UNIVERSITY OF COPENHAGEN
Niels Bohr Institute

A Study of Top Anomalous Couplings at a Future e^+e^- Collider

Julie Munch Torndal

Supervisor: Jørgen Beck Hansen

September 20th, 2021

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4} O_j^{(8)} + \ldots$$

- 7 independent dim-6 operators contributing to top couplings
  $\rightarrow$ 10 anomalous couplings which depend on 7 operator coefficients

$$\mathcal{L}_{Wtb} = -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- - \frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^- + H.c.$$

$$\mathcal{L}_{Ztt} = -\frac{g}{2c_W} \bar{t} \gamma^\mu (X_{tt}^L P_L + X_{tt}^R P_R - 2s_W^2 Q_t) t Z_\mu - \frac{g}{2c_W} \bar{t} \frac{i\sigma^{\mu\nu} q\nu}{M_Z} (d_V^Z + i d_A^Z \gamma_5) t Z_\mu$$

$$\mathcal{L}_{\gamma tt} = -eQ_t \bar{t} \gamma^\mu A_\mu - e\bar{t} \frac{i\sigma^{\mu\nu} q\nu}{m_t} (d_V^\gamma + i d_A^\gamma \gamma_5) t A_\mu$$

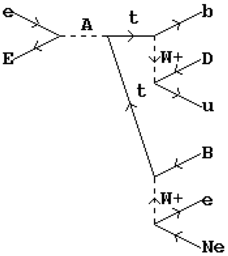
Monte Carlo Samples

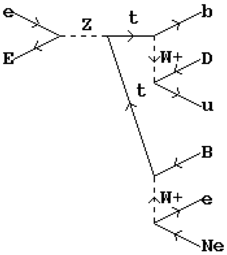
Signal:

Semileptonic channel

$$e^+e^- \to t\bar{t} \to bW^+\bar{b}W^- \to b\bar{b}q\bar{q}\ell\nu_\ell$$

Planned phase of FCC-ee @ $\sqrt{s} = 365$ GeV





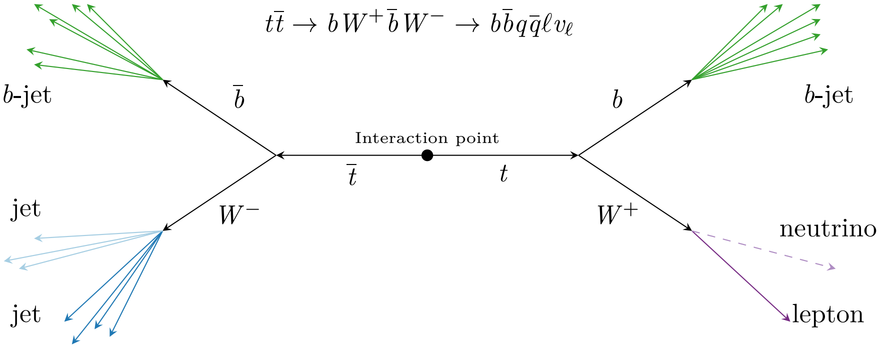
Backgrounds:

Process	σ [pb]
$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu_\ell$	0.1933
$\mu\mu$	0.7942
$\tau\tau$	0.7937
$\sum q\bar{q}$	4.143
$b\bar{b}$	0.7448
γZ	3.386
WW	10.72
ZZ	0.6428
ZH	0.1173
ZWW	$15.91 \cdot 10^{-3}$
ZZZ	$0.7633 \cdot 10^{-3}$
Single top	$3.337 \cdot 10^{-3}$

- All MC files are generated in the FCCSW framework with DelphesPythia8_EDM4HEP and IDEA Delphes Card
- Anomalous couplings available with Whizard

Signature of semileptonic channel:

$$t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow b\bar{b}q\bar{q}\ell v_\ell$$

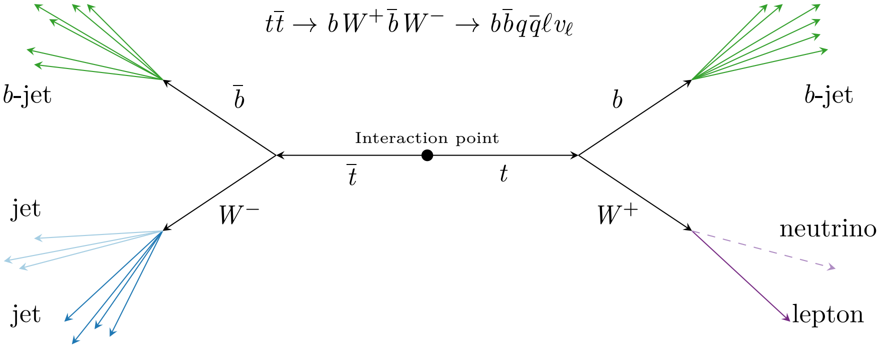


Signature objects:

- 1 lepton
- 1 neutrino
- 4 jets
- (2 b-tags)

Signature of semileptonic channel:

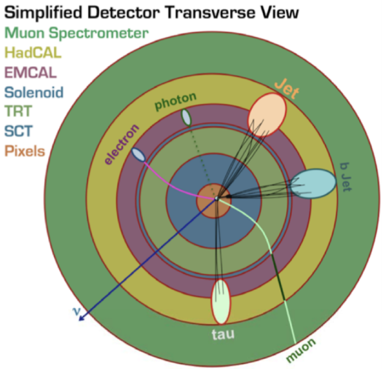
$$t\bar{t} \to bW^+\bar{b}W^- \to b\bar{b}q\bar{q}\ell v_\ell$$



Signature objects:

- 1 lepton
- 1 neutrino ✓
- 4 jets
- (2 b-tags)

Missing momentum



Lepton Identification:

$$\ell = \{e, \mu\}$$

- Assumes perfect PID

Main tau decay modes:

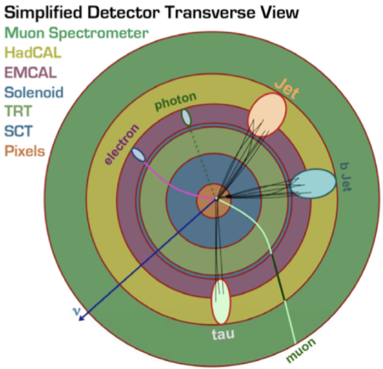
- $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ (17.8 %),
- $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ (17.4 %),
- $\tau^- \to \pi^- (n\pi^0) \nu_\tau$ (48 %)
- $\tau^- \to \pi^- \pi^+ \pi^- (n\pi^0) \nu_\tau$ (15 %)

Highest energy lepton:

Selecting highest energy lepton as signature lepton has an acceptance of

96.6±0.7 %

→ Find RP highest energy lepton
→ Match to MC particle
→ Parent history (EDM4Hep gives parent and daughter history for MCParticleData)
→ Stopping criteria with PDG and status code



Lepton Identification:

$$\ell = \{e, \mu\}$$

- Assumes perfect PID

Main tau decay modes:

- $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ (17.8 %),
- $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ (17.4 %),
- $\tau^- \to \pi^-(n\pi^0)\nu_\tau$ (48 %)
- $\tau^- \to \pi^-\pi^+\pi^-(n\pi^0)\nu_\tau$ (15 %)

Highest energy lepton:

Selecting highest energy lepton as signature lepton has an acceptance of

96.6±0.7 %

→ Find RP highest energy lepton
→ Match to MC particle
→ Parent history (EDM4Hep gives parent and daught
→ Stopping criteria with PDG and status code

Signature objects:

- 1 lepton ✓
- 1 neutrino ✓
- 4 jets
- (2 b-tags)

FCCAnalyses / JetClustering

- analyzers/dataframe/**JetClustering**.h
- analyzers/dataframe/**JetClustering**.cc
- analyzers/dataframe/**JetClustering**Utils.h
- analyzers/dataframe/**JetClustering**Utils.cc

Jet Clustering Interface:

- flexible "after burner" using FastJet
- run multiple jet reconstructions at once
- select input particles
- access to jet constituents

Jet Reconstruction:

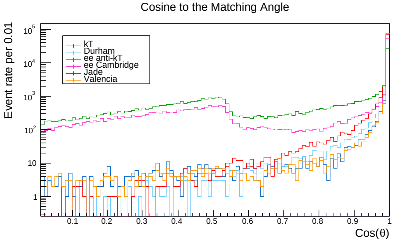
Jet Algorithms
$k_\perp$
Durham (e^+e^- $k_\perp$)
e^+e^- Anti-$k_\perp$
e^+e^- Cambridge/Aachen
Valencia
Jade

$\otimes$

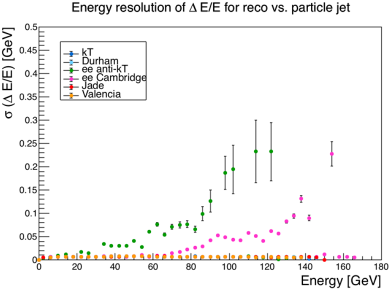
Recombination Schemes
E-scheme
$E0$-scheme
p-scheme

- sequential recombination
- exclusive clustering with *exactly* 4 jets
- highest energy lepton excluded from the clustering

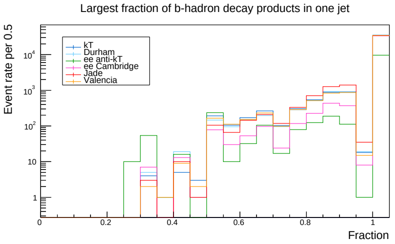
Matching angle between reco and particle jets



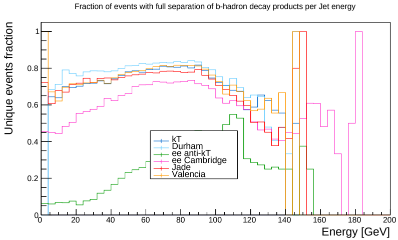
Energy resolution of jets



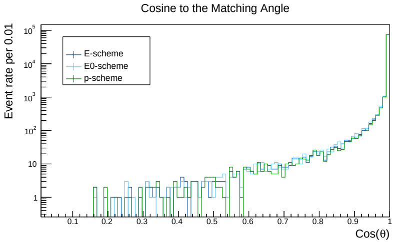
Largest fraction of b-hadron decay products in one jet



