Mapping Processes with Operators in the SMEFT

Juan Rojo
VU Amsterdam & Theory group, Nikhef

on behalf of the Area 3 subgroup: *Experimental Measurements and Observables*

summary of contributions from FitMaker and SMEFiT groups

3rd General Meeting of the LHC Effective Field Theory Working Group

our goal

To establish the **correspondence** between **processes** and **directions in the SMEFT parameter space** (``operators'') via the measurement of **specific observables**

a general LHC cross-section in the SMEFT is written as:

$$\sigma_{\rm LHC}\left(c/\Lambda^2\right) \simeq \sigma_{\rm SM} \times \left(1 + \sum_{m=1}^{N_6} c_m \frac{\kappa_m}{\Lambda^2} + \sum_{m,n=1}^{N_6} c_m c_n \frac{\kappa_{mn}}{\Lambda^4}\right)$$

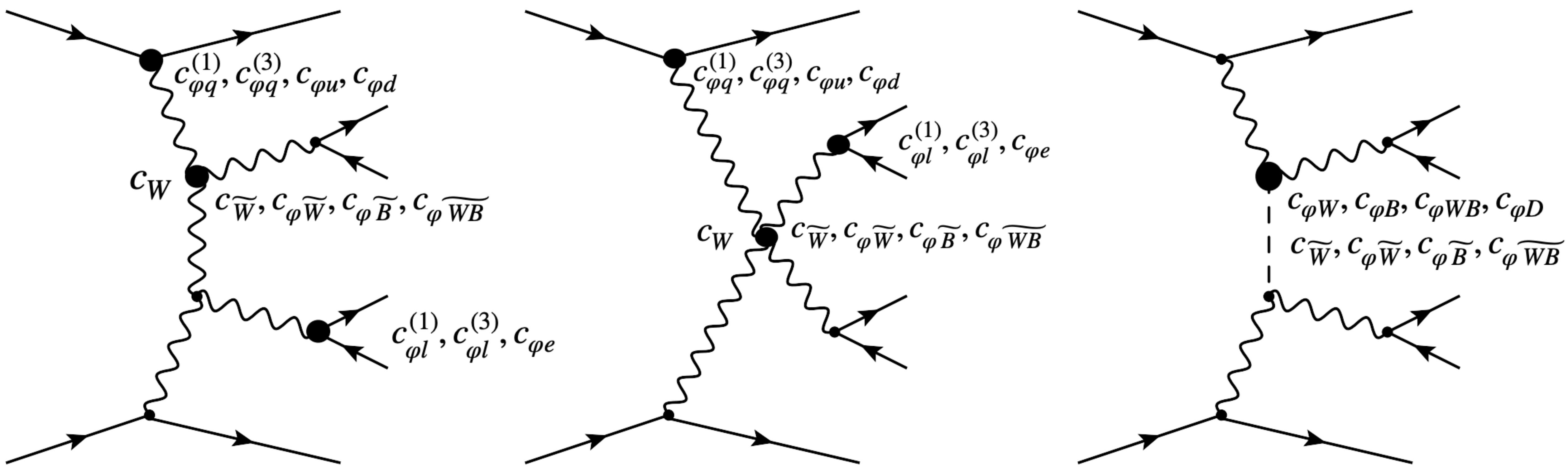
EFT coefficients

where the sum runs over all SMEFT operators contributing to this process

- ☑ Given a physical process (*e.g.* Higgs production in gluon fusion), **which SMEFT operators** enter the theory predictions?
- ☑ Can we quantify the **relative importance** of each of those operators?
- ☑ How to assess the **information** that a given measurement brings in the EFT parameter space?

for example

Consider the vector boson scattering (VBS) process



CP properties	Operator	Coefficient	Definition
	$\mathcal{O}_W$	c_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
	$\mathcal{O}_{\varphi W}$	$c_{\varphi W}$	$(\varphi^\dagger \varphi - \frac{v^2}{2}) W_{\mu\nu}^I W^{I\mu\nu}$
CP-even	$\mathcal{O}_{\varphi B}$	$c_{\varphi B}$	$(\varphi^\dagger \varphi - \frac{v^2}{2}) B_{\mu\nu} B^{\mu\nu}$
	$\mathcal{O}_{\varphi WB}$	$c_{\varphi WB}$	$\left(\varphi^\dagger \sigma_I \varphi\right) W_{\mu\nu}^I B^{\mu\nu}$
	$\mathcal{O}_{\varphi D}$	$c_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$

Inspection of **Feynman diagrams** reveals how SMEFT operators enter the VBS process…

however quantifying the **relative importance** of the various operators is much more subtle

why this is not easy

- The mapping between processes and operators depends sensitively on the **settings of the theory calculation**, e.g. linear vs quadratic EFT

$$\sigma_{\rm LHC}\left(c/\Lambda^2\right) \simeq \sigma_{\rm SM} \times \left(1+\sum_{m=1}^{N_6} c_m \frac{\kappa_m}{\Lambda^2}\right) \qquad \sigma_{\rm LHC}\left(c/\Lambda^2\right) \simeq \sigma_{\rm SM} \times \left(1+\sum_{m=1}^{N_6} c_m \frac{\kappa_m}{\Lambda^2}+\sum_{m,n=1}^{N_6} c_m c_n \frac{\kappa_{mn}}{\Lambda^4}\right)$$

- But also in the **perturbative order** of the EFT calculation: LO QCD vs NLO QCD

$$\sigma_{\rm LHC}\left(c/\Lambda^2\right) \simeq \sigma_{\rm SM} \times \left(1+\sum_{m=1}^{N_6} c_m \frac{\kappa_m^{\rm LO}}{\Lambda^2}\right) \qquad \sigma_{\rm LHC}\left(c/\Lambda^2\right) \simeq \sigma_{\rm SM} \times \left(1+\sum_{m=1}^{N_6} c_m \frac{\kappa_m^{\rm NLO}}{\Lambda^2}\right)$$

- Furthermore, each **bin** (or choice of selection cuts) has associated a different combination of EFT operators: cannot discuss ``**processes**'' without the corresponding ``**observables**''

- Needless to say, the mapping depends on the operator basis, **flavour assumptions**, ….

Information geometry

One useful estimator to quantify the **relative amount of information** that is provided by a given measurement on a given EFT coefficient is provided by the **Fisher information matrix**

$$I_{ij}(\boldsymbol{c}) = -\mathrm{E}\left[\frac{\partial^2 \ln f(\boldsymbol{\sigma}_{\exp}|\boldsymbol{c})}{\partial c_i \partial c_j}\right], \qquad i,j = 1,\ldots,n_{\rm op}\,,$$

for gaussian, uncorrelated measurements:

$$f(\boldsymbol{\sigma}_{\exp})|\boldsymbol{c}) = \prod_{m=1}^{n_{\rm dat}} \frac{1}{\sqrt{2\pi\delta^2_{\exp,m}}} \exp\left(-\frac{\left(\sigma_m^{(\exp)} - \sigma_m^{(\rm th)}(\boldsymbol{c})\right)^2}{2\delta^2_{\exp,m}}\right)$$

linear

$$I_{ij} = \sum_{m=1}^{n_{\rm dat}} \frac{\sigma^{(\rm eft)}_{m,i}\sigma^{(\rm eft)}_{m,j}}{\delta^2_{\exp,m}}$$

