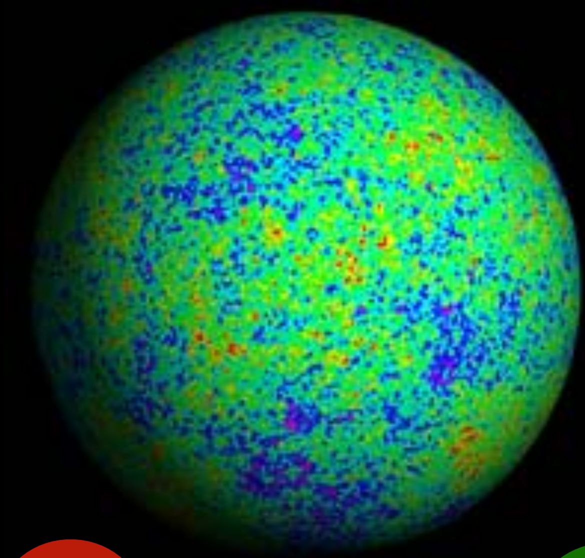
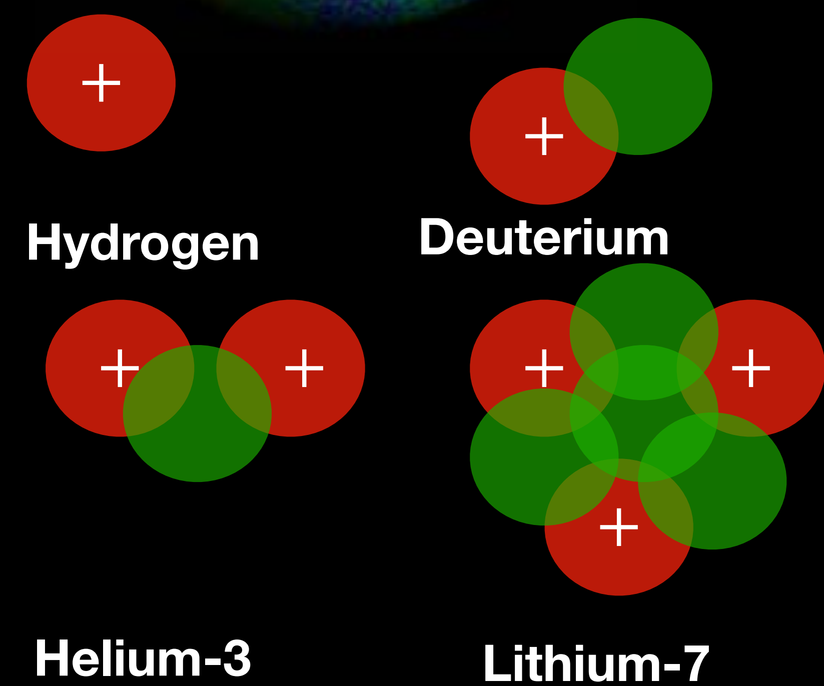
Overview of cosmological tools for phase transitions

Graham White

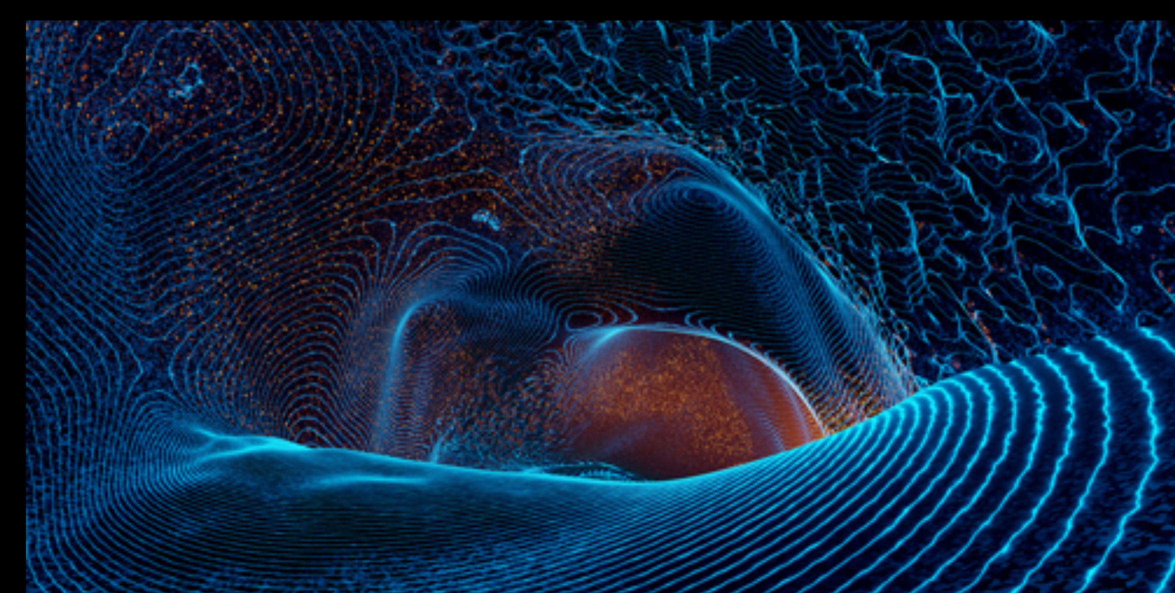
Why care about phase transitions?

