Muon $g-2$ from lattice QCD

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Introduction

- Standard Model : the muon is an elementary particle
- Same charge but 200 heavier than the electron
- Spin $1/2$ particle
- The magnetic moment of the muon is proportional to the spin $\vec{\mu} = g\left(\frac{Qe}{2m}\right)\vec{s}$

$$a_\mu = \frac{g-2}{2}$$

Why is this observable so interesting ?

1) can be measured very precisely : < 0.5 ppm !

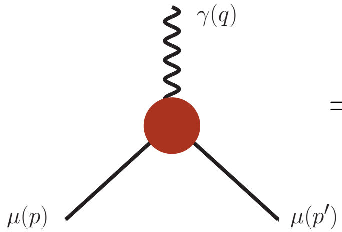
2) can also be predicted very precisely in the SM

3) sensitive to new physics

Introduction

- Corrections to the vertex function : Dirac and Pauli form factors

  Assuming Lorentz invariance and P and T symmetries, the vertex function can be decomposed into 2 form factors



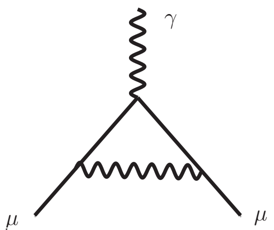
$$= -ie\,\overline{u}(p',\sigma')\Gamma_\mu(p',p)u(p,\sigma)$$

$$= -ie\,\overline{u}(p',\sigma')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p,\sigma)$$

$F_1(0) = 1$ (charge conservation)

$$F_2(0) = a_\mu = \frac{g-2}{2}$$

- Classical result : $g = 2$ for elementary fermions (Dirac equation)



Quantum field theory : $a_\mu = \frac{g-2}{2} \neq 0$

$\hookrightarrow$ quantum effects

$$a_\mu^{(1)} = \frac{\alpha}{2\pi} \approx 0.00116 \quad \text{[Schwinger '48]}$$

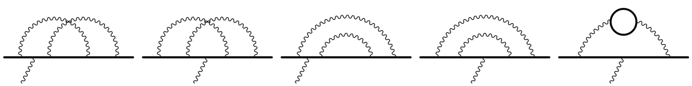
Standard model contributions : QED

- **QED accounts for more than 99.99% of the final result** [Aoyama et al. '12 '19]

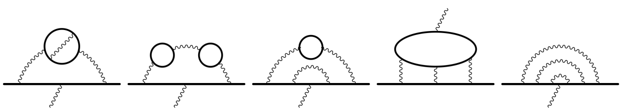
$$a_\mu^{\rm QED} = \left(\frac{\alpha}{\pi}\right) a_\mu^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a_\mu^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 a_\mu^{(3)} + \cdots$$

→ 5-loop contributions are known !

Order α^4 (7 diagrams)

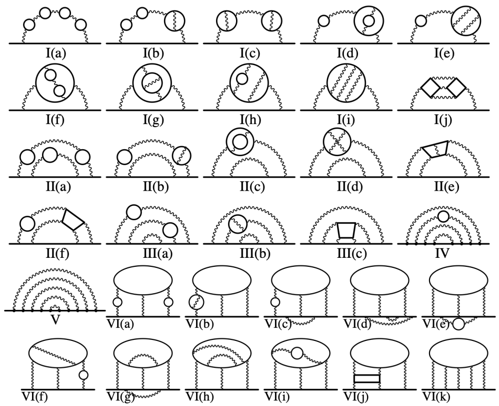


Order α^6 (72 diagrams)



Order α^8 (891 diagrams) ...

Order α^{10} (12 672 diagrams)



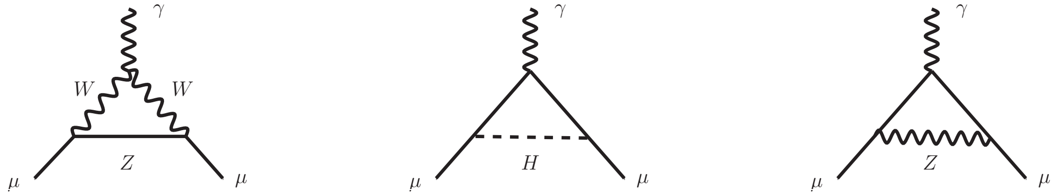
n	$a_\mu^{(1)} \times 10^{11}$	n	$a_\mu^{(1)} \times 10^{11}$
1	116 140 973.321(23)	4	381.004(17)
2	413 217.6258(70)	5	5.0783(59)
3	30 141.90233(33)		

→ Uncertainty far below Δa_μ. Strong test of QED.

$$a_\mu^{\rm QED} = 116\ 584\ 718.931(104) \times 10^{-11}$$
$$a_\mu^{\rm SM} = 116\ 591\ 810(43) \times 10^{-11}$$

Standard model contributions

- **Electroweak corrections** [Czarnecki '02] [Gnendiger '13]



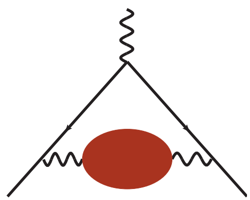
→ Two-loop contributions are known : $a_\mu^{\mathrm{EW}} \times 10^{11} = 153.6(1.0)$

→ Contributes to only 1.5 ppm ($\sim$ 4 $\times$ exp. error) $\Rightarrow$ under control

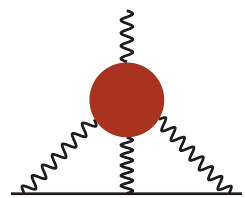
- **QCD corrections**

→ Quarks and gluons do not directly couple to the muon : contribution via loop diagrams

→ The two relevant contributions (to reduce the error) are



Hadronic Vacuum Polarisation (LO-HVP, α^2)



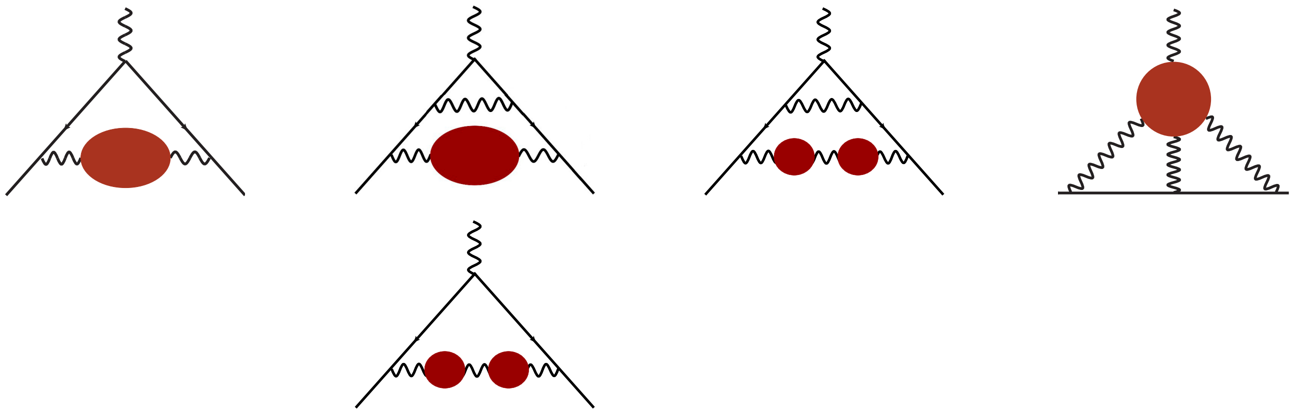
Hadronic Light-by-Light scattering (HLbL, α^3)

- **Contribution from unknown particles / interactions (?)** $a_\ell^{\mathrm{NP}} = \mathcal{C}\,\frac{m_\ell^2}{\Lambda^2}$

→ Talk tomorrow morning by Martin Hoferichter

Other hadronic contributions

LO HVP (α^2) **NLO HVP** (α^3) **NNLO HVP** (α^4) **HLbL** (α^3)



