Current status of the vFFA studies

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Vertical excursion FFA (VFFA)

- Invented in 1955, rediscovered in 2013.
- Orbit moves vertically when the beam is accelerated.
- Constant path length over whole momentum range (zero momentum compaction factor for all orders).
- Isochronism for ultra-relativistic energies (slippage factor only dependent of Lorentz gamma, like a Linac).

Advantages of FFAs

- **Flexibility**: Beam pulse only controlled by RF, allowing fast and sophisticated patterns
- **Sustainability**: energy efficient operation, enhanced with SC or permanent magnets, reduced operating cost
- **Reliability**: DC power supply simple and cheap, low failure rate and higher redundancy

Particular case of vFFA

- Fixed RF frequency scheme for any momentum range at relativistic energies
- Rectangular magnet considered, potentially **easier to manufacture** than spiral HFFA
- Tall magnet, but **smaller footprint** than HFFA

Disadvantages of FFAs

- Reverse bend:
  - Pros: Orbit oscillations could reduce problem of neutrino radiation
  - Cons: Big circumference of the machine

Mitigation: ➡SC magnets

➡Minimisation of reverse bend, addition of edge focusing

- Orbit excursion too large for high gradient cavities

Mitigation: ➡Maximisation of field gradient

➡Insertion of dispersion suppressor

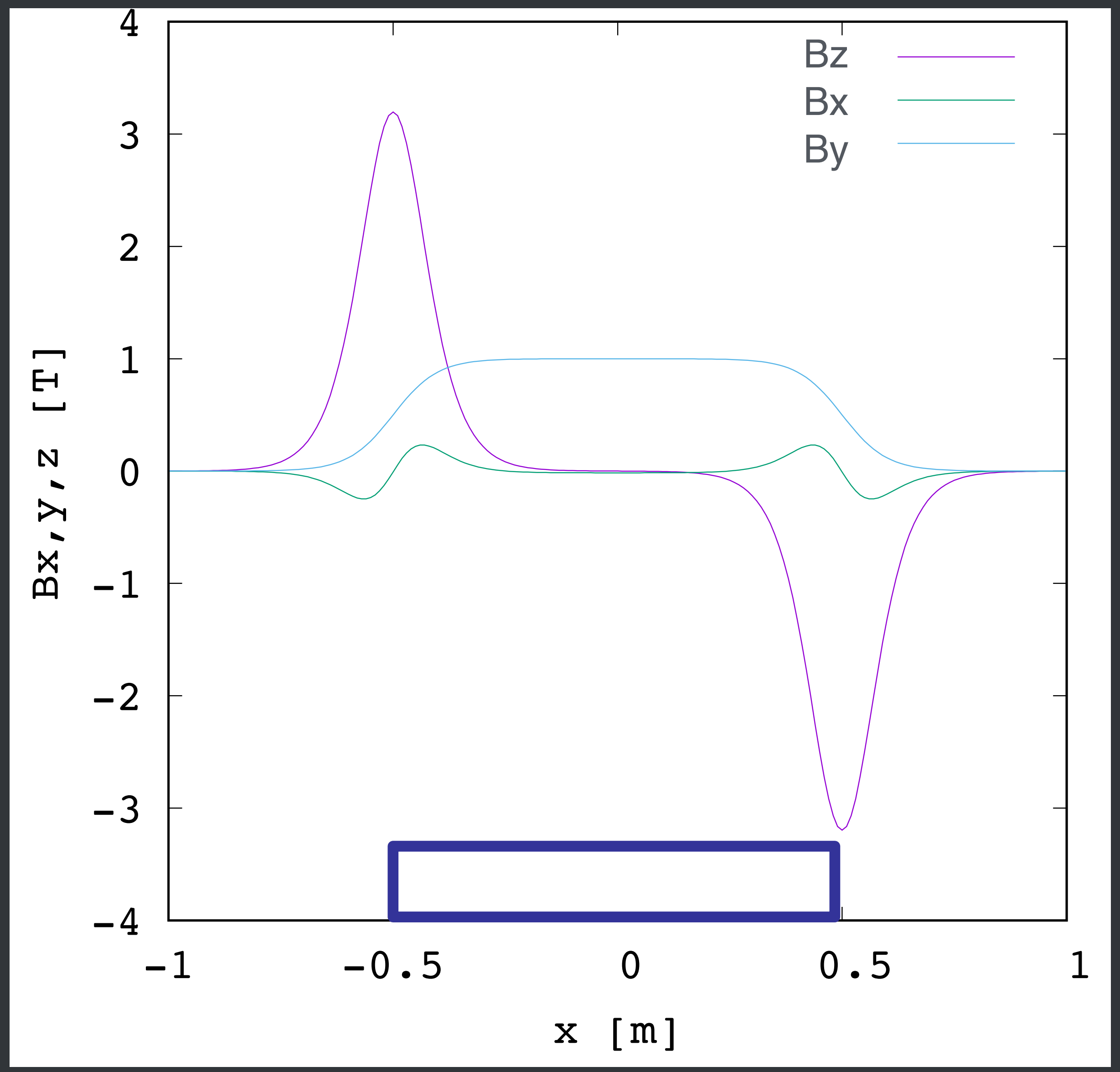
➡Reduction of momentum range

Magnetic field in VFFA

- Exponentially increasing magnetic field to satisfy zero-chromatic conditions.

  Cartesian coordinates x (hor.),y (vert.),z (long.)

$$\begin{cases} B_x(x,y,z) = B_0 e^{m(y-y_0)} \sum_i b_{xi}(z)(x-x_0)^i \\ B_y(x,y,z) = B_0 e^{m(y-y_0)} \sum_i b_{yi}(z)(x-x_0)^i \\ B_z(x,y,z) = B_0 e^{m(y-y_0)} \sum_i b_{zi}(z)(x-x_0)^i \end{cases}$$



- Non-zero longitudinal field on median plane.
- Importance of fringe field modelling, (more in small machines).
- Expansion of the field in the magnet shows alternance of normal and skew components.