$t = 10^6$ years





$t = 1$ min

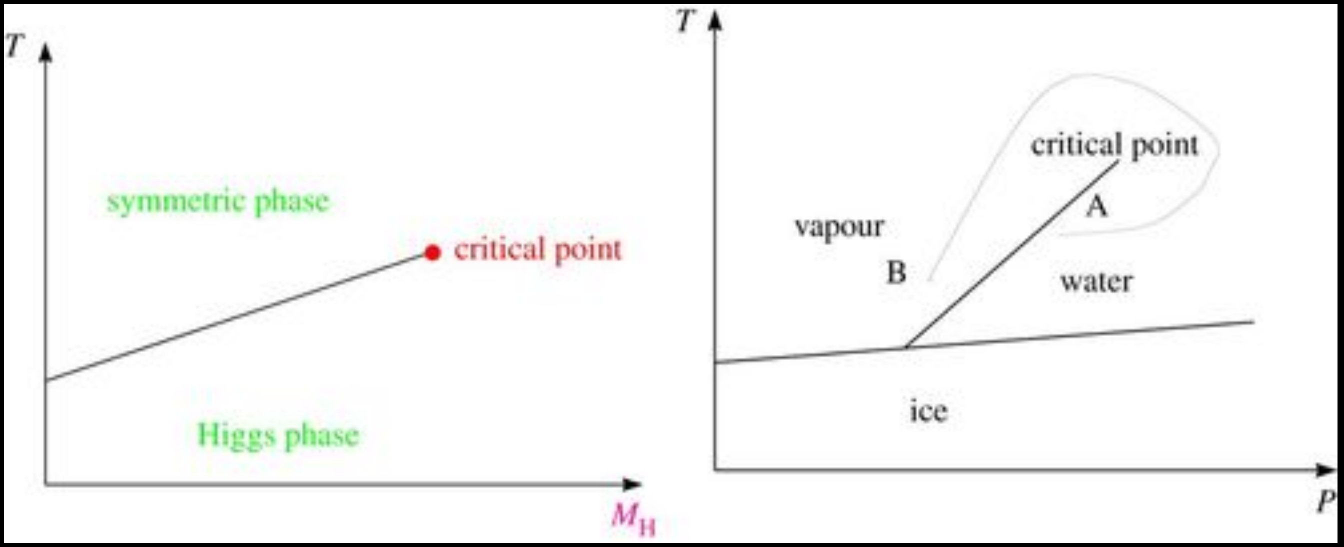


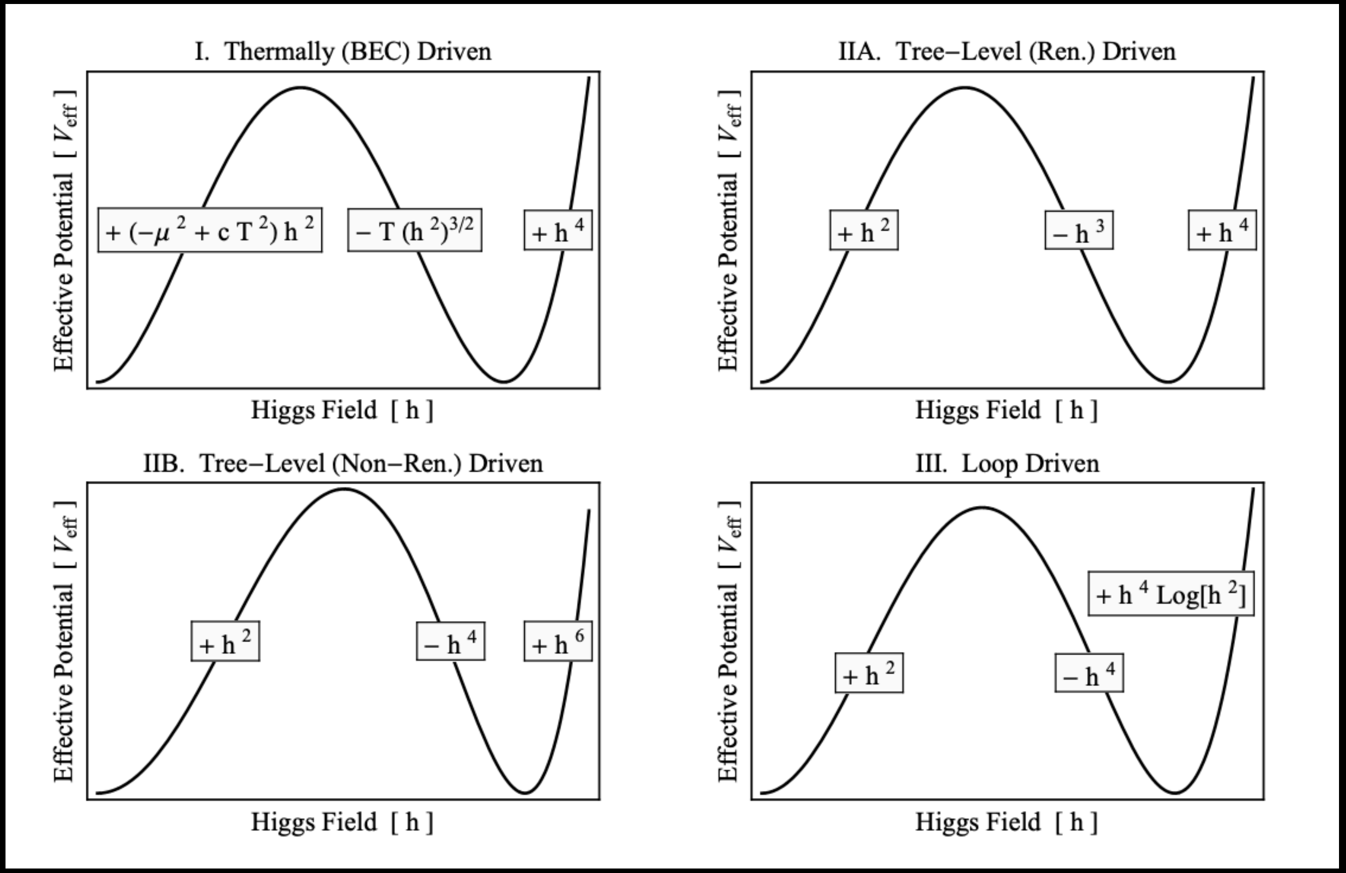
$t = 0!$

Image credits: WMAP,
Phys. Rev. Lett. 124, 041804

Why care about phase transitions?

2. Electroweak phase diagram is *the* question we can answer at next generation colliders





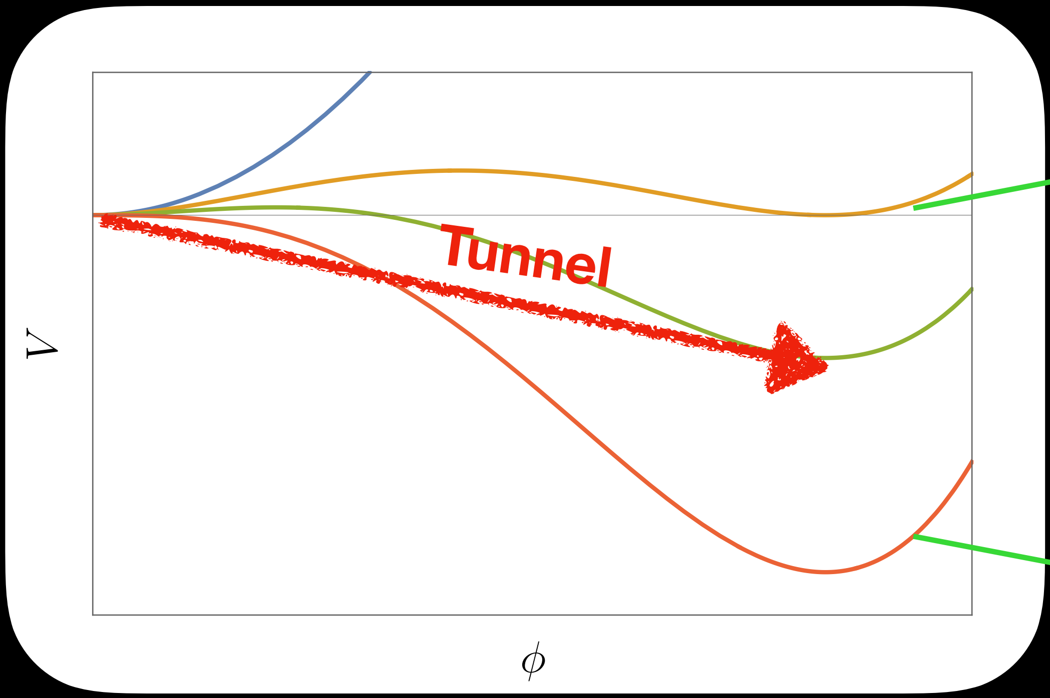
Figures taken from 1209.1819 and “The Higgs Boson and cosmology

Why care about Pts in the early universe?

