Lecture: Dark Matter-III
Freeze-In and Freeze-Out Scenarios

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BCVSPIN Online

Probing the Mysteries of the Universe

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1. Introduction

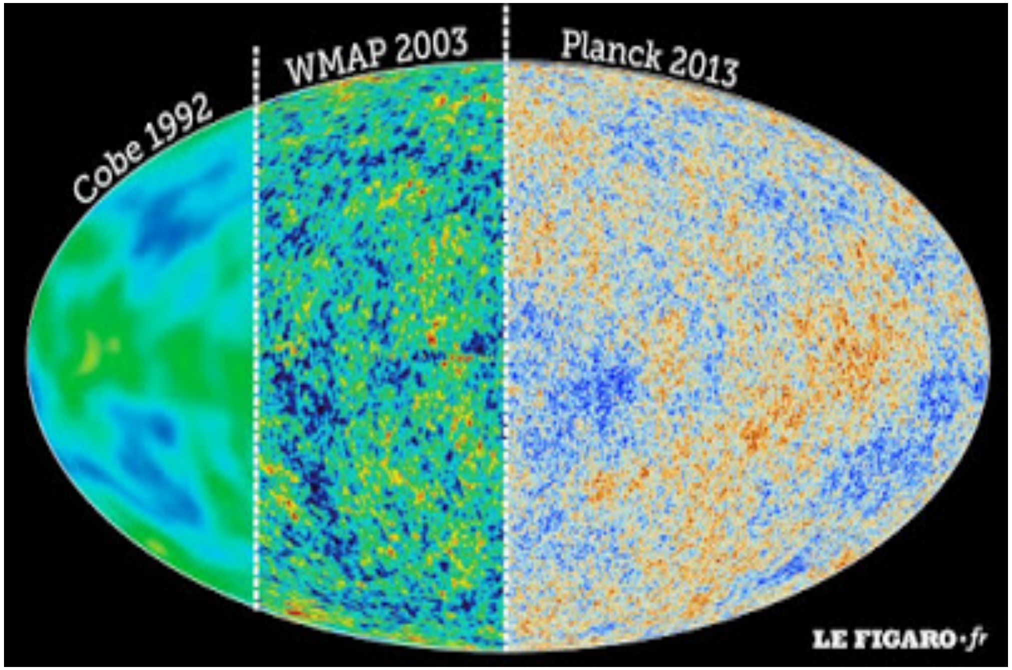
Problems in the Standard Model

The Standard Model (SM) is the best theory in describing the nature of elementary particle physics, which is in excellent agreement with almost of all current experimental results (including LHC Run-2 results) as of TODAY

However,
New Physics beyond SM is strongly suggested by both experimental & theoretical points of view

Cosmological Dark Matter Problem

CMB measurements: COBE → WMAP → Planck



Planck 2015 arXiv:1502.01589

Parameter	[1] *Planck* TT+lowP
$\Omega_b h^2$	0.02222 ± 0.00023
$\Omega_c h^2$	0.1197 ± 0.0022
$100\theta_{MC}$	1.04085 ± 0.00047
τ	0.078 ± 0.019
$\ln(10^{10} A_s)$	3.089 ± 0.036
n_s	0.9655 ± 0.0062
H_0	67.31 ± 0.96
Ω_m	0.315 ± 0.013
σ_8	0.829 ± 0.014
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014

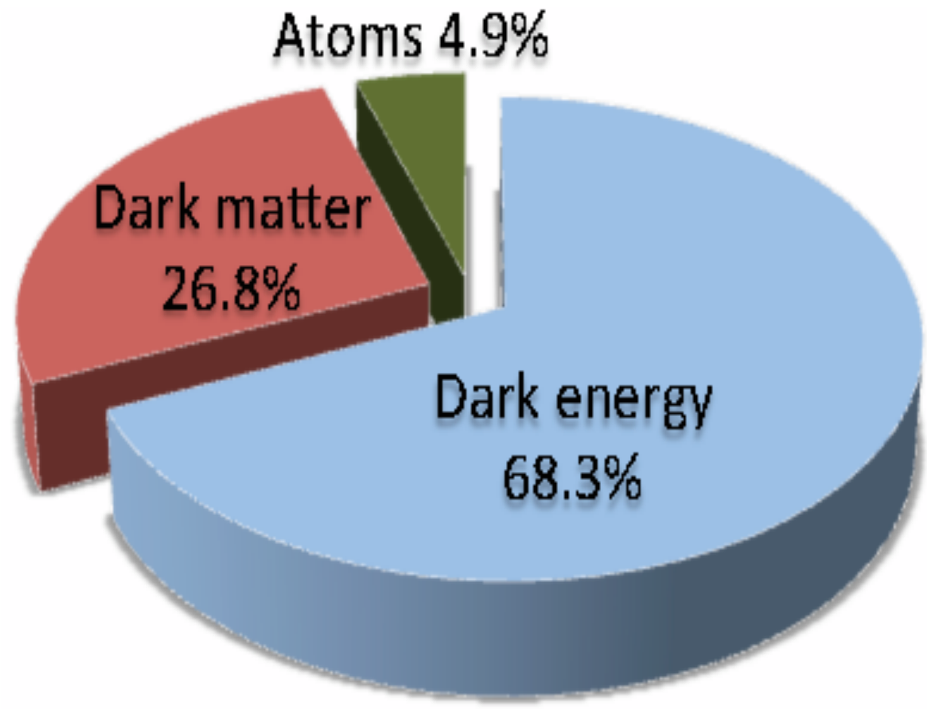
The observational cosmology is now a precision science!

$$T = 2.725 \text{ K}$$

$$\frac{\delta T}{T} \sim 10^{-5}$$

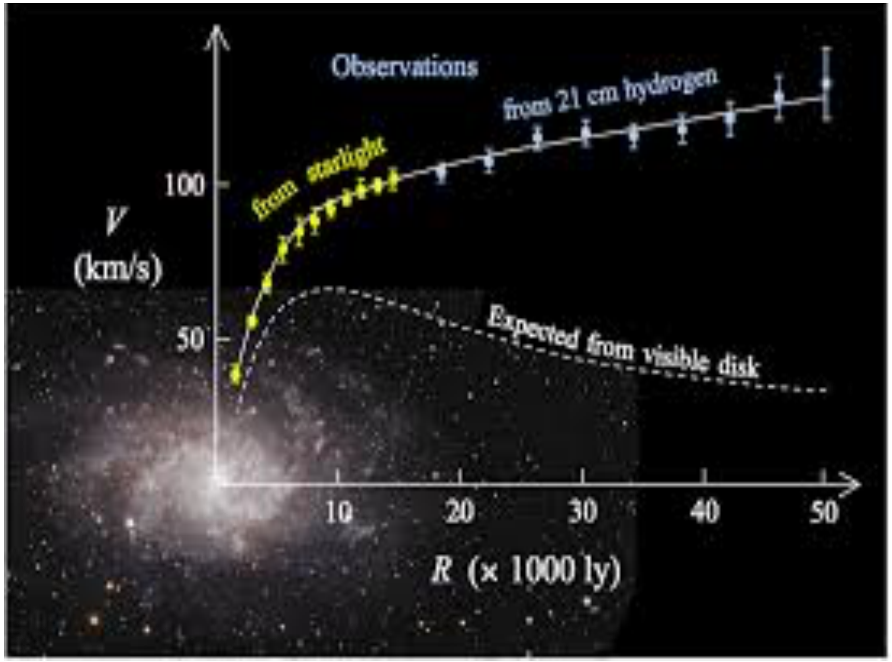
Existence of Dark Matter has been established!

Energy budget of the Universe is precisely determined by the recent CMB anisotropy observations (WMAP & Planck)



Other evidences of dark matter

Galaxy rotation curve



Bullet Cluster

Dark Matter problem is one of the major problems of the Standard Model

Basic properties of the DM

1. Non-baryonic (not the ordinary matter)
2. Electrically neutral
3. Stable

1. No candidates in the Standard Model (SM)
   —> motive us to go beyond the Standard Model
2. Identity of the DM particle is still a big problem in particle physics and cosmology
3. Many proposed candidates: axion, Weakly Interacting Massive Particle (WIMP), Strongly Interacting Massive Particle, Self Interacting Massive Particle, Feebly Interacting Massive Particle, etc.