- LO HVP : includes photons in the QCD blob
  $\rightarrow$ strictly speaking, not an expansion in α, but consistent !
- NLO HVP and NNLO HVP differ by the QED kernel functions
  $\rightarrow$ NLO HVP : same order as HLbL (but negative contribution)
  $\rightarrow$ Not negligible, but error under control (the required relative precision is smaller)

Theory status just after the white-paper (2020)

The Muon $g-2$ Theory Initiative :

- website : https ://muon-gm2-theory.illinois.edu/
- White Paper posted 10 June 2020

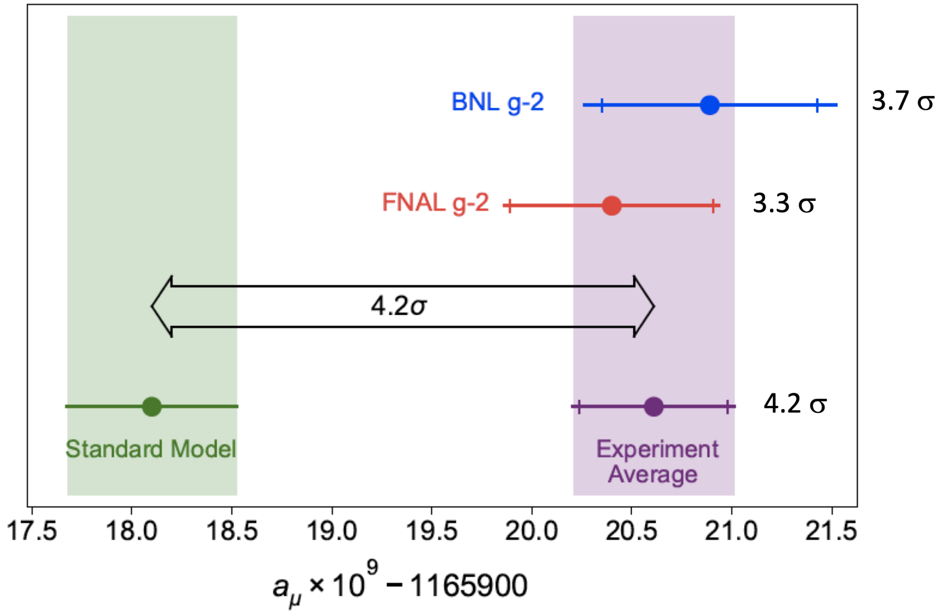
  The anomalous magnetic moment of the muon in the Standard Model [Phys.Rept. 887 (2020) 1-166]

Contribution	$a_\mu \times 10^{11}$	
- QED (leptons, 10^{th} order)	$116\ 584\ 718.931 \pm 0.104$	[Aoyama et al. '12 '19]
- Electroweak	153.6 ± 1.0	[Gnendiger et al. '13]
- Strong contributions		
HVP (LO)	$6\ 931 \pm 40$	[DHMZ '19, KNT '20]
HVP (NLO)	-98.3 ± 0.7	[Hagiwara et al. '11]
HVP (NNLO)	12.4 ± 0.1	[Kurtz et al. ''14]
HLbL	92 ± 18	[See WP]
Total (theory)	$116\ 591\ 810 \pm 43$	

→ The error budget is totally dominated by hadronic contributions !

→ Lattice calculations can play a major role there.

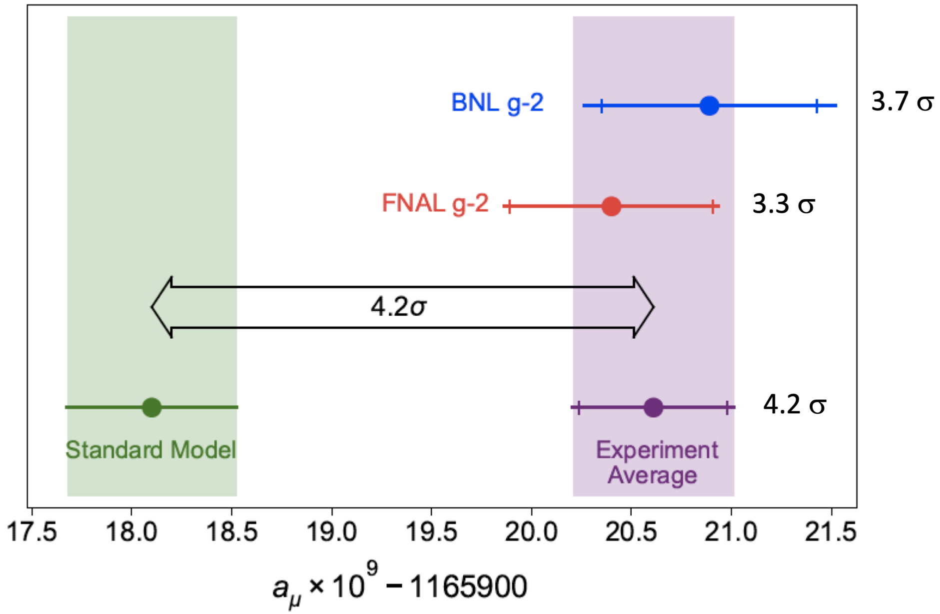
Status after the first run of the E989 experiment at Fermilab



- Remarkable confirmation of the Brookhaven result (2004)
- Similar precision for both theory and experiment
- This is a large discrepancy ($2\times$ electroweak contribution !)
- Theory error is dominated by hadronic contributions

  $\rightarrow$ reduction of the theory error by a factor of 3-4 needed to match upcoming experiments

Status after the first run of the E989 experiment at Fermilab

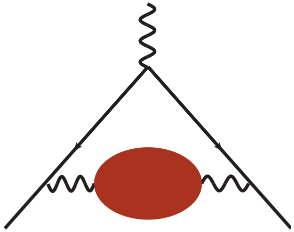


BUT :

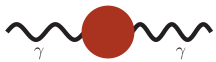
- does not include the most recent lattice results
  - → complete lattice calculation of the HLbL contribution by Mainz [Eur.Phys.J.C 81 (2021) 7, 651]
  - → first sub-percent calculation of the HVP contribution by BMW [Nature 593 (2021) 7857]

Outline of the talk : hadronic contributions

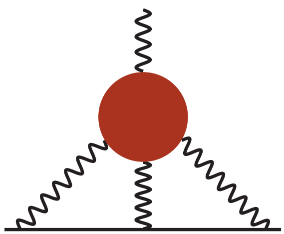
- **Hadronic Vacuum Polarisation** (HVP, α^2)



- Blobs : all possible intermediate hadronic states ($\pi\pi, \cdots$)
- Precision physics (Goal : precision < 0.3%)

$\Pi_{\mu\nu}(Q) =$  $= \int \mathrm{d}^4x\, e^{iQ\cdot x} \, \langle V_\mu(x) V_\nu(0) \rangle$

- **Hadronic Light-by-Light scattering** (HLbL, α^3)



Hadronic light-by-light tensor $\Pi_{\mu\nu\lambda\sigma}(p_1, p_2, p_3)$

- Small but contributes to the total uncertainty !
- 4-point correlation function
- More difficult, but 10% precision is enough

Standard model prediction of hadronic contributions

- Perturbative QCD not applicable : we need non-perturbative methods

- Two first-principle approaches :

The dispersive framework (data-driven)

→ based on analyticity, unitarity ...
→ ... but relies on experimental data
→ several group have published results for the HVP [Davier et al. '19] [Keshavarzi et al. '20]
→ more difficult for the Light-by-Light, but a lot of progress recently
(analytic structure of the 4-point function more difficult, exp. data sometimes missing)

Lattice QCD

→ ab-initio calculations (it is not a model !)
→ need to control all sources of error (challenging at this level of precision)
→ many groups : so cross-checks are possible

