Gravitational wave probes of new physics

Overview:

- GW introduction
- GW's from new physics in the early universe
  - Phase transitions
  - Scalar field dynamics (axions)
- GW's as indirect probes of NP
  - superradiance - BH spindown
  - new forces in mergers

What are GW's?

Any GR textbook, e.g. Carroll

Einstein equations:

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

metric

matter

$g_{\mu\nu}$ and its derivatives

EOM for $g_{\mu\nu}$

Expand around empty, flat space-time:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \eta_{\mu\nu} = \pm \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with $|h_{\mu\nu}| \ll 1$.

Generic, weak field limit $\rightarrow$ contains more than just GW's

Example: gravitational field outside of star:

$$h_{00} = -2\phi \qquad \text{with } \Delta\phi(\vec{x}) = 4\pi G \rho(\vec{x})$$

(Poisson equation)

Analogy with E&M: Not every electric field is an EM-wave

Q: What are sources of EM-waves?

[A: Accelerated charges, dipole-radiation]

Gauge invariance: In GR, have general coordinate invariance

$g_{\mu\nu}, h_{\mu\nu}$ not unique

↳ can choose a gauge for $h_{\mu\nu}$

Useful for GW's: Transverse, traceless gauge

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & 2s_{ij} & \\ 0 & & & \end{pmatrix}$$

↳ Simplifies vacuum equations of motion

$$\Box h^{TT}_{\mu\nu} = 0$$

Q: How do I solve this?

Plane wave ansatz: $h^{TT}_{\mu\nu} = C_{\mu\nu} e^{ik_\sigma x^\sigma}$

with $k_\sigma k^\sigma = 0$. For $k_\mu = (\omega, 0, 0, \omega)$ have transverse polarisation tensor

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & C_{12} & 0 \\ 0 & C_{12} & -C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$C_{11} = h_+$
$C_{12} = h_\times$

Q: Why $+$, $\times$?

GW sources:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

now keep this part

find $\Box \bar{h}^{TT}_{\mu\nu} = -16\pi G_N T^{TT}_{\mu\nu}$ where $\bar{h}^{TT}_{\mu\nu} = h^{TT}_{\mu\nu} - \frac{1}{2} h^{TT\sigma}_{\sigma} \eta_{\mu\nu}$

inhomogeneous wave equation

Formal solution using Green's function of d'Alembert operator, G:
(drop ^{TT} superscripts)

$$\Box_x G(x-y) = \delta^4(x-y)$$

$$\hookrightarrow \quad \bar{h}_{\mu\nu}(x) = -16\pi G \int d^4y \, G(x-y)\, T_{\mu\nu}(y)$$

Again in full analogy with E&M.

Typically interested in the field far away from the source. In E&M, this is the dipole component. Here:

retarded time $t_r = t - r$

$$\bar{h}_{ij}(t,\vec{x}) = \frac{2G}{r} \frac{d^2}{dt^2} I_{ij}(t_r)$$

Quadrupole moment $\quad I_{ij}(t) = \int y^i y^j \, T^{00}(t,\vec{y})\, d^3\vec{y}$

Notes:
- $\frac{d^2}{dt^2} \leftrightarrow$ accelleration
- T^{00} appears because we assume a non-relativistic source
- quadrupole radiation (vs. dipole in E&M case).

Astrophysical sources:

- Binary systems of black holes, neutron stars, white dwarfs etc. Also supermassive BHs!

- historically important: Hulse Taylor binary

  ↳ Change in orbital frequency due to energy loss from GW emission → indirect detection of GWs.

- these systems are now becoming laboratories for probing deviations from SM expectations → last part of lecture.

<u>Cosmological sources:</u>

- Inflation
- Cosmic strings
- Phase transitions
- Axions

focus on these

Note: Cosmological sources produce a stochastic signal (i.e. noise) while e.g. binaries produce a coherent signal - though there is also a stochastic BG from unresolved binaries