We may categorize Dark Matters into

1. Thermal Dark Matter

- DM particle has been in thermal equilibrium with the SM particles plasma in the early universe
- At some point, it decoupled from the plasma (Freeze-Out)

Example: WIMP DM

2. Non-Thermal Dark Matter

- DM particle has never been in thermal equilibrium with the SM particles plasma
- In a simple scenario, DM particles have been produced by the thermal plasma (Freeze-In)

Example: gravitino DM in SUSY models

In this lecture, I will review two popular DM scenarios, Freeze-In & Freeze-Out scenarios, and address answers to the following questions:

- What DM property distinguishes two scenarios?
- In each scenario, how is the observed DM relic density reproduced?
- Is it possible to probe DM matter/Dark sector?

2. Basics of Particle Cosmology

Natural Units

$$\hbar = c = k_B = 1$$

$$[\text{Energy}] = [\text{Temperature}] = [\text{Mass}]$$

$$[\text{Length}] = [\text{Time}] = [\text{Mass}]^{-1}$$

Examples:

$$[\text{Particle Number}] = [\text{Mass}]^0 : \text{dimensionless}$$

$$[\text{Number Density}] = \frac{[\text{Number}]}{[\text{Volume}]} = [\text{Mass}]^3$$

$$[\text{Energyr Density}] = \frac{[\text{Energy}]}{[\text{Volume}]} = [\text{Mass}]^4$$

$$[\text{Entropy Density}] = [\text{Mass}]^3$$

Big Bang Cosmology

Solving Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_P^2} T_{\mu\nu}$$

(Reduced Planck Mass : $M_P = 2.4 \times 10^{18}$ GeV)

with Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a(t)^2 (dr^2 + r^2 d\Omega^2)$$

(homogeneous & isotropic universe)

Perfect fluid approximation:

$$T^{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

ρ : Energy Density

p : Pressure

Friedmann equation from (0,0)-component

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{M_P^2}$$

Continuity equation from combination of (0,0) and (i,i) -components

$$\dot{\rho} + 3H(\rho + p) = 0$$

This equation is equivalent to the 1st law of thermodynamics: total entropy of the universe is conserved:

$$dS = 0$$

Equation of state: $p = w\rho$

- Radiation (relativistic particles):

$$w = \frac{1}{3} \to \rho_R \propto a^{-4}$$

- Matter (non-relativistic particles):

$$w = 0 \to \rho_M \propto a^{-3}$$

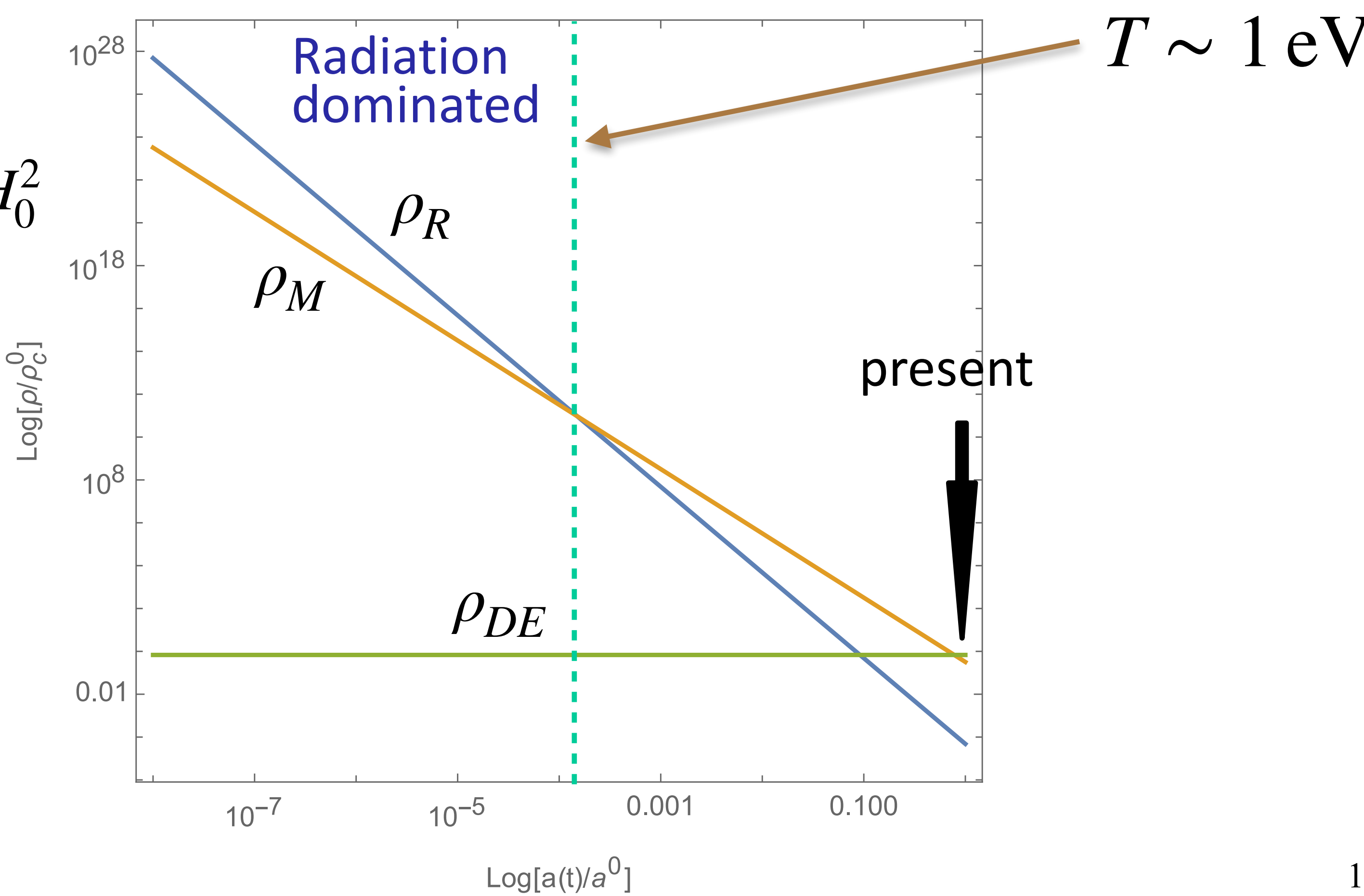
- Dark Energy (Cosmological Constant):

$$w = -1 \to \rho_{DE} = \text{constant}$$

Energy budget of the universe as a function of the scale factor (evolution of the universe)

$$\rho_R^0 : \rho_M^0 : \rho_{DE}^0 \simeq 4.8 \times 10^{-5} : 0.32 : 0.68$$

$$\rho_c = 3M_P^2 H_0^2$$



In commonly discussed DM scenarios, creations/annihilations of DM particles occur in the radiation dominated era, in which all the SM particles are in thermal equilibrium. $a \propto t^{1/2}, \ H \propto t^{-1}$

Equilibrium thermodynamics:if a particle X is in thermal equilibrium, its properties are determined by only its mass and temperature of the plasma.

Thermal distribution: $f(\vec{P}) = \dfrac{1}{e^{E/T} \pm 1}, \quad E = \sqrt{|\vec{P}|^2 + m^2}$

	$T \gg m$	$T \lesssim m$
$n_X = \dfrac{g_X}{(2\pi)^3} \int d^3P \, f(\vec{P})$	$\sim T^3$	$\sim (mT)^{3/2} \, e^{-m/T}$
$\rho_X = \dfrac{g_X}{(2\pi)^3} \int d^3P \, E \, f(\vec{P})$	$\sim T^4$	$\sim m n_X$
$p_X = \dfrac{g_X}{(2\pi)^3} \int d^3p \, \dfrac{\lvert\vec{P}\rvert^2}{E} \, f(\vec{P})$	$\sim T^4$	~ 0

3. Thermal & Non-Thermal DM

Freeze-Out & Freeze-In scenarios

We may categorize Dark Matters into

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Example: WIMP DM

2. Non-Thermal Dark Matter

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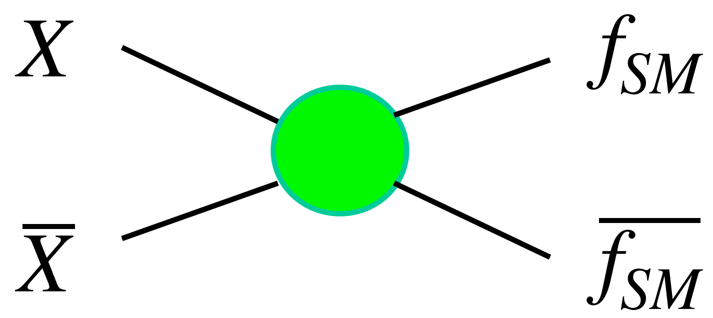
Example: gravitino DM in SUSY models

How to judge a DM particle is in thermal equilibrium or not?

