PARTON DISTRIBUTIONS:

DETERMINING PROBABILITIES IN A SPACE OF FUNCTIONS

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PROLOGUE: THE DISCOVERY OF THE W

LAST TIME WE LOOKED FOR "NEW" PHYSICS AT A HADRON COLLIDER

THEORETICAL PREDICTION...

42 G. Altarelli et al. / Vector boson production

TABLE 2
Values (in nb) of the total cross sections for $W^{\pm}$ and Z^0 production

	$W^+ + W^-$	$W^+ + W^-$	$W^+ + W^-$	Z^0	Z^0	Z^0	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$
$\sqrt{S}$ (GeV)	GHR	DO1	DO2	GHR	DO1	DO2	GHR	DO1	DO2
540	4.2	4.3	4.1	1.3	1.3	1.2	3.1	3.4	3.5
700	6.2	6.3	6.1	2.0	1.9	1.8	3.1	3.3	3.4
1000	9.5	9.5	9.6	3.1	3.0	2.9	3.1	3.2	3.3
1300	12.5	12.5	12.9	4.0	3.9	3.9	3.1	3.2	3.3
1600	15.5	15.6	16.5	5.0	4.8	5.0	3.1	3.2	3.3

ALTARELLI, ELLIS, GRECO, MARTINELLI, 1984

...AND EXPERIMENTAL DISCOVERY

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/85-108
11 July 1985

W PRODUCTION PROPERTIES AT THE CERN SPS COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

Aachen[1] - Amsterdam (NIKHEF)[2] - Annecy (LAPP)[3] - Birmingham[4] - CERN[5] - Harvard[6] - Helsinki[7] - Kiel[8] - London (Imperial College[9] and Queen Mary College[10]) - Padua[11] - Paris (Coll. de France)[12] - Riverside[13] - Rome[14] - Rutherford Appleton Lab.[15] - Saclay (CEN)[16] - Victoria[17] - Vienna[18] - Wisconsin[19] Collaboration

The corresponding experimental result for the 1984 data at $\sqrt{s}$ = 630 GeV is

$$(\sigma \cdot B)_W = 0.63 \pm 0.05\ (\pm 0.09)\ \text{nb}.$$

This is in agreement with the theoretical expectation [14] of $0.47^{+0.14}_{-0.08}$ nb. We note that the 15%

- AGREEMENT AND UNCERTAINTIES AT 20% CONSIDERED TO BE SATISFACTORY
- RESULTS FROM DIFFERENT PDF SETS DIFFER BY AT LEAST 5%
- NO WAY TO ESTIMATE PDF UNCERTAINTIES

PDFs: THEN AND NOW

PDFs IN 1984

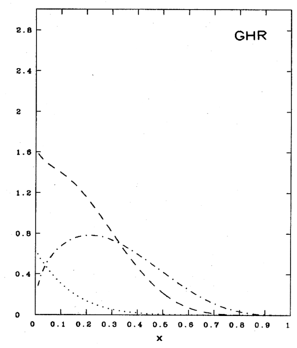


FIG. 25. Parton distributions of Glück, Hoffmann, and Reya (1982), at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and q_v (dotted line).

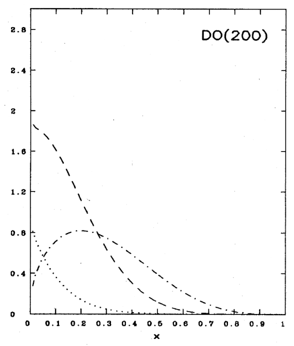


FIG. 27. "Soft-gluon" ($\Lambda=200$ MeV) parton distributions of Duke and Owens (1984) at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

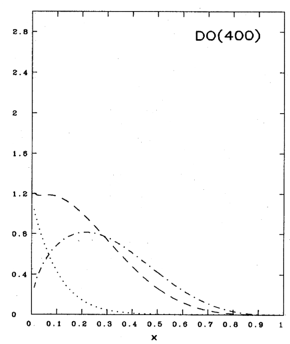
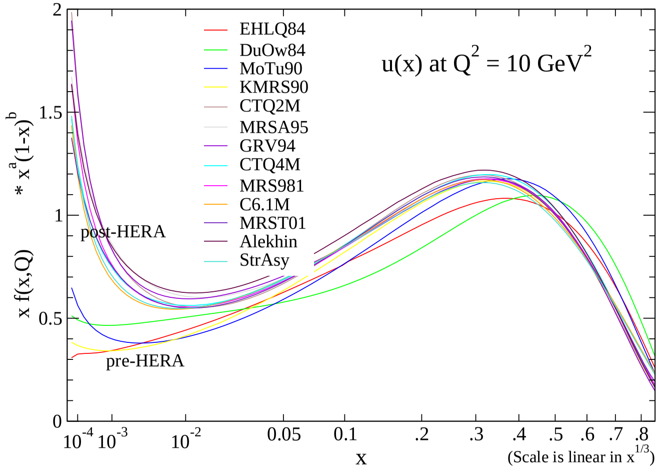


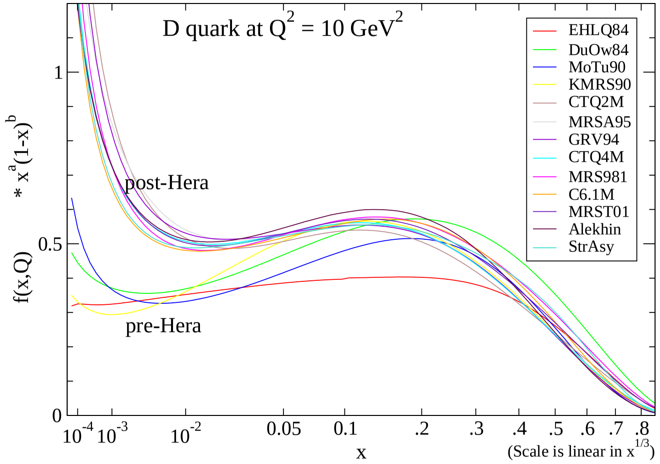
FIG. 26. "Hard-gluon" ($\Lambda=400$ MeV) parton distributions of Duke and Owens (1984) at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

Rev. Mod. Phys., Vol. 56, No. 4, October 1984

GHR VS DUKE-OWENS

20 YEARS OF VALENCE PDFs





W.-K. TUNG, 2004

- HADRON COLLIDERS CIRCA 1985 $\Rightarrow$ QUALITATIVE QCD: DISCOVERY PHYSICS
- DIS AT NMC AND HERA 1995-2005 $\Rightarrow$ QUANTITATIVE QCD: PRECISION PHYSICS
- HADRON COLLIDERS CIRCA 2010 $\Rightarrow$ PRECISION QCD $\leftrightarrow$ NEW PHYSICS

SUMMARY

- THE PROBLEM, AND A SOLUTION

  THE NNPDF APPROACH

- RESULTS

  CONSISTENCY TESTS

- DATA

  IMPACT, CONSISTENCY & INCONSISTENCY

- APPLICATIONS

  SENSITIVITY TO DISCOVERY

- PDF UNCERTAINTIES

  WHERE DO THEY COME FROM?

