new results are closer to those of other groups, mainly due to some bug fixes in the analysis program that combined the farm runs.

2. We will discuss some issues related to the presence of QED in the PDFs and our present ideas on how the KKMC-hh ISR correction should be interpreted.

KKMC-hh includes soft photon exponentiation to all orders, with exact residuals through order $\alpha^2 L$. (NNLO, NLL).

KKhhFoam is not exact, but accurate when hard photon corrections are relatively small. It is an integrator, not a generator, but can be used for rapidly generating inclusive results. It has the advantage that ISR, IFI, and FSR corrections can be calculated completely independently and transparently.

KKhhFoam: 7-dimensional adaptive MC integral

Let $z \equiv 1 - v, z' \equiv 1 - v', w \equiv 1 - r$ and $w' \equiv 1 - r'$ and define scales

$\hat{s} \equiv x_1 x_2 s$ (quarks before ISR), $\bar{s} \equiv z\hat{s}$, $s' = M_{ll}^2 = zz'ww'\hat{s}$ (leptons after FSR),

the integral over z can be swapped for one over $\bar{s}$ and the s' constraint for one on z' giving

$$\sigma = \sum_q \int ds' \int_{x_1 x_2 \geq s'/s}^{1} dx_1 dx_2 \;\; f_q^{h_1}(x_1, \hat{s}) f_{\bar{q}}^{h_2}(x_2, \hat{s}) \int_{s'/\hat{s}}^{1} dz \; \underbrace{\boxed{\rho(\gamma_I(\hat{s}), z)}}_{\text{ISR}}$$

$$\times \int_{ww' \geq s'/\bar{s}}^{1} \frac{dwdw'}{ww'} \int_{-1}^{1} d\cos\theta \; \overbrace{\boxed{\rho(\gamma_X(\cos\theta), w)\, \rho(\gamma_X(\cos\theta), w')}}^{\text{IFI factors}} \underbrace{\boxed{\rho\left(\gamma_F(s'), \frac{s'}{ww'\bar{s}}\right)}}_{\text{FSR}}$$

$$\times \frac{1}{4} \mathrm{Re} \sum_{\{\lambda\}} e^{\alpha \Delta B_4^V(\bar{s}w)} \mathfrak{M}_{\{\lambda\}}^V(\bar{s}w, \cos\theta) \left[e^{\alpha \Delta B_4^V(\bar{s}w')} \mathfrak{M}_{\{\lambda\}}^{V'}(\bar{s}w', \cos\theta) \right]^*$$

$$\rho(\gamma, z) = \frac{e^{-C_E \gamma}}{\Gamma(1+\gamma)} \gamma (1-z)^{\gamma - 1}, \quad \gamma_I = Q_I^2 \frac{\alpha}{\pi} \left[\ln\left(\frac{\hat{s}}{m_I^2}\right) - 1 \right], \quad \gamma_F = Q_F^2 \frac{\alpha}{\pi} \left[\ln\left(\frac{s'}{m_F^2}\right) - 1 \right], \quad \gamma_X = Q_I Q_F \frac{\alpha}{\pi} \ln\left(\frac{1 - \cos\theta}{1 + \cos\theta}\right)$$