➡ Strongly coupled optics

VFFA Optics

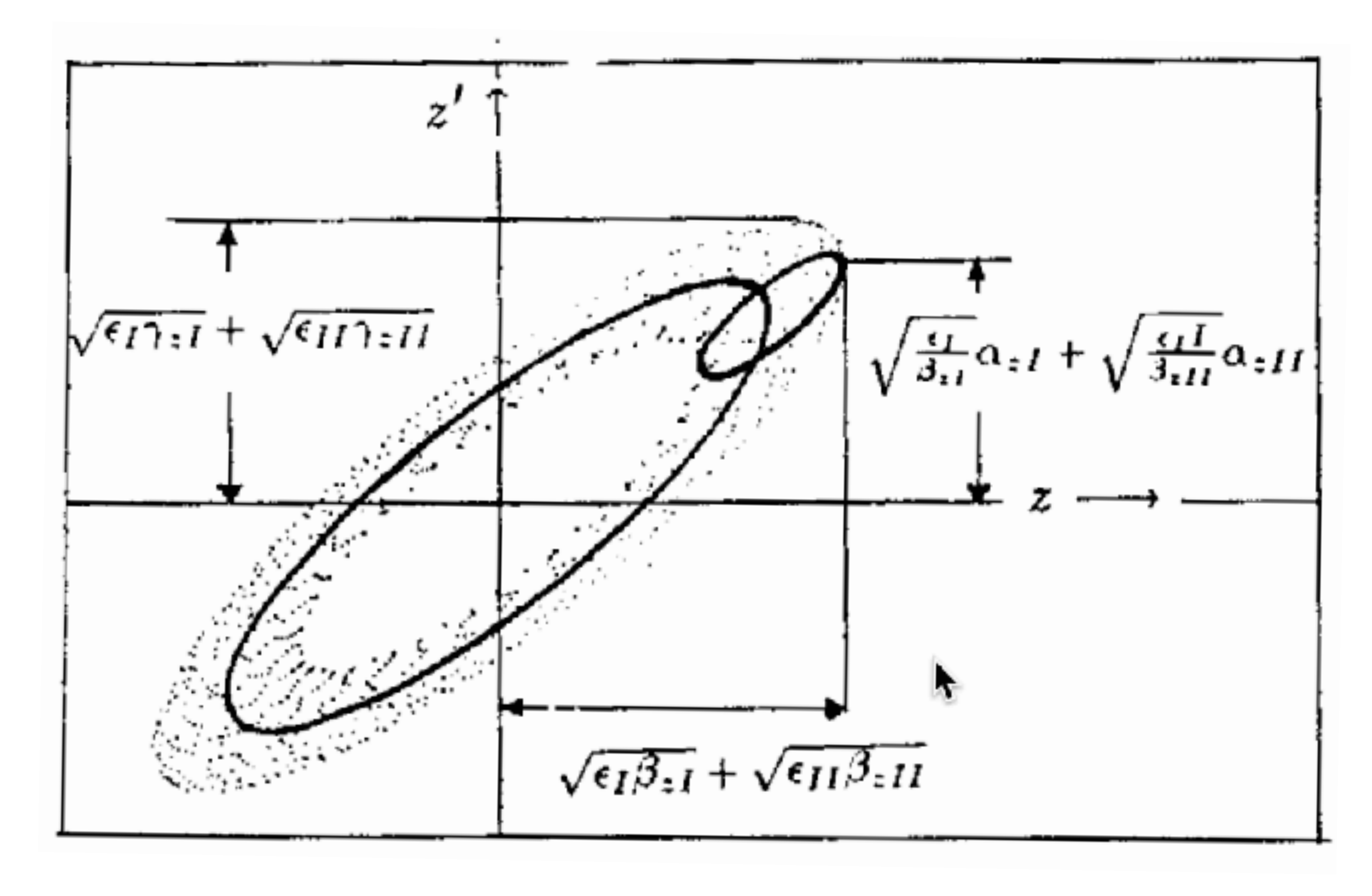
Tunes

- Tunes obtained from Eigenvalues of computed transfer matrices
- Clarification of the tunes between *Q* and *(1-Q)*:
    - From integration of the beta-functions in decoupled space (Parzen procedure)
    - from rotation in phase space per cell

Beam envelope

beta-functions from Willeke-Ripken procedure:

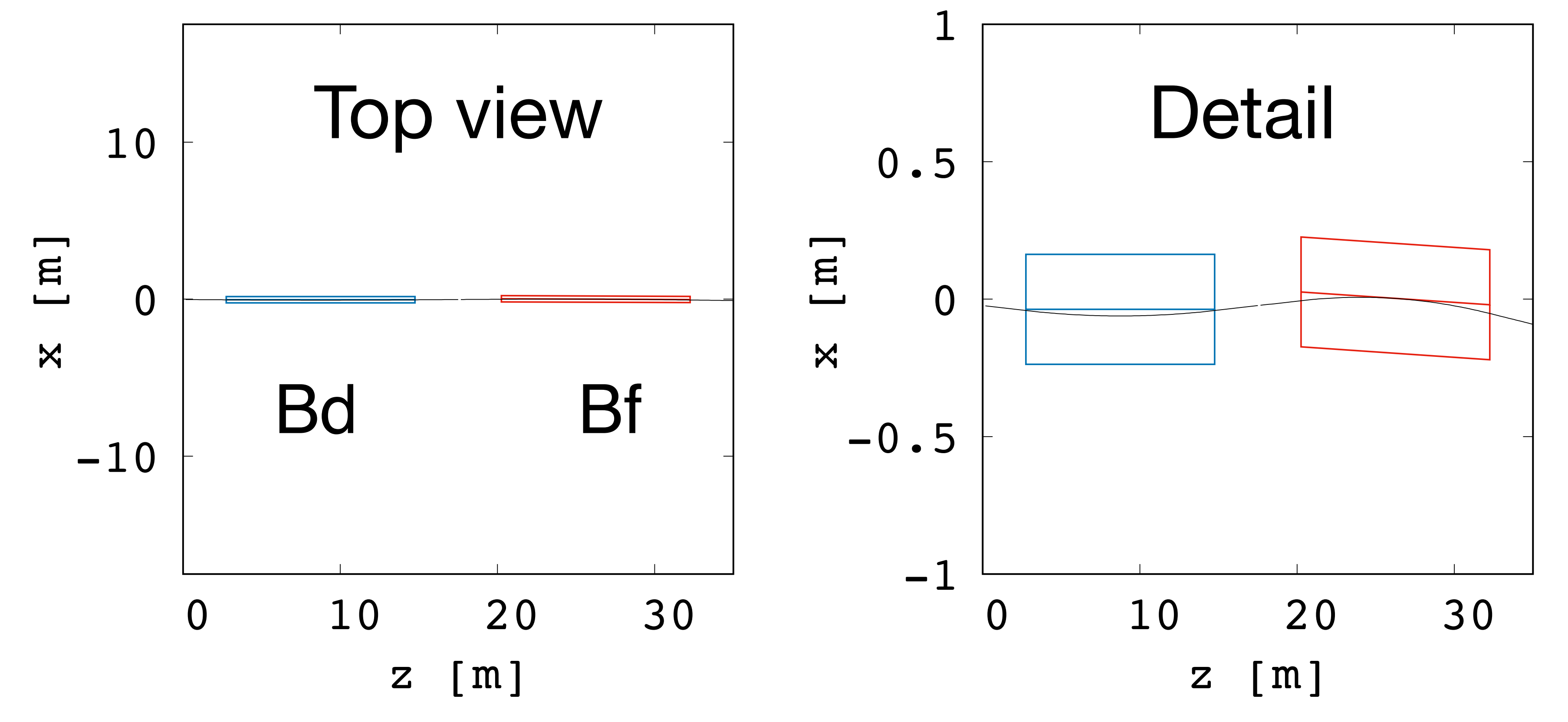
$$z_{env} = \begin{pmatrix} x_{env} \\ x'_{env} \\ y_{env} \\ y'_{env} \end{pmatrix} = \begin{pmatrix} \sqrt{\varepsilon_I}\sqrt{\beta_{xI}} + \sqrt{\varepsilon_{II}}\sqrt{\beta_{xII}} \\ \sqrt{\varepsilon_I}\sqrt{\gamma_{xI}} + \sqrt{\varepsilon_{II}}\sqrt{\gamma_{xII}} \\ \sqrt{\varepsilon_I}\sqrt{\beta_{yI}} + \sqrt{\varepsilon_{II}}\sqrt{\beta_{yII}} \\ \sqrt{\varepsilon_I}\sqrt{\gamma_{yI}} + \sqrt{\varepsilon_{II}}\sqrt{\gamma_{yII}} \end{pmatrix}$$