WHAT'S THE PROBLEM? D. Kosower, 1999

- FOR A SINGLE QUANTITY, WE QUOTE 1 SIGMA ERRORS: VALUE± ERROR
- FOR A PAIR OF NUMBERS, WE QUOTE A 1 SIGMA ELLIPSE
- FOR A FUNCTION, WE NEED AN "ERROR BAR" IN A SPACE OF FUNCTIONS

MUST DETERMINE THE PROBABILITY DENSITY (MEASURE) $\mathcal{P}[F_2]$

IN THE SPACE OF FUNCTIONS $F_2(x, Q^2)$

EXPECTATION VALUE OF $\mathcal{F}\left[F_2(x, Q^2)\right] \Rightarrow$ FUNCTIONAL INTEGRAL

$$\left\langle \mathcal{F}\left[F_2(x, Q^2)\right] \right\rangle = \int \mathcal{D}F_2\, \mathcal{F}\left[F_2(x, Q^2)\right]\, \mathcal{P}[F_2],$$

MUST DETERMINE AN INFINITE–DIMENSIONAL OBJECT

FROM A FINITE SET OF DATA POINTS

THE STANDARD (HESSIAN) SOLUTION

(MSTW, CTEQ):

FUNCTIONAL PARTON FITTING

- CHOOSE A FIXED FUNCTIONAL FORM:
  - MSTW: 20 PARMS.

    $xq(x,Q_0^2) = A(1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta$, (5 indep. fns.); $x[\bar{u}-\bar{d}](x,Q_0^2) = A(1-x)^\eta(1+\gamma x+\delta x^2)x^\delta$;

    $x[s-\bar{s}](x,Q_0^2) = A_-(1-x)^{\eta_s}x^{\delta_-}(1-x/x_0)$; $xg(x,Q_0^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5}+\gamma_g x)x^{\delta_g}+A_{g'}(1-x)$
  - CTEQ/TEA: 22→ 26 PARMS.

    $$x\,f(x,Q_0) = a_0\,x^{a_1}\,(1-x)^{a_2}\,\exp\left(a_3x+a_4x^2+a_5\sqrt{x}+a_6x^{-a_7}\right)$$ (7 indep. functions)

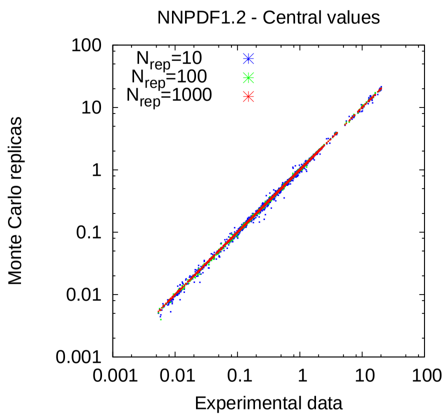
    a_6, a_7 ONLY USED FOR GLUON
  - BASIS FUNCTIONS: $u_v \equiv u-\bar{u}$, $d_v \equiv d-\bar{d}$, $\bar{u}\pm\bar{d}$ (MSTW) OR $\bar{u}$, $\bar{d}$ (CTEQ), $s^\pm \equiv s\pm\bar{s}$ (CTEQ $\bar{s}+s$ ONLY), g.
- EVOLVE TO DESIRED SCALE & COMPUTE PHYSICAL OBSERVABLES
- DETERMINE BEST-FIT VALUES OF PARAMETERS
- DETERMINE ERROR BY PROPAGATION OF ERROR ON PARMS. ('HESSIAN METHOD') PARM. SCANS ALSO POSSIBLE ('LAGR. MULTIPLIER METHOD')

THE NNPDF APPROACH:
THE NEURAL MONTE CARLO

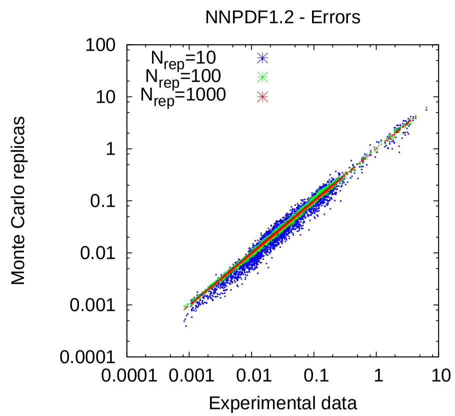
BASIC IDEA: MONTE CARLO SAMPLING
OF THE PROBABILITY MEASURE IN THE (FUNCTION) SPACE OF PDFs

- START FROM MONTE CARLO SAMPLING OF DATA SPACE
- SPACE OF FUNCTIONS HUGE
  5 BINS FOR 10 PTS× 7 FCTNS $\rightarrow 5^{70} \sim 10^{49}$ BINS
- IMPORTANCE SAMPLING: DATA TELL US WHICH BINS ARE POPULATED