Results Presented in the Following Table

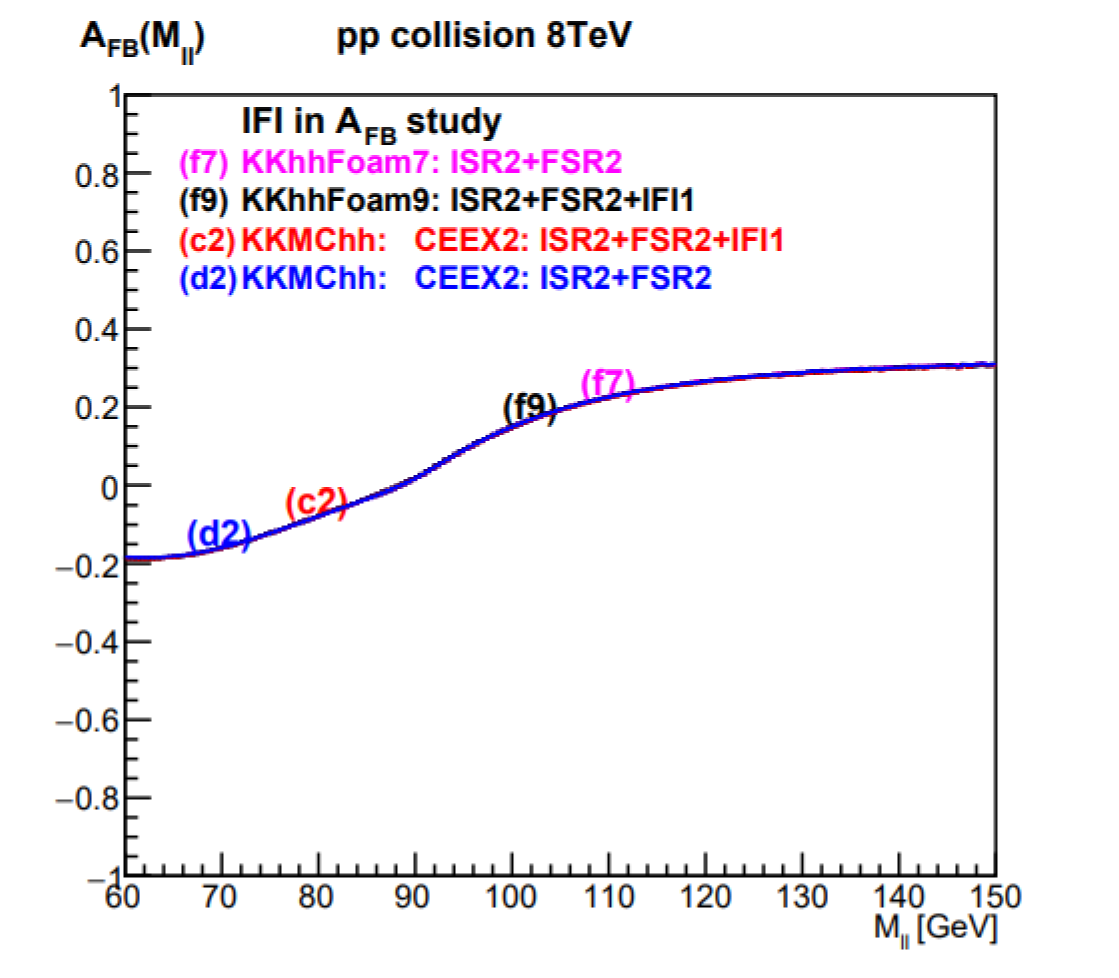
- All KKMChh results are for muon pair final states with proton collisions with $\sqrt{s} = 8000\ \mathrm{GeV}$.
- All other results are from Stefan Dittmaier's summary table, May 2021.
- Our tabulated results all include a dilepton mass cut in all cases:
$$60\ \mathrm{GeV} < M_{ll} < 150\ \mathrm{GeV}.$$
- The following table shows only A_4 calculated from A_{FB} in the full phase space without fermion cuts for 32 billion events.
- After the tables, I have included 9 billion event distributions for A_4 calculated as $\frac{8}{3}A_{FB}$ with 1 GeV binning for the full range 60 GeV < M_{ll} < 150 GeV. These are new.
- These results use NNPDF3.1-LuxQED PDFs
- Runs will soon be available with regular NNPDF3.1 and MMHT as well. Preliminary results suggest that the IFI contribution will have a very weak dependence on the PDF version used.

