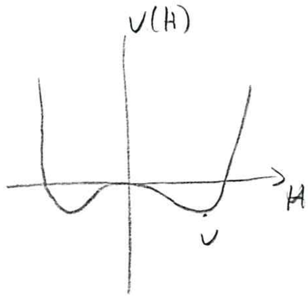
Phase transitions in the early universe



The Higgs potential of the SM is given by $V(H) = -\mu^2 |H|^2 + \lambda |H|^4$

At tree level the minimum $\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ is given by

$$v^2 = \frac{\mu^2}{\lambda} \quad (\text{if } \mu^2 > 0).$$

In QFT the potential receives radiative corrections. To incorporate them one introduces the effective potential $V_{eff}(\varphi_{cl})$, where φ_{cl} is the classical background field. The minimum is then found by imposing

$$\frac{\partial V_{eff}}{\partial \varphi_{cl}} = 0 .$$

Methods for computing the one loop corrected eff. potential can be found e.g. in Peskin.

For the SM, the renormalized 1-loop result is

$$V(\varphi_{cl}) = V_0(\varphi_{cl}) + \frac{1}{64\pi^2} \sum_{i=W,Z,h,t,\chi} \underbrace{n_i}_{\#dof} m_i^4(\varphi_{cl}) \left[\log \frac{m_i^2(\varphi_{cl})}{\mu^2} - \underbrace{C_i}_{\substack{\frac{5}{6} \text{ gauge bosons} \\ \frac{3}{2} \text{ else}}} \right]$$

Literature:
- M. Quiros: Finite T field theory and PT's (hep-ph/9901312)
- Laine, Vuorinen: 1701.01554
- Books by Kapusta, Le Bellac
- Arxiv: 2008.09136 Hindmarsh et al

This is the vacuum case. The early universe is a messy place however, a hot plasma with all SM particles in thermal equilibrium. This induces additional, <u>finite temperature</u> contributions to V_{eff}.

Intuitive picture: A particle moving through the plasma is constantly hit and scattered. This makes it more difficult to accelerate it $\Leftrightarrow$ the particle gets an additional <u>thermal mass</u>.

The contributions to V_{eff} separate into a $T=0$ and a finite temperature part. One finds:

$$\Delta V^{(1\text{-loop})}(\varphi_{cl}, T) = \frac{T^4}{2\pi^2}\left[\sum_{i=W,Z} n_i \underbrace{J_B\left(\frac{m_i^2(\varphi_{cl})}{T^2}\right)}_{\to -\frac{\pi^4}{45}+\frac{\pi^2}{12}\frac{m^2}{T^2}+\dots} + n_t \underbrace{J_F\left(\frac{m_t^2(\varphi_{cl})}{T^2}\right)}_{\to \frac{7\pi^4}{360}-\frac{\pi^2}{24}\frac{m^2}{T^2}+\dots}\right]$$

Finally the total eff. potential at finite T can be written as

$$V(\varphi_{cl}, T) = D(T^2 - T_0^2)\varphi_{cl}^2 - ET\varphi_{cl}^3 + \frac{\lambda(T)}{4}\varphi_{cl}^4$$

$$D = \frac{1}{8v^2}(2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v^3}(2m_W^3 + m_Z^3) \qquad \text{[no fermion contribution]}$$

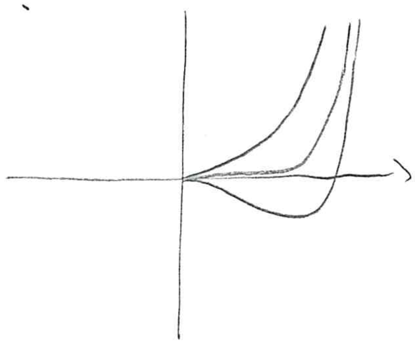
$$T_0^2 = \frac{m_h^2 - 8Bv^2}{4D} \qquad B = \frac{3}{64\pi^2 v^4}(2m_W^4 + m_Z^4 - 4m_t^4)$$

The important point here is that at $T > T_0$, the global minimum of $U_{eff}(T)$ is at $\varphi_{cl} = 0$, i.e. the electroweak symmetry is restored.

In cosmology, the universe after the big bang (and inflation) reheats to high temperatures ($T \gg T_0$) so that EW symmetry is unbroken initially. The universe undergoes a transition from $\varphi_{cl} = 0$ to $\varphi_{cl} \sim v$ around $T_0 \sim 10^2$ GeV

[also QCD PT near GeV scale]

Consider $U(\phi, T) = D(T^2 - T_0^2)\,\phi^2 + \frac{\lambda(T)}{4}\phi^4$.



stationary points: $\frac{dU}{d\phi} = 0$

$$\phi_1(T) = 0$$

$$\phi_2(T) = \sqrt{\frac{2D(T_0^2 - T^2)}{\lambda(T)}} \qquad (T < T_0)$$

At $T > T_0$, $\phi = 0$ is the only solution.

At $T = T_0$, both solutions are at $\phi = 0$. For $T < T_0$, ϕ_2 is the global minimum and ϕ_1 becomes a local maximum. There is no barrier between the minima, the field can adiabatically follow the vacuum state.

$\rightarrow$ ~~2nd order PT.~~

Crossover.

For more complex potentials, a barrier might exist. E.g.

$$U(\phi, T) = D(T^2 - T_0^2)\,\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4$$

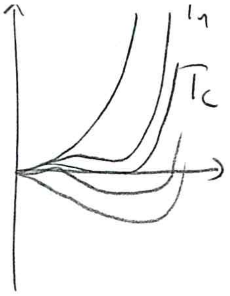
$T > T_1$: $\phi = 0$ is min.

At T_1 : Second min. appears

At T_c : Both minima have equal value

$T < T_c$: $\phi = 0$ becomes meta-stable

$T < T_0$: barrier goes away, $\phi = 0$ is max.



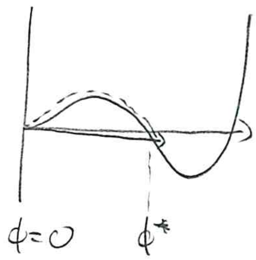
The PT can start at $T \lesssim T_c$. If the tunneling probability is small, $T < T_c$. Also models with tree level barriers exist $\Rightarrow T_0 = 0$.

Thermal tunneling

Bubble formation

Tunneling rate : $\frac{\Gamma}{V} \sim A(T)\, e^{-S_3/T}$

$$S_3 = \int d^3x \left(\frac{1}{2} (\nabla\phi)^2 + U(\phi, T) \right)$$



with ϕ being the bounce solution

path from $\phi = 0$ to ϕ^* that minimises action.

Explain bubble expansion

↳ energy gain vs. surface tension

Compete with expansion of universe $\rightarrow \frac{S_3}{T} \sim 140$

Motivations:

1. First order PT = deviation from th. eqo $\Rightarrow$ Baryogenesis
2. GWs in range of planned expts.