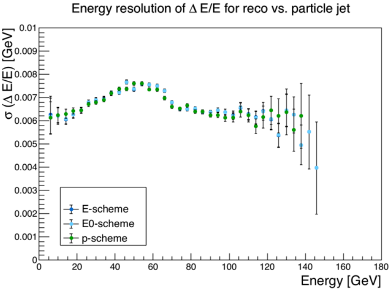
Fraction of events with full separation of b-hadron decay products



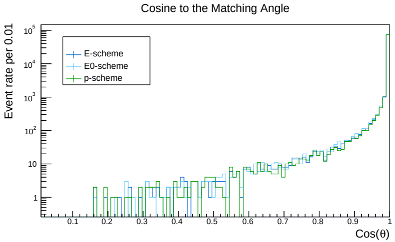
Matching angle between reco and particle jets



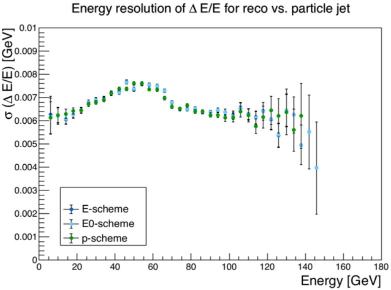
Energy resolution of jets



Matching angle between reco and particle jets



Energy resolution of jets



→ **Jet Definition = Durham + E-scheme**

Signature objects:

- 1 lepton ✓
- 1 neutrino ✓
- 4 jets ✓
- (2 b-tags)

Jet Tagging Interface

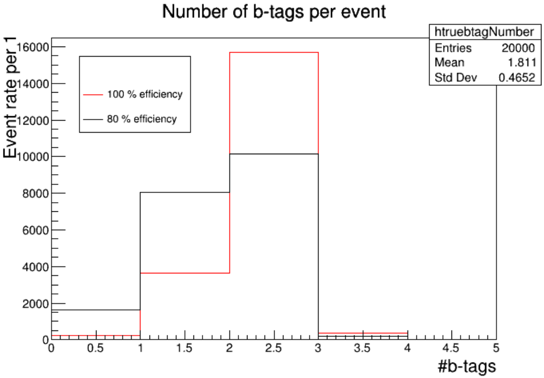
- Jet is matched to parton by

  $Angle(\text{jet,parton}) < 0.3$ rad

- Flavour priority: $b > c >$light flavour
- Flat efficiency for individual flavours

FCCAnalyses / JetTagging

- analyzers/dataframe/JetTaggingUtils.h
- analyzers/dataframe/JetTaggingUtils.cc

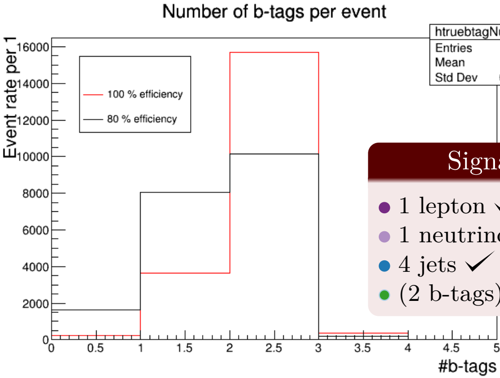


Jet Tagging Interface

- Jet is matched to parton by
  $Angle(\text{jet,parton}) < 0.3$ rad
- Flavour priority: $b > c >$light flavour
- Flat efficiency for individual flavours

FCCAnalyses / JetTagging

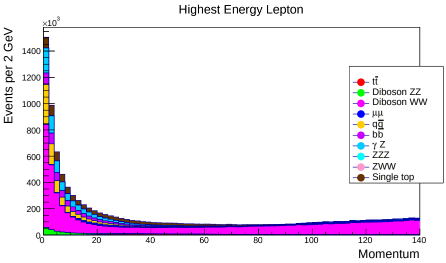
- analyzers/dataframe/JetTaggingUtils.h
- analyzers/dataframe/JetTaggingUtils.cc

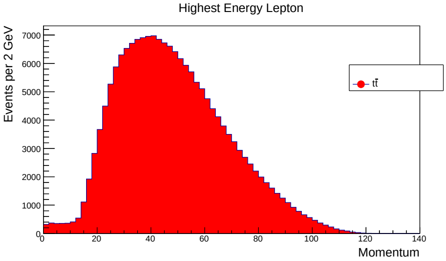


Signature objects:

- 1 lepton ✓
- 1 neutrino ✓
- 4 jets ✓
- (2 b-tags) ✓

Event Selection

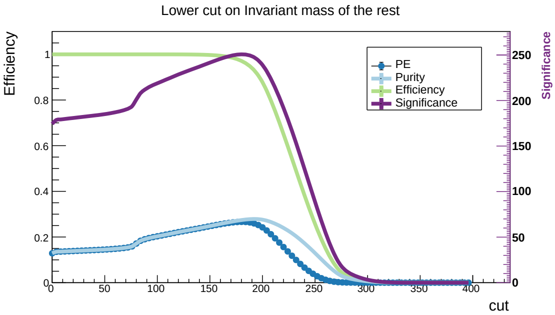


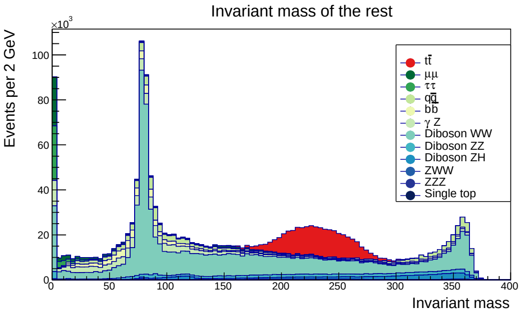


Process	σ [pb]
$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu_\ell$	0.1933
$\mu\mu$	0.7942
$\tau\tau$	0.7937
$\sum q\bar{q}$	4.143
$b\bar{b}$	0.7448
γZ	3.386
WW	10.72
ZZ	0.6428
ZH	0.1173
ZWW	$15.91 \cdot 10^{-3}$
ZZZ	$0.7633 \cdot 10^{-3}$
Single top	$3.337 \cdot 10^{-3}$

→ Signal region!

Event Selection





$$\text{Significance} = \frac{\text{sig}}{\sqrt{\text{sig} + \text{bkg}}}$$

$$\text{Efficiency} = \frac{\text{sig}}{\text{sig}_{\text{tot}}}$$

$$\text{Purity} = \frac{\text{sig}}{\text{sig} + \text{bkg}}$$

Signal region cuts

- At least 1 lepton
- Thrust < 0.85
- $M(rest) > 160$ GeV
- $M(\ell_{HE}, \not{E}) > 50$ GeV
- $p_{\ell_{HE}} < 100$ GeV
- $p_{\ell_{HE}} > 15$ GeV
- $p_{\ell_{2nd\,HE}} < 40$ GeV
- Exactly 4 jets
- At least 1 b-tag

Cut flow

Selection	Signal/10^3	Background/10^3	ϵ_{sig}	ϵ_{bkg}	Signif
Initial	262.89 ± 0.24	$29\,052 \pm 22$	1.0	1.0	48.6
At least 1 lepton	246.01 ± 0.23	$13\,565 \pm 14$	0.94	0.47	66.2
Thrust < 0.85	236.07 ± 0.23	1599.1 ± 4.7	0.90	0.055	174.3
$M(rest) > 160$ GeV	234.61 ± 0.23	676.4 ± 2.9	0.89	0.023	245.8
$M(\ell_{HE}, \not{E}) > 50$ GeV	181.21 ± 0.20	178.3 ± 1.5	0.69	0.0061	302.2
$p_{\ell_{HE}} < 100$ GeV	179.72 ± 0.20	133.6 ± 1.3	0.68	0.0046	321.0
$p_{\ell_{HE}} > 15$ GeV	176.01 ± 0.20	117.8 ± 1.2	0.67	0.0041	324.7
$p_{\ell_{2nd\,HE}} < 40$ GeV	175.07 ± 0.20	103.5 ± 1.1	0.67	0.0036	331.7
Exactly 4 jets	175.07 ± 0.20	93.4 ± 1.1	0.67	0.0032	337.9
At least 1 b-tag	160.62 ± 0.19	11.14 ± 0.25	0.61	0.000 38	387.6

15 % impr.

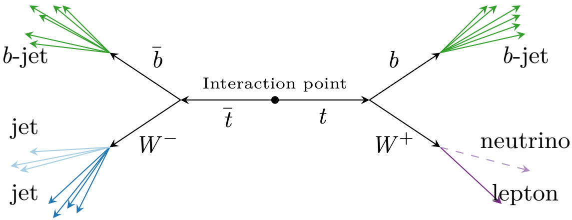
- $\epsilon_{sig}(e, \mu) = 84$ % and $\epsilon_{sig}(\tau) = 15$ %
- Largest background contributions: ZH, ZZ and single top

Angular distributions:

$t\bar{t}$ production	$t \to Wb$	$W \to \ell\nu$
θ_{et}	$\theta^*_{tb},\ \phi^*_{tb}$	$\theta^*_{W\ell},\ \phi^*_{W\ell}$

Total cross section:

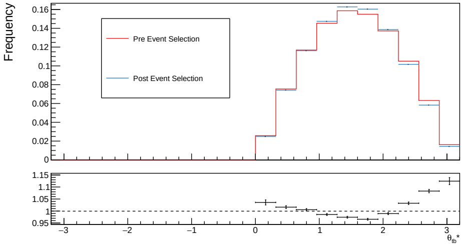
$t\bar{t}$ production
σ_{tot}

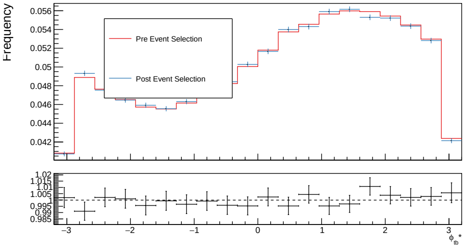


ta__ttA	δd_A^{γ}	🔓
tv__ttA	δd_V^{γ}	🔓
ta__ttZ	δd_A^{Z}	🔒
tv__ttZ	δd_V^{Z}	🔒
vl__ttZ	δX_{tt}^{L}	🔒
vr__ttZ	δX_{tt}^{R}	🔓
tl__tbW__Re	δg_L	🔓
tr__tbW__Re	δg_R	🔓
vl__tbW__Re	δV_L	🔓
vr__tbW__Re	δV_R	🔓