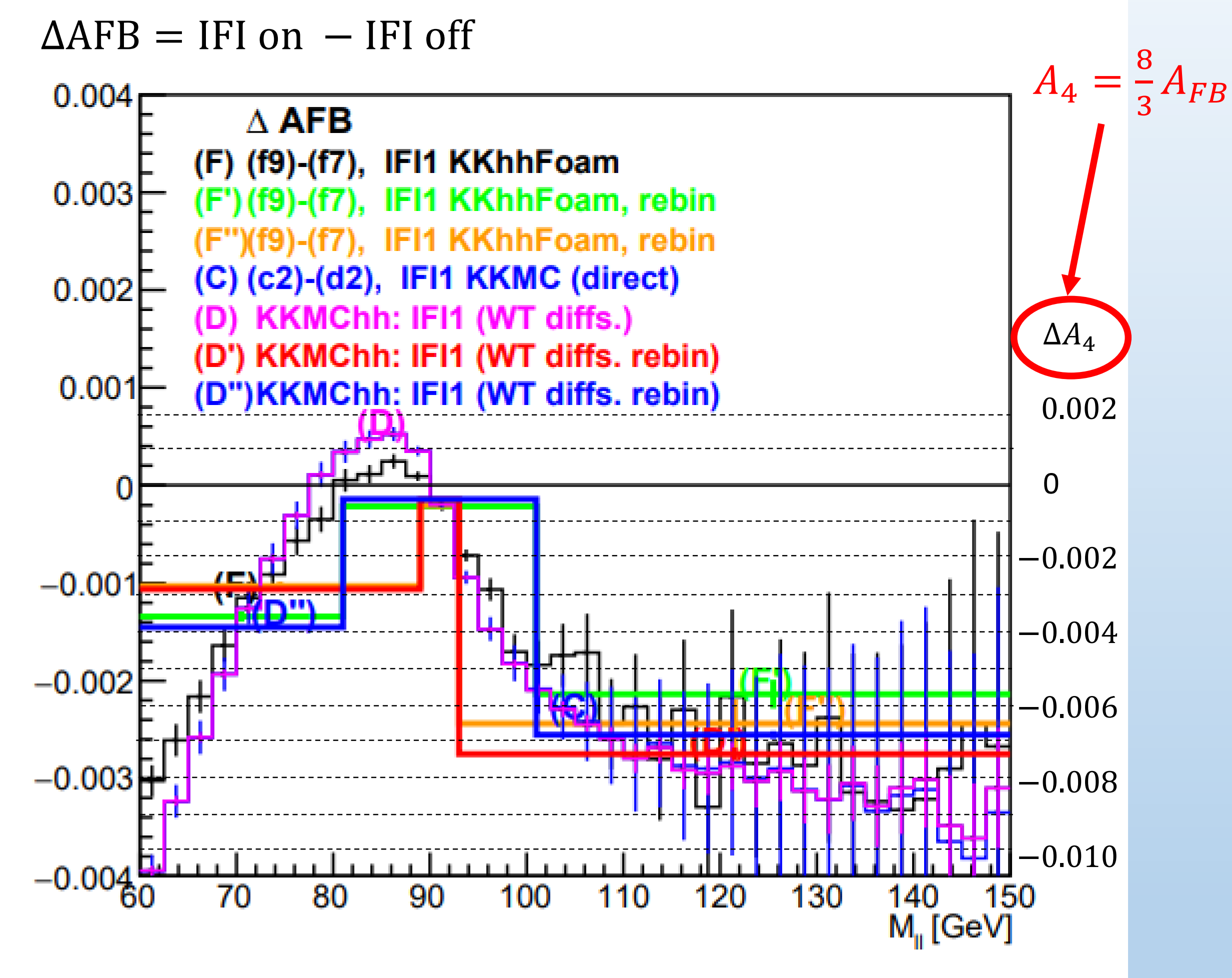
IFI Contribution to "A_4" $\equiv \frac{8}{3} A_{FB}$

The table shows the difference in $A_4 \times 10^4$ with IFI on minus IFI off. The numbers are from Stefan Dittmaier's May presentation with new KKMChh results for 32 billion events.

version	$89 < M_{ll} < 93$	$60 < M_{ll} < 81$	$81 < M_{ll} < 101$	$101 < M_{ll} < 150$
KKMChh	$\mathbf{-3.8(2)}$	$\mathbf{-38.7(4)}$	$\mathbf{-3.7(1)}$	$\mathbf{-68(1)}$
KKhhFoam	$-5.4(7)$	$-35.8(1)$	$-5.7(6)$	$-57(4)$
MCSANC	$-2.7(4)$	$-34(2)$	$-3.9(3)$	$-60(2)$
WZGRAD2	$-1.1(5)$	$-37(3)$	$-2.3(5)$	$-51(4)$
POWHEG_ew	$1.6(1.5)$	$-64(8)$	$1.2(1.2)$	$-54(10)$
RADY (CMS)	$-1.5(1)$	$-33.6(4)$	$-2.49(7)$	$-59.5(1)$
A. Huss	$-1.42(6)$	$-33.9(6)$	$-2.57(7)$	$-58.7(3)$