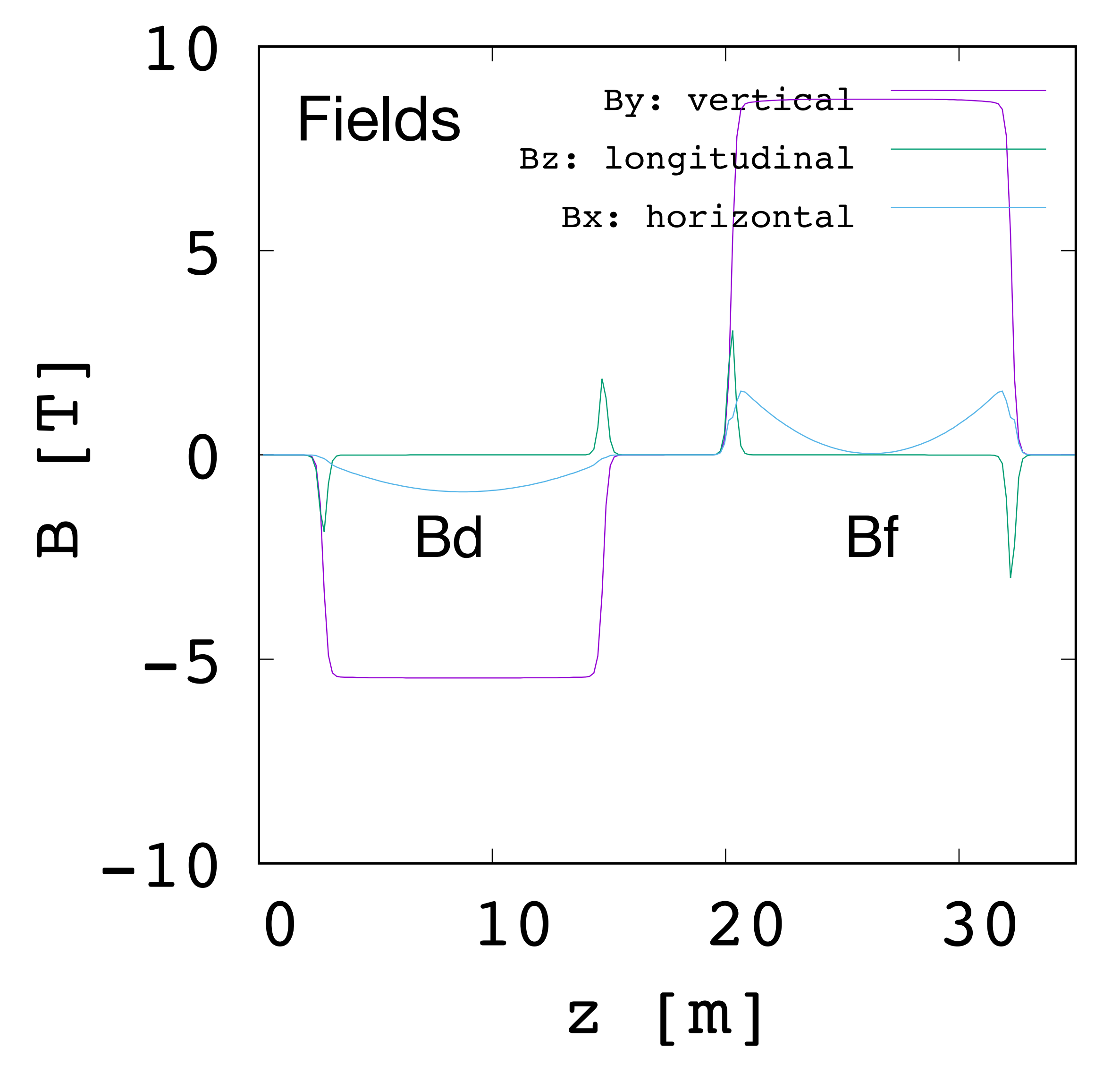


VFFA lattice for muon acceleration

Design constraints:

- LHC circumference,
- Final energy 1.5 TeV,
- Momentum multiplicator is 30,
- Maximum magnetic field is 10 T,
- Orbit excursion less than 0.5 m.



