Muon $g-2$ puzzle

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Muon $g-2$ puzzles

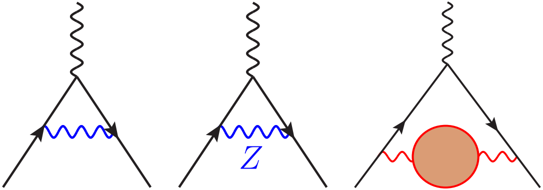
What is the **muon $g-2$ puzzle**?

- 4.2σ tension between experiment talk by H. Binney and SM prediction based on $e^+e^- \to$ hadrons data Aoyama et al. 2020
- 2.1σ (3.7σ) tension between e^+e^- data and lattice-QCD calculation by BMWc 2020
  $\hookrightarrow$ talk by A. Gérardin
- Tensions in electroweak fit and low-energy hadron phenomenology if HVP is changed substantially
- BSM implications

This talk:

- Review of data-driven SM prediction
- Discussion of all these "puzzles"

Anomalous magnetic moment of the electron



- **SM prediction for $(g-2)_\ell$**

$$a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{EW}} + a_\ell^{\text{had}}$$

- For electron: electroweak and hadronic contributions under control
- For a precision calculation need:
  - Independent input for α
  - Higher-order QED contributions

Anomalous magnetic moment of the electron: QED

- QED expansion

$$a_e^{\text{QED}} = A_1 + A_2\left(\frac{m_e}{m_\mu}\right) + A_2\left(\frac{m_e}{m_\tau}\right) + A_3\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right)$$

$$A_i = \left(\frac{\alpha}{\pi}\right)A_i^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A_i^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A_i^{(6)} + \cdots$$

- Numerical calculation up to five loops Aoyama, Kinoshita, Nio
- Recent developments
  - Analytic cross check of $A_{2,3}$ at 4 loops Kurz et al. 2014
  - Semi-analytic calculation of A_1 at 4 loops Laporta 2017
  - Independent calculation of 5-loop coefficient Volkov 2019

$$A_1^{(10)}\Big|_{\text{no lepton loops, AKN}} = 7.668(159) \qquad A_1^{(10)}\Big|_{\text{no lepton loops, Volkov}} = 6.793(90)$$

$\hookrightarrow$ 4.8σ difference

- Five-loop coefficient not an issue right now, but will become important in the future

Anomalous magnetic moment of the electron: fine-structure constant

- Input from **atom interferometry**

$$\alpha^2 = \frac{4\pi R_\infty}{c} \times \frac{m_\text{atom}}{m_e} \times \frac{\hbar}{m_\text{atom}}$$

- With **Rb measurement** LKB 2011

$$a_e^\text{exp} = 1{,}159{,}652{,}180.73(28) \times 10^{-12}$$

$$a_e^\text{SM} = 1{,}159{,}652{,}182.03(1)_\text{5-loop}(1)_\text{had}(72)_{\alpha(\text{Rb})} \times 10^{-12}$$

$$a_e^\text{exp} - a_e^\text{SM} = -1.30(77) \times 10^{-12}[1.7\sigma]$$

$\hookrightarrow$ α limiting factor, but more than an order of magnitude to go in theory

Anomalous magnetic moment of the electron: fine-structure constant

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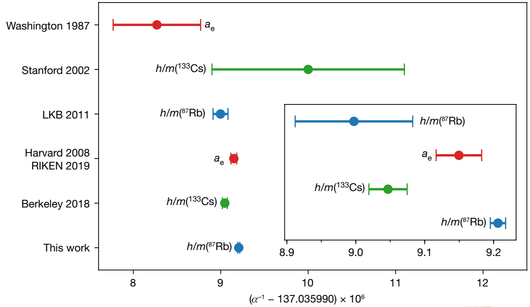
$\hookrightarrow$ α limiting factor, but more than an order of magnitude to go in theory

- With **Cs measurement** Berkeley 2018, Science 360 (2018) 191

$$\begin{aligned} a_e^{\text{SM}} &= 1{,}159{,}652{,}181.61(1)_{\text{5-loop}}(1)_{\text{had}}(23)_{\alpha(\text{Cs})} \times 10^{-12} \\ a_e^{\text{exp}} - a_e^{\text{SM}} &= -0.88(36) \times 10^{-12}[2.5\sigma] \end{aligned}$$

$\hookrightarrow$ for the first time a_e^{exp} limiting factor

Anomalous magnetic moment of the electron: fine-structure constant



LKB 2020

- Tensions
  - Berkeley 2018 vs. LKB 2020: 5.4σ
  - LKB 2011 vs. LKB 2020: 2.4σ

- With new **Rb measurement** LKB 2020, Nature 588 (2020) 61

$$a_e^{\text{SM}} = 1{,}159{,}652{,}180.25(1)_{\text{5-loop}}(1)_{\text{had}}(9)_{\alpha(\text{Rb})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = 0.48(30) \times 10^{-12} [1.6\sigma]$$

$\hookrightarrow$ on the opposite side of a_e^{exp}!

What does this mean for BSM?

- There seems to be an experimental issue in the determination of α
- Expectations from a_μ, depending on **mass scaling**:
  - m_ℓ^2: $a_e^{\text{BSM}} \sim 0.065(18) \times 10^{-12}$
  - m_ℓ: $a_e^{\text{BSM}} \sim 13.5(3.7) \times 10^{-12}$
- Compare to
  - LKB 2020 sensitivity: 0.095×10^{-12}
  - LKB 2020 vs. Berkeley 2018: $1.36(25) \times 10^{-12}$
  - LKB 2020 vs. a_e^{exp}: $0.48(30) \times 10^{-12}$

↪ LKB 2020 close to quadratic regime, but the tensions start much earlier

- Situation unclear, improved a_e^{exp} all the more important Gabrielse

The Standard Model prediction for $(g-2)_\mu$: QED

- **5-loop QED** result Aoyama, Kinoshita, Nio 2018:

$$a_\mu^{\text{QED}} = 116\,584\,719.0(1) \times 10^{-11}$$

$\hookrightarrow$ insensitive to input for α (at this level)