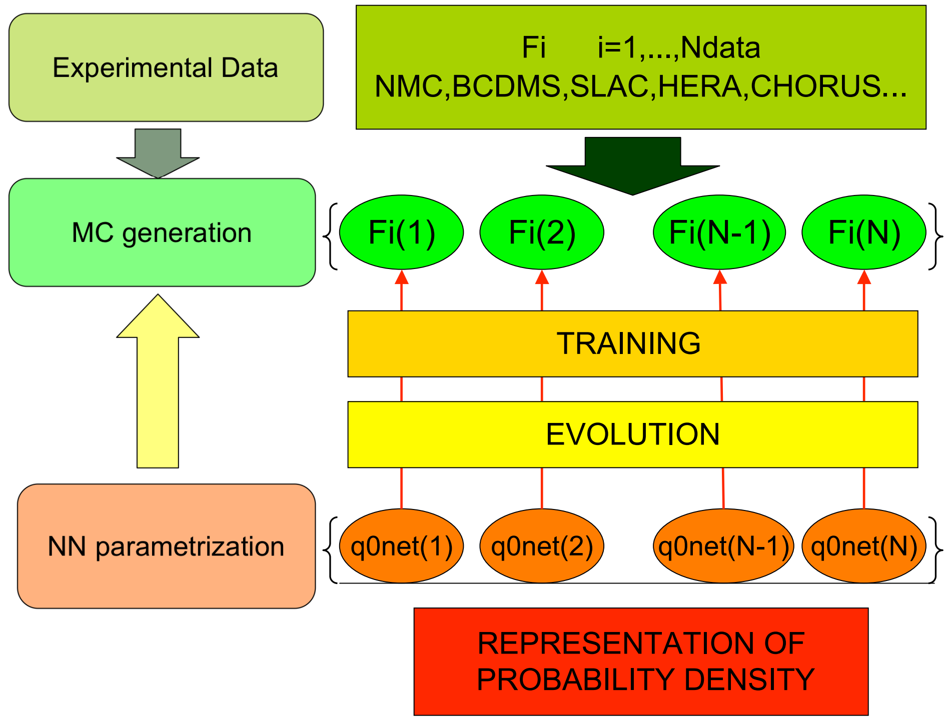
replica averages vs. central values



replica standard dev. vs. uncertainties

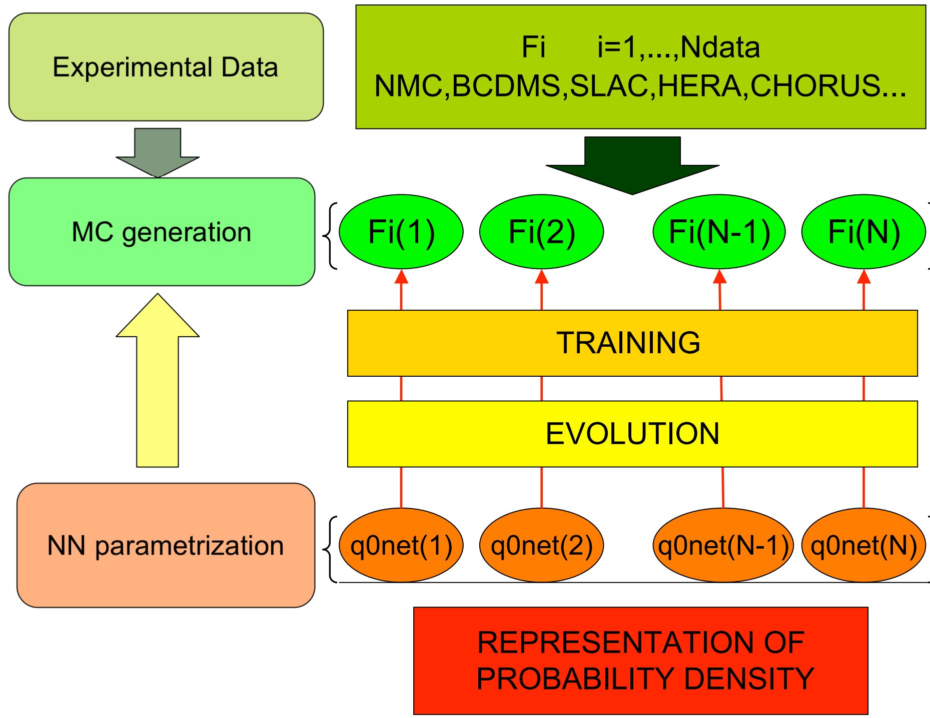


10 REPLICAS ENOUGH FOR CENTRAL VALS, 100 FOR UNCERTAINTIES, 1000 FOR CORRELNS

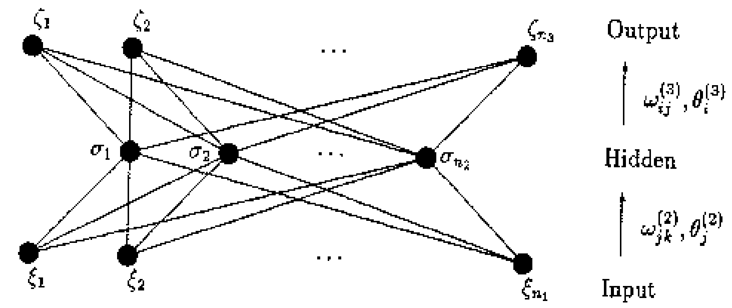


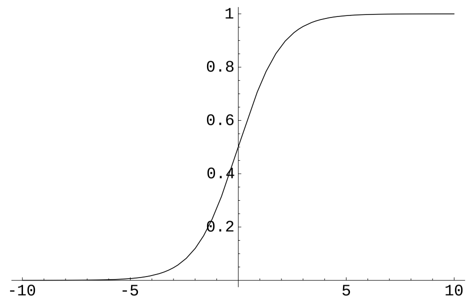
DATA MONTE CARLO ⇒ PDF MONTE CARLO
NEURAL NETWORK PARM+ CROSS-VALIDATION METHOD

- EACH PDF↔ NEURAL NETWORK PARAMETRIZED BY 37 PARAMETERS
- NNPDF1.X, NNPDF2.X: $37 \times 7 = 259$ PARMS (RECALL MSTW, CTEQ → 20 FREE PARAMETERS) "INFINITE" NUMBER OF PARAMETERS⇒ CAN REPRESENT ANY FUNCTION
- COMPLEX SHAPES (LARGE NO.OF PARAMETERS) REQUIRE LONGER FITTING
- FIT STOPS WHEN QUALITY OF FIT TO RANDOMLY SELECTED "VALIDATION" DATA (NOT FITTED) STOPS IMPROVING
- CAN OBTAIN A FIT WITH χ^2 LOWER THAN BEST FIT ("OVERLEARNING")



NEURAL NETWORKS
A NONLINEAR FUNCTIONAL FORM





MULTILAYER FEED-FORWARD NETWORKS

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by weights and thresholds
  $$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right)$$
- Sigmoid activation function
  $g(x) = \frac{1}{1+e^{-\beta x}}$

EXAMPLE: A 1-2-1 NN

$$f(x) = \frac{1}{1+e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}}}}$$

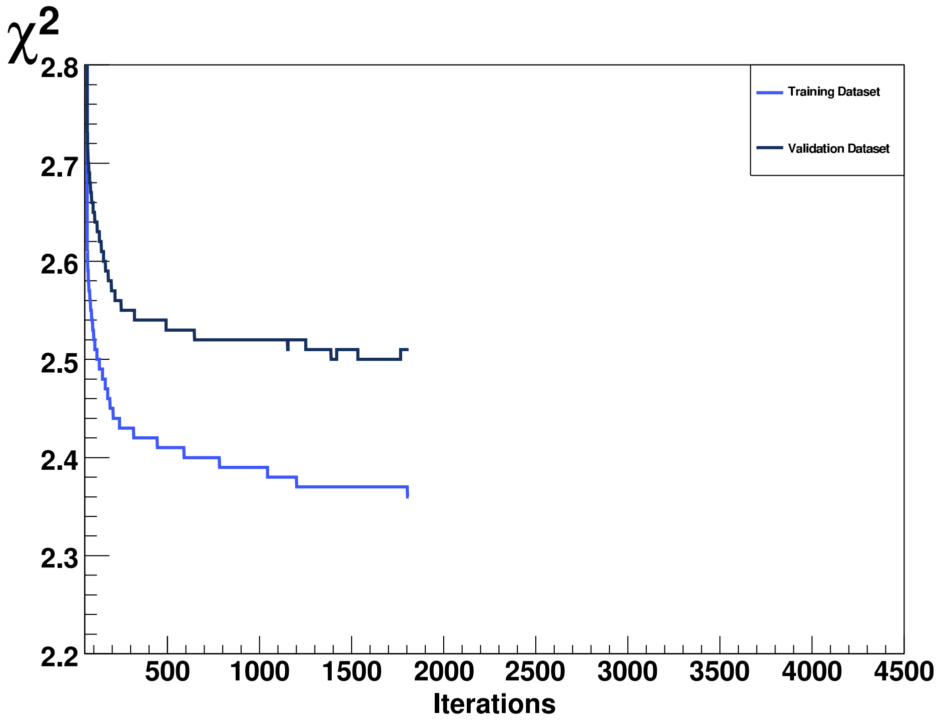
- ANY FUNCTION CAN BE REPRESENTED BY A SUFFICIENTLY BIG NEURAL NETWORK ⇒ CAN CHECK INDEPENDECE OF SIZE
- MINIMZATION PERFORMED BY GENETIC ALGORITHM
- INFORMATION CONTAINED ("COARSENESS") IN NN INCREASES DURING MINIMIZATION