IFI contributions to A_{FB} , no lepton cuts





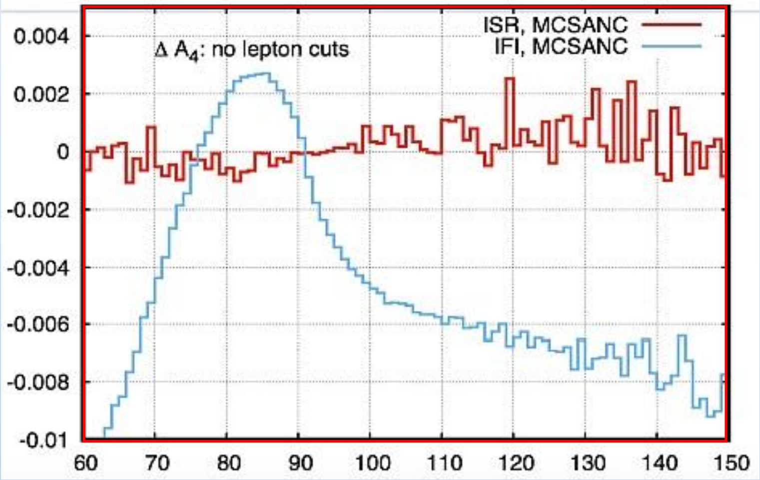
$\mathbf{\Delta AFB}$: **D** is the best KKMChh result. Multiply by 8/3 for A_4. **C** is the same, calculated differently. **F** is KKhhFoam.
D' , **F'** are rebinned for $60 - 89$ GeV, $89 - 93$ GeV, $93 - 150$ GeV (tabulated on previous slide with 8/3 factor)
D'', **F''** are rebinned for $60 - 81$ GeV, $81 - 101$ GeV, $101 - 150$ GeV (tabulated on previous slide with 8/3 factor)