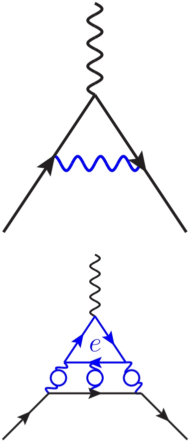
- QED coefficients enhanced by $\log m_\mu/m_e$
- Enhancement from naive RG expectation for 6-loop QED

$$10 \times \frac{2}{3}\pi^2 \log\frac{m_\mu}{m_e} \times \left(\frac{2}{3}\log\frac{m_\mu}{m_e}\right)^3 \sim 1.6 \times 10^4$$

$\hookrightarrow$ would imply $a_\mu^{\text{6-loop}} \sim 0.2 \times 10^{-11}$

- Refined RG estimate Aoyama, Hayakawa, Kinoshita, Nio 2012

$$a_\mu^{\text{6-loop}} \sim 0.1 \times 10^{-11}$$



The Standard Model prediction for $(g-2)_\mu$: electroweak

- Electroweak contribution Gnendiger et al. 2013

$$a_\mu^{\text{EW}} = (194.8 - 41.2) \times 10^{-11} = 153.6(1.0) \times 10^{-11}$$

- Remaining uncertainty dominated by $q = u, d, s$ loops

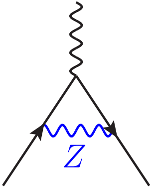
  ↪ nonperturbative effects Czarnecki, Marciano, Vainshtein 2003

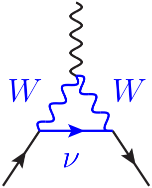
- Two-loop calculation recently revisited without asymptotic expansion Ishikawa, Nakazawa, Yasui 2019

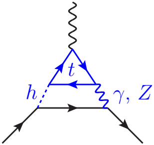
$$a_\mu^{\text{EW}} = 152.9(1.0) \times 10^{-11}$$

- 3-loop corrections?
  - 3-loop RG estimate accidentally cancels in scheme chosen by Gnendiger et al. 2013, with an error of 0.2×10^{-11}
  - α_s corrections to t-loop should scale as

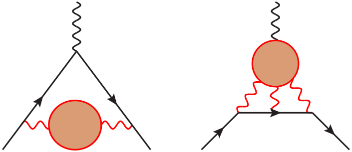
$$a_\mu^{t\text{-loop}}\big|_{\text{2-loop}} \times \frac{\alpha_s}{\pi} \lesssim 0.3 \times 10^{-11}$$







The Standard Model prediction for $(g-2)_\mu$: hadronic effects



- **Hadronic vacuum polarization**: need hadronic two-point function

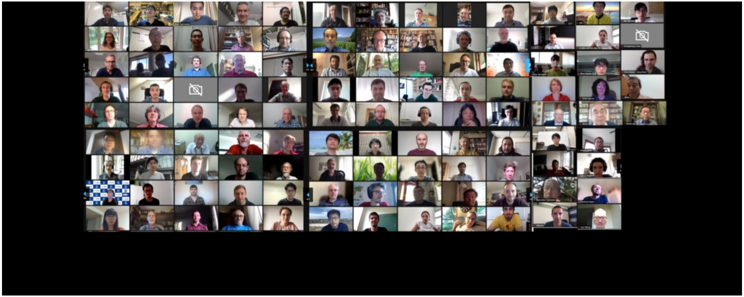
$$\Pi_{\mu\nu} = \langle 0|T\{j_\mu j_\nu\}|0\rangle$$

- **Hadronic light-by-light scattering**: need hadronic four-point function

$$\Pi_{\mu\nu\lambda\sigma} = \langle 0|T\{j_\mu j_\nu j_\lambda j_\sigma\}|0\rangle$$

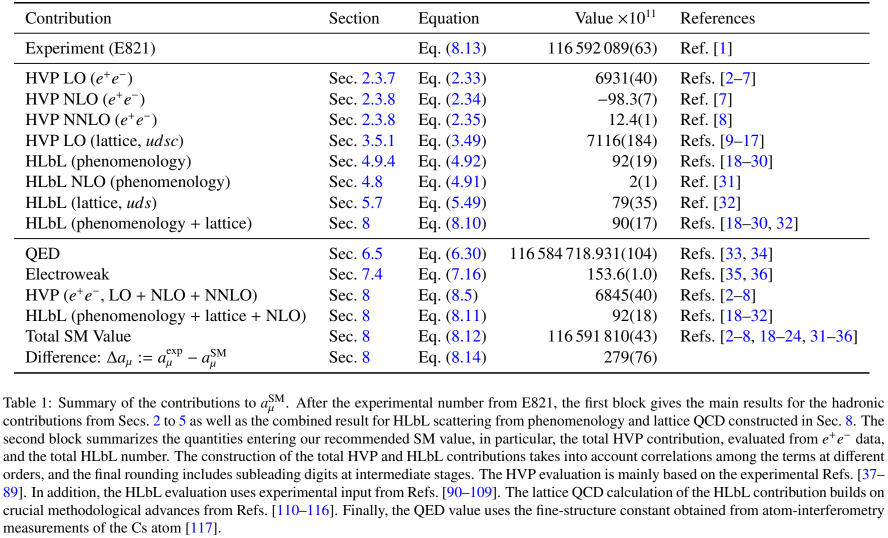
- Main challenge: how to evaluate the hadronic contributions

The Muon $g-2$ Theory Initiative



- Formed in 2017, series of workshops since (last plenary one virtually at KEK in June 2021) `https://www-conf.kek.jp/muong-2theory/`
- Map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results
- White paper 2006.04822: **The anomalous magnetic moment of the muon in the Standard Model** `https://muon-gm2-theory.illinois.edu/`

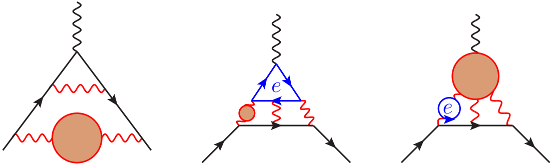
The anomalous magnetic moment of the muon in the Standard Model



Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

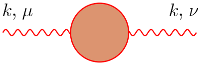
The Standard Model prediction for $(g-2)_\mu$: higher-order hadronic effects



- Once $\Pi_{\mu\nu}$ and $\Pi_{\mu\nu\lambda\sigma}$ known, higher-order iterations determined
- Standard for NLO HVP Calmet et al. 1976
- NNLO HVP found to be relevant recently Kurz et al. 2014
- NLO HLbL already further suppressed Colangelo et al. 2014
- Mixed leptonic and hadronic corrections at $\mathcal{O}(\alpha^4)$ small MH, Teubner 2021

