Improved study of the ionization efficiency for nuclear recoils in Si and Ge at low energies[1]

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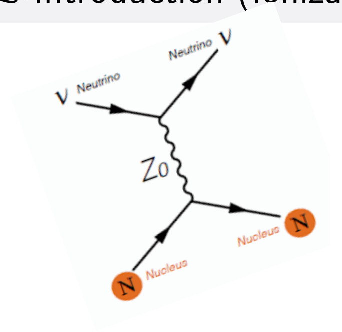
[1] **Sarkis Y.**, Aguilar-Arevalo A., and D'Olivo, J.C., Phys. Atom. Nuclei 84, 590–594 (2021),

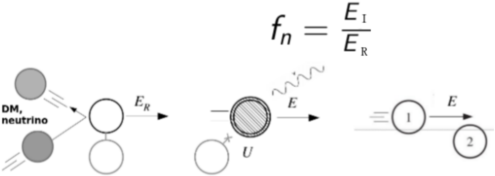
Sarkis Y., Aguilar-Arevalo A., and D'Olivo, J.C.,PhysRevD.101.102001

1. Description of the energy given to atomic motion by a nuclear recoil at low energies (Lindhard+).
2. Electron straggling contribution.
3. Energy dependent binding energy.
4. Modeling Coulomb repulsion effects at low energies.
5. Results fitting Si and Ge QF data.
6. Conclusions.

Useful links:
(A) Magnificent CEνNS(2020)
(B) XXXV DPC-SMF meeting (2021)



E_R : Nuclear Recoil energy.
E_I : Ionization (visible) energy $[\mathrm{keV_{ee}}]$.



- Using dimensionless units ($\varepsilon = 11.5E(\mathrm{keV})/\mathrm{Z}^{7/3}$),

$$quenching = \frac{\text{total ionization energy}}{\text{total deposited energy}} = f_n = \frac{\bar{\eta}}{\varepsilon_R}$$

where $\bar{\eta}$ and ε_R are the ionization energy and the total recoil energy.

- Energy u is lost to some disruption of the atomic bonding: $\varepsilon_R = \varepsilon + u$. The ion moves with a kinetic energy ε.

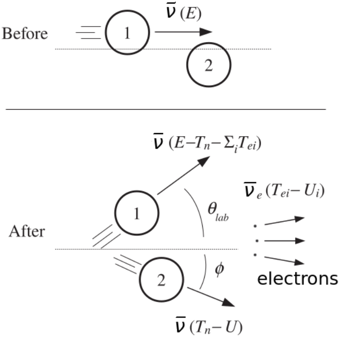
Basic integral equation and approximations

(T_n : Nuclear kinetic energy and T_{ei} electron kinetic energy.)

$$\underbrace{\int d\sigma_{n,e}}_{\textit{total cross section}} \left[\underbrace{\bar{\nu}\left(E - T_n - \sum_i T_{ei}\right)}_{\text{A}} + \underbrace{\bar{\nu}\,(T_n - U)}_{\text{B}} + \underbrace{\bar{\nu}(E)}_{\text{C}} + \underbrace{\sum_i \bar{\nu}_e\,(T_{ei} - U_{ei})}_{\text{D}} \right] = 0 \qquad (1)$$

Lindhard's (five) approximations

1. Neglect contribution to atomic motion coming from electrons.
2. **Neglect the binding energy, U = 0. (Now taken into account)**
3. Energy transferred to electrons is small compared to that transferred to recoil ions.
4. Effects of electronic and atomic collisions can be treated separately.
5. T_n is also small compared to the energy E.



Simplified equation with binding energy

- Relaxing approximations II, III and V.
- Considering U constant, Lindhard $S_e = k\varepsilon^{1/2}$.
- Nuclear stopping $d\sigma_n(t)$ with $t = \varepsilon^2 \sin^2(\theta/2)$.
- We solve for $\bar{\nu}$ then $\bar{\eta} = \varepsilon_R - \bar{\nu}$ and, $f_n = \bar{\eta}/\varepsilon_R$.