*n.b. operator normalisation is arbitrary, thus **absolute values of Fisher unphysical**. Normalise to the sum over a given operator: relative Fisher is physical*

quadratic

$$I_{ij} = \mathrm{E}\left[\sum_{m=1}^{n_{\rm dat}} \frac{1}{\delta^2_{\exp,m}}\left(\sigma_{m,ij}\left(\sigma_m^{(\rm th)} - \sigma_m^{(\exp)}\right) + \left(\sigma^{(\rm eft)}_{m,i} + \sum_{l=1}^{n_{\rm op}} c_l \sigma^{(\rm eft)}_{m,il}\right)\left(\sigma^{(\rm eft)}_{m,j} + \sum_{l'=1}^{n_{\rm op}} c_{l'}\sigma_{m,jl'}\right)\right)\right]$$

at the linear level, Fisher information independent of fit result (but not at the quadratic level)

Information geometry

One useful estimator to quantify the **relative amount of information** that is provided by a given measurement on a given EFT coefficient is provided by the **Fisher information matrix**

$$I_{ij}(\boldsymbol{c}) = -\mathrm{E}\left[\frac{\partial^2 \ln f(\boldsymbol{\sigma}_{\rm exp}|\boldsymbol{c})}{\partial c_i \partial c_j}\right], \qquad i,j = 1,\ldots,n_{\rm op}\,,$$

for gaussian, uncorrelated measurements:

$$f(\boldsymbol{\sigma}_{\rm exp})|\boldsymbol{c}) = \prod_{m=1}^{n_{\rm dat}} \frac{1}{\sqrt{2\pi\delta^2_{{\rm exp},m}}} \exp\left(-\frac{\left(\sigma_m^{(\rm exp)} - \sigma_m^{(\rm th)}(\boldsymbol{c})\right)^2}{2\delta^2_{{\rm exp},m}}\right)$$

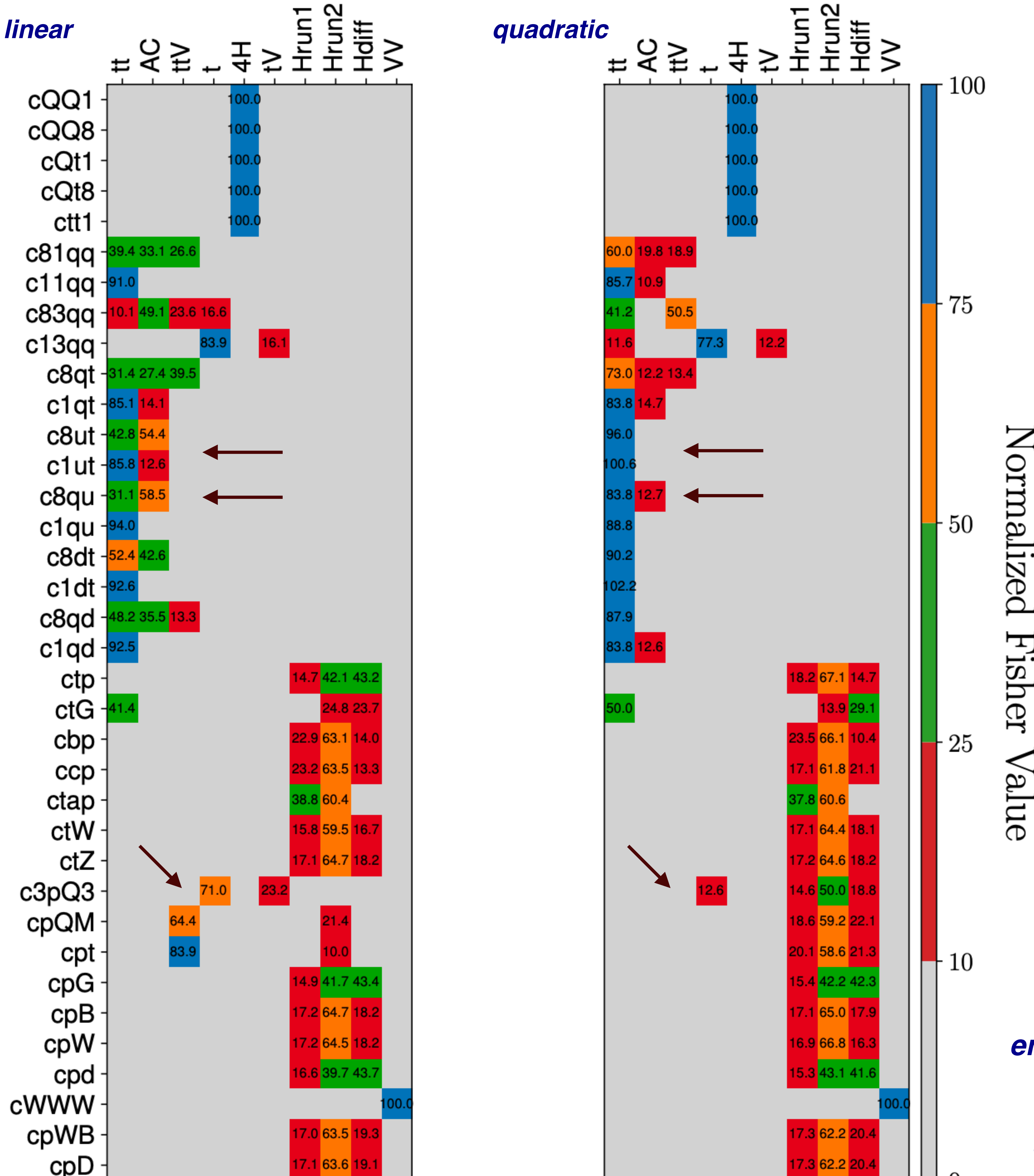
linear

$$I_{ij} = \sum_{m=1}^{n_{\rm dat}} \frac{\sigma^{(\rm eft)}_{m,i}\sigma^{(\rm eft)}_{m,j}}{\delta^2_{{\rm exp},m}}$$

*n.b. operator normalisation is arbitrary, thus **absolute values of Fisher unphysical**. Normalise to the sum over a given operator: relative Fisher is physical*

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_{k=1}^{n_{\rm op}} \frac{c_k}{\Lambda^2}\mathcal{O}_k = \mathscr{L}_{\rm SM} + \sum_{k=1}^{n_{\rm op}} \frac{\widetilde{c_k}}{\Lambda^2}\widetilde{\mathcal{O}}_k$$

$$\widetilde{\mathcal{O}}_k = A_k \mathcal{O}_k\,, \quad \widetilde{c}_k = A_k^{-1} c_k$$