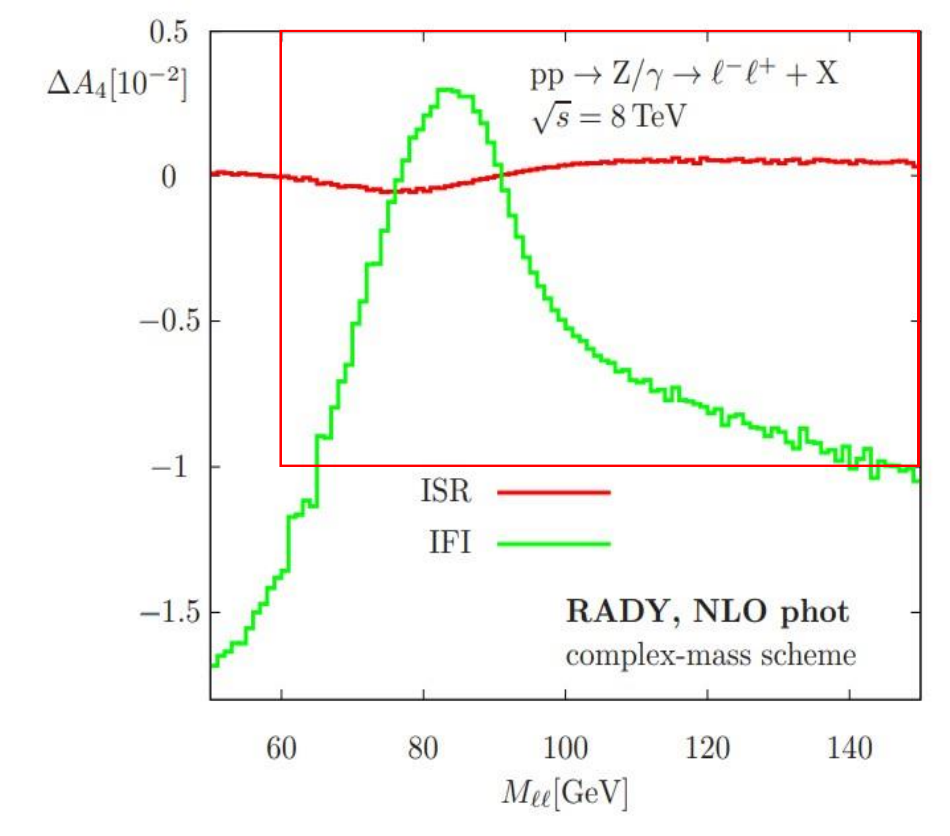
Comparison to MCSANC and RADY

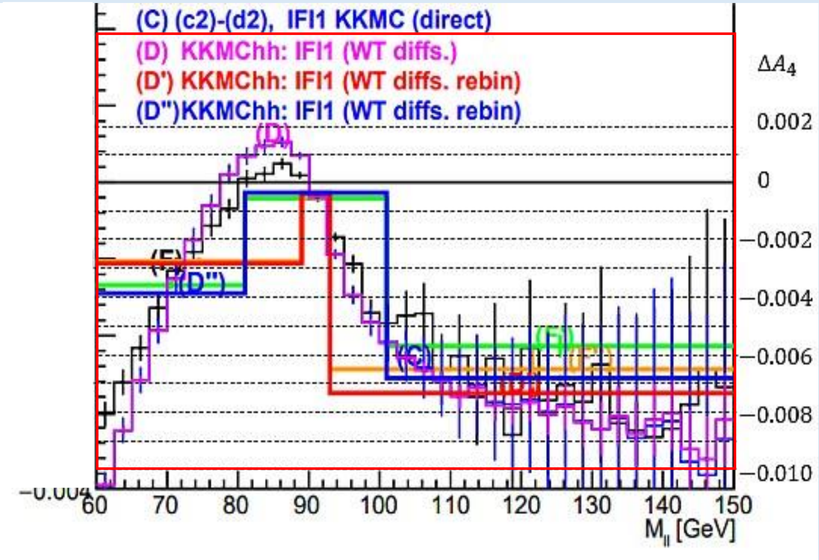
The following figures have been resized to the same scale to show the comparison clearly.

ISR is not shown for KKMChh: some ideas on ISR will be discussed in the following slides.

MCSANC **RADY** **KKMChh**







The red box on each figure is drawn to the same scale.

ISR and PDFs: Conceptual Issues for KKMChh

- KKMC-hh includes ab-initio QED calculations without cutoffs. Quark masses are taken to be physical parameters, observable via their effect on the measurable radiated photon distributions, as are electron masses in e^+e^- collisions. (KKMC-hh assumes current masses.)
- This is only correct for "pruned" PDFs describing the quarks have been fit to data with the QED backed out, so that the data is accurately represented by QCD evolution alone.
- "Standard" PDF sets are based on input data which may or may not back out QED to various degrees. The assumption has normally been that the QED contamination is small compared to the PDF errors, and the data is fit to a pure QCD evolution.
- Fitting data containing QED to pure QCD evolution is incorrect unless the resulting error is shown to be small enough to neglect. Nevertheless, we have used them so far as the best available option – leaving open the question of how big an error is made in neglecting the QED contamination.