(Y. Sarkis et al, Phys. Rev. D 101, 102001 (2020))

$$-\frac{1}{2}k\varepsilon^{3/2}\bar{\nu}''(\varepsilon) + \underbrace{k\varepsilon^{1/2}}_{S_e\,:\,Lindhard}\bar{\nu}'(\varepsilon) = \int_{\varepsilon u}^{\varepsilon^2}\underbrace{dt\frac{f\left(t^{1/2}\right)}{2t^{3/2}}}_{d\sigma_n}[\bar{\nu}(\varepsilon - t/\varepsilon) + \bar{\nu}(t/\varepsilon - u) - \bar{\nu}(\varepsilon)] \quad (2)$$

- Threshold at $\varepsilon_R^{threshold} = 2u$.
- Inter-atomic potential dependent $f(t)$.
- e.g T.F., Moliere, AVG and Ziegler

This work includes improvements to S_e, U and $\bar{\nu}-$equation.

Straggling

- Straggling $\Omega^2 = \langle \delta E - \langle \delta E \rangle \rangle^2$, is an inherent feature of stopping.
- In adimensional units[2]: $\frac{d\Omega^2}{d\rho} \equiv W = \frac{C^2}{\pi a^2} \int_0^E T_n^2 \sigma(T_n) dT_n$.
- Straggling appears when approximation (III) is relaxed up to second order in $(\Sigma_i T_{ei})$.
- Assuming a general electronic stopping power $S_e(\varepsilon)$, the integro-differential equation can be written,

$$-\frac{1}{2}\varepsilon S_e(\varepsilon)\left(1+\frac{W(\varepsilon)}{S_e(\varepsilon)\varepsilon}\right)\bar{\nu}''(\varepsilon)+S_e(\varepsilon)\bar{\nu}'(\varepsilon)= \int_{\varepsilon u}^{\varepsilon^2} dt \frac{f\left(t^{1/2}\right)}{2t^{3/2}}\left[\bar{\nu}(\varepsilon - t/\varepsilon)+\bar{\nu}(t/\varepsilon - u)-\bar{\nu}(\varepsilon)\right], \tag{3}$$

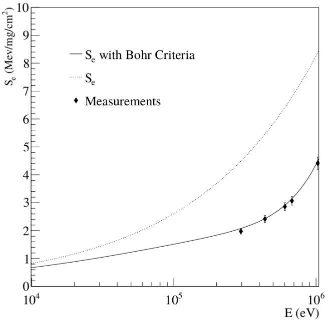
Beyond than using the ratio $\xi(\varepsilon) = S_e(\varepsilon)/S_n(\varepsilon)$ as a measure of the energy dissipation, consider by Lindhard and Bezrukov.

[2] $C = 11.5/Z^{7/3}[\frac{1}{\mathrm{keV}}]$

High energy effects (> 10 keV) for $S_e(\varepsilon)$

§ *Bohr Stripping*

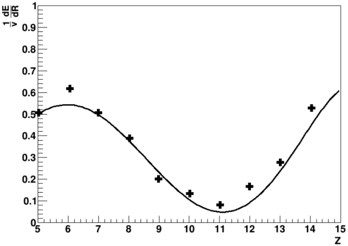
- Electrons can be lost according to momentum transferred.
- The effective number of electrons obeys $Z^\dagger \approx Ze^{-v/Z^{2/3}v_0}$.
- $S_e \propto Z^\dagger$, this leads to damping.



▫ S_e vs data

§ *Z oscillations*

- When the ion charge changes, the transport cross section changes.
- Phase shift is needed appear to maintain neutrality of electron Fermi gas.



▫ Z oscillation for Si.

§ Coulomb repulsion effects

- At low energies S_e departures from velocity proportionality.
- Colliding nuclei will partially penetrate the electron clouds.

$$S_e = Nmv \int_0^{\infty} v_F \sigma_{tr}(v_F) N_e dV \to Nmv \int_R^{\infty} v_F \sigma_{tr}(v_F) N_e dV$$

R distance closest approach

- Three models will be considered; **Tilinin**[3]**, Kishinevsky**[4] **and Arista**[5]
- Models change details of the inter-atomic potential.
- Hence affect $f(t^{1/2})$ and S_e at low energies.