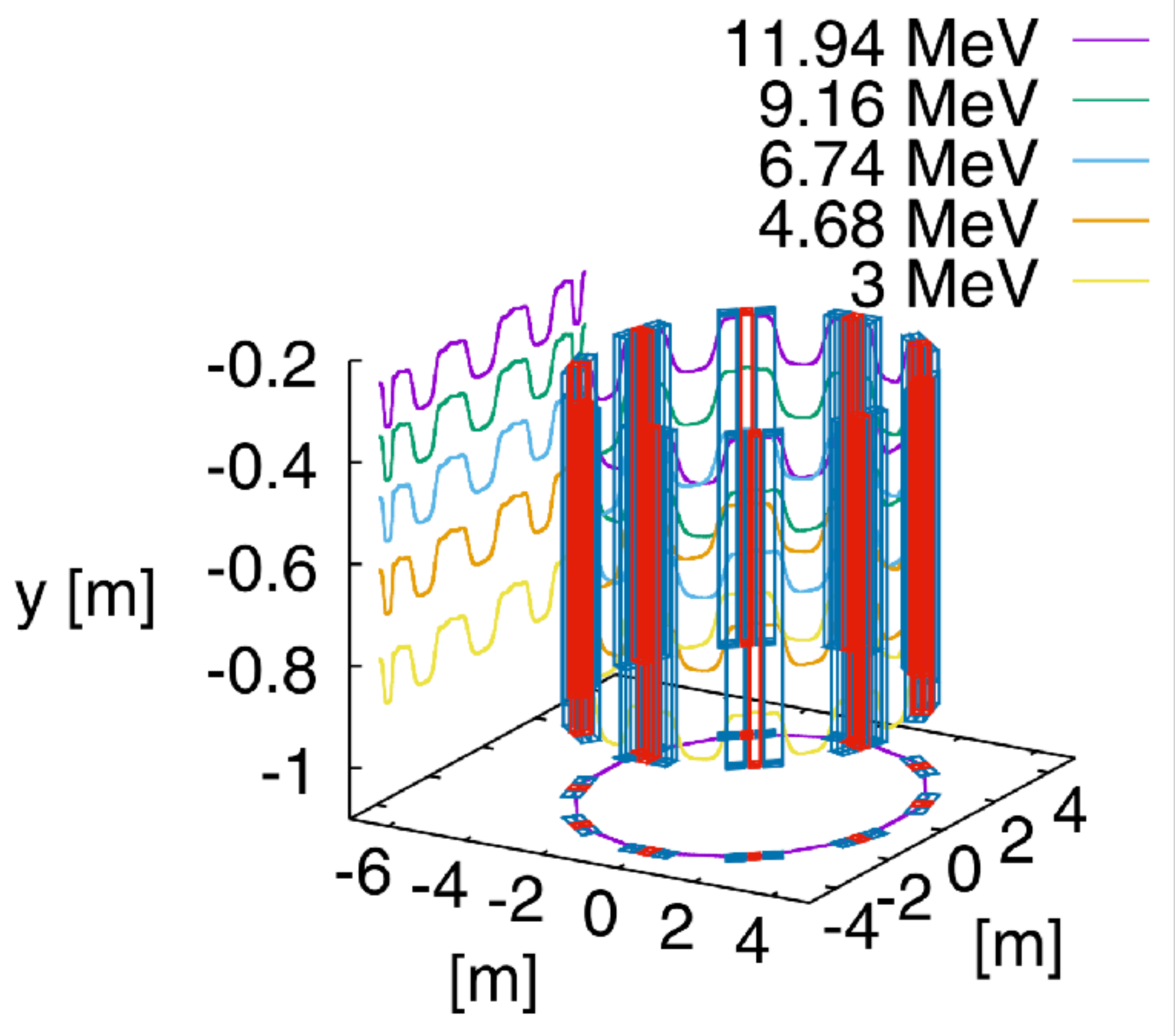


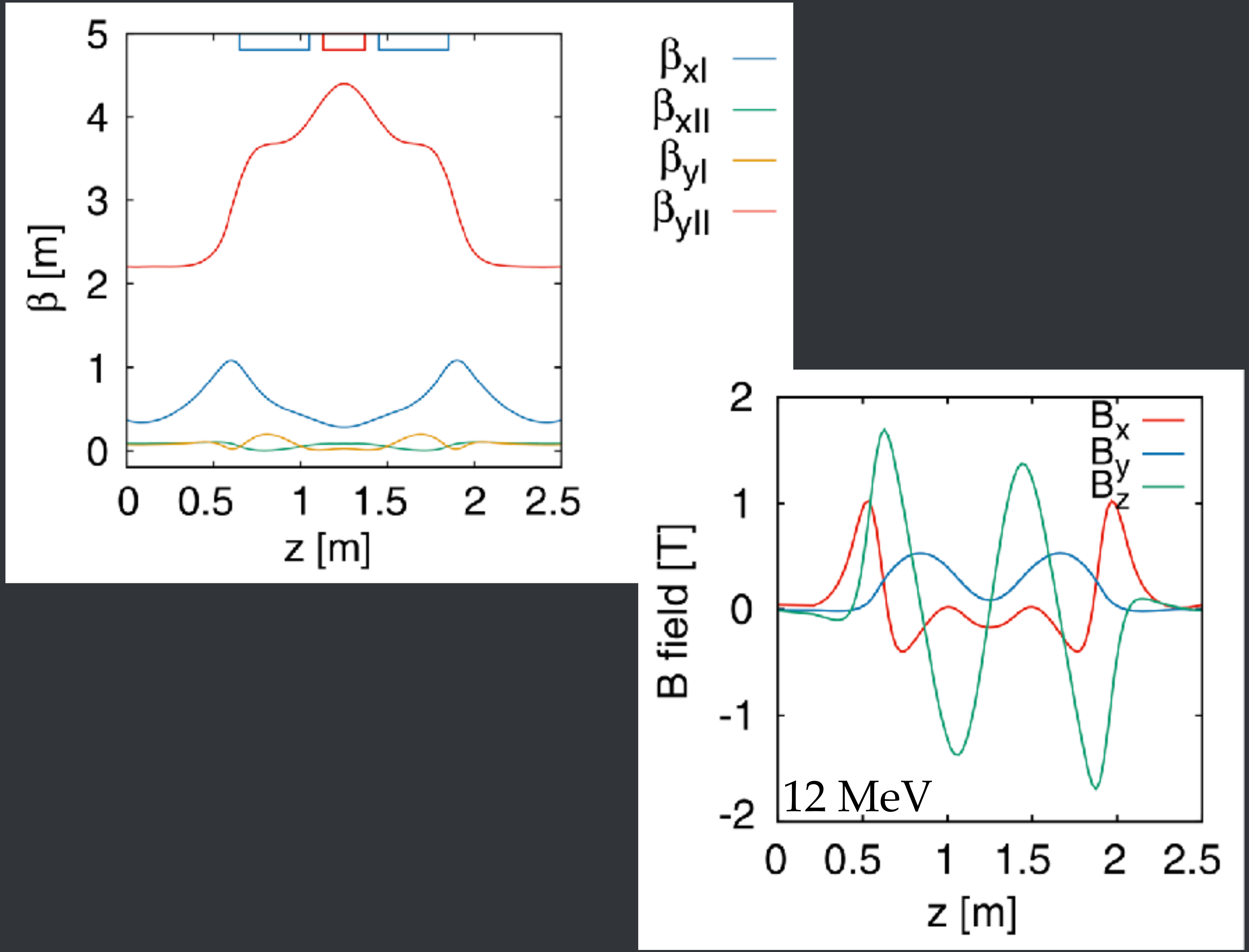
	FODO design
Energy	50 GeV to 1.5 TeV
Cell length	35 m
Number of cells	810
Packing factor	86%
Maximum field	8.7 T
Normalised gradient m^*	6.8 m^{-1}
Orbit excursion	0.50 m
Cell tune	0.3957/0.0861

$$^*m = \frac{1}{B}\frac{dB}{dy}$$ (y: vertical direction)

VFFA test ring

Proof-of-principle ring (3-12 MeV proton) to be built by 2027.





Coil configuration design

- Coil designed based on Reverse Biot-Savart law
- Biot-Savart law: $B = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \vec{r}}{|r|^3}$
- Starting from a field model in the form of $B_y = B_0 e^{m(y-y_0)} g(z)$
- Passing in the frequency domain $B_y = \left(\sum_i a_i e^{j\omega_{yi} y}\right) \left(\sum_k b_k e^{j\omega_{zi} z}\right)$
- The current density Jy and Jz of 2 infinite parallel current sheets separated by a gap ±g are

$$J_z = \sum_i \left(a_i e^{j\omega_{yi} y} \sum_k b_k e^{j\omega_{zi} z} e^{g\sqrt{\omega_{yi}^2 + \omega_{zi}^2}} \right) \quad \text{and} \quad J_y = \int \frac{\partial J_z}{\partial z} dy$$

- A 2D-FFT of B_y can then compute the current density.