For simplicity, let us consider a Dirac fermion DM (X) interacting the SM particles (quarks or leptons).

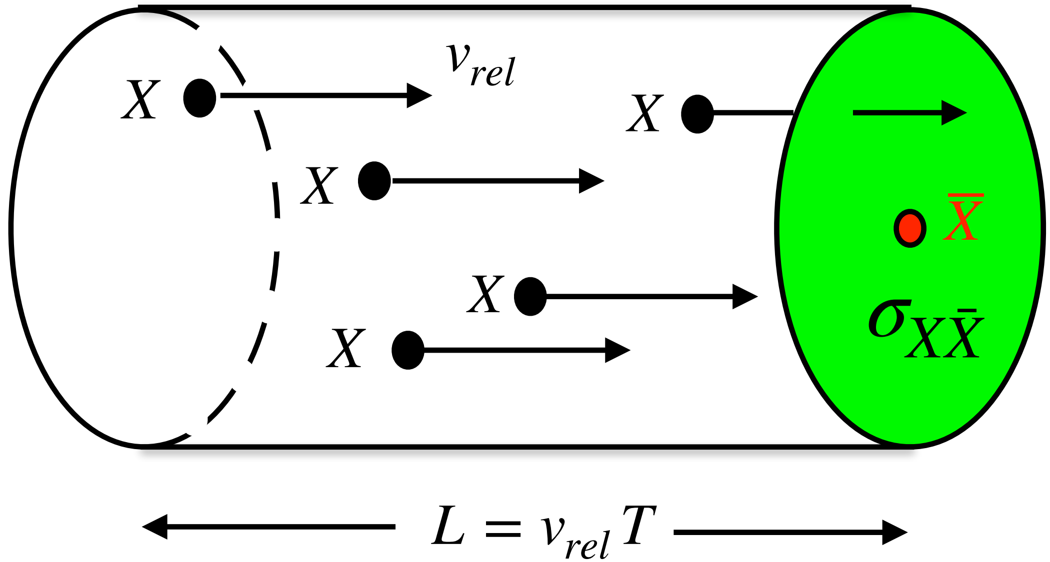
For evaluating DM density in the present Universe, we are inserted in the process which changes the number of DM particles:

DM pair annihilation/creation processes:



DM pair annihilation/creation cross section: $\sigma_{X\bar{X}}$

Number of DM pair annihilations



In rest frame of one target $\bar{X}$, we consider a beam of X particles with relative velocity v_{rel}

In a time interval *T*, all *X* particles inside the cylinder can scatter with the target

$$N_{coll} = n_X \times V_{cyl} = n_X\,\sigma_{XX}\,v_{rel}\,T$$

We define the annihilation rate, which is the number of annihilations happening per unit time, as

$$\Gamma_{ann} = n_X \, \sigma_{XX} v_{rel}$$

For the expanding universe, the time scale (age of the universe) is determined by the Hubble parameter.
In the radiation dominated era,

$$H = \frac{\dot{a}}{a} \propto \frac{1}{t}$$

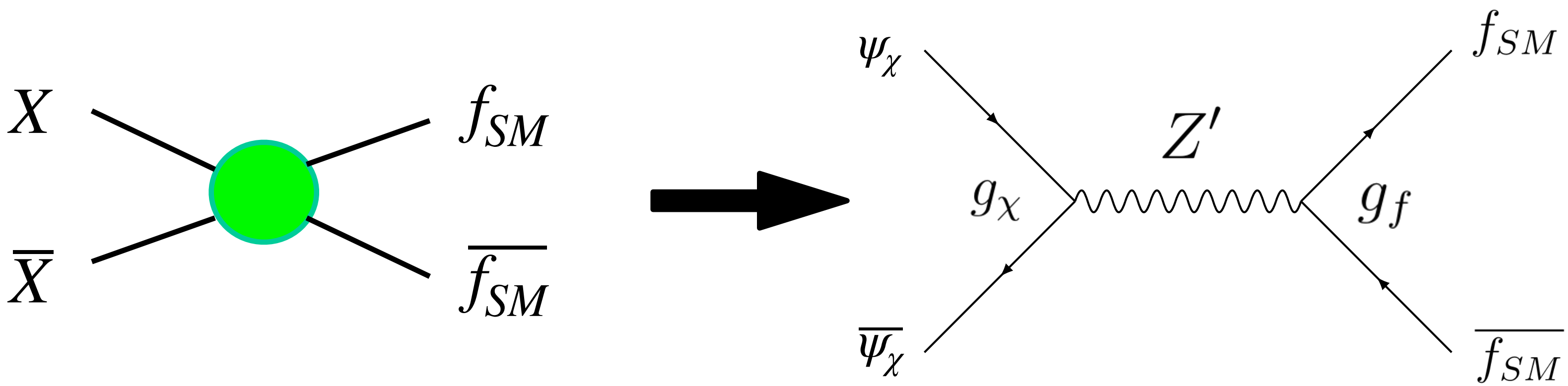
So, we judge the DM X is in thermal equilibrium or not by

$$\frac{\Gamma_{ann}}{H} > 1 \text{ or } \frac{\Gamma_{ann}}{H} < 1$$

Here, thermally averaged value: $\Gamma_{ann} = n_X^{EQ} \langle \sigma_{XX} v_{rel} \rangle$

For understanding physics, discussion based on a concrete model is very helpful.
Here let us consider a simple Dirac fermion DM model, which is called "Z'-portal Fermion DM" scenario

DM particle communicates with the SM particles through a new gauge boson Z'



For simplicity, we set $m_{Z'} \ll m_\chi$

Thermal DM or Non-Thermal DM?

We parameterize the DM pair annihilation as

$$\sigma(s) = \frac{g_\chi^2 \, g_f^2}{4\pi \, s}$$

DM annihilation rate: $n_\chi \langle \sigma v_{rel} \rangle \sim \frac{g_\chi^2 \, g_f^2}{4\pi} T$

Expansion rate of the universe: $H \sim \frac{T^2}{M_P}$

At Temperature ~ DM mass, if $n_\chi \langle \sigma v_{rel} \rangle > H$

→ Thermal DM

→

Thermal DM: $g_\chi \, g_f \gtrsim \sqrt{\frac{m_\chi}{M_P}}$

Non-Thermal DM: $g_\chi \, g_f \lesssim \sqrt{\frac{m_\chi}{M_P}}$

Evaluation of the DM relic density (thermal DM case)

For well-studied thermal DM scenario, we evaluate DM relic density by solving the Boltzmann equation $(N_X = N_{\bar{X}})$

$$\frac{dN_X}{dt} = \text{creation} - \text{anihilation}$$
$$= \text{creation} - \Gamma_{ann} N_X$$

$$\frac{d}{dt}N_X = \frac{d}{dt}(n_X V) = \frac{dn_X}{dt}V + n_X\frac{dV}{dt} = V\left(\frac{dn_X}{dt} + n_X\frac{\frac{dV}{dt}}{V}\right)$$
$$= V\left(\frac{dn_X}{dt} + 3Hn_X\right) \qquad (V \propto a(t)^3)$$

$$\Gamma_{ann} N_X = n_X \langle \sigma_{X\bar{X}} v_{rel} \rangle (n_X V) = \langle \sigma_{X\bar{X}} v_{rel} \rangle (n_X)^2 V$$

If X is in thermal equilibrium, creation = annihilation:

$$\text{creation} = \text{anihilation}\,|_{\text{thermal}} = \langle \sigma_{X\bar{X}} v_{rel} \rangle (n_X^{EQ})^2 V$$

Then, the Boltzmann equation is expressed as

$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma_{X\bar{X}} v_{rel} \rangle \left(n_X^2 - \left(n_X^{EQ} \right)^2 \right)$$

- 2nd term in LHS: dilution by expansion of the universe
- 1st term in RHS: DM pair annihilation rate
- 2nd term in RHS: DM pair creation rate from the SM thermal plasma

It is more convenient to express the Boltzmann equation in terms of "Yield":

$$Y \equiv \frac{n}{s} = \frac{nV}{sV} = \frac{N_X}{S}$$

Since the total entropy S is conserved, the Yield is the total number of X particles in the Universe normalized by a constant S.