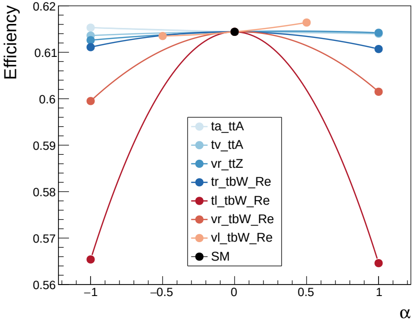
Selection on Observables

Angular distributions:



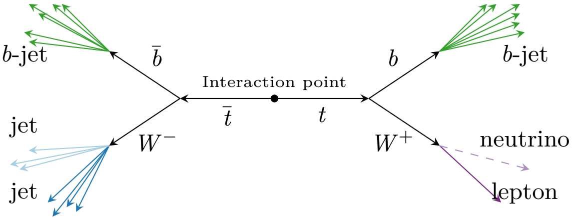


Total cross section:



Event Reconstruction:

- Combinatorics
- Measurement corrections



χ^2-function to minimise:

$$L(y) = S(y) + g(x) + 2\sum_{k=1}^{m} \lambda_k f_k(a, y, x)$$

- **Least Squares Principle:** $S(y) = \Delta y^T \mathbf{V}(y)^{-1} \Delta y = \min$
- **Lagrange multipliers:** $2\sum_{k=1}^{m} \lambda_k f_k(a, y)$
- **Model expressed as m constraints:** $f_k(\bar{a}, \bar{y}) = 0 \quad , \quad k = 1, \ldots, m$
- **Penalty function:** $g(x) = -2\ln(pdf(x))$

χ^2-function to minimise:

$$L(y) = S(y) + g(x) + 2\sum_{k=1}^{m} \lambda_k f_k(a, y, x)$$

Solution:

$$\begin{pmatrix} y^{n+1} \\ x^{n+1} \end{pmatrix} = \begin{pmatrix} y_0 \\ x^n \end{pmatrix} - \tilde{\mathbf{V}} \begin{pmatrix} 0 \\ \frac{1}{2}\frac{\mathrm{d}^2 g}{\mathrm{d}x^2}\big|^{-1}_{x=x^n} \end{pmatrix} + \tilde{\mathbf{V}} B^T (B\tilde{\mathbf{V}}B^T)^{-1}$$
$$\times \left[A(a^n - a_0) + B \begin{pmatrix} y^n - y_0 \\ \frac{\mathrm{d}g}{\mathrm{d}x}\big|^{-1}_{x=x^n} / \frac{\mathrm{d}^2 g}{\mathrm{d}x^2}\big|_{x=x^n} \end{pmatrix} - f(a^n, y^n, x^n) \right]$$

$$a^{n+1} = a_0 + W_A^{-1} A^T W_B$$
$$\times \left[A(a^n - a_0) + B \begin{pmatrix} y^n - y_0 \\ \frac{\mathrm{d}g}{\mathrm{d}x}\big|^{-1}_{x=x^n} / \frac{\mathrm{d}^2 g}{\mathrm{d}x^2}\big|_{x=x^n} \end{pmatrix} - f(a^n, y^n, x^n). \right]$$

with

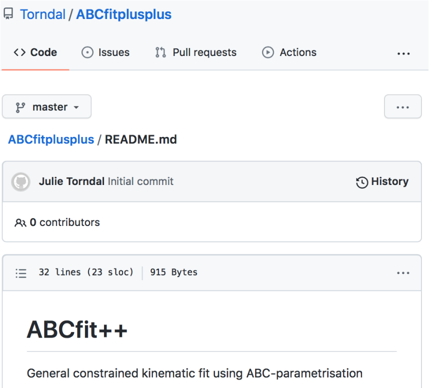
$$\tilde{\mathbf{V}} = \begin{pmatrix} \mathbf{V} & 0 \\ 0 & \left(\frac{1}{2}\frac{\mathrm{d}^2 g}{\mathrm{d}x^2}\big|_{x=x^n}\right)^{-1} \end{pmatrix}, \quad B = \frac{\partial f(a, y, x)}{\partial (y, x)}, \quad A = \frac{\partial f(a, y, x)}{\partial a},$$

$$W_B = (BVB^T)^{-1} \quad \text{and} \quad W_A^{-1} = (A^T W_B A)^{-1}$$

General software package for constrained fitting

Base Classes:

- Coordinate Representation
- Particle Object
- Constraint
- Composite constraint
- Probability distribution functions
- Matrix Algebra
- ABC Fit

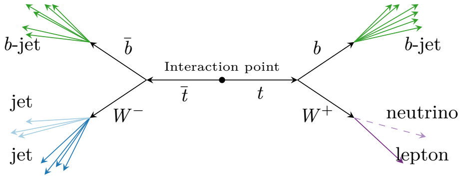


Link to GitHub

Constraints:

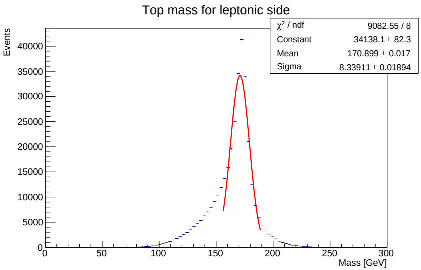
- Energy and momentum conservation

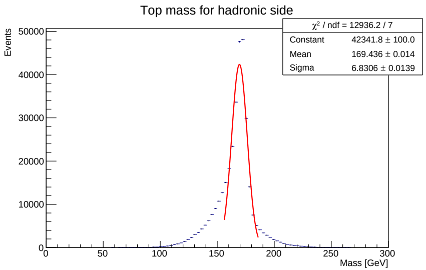
$$\sum \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} 365 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{GeV}$$

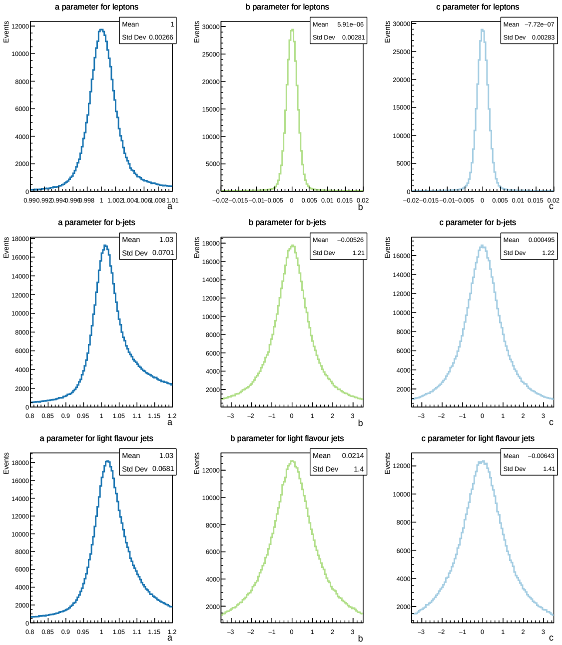


- Mass constraints for the two top systems

$$\mu = 173 \text{ GeV}, \quad \sigma_{\text{lep}} = 10 \text{ GeV} \quad \text{and} \quad \sigma_{\text{had}} = 9 \text{ GeV}$$







Gaussian ABC-parameters:

$$\vec{p}_j^{\,r} = a_j|\vec{p}_j^{\,m}|\vec{p}_j^{\,a} + b_j\vec{p}_j^{\,b} + c_j\vec{p}_j^{\,c}$$

Cartesian unit vectors:

$$\vec{p}_j^{\,a} = \frac{\vec{p}_j^{\,m}}{|\vec{p}_j^{\,m}|},$$

$$\vec{p}_j^{\,a} \cdot \vec{p}_j^{\,b} = 0,$$

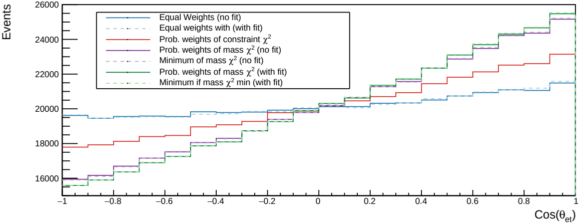
$$\vec{p}_j^{\,c} = \vec{p}_j^{\,a} \times \vec{p}_j^{\,b}.$$

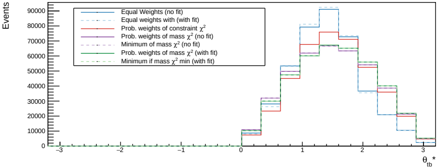
$$(a, b, c)_{\text{lepton}} = (1, 0, 0)$$

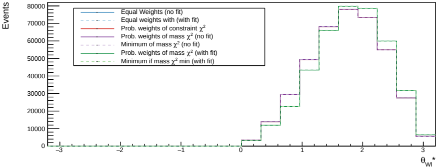
$$(a, b, c)_{\text{b-jet}} = (1.03, 0, 0)$$

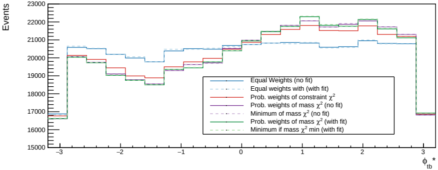
$$(a, b, c)_{\text{lf-jet}} = (1.03, 0, 0)$$

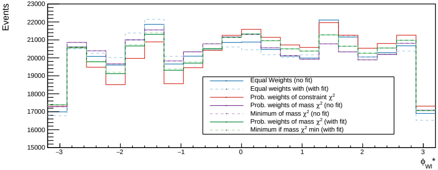
Event Reconstruction











Fitting procedure

Parabolic dependence on a single coupling:

$$\begin{aligned}|\mathcal{M}|^2 =&|\mathcal{M}_{SM} + \alpha \mathcal{M}'_{D=6}|^2\\ =&|\mathcal{M}_{SM}|^2 + \alpha(\mathcal{M}^*_{SM}\mathcal{M}'_{D=6} + \mathcal{M}_{SM}\mathcal{M}'^*_{D=6})\\ &+ \alpha^2|\mathcal{M}'_{D=6}|^2\end{aligned}$$