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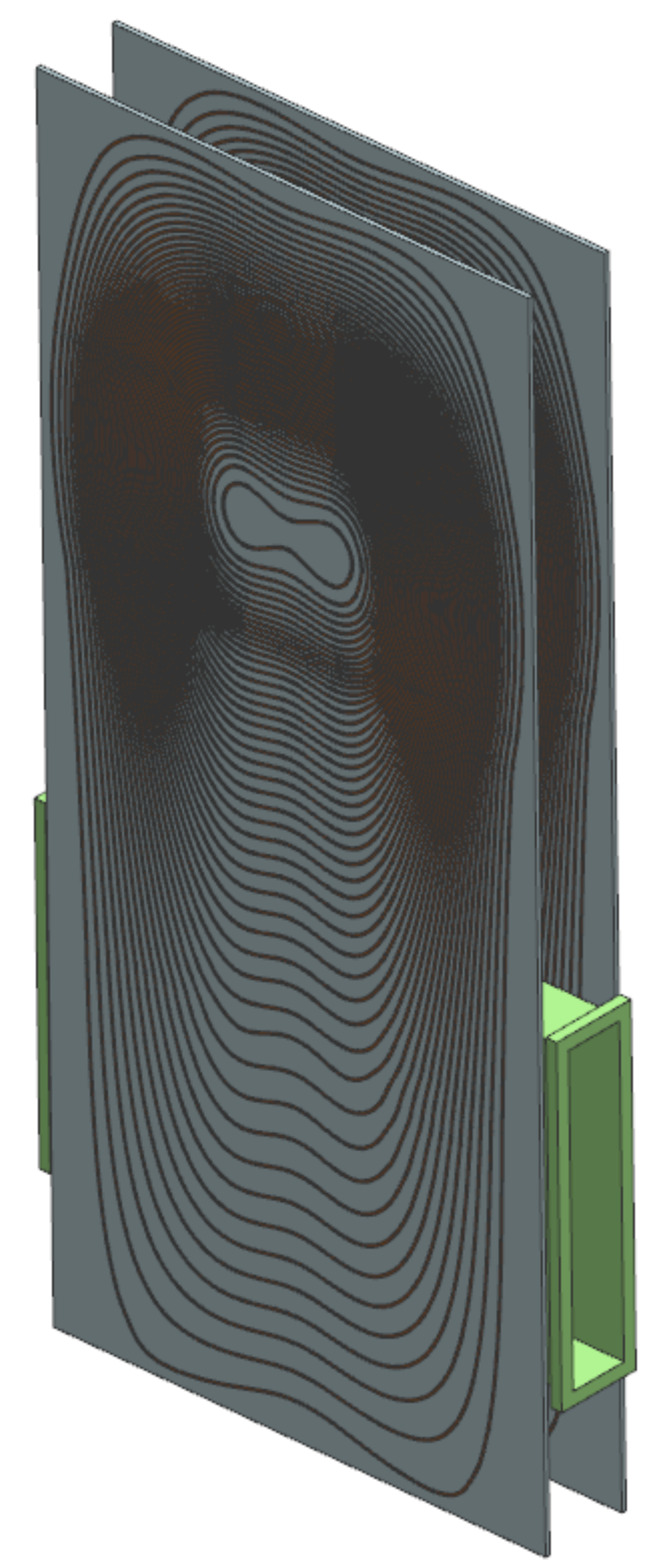
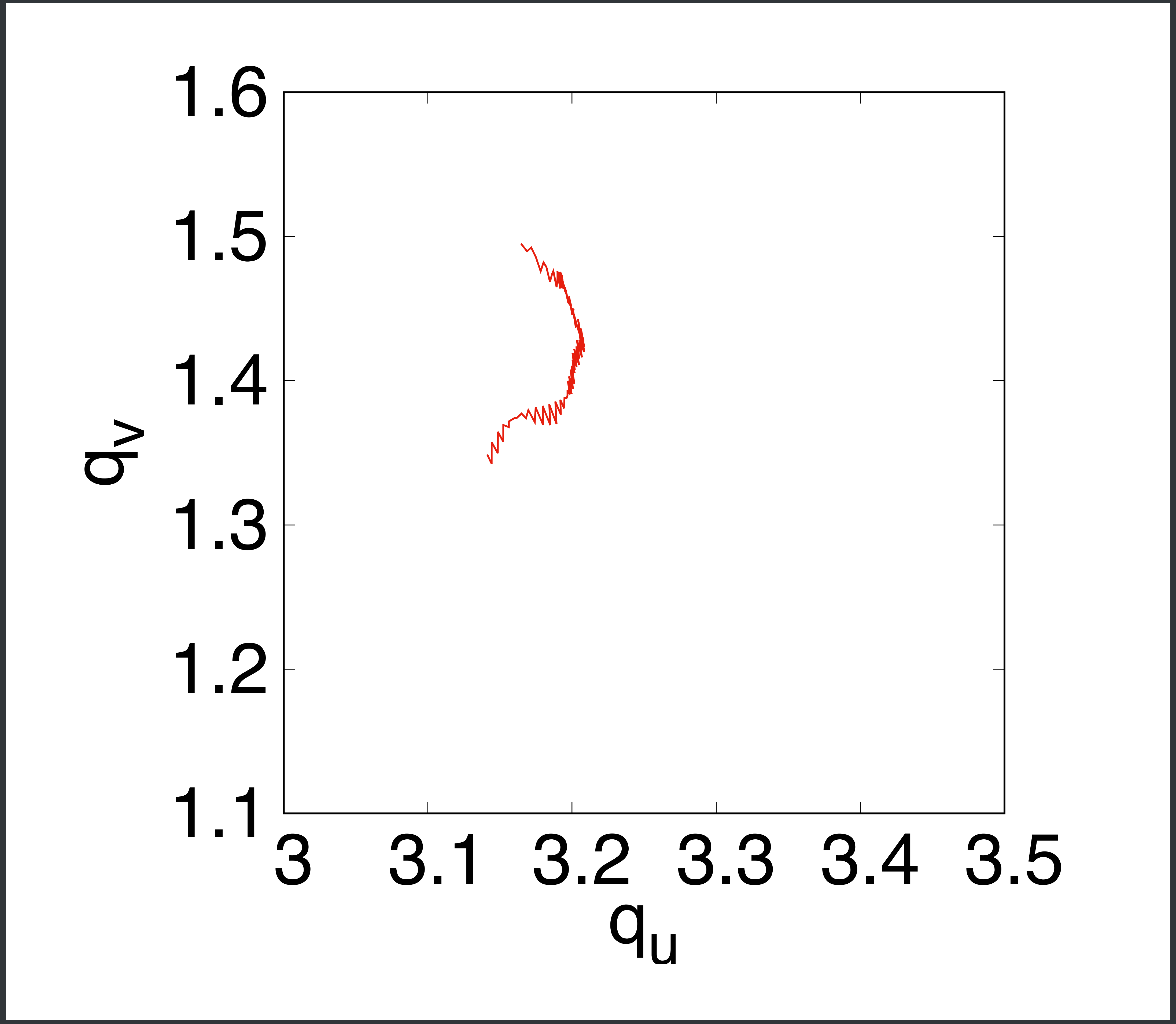
First prototype coil configuration

Prototype parameters:

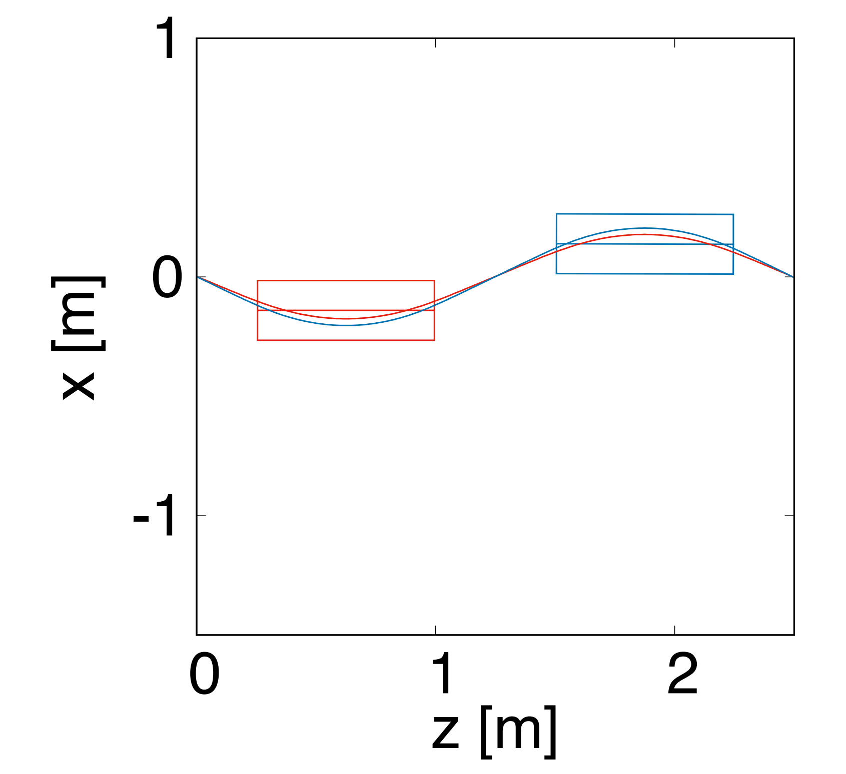
- Normal conducting with SC winding method
- 1 m-long magnet
- Normalised gradient m=1.3 m^{-1}.
- 0.6 m vertical good field region
- 22 cm full gap size
- Coil made of 50 contours, each contour made of 16 turns
- 4.7 mm minimum spacing (centre coil to centre coil)