- It provides two completely independent determinations

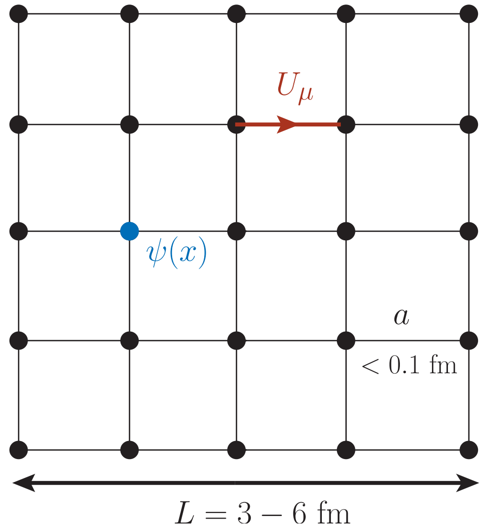
Lattice QCD

- Rigorous calculation : specific regularization of the path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \int \mathcal{D}[\overline{\psi}]\, \mathcal{D}[\psi]\ \mathcal{O}[\overline{\psi}, \psi, U]\ e^{-S_E[U,\overline{\psi},\psi]}$$

- Use an hypercubic lattice to regularize the theory :

  - Lattice spacing : UV regulator
  - Finite volume : IR regulator
  - Different discretizations are possible
    $\Rightarrow$ Different properties, numerical costs
  - Finite number of degrees of freedom
    $\Rightarrow$ numerical simulations



- Very large number of degrees of freedom $\Rightarrow$ Stochastic evaluation using Monte-Carlo
  $\rightarrow$ generate n gauge configurations $\{U_\mu^{(i)}\}$) with probability weight given by the action

$$\overline{\mathcal{O}} = \sum_{i=1}^{n} \langle \mathcal{O} \rangle_F \, [U_\mu^{(i)}] = \langle \mathcal{O} \rangle + \delta\mathcal{O} \ \rightarrow\ \text{statistical error}$$

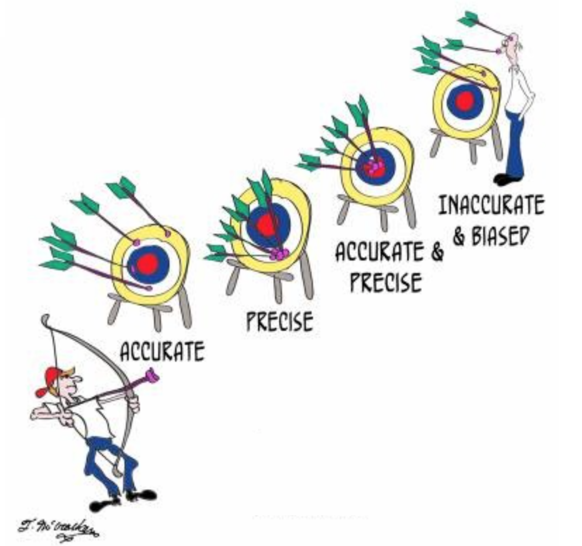
Lattice QCD : sources of errors

Statistical error

- Monte-Carlo algorithm : statistical error $\rightarrow \sim 1/\sqrt{N_{\text{meas}}}$

Systematic errors

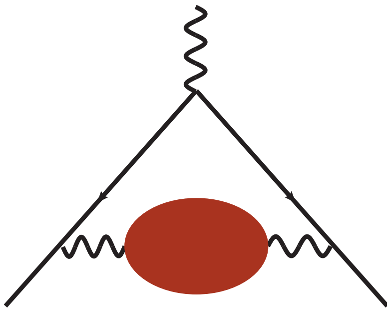
- Finite lattice spacing : continuum extrapolation
- Finite volume
  $\rightarrow$ one should take the infinite volume limit
- Isospin-breaking and QED corrections
  $\rightarrow$ Need to be included at this level of precision

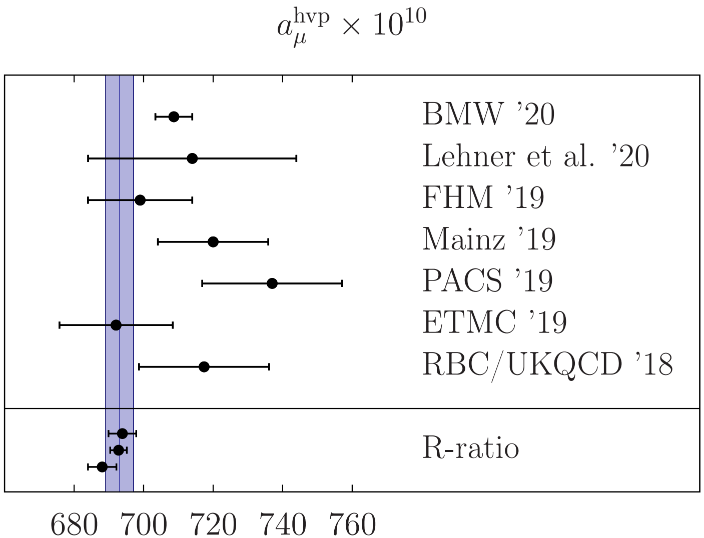


Effective field theories are helpful

- Symanzik Effective Field Theory : behavior of the continuum extrapolation
- Chiral perturbation theory : quark mass dependence, volume effects
- They are valuable guide to reach the physical point !

Hadronic vacuum polarization





→ Many lattice collaborations (with different systematic errors)

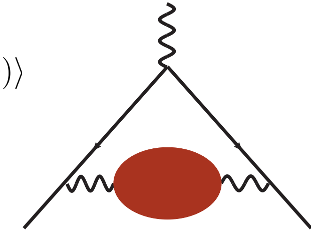
→ Precision of about 2% for lattice, 0.6% for the data driven approach

→ Recent lattice calculation below 1% by the Budapest-Marseille-Wuppertal collaboration

Lattice QCD approach to the hadronic vacuum polarization (HVP)

$\Pi_{\mu\nu}(Q) =$ 

$= \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\right) \Pi(Q^2) = \int \mathrm{d}^4 x \, e^{iQ\cdot x} \, \langle V_\mu(x) V_\nu(0) \rangle$



EM current : $V_\mu(x) = \frac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma_\mu d(x) - \frac{1}{3}\overline{s}(x)\gamma_\mu s(x) + \frac{2}{3}\overline{c}(x)\gamma_\mu c(x) + \cdots$

- Integral representation over Euclidean momenta ($\rightarrow$ accessible from lattice !)

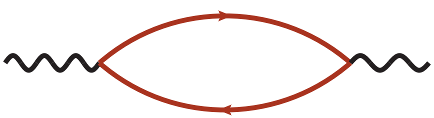
$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2 \; f(Q^2) \; \left(\Pi(Q^2) - \Pi(0)\right)$$

- Time-momentum representation [Blum '02] [Bernecker, Meyer '11]

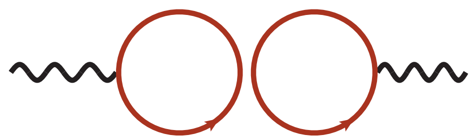
$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}x_0 \; K(x_0) \; G(x_0) \,, \qquad G(x_0) = -\frac{1}{3} \sum_{k=1}^{3} \sum_{\vec{x}} \langle V_k(x) V_k(0) \rangle$$

- Start with iso-symmetric QCD without QED : two sets of Wick contractions

Connected contribution



(quark) disconnected contribution



$\mathrm{O}(1-2\%)$

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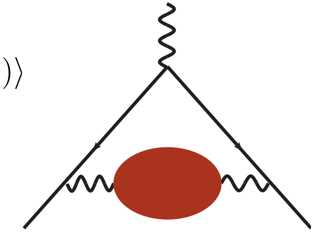

EM current : $V_\mu(x) = \frac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma_\mu d(x) - \frac{1}{3}\overline{s}(x)\gamma_\mu s(x) + \frac{2}{3}\overline{c}(x)\gamma_\mu c(x) + \cdots$

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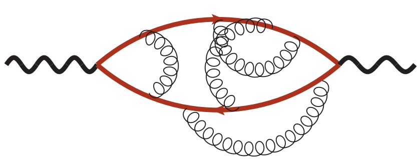
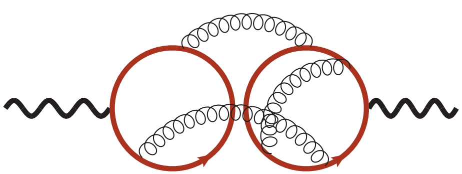