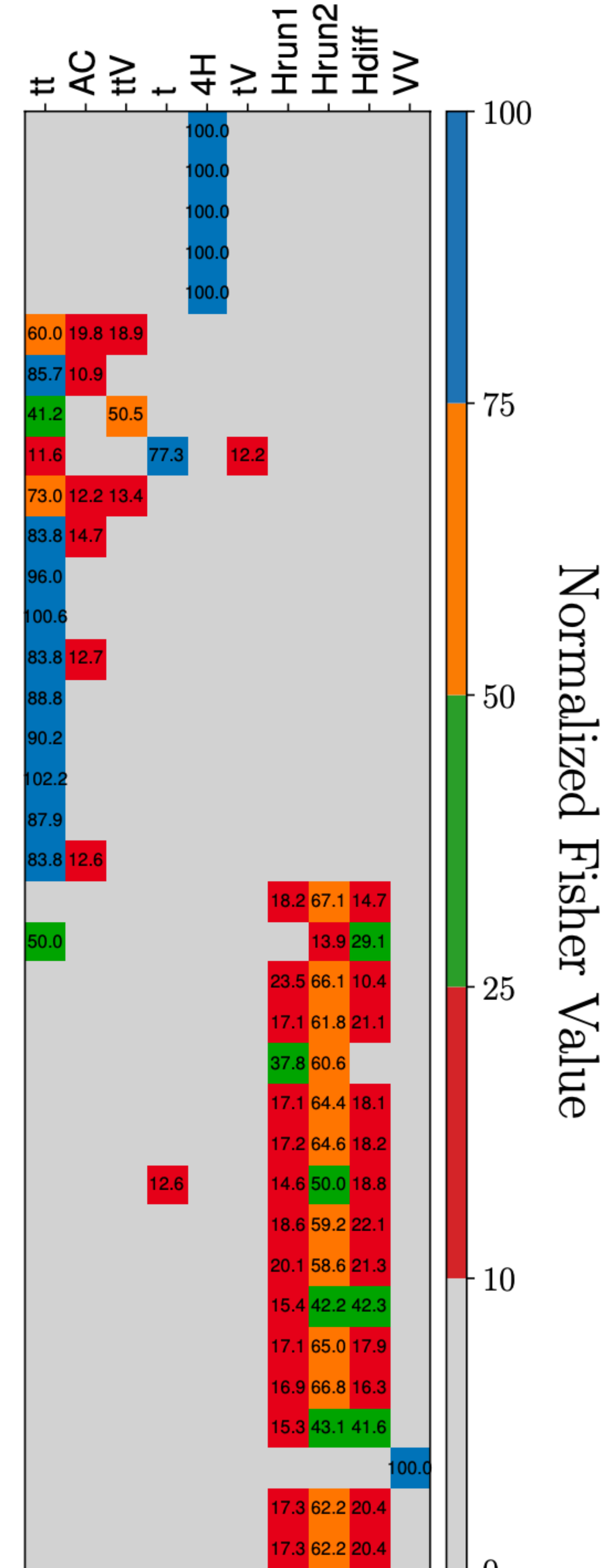


- Clean **mapping** between EFT coefficients and input data, quantifying **relative impact of each process** on a given coefficient
- Depends on **linear vs quadratic**, and also LO vs NLO EFT
- Much more useful than just ``**listing**'' which EFT coeffs enter the theory prediction for a given process

entries of each row normalised to 100

comparison between "rows" not physical!

Class	DoF	$t\bar{t}$	$t\bar{t}V$	t	tV	$t\bar{t}Q\bar{Q}$	h (μ_i^f, Run-I)	h (μ_i^f, Run-II)	h (STXS, Run-II)	VV
2-heavy-2-light	$c_{Qq}^{1,8}$	✓	✓			✓	✓	✓	✓	
	$c_{Qq}^{1,1}$	(✓)	(✓)			✓	(✓)	(✓)	(✓)	
	$c_{Qq}^{3,8}$	✓	✓	(✓)	(✓)	✓	✓	✓	✓	
	$c_{Qq}^{3,1}$	(✓)	(✓)	✓	✓	✓	(✓)	(✓)	(✓)	
	c_{tq}^{8}	✓	✓			✓	✓	✓	✓	
	c_{tq}^{1}	(✓)	(✓)			✓	(✓)	(✓)	(✓)	
	c_{tu}^{8}	✓	✓			✓	✓	✓	✓	
	c_{tu}^{1}	(✓)	(✓)			✓	(✓)	(✓)	(✓)	
	c_{Qu}^{8}	✓	✓			✓	✓	✓	✓	
	c_{Qu}^{1}	(✓)	(✓)			✓	(✓)	(✓)	(✓)	
	c_{td}^{8}	✓	✓			✓	✓	✓	✓	
	c_{td}^{1}	(✓)	(✓)			✓	(✓)	(✓)	(✓)	
	c_{Qd}^{8}	✓	✓			✓	✓	✓	✓	
	c_{Qd}^{1}	(✓)	(✓)			✓	(✓)	(✓)	(✓)	
4-heavy	c_{QQ}^{1}					✓				
	c_{QQ}^{8}					✓				
	c_{Qt}^{1}					✓				
	c_{Qt}^{8}					✓				
	c_{tt}^{1}					✓				
4-lepton	c_{ll}			✓	✓		✓	✓	✓	✓
2-fermion +bosonic	$c_{t\varphi}$						✓	✓	✓	
	c_{tG}	✓	✓			✓	✓	✓	✓	
	$c_{b\varphi}$						✓	✓	✓(b)	
	$c_{c\varphi}$						✓	✓		
	$c_{\tau\varphi}$						✓	✓		
	c_{tW}	✓		✓	✓		✓	✓		
	c_{tZ}		✓		✓		✓	✓		
	$c_{\varphi Q}^{(3)}$		✓(b)	✓	✓		✓(b)	✓(b)	✓(b)	
	$c_{\varphi Q}^{(-)}$		✓		✓		✓	✓	✓(b)	
	$c_{\varphi t}$		✓		✓		✓	✓		
	$c_{\varphi l_i}^{(1)}$						✓	✓		✓
	$c_{\varphi l_i}^{(3)}$			✓	✓		✓	✓	✓	✓
	$c_{\varphi e}$						✓	✓		✓
	$c_{\varphi\mu}$						✓	✓		
	$c_{\varphi\tau}$						✓	✓		
	$c_{\varphi q}^{(3)}$		✓	✓	✓		✓	✓	✓	✓
	$c_{\varphi q}^{(-)}$		✓		✓		✓	✓	✓	✓
	$c_{\varphi u}$		✓		✓		✓	✓	✓	✓
	$c_{\varphi d}$		✓		✓		✓	✓	✓	✓
purely bosonic	$c_{\varphi G}$						✓	✓	✓	
	$c_{\varphi B}$						✓	✓	✓	
	$c_{\varphi W}$						✓	✓	✓	
	$c_{\varphi d}$						✓	✓	✓	
	$c_{\varphi D}$		✓	✓	✓		✓	✓	✓	✓
	$c_{\varphi WB}$		✓	✓	✓		✓	✓	✓	✓
	c_{WWW}									✓



Information geometry

At a finer level, compare information provided by **different datasets for the same processes**, or different bins within a given dataset, here inclusive top-quark pair in SMEFiT