DATA MONTE CARLO ⇒ PDF MONTE CARLO
CROSS-VALIDATION

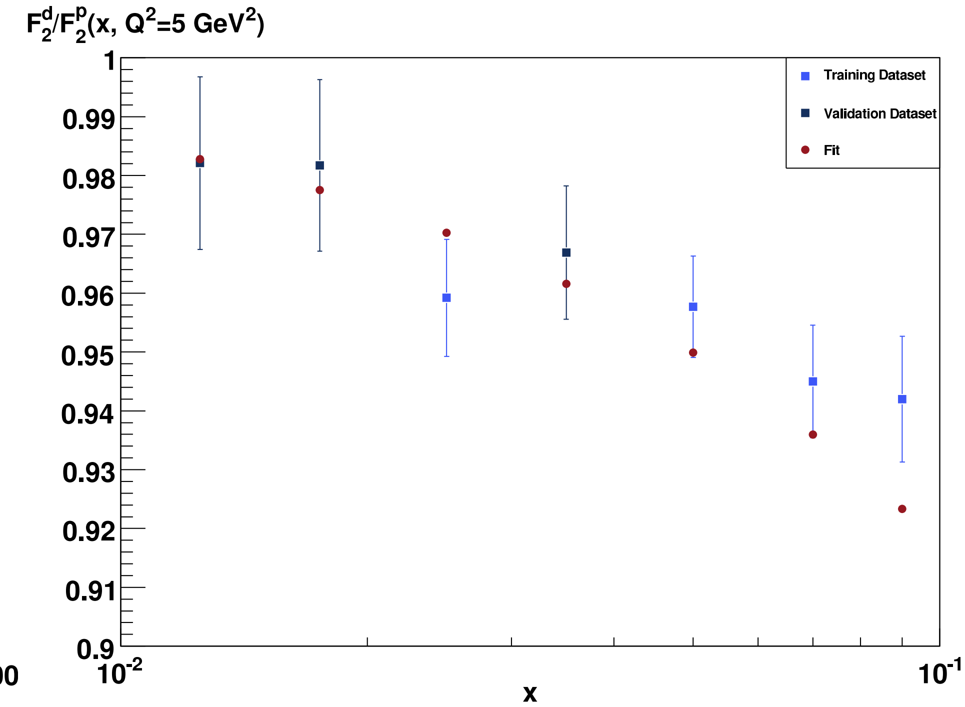
- REPLICAS ARE FITTED TO A DATA SUBSET
- A DIFFERENT SUBSET OF DATA USE FOR EACH REPLICA
- OPTIMAL FIT WHEN FIT TO VALIDATION (CONTROL) DATA STOPS IMPROVING
-