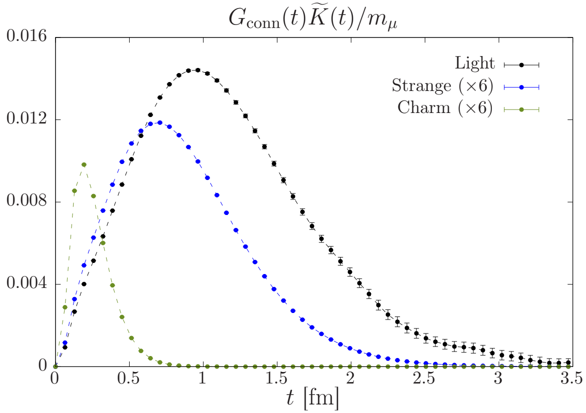
(quark) disconnected contribution



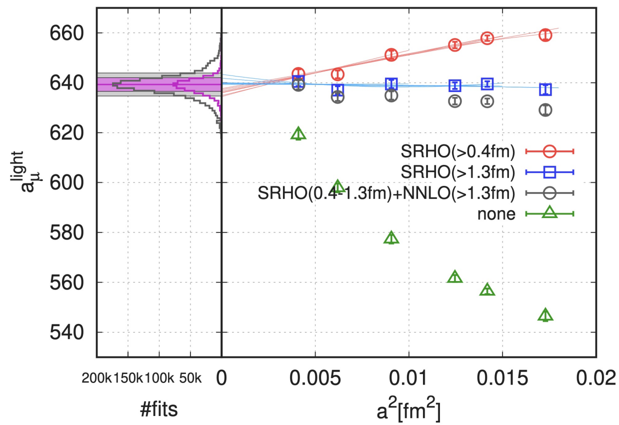
$\mathrm{O}(1-2\%)$

Challenges for sub-percent precision

- Noise problem (light-quark contribution)



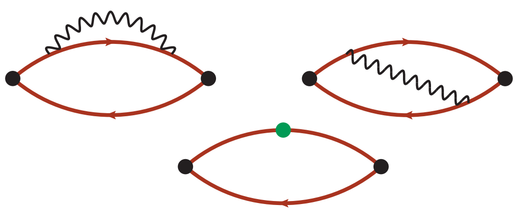
- Continuum extrapolation [BMW '20]



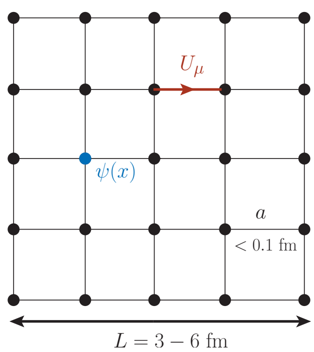
- QED / strong isospin breaking corrections

$m_u \neq m_d$: $\mathrm{O}(\frac{m_u - m_d}{\Lambda_{\rm QCD}}) \approx 1/100$

$Q_u \neq Q_d$: $\mathrm{O}(\alpha_{\rm em}) \approx 1/100$



- Finite-volume effects $\mathcal{O}(3\%)$



Solution to the noise problem

Signal / noise problem

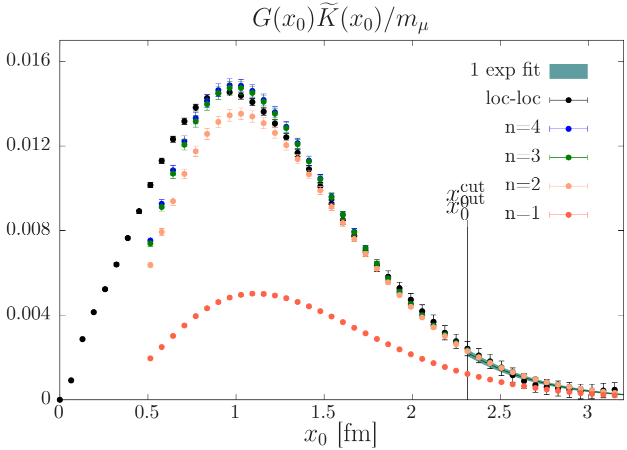
Solution to the noise problem

The vector correlators admits a spectral decomposition :

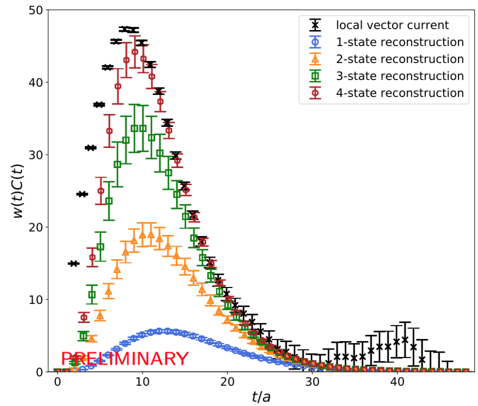
$$\langle V(x_0)V(0)\rangle = \sum_n \langle 0|V|n\rangle \, \frac{1}{2E_n} \, \langle n|V(0)|0\rangle \, e^{-E_n x_0}$$

- $|n\rangle$ are the eigenstates in finite volume
- E_n and $\langle 0|V|n\rangle$ can be computed on the lattice using sophisticated spectroscopy methods

[Mainz and RBC/UKQCD Collaborations]



[A. Gerardin et al, Phys.Rev. D100 (2019), 014510]



[Plot by A. Meyer (RBC/UKQCD) @ Lattice 2019]

→ Only a few number of states are needed (but more states needed at the physical pion mass)

→ Noise now grows linearly with x_0 (not exponentially)

→ Can be combined with powerful algorithmic improvements.

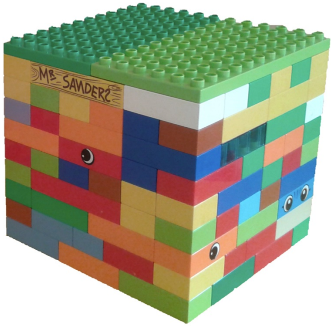
Corrections for finite-size effects

Finite volume effects

Corrections for finite-size effects

- Direct lattice calculation [Budapest-Marseille-Wuppertal '21]

$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$



$L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

$\rightarrow$ Finite size effects correction : about 3% with $L = 6$ fm

$\rightarrow$ Very expensive calculation : volume effects are most important at large distances

- Effective field theories can also be used

$\rightarrow$ analytical results

$\rightarrow$ better understanding of the volume dependance

  $\rightarrow$ NNLO-ChiPT : [C. Aubin et al, arXiv :1905.09307], [J. Bijnens et al, JHEP 1712 (2017) 114]

  $\rightarrow$ Correction based on the time-like pion form factor [H. Meyer, Phys.Rev.Lett. 107 (2011)]

  $\rightarrow$ Hamiltonian approach in [M. Hansen, A. Patella, arXiv :1904.10010]

$\rightarrow$ In very good agreement with the direct lattice calculation

Isospin-breaking corrections

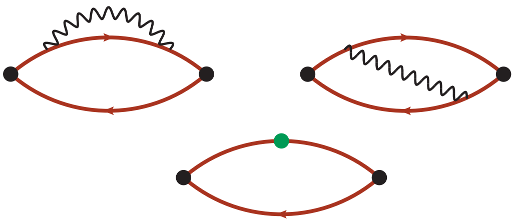
QED + strong isospin-breaking effects

Isospin-breaking corrections

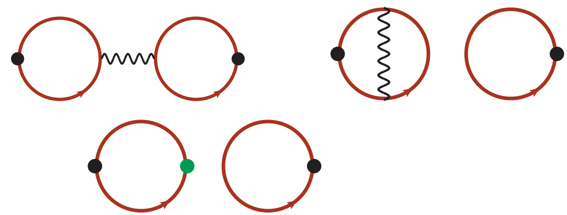
- Lattice simulations are usually performed with QCD only and assuming $m_u = m_d$

