Unfolding in ALICE

Jan Fiete Grosse-Oetringhaus, CERN
for the ALICE collaboration

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Content

- Unfolding methods used in ALICE
- Applications of unfolding in ALICE
  - Observations, Issues and remarks

Unfolding Methods

- χ^2-Minimization with regularization (only 1D)

$$\chi^2(T^*) = \sum_m \left(\frac{M_m^* - \sum_t R_{mt} T_t^*}{e_m} \right)^2 + \beta R(T^*) \qquad R(T) = \sum_t (a_t)^2$$

β weight

- Available regularizations
  - Mostly used:
    linear and constant

$$a_t = \frac{T_t'}{\sqrt{T_t}} = \frac{T_t - T_{t\text{-}1}}{\sqrt{T_t}}$$

prefer constant

$$a_t = \frac{T_t''}{\sqrt{T_t}} = \frac{T_{t-1} + 2T_t - T_{t+1}}{\sqrt{T_t}}$$

prefer linear least curvature

$$a_t = \frac{\hat{T}_t''}{\sqrt{\hat{T}_t}} \qquad \hat{T}_t = \ln T_t$$

prefer exp

$$R(T) = \sum_t T_t \ln \frac{T_t}{\varepsilon_t}$$

reduced cross-entropy

- Implementation uses Minuit, Migrad
  - Uncertainties from Minuit

Unfolding Methods (2)

- Iterative Bayesian Unfolding (1D + nD)

$$\tilde{R}_{tm} = \frac{R_{mt} P_t}{\sum_{t'} R_{mt'} P_{t'}} \qquad U_t = \sum_m \tilde{R}_{tm} M_m$$

d‘Agostini

- Optionally smoothing can be applied

$$\hat{U}_t = (1-\alpha) U_t + \frac{\alpha}{3} (U_{t-1} + U_t + U_{t+1})$$

- Uncertainty on unfolded distribution by randomization of input spectrum
  - Poisson distribution used per bin

Unfolding Methods (3)

- Unfolding methods part of the software framework
  - Main interface function (for 1D case):
    Unfold(TH2* correlation, TH1* efficiency, TH1* measured, TH1* initialConditions, TH1* result)
  - nD case uses THnSparse
- Functions to evaluate bias b_t due to regularization

$$b_t = \sum_m \frac{\partial T_t}{\partial M_m}((RT)_m - M_m)$$

(see e.g. Cowan)

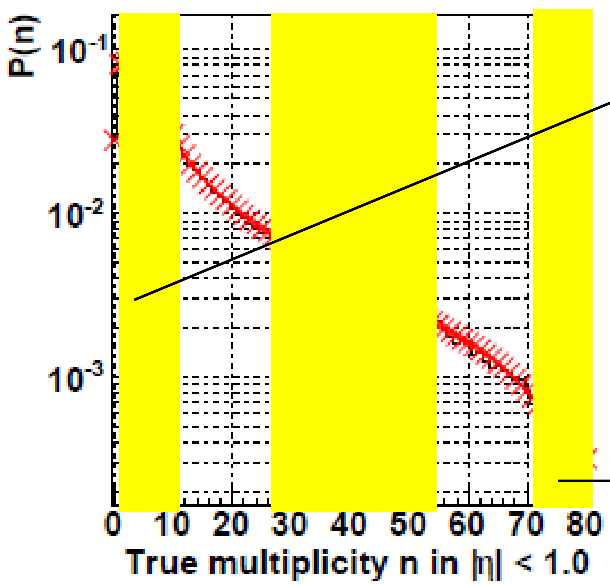
  - Derivate calculated numerically

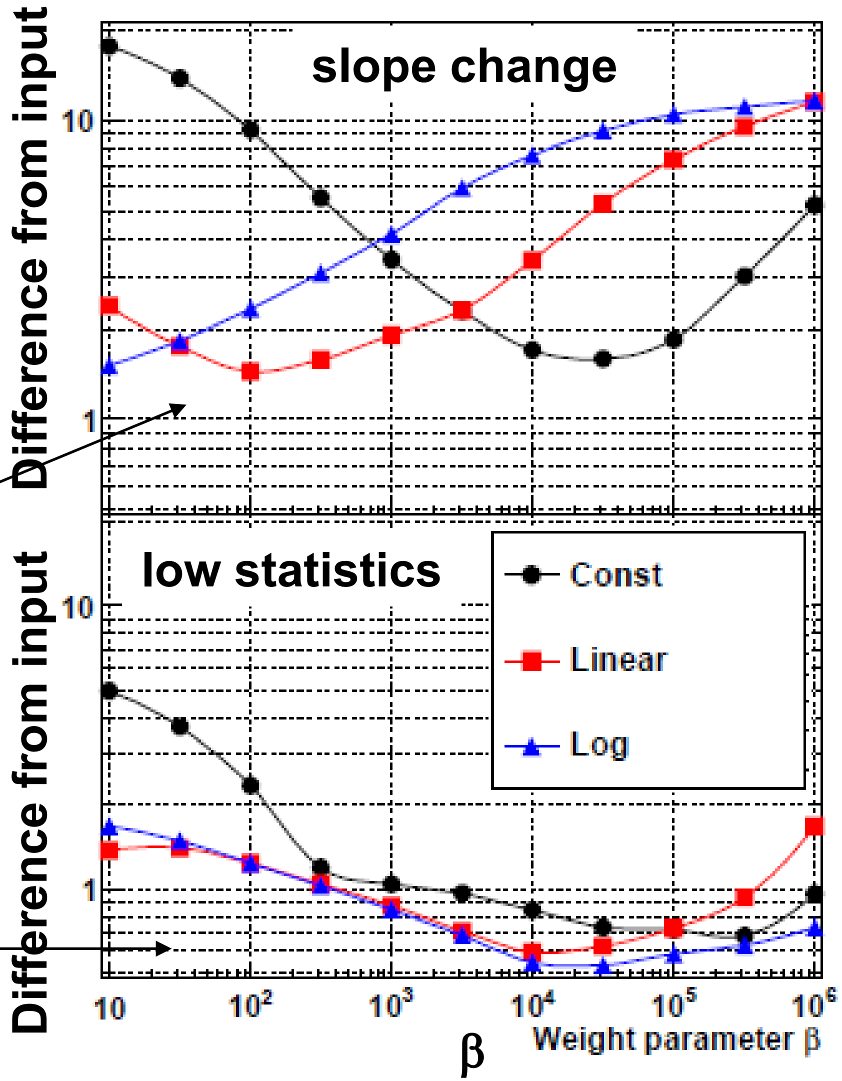
$$\frac{\partial T_t}{\partial M_m} = \frac{1}{6d}\left[8\left(f\left(\frac{d}{2}\right) - f\left(-\frac{d}{2}\right)\right) - \left(f(d) - f(-d)\right)\right]$$

$$f(x) = T_t(M \mid M_m = M_m + x\sqrt{M_m})$$

Example: Evaluate Regularization Weight with MC

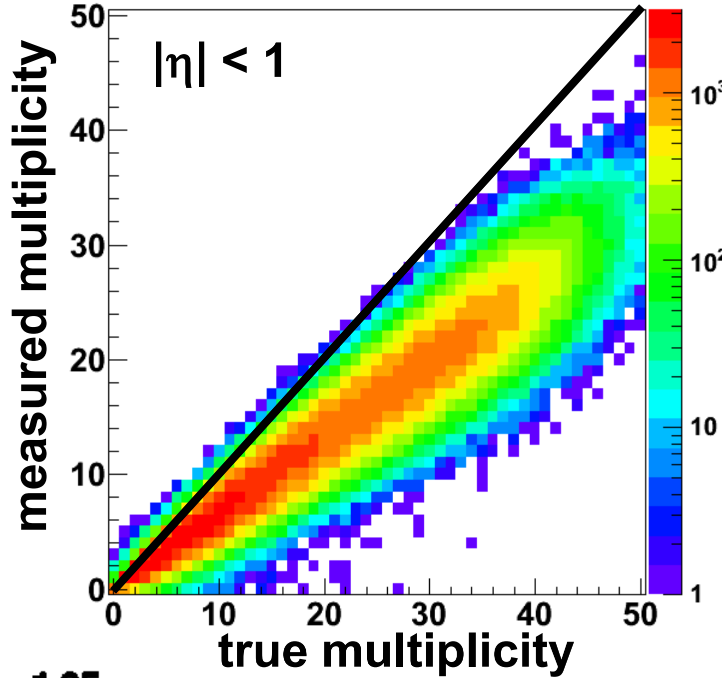
- Cooking down the consistency of the unfolded solution with the MC to one number is tricky
  – Different regions, different qualitative shape

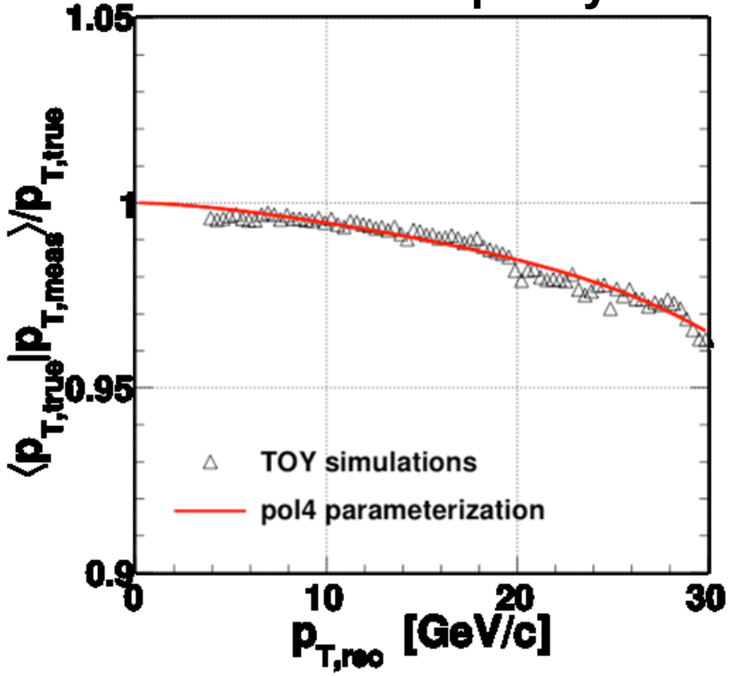




Applications

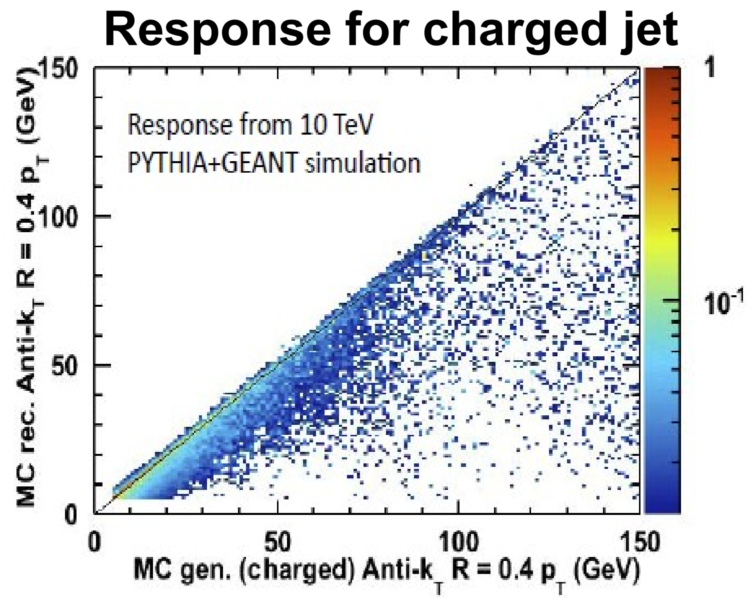
- Multiplicity distribution
  - Efficiency ~ 70% (inactive modules in pixel detector due to cooling issues)
    → far off the diagonal
  - Significant spread in response matrix → wide correlations in unfolded spectrum
- p_T spectrum
  - Shift is 2-4% for 20-30 GeV
  - Unfolding needed, response matrix not very wide

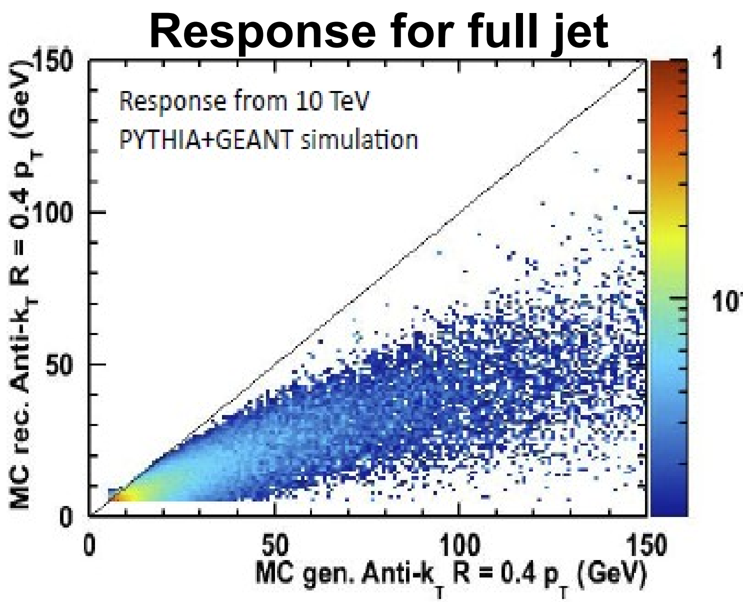




Applications (2)

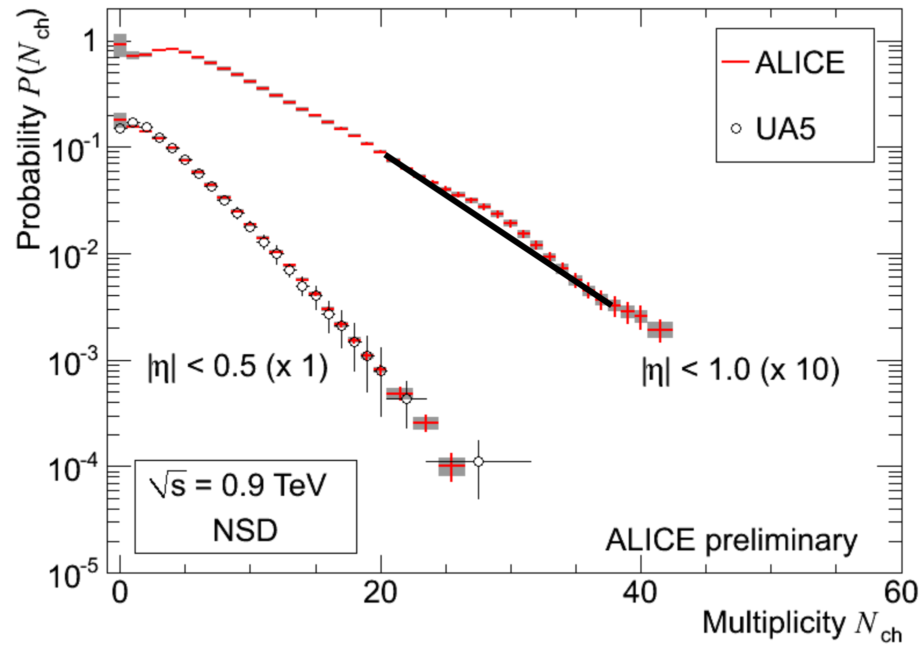
- Jet spectrum
  - Wide response matrix σ~20%
  - Corrections to full (charged+neutral) jet → significant shift because all neutrals missing (1/3)
  - Caveat: MC simulations usually done in $p_{T,hard}$ bins
    - Jet yield at low p_T >> MC at low p_T

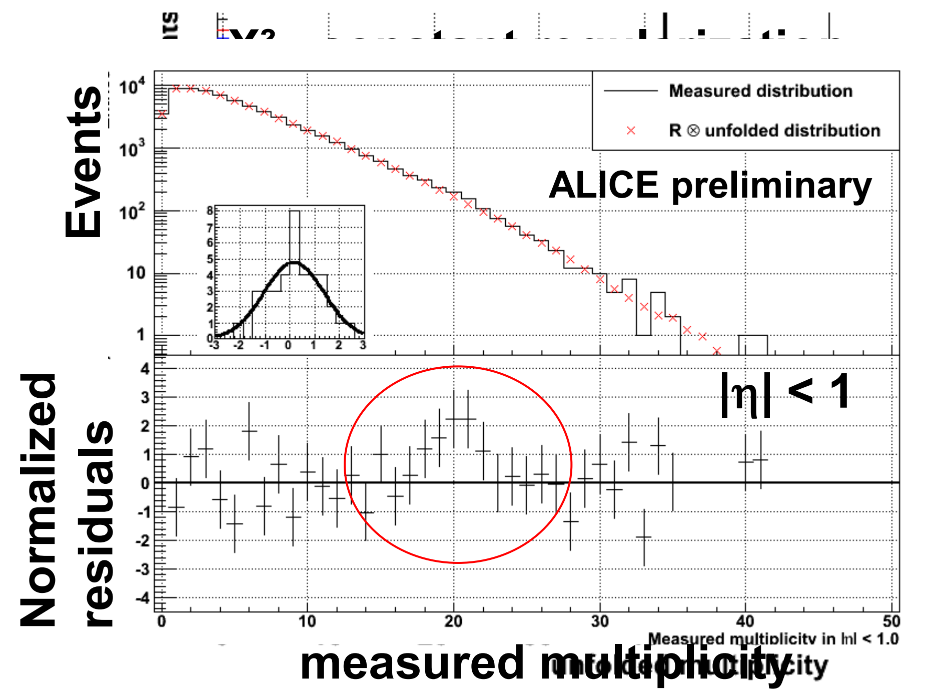




Example Issue

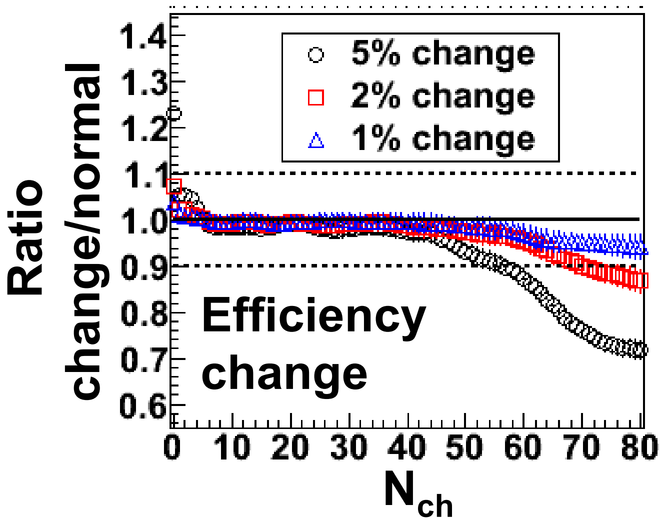
- Observation: nice hump in unfolded distribution (which got people immediately excited)
- Usual cross-checks
  → independent of
  - Regularization scheme
  - Unfolding method
- Forcing the distribution to be exponential
  - Moves the structure to the residuals
  - However, in fewer bins and (visually) much less significant
- Uncertainties of unfolded spectrum and wide response matrix can enhance fluctuations in the data significantly
- Slope change visible in measured data → transition gets spread out by regularization





Systematic Uncertainties

- From the unfolding procedure
  - Bias
  - Some fluctuations (1-5%) remain
  - Use MC to get a feeling
- Uncertainties are correlated between the bins
- Due to the response matrix
  - Uncertainties on the response matrix "propagate through" the unfolding into the unfolded distribution
  - Create different response matrices resembling the uncertainty
    - E.g. detector efficiency uncertainty of 1% → Create three response matrices with -1%, 0, +1% efficiency w.r.t. nominal
    - Use difference in unfolded distribution as systematic uncertainty → tricky because overlaid by "typical" fluctuations, small effects cannot be disentangled



Summary

- ALICE uses unfolding (up to now) for multiplicity distribution, p_T spectrum and jet spectrum
- The ALICE software framework provides
  - χ^2-minimization with regularization
  - Iterative Bayesian unfolding
- Unfolding usually requires a lot of tuning
- Systematic uncertainties sometimes tricky
- Uncertainties on unfolded distributions are always understood (by others) as single bin uncertainties
  - Can we invent a nice way to visualize that this is not the case?

Thanks for help with preparing this talk to Christian Klein-Boesing, Jan Rak

Unfolding tutorial (introductory slides, exercises and solutions):
www.cern.ch/jgrosseo/permanent/unfolding
(prepared for a Helmholtz power week)
Feel free to use it!