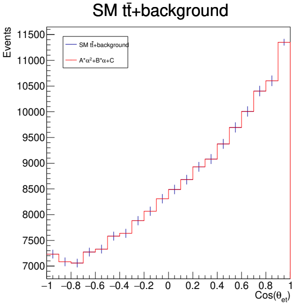
Constructing the parabola:

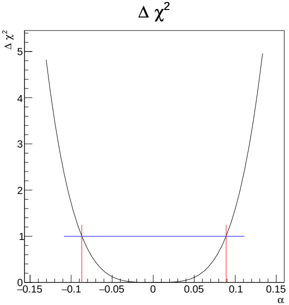
$$\begin{aligned}f(\alpha^0) &= C\\ f(\alpha^+) &= A\alpha^2 + B\alpha + C\\ f(\alpha^-) &= A\alpha^2 - B\alpha + C\end{aligned} \quad \Rightarrow \quad \begin{aligned}A &= \frac{f(\alpha^+) + f(\alpha^-)}{2\alpha^2} - \frac{f(SM)}{\alpha^2}\\ B &= \frac{f(\alpha^+) - f(\alpha^-)}{2\alpha}\\ C &= f(SM)\end{aligned}$$

Fit model:

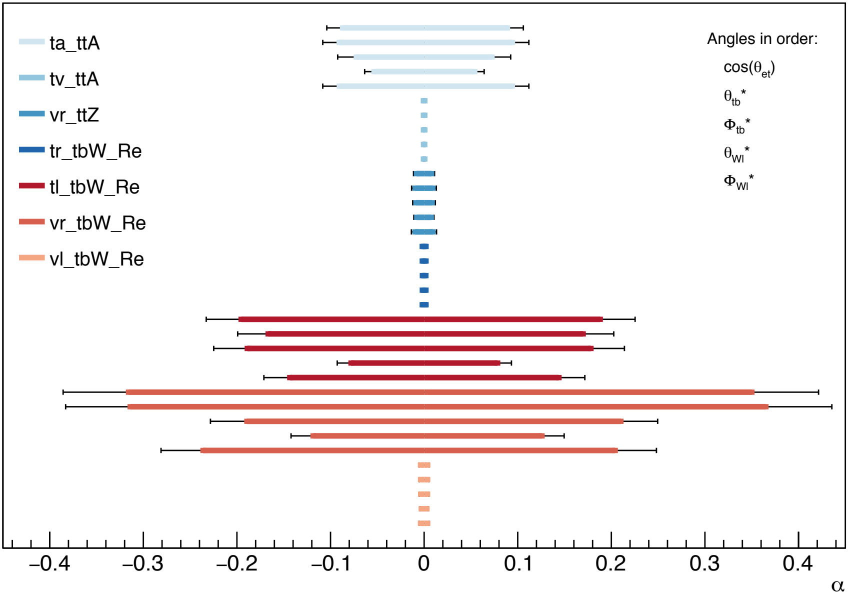
$$f(x) = A \cdot x^2 + B \cdot x + f(SM) + f(Bkg)$$

$$\chi^2 = \sum_{i=1}^{n} \frac{(y_i - f(x_i;\alpha))^2}{\sigma^2_{y_i} + \sigma^2_{f(x_i;\alpha)}}$$





Gauging the sensitivity



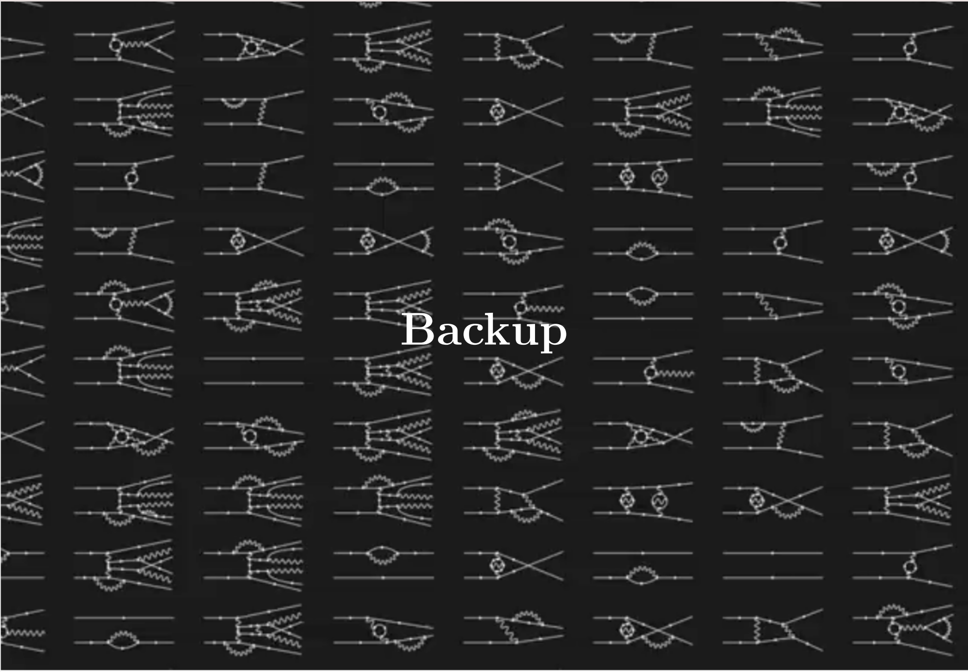
- **Sensitivity to top anomalous couplings** determined from 1σ confidence intervals in semileptonic channel for $t\bar{t}$ @ FCC-ee