$m_u \neq m_d$: $\mathrm{O}(\frac{m_u - m_d}{\Lambda_{\mathrm{QCD}}}) \approx 1/100$	Strong isospin breaking
$Q_u \neq Q_d$: $\mathrm{O}(\alpha_{\mathrm{em}}) \approx 1/100$	Electromagnetic isospin breaking

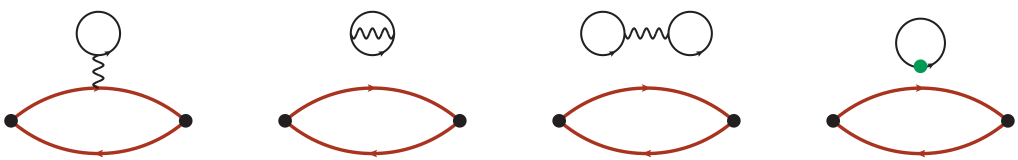
▶ Corrections to the connected part :



▶ Corrections to the disconnected part :

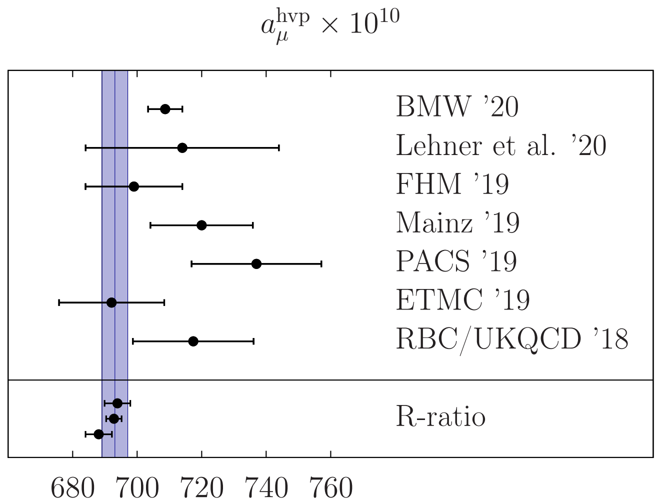


▶ Challenging : beyond the electro-quenched approximation (diagrams are $1/N_c$ suppressed)



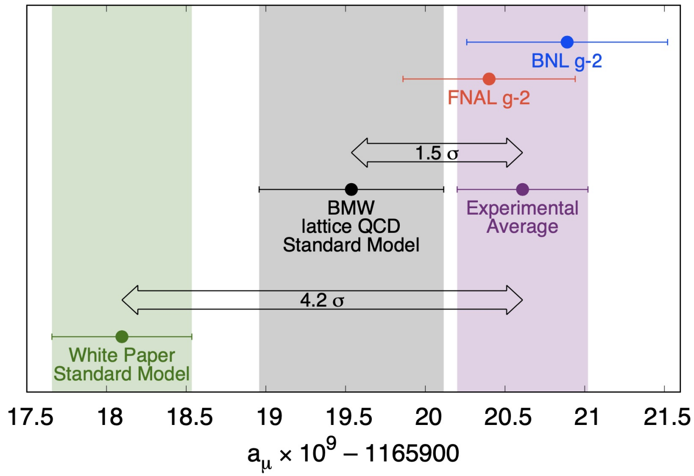
▶ BMW '21 : first calculation that includes all diagrams. About 1% of the full contribution.

Summary : current status for the lattice HVP calculations



- First sub-percent lattice calculation by BMWc (competitive with the data-driven approach)
- If confirmed, would reduce the discrepancy with experiment to $< 2\sigma$
- Need confirmation by other lattice groups
- **Ultimate Goal** : 0.2%
  - average between lattice and dispersive might help ...
  - ... but only if they agree
  - it is probably too soon to quote a "SM estimate of the LO-HVP" with $< 0.5\%$ precision

Summary : current status for the lattice HVP calculations



- First sub-percent lattice calculation by BMWc (competitive with the data-driven approach)
- If confirmed, would reduce the discrepancy with experiment to $< 2\sigma$
- Need confirmation by other lattice groups
- **Ultimate Goal** : 0.2%
  - average between lattice and dispersive might help ...
  - ... but only if they agree
  - it is probably too soon to quote a "SM estimate of the LO-HVP" with $< 0.5\%$ precision

What could go wrong on the lattice ?

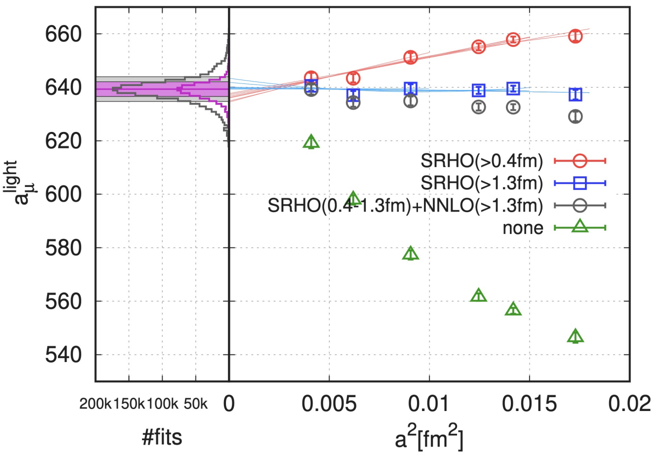
- Is the continuum extrapolation under control ?

  → Based on Symanzik Effective Field Theory : one expects a^2 scaling ...

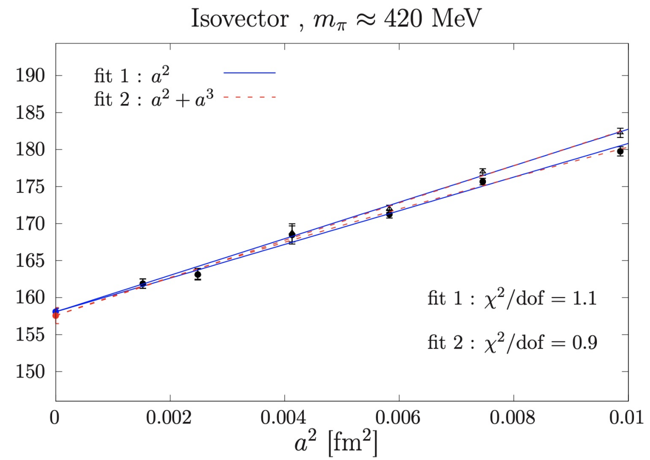
  → ... up to logarithms $a^2/\log(a)^\Gamma$!

  → For pure Yang-Mills one has $\Gamma > 0$. Might also be true for QCD [Husung et al '19]

  → Often logarithms are neglected (2 reasons : Γ are not known + lack of data)



[BMW collaboration]



[Mainz group]

What could go wrong on the lattice ?

- What can be done ?
  - comparison between different collaborations :
    - different discretization of the action (Wilson, Domain Wall, staggered)
    - with different approach to the continuum limit.
  - it is extremely important to have many (small !) lattice spacings
    - Are we in the scaling regime ? Are logarithms under control ?