Boltzmann equation in terms of "yield"

$$Y = \frac{n}{s}$$

$$\frac{dY}{dx} = -\frac{\langle \sigma v \rangle}{x^2} \frac{s(m_\chi)}{H(m_\chi)} \left(Y^2 - Y_{EQ}^2\right),$$

$$x = m_\chi / T$$

$$H(m_\chi) = \sqrt{\frac{\pi^2}{90} g_*} \frac{m_\chi^2}{M_P},$$

$$s(m_\chi) = \frac{2\pi^2}{45} g_* m_\chi^3,$$

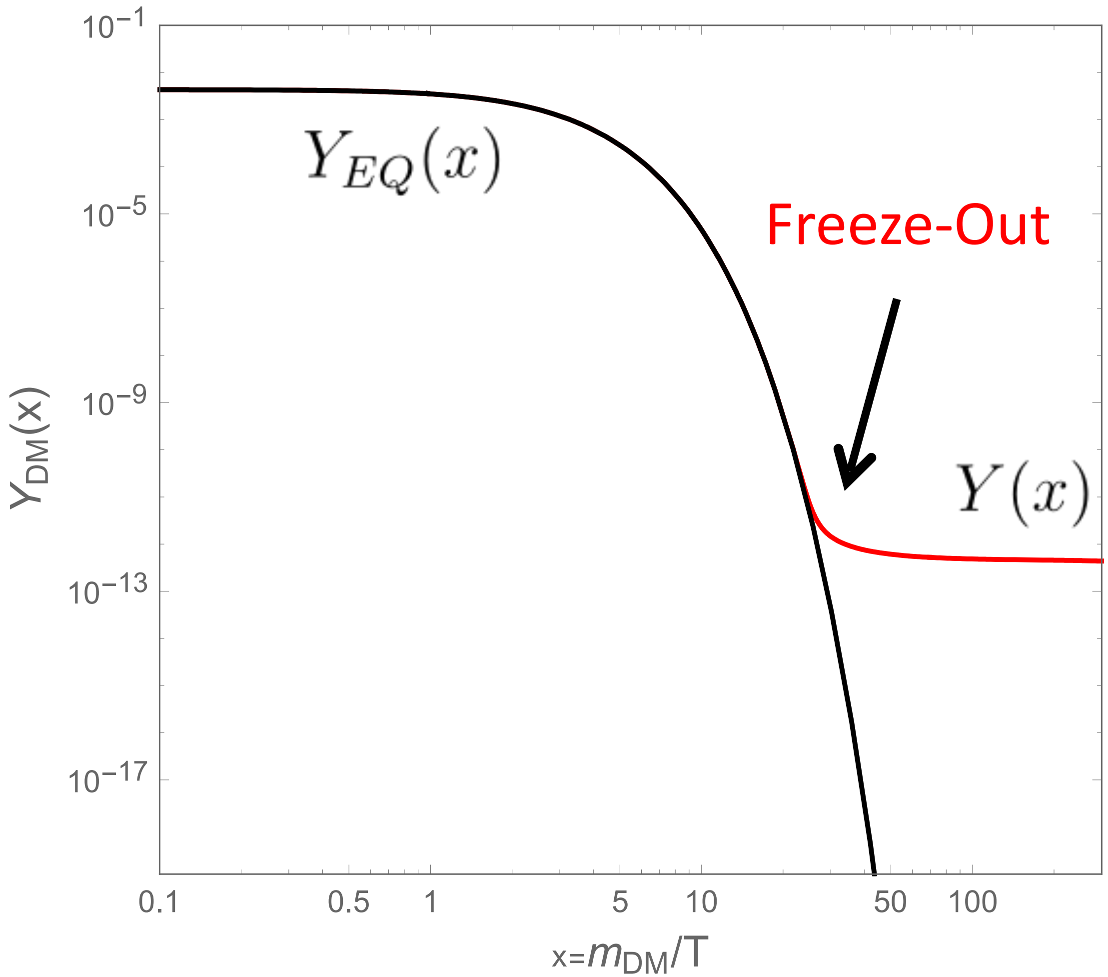
$$Y_{EQ}(x) = \frac{g_{DM}}{2\pi^2} \frac{x^2 m_\zeta^3}{s(m_\chi)} K_2(x)$$

$$\langle \sigma v \rangle = \frac{g_{DM}^2}{64\pi^4} \left(\frac{m_\chi}{x}\right) \frac{1}{n_{EQ}^2} \int_{4m_\chi^2}^{\infty} ds \; (\sigma v)\, s \sqrt{s - 4m_\chi^2}\, K_1\left(\frac{x\sqrt{s}}{m_\chi}\right)$$

g_{DM} : number of DM d.o.f

For a thermal DM, we solve the Boltzmann equation with an initial condition

$$Y(x) = Y_{EQ}(x) \quad \text{(for x << 1)}$$



$$\sigma(s) = \frac{g_\chi^2 \, g_f^2}{4\pi \, s}$$

$$m_\chi = 1\,\text{TeV} \gg m_{Z'}$$

$$g_\chi = g_f = 1.1$$

The DM relic density: $$\Omega_{DM} h^2 = \frac{m_\chi s_0 Y(\infty)}{\rho_c / h^2},$$

$$\text{where } s_0 = 2890 \text{ cm}^{-3}$$

$$\rho_c / h^2 = 1.05 \times 10^{-5} \text{ GeV/cm}^3$$

This should reproduce the observed DM density measured by Planck 2018

$$\Omega_{DM} h^2 = 0.12$$

We find a solution: $$m_\chi = 1\,\text{TeV} \gg m_{Z'}$$

$$g_\chi = g_f = 1.1$$

``WIMP DM Miracle"

WIMP (Weakly Interacting Massive Particle) is a primary candidate for the thermal DM in our Universe.

With a given annihilation cross section, the Boltzmann equation is easily solved, and we can find a good proximation formula to derive the observed DM density:

$\Omega_{DM} h^2 \sim 0.1$ is obtained if $\langle \sigma v_{rel} \rangle \sim 1\,\mathrm{pb}$

We may parametrize $\langle \sigma v_{rel} \rangle = \frac{e^4}{4\pi} \frac{1}{m_\chi^2} = 4\pi \alpha_{em}^2 \frac{1}{m_\chi^2}$

For $\alpha_{em} = \frac{1}{128}$, we find $m_\chi \sim 500\,\mathrm{GeV}$ leads to $\langle \sigma v_{rel} \rangle \sim 1\,\mathrm{pb}$

The mass (physics) scale of WIMP to be around 100 GeV-1 TeV is suggested by the observation!

Evaluation of the DM relic density (Non-Thermal DM case)

For Non-Thermal DM case, we can also use the same Boltzmann equation

$$\frac{dY}{dx} = -\frac{\langle \sigma v \rangle}{x^2} \frac{s(m_\chi)}{H(m_\chi)} \left(Y^2 - Y_{EQ}^2\right)$$

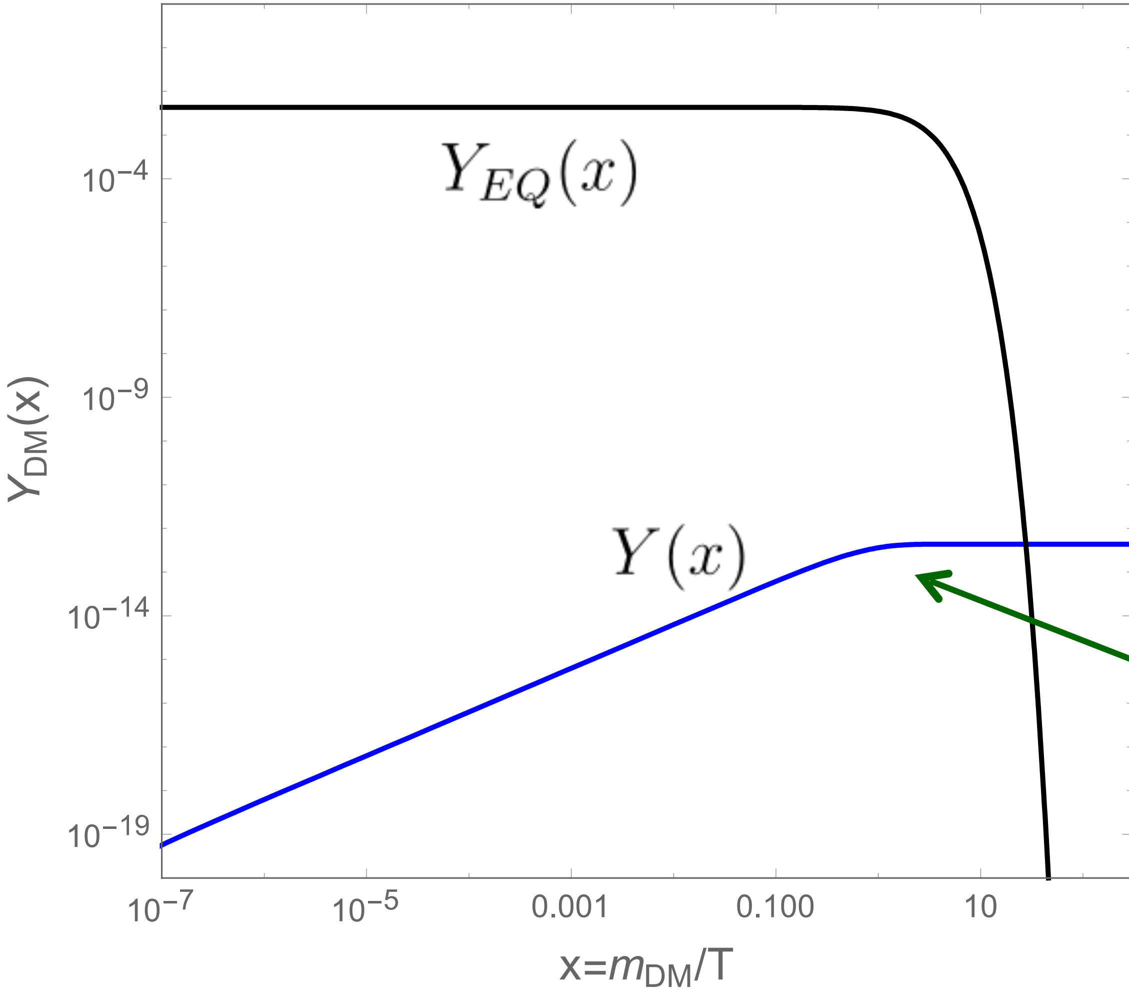
A crucial difference from thermal DM case lies in the initial condition:

$$Y(x_{RH}) = 0,$$

where $x_{RH} = \frac{m_{DM}}{T_{RH}}$ is the reheat temperature after inflation

We can easily solve the Boltzmann equation: $\frac{dY}{dx} \simeq const$

Freeze-In mechanism



$$m_\chi = 1\,\mathrm{TeV} \gg m_{Z'}$$

$$g_\chi = g_f = 3.2 \times 10^{-6}$$

DM production from thermal plasma stops at T ~ DM mass

$\Omega_{DM} h^2 = 0.12$ is reproduced

* Result is independent of T_RH.

4. Experimental prob of DM/Dark sector

Example:
Freeze-In/Out Dirac Fermion Dark Matter scenario and Lifetime Frontier Experiments

- In Z'-portal DM scenario, the couplings of the mediator Z' with the SM particles are very small, the mediator (Z') can be long-lived

- Many experiments to search for a long-lived charge neutral particles are planned and proposed (Lifetime Frontier)

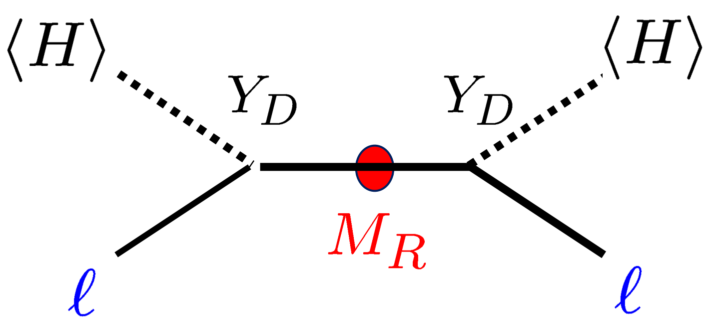
Z' boson in the gauged B-L (Baryon-Lepton number) extended Standard Models (Minimal B-L Model) is one of the search targets

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	**3**	**2**	$1/6$	$1/3$
u_R^i	**3**	**1**	$2/3$	$1/3$
d_R^i	**3**	**1**	$-1/3$	$1/3$
l_L^i	**1**	**2**	$-1/2$	-1
N_R^i	**1**	**1**	0	-1
e_R^i	**1**	**1**	-1	-1
H	**1**	**2**	$-1/2$	0
Φ	**1**	**1**	0	2

Properties of gauged B-L extended SMs

- It is easy (well-motivated) to gauge the global B-L symmetry in the SM
- All the gauge anomalies are cancelled in the presence of 3 right-handed neutrinos (RHNs)
- New B-L gauge boson mass & RHNs' Majorana masses are generated by the B-L gauge symmetry breaking
- The seesaw mechanism for generating tiny neutrino masses is implemented automatically.