[3] I.S.Tilinin Phys. Rev. A 51, 3058 (1995)

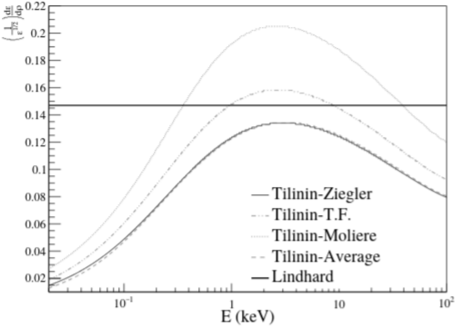
[4] Kishinevsky, L.M., 1962, Izv. Akad. Nauk SSSR, Ser. Fiz. 26, 1410.

[5] J.M. Fernández-Varea, N.R. Arista, Rad. Phy. and C.,V 96, 88-91, (2014),

Low energy effects for S_e

§ *Electronic stopping power*

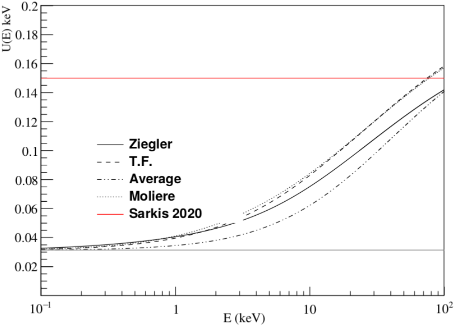
- Computed for T.F., Ziegler, Moliere and Average inter-atomic potentials.



▫ $S_e/\sqrt{\varepsilon}$ for Tilinin model computed with four different inter-atomic potentials

§ *Binding energy*

- Frenkel pair creation energy.
- Atomic binding with T.F theory.



▫ Variable binding energy

Results (Si)

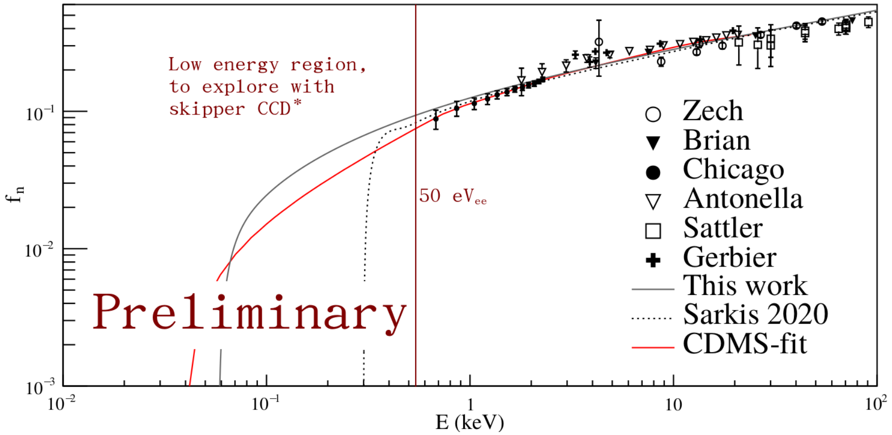


Figure: Silicon QF model compare between Sarkis 2020 with constant binding and with variable binding, straggling low and high energy effects.

* *For more details see G. Ferández Moroni & B. Cervantes Vergara* (CONNIE) talk at session CEνNS Experiments.

Results (Ge) with Collar recent data

For Ge study we have to consider a geometrical factor, mentioned by Tilinin and only significant for high Z ($Z > 20$).

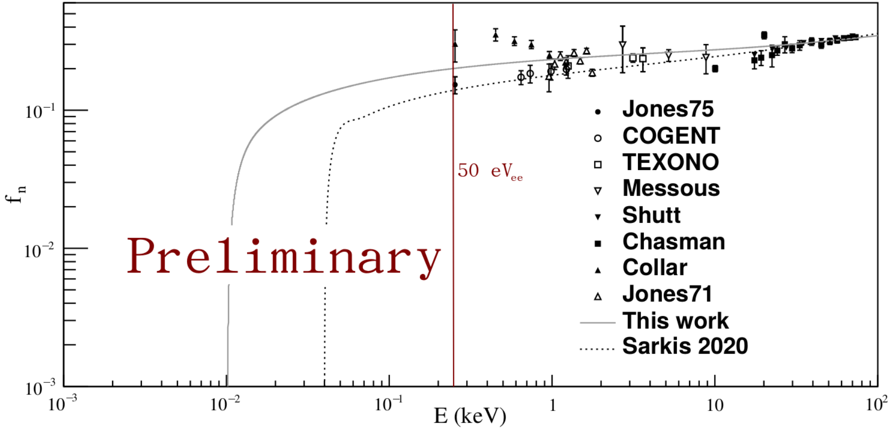


Figure: Germanium QF model with straggling, **geometrical factor**, low and high energy effects.