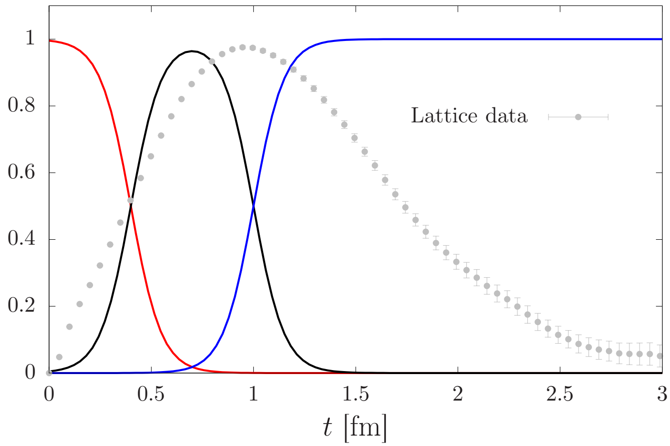
- Time momentum representation (TMR)

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}x_0 \; K(x_0) \; G(x_0) \; , \qquad G(x_0) = -\frac{1}{3} \sum_{k=1}^{3} \sum_{\vec{x}} \langle V_k(x) V_k(0) \rangle$$

  - most collaborations use the TMR method (time in treated differently)
  - in principle it is perfectly fine, but we might miss a sytematic error
  - different approaches have been proposed [Meyer '18], not yet used in practice

- Cross check using a second observable :
  - based on the same lattice data (vector correlator)
  - easier to calculate

Window observables and cross-checks



$$a_\mu^{\rm win} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t)\, K(t)\, W(t; t_0, t_1)$$

→ Short distances (SD)

→ Intermediate distances (ID)

→ Long distances (LD)

- By construction, the sum over the 3 windows gives the full contribution

$$a_\mu^{\rm LO-HVP} = a_\mu^{\rm win,SD} + a_\mu^{\rm win,ID} + a_\mu^{\rm win,LD}$$

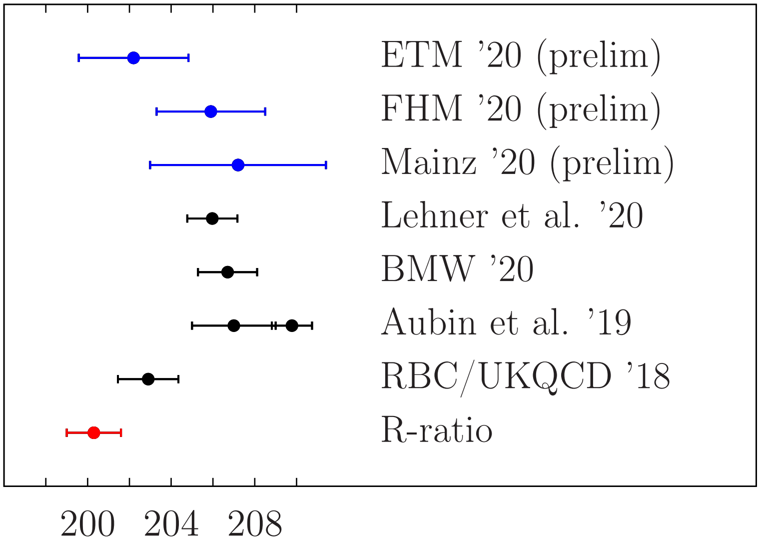
- Each window observakbe is subject to very different systematic errors

Short-distance	Intermediate-distance	Long-distance
stat. precise	stat. precise	noise problem
discretization effects	small finite volume effect	finite volume corrections
		large taste breaking (staggered)

Cross-checks : window quantities

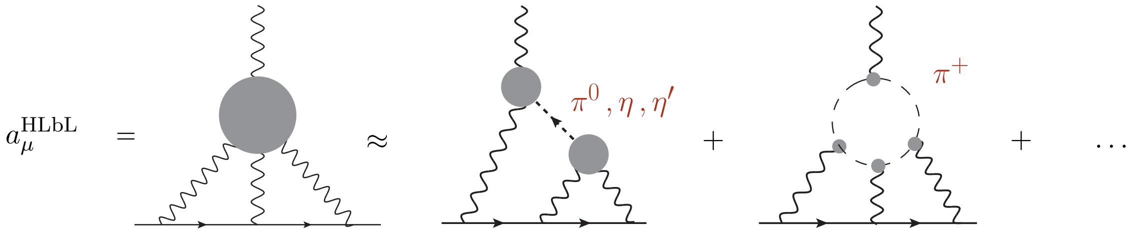
- workshop organized in November 2020 to discuss those issues [https ://indico.cern.ch/event/956699/]

$$\left(a_\mu^{\mathrm{hvp}}\right)^{\mathrm{win},l} \times 10^{10}$$



- lattice results systematically above the R-ratio
- agreement between lattice calculations not yet satisfactory : need to be improved !

Hadronic light-by-light scattering contribution

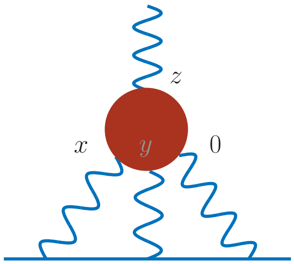


Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
total HLbL	92 ± 19
LO HVP	6931 ± 40

- results from [Phys. Rept. 887 (2020) 1-166]
- Enters at $O(\alpha^3)$
- $\Delta a_\mu^{\rm exp} = 28 \times 10^{-10} \approx 3 \times a_\mu^{\rm hlbl}$

Direct lattice calculation of the Hadronic light-by-light contribution

- Two collaborations : RBC/UKQCD and Mainz, both using position space approaches
- Mainz approach : position-space [Eur.Phys.J.C 81 (2021)] [Eur.Phys.J.C 80 (2020)3]



$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int \mathrm{d}^4 y \int \mathrm{d}^4 x \, \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

$$i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) = -\int \mathrm{d}^4 z \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle$$

→ $\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ is the QED kernel, computed semi-analytically in infinite volume

→ Avoid $1/L^2$ finite-volume effects from the massless photons $\Rightarrow \sim e^{-m_\pi L}$

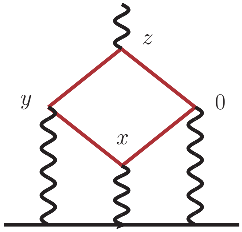
- RBC/UKQCD : [T. Blum et al, Phys.Rev. D93 (2016)] [arXiv :1911.08123]

→ Similar strategy

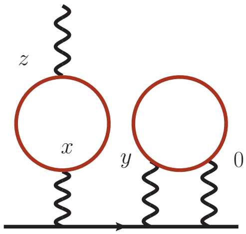
→ QED_L : photon in finite volume ⇒ power-law volume corrections

Wick contractions : 5 classes of diagrams

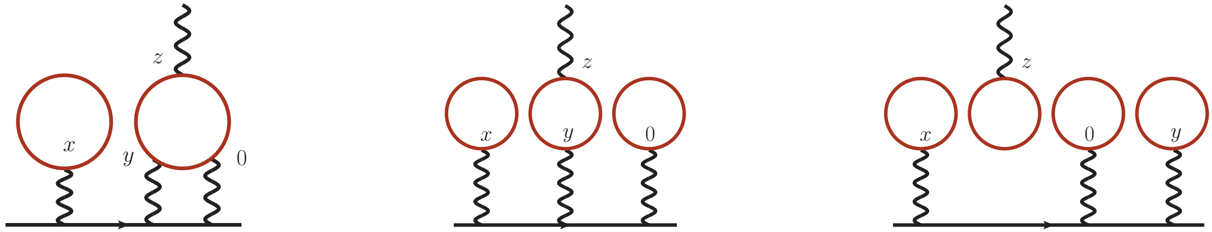
- Fully connected contribution



- Leading 2+2 (quark) disconnected contribution



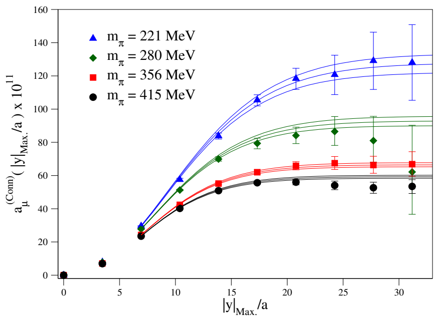
- Sub-dominant disconnected contributions (3+1, 2+1+1, 1+1+1+1)

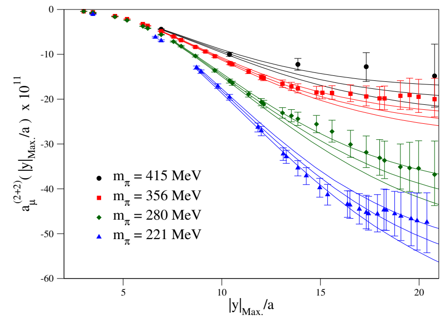


- Second set of diagrams vanish in the SU(3) limit (at least one quark loop which couple to a single photon)
  → Smaller contributions, have been shown to be irrelevant at the 10% level [Mainz '21 : 2104.02632]
- 2+2 disconnected diagrams are not negligible !
  → Large-N_c prediction : 2+2 disc ≈ - 50 % × connected [Bijnens '16]
  → Cancellation ⇒ more difficult (correlations does not seems to help in practice ...)