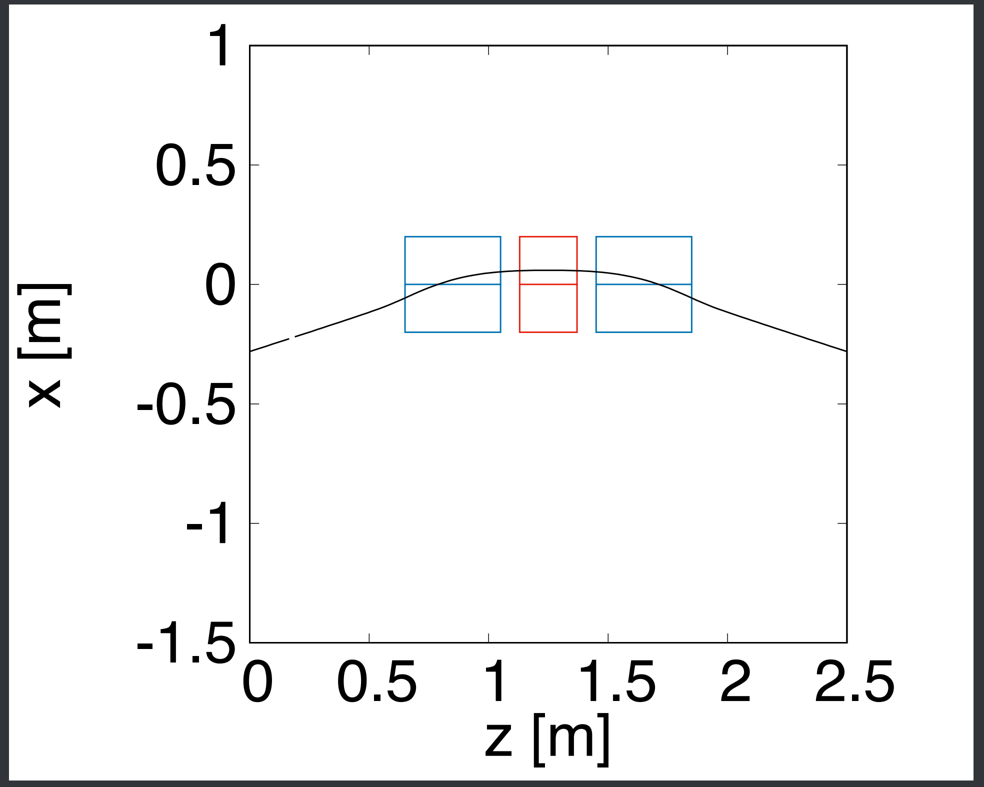
Optics in more realistic magnet model



- Magnetic field model become available and optics is calculated based on 3-D field map.
- FODO cell lattice is taken to see how accurately magnetic field is created with realistic coil configuration.
- Tune should be constant during acceleration (scaling optics). Not fixed at the current magnet design.



Lattice to check magnet accuracy



Lattice baseline

R&D for VFFA magnet

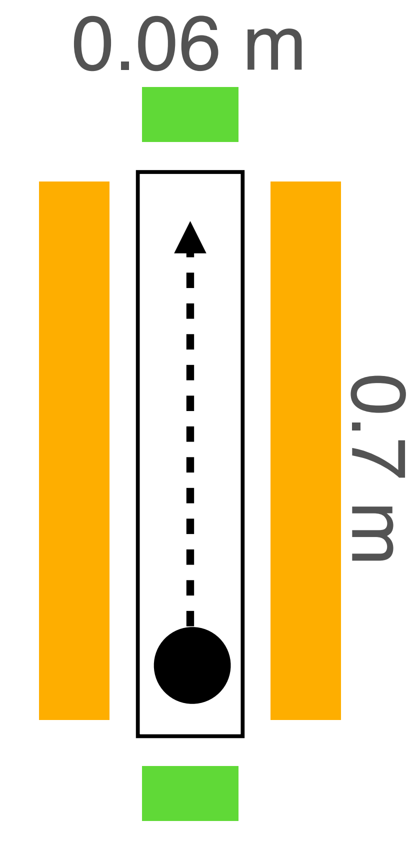
	1st NC prototype	12 MeV proton	1.2 GeV proton	1.5 TeV muon
Aperture H [mm] x D [mm]	600 x 220	700 x 300	900 x 300	700 x 200
Length [m]	1	0.5 ~ 1	2 ~ 3	10 ~ 20
Max Field [T]	0.01	3	6	9
Normalised gradient m^* [m^{-1}]	1.3	1.3 ± 25 %	0.9 ± 25 %	6.8
Momentum ratio	2	2	2	30