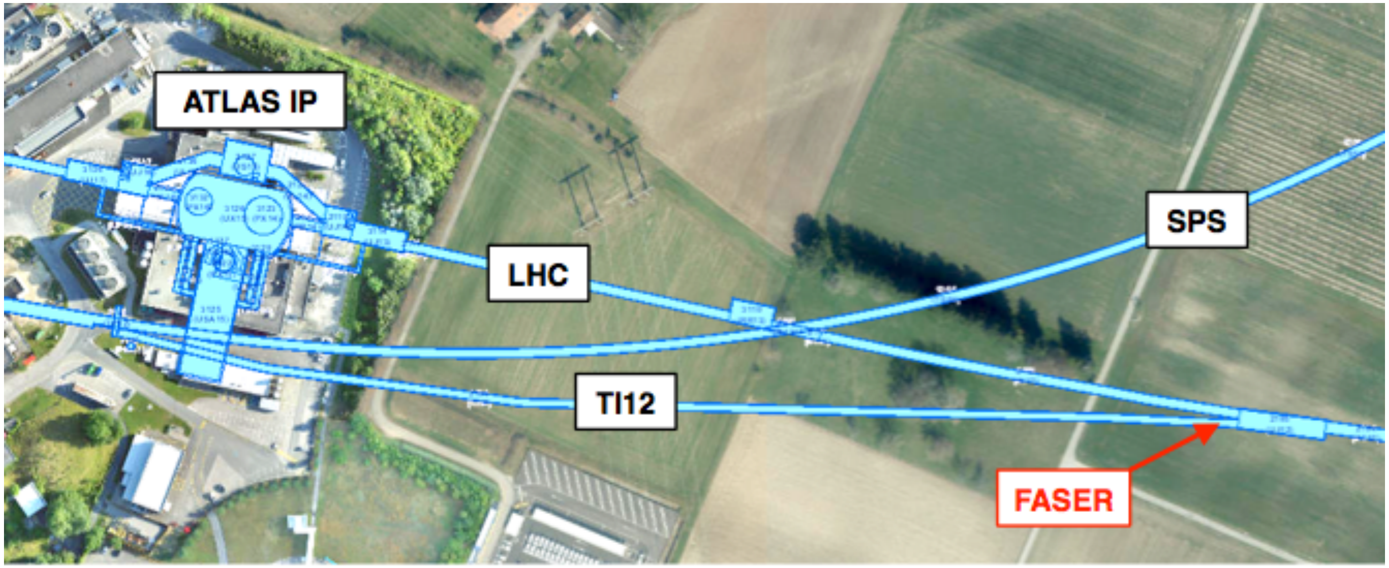
Seesaw mechanism



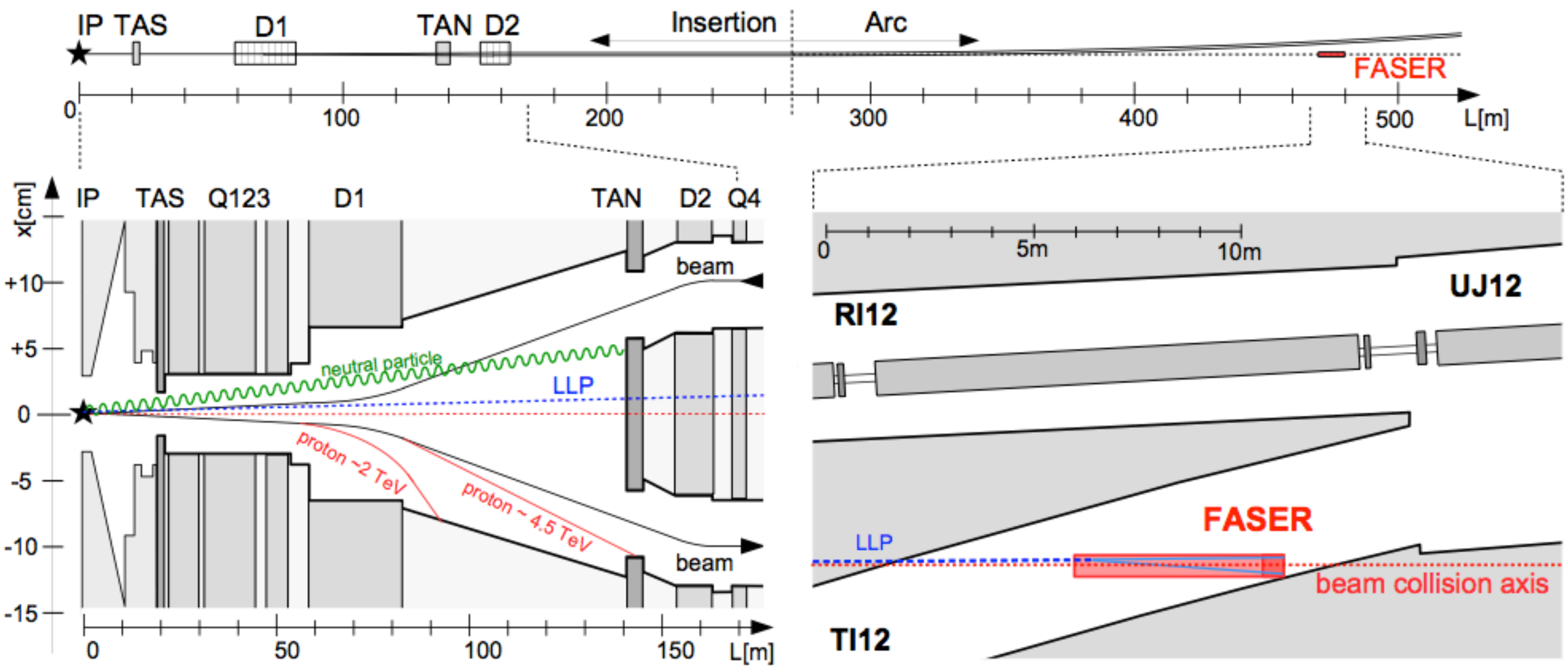
$$m_\nu = \frac{(Y_D\langle H\rangle)^2}{M_R} = Y_D\langle H\rangle\left(\frac{Y_D\langle H\rangle}{M_R}\right) \ll Y_D\langle H\rangle$$

ForwArd Search ExpeRiment (FASER)

- Recently approved (March, 2019) new experiment at CERN to search for long-lived exotic particles
- The FASER detector will be installed in a tunnel near the ATLAS detector about 480 m away



- Long-lived, electrically neutral particles can be created from rare decays of hadrons and propagate to the detector and leave ``displaced vertex” signature

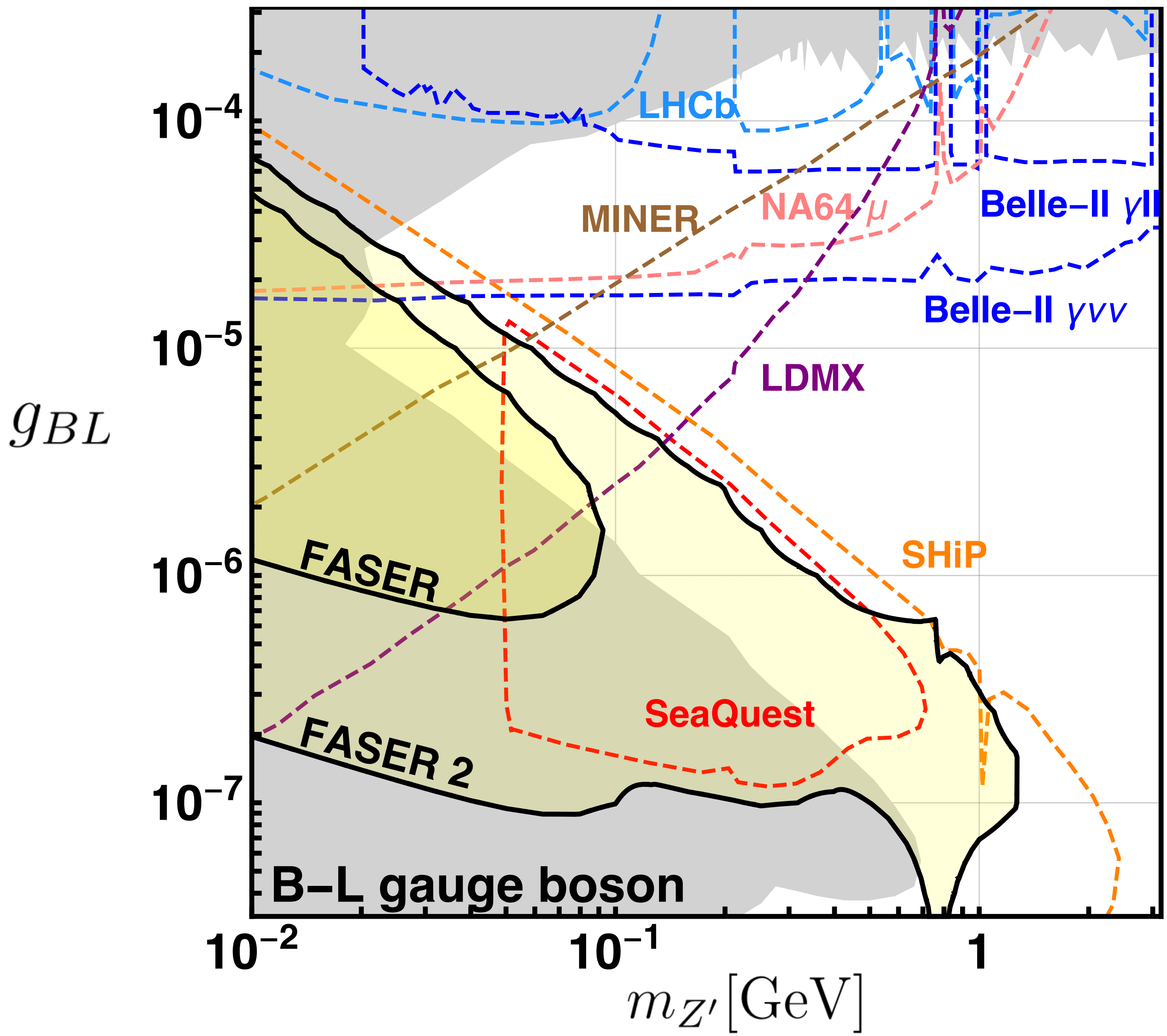


➢ Various long-lived neutral particles involved in new physics models beyond the Standard Model can be searched

``FASER's Physics Reach for Long-Lived Particles''
FASER Collaboration
arXiv: 1811.12522

- Dark Vectors
- Dark Scalars
- Dark pseudo-Scalars
- Heavy Neutral leptons
- Axion-like particles

Search reach by FASER + others for a long-lived B-L gauge boson



The minimal B-L model + Dirac Fermion Dark Matter

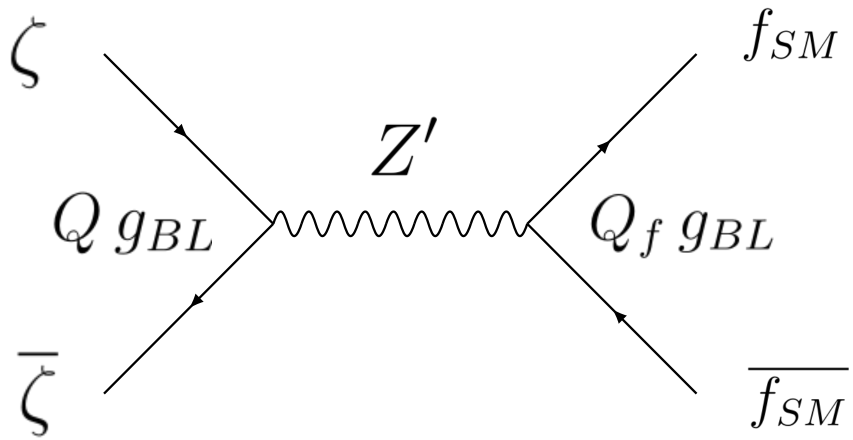
- ➢ Although the minimal B-L model is a simple, well-motivated Beyond the SM, a DM candidate is still missing.