- **Event selection** and **event reconstruction** performed

- **Jet studies** shed light on reevaluating jet definitions at future circular colliders

- **Jet Clustering and Jet Tagging tools** included in `FCCAnalyses`

- **`ABCfit++`** written as a general package for kinematic fitting



Backup

Jet Algorithms		
k_t	clustering_kt	suitable for pp collisions
Anti-k_t	clustering_antikt	
Cambridge/Aachen	clustering_cambridge	
Generalised-k_t	clustering_genkt	
Durham	clustering_ee_kt	suitable for e^+e^- collisions
Generalised-k_t for e^+e^-	clustering_ee_genkt	
Valencia (Plugins)	clustering_Valencia	
Jade (Plugins)	clustering_Jade	

Arguments for jet definition:

1. Jet cone radius
2. Clustering
   - 0=inclusive clustering,
   - 1=exclusive clustering with dcut,
   - 2=exclusive clustering to exactly njets,
   - 3=exclusive clustering up to exactly njets,
   - 4=exclusive clustering with ycut.
3. Cut-value depending on clustering
4. Ordering of returned jets
   - 0=sorted by p_t,
   - 1=sorted by E.
5. Recombination scheme

+. Additional input parameters specific to jet algorithm
   - see JetClustering.h

Recombination schemes

Recombination Schemes
E-scheme
p_t-scheme
p_t^2-scheme
E_t-scheme
E_t^2-scheme
Boost-invariant p_t-scheme
Boost-invariant p_t^2-scheme
$E0$-scheme
p-scheme

- FastJet is focused towards hadron colliders and besides the E-scheme it does not have dedicated schemes for e^+e^- collisions.
- $E0$- and p-scheme are external recombination schemes

E-scheme: Parton i and j are replaced by a pseudojet k with four-momentum

$$\mathbf{p}_k = \mathbf{p}_i + \mathbf{p}_j$$

- Lorentz invariant, energy and momentum conserved, non-zero mass for pseudojet k.

E0-scheme: The four-momentum of pseudojet k is rescaled to have zero invariant mass

$$E_k = E_i + E_j \quad , \quad \vec{p_k} = \frac{E_k}{|\vec{p_i} + \vec{p_j}|} \cdot (\vec{p_i} + \vec{p_j})$$

- Not Lorentz invariant, only conserves energy.

p-scheme: The four-momentum is constructed to have zero invariant mass

$$\vec{p_k} = \vec{p_i} + \vec{p_j} \quad , \quad E_k = |\vec{p_k}|$$

- Not Lorentz invariant, only conserves momentum.

List of observables

- Highest energy lepton
- 2nd highest energy lepton
- Lepton momentum
- Lepton momentum excluding highest energy lepton
- Momentum difference between highest and second highest energy lepton
- Missing momentum
- Invariant mass of lepton-neutrino pair
- Invariant mass of 1st and 2nd highest energy leptons
- Invariant mass of event excluding highest energy lepton
- Invariant mass of entire event
- Thrust of event excluding highest energy lepton
- Thrust of entire event
- Mass of jet
- Energy of jet
- Number of b-tagged jets
- Significance distribution
- Minimum of distance measure, d_{min}
- Invariant mass of one jet, $\Delta(m_i - m_W/2)$
- Invariant mass of di-jet system, $\Delta(m_ij - m_W)$
- Invariant mass of tri-jet system, $\Delta(m_ijk - m_t)$
- Invariant mass of lepton-neutrino-jet system, $\Delta(m_{l,nu,i} - m_t)$

$$\vec{p}_j^{\,r} = a_j|\vec{p}_j^{\,m}|\vec{p}_j^{\,a} + b_j\vec{p}_j^{\,b} + c_j\vec{p}_j^{\,c} \tag{1}$$

Parameters: a_j, b_j and c_j
Unit vectors forming a Cartesian system: $\vec{p}_j^{\,a}$, $\vec{p}_j^{\,b}$ and $\vec{p}_j^{\,c}$
Measured jet momentum: $\vec{p}_j^{\,m}$,

$$\vec{p}_j^{\,a} = \frac{\vec{p}_j^{\,m}}{|\vec{p}_j^{\,m}|},$$

$$\vec{p}_j^{\,b} = \frac{1}{\sqrt{p_{x,m}^2 + p_{y,m}^2}}(p_y^m, -p_x^m, 0),$$

$$\vec{p}_j^{\,c} = \frac{1}{\sqrt{|\vec{p}_j^{\,m}|^2(p_{x,m}^2 + p_{y,m}^2)}}(-p_x^m p_z^m, -p_y^m p_z^m, p_{x,m}^2 + p_{y,m}^2).$$