3. Can explain why we are here

$$Y_B = \frac{n_B - \overline{n}_B}{s} \approx \frac{n_B}{s} = \begin{cases} (7.3 \pm 2.5) \times 10^{-11}, \mathrm{BBN} \\ (9.2 \pm 1.1) \times 10^{-11}, \mathrm{WMAP} \\ (8.59 \pm 0.11) \times 10^{-11}, \mathrm{Planck}\,. \end{cases}$$

The problem:
1) Evolution of the minima

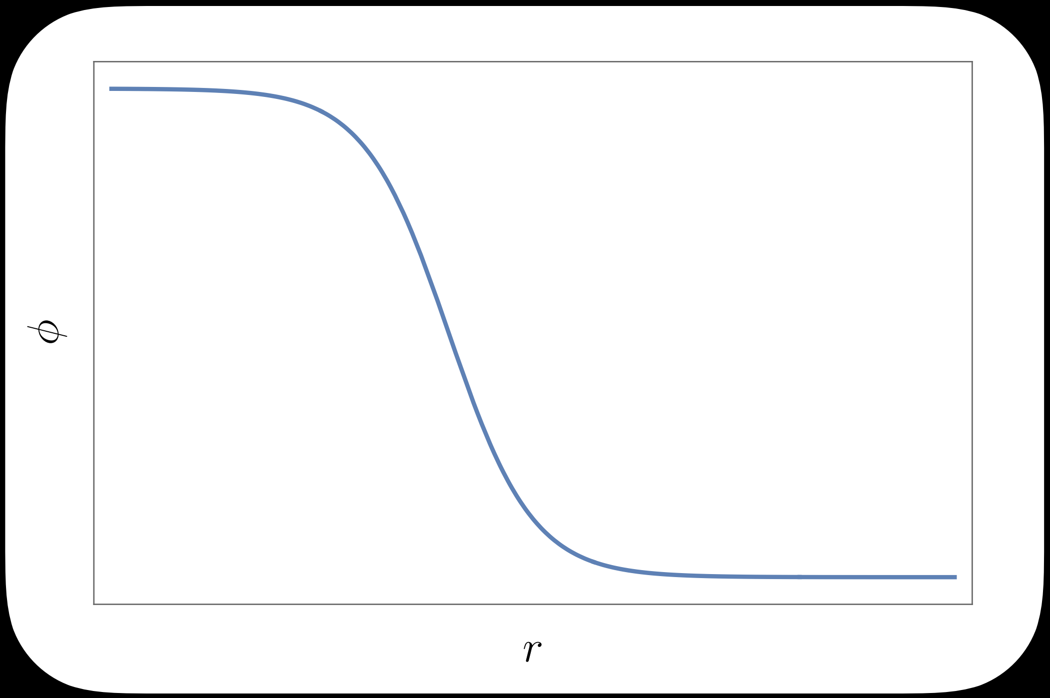


Critical temperature

T=0

Need to calculate the temperature evolution of the potential and the minima

The problem:
1) Tunneling rate

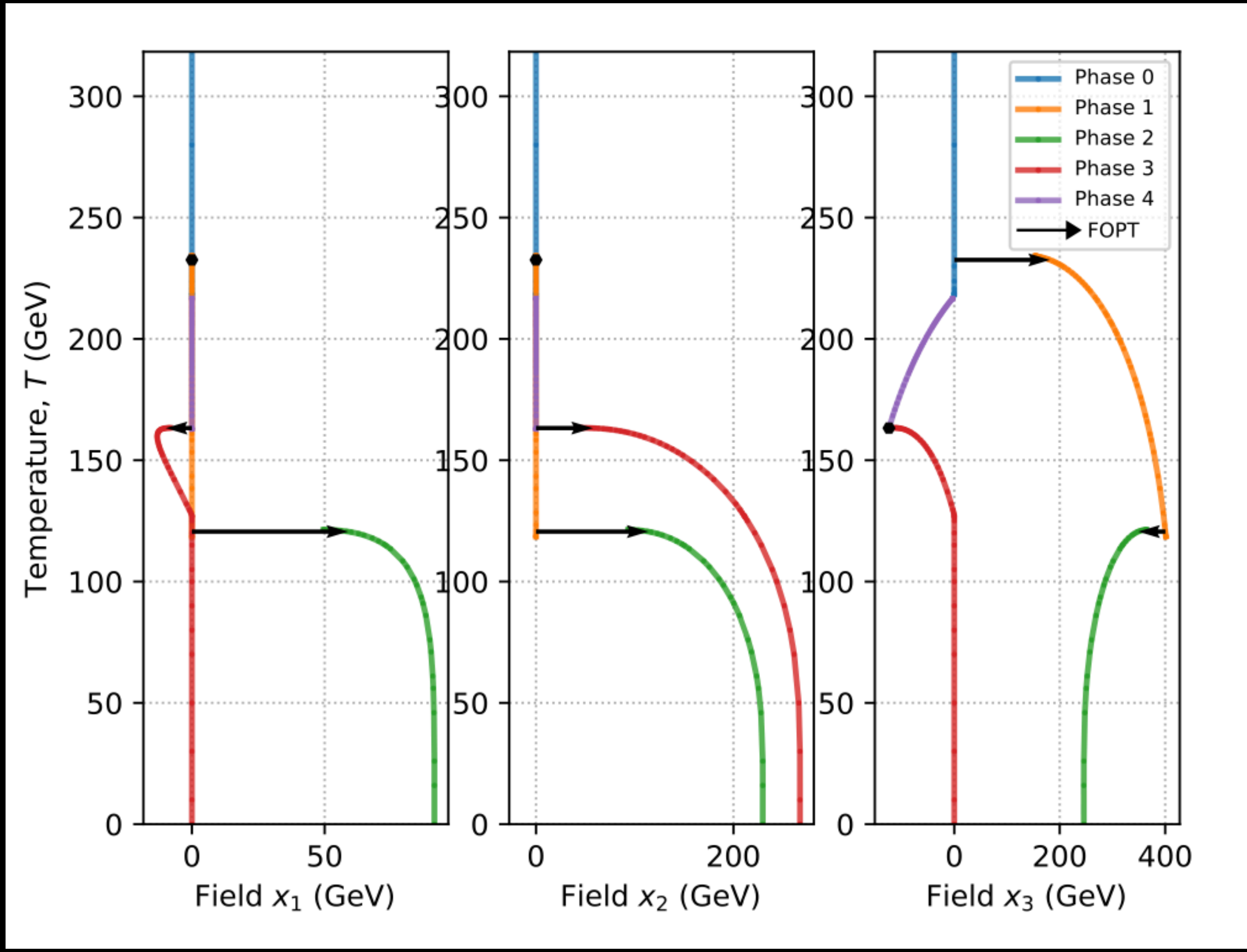


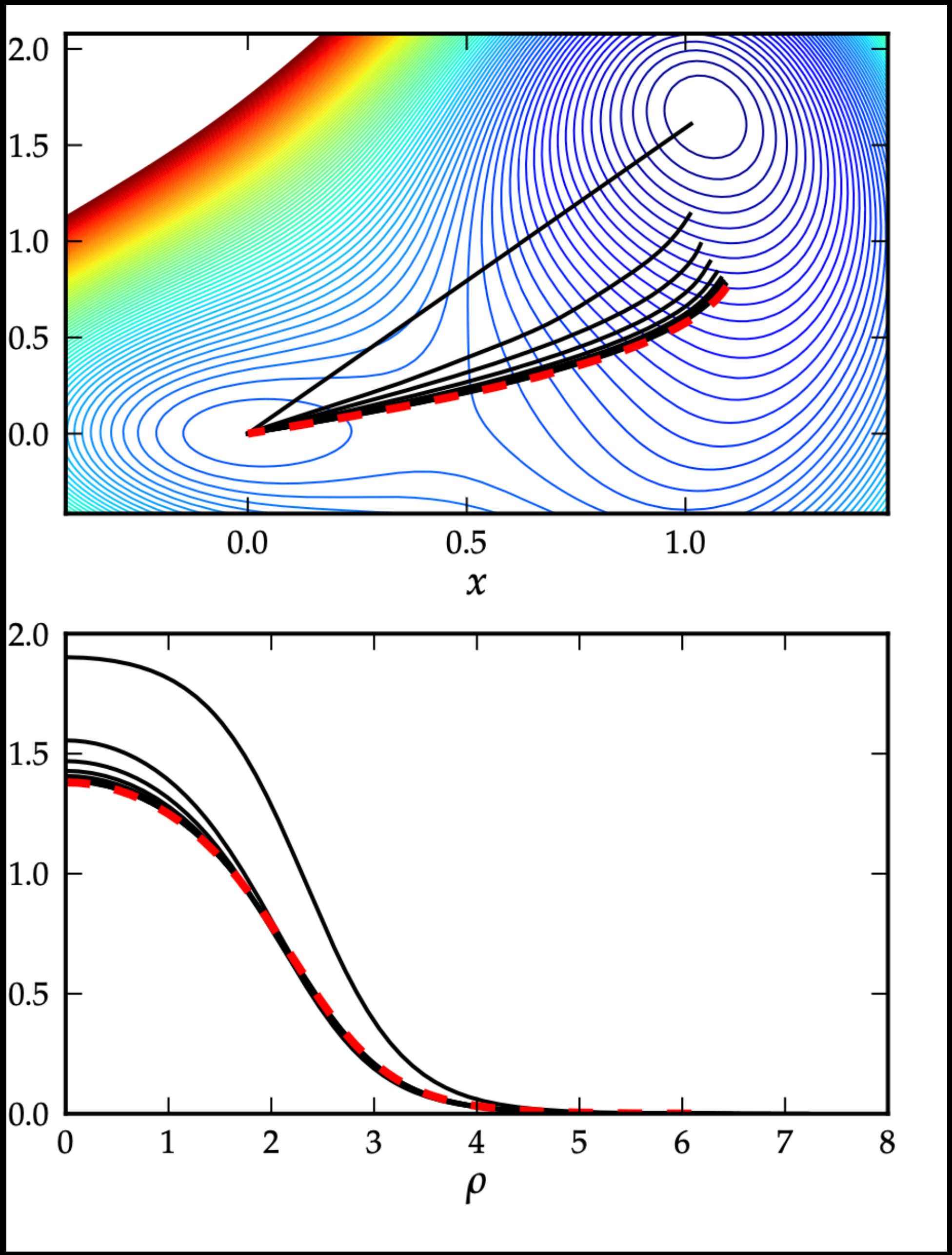
$$p \sim e^{-S_E/T}$$

$$S_E = 4\pi \int r^2 dr L(\phi(r))$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = \frac{\partial V}{\partial \phi}$$

The problem gets much harder in N dimensions





From cosmo trans and PT manuals

N dimensional problem in tracking minima

Step 1: Describe the effective potential at finite temperature

- **CT and PT both require tree level potential**