- ➢ A simple way to supplement the model with a DM candidate we introduce an SM singlet Dirac fermion

	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{B-L}$
$\zeta = \zeta_L + \zeta_R$	$\mathbf{1}$	$\mathbf{1}$	0	Q

- ➢ Arbitrary $|Q| \neq 1, 3$ → stability is ensured

DM particle communicates with the SM particles through the B-L gauge boson Z'



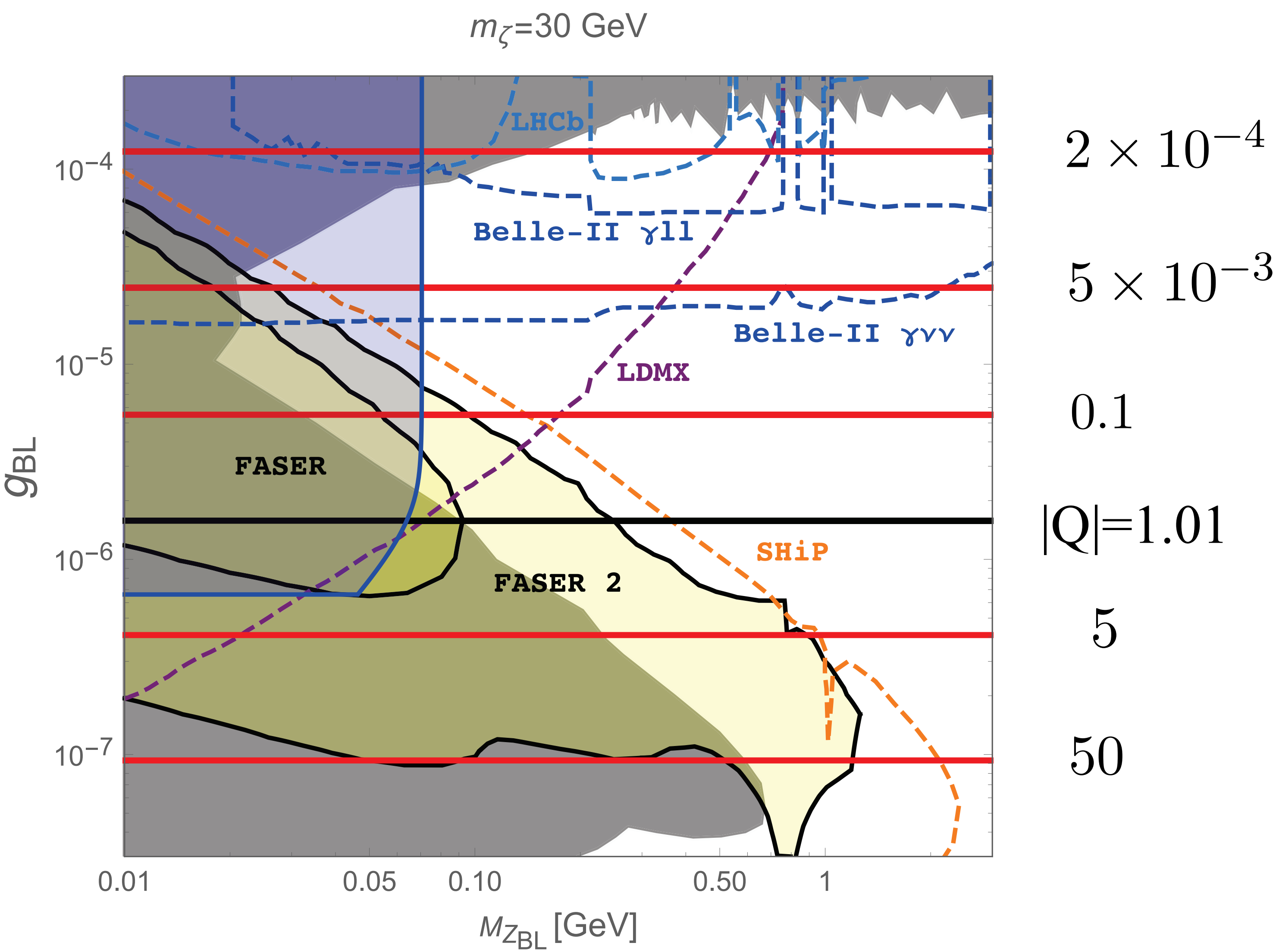
(1) Freeze-In DM case with $m_{Z'} \ll m_\chi$

Ref: Mohapatra & NO, PRD 102 (2020) 3, 035028

The toy Z'-portal model analysis is a good approximation

$$\Omega_{DM} h^2 = 0.12 \rightarrow Q\, g_{BL}^2 \simeq 10^{-11}$$

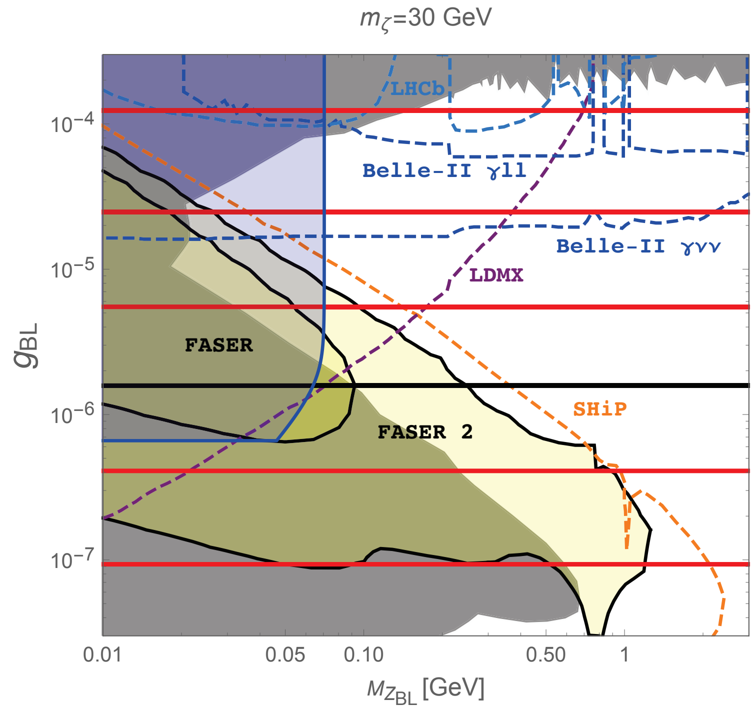
Testing the scenario by Lifetime Frontier Experiments

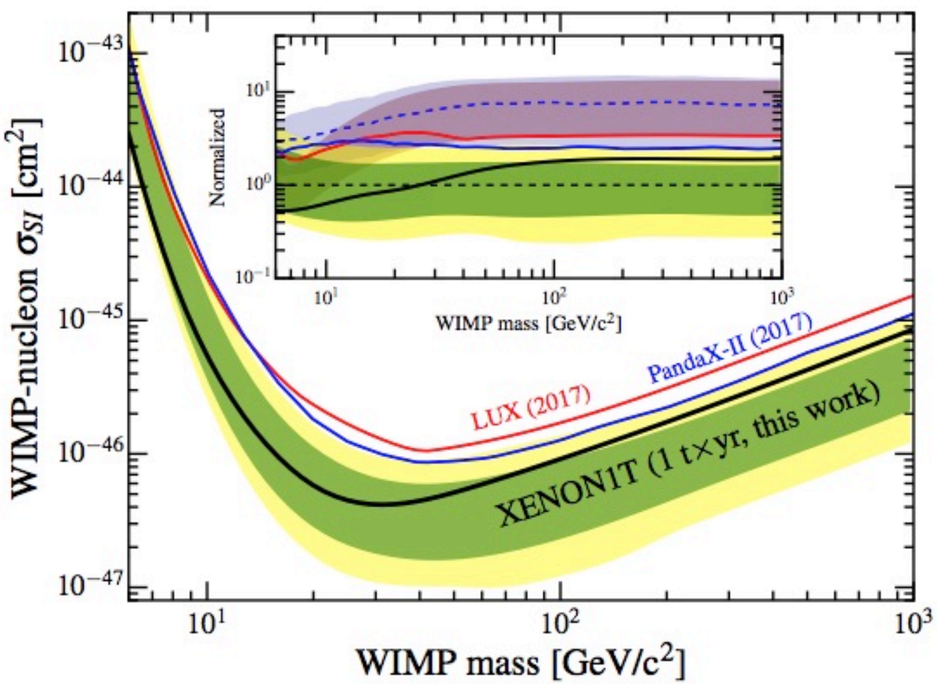


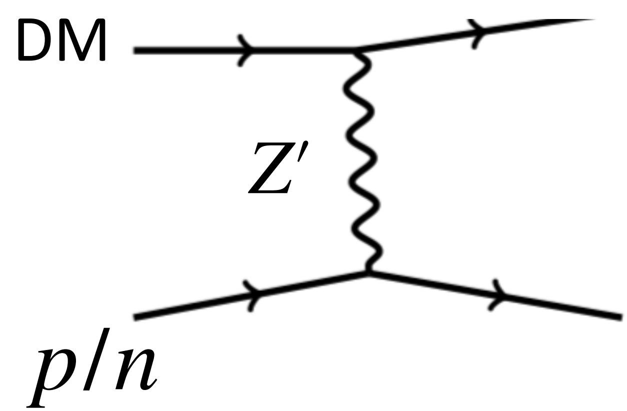
The result shifts downward as Q becomes larger