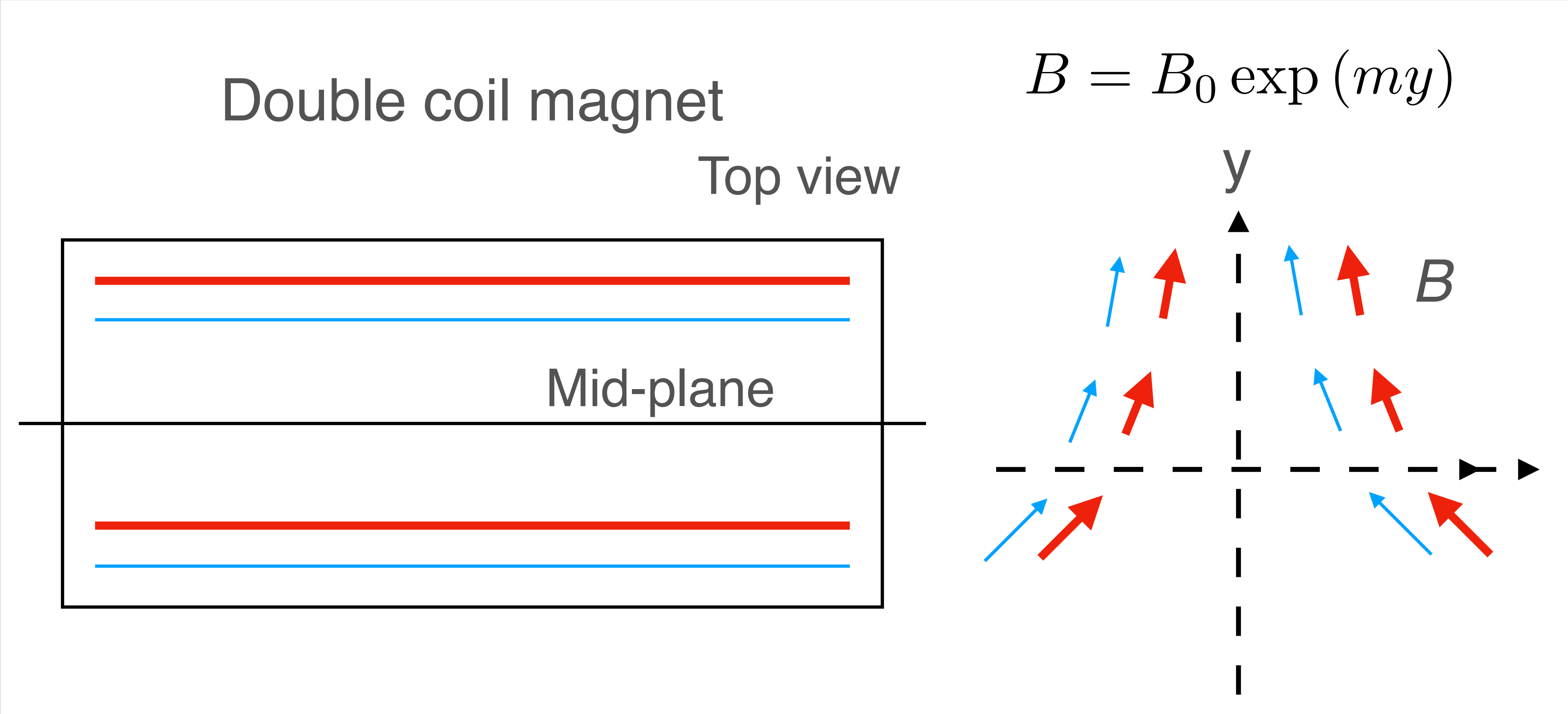
$^*m = \frac{1}{B}\frac{dB}{dy}$ (y: vertical direction)

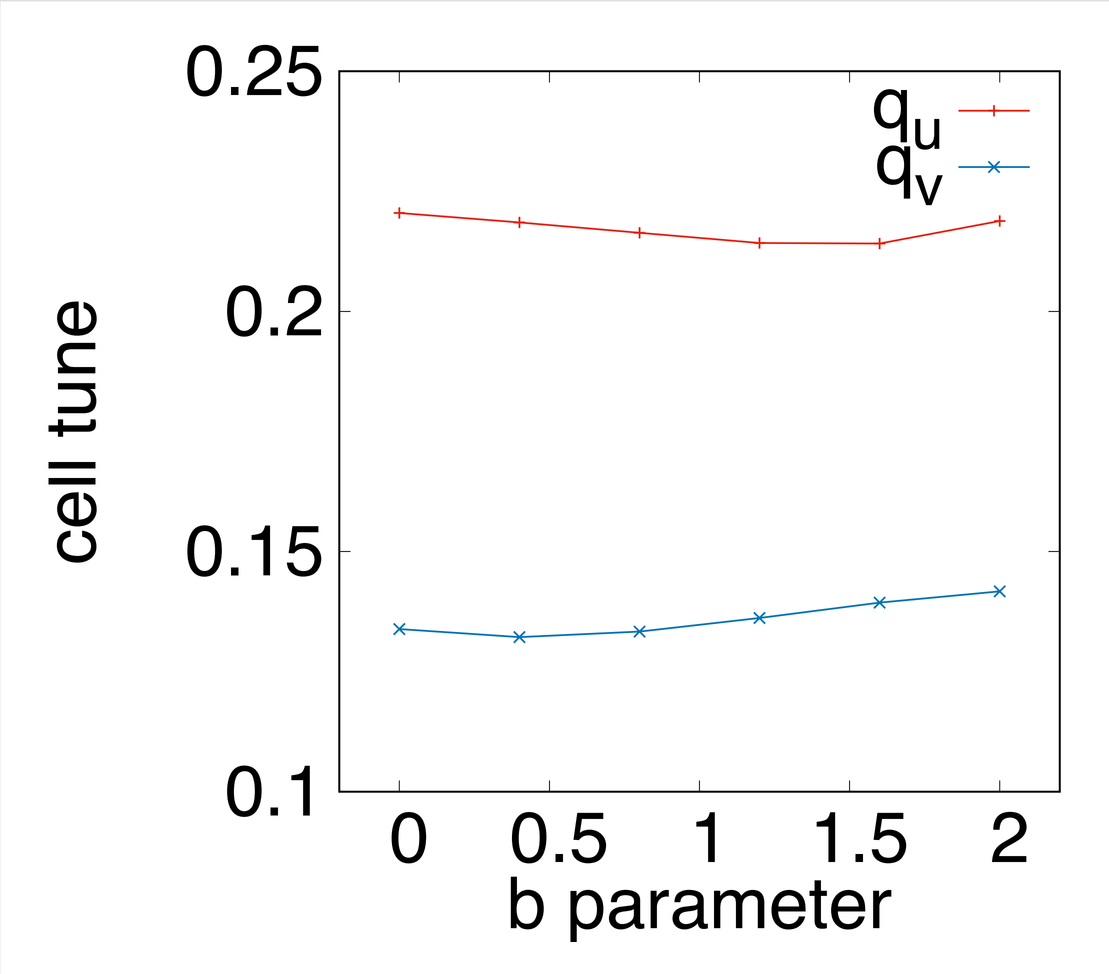
(PRELIMINARY NUMBERS)