Hadronic vacuum polarization

- General principles yield **direct connection with experiment**
    - **Gauge invariance**



$$= -i\big(k^2 g^{\mu\nu} - k^\mu k^\nu\big)\Pi\big(k^2\big)$$

-
    - **Analyticity**

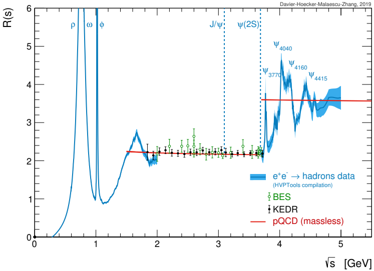
$$\Pi_{\text{ren}} = \Pi\big(k^2\big) - \Pi(0) = \frac{k^2}{\pi}\int\limits_{4M_\pi^2}^{\infty} \mathrm{d}s \frac{\operatorname{Im}\Pi(s)}{s\big(s-k^2\big)}$$

-
    - **Unitarity**

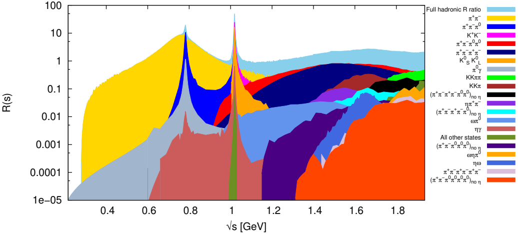
$$\operatorname{Im}\Pi(s) = -\frac{s}{4\pi\alpha}\sigma_{\text{tot}}\big(e^+e^- \to \text{hadrons}\big) = -\frac{\alpha}{3}R(s)$$

- 1 Lorentz structure, 1 kinematic variable, no free parameters
- **Dedicated e^+e^- program** under way, new results from SND (published), CMD3, BaBar, BESIII, Belle II soon

Hadronic vacuum polarization from e^+e^- data



Davier, Hoecker, Malaescu, Zhang 2019



Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
    - up to about 2 GeV: sum of exclusive channels
    - above: inclusive data + narrow resonances + pQCD
- **Tensions in the data**: most notably between KLOE and BaBar 2π data

Breakdown of the HVP error

HVP from e^+e^- data

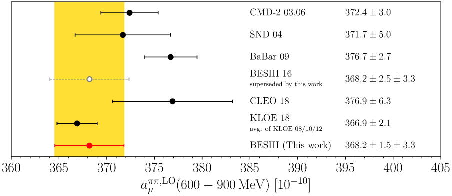
$$a_\mu^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error

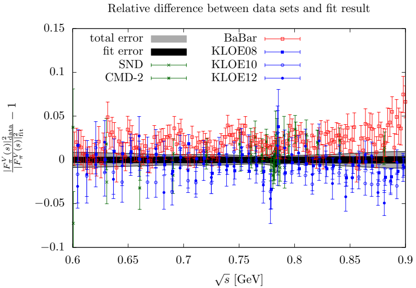
  $\hookrightarrow$ would get $4.2\sigma \to 4.8\sigma$ when ignoring additional systematic error
- There was broad consensus to adopt **conservative error estimates**

  $\hookrightarrow$ **merging procedure** in WP20 covers tensions in the data and different methodologies for the combination of data sets
- Systematic effect dominated by [fit w/o KLOE - fit w/o BaBar]/2

Cross checks from analyticity and unitarity



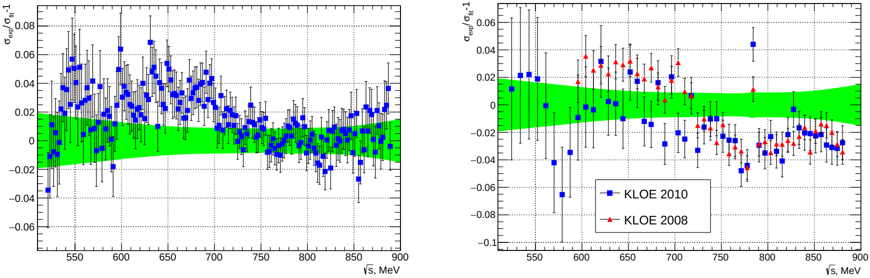
BESIII 2009.05011



Colangelo, MH, Stoffer 2018

- For “simple” channels $e^+e^- \to 2\pi, 3\pi$ can derive form of the cross section from **general principles of QCD** (analyticity, unitarity, crossing symmetry)
  ↪ strong cross check on the data sets (covering about 80% of HVP)
- Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests

New data since WP20



BaBar vs. SND 20 2004.00263 KLOE vs. SND 20

- New data from SND experiment not yet included in WP20 number

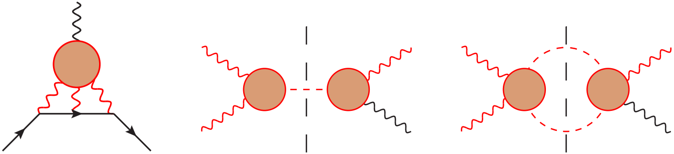
  $\hookrightarrow$ lie between BaBar and KLOE

- **More $\pi\pi$ data to come** from: CMD3, BESIII, BaBar, Belle II

- New data on 3π: BESIII, BaBar

- **MUonE project**: extract space-like HVP from μe scattering

HLbL scattering: white paper



- **Uncertainty due to HLbL scattering** arguably played a major role in BNL experiment being discontinued

  ↪ new development: **data-driven dispersive methods** in analogy to HVP
- Strategy in the white paper
  - Take well-controlled results for the dominant low-energy contributions
  - Generous estimate for uncertainty due to subleading contributions
- **Recommended value**: $a_\mu^{\text{HLbL}}\text{(phenomenology)} = 92(19)\times 10^{-11}$
- **Lattice QCD** RBC/UKQCD 2019: $a_\mu^{\text{HLbL}}(\text{lattice}, \mathit{uds}) = 79(35)\times 10^{-11}$

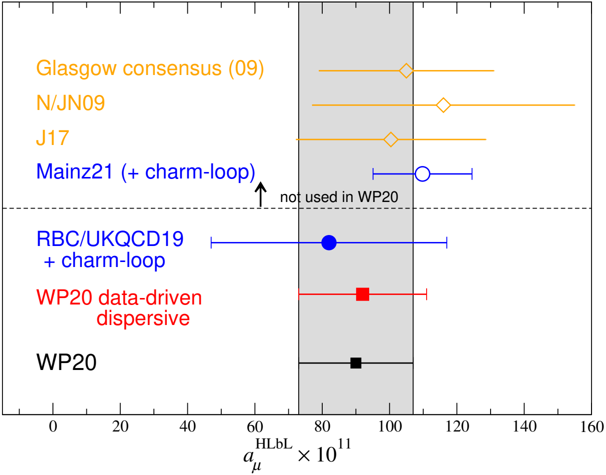
  ↪ can combine with phenomenological value more recent calculation Mainz 21