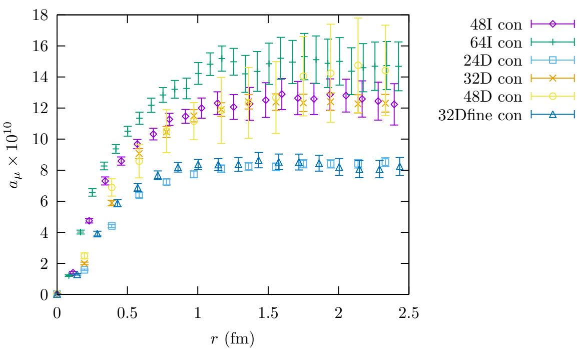
Large cancellation between the two leading contributions

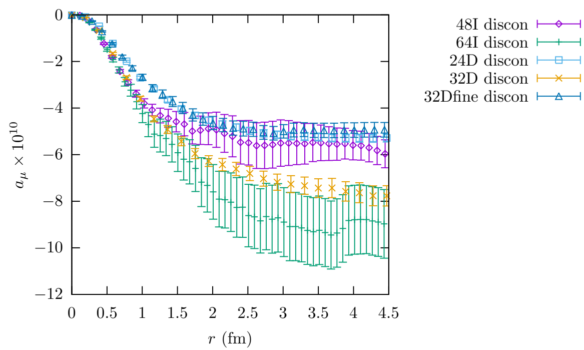
- Connected and disconnected contributions from Mainz ($m_\pi = 200$ MeV)





- Connected and disconnected contribution from RBC/UKQCD at the physical pion mass

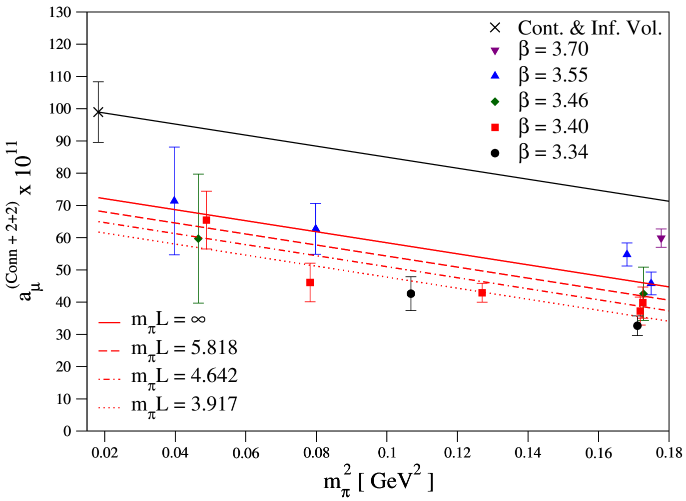




- signal/noise problem even more difficult than HVP

Which errors are relevant

Mainz group [2104.02632]

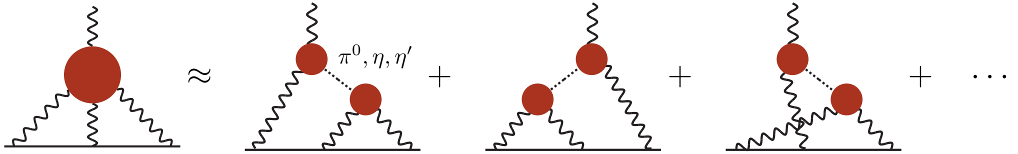


- Statistical noise at long distances
- Finite-volume effects are large
- Continuum extrapolation

$\rightarrow$ Chiral extrapolation milder than expected (based on π^0-pole contribution)

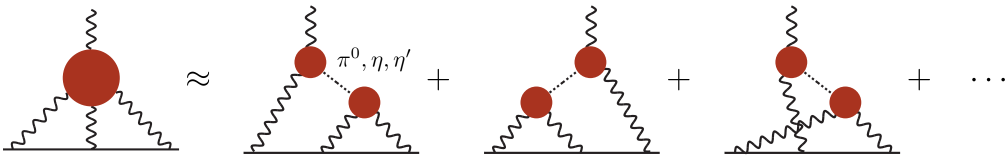
$\rightarrow$ Isospin-breaking corrections are not relevant here

- Both long-range contribution and finite-volume effects are well describe by π^0-exchange :

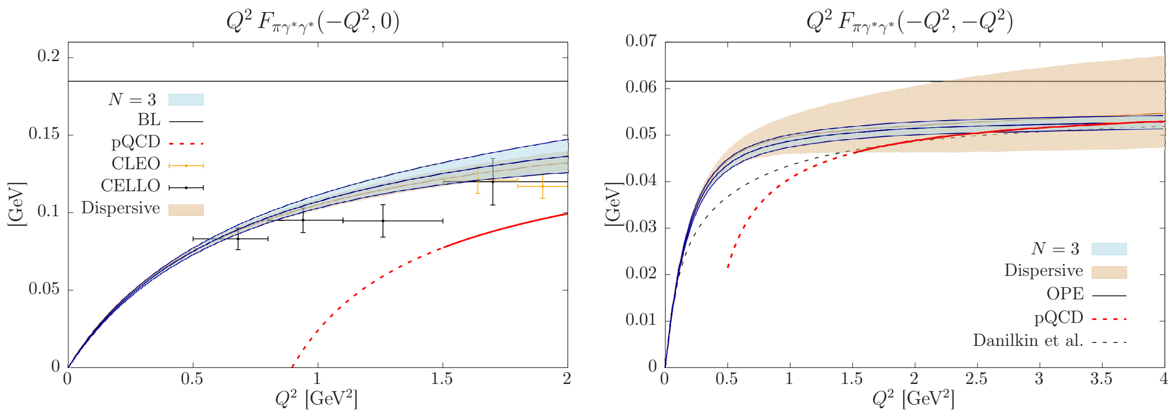


$\rightarrow$ the key ingredient is the pion transition form factor $\mathcal{F}_{\pi^0\gamma\gamma}(Q_1^2, Q_2^2)$

Lattice inputs for the dispersive framework : the pion-pole contribution



- Pion transition form factor



- Fully model independant

$$a_\mu^{\mathrm{HLbL};\pi^0} = (59.9 \pm 3.6) \times 10^{-11}$$

[A. G et al, Phys.Rev. D100 (2019)]

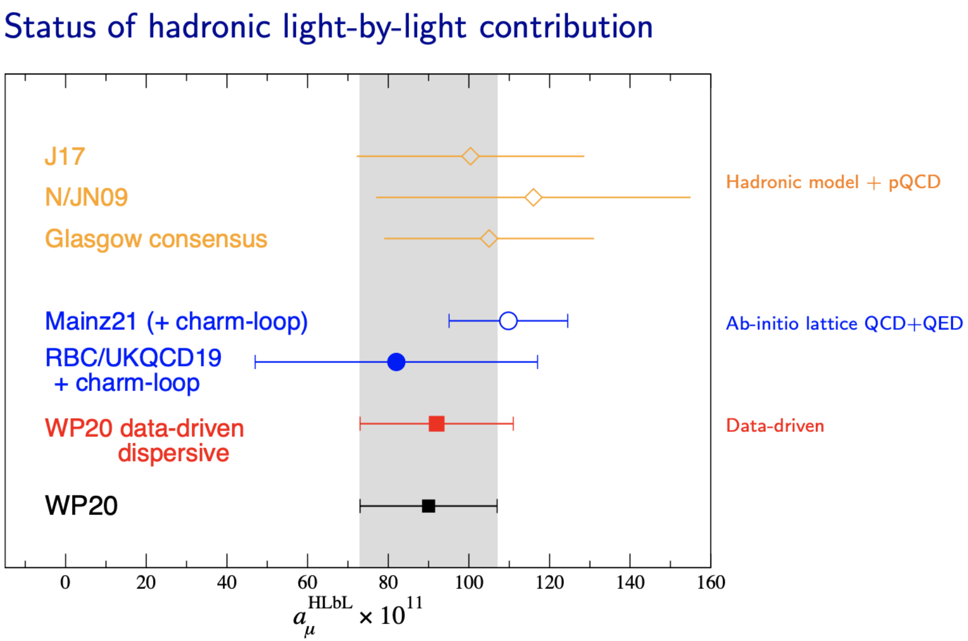
$\rightarrow$ Compatible with the dispersive result

$$a_\mu^{\mathrm{HLbL};\pi^0} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$