- **Both can accept mass eigenvalues as input though in principle you can get around needing the analytic form in PT (see NMSSM model)**
- **CT requires minima at zero T as input**

Step 2: track the minima

$$\frac{\partial \overrightarrow{\phi}_{\min}}{\partial T} = - M^{-1} \frac{\partial \overrightarrow{b}}{\partial T}$$

$$b_i = \frac{\partial V}{\partial \phi_i}, \quad M_{ij} = \frac{\partial^2 V}{\partial \phi_i} \partial \phi_j$$

Algorithm for tracing minima originally in CT

PT has a few advantages:

1) maintained
2) Easier input
3) C++ (fast!)
4) Less buggy and handles discrete symmetries
5) CT misses some minima (saddle points)
6) Both PT and CT find some spurious minimum but PT lets the user decide

The problem:
Solving the eqns of motion

$$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} = \frac{dV}{d\phi} \to \phi(r) = v$$

HUGE basin of attraction toward trivial solution!

Overshoots oscillate between $\pm\infty$ (for who knows what reason)

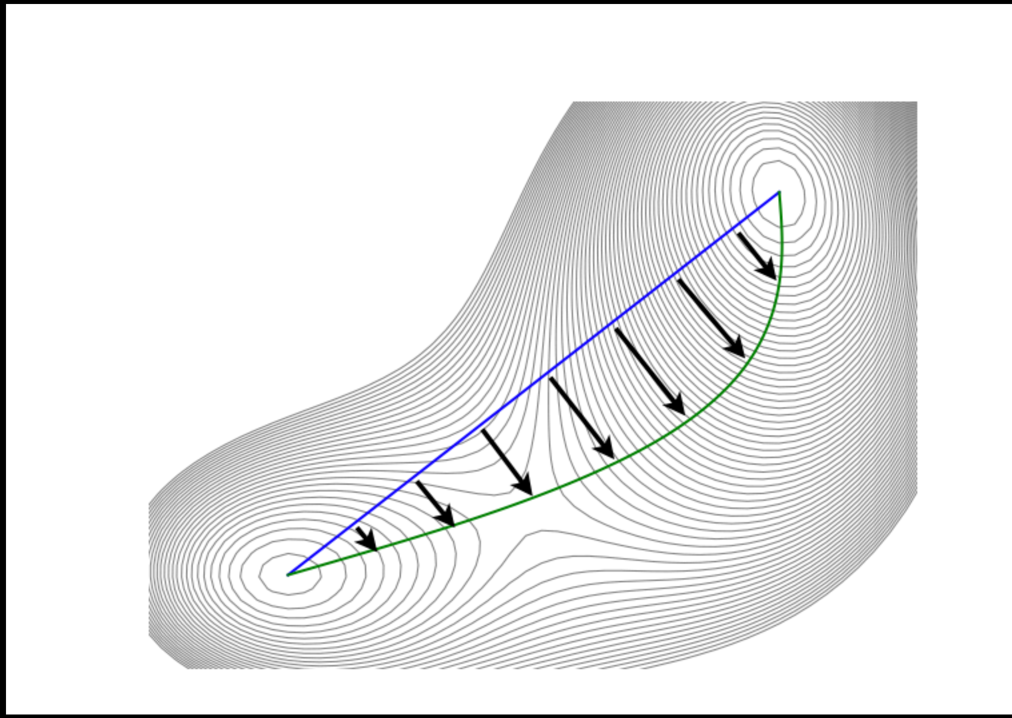
Solving the multi-field bounce

Path deformation algorithm (cosmotransitions)