Conclusions

1. *Considering a variable binding energy and Coulomb repulsion effects allow us to compute the QF an order of magnitude lower than our previous work (down to* ≈ 40 *eV).*
2. *We incorporate corrections due to electronic straggling in the Int. Diff. Eq.*
3. *For silicon Coulomb effects allow us to fit the data and have a threshold near Frenkel-pair creation energy.*
4. *For germanium our model shows potential to explain recent measurements* [6]*.*
5. *Much work can be done from here, e.g directional quenching factor, straggling for* $\bar{\nu}$*, higher moments study, etc.*

[6] J.I.Collar, et al, PRD 103,122003 (2021)

Thanks

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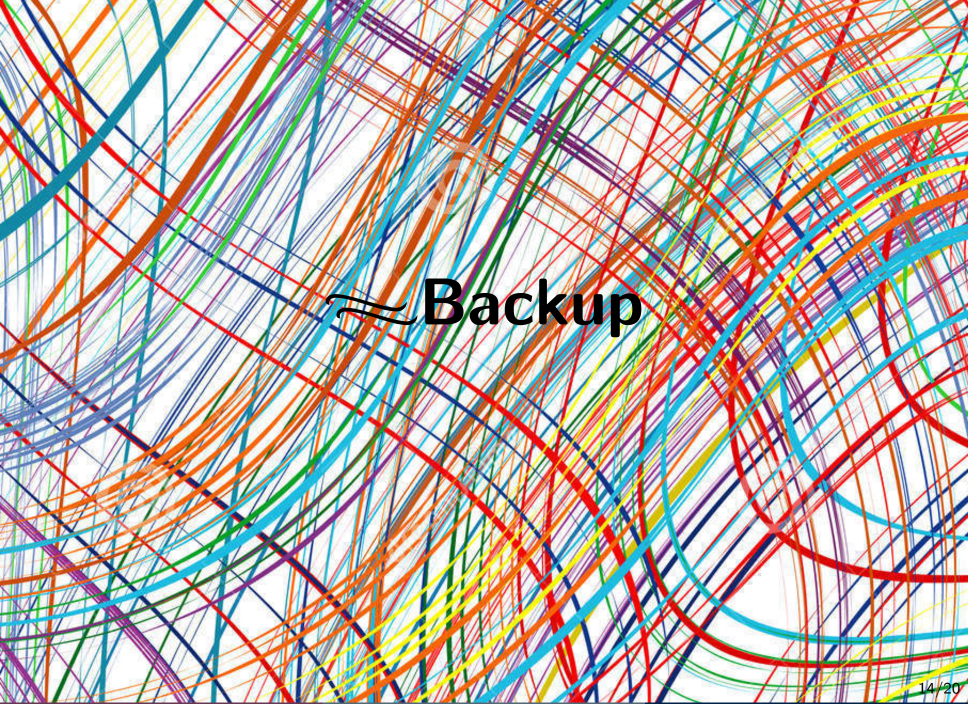
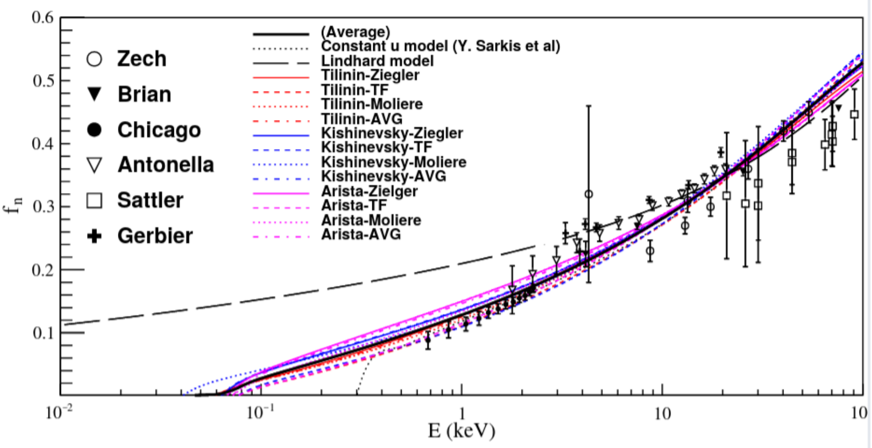
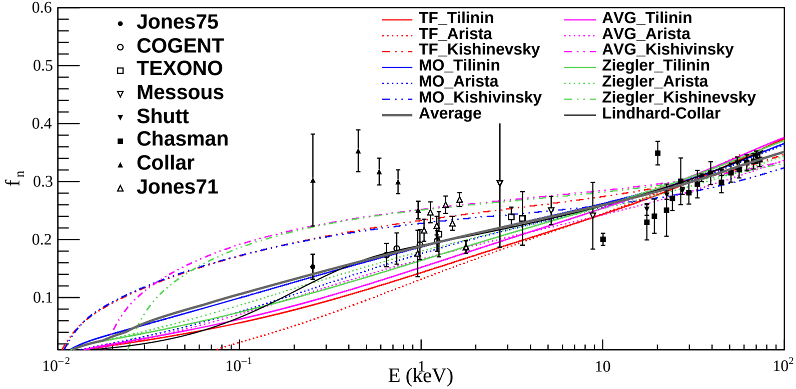