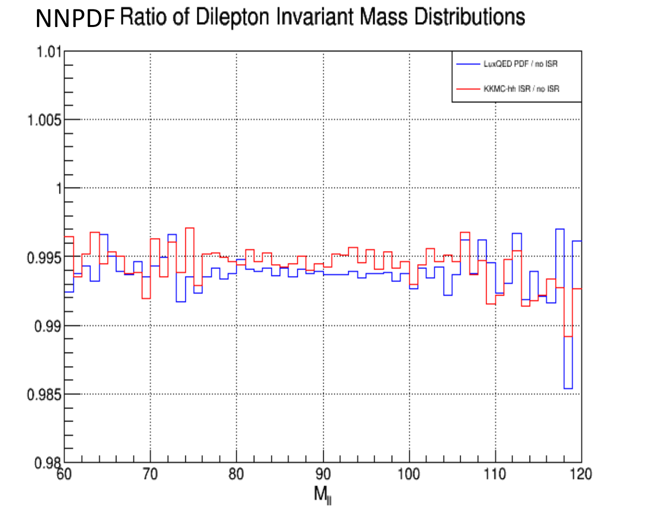
Suggestive Comparisons

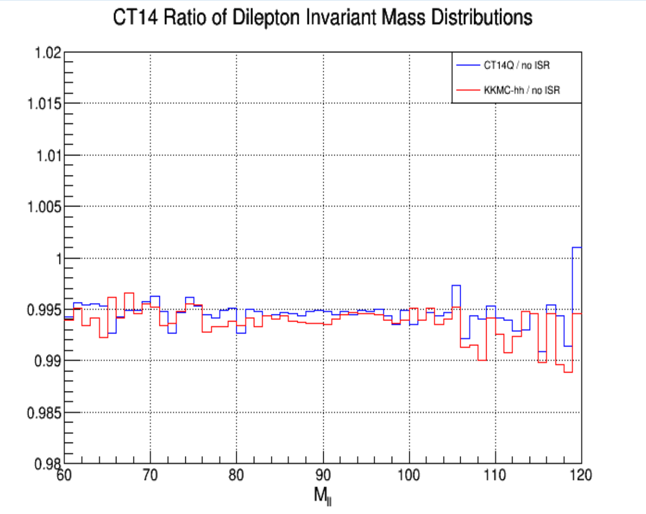
The idea that the standard PDFs might be an adequate starting point can be partially checked by comparing to PDFs that do include a combination of QCD and QED evolution.

If the assumption that these PDFs are adequate is good, adding ISR via KKMChh should produce inclusive cross sections that agree with cross sections obtained via the QED-corrected version of the same PDFs with no ISR from KKMChh.

A mismatch between the two calculations would suggest the degree of error in the approach.

The graphs show ISR on/off ratios of M_{ll} distributions in red and QED PDF / normal PDF in blue, for NNPDF (top) and CT14 (bottom) for 10^9 events without FSR. The agreement is close enough to suggest an equivalence of the approaches, but only circumstantially – we would like to do better.





Matching KKMChh to QED-Corrected PDFs

Since QED-corrected PDFs fit data containing QED effects to a combined QCD and QED evolution, they seem to provide a firmer starting point, but using them directly and adding the full ISR correction from KKMChh would obviously be double counting.

One approach would be to take the QED corrected PDFs as a starting point and back out the part of the QED evolution in KKMChh that is below a scale Q_0 where data determines the PDFs. This has the added advantage that this also removes the part of the evolution influenced by the quark masses, which have been a source of controversy.

Another approach would be to modify the PDFs themselves – starting with QED-corrected ones so they already represent data as well as possible, acknowledging both QCD and QED evolution, and then "pruning" the QED contribution by removing what will be later added back by KKMChh. This is essentially the same solution, but the backward evolution step removing the QED can be once and for all.

The resulting pruned PDFs would be the ideal starting point KKMChh has been assuming. If they turn out to give results compatible with standard non-QED PDFs within the desired error budget, that would justify the assumptions originally made in KKMChh – that any "double counting" is negligible. If not, the pruned PDFs can be used to remove this source of error.

QED and PDFs in KKMChh

$$\rho(\gamma, z) = \frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}\gamma(1-z)^{\gamma-1}$$

at leading order.

In simplified form, the cross-section

$$\sigma(s) = \frac{3\pi}{4}\sum_q \int d\hat{s}\, dx_1 dx_2\, \delta(\hat{s} - s x_1 x_2) f_q^{h_1}(x_1, \hat{s}) f_{\bar{q}}^{h_2}(x_2, \hat{s}) \int dz\;\; \rho(\gamma_I(\hat{s}), z)\;\; \sigma_{q\bar{q}}\,(z\hat{s})$$

can be rewritten using $\rho(\gamma_1 + \gamma_2, z) = \int dz_1 dz_2 \delta(z - z_1 z_2)\rho(\gamma_1, z_1)\rho(\gamma_2, z_2)$ as

$$\sigma(s) = \frac{3\pi}{4}\sum_q \int d\,\bar{s}\, dx_1' dx_2'\, \delta(\bar{s} - s x_1' x_2') \hat{f}_q^{h_1}(x_1', \bar{s}) \hat{f}_{\bar{q}}^{h_2}(x_2', \bar{s})\;\; \sigma_{q\bar{q}}\,(\bar{s})$$

with $\bar{s} = z\hat{s}$, $\;\hat{f}_q^{h_1}(x_1', \bar{s}) \equiv \int dx_1 dz_1 \delta(x_1' - z_1 x_1)\, f_q^{h_1}(x_1, \hat{s}) \rho\left(\frac{\gamma_I(\hat{s})}{2}. z_1\right)$, ...