HLbL scattering: white paper details

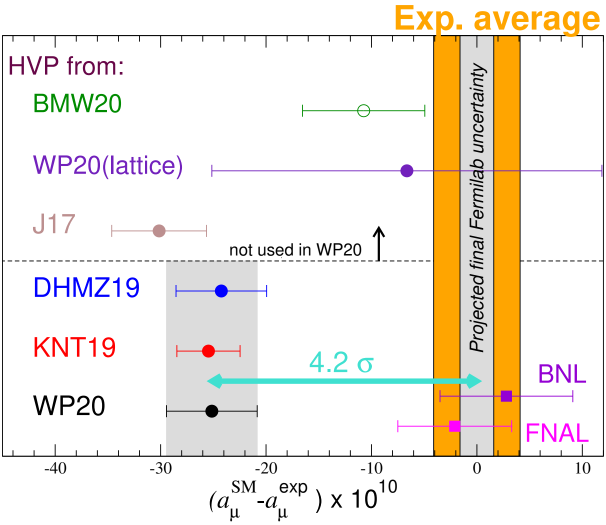
Contribution	PdRV(09)	N/JN(09)	J(17)	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	$-19(19)$	$-19(13)$	$-20(5)$	$-16.4(2)$
S-wave $\pi\pi$ rescattering	$-7(7)$	$-7(2)$	$-5.98(1.20)$	$-8(1)$
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	$-1(3)$
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

All to be compared to projected **final E989 precision**: $\Delta a_\mu^{\text{E989}} = 16 \times 10^{-11}$

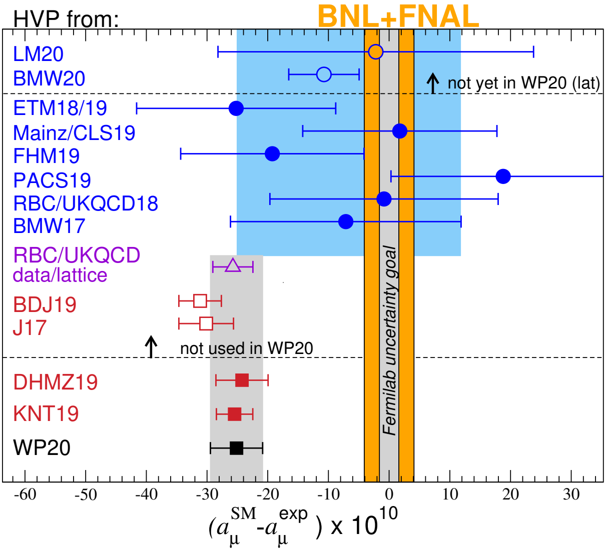
Status of HLbL scattering



The situation after the Fermilab announcement



The situation after the Fermilab announcement: a closer look



HVP from lattice QCD

	e^+e^- from WP	lattice average from WP	BMWc v3
$a_\mu^{\text{HVP, LO}} \times 10^{11}$	6 931(40)	7 116(184)	7 075(55)
difference to e^+e^-		1.0σ	2.1σ
tension with experiment	4.2σ	0.4σ	1.5σ

- Calculation from BMWc in tension with e^+e^- data
- How can we test this result?
  - Independent lattice calculations at same level of accuracy
  - Hadronic running of α
  - Correlations with low-energy hadron phenomenology

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

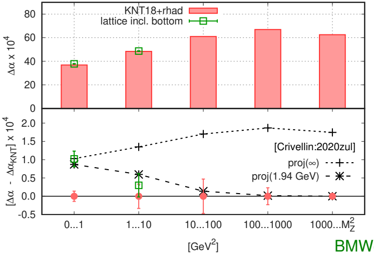
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

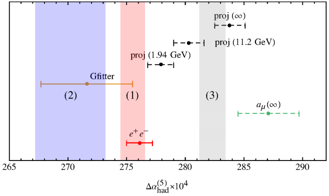
- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\big(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\big)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**

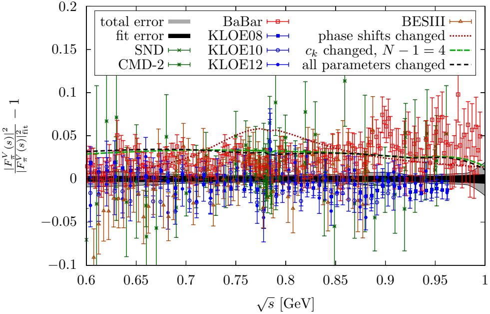


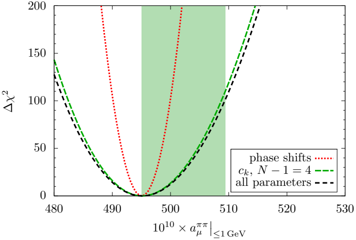
BMWc 2020



Crivellin, MH, Manzari, Montull 2020

Changing the $\pi\pi$ cross section below 1 GeV



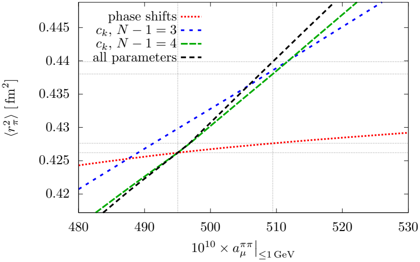


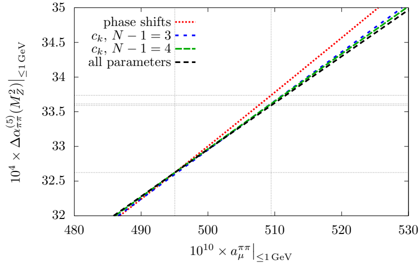
Colangelo, MH, Stoffer 2020

- Changes in 2π cross section **cannot be arbitrary** due to analyticity/unitarity constraints, but increase is actually possible
- Three scenarios:
  1. "Low-energy" scenario: **$\pi\pi$ phase shifts**
  2. "High-energy" scenario: **conformal polynomial**
  3. Combined scenario