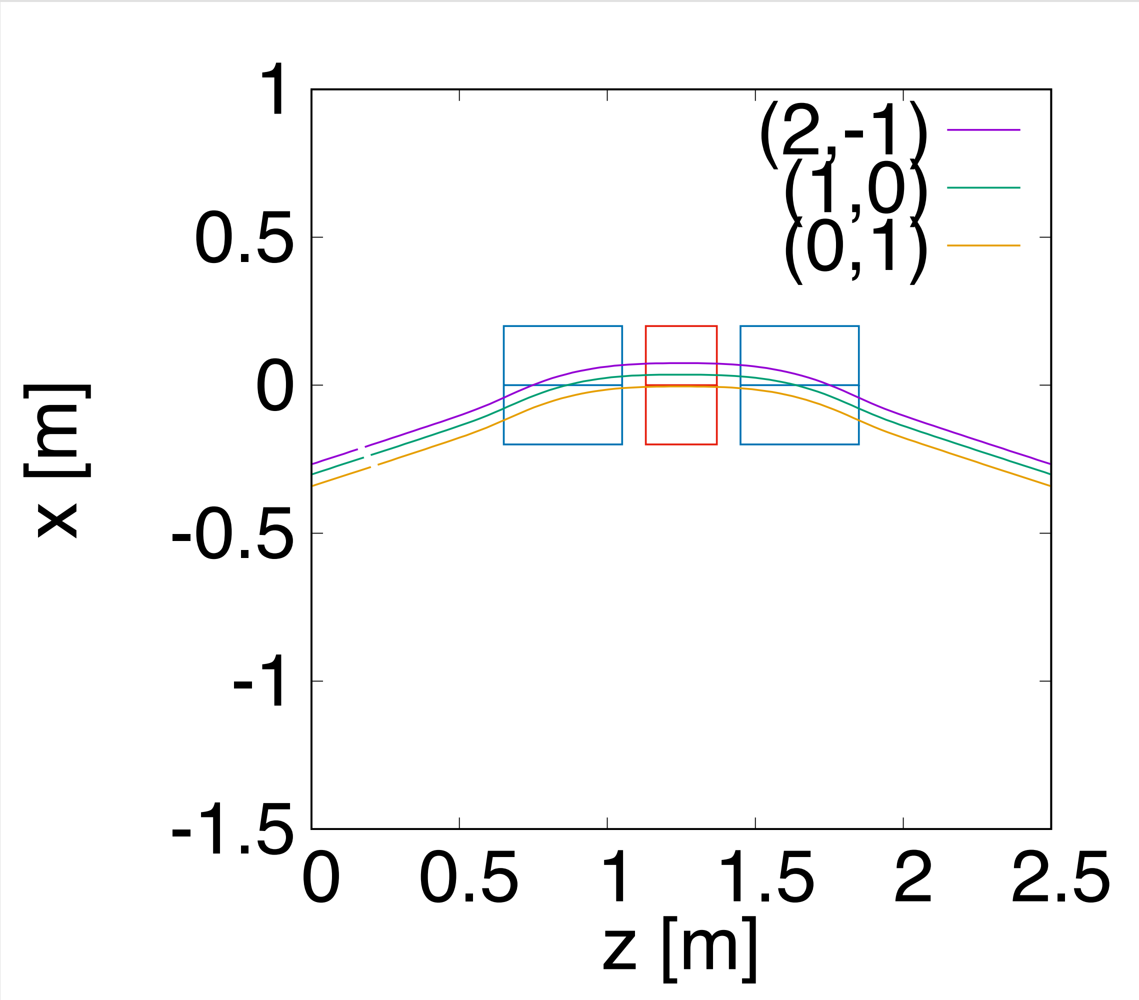
Double coil design

COD corrector





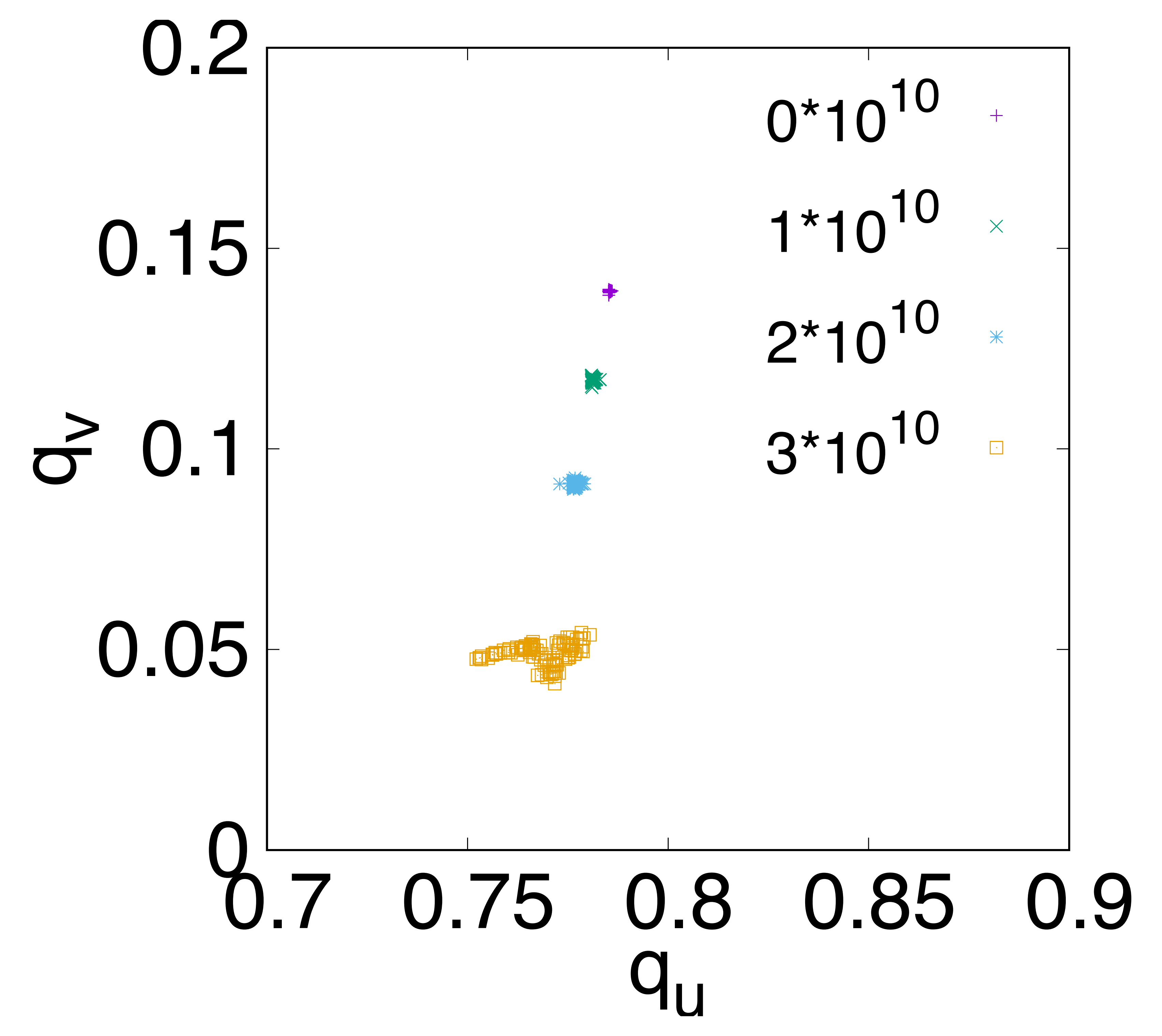




- Horizontal orbit shift with double coil design
- Several cm orbit movement with marginal tune change

High intensity effects (proton driver)

space charge tune shift



- Space charge tune shift from a simple model (uniform charge distribution, no longitudinal bunch structure): ΔQ_u~-0.07, ΔQ_v~-0.30

- Apply formula of tune shift per ring for decoupled optics:
  - a = horizontal beam size
  - b = vertical beam size
  - Emittance = 0.25 pi mm mrad

$$\Delta Q_u = -\frac{n_t r_p R/Q_u}{\pi a\,(a+b)\,\beta^2\gamma^3} = -0.23$$

$$\Delta Q_v = -\frac{n_t r_p R/Q_v}{\pi b\,(a+b)\,\beta^2\gamma^3} = -0.43$$

➡ Reasonable tune shift, but needs more theoretical understanding.

High intensity effects (muon)

space charge tune shift

Muons per bunch	2.83 10^{12}
Bunch length	23.1 mm
Normalised emittance	43 μm
Muon energy	50 GeV

- Apply same formula of tune shift per ring for decoupled optics in muon acceleration lattice:

➡ ΔQ_u=-0.49

➡ ΔQ_v=-1.06

Summary

- Development at RAL of VFFA as a proton driver for spallation neutron source
- Proof of principle ring (3-12 MeV proton) planned by 2027
- Coil-based prototype magnet designed
- Strong synergy with muon collider study, preliminary design for muon acceleration from 50 GeV to 1.5 TeV
- Concern over space charge in current lattice for muon acceleration

Thank you for your attention