absorbing the ISR into modified PDFs. If the original PDFs were "pruned" to have no QED content, this would generate ones that had QED corrections added via KKMChh.

However, if the original PDFs already account for QED evolution, a similar transformation could be used to back the QED out. This could be done up to a scale Q_0 giving a cross section of the form

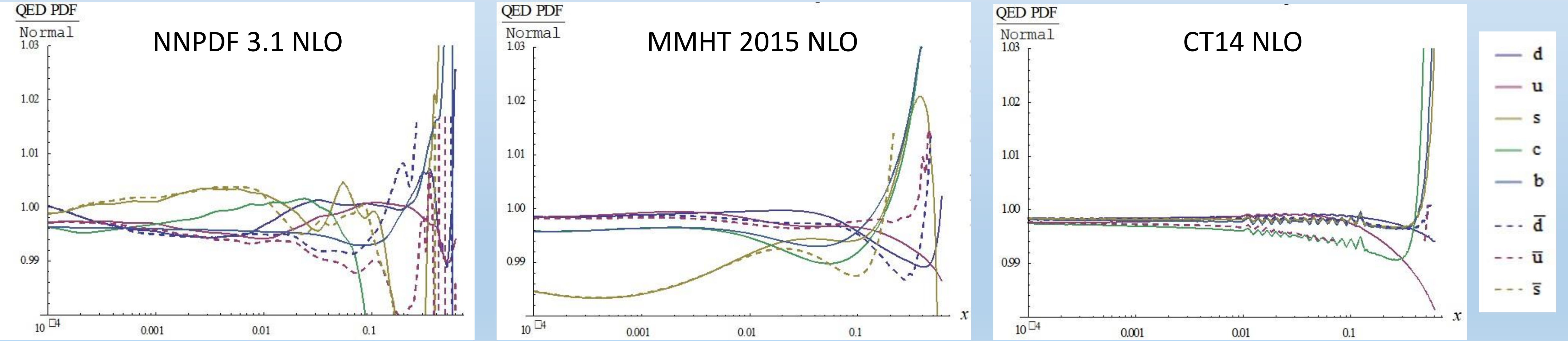
$$\sigma(s) =$$

$$= \frac{3\pi}{4}\sum_q \int d\hat{s}\, dx_1 dx_2\, \delta(\hat{s} - s x_1 x_2) f_q^{h_1}(x_1, \hat{s}) f_{\bar{q}}^{h_2}(x_2, \hat{s}) \int dz\;\, dz'\, \rho(\gamma_I(\hat{s}), z) \boxed{\rho(-\gamma_I(Q_0), z')} \sigma_{q\bar{q}}\,(z\hat{s})$$

Choice of QED-Corrected PDFs

Will pruning QED-corrected PDFs or backing out QED evolution below some scale Q_0 on the fly give more accurate results? This will depend on the accuracy of the QED-corrected PDFs.

Several choices are available, and all make some different assumptions. The following graphs show the ratios of QED to non-QED PDFs at $Q = M_Z$. There are clear differences in how taking account of the QED affected the PDFs.



Summary

The latest results for IFI contributions to AFB have converged to a point where there is close agreement with most of the other groups' results.

To be compared to other groups, the ISR result needs to be specified relative to a clear baseline. What is the definition of "without ISR"? Calculating it relative to a standard QED PDF as we have clearly isn't the same thing.

The ability to reverse QED evolution in KKMChh opens up the possibility to compare directly to factorized calculations, and also provides a way to completely eliminate any dependence on the quark masses (trading it for dependence on a scale Q_0).

A version of KKhhFoam with the backward evolution implemented is presently being tested. Once it is ready, it can be implemented in the full KKMChh as well.