≃Backup

Si QF models-potentials



Ge QF models-potentials



Tilinin geometrical factor

Tilinin argue that for non small angles or a general trajectory of the ion there should exist a geometrical factor: $d\chi\rho d\rho dz' = \left[1 + (f_z'(\theta))^2\right]^{1/2} d\chi\rho d\rho dz$. Tilinin made a raw approximation to evaluate the angle

$$\theta \sim Z_1 Z_2 \left(Z_1^{2/3} + Z_2^{2/3}\right)^{-1/2} \left(2e^2/a_0\right)/E,$$

with $\theta < 1$. Is full fill when the energy is $E < 67$ eV for Si and for Ge $E < 190$ eV. As expected for Si the limit is very reasonable and in the order of magnitude of the binding and Tilinin model can be justified to be use.

Many experiments that rely on quenching factors

Ge

CONUS

MIνER

TEXONO

Liquid scintillators (Xe and Ar)

The RED-100 experiment

XENON Dark Matter Project

Coherent Captain-Mills (CCM) Experiment

Si

CONNIE

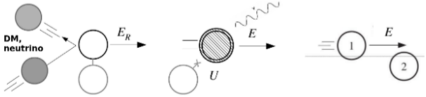
COHERENT NEUTRINO NUCLEUS INTERACTION EXPERIMENT

SuperCDMS

Nuclear recoil in a pure material

- Suppose that the ion recoils from the interaction with an energy E_R, after recoiling with an incident particle (e.g., a **neutrino**).
- Energy U is lost to some disruption of the atomic bonding, then $E_R = E + U$, then the ion moves with a kinetic energy E.
- The moving ion sets off a cascade of slowing-down processes that dissipate the energy E throughout the medium.

$$f_n = \frac{E_{\mathrm{I}}}{E_{\mathrm{R}}}$$



Lindhard's model

- Lindhard's theory concerns with determining the fraction of E_R which is given to electrons, H, and that which is given to atomic motion, N, with $E_R = N + H$.
- Defining reduced dimensionless quantities,
  $\varepsilon_R = c_Z E_R, \eta = c_Z H, \nu = c_Z N$ where $c_Z = 11.5/Z^{7/3}$keV.
- This separation is written as $\varepsilon_R = \bar{\eta} + \bar{\nu}$ ("average").
- The quenching factor (f_n) for a nuclear recoil is then defined as the fraction of E_R which is given to electrons ($u = c_Z U$):

$$\boxed{f_n = \frac{\bar{\eta}}{\varepsilon_R} = \frac{\varepsilon + u - \bar{\nu}}{\varepsilon + u}} \qquad (4)$$

When u=0 one recovers the usual definition.