		ATLAS_tt_8TeV_ljets_Mtt	ATLAS_tt_8TeV_dilep_Mtt	CMS_tt_8TeV_ljets_Ytt	CMS_tt2D_8TeV_dilep_MttYtt
4H	OQQ1	×(×)	×(×)	×(×)	×(×)
	OQQ8	×(×)	×(×)	×(×)	×(×)
	OQt1	×(×)	×(×)	×(×)	×(×)
	OQt8	×(×)	×(×)	×(×)	×(×)
	Ott1	×(×)	×(×)	×(×)	×(×)
2L2H	O81qq	0.99(0.10)	0.91(0.93)	3.94(5.00)	2.91(0.46)
	O11qq	2.61(-2.01)	2.83(1.15)	2.08(7.52)	7.72(-5.87)
	O83qq	0.57(-0.79)	0.55(0.50)	2.64(3.71)	1.82(-2.42)
	O13qq	0.00(-0.28)	0.00(0.17)	0.00(1.06)	0.00(-0.87)
	O8qt	0.97(0.01)	1.18(0.83)	6.10(4.36)	3.92(0.34)
	O1qt	3.93(-1.78)	3.27(1.07)	2.27(7.19)	2.81(-5.91)
	O8ut	1.32(-0.02)	1.26(0.76)	4.81(1.12)	3.46(-0.17)
	O1ut	1.99(-2.38)	2.54(1.59)	3.48(9.30)	4.28(-6.88)
	O8qu	1.74(-0.31)	1.89(2.19)	10.61(4.38)	6.91(-0.29)
	O1qu	1.26(-1.00)	1.21(0.49)	2.12(3.83)	3.43(-2.91)
	O8dt	1.72(-0.54)	1.77(15.63)	7.79(6.35)	5.87(-2.21)
	O1dt	1.78(-1.66)	1.58(1.36)	5.83(6.20)	5.98(-5.80)
	O8qd	2.68(3.54)	3.06(2.98)	13.19(4.50)	8.83(-1.15)
	O1qd	1.45(-1.84)	1.64(1.14)	3.55(6.90)	6.78(-6.56)
2FB	Otp	×(×)	×(×)	×(×)	×(×)
	OtG	1.00(1.14)	3.02(3.60)	17.98(21.66)	10.35(11.74)
	Obp	×(×)	×(×)	×(×)	×(×)
	Ocp	×(×)	×(×)	×(×)	×(×)
	Otap	×(×)	×(×)	×(×)	×(×)
	OtW	×(×)	×(×)	×(×)	×(×)
	OtZ	×(×)	×(×)	×(×)	×(×)
	O3pQ3	×(×)	×(×)	×(×)	×(×)
	OpQM	×(×)	×(×)	×(×)	×(×)
	Opt	×(×)	×(×)	×(×)	×(×)
B	OpG	×(×)	×(×)	×(×)	×(×)
	OpB	×(×)	×(×)	×(×)	×(×)
	OpW	×(×)	×(×)	×(×)	×(×)
	Opd	×(×)	×(×)	×(×)	×(×)
	OWWW	×(×)	×(×)	×(×)	×(×)
	OpWB	×(×)	×(×)	×(×)	×(×)
	OpD	×(×)	×(×)	×(×)	×(×)

		ATLAS_CMS_tt_AC_8TeV	ATLAS_tt_AC_13TeV
4H	OQQ1	×(×)	×(×)
	OQQ8	×(×)	×(×)
	OQt1	×(×)	×(×)
	OQt8	×(×)	×(×)
	Ott1	×(×)	×(×)
2L2H	O81qq	3.66(1.78)	29.42(18.18)
	O11qq	1.06(-2.29)	6.57(14.86)
	O83qq	5.42(-0.69)	43.64(5.09)
	O13qq	0.00(-0.40)	0.00(-0.72)
	O8qt	3.61(2.05)	23.81(10.31)
	O1qt	1.67(2.99)	12.47(11.22)
	O8ut	5.78(0.30)	48.66(3.34)
	O1ut	1.92(-3.34)	10.71(-1.55)
	O8qu	7.16(2.07)	51.37(10.53)
	O1qu	0.92(1.65)	4.76(6.90)
	O8dt	4.62(-0.36)	38.02(7.16)
	O1dt	0.73(-2.09)	5.79(-2.69)
	O8qd	4.44(1.59)	31.05(6.77)
	O1qd	1.09(2.65)	5.95(9.48)
2FB	Otp	×(×)	×(×)
	OtG	0.00(-0.00)	0.00(0.00)
	Obp	×(×)	×(×)
	Ocp	×(×)	×(×)
	Otap	×(×)	×(×)
	OtW	×(×)	×(×)
	OtZ	×(×)	×(×)
	O3pQ3	×(×)	×(×)
	OpQM	×(×)	×(×)
	Opt	×(×)	×(×)
B	OpG	×(×)	×(×)
	OpB	×(×)	×(×)
	OpW	×(×)	×(×)
	Opd	×(×)	×(×)
	OWWW	×(×)	×(×)
	OpWB	×(×)	×(×)
	OpD	×(×)	×(×)

		CMS_tt_13TeV_ljets_2015_Mtt	CMS_tt_13TeV_dilep_2015_Mtt	CMS_tt_13TeV_ljets_2016_Mtt	CMS_tt_13TeV_dilep_2016_Mtt	ATLAS_tt_13TeV_ljets_2016_Mtt
4H	OQQ1	×(×)	×(×)	×(×)	×(×)	×(×)
	OQQ8	×(×)	×(×)	×(×)	×(×)	×(×)
	OQt1	×(×)	×(×)	×(×)	×(×)	×(×)
	OQt8	×(×)	×(×)	×(×)	×(×)	×(×)
	Ott1	×(×)	×(×)	×(×)	×(×)	×(×)
2L2H	O81qq	1.00(3.36)	0.33(1.06)	8.04(18.02)	13.74(28.04)	7.52(2.90)
	O11qq	5.04(8.34)	0.79(2.93)	33.94(40.43)	23.09(33.84)	12.87(-2.19)
	O83qq	0.43(4.09)	0.13(1.37)	2.44(19.41)	0.94(16.31)	0.56(-0.87)
	O13qq	0.00(1.18)	0.00(0.42)	0.01(5.59)	0.00(4.52)	0.00(-0.27)
	O8qt	1.10(3.34)	0.40(0.84)	12.94(40.12)	3.04(17.58)	1.73(4.95)
	O1qt	4.47(7.75)	0.84(2.80)	40.98(38.12)	17.01(35.94)	9.56(-0.78)
	O8ut	1.41(2.22)	0.38(0.29)	17.86(56.00)	7.93(24.76)	4.40(10.81)
	O1ut	3.75(9.56)	0.66(3.50)	29.79(46.20)	25.36(39.21)	13.98(-0.16)
	O8qu	1.06(2.82)	0.67(1.02)	4.89(60.40)	2.07(10.54)	1.29(3.18)
	O1qu	3.71(3.80)	0.35(1.41)	50.44(54.88)	20.38(23.22)	11.16(5.26)
	O8dt	2.27(6.27)	0.78(2.12)	17.03(36.48)	9.66(25.51)	5.48(0.07)
	O1dt	4.03(7.73)	0.75(2.68)	35.61(50.76)	23.88(39.51)	13.15(1.39)
	O8qd	1.60(2.37)	1.21(1.41)	6.38(59.68)	7.08(13.43)	4.16(0.95)
	O1qd	5.79(7.93)	0.97(2.96)	44.03(40.75)	18.05(34.96)	10.21(-1.85)
2FB	Otp	×(×)	×(×)	×(×)	×(×)	×(×)
	OtG	0.94(1.22)	1.42(1.81)	3.43(4.66)	2.07(2.68)	1.21(1.46)
	Obp	×(×)	×(×)	×(×)	×(×)	×(×)
	Ocp	×(×)	×(×)	×(×)	×(×)	×(×)
	Otap	×(×)	×(×)	×(×)	×(×)	×(×)
	OtW	×(×)	×(×)	×(×)	×(×)	×(×)
	OtZ	×(×)	×(×)	×(×)	×(×)	×(×)
	O3pQ3	×(×)	×(×)	×(×)	×(×)	×(×)
	OpQM	×(×)	×(×)	×(×)	×(×)	×(×)
	Opt	×(×)	×(×)	×(×)	×(×)	×(×)
B	OpG	×(×)	×(×)	×(×)	×(×)	×(×)
	OpB	×(×)	×(×)	×(×)	×(×)	×(×)
	OpW	×(×)	×(×)	×(×)	×(×)	×(×)
	Opd	×(×)	×(×)	×(×)	×(×)	×(×)
	OWWW	×(×)	×(×)	×(×)	×(×)	×(×)
	OpWB	×(×)	×(×)	×(×)	×(×)	×(×)
	OpD	×(×)	×(×)	×(×)	×(×)	×(×)

mtt distributions from Run II (2016 dataset) and charge-asymmetry dominate constraints in ttbar inclusive dataset

Linear EFT cross-sections

In the linear EFT approximation, one can express the dependence on the Wilson coeffs as

$$\mu_X\left(c/\Lambda^2\right) = \sigma_{\mathrm{X,EFT}}\left(c/\Lambda^2\right)/\sigma_{\mathrm{X,SM}} = 1 + \sum_{m=1}^{N_6} c_m \frac{\kappa_m^X}{\Lambda^2}$$

As mentioned before, overall normalisation of EFT coeffs (or EFT cross-section) is arbitrary

to quantify which processes have the strongest linear dependencies in which EFT coefficients, plot:

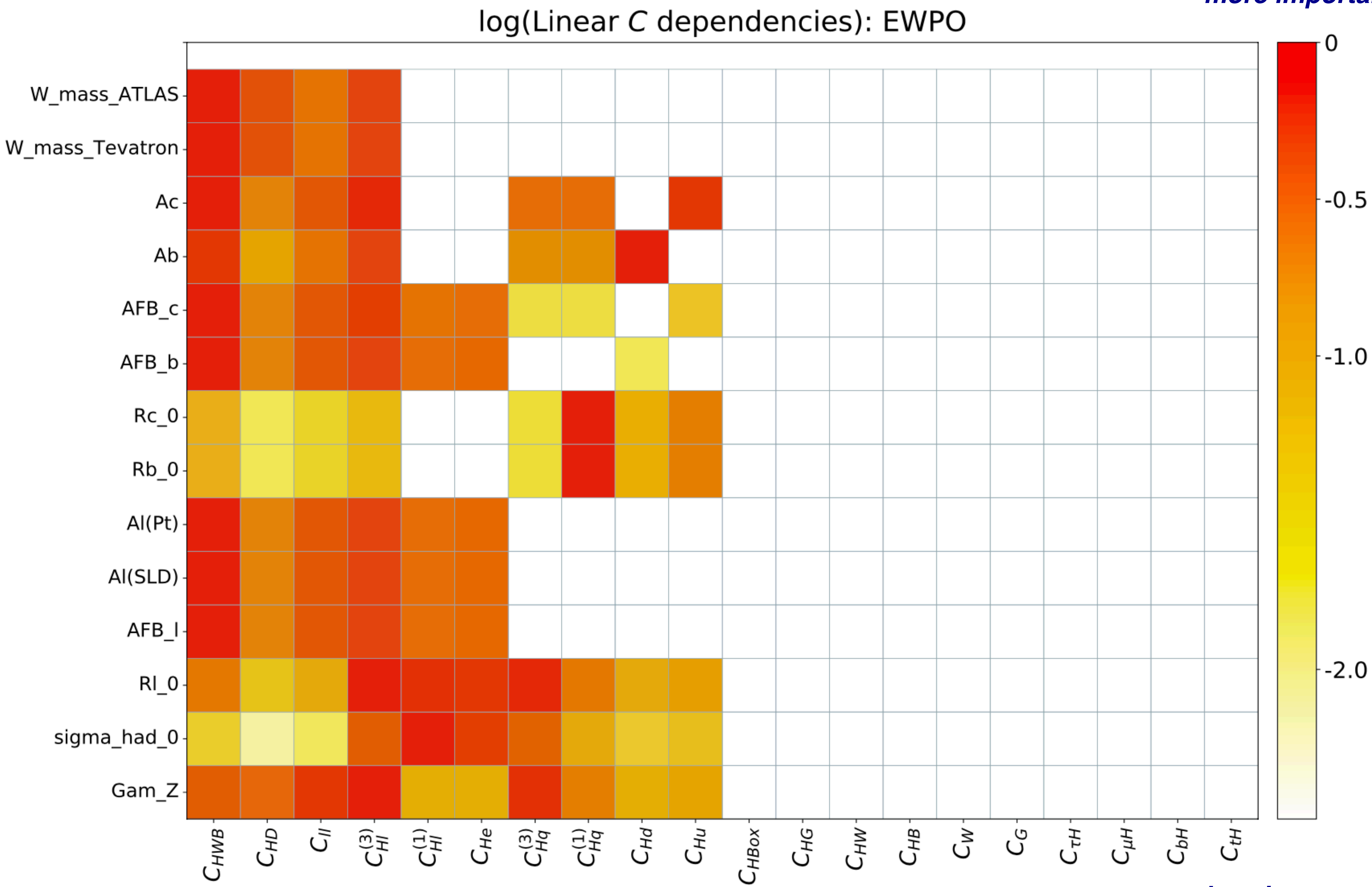
$$\ln\left(\kappa_m^X/\max_{\in X}\left(\kappa_m^X\right)\right)$$

normalised to largest linear EFT cross-section within the given measurement X

This estimator is closely related to the **Fisher information** but without the experimental covariance matrix accounted for. It measures the **size of the EFT cross-section** in a given process, normalised to the largest one

Linear EFT analysis

more important

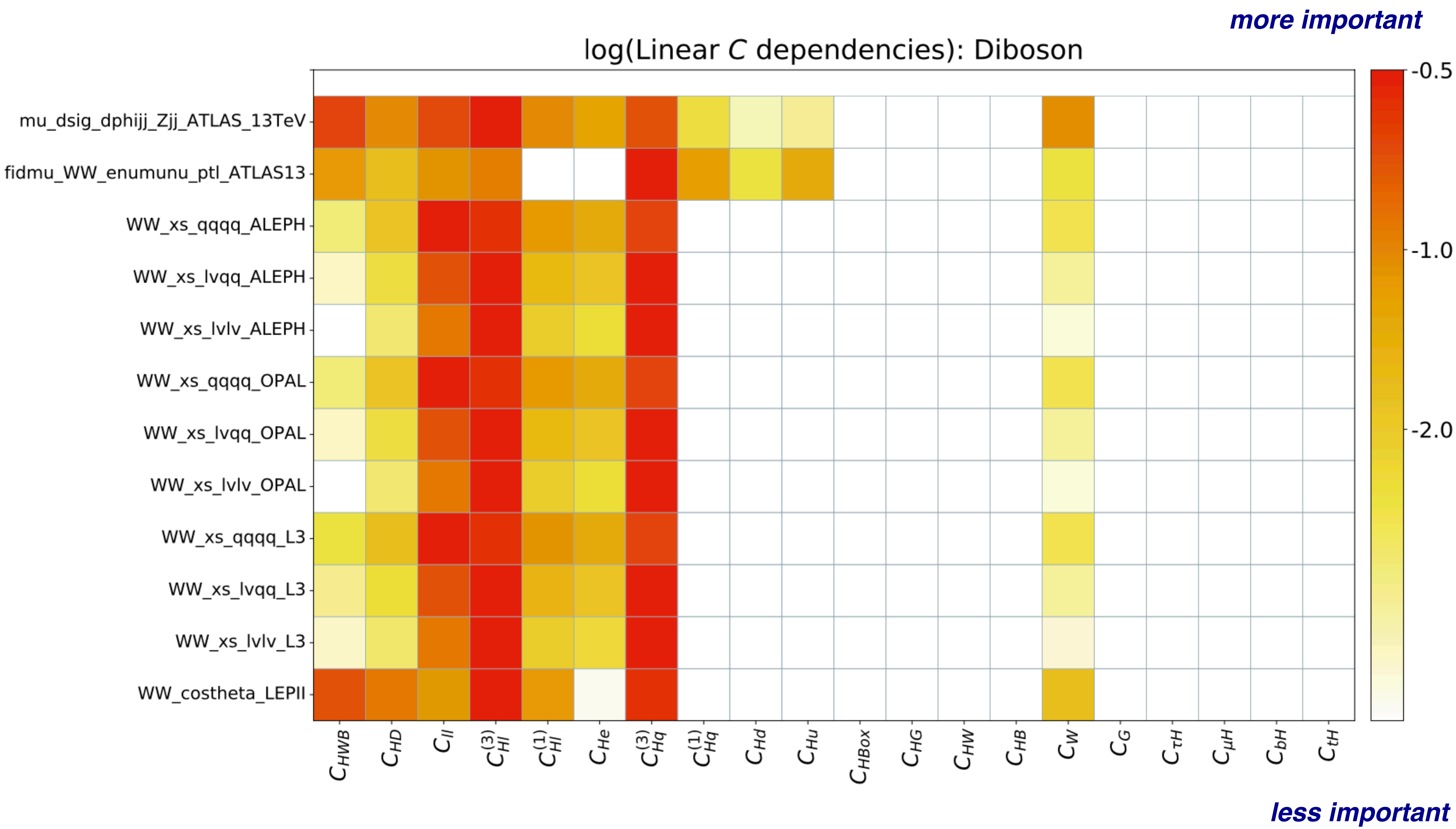


less important

As for the Fisher, the information is the change of colour ("sensitivity") in each column

comparison between ``columns" unphysical

Linear EFT analysis



As for the Fisher, the information is the change of colour ("sensitivity") in each column

Linear EFT analysis

As for the Fisher, the information is the change of colour ("sensitivity") in each column

Principal Component Analysis

PCA provides another useful handle to understand the relationship between **processes/ observables** and **directions in the EFT parameter space**

restricted to the linear EFT approximation

$$\sigma_m^{(\mathrm{th})}(\boldsymbol{c}) = \sigma_m^{(\mathrm{sm})} + \sum_{i=1}^{n_{\mathrm{op}}} c_i \sigma_{m,i}^{(\mathrm{eft})}$$

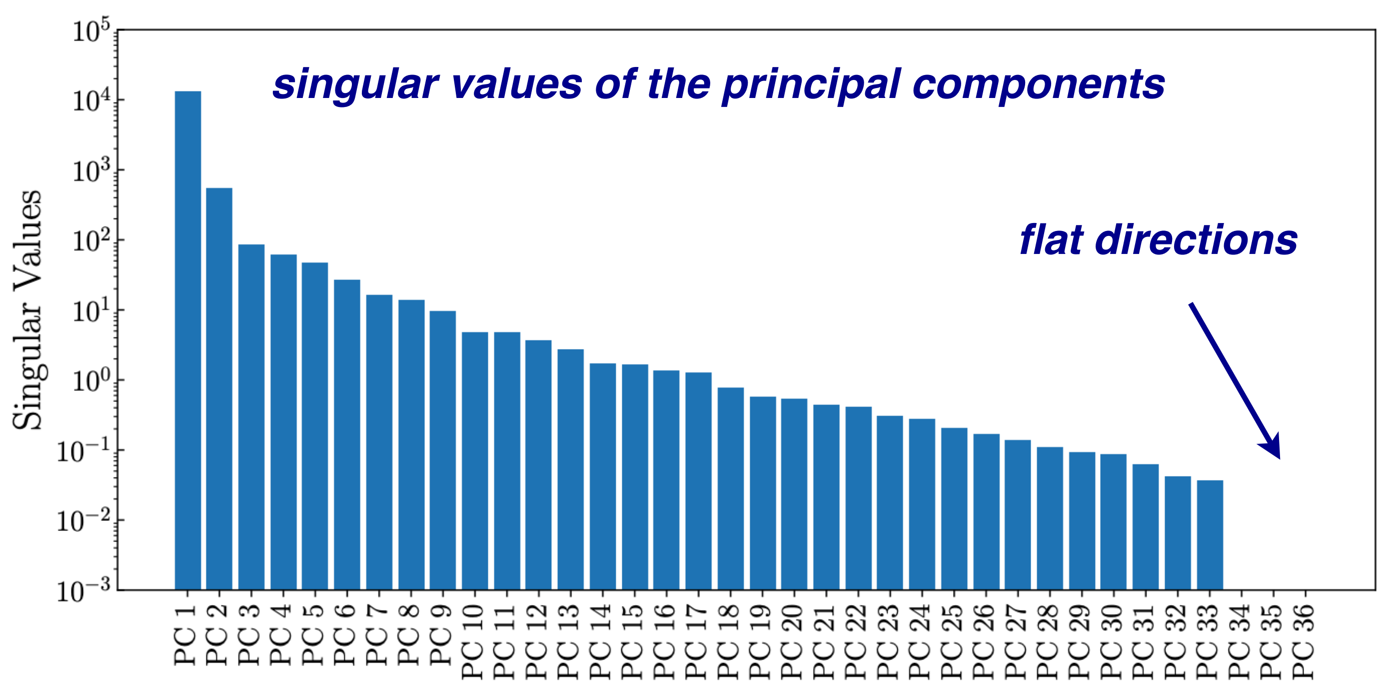
$$K = UWV^\dagger$$

$$K_{mi} = \sigma_{m,i}^{(\mathrm{eft})} / \delta_{\mathrm{exp,m}};$$

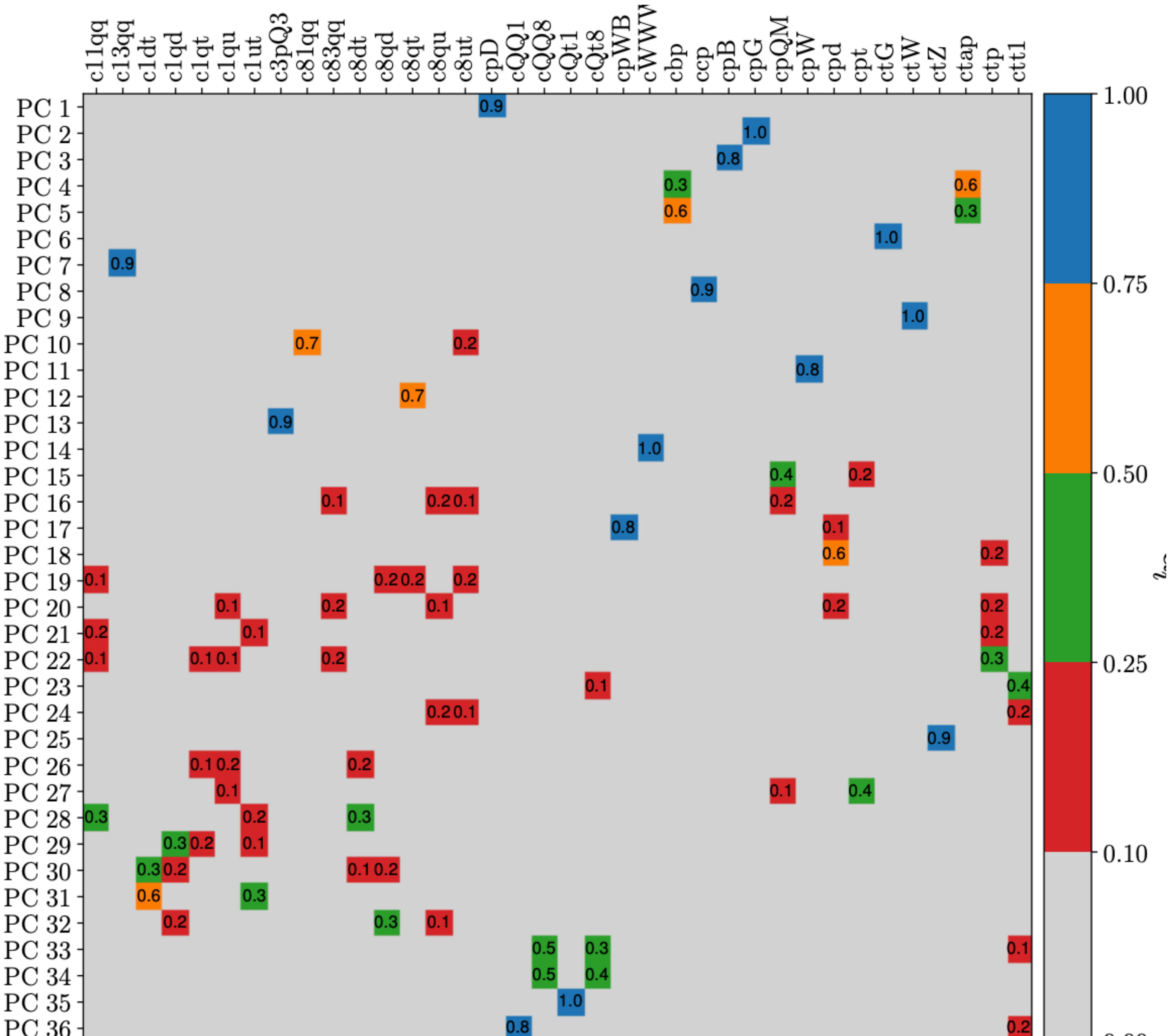
$$\mathrm{PC}_k = \sum_{i=1}^{n_{\mathrm{op}}} a_{ki} c_i\,, \quad k = 1, \ldots, n_{\mathrm{op}}\,, \qquad \left(\sum_{i=1}^{n_{\mathrm{op}}} a_{ki}^2 = 1 \;\; \forall k \right)$$

determine **most sensitive directions** and identify possible flat directions using PCA

n.b. flat directions are not necessarily a problem!



Principal Component Analysis



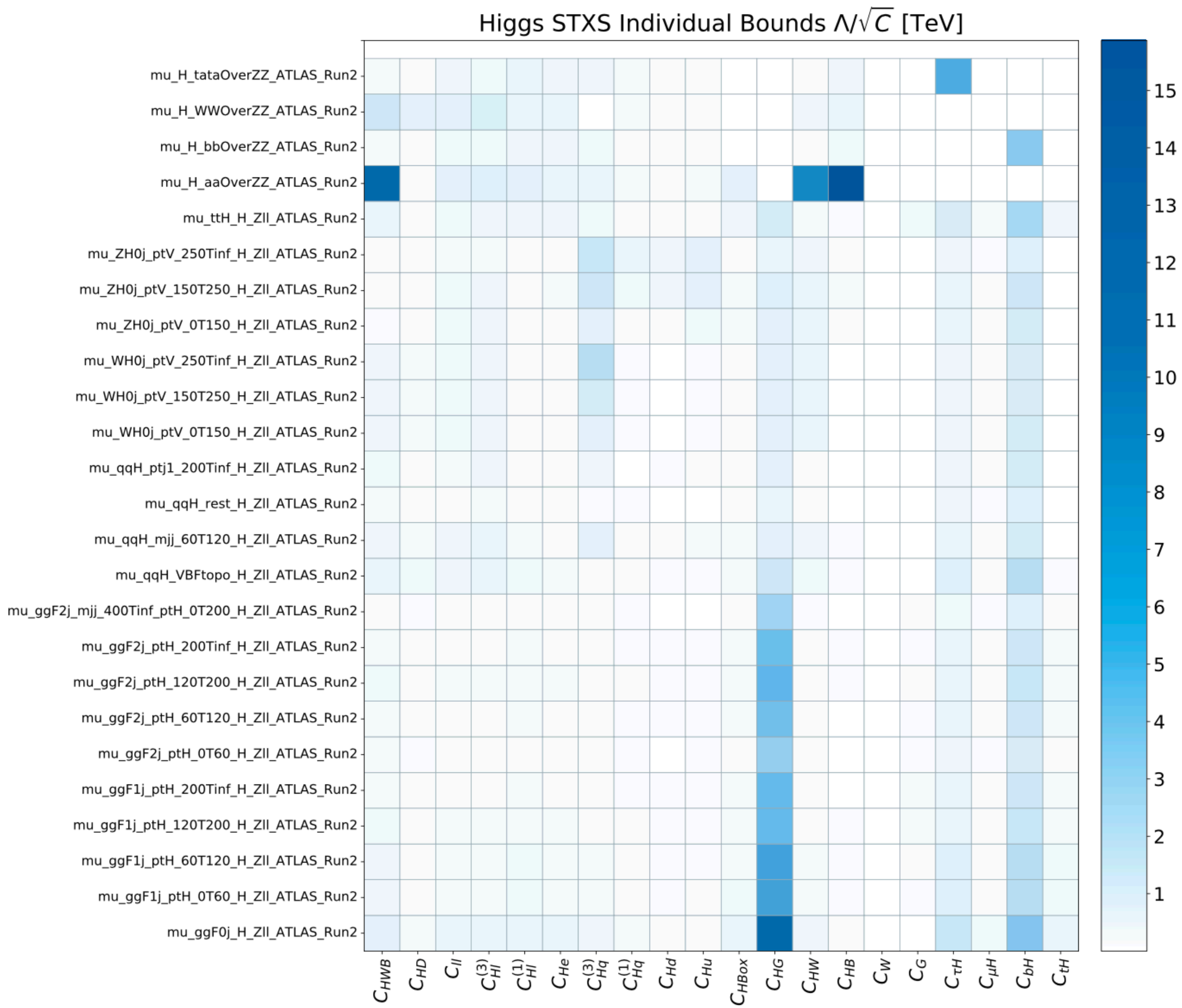
- Determine which **coefficients are determine by one or a few processes**, and which ones only enter at the level of linear combinations of many coefficients

  e.g. most four-fermion operators in Warsaw are not ``natural” directions

- Powerful tool to understand fit results, eventually could be used to **fit in the PCA basis** (though this is not required)