Step 1. Solve 1-d problem

$$\frac{d^2x}{dr^2} + \frac{2}{r}\frac{dx}{dr} = \frac{dV}{dx}$$

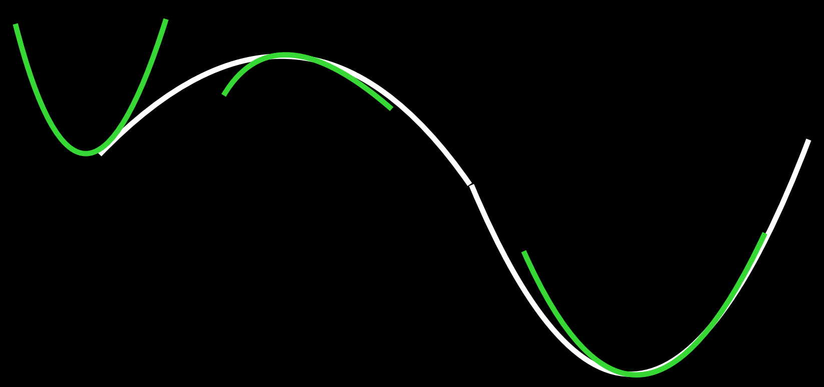
$$\frac{d^2\phi_i}{dx^2}\left(\frac{dx}{dr}\right)^2 = \nabla_\perp V$$



The problem:
Solving the eqns of motion

Any bubble's algorithm:

Solve bounce for small pieces of Δr where the potential is approximated by a quadratic



Mathematica interface (feature/bug depending on how it interfaces with a scan)
Super slow but pretty stable in my experience (perhaps the most stable)
Inaccurate in thin wall limit

The problem:
Solving the eqns of motion

4 algorithms (BP1)

Similar to PD start with a 1-field solution

$$\phi_i = A_i + \epsilon_i$$

$$\frac{\partial^2 \epsilon_i}{\partial r^2} + \frac{2}{r}\frac{\partial \epsilon_i}{\partial r} = \sum_j \frac{\partial^2 V}{\partial_i \partial_j}\bigg|_A \epsilon_j + f(A) \qquad f(A) = \frac{\partial V}{\partial \phi_i}\bigg|_A - \frac{\partial^2 A_i}{\partial r^2} - \frac{2}{r}\frac{\partial A_i}{\partial r}$$

Can solve analytically, easy to solve numerically as it has a weaker basin of attraction to the trivial solution

Time comparison

	Action			Time (s)		
# fields	BP	CT	AB	BP	CT	AB
1	54.1	52.6	52.4	0.051	0.066	1.285
2	20.8	21.1	20.8	0.479	0.352	7.473
3	22.0	22.0	22.0	0.964	0.215	25.209
4	55.9	56.4	55.9	1.378	0.255	54.258
5	16.3	16.3	16.3	2.958	0.367	305.531
6	24.5	24.5	24.4	4.853	0.337	830.449
7	36.7	36.6	36.7	6.754	0.375	1430.892
8	46.0	46.0	46.0	10.014	0.409	1805.713

Path Deformation is weirdly good… v2 will have path deformation

The problem: Calculate GWs:

Need to design your own: how to

Easy: $\Omega_{GW} = \Omega(\beta/H, T_*, \alpha, v_w)$

$$\beta/H = T\frac{d(S_E/T)}{dT}$$

$$\alpha = \frac{\Delta V - \frac{T}{4}(d\Delta V/dT)}{\rho_{\rm rad}}$$

$$\frac{S_E}{T_*} = 131 - 14 - 4\log\left(\frac{T}{100\mathrm{GeV}}\right) - \log\left(\frac{\beta/H}{100}\right)$$

The problem: Calculate GWs:

Need to design your own: how to

Slightly harder:

1. Solve hydrodynamic equations to obtain the fluid velocity using a variable speed of sound (see 2004.06995)
2. Solve for number density of bubbles to obtain mean bubble separation
3. Use the evolution of number density to properly estimate percolation temperature
4. Include fits for energy lost due to vorticity (see 1906.00480)
5. Include the effect of temperature fluctuations on nucleation rate (see 2108.11947)

Can make quite a large difference (2103.06933)

Guo, Sinha, Vagie White: 2103.06933

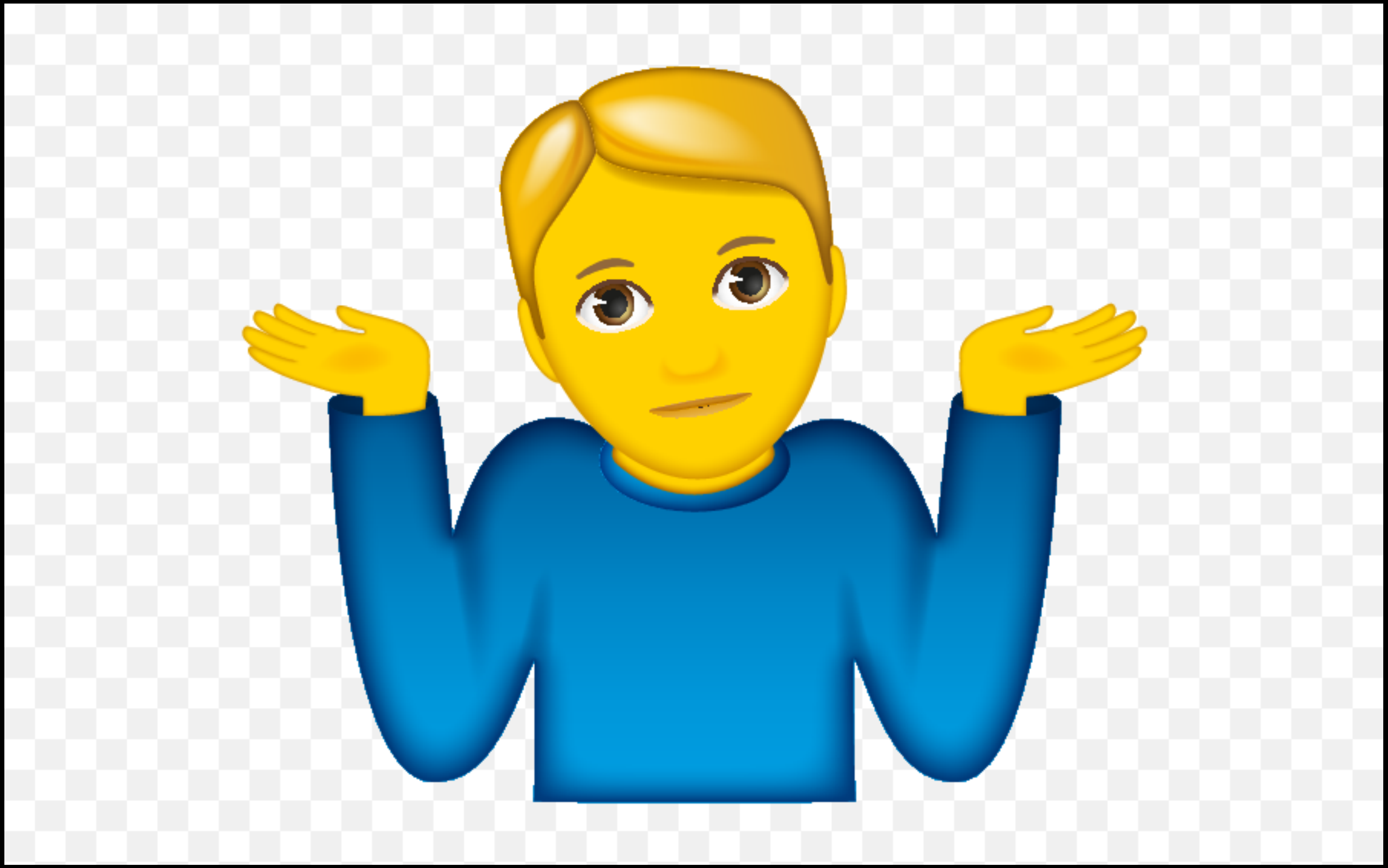
Effect	Range (medium)	Range (low)	Type
Tracking bubble distribution	$\mathcal{O}(10^{-2} - 10^{0})$	$\mathcal{O}(10^{-4} - 10^{0})$	Random
Solve hydrodynamic eqns	$\mathcal{O}(10^{-4} - 10^{1})$	$\mathcal{O}(10^{-2} - 10^{0})$	Random
Finite lifetime	$\mathcal{O}[(10^{-3}), \frac{1}{3}]$	$\mathcal{O}(10^{1} - 10^{3})$	Overestimate
Vorticity/ reheating	$\mathcal{O}(10^{-1} - 10^{0})$		Random