[Hoferichter et al. '18]

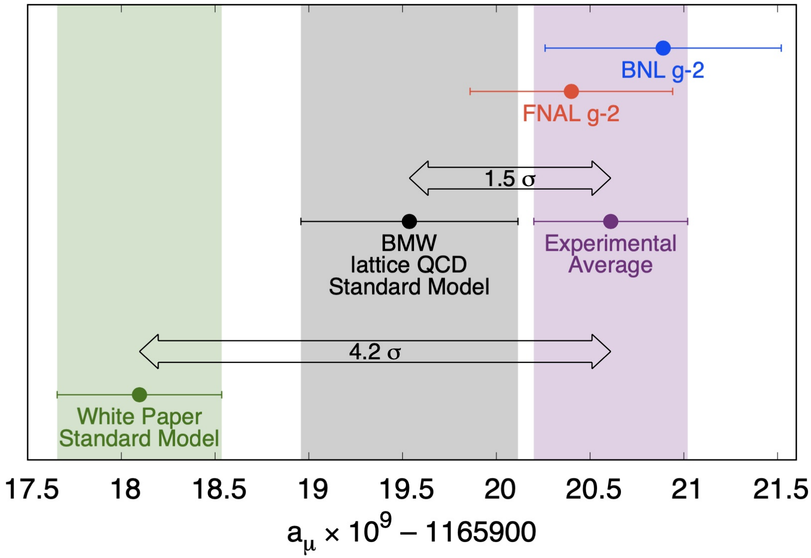
- The BMW and ETM collaborations have presented preliminary results for the η and η'

Conclusion HLbL : Summary of lattice results



- First lattice QCD results are now published
  - → In good agreement with the dispersive framework
  - → But systematic errors are sizeable, cross-checks would be welcome
- Lattice can also provide valuable inputs to the dispersive framework
  - → pseudoscalar-pole contribution (π^0, η, η')
- Close, but not yet at the target precision ($< 10\%$)

Conclusion

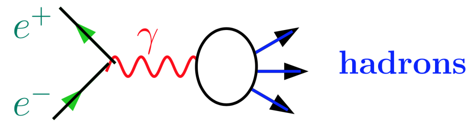


- First run $\approx 1/20$ of the expected total statistics
  - → are we close to NP discovery ?
  - → non-perturbative hadronic contributions dominate the error
  - → the recent BMW lattice result reduces the tension with the measurement !
  - → ... but there is now a tension with the dispersive framework : need confirmation !
- Rapid progress on the lattice
  - → first sub-percent lattice calculation by BMW, but in tension with R-ratio estimates
  - → first complete calculation by Mainz : confirm the size of the HLbL contribution

Hadronic vacuum polarization : dispersive framework

- Use analyticity + optical theorem

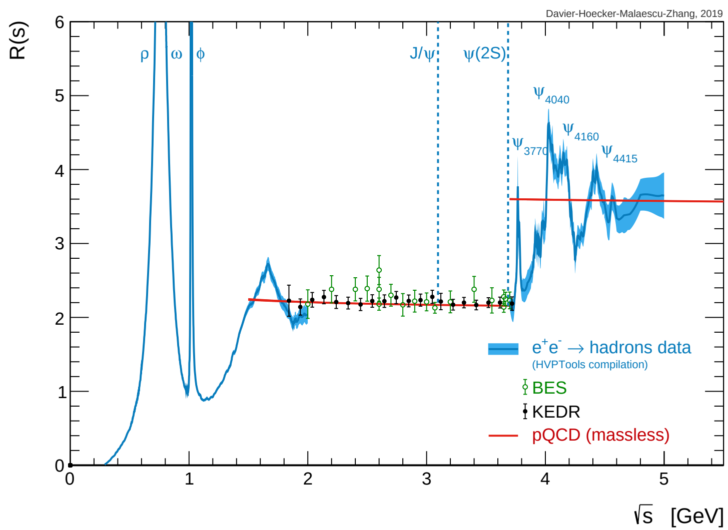
$$R_{\rm had}(s) = \frac{\sigma^0(e^+e^- \to \gamma^* \to \text{hadrons})}{(4\pi\alpha^2/3s)}$$



- $\widehat{K}(s)$ is a known kernel function

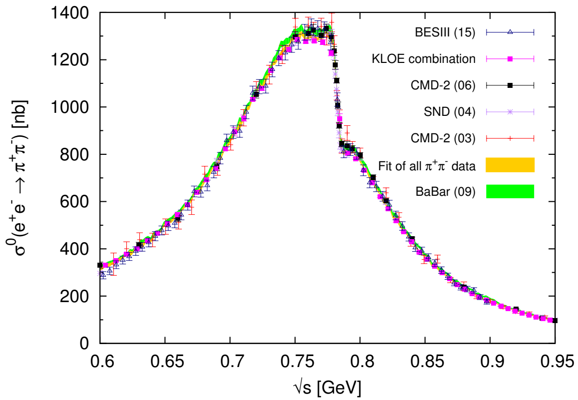
$$a_\mu^{\rm HVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left\{ \int_{m_\pi^2}^{E_{\rm cut}^2} {\rm d}s \frac{R_{\rm had}^{\rm data}(s)\widehat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} {\rm d}s \frac{R_{\rm had}^{\rm pQCD}(s)\widehat{K}(s)}{s^2} \right\}$$

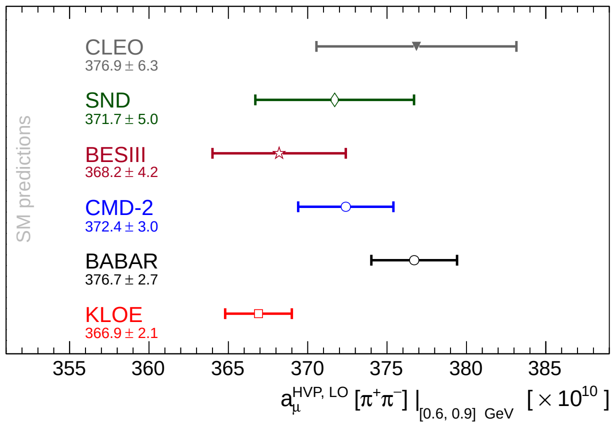
- Compilation of experimental data from many experiments



Hadronic vacuum polarization and dispersive theory

- Subject to experimental uncertainties : careful propagation of experimental uncertainties
  - $\rightarrow$ Groups with $\neq$ methodologies are in good agreement [Davier et al. '19] [Keshavarzi et al. '20]
  - $\rightarrow$ But local discrepancies (tensions already there in the experimental data) !
  - $\rightarrow$ Problematic for the dominant $\pi\pi$ channel





Difference between BABAR and KLOE : $\Delta a_\mu = 9.8(3.4) \times 10^{-10}$

Difference pheno / exp for the $g-2$: $\Delta a_\mu = 28(8) \times 10^{-10}$

- White paper average for the dispersive approach

$$a_\mu^{\rm hvp} = 693.1(2.8)_{\rm stat}(0.7)_{\rm DV+QCD}(2.8)_{\rm KLOE/BABAR} \times 10^{-10} \quad [0.58\%]$$