Testing the scenario by Lifetime Frontier Experiments

Blue-shaded region is excluded by XENON1T experiment for direct DM search





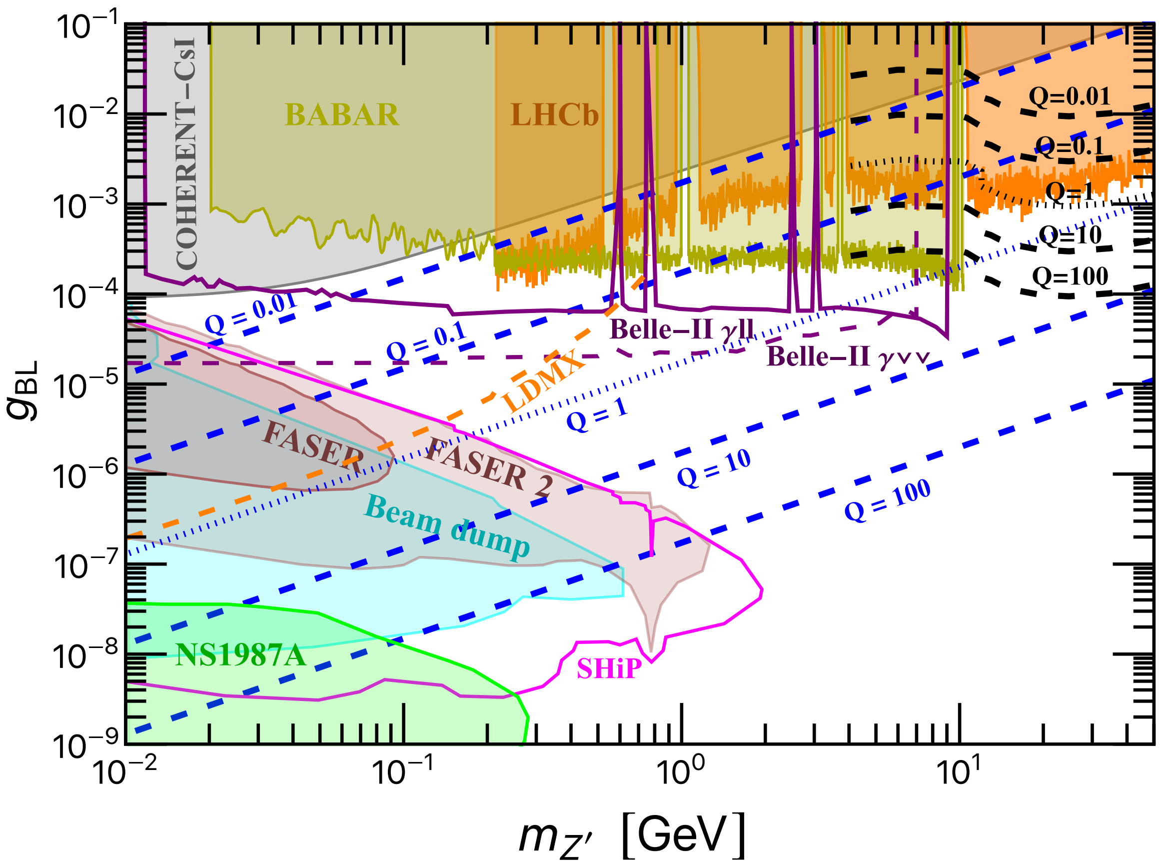


(2) Freeze-In/Out DM case with a light DM $m_\chi < m_{Z'}$

Ref: Nath, NO, Okada, Raut & Shafi: arXiv: 2112.08960

(i) Freeze-Out DM case

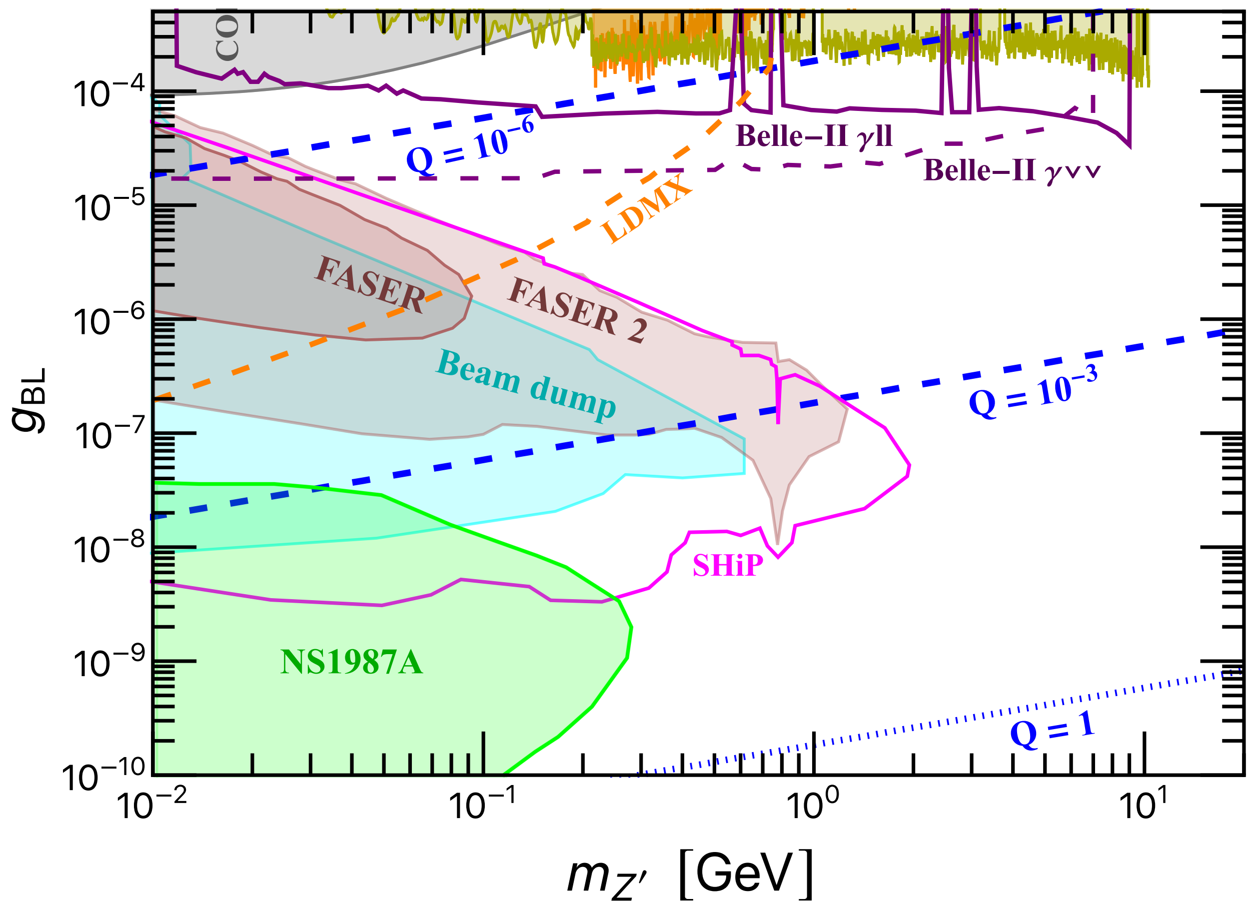
Along the blue lines, the observed DM density is reproduced.



(ii) Freeze-In DM case

DM mass =10 keV

Along the blue lines, the observed DM density is reproduced.



5. Conclusion

- The existence of DM requires us to extend the SM for incorporating a DM candidate
- Many DM scenarios have been proposed. We categorize scenario into Thermal & Non-Thermal DM scenarios
- For both scenarios, we have discussed a way to reproduce the observed DM relic density, Freeze-In/Out mechanism
- We have discussed the scenarios based on a Z'-portal Dirac fermion DM scenario, which can be realized by a simple extension of the minimal B-L model
- In the model, Z' gauge boson can be long-lived. Its existence will be explored by a variety of future ``Lifetime Frontier'' experiments.