Bubble wall velocity

As yet no code exists

Requires solving integral-differential equations as discussed here

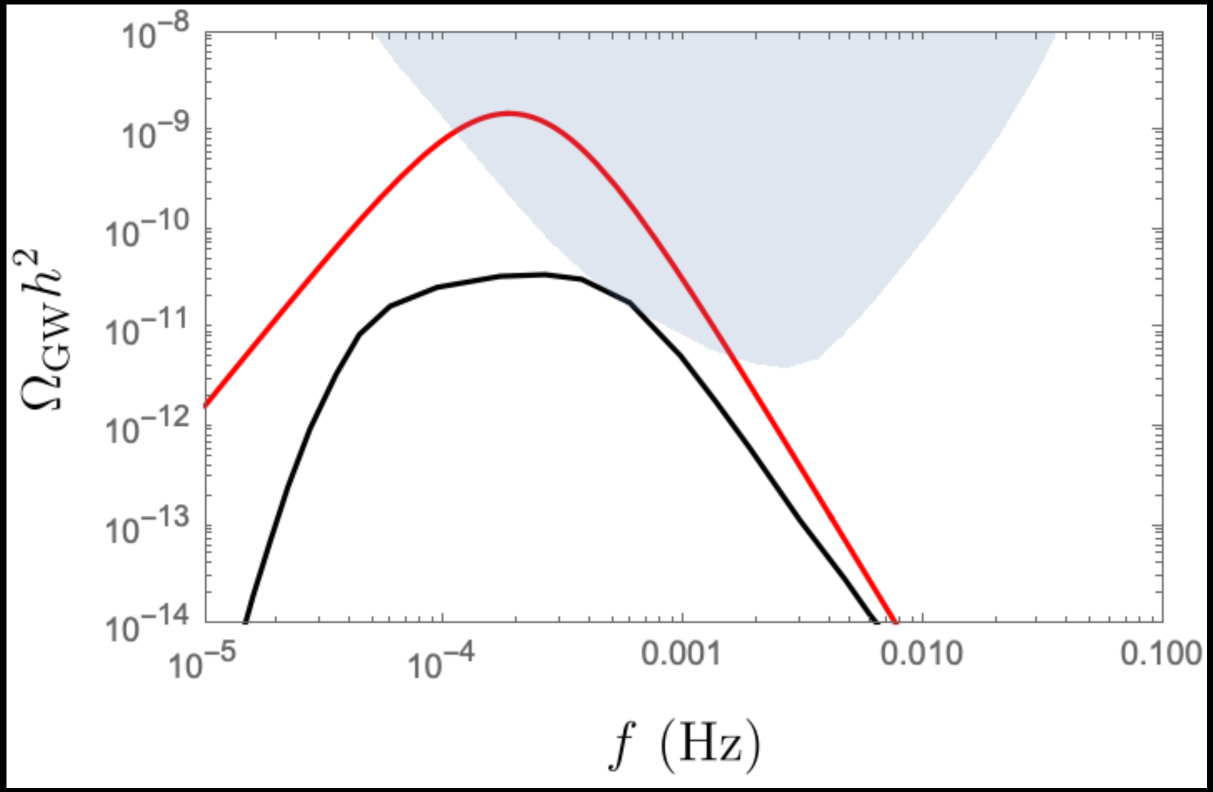
1506.04741



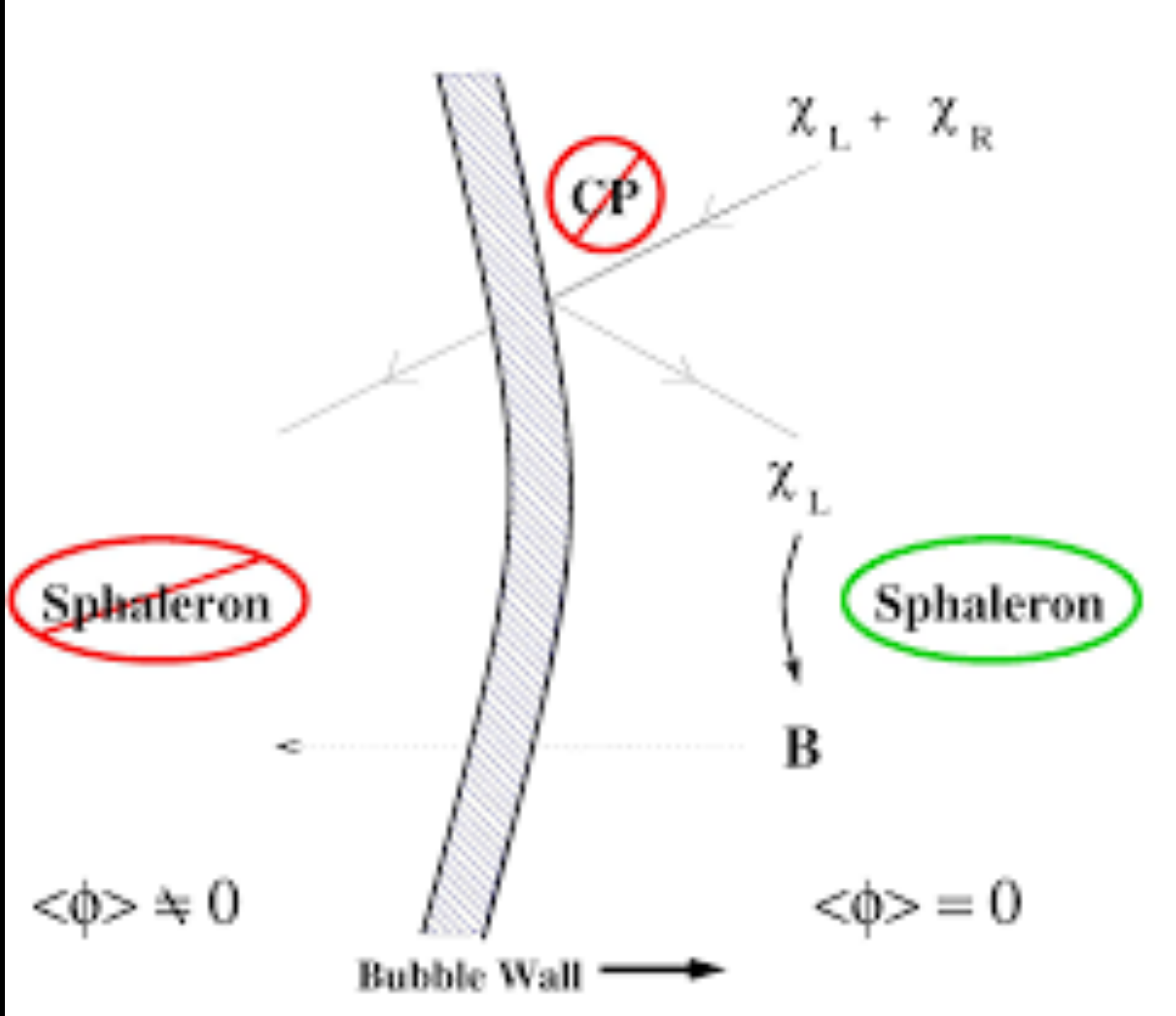
The problem: Calculate GWs:

Need to design your own: how to

Much harder: Do a simulation



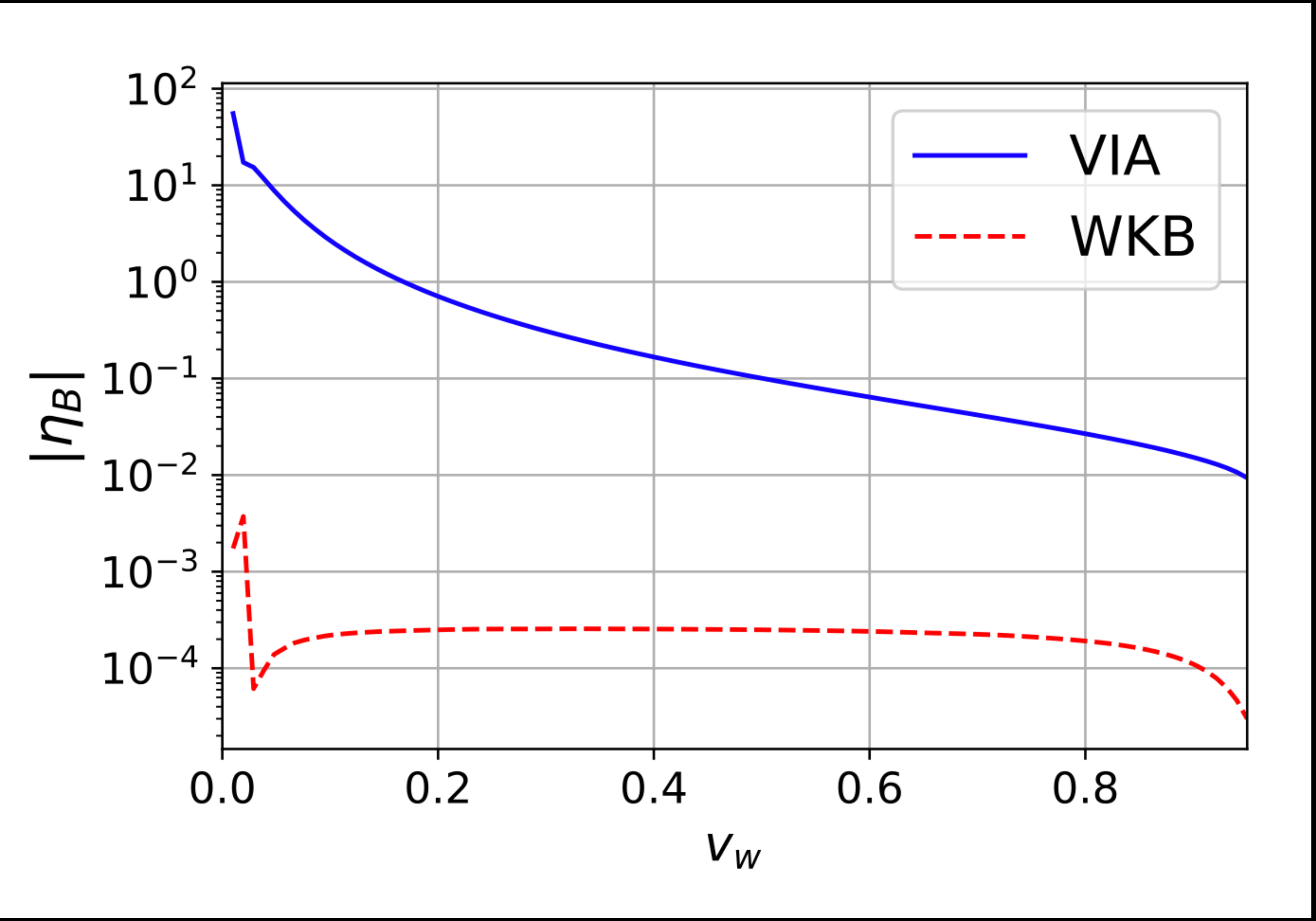
The problem: Calculating the baryon asymmetry



1206.2942

The problem: Calculating the baryon asymmetry

Two methods differ greatly!



2108.04249

The problem: Calculating the baryon asymmetry

WKB : $\left(\partial_\mu \gamma^\mu - m(x) e^{i\phi(x)}\right) \psi = 0, \quad f = \frac{1}{e^{\beta E + \delta} + 1}$