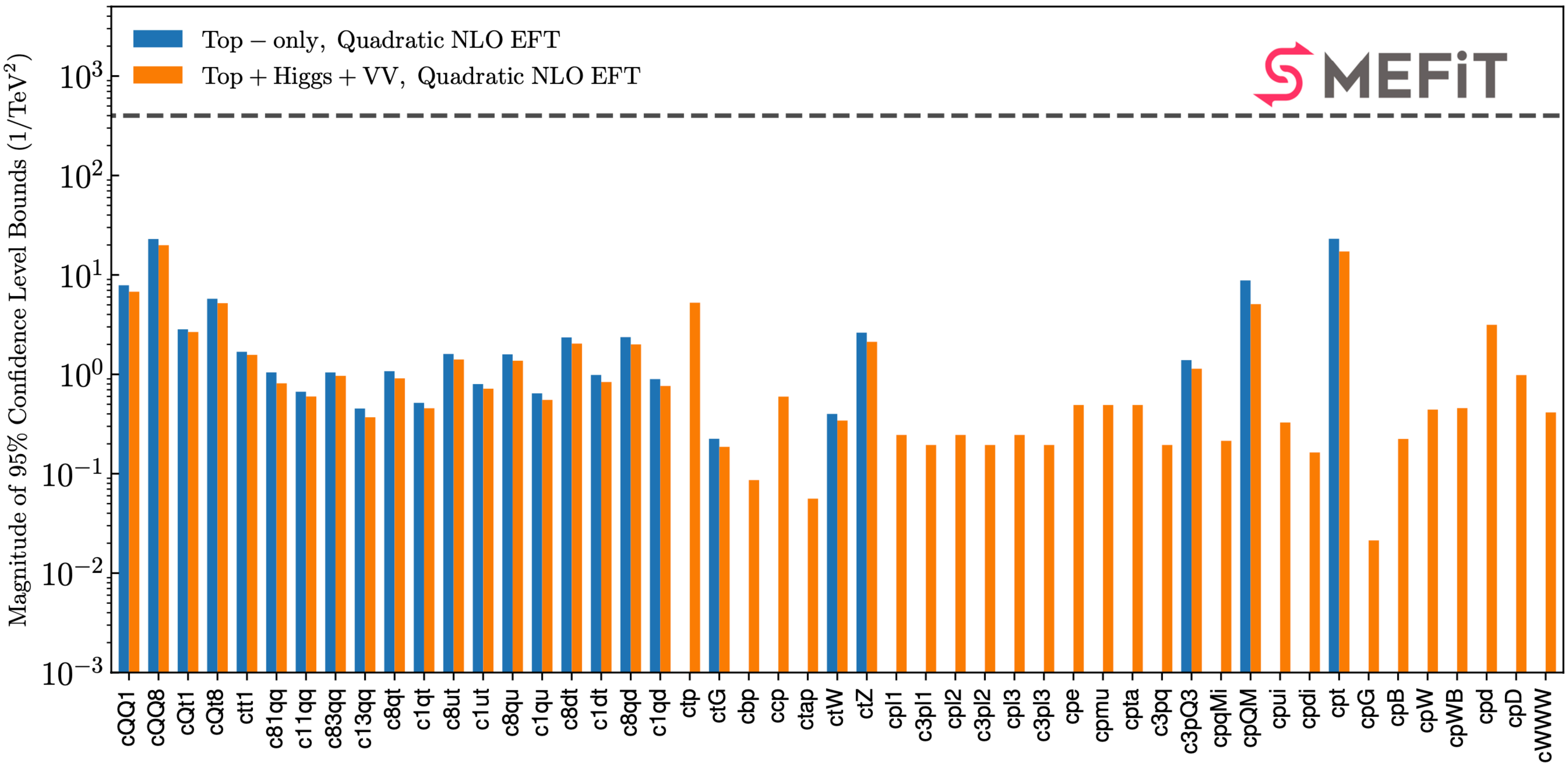
One-parameter fits

The sensitivity of a given dataset to specific EFT coefficient can also be quantified by **individual (1-parameter) fits,** which provide by construction, maximal sensitivity on a specific direction on the EFT parameter space



Global fits

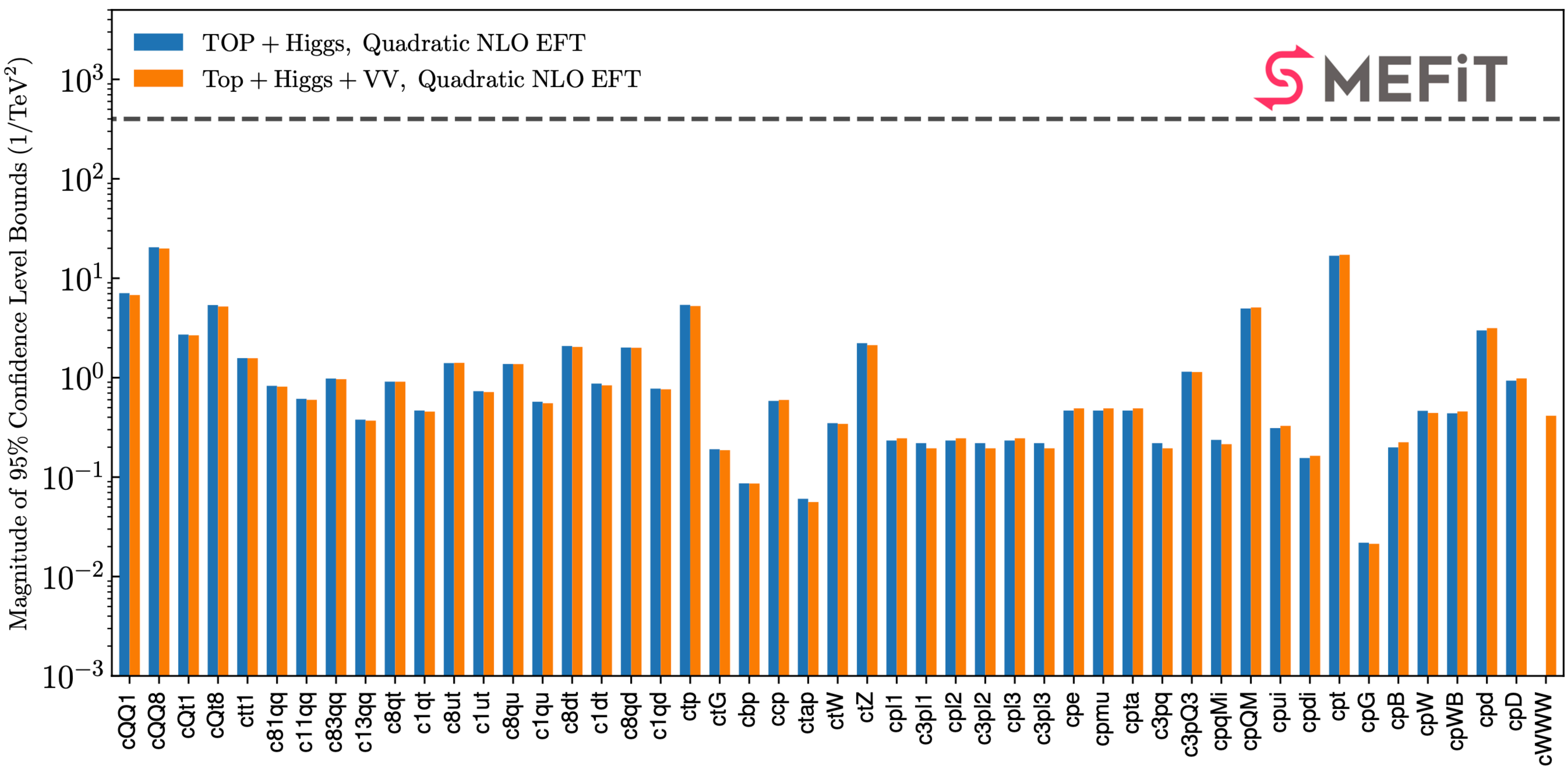
a complementary method to assess the information on the EFT parameter space provided by different processes/observables is **carrying out the fit with and without them** and comparing results



impact of Higgs & diboson data when added to the top-quark dataset

Global fits

a complementary method to assess the information on the EFT parameter space provided by different processes/observables is **carrying out the fit with and without them** and comparing results



impact of diboson data when added to the top-quark & Higgs dataset

Summary and outlook

- Establishing a clean map between **processes/operators and EFT parameters** is non-trivial

- Several statistical estimators and tools available, their combination is necessary to determine the complete picture

- These estimators can be evaluated at different levels, from processes to datasets or even to individual bins, depending on the information we are searching for

- Should be careful and evaluate only quantities which are **model-independent** (in particular, on the overall normalisation of the EFT operators)

- The more global the dataset and the more general the EFT basis, the more interesting information one can obtain from such analysis!