Nuisance Parameters in Likelihoods for Searches

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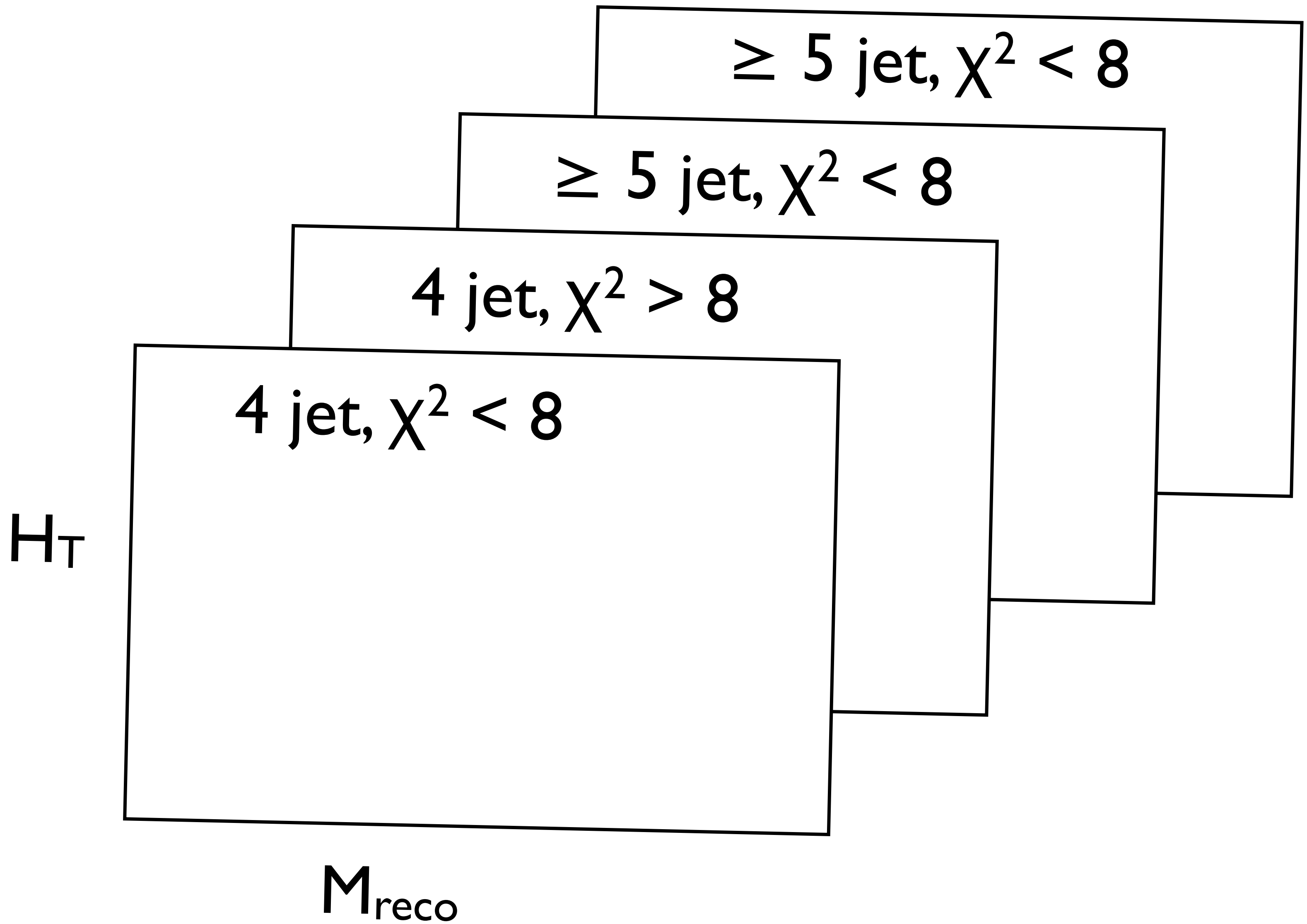
PhyStat 2011 - 17-20 January
CERN

Likelihood for Multisource Spectrum

- very common analysis: use binned spectrum in one or more dimensions to gain statistical sensitivity
- typically several backgrounds contribute
- focus here is how to represent all types of systematic uncertainties in likelihood as nuisance parameters
- example problem: t′ search from CDF
- methods here are beginning to be used in CMS, and could easily be incorporated into RooStats
- useful for limits, discovery, and measurements of new physics processes