$\hookrightarrow$ 2. and 3. lead to uniform shift, 1. concentrated in ρ region

Correlations





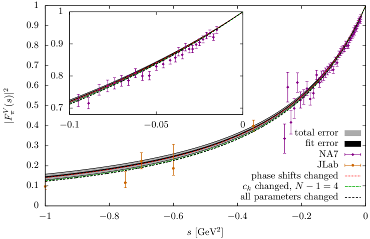
Correlations with other observables:

- **Pion charge radius $\langle r_\pi^2\rangle$**

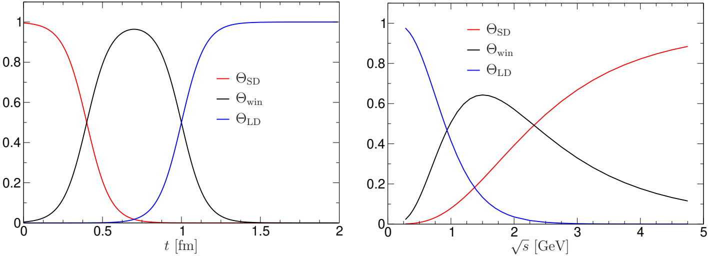
  ↪ significant change in scenarios 2. and 3.

  ↪ can be tested in lattice QCD

- **Hadronic running of α**
- **Space-like pion form factor**



Window quantities



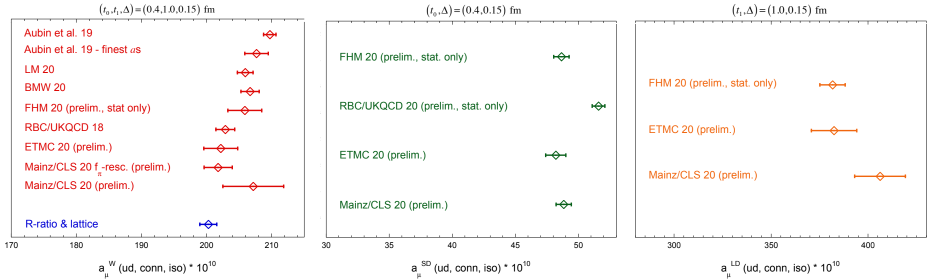
- **Weight functions in Euclidean time** proposed by RBC/UKQCD 2018, see talk by A. Gérardin

  ↪ long-distance, intermediate, and short-distance window
- For intermediate window $a_\mu^{\text{int}}[\text{RBC/UKQCD}] = 231.9(1.5) \times 10^{-10}$ and $a_\mu^{\text{int}}[\text{BMWc}] = 236.7(1.4) \times 10^{-10}$ **differ by 2.3σ**
- Difference between BMWc and e^+e^- in intermediate window is **3.7σ**, but $\pi\pi$ channel below 1 GeV split 69 : 28 : 3, relevant changes above 1 GeV?
- Detailed study of windows key tool for **comparison among lattice and with e^+e^-**

Crosschecks

“Window” quantities

(Plots from Davide Giusti)



- Straightforward reference quantities
- Can be applied to individual contributions (light, strange, charm, disconnected,…)
- Comparison with e^+e^-/ R-ratio may require tuning of the window

Summary talk by H. Wittig at Muon $g-2$ Theory Initiative virtual workshop, Nov 2020
“The hadronic vacuum polarization from lattice QCD at high precision” `https://indico.cern.ch/event/956699/`

BSM: general remarks

- **BSM effect sizable**

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11} > a_\mu^{\text{EW}}$$

- Requires some form of enhancement:
  - **Chiral enhancement**: chirality flip $\propto m_\mu^2$ in SM

    $\hookrightarrow$ enhancement by $\tan\beta \sim 50$ in SUSY, $m_t/m_\mu \sim 1600$ in leptoquark models
  - **Light BSM**: axion-like particles, Z', $L_\mu - L_\tau$, light scalars
- Connections to other recent hints for the **violation of lepton flavor universality**?
  - B anomalies: $b \to s\ell\ell$ ($R(K^{(*)}$, P_5', $\ldots$), $b \to c\tau\nu$ ($R(D^{(*)}$)
  - First-row CKM unitarity, CMS dilepton data
  - Anomalous magnetic moment of the electron (?)

BSM: many possible models

13:40 CEST
14:25 CEST
15:00 CEST
15:10 CEST
15:45 CEST
15:55 CEST
16:30 CEST
16:40 CEST

Monday (31/05)

Data-driven evaluations of a_μ^{HVP} : Introduction, basics and main features
T. Teubner
15:00 - 15:45
SM/HVP session
slides

Aspects of the data-driven evaluation of HVP
M. Hoferichter
15:45 - 16:30
SM/HVP session
slides

Muon g-2 and $\Delta\alpha$ connection
M. Passera
16:30 - 17:15
SM/HVP session
slides

Tuesday (01/06)

g-2 and lepton flavour universality violation
A. Crivellin
13:40 - 14:25
B physics session
slides

Muon g-2 and B anomalies from Dark Matter
L. Calibbi
14:25 - 15:10
B physics session
slides

A model of muon anomalies
A. Greljo
15:10- 15:55
B physics session
slides

Leptoquark for (g-2) and B-meson anomalies
S.C. Park
15:55 - 16:40
B physics session
slides

TU Dresden colloquium

Wednesday (02/06)

g-2 in the general MSSM
D. Stöckinger
15:00 - 15:45
SUSY session
slides

The Tiny (g-2) Muon Wobble from Small-μ Supersymmetry
N. Shah
15:45 - 16:30
SUSY session
slides

Anomalous muon magnetic moment, supersymmetry,naturalness, LHC search limits and the landscape

Thursday (03/06)

Minimal models for g-2 and dark matter confront asymptotic safety
K. Kowalska
15:00 - 15:45
non-SUSY session
slides

Leptophilic bosons and muon g-2 in 2HDM
E.J. Chun
15:45 - 16:30
non-SUSY session
slides

Naturalness, the deus-ex-machina of the muon g-2 anomaly and lepton non-universality

Friday (04/06)

Constraints on “invisible” Feebly-Interacting Particles and g-2 anomalies
L. Darmé
15:00 - 15:45
low-energy session
slides

Challenges for an axion explanation for muon g-2
J. Fan
15:45 - 16:30
low-energy session
slides

Muon and electron g-2, proton and cesium weak charges implications on dark Z_d models

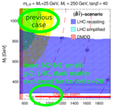
`http://pheno.csic.es/g-2Days21/program/`

BSM: landscape of models

There are many more examples. . .

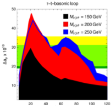
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



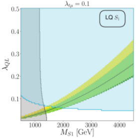
Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



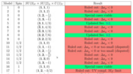
Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)

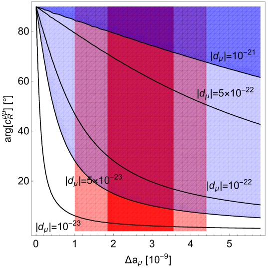


Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result
1	0	$(\mathbf{1},\mathbf{1},1)$	Ruled out: $\Delta a_\mu < 0$
2	0	$(\mathbf{1},\mathbf{1},2)$	Ruled out: $\Delta a_\mu < 0$
3	0	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	Special cases viable
4	0	$(\mathbf{1},\mathbf{3},-1)$	Ruled out: $\Delta a_\mu < 0$
5	0	$(\overline{\mathbf{3}},\mathbf{1},1/3)$	Updated Sec. 3.2
6	0	$(\overline{\mathbf{3}},\mathbf{1},4/3)$	Ruled out: $\Delta a_\mu < 0$
7	0	$(\overline{\mathbf{3}},\mathbf{3},1/3)$	Ruled out: $\Delta a_\mu < 0$
8	0	$(\mathbf{3},\mathbf{2},7/6)$	Updated Sec. 3.2
9	0	$(\mathbf{3},\mathbf{2},1/6)$	Ruled out: $\Delta a_\mu < 0$
10	1/2	$(\mathbf{1},\mathbf{1},0)$	Ruled out: $\Delta a_\mu < 0$
11	1/2	$(\mathbf{1},\mathbf{1},-1)$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
12	1/2	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
13	1/2	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	Ruled out: $\Delta a_\mu < 0$
14	1/2	$(\mathbf{1},\mathbf{3},0)$	Ruled out: $\Delta a_\mu < 0$
15	1/2	$(\mathbf{1},\mathbf{3},-1)$	Ruled out: $\Delta a_\mu < 0$
16	1	$(\mathbf{1},\mathbf{1},0)$	Special cases viable
17	1	$(\mathbf{1},\mathbf{2},-3/2)$	Ruled out: UV compl. $M_{Z'}$ limit

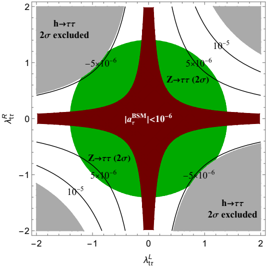
[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

Possibly related measurements

- **Muon EDM**
  - Sizable for $\mathcal{O}(1)$ BSM phase
  - Could be accessible at dedicated PSI experiment, projected to reach $|d_\mu| \simeq 5 \times 10^{-23}\, e\,\text{cm}$ Adelmann et al. 2021
- **Electron/tau $g-2$**
  - Electron $g-2$ requires resolution of conflicting α measurements from Rb and Cs
  - Tau difficult, best bet polarization upgrade at Belle II $\hookrightarrow$ interesting parameter space starts at $|a_\tau^{\text{BSM}}| \lesssim 5 \times 10^{-6}$
  - Strategy via asymmetry measurements Bernabéu et al. 2008, see talk by Caleb Miller on Fr., 19:15

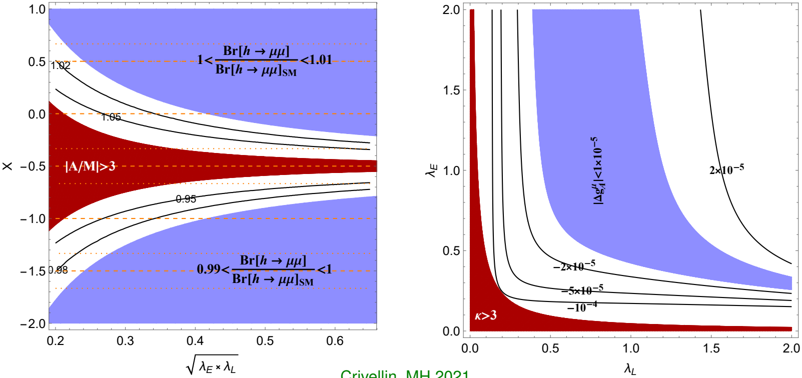


Crivellin, MH, Schmidt-Wellenburg 2018



Crivellin, MH, Roney 2021

Possibly related measurements



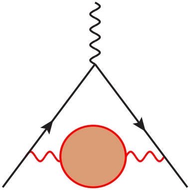
Crivellin, MH 2021

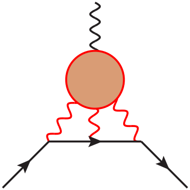
- ***h* → $\mu\mu$ and *Z* → $\mu\mu$**
  - If a_μ^{BSM} due to chiral enhancement, also $h \to \mu\mu$ and $Z \to \mu\mu$ affected
  - Effect mainly depends on $SU(2)_L$ representations and hypercharge of new particles

    ↪ simplified models
  - Could be tested at future colliders

Conclusions

- **Electron $g-2$**
  - Quo vadis α?
- **Hadronic vacuum polarization**
  - Presently largest systematic uncertainty in $\pi\pi$ channel
  - Dispersive analysis to consolidate error estimate
  - Ultimately new data required: CMD3, BaBar, BESIII, Belle II
  - New lattice calculations soon
- **Hadronic light-by-light scattering**
  - Use dispersion relations to remove model dependence as far as possible
  - Good agreement between phenomenology and lattice
- **BSM implications**
  - Possible mechanisms: chiral enhancement, light BSM
  - Connection to B anomalies?





Lepton dipole moments: experimental status

- Dipole moments: definition

$$\mathcal{H} = -\boldsymbol{\mu}_\ell \cdot \boldsymbol{B} - \boldsymbol{d}_\ell \cdot \boldsymbol{E}$$

$$\boldsymbol{\mu}_\ell = -g_\ell \frac{e}{2m_\ell}\boldsymbol{S} \qquad \boldsymbol{d}_\ell = -\eta_\ell \frac{e}{2m_\ell}\boldsymbol{S} \qquad a_\ell = \frac{g_\ell - 2}{2}$$

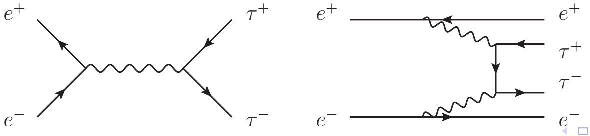
- Anomalous magnetic moments Hanneke et al. 2008, Bennett et al. 2006, Abi et al. 2021

$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.73(28) \times 10^{-12} \qquad a_\mu^{\text{exp}} = 116{,}592{,}061(41) \times 10^{-11}$$

- Electric dipole moments Andreev et al. 2018, Bennett et al. 2009

$$|d_e| < 1.1 \times 10^{-29}\, e\,\text{cm} \qquad |d_\mu| < 1.5 \times 10^{-19}\, e\,\text{cm} \qquad 90\%\,\text{C.L.}$$

- Not much known about τ dipole moments, some limits from



EFT analysis

- **Effective dipole operators** $\mathcal{H}_{\text{eff}} = c_R^{\ell_f \ell_i}\, \bar{\ell}_f \sigma_{\mu\nu} P_R \ell_i F^{\mu\nu} + \text{h.c.}$

$$a_\ell = -\frac{4m_\ell}{e}\,\text{Re}\, c_R^{\ell\ell} \qquad d_\ell = -2\,\text{Im}\, c_R^{\ell\ell} \qquad \text{Br}[\mu \to e\gamma] = \frac{m_\mu^3}{4\pi\,\Gamma_\mu}\left(|c_R^{e\mu}|^2 + |c_R^{\mu e}|^2\right)$$

$\hookrightarrow$ in general only one power in m_ℓ for a_ℓ

- Consequences
  - Phase of c_R^{ee} much better constrained than phase of $c_R^{\mu\mu}$

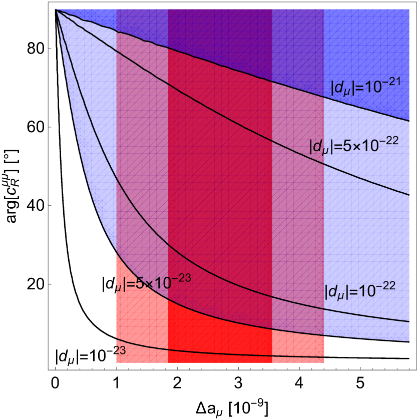
$$\left|\frac{\text{Im}\, c_R^{ee}}{\text{Re}\, c_R^{ee}}\right| \lesssim 6 \times 10^{-7} \qquad \left|\frac{\text{Im}\, c_R^{\mu\mu}}{\text{Re}\, c_R^{\mu\mu}}\right| \lesssim 600$$

  - If $c_R^{e\mu} = \sqrt{c_R^{ee} c_R^{\mu\mu}}$, e.g. for single-particle solutions with chiral enhancement

$$\text{Br}[\mu \to e\gamma] = \frac{\alpha m_\mu^2}{16 m_e \Gamma_\mu}|\Delta a_\mu \Delta a_e| \sim 8 \times 10^{-5}$$

$\hookrightarrow$ violates **MEG bound** $\text{Br}[\mu \to e\gamma] < 4.2 \times 10^{-13}$ by 8 orders of magnitude!

Future measurements of the muon EDM

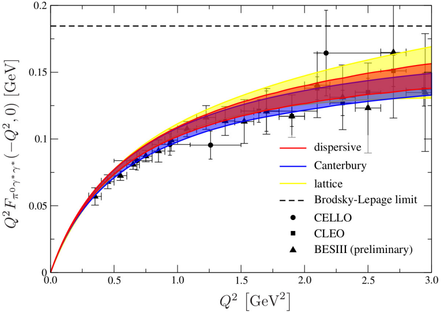


Crivellin, MH, Schmidt-Wellenburg 2018

- Current limit from E821: $|d_\mu| < 1.5 \times 10^{-19}\, e\,\text{cm}$
- Fermilab/J-PARC $(g-2)_\mu$ experiments will be sensitive to $|d_\mu| \sim 10^{-21}\, e\,\text{cm}$
- Proposal for a dedicated muon EDM experiment at PSI, could reach

  $|d_\mu| \sim 5 \times 10^{-23}\, e\,\text{cm}$

HLbL scattering: pion pole



- Pion pole from data MH et al. 2018, Masjuan, Sánchez-Puerto 2017 and lattice QCD Gérardin et al. 2019

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{dispersive}} = 63.0^{+2.7}_{-2.1}\times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{Canterbury}} = 63.6(2.7)\times 10^{-11}$$

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice+PrimEx}} = 62.3(2.3)\times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice}} = 59.7(3.6)\times 10^{-11}$$

↪ agree within uncertainties well below Fermilab goal

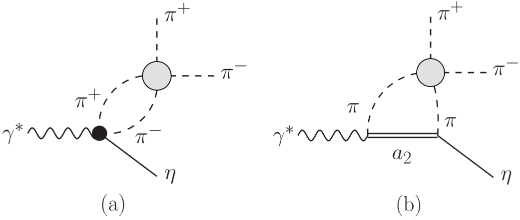
- Singly-virtual results agree well with BESIII measurement

HLbL scattering: subleading contributions

- **Subleading contributions**
  1. η, η' poles
  2. Subleading two-pion and multi-hadron intermediate states

     $\hookrightarrow$ narrow-resonance description
  3. Short-distance constraints and their implementation
- In the following: brief review of status and prospects
- For more details: see talks by J. Bijnens, G. Colangelo, B. Kubis, A. Rebhan, P. Stoffer at recent meeting Muon $g-2$ Theory Initiative meeting (virtual at KEK) `https://www-conf.kek.jp/muong-2theory/`

HLbL scattering: η, η' poles



Holz et al. 2021

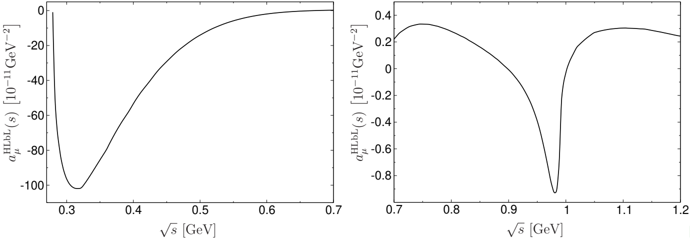
- So far only based on **Canterbury approximants** Masjuan, Sánchez-Puerto 2017

$$a_\mu^{\eta\text{-pole}}\Big|_{\text{Canterbury}} = 16.3(1.4)\times 10^{-11} \qquad a_\mu^{\eta'\text{-pole}}\Big|_{\text{Canterbury}} = 14.5(1.9)\times 10^{-11}$$

- Impact of **factorization-breaking terms** not well understood: in general $F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) \neq F(q_1^2)F(q_2^2)$
- Can be cross checked with data on $e^+e^- \to \eta\pi\pi$ Holz et al. 2021

  $\hookrightarrow$ need more differential data to ascertain role of **left-hand cut from a_2 diagram**

HLbL scattering: scalar contributions



Danilkin, MH, Stoffer 2021

- Single-particle poles in general depend on the **choice of tensor basis**

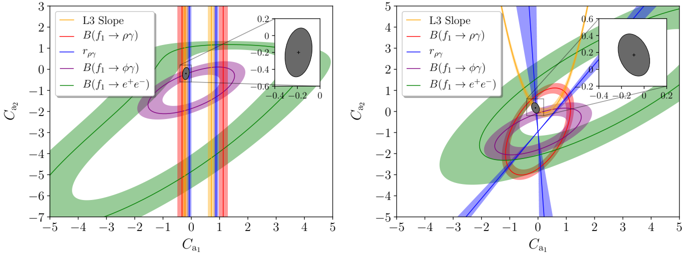
  $\hookrightarrow$ basis independence only ensured by **sum rules** for entire HLbL tensor

- Exception: pseudoscalar poles
- Scalar contributions first non-trivial test case
- For $f_0(500)$ and $f_0(980)$ implementation in terms of $\gamma^*\gamma^* \to \pi\pi/\bar{K}K$

  $\hookrightarrow$ can **compare full and narrow-resonance description** for $f_0(980)$

$$a_\mu^{\text{HLbL}}[f_0(980)]\big|_{\text{rescattering}} = -0.2(1) \times 10^{-11} \qquad a_\mu^{\text{HLbL}}[f_0(980)]\big|_{\text{NWA}} = -0.37(6) \times 10^{-11}$$

HLbL scattering: axial-vector contributions



Zanke, MH, Kubis 2021

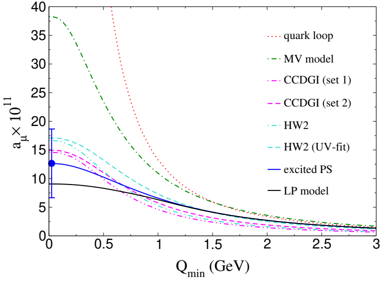
- Challenges regarding axial-vector states
  - Require multi-hadron channels: $a_1 \to 3\pi$, $f_1 \to \eta\pi\pi, \dots$

    $\hookrightarrow$ **narrow-resonance approximation**
  - Limited information on transition form factors

    $\hookrightarrow$ global analysis of f_1 decays Zanke, MH, Kubis 2021, asymptotic constraints MH, Stoffer 2020

    $\hookrightarrow$ **improved measurement of $f_1 \to e^+e^-$ would be valuable**
  - Need tensor basis in which kinematic singularities are manifestly absent

HLbL scattering: short-distance constraints



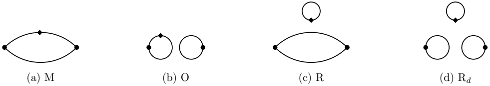
Colangelo et al. 2021

- Open issue how to best **implement the short-distance constraints**
  1. Melnikov–Vainshtein model: anomaly exact in chiral limit, low-energy 2π and 3π cuts missing
  2. Holographic QCD Leutgeb–Rebhan, Cappiello et al. 2019: model for QCD, implementation in terms of axial-vector states
  3. Regge model for excited pseudoscalars Colangelo et al. 2019: individual pseudoscalar contributions not affected by sum rules, but works only away from chiral limit
  4. Interpolation between low- and high-energy constraints Lüdtke, Procura 2020

$\hookrightarrow$ good agreement among 2.–4. for the effect on HLbL

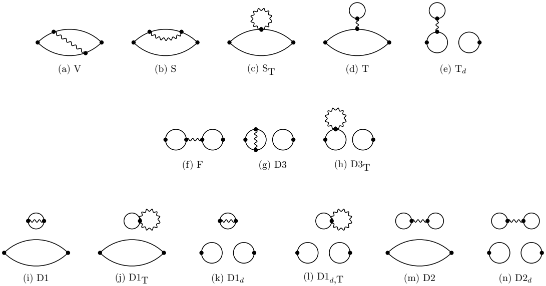
Isospin breaking on the lattice

- **Strong isospin breaking** $\propto m_u - m_d$



(a) M (b) O (c) R (d) R_d

- **QED effects** $\propto \alpha$



(a) V (b) S (c) S_T (d) T (e) T_d

(f) F (g) D3 (h) $D3_T$

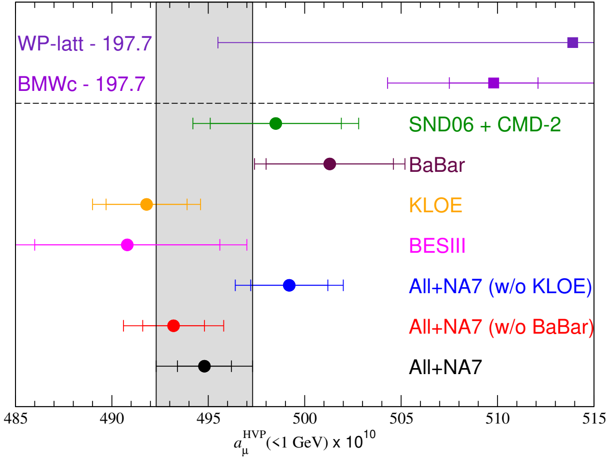
(i) D1 (j) $D1_T$ (k) $D1_d$ (l) $D1_{d,T}$ (m) D2 (n) $D2_d$

plots from Gülpers et al. 2018

- **Matches data-driven convention for leading-order HVP**

  ↪ diagram (f) F without additional gluons is subtracted

$\pi\pi$ contribution below 1 GeV



Assumption: suppose all changes occur in $\pi\pi$ channel below 1 GeV

$\hookrightarrow a_\mu^{\text{total}}[\text{WP20}] - a_\mu^{2\pi,<1\,\text{GeV}}[\text{WP20}] = 197.7 \times 10^{-10}$