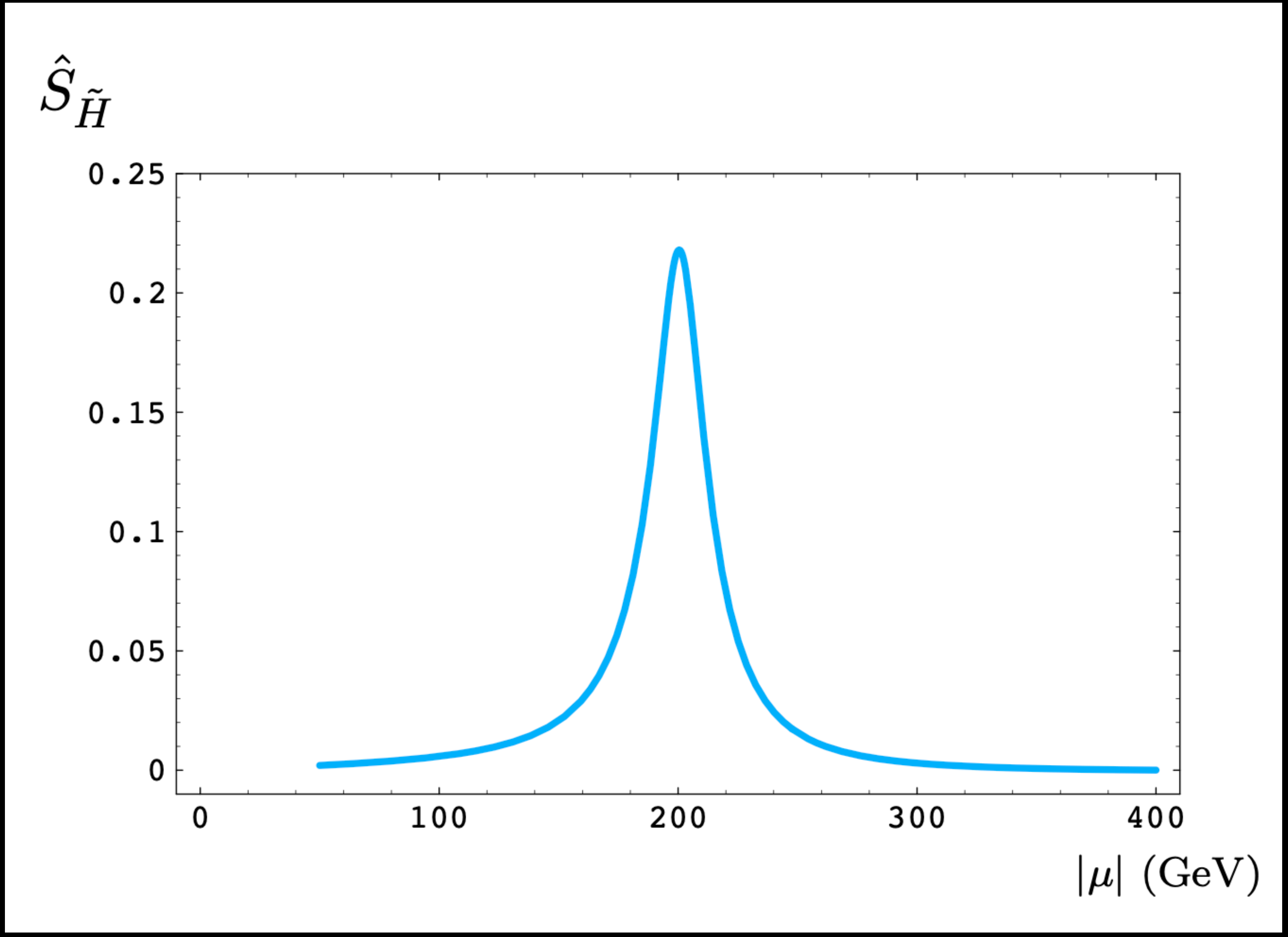
VIA: $G(x,y) = G^0(x,y) + G^0(x,z) \odot \Sigma(z,y) \odot G(z,x)$

$$G(x,y) = G^0(x,y) + G(x,z) \odot \Sigma(z,y) \odot G^0(z,x)$$

$$\lim_{x \to y} \left(\mathscr{E}\,.\,\mathscr{O}\,.\,\mathscr{M}\,._x G(x,y) - G(x,y) \mathscr{E}\,.\,\mathscr{O}\,.\,\mathscr{M}_y\right) \quad \partial_\mu J^\mu = f(\Sigma)$$

The problem: Calculating the baryon asymmetry

Reason for difference: resonance



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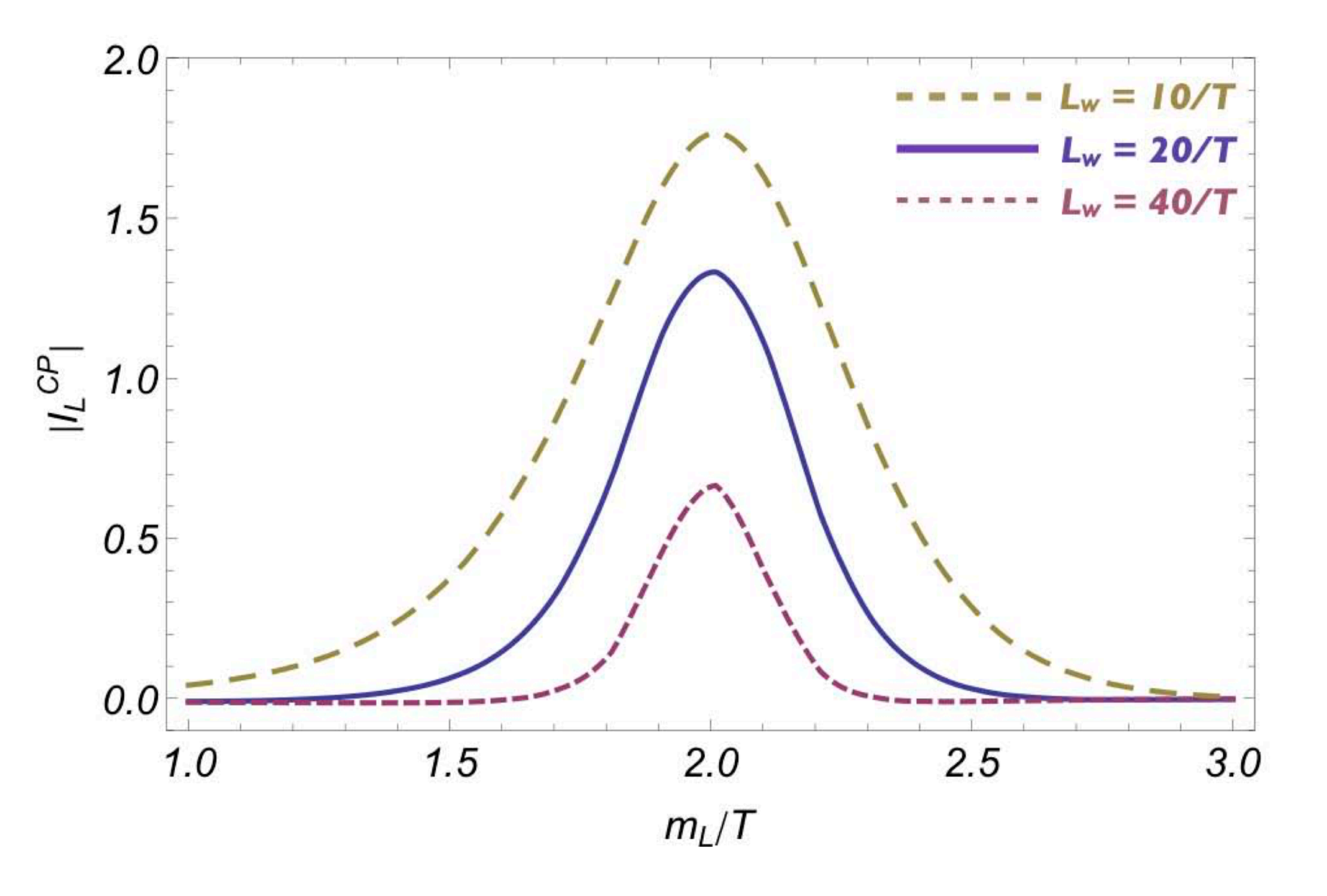
The problem: Calculating the baryon asymmetry

Resonance appears to be real, even if calculated in a questionable way



1106.0747

The problem:
BAU:

Only candidate is BSMPT 2007.01725

1) Allows calculation in 2 scenarios
2) Also calculates details of the phase transition
3) c++ code

Mea cupla: Found recently and couldn’t test or even get to run before this talk!
Therefore can’t make comments on bugs, ease of use* and speed, but look exciting!

*Except I couldn’t get it to work in a brief amount of time of a few hours, but that is typical

Summary and conclusion

1) Understanding the properties of phase transitions in the early Universe has wide application
2) Codes currently cover a lot of parts of the problem but each with its own strengths and weaknesses
3) Bubble wall velocities and GW calculators are currently a hole in the landscape of codes
4) BAU calculation currently has only one candidate and could use more