OPTIMAL FITTING

χ^2



FIT TO DATA

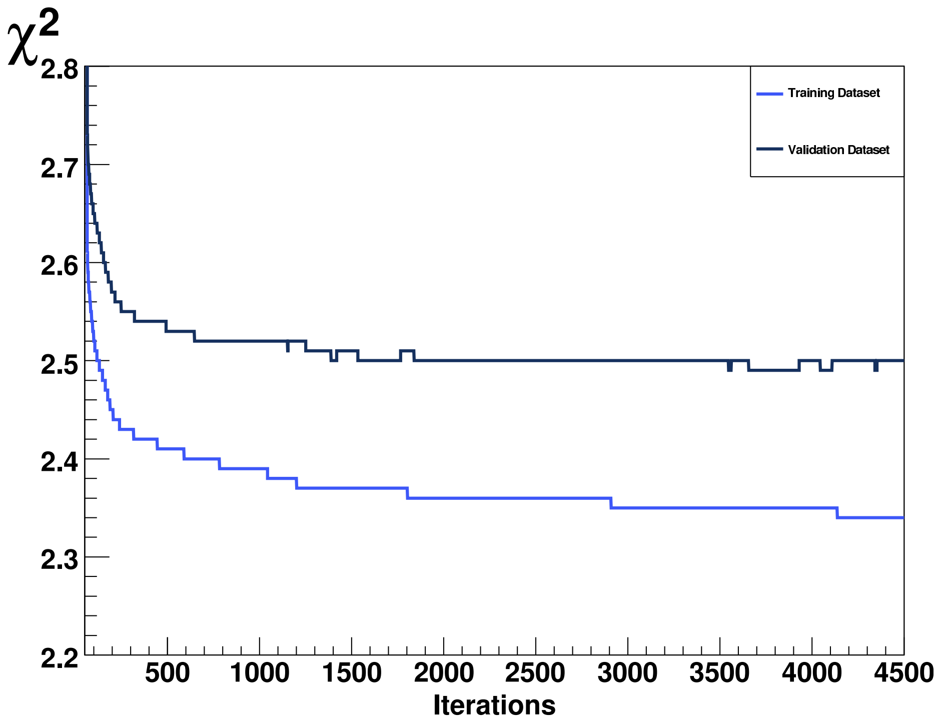


DATA MONTE CARLO ⇒ PDF MONTE CARLO
CROSS-VALIDATION

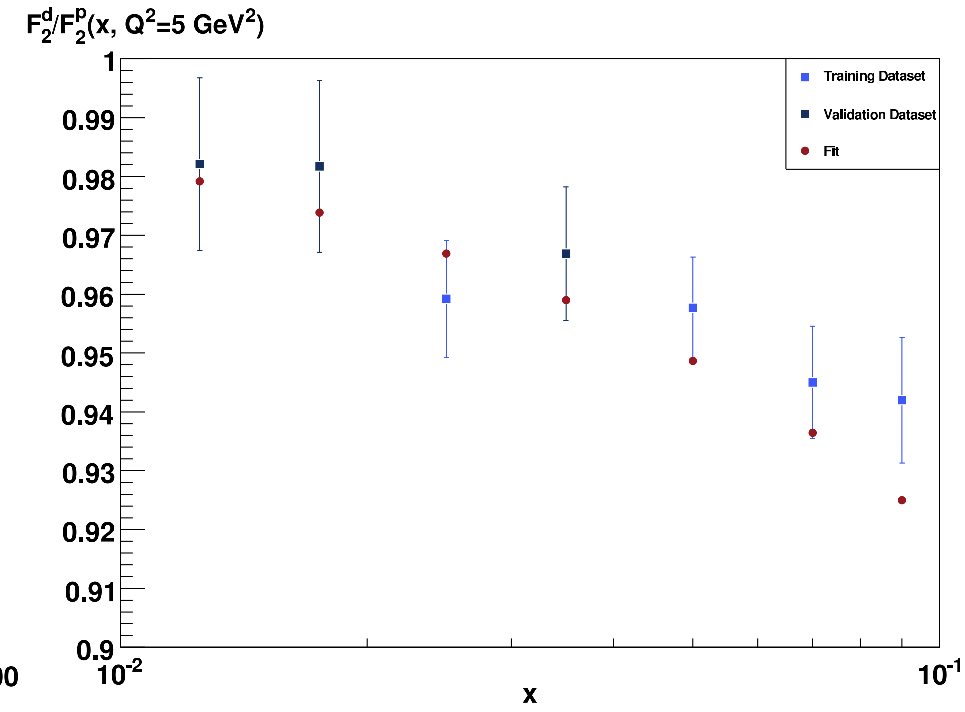
- REPLICAS ARE FITTED TO A DATA SUBSET
- A DIFFERENT SUBSET OF DATA USE FOR EACH REPLICA
- OPTIMAL FIT WHEN FIT TO VALIDATION (CONTROL) DATA STOPS IMPROVING
- THE BEST FIT IS NOT AT THE MINIMUM OF THE χ^2

OVERFITTING

χ^2



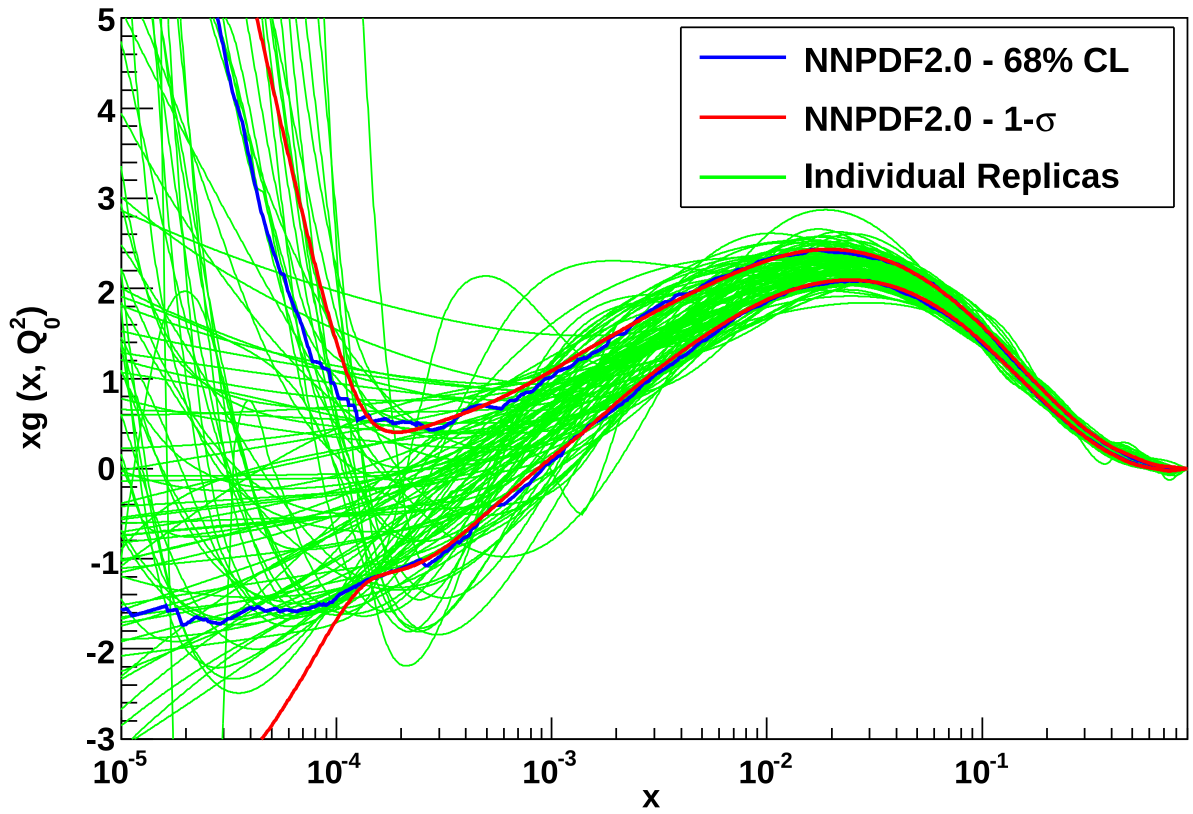
FIT TO DATA



LIKELIHOOD CONTOURS AND UNCERTAINTIES

NNPDF: ONE σ VS. CENTRAL 68% FOR THE MC DISTRIBUTION OF PDFS

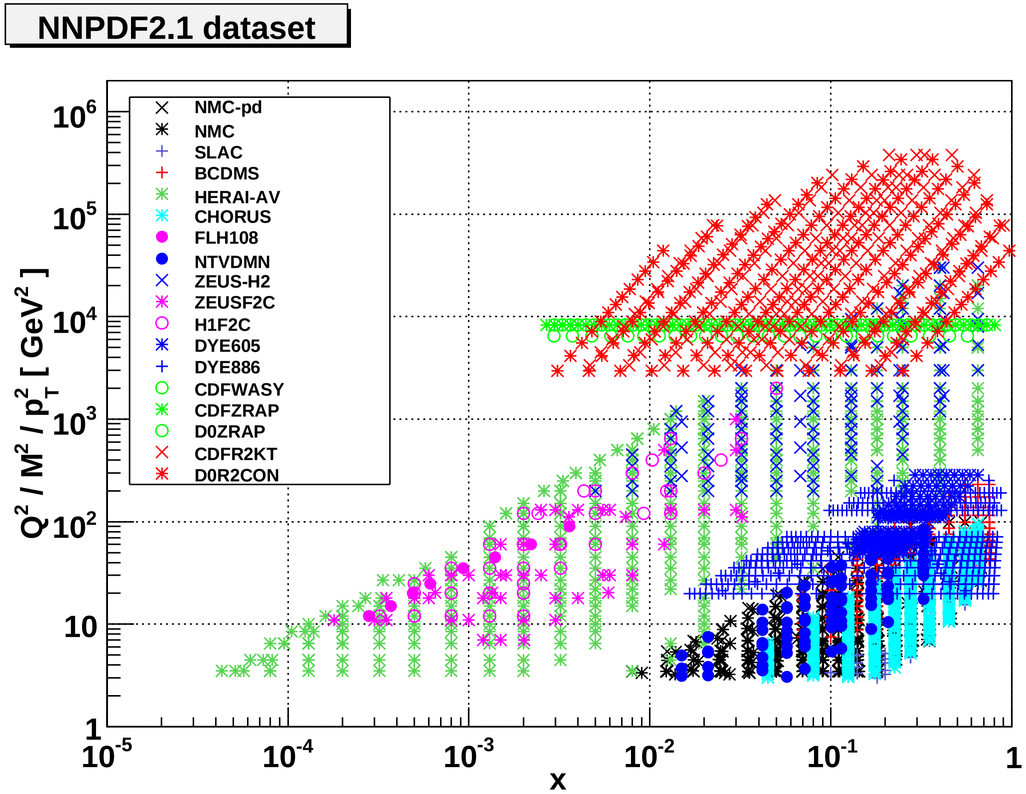
Example: the gluon distribution in the NNPDF2.0 set