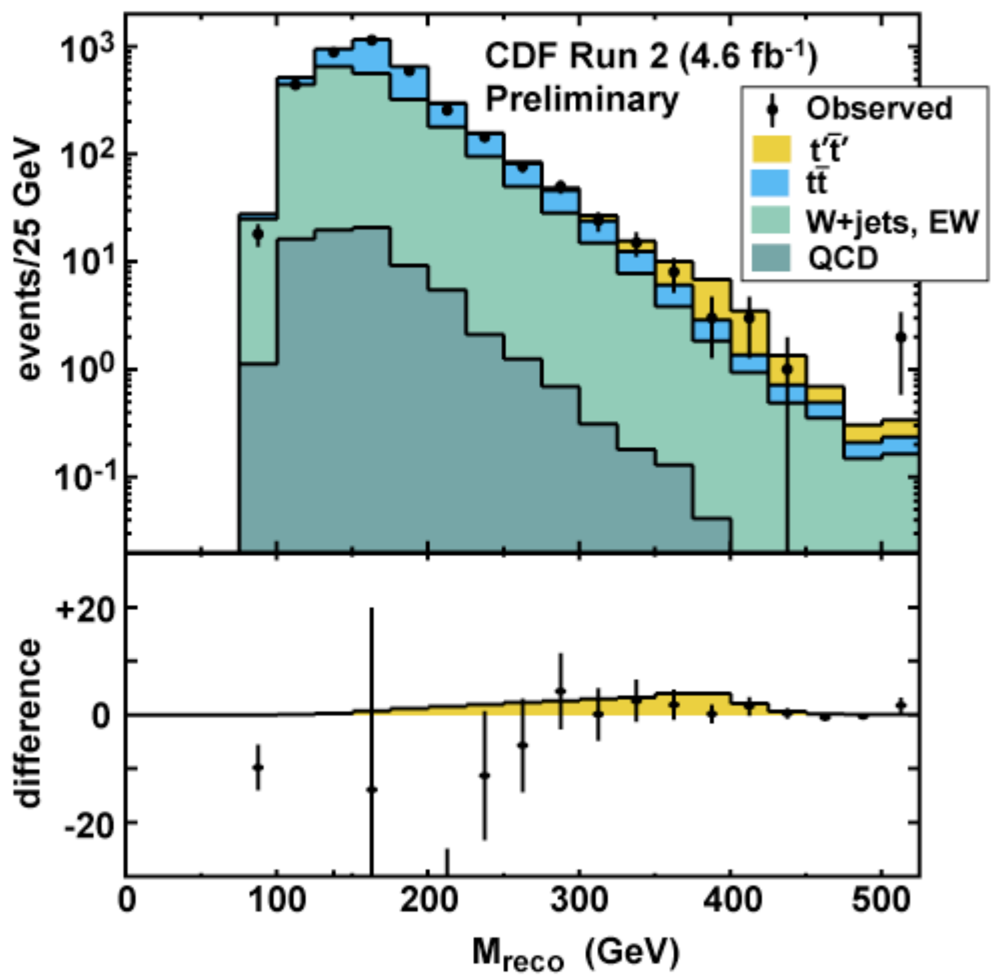
Example: t′ Search in CDF

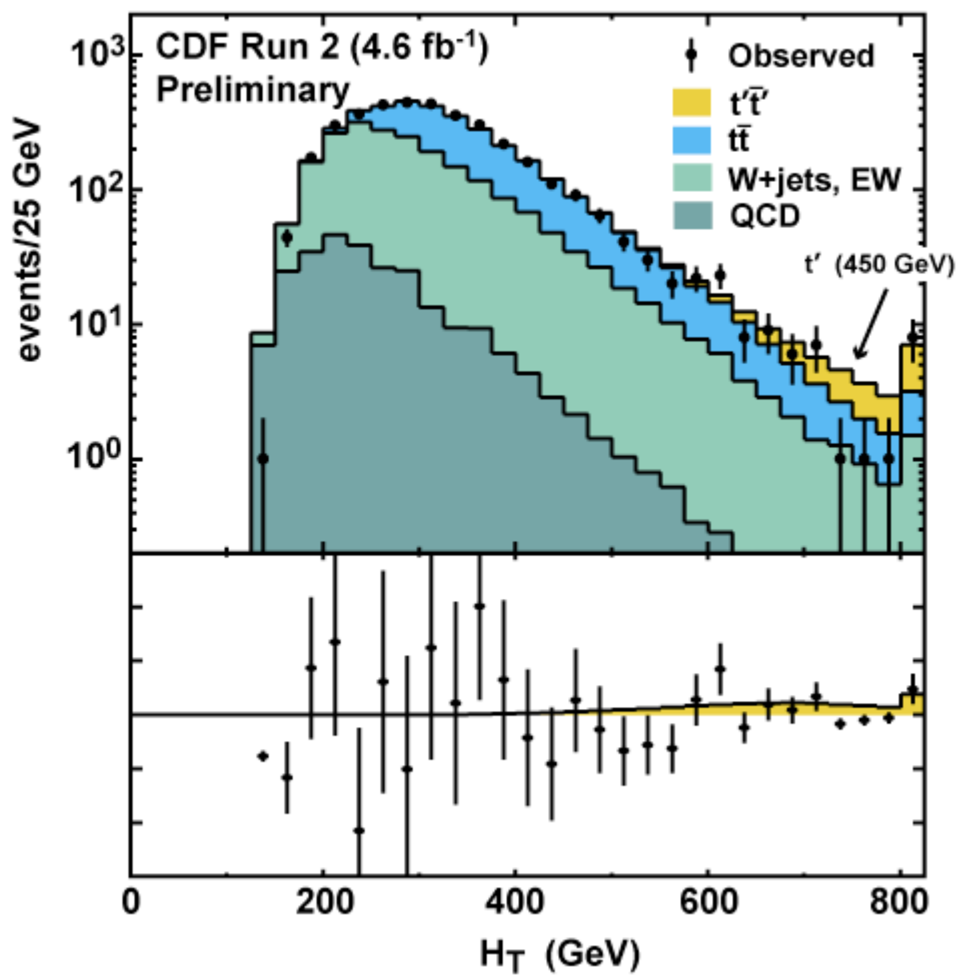
- use 3D binned likelihood in H_T, M_{reco}, N_{jet}/χ^2



Example: t′ Search in CDF

- main backgrounds: tt, W+jets, QCD





Systematic Uncertainties

types of nuisance parameters

- integrated luminosity
- background normalizations
- lepton ID data/MC ratios

} multiplicative

- jet energy scale
- initial/final state radiation
- Q^2 scale

} template morphing

- Monte Carlo statistics — "Barlow-Beeston light"

Multiplicative nuisance parameters

- likelihood is Poisson at its core, with bin contents coming from simple sums over sources
  - parameter(s) of interest: α
  - nuisance parameters: β
- nuisance parameters are gaussian-constrained:

$$-\ln \mathcal{L}(\bar{x};\bar{\alpha},\bar{\beta}) = \sum_{i=1}^{n}(\mu_i - n_i \ln \mu_i) + \frac{(\beta_1' - \beta_1)^2}{2\sigma_1^2} + \frac{(\beta_2' - \beta_2)^2}{2\sigma_2^2} + \ldots$$

$$\mu_i = L\sigma_1\beta_1\beta_2\epsilon_{1i} + L\sigma_2\beta_2\beta_3\epsilon_{2i} + \ldots$$

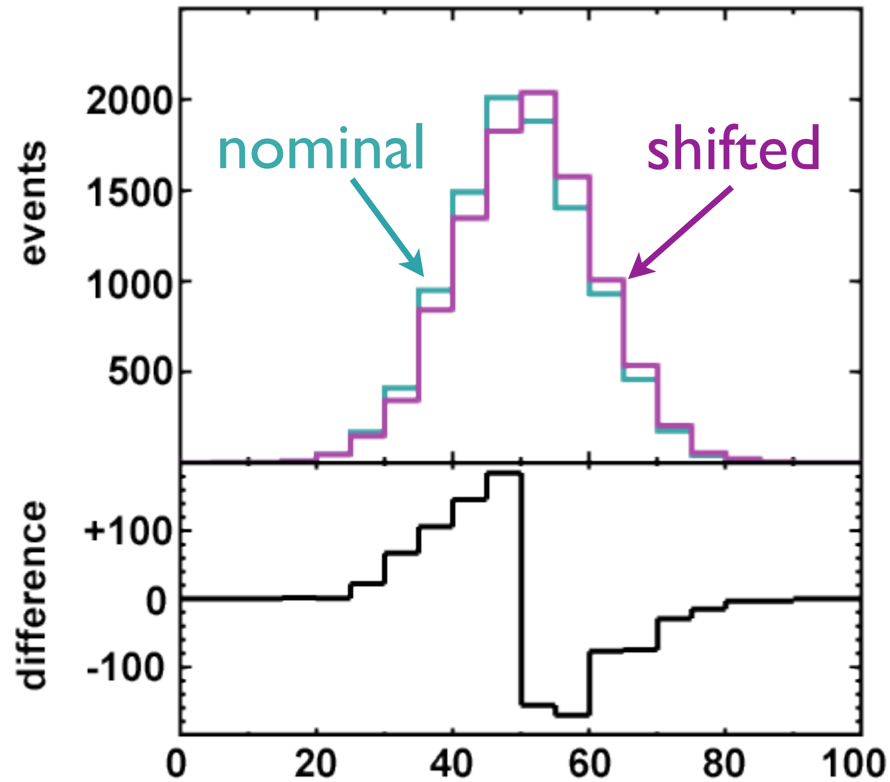
correlations!

Pitfalls of multiplicative parameters

- must avoid normalization of any source going negative...MINUIT bounds or
- must avoid bins with just one source vanishing or likelihood will suddenly jump
- especially true of signal source, near zero signal

Template "morphing"

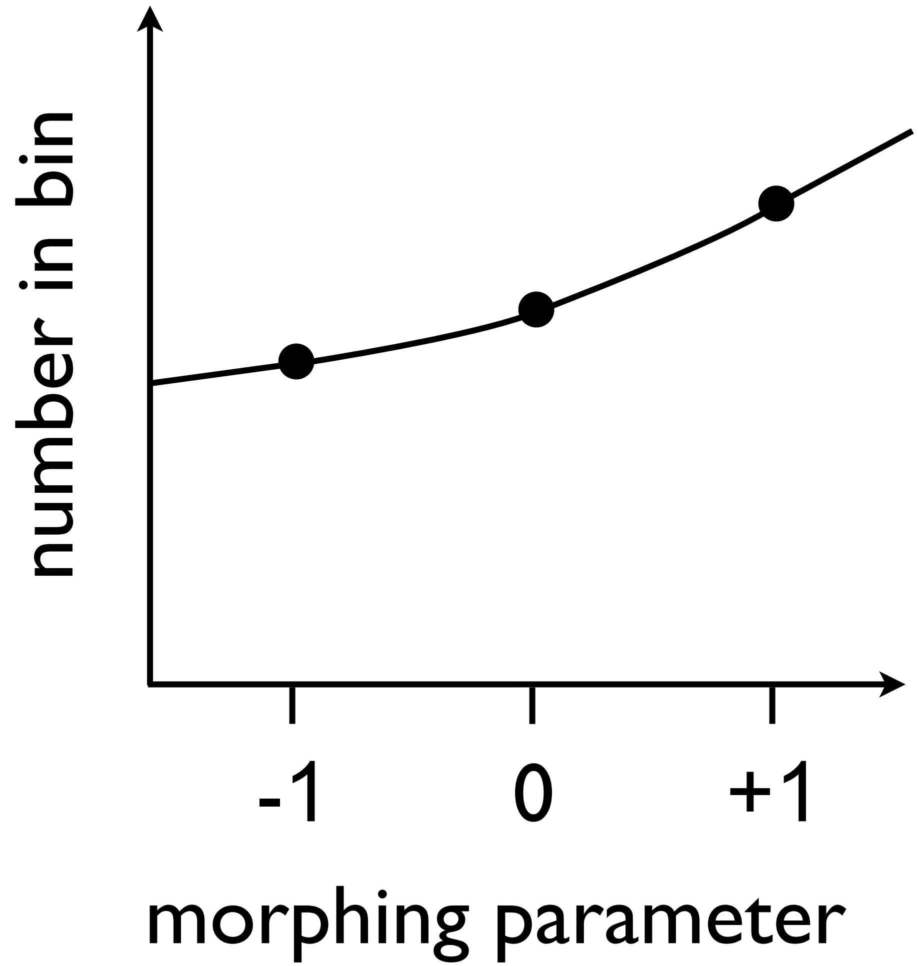
- energy scales in particular can cause coherent shifts in shape (and normalization!) of template
- typically evaluate template shapes at three values of systematic effect corresponding to -1σ, 0, +1σ shifts in the base effect
- can use more evaluations, or other points
- define morphing nuisance parameter which allows one to continuously "morph" shape as parameter changes



$$\mu_i = \mu_i^0 + f\mathcal{M}(\mu_i^-, \mu_i^0, \mu_i^+)$$

Template morphing

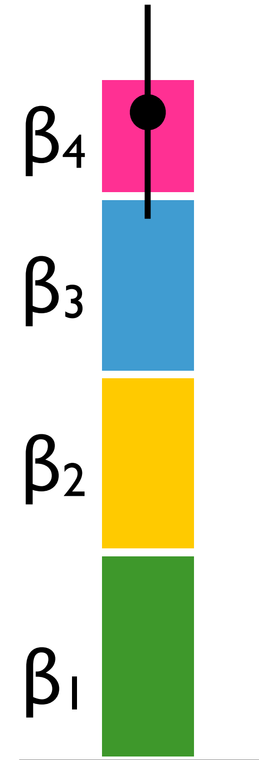
- sophisticated "horizontal" treatment* of Read works in 1D only
- use "vertical morphing" for t′ analysis in 3D
- interpolate quadratically within range
- extrapolate linearly outside range
- avoids "kink" at zero; no symmetrization needed
- multiple morphing parameters work well



*A. Read, Nucl. Instrum. Meth. Res. A 425 (1999) 357-369, encoded in class RootLinearMorph.

MC Statistics: Barlow-Beeston method

- to treat bin statistical uncertainties, Barlow and Beeston suggested* introducing a nuisance parameter β_i for each source i in each bin
- implemented Newton-type iterative method to solve simultaneous nonlinear equations for β_i which maximize likelihood
- each bin is independent, so this can be done inside our MINUIT fit for profile likelihood
- problem: MINUIT fails due to small jumps in β_i as expected number in source changes



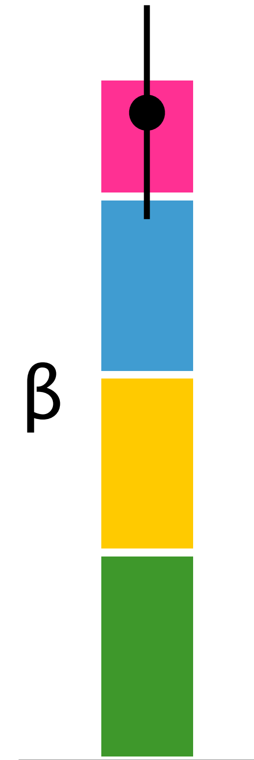
*Comp. Phys Comm. 77 (1993) 219

MC Statistics: Barlow-Beeston "lite"

- only one overall B-B parameter β is needed per bin since the source uncertainties are independent!
- constrain β to 1 within total statistical uncertainty in bin (add in quadrature)
- can solve analytically for β_i in bin i:

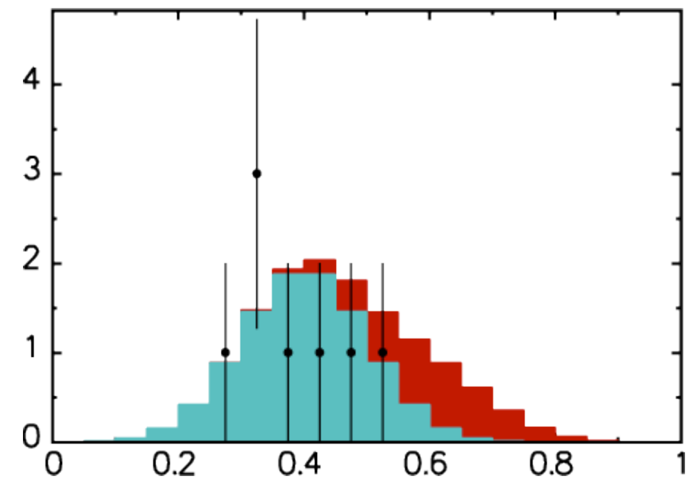
$$\frac{-\ln \mathcal{L}_i}{\partial \beta_i} = 0 \quad \Rightarrow \quad \beta_i^2 + (\mu_i \sigma_i^2 - 1)\beta_i - n_i \sigma_i^2 = 0$$

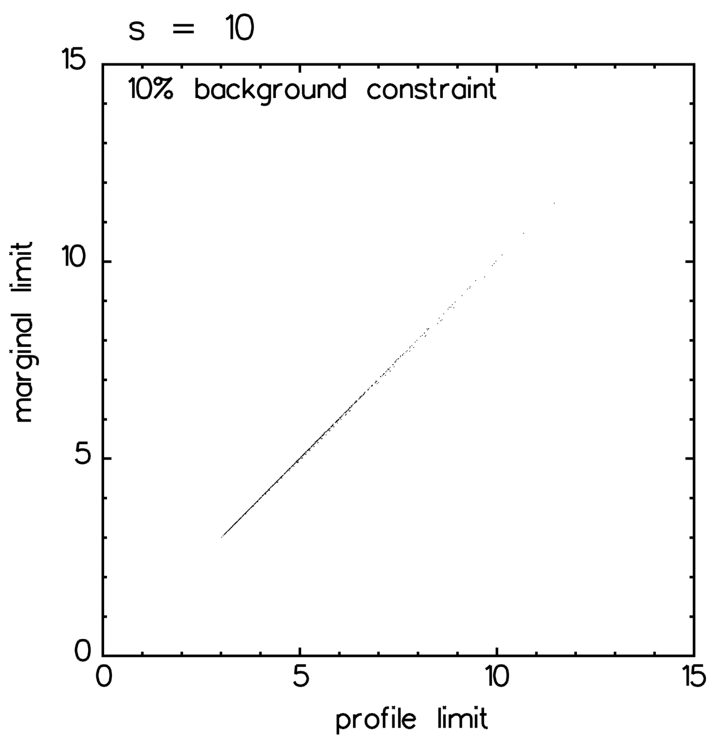
- this <u>does</u> represent fairly the effect of the overall statistical uncertainty in each bin
- works for non-MC sources
- method works well! MINUIT is stable!

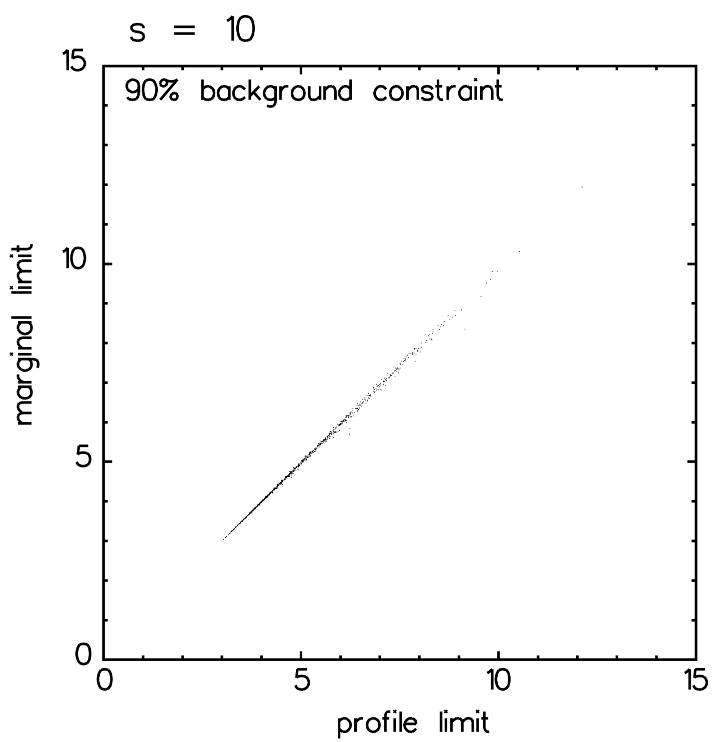


Profiling vs. Marginalization

In binned likelihood fits to multisource spectra, the result of using a profile treatment of nuisance parameters gives nearly identical results to marginalization







Summary

- three basic types of nuisance parameters are sufficient for almost all binned Poisson likelihood problems:
  - normalization
  - morphing (correlated shape and normalization)
  - statistical uncertainties
- in the context of a binned multisource spectrum likelihood, have shown straightforward, stable methods to implement these
- works for profiled or marginalized likelihood
- could/should be standardized within RooStats