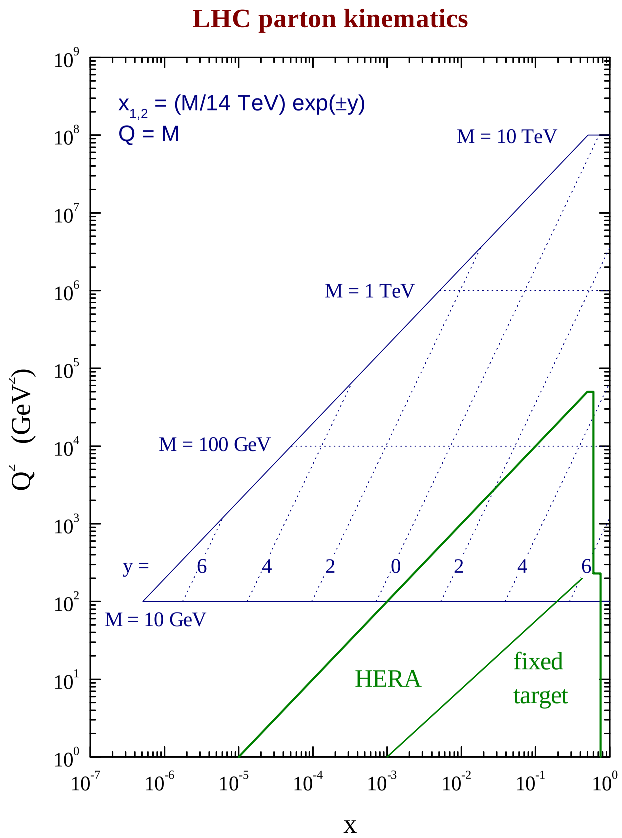


- ENSEMBLE OF REPLICAS $\leftrightarrow$ PROBABILITY DISTRIBUTION OF PDFS
- EXPECTED CENTRAL VALUE $\leftrightarrow$ MEAN; UNCERTAINTY $\leftrightarrow$ STANDARD DEVIATION
- ANY FEATURES OF DISTRIBUTION CAN BE DETERMINED (C.L., CORRELATIONS...)
- DISTRIBUTION NEED NOT BE GAUSSIAN $\rightarrow$ STANDARD DEVIATION $\neq$ 68% C.L. (GLUON $\Leftrightarrow$ STRUCTURE FUNCTION POSITIVITY CONSTRAINTS)

RESULTS

THE NNPDF2.1 PDF DETERMINATION

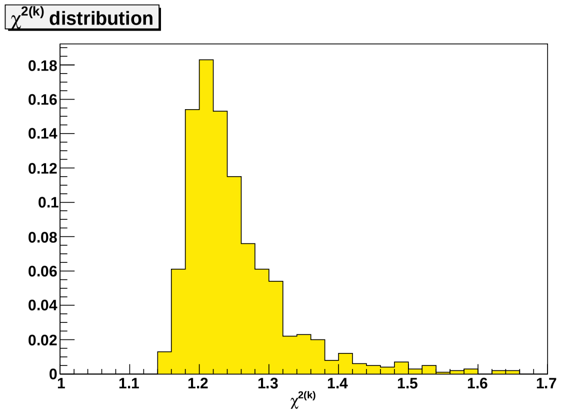




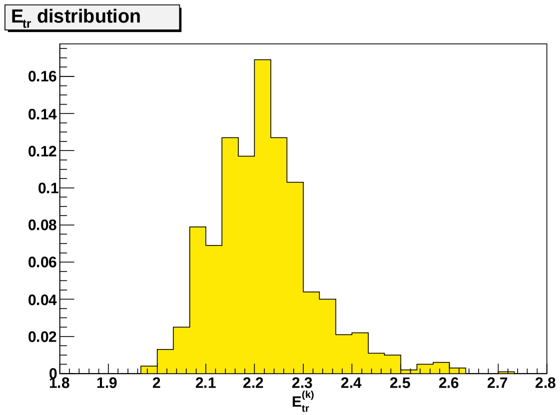
- GLOBAL PDF FIT, INCLUDES
  - DIS: NEUTRAL AND CHARGED CURRENT, CHARGED LEPTON AND NEUTRINO BEAMS, INCLUSIVE AND CHARM-TAGGED
  - DRELL-YAN: FIXED TARGET AND COLLIDER, NEUTRAL γ^* AND Z AND CHARGED CURRENT W PRODUCTION
  - INCLUSIVE JETS
- NLO QCD, HQ MASSES INCLUDED TO $O(\alpha_s)$
- 7 PDFS PARAMETRIZED INDEPENDENTLY, HQ GENERATED DYNAMICALLY
- SETS WITH DIFFERENT VALUES OF α_s, m_c, m_b AVAILABLE

STATISTICAL FEATURES

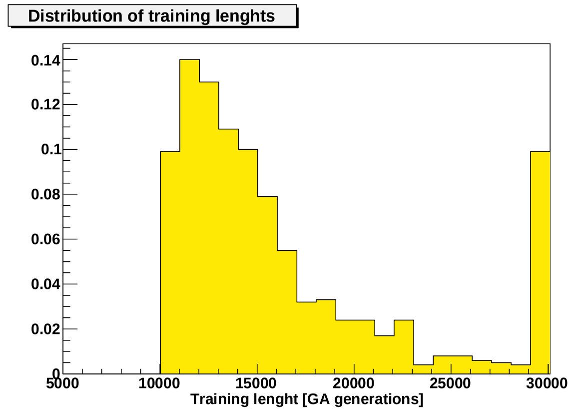
χ^2 TO DATA



χ^2 TO REPLICA



TRAINING LENGTHS



- χ^2 TO REPLICA PEAKED AROUND 2,
  χ^2 TO DATA PEAKED AROUND 1
- χ^2 OF AVERAGE SMALLER THAN AVERAGE OF χ^2
- AVERAGE UNCERTAINTY OF PREDICTION
  SMALLER THAN AVERAGE UNCERTAINTY ON DATA

$\Rightarrow$ FIT "LEARNS" UNDERLYING LAW

χ^2_{tot}	1.16
$\langle E \rangle \pm \sigma_E$	2.24 ± 0.09
$\langle E_{tr} \rangle \pm \sigma_{E_{tr}}$	2.22 ± 0.11
$\left\langle E_{val} \right\rangle \pm \sigma_{E_{val}}$	2.28 ± 0.12
$\langle TL \rangle \pm \sigma_{TL}$	$(1.6 \pm 0.6)\ 10^4$
$\left\langle \chi^{2(k)} \right\rangle \pm \sigma_{\chi^2}$	1.25 ± 0.09
$\left\langle \sigma^{(exp)} \right\rangle_{dat}$ (%)	11.3%
$\left\langle \sigma^{(net)} \right\rangle_{dat}$ (%)	4.4%
$\left\langle \rho^{(exp)} \right\rangle_{dat}$	0.18
$\left\langle \rho^{(net)} \right\rangle_{dat}$	0.56

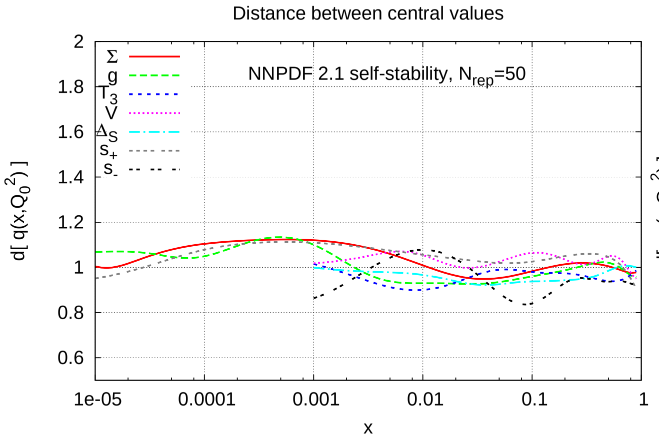
SCALING & STABILITY

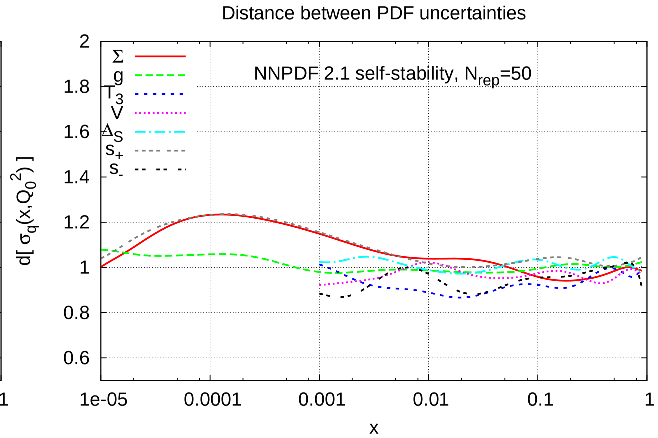
- COMPARE RESULTS BETWEEN DIFFERENT SETS OF REPLICAS ⇒ STATISTICALLY EQUIVALENT

DISTANCE

$$d^2\left(\langle q^{(1)}\rangle,\langle q^{(2)}\rangle\right)=\frac{\left(\langle q^{(1)}\rangle_{(1)}-\langle q^{(2)}\rangle_{(2)}\right)^2}{\sigma^2_{(1)}[\langle q^{(1)}\rangle]+\sigma^2_{(2)}[\langle q^{(2)}\rangle]}\quad \text{WITH } \sigma^2_{(i)}[\langle q^{(i)}\rangle]=\frac{1}{N^{(i)}_{\rm rep}}\sigma^2_{(i)}[q^{(i)}]$$

& similarly for uncertainties





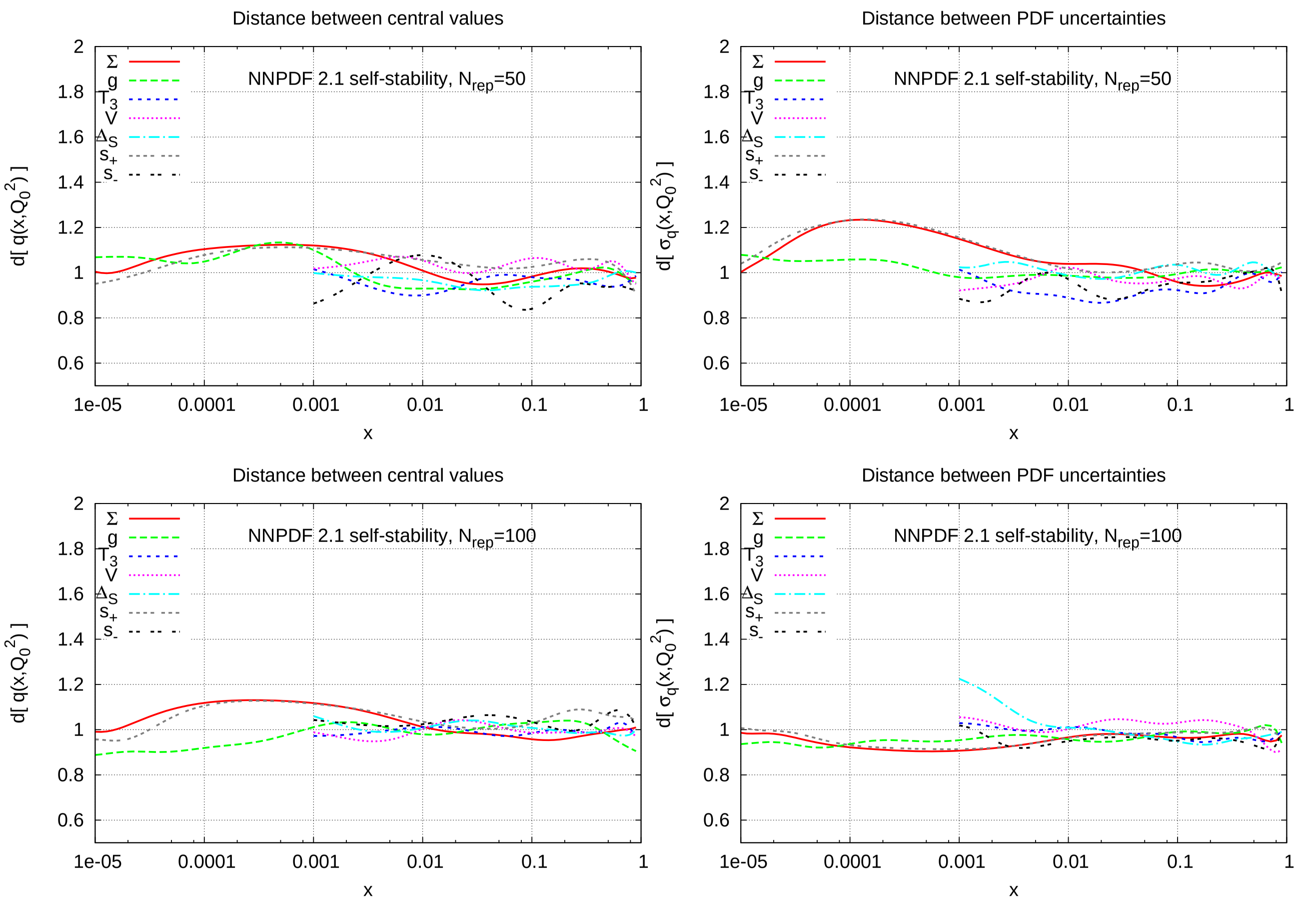
SCALING & STABILITY

- COMPARE RESULTS BETWEEN DIFFERENT SETS OF REPLICAS $\Rightarrow$ STATISTICALLY EQUIVALENT
- REPEAT COMPARISON FOR DIFFERENT $N_{\rm rep}$ $\Rightarrow$ FLUCTUATIONS SCALE WITH $N_{\rm rep}$!

DISTANCE

$$d^2\left(\langle q^{(1)}\rangle,\langle q^{(2)}\rangle\right)=\frac{\left(\langle q^{(1)}\rangle_{(1)}-\langle q^{(2)}\rangle_{(2)}\right)^2}{\sigma^2_{(1)}[\langle q^{(1)}\rangle]+\sigma^2_{(2)}[\langle q^{(2)}\rangle]}\quad \text{WITH } \sigma^2_{(i)}[\langle q^{(i)}\rangle]=\frac{1}{N^{(i)}_{\rm rep}}\sigma^2_{(i)}[q^{(i)}]$$

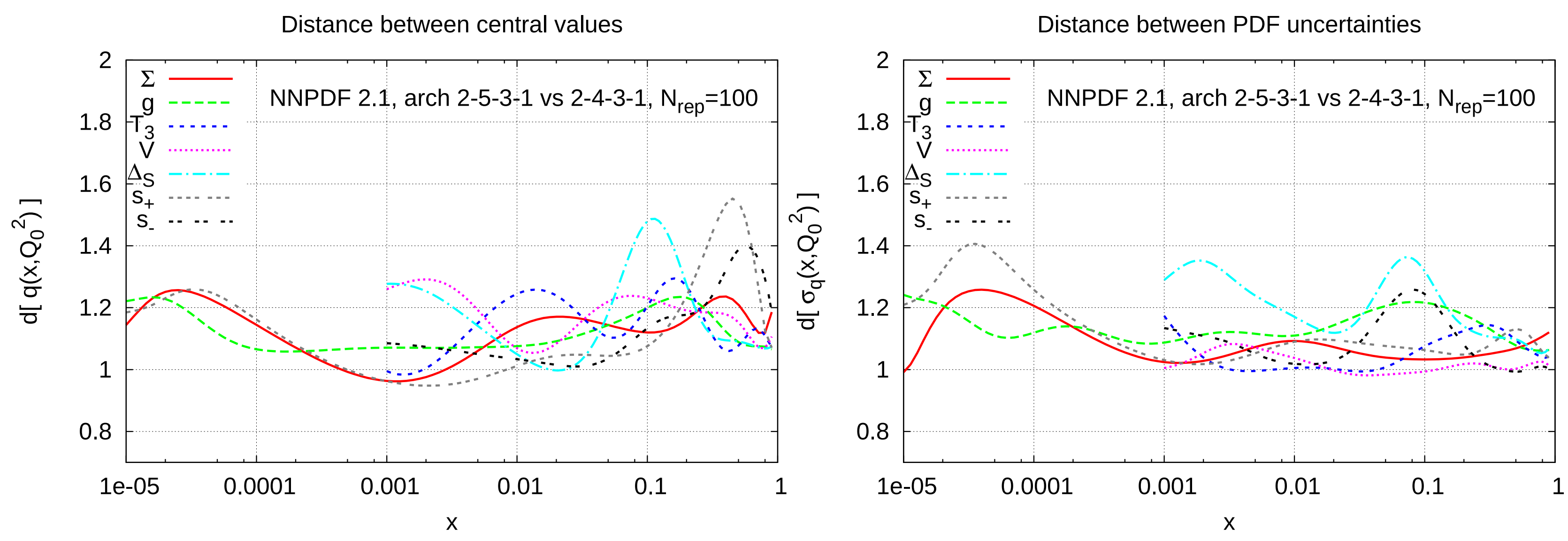
& similarly for uncertainties



PARAMETRIZATION INDEPENDENCE

COMPARE RESULTS OBTAINED WITH DIFFERENT ARCHITECTURE

OF NEURAL NETWORK: 2-4-3-1 VS 2-5-3-1 (31 PARAMETERS VS 37)



STATISTICALLY EQUIVALENT!

DESPITE USING $6 \times 7 = 42$ LESS PARAMETERS

CONSISTENT INFORMATION PROCESSING

NEW DATA $\Rightarrow$ BAYES' THEOREM

$$\langle \mathcal{O} \rangle_{\rm new} = \int \mathcal{O}[f]\, \mathcal{P}_{\rm new}(f)\, Df, = \mathcal{N}_\chi \int \mathcal{O}[f]\, \mathcal{P}(\chi^2|f)\mathcal{P}_{\rm old}(f)\, Df,$$

IN A MONTE CARLO APPROACH...

$$\langle \mathcal{O} \rangle_{\rm new} = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{N}_\chi \mathcal{P}(\chi^2|f_k)\mathcal{O}[f_k] = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} w_k\, \mathcal{O}[f_k], \quad w_k = \mathcal{N}(\chi^2_k)^{n/2-1} e^{-\frac{1}{2}\chi^2_k}$$

$\Rightarrow$ EFFECT OF NEW DATA IS ACCOUNTED FOR BY

REWEIGHTING MONTE CARLO AVERAGES

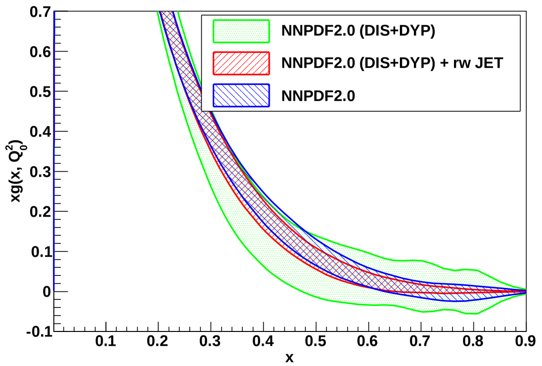
- DETERMINE PDFS INCLUDING SOME DATA BY BAYES' THEOREM (REWEIGHTING)
- DETERMINE PDFS BY ENLARGING THE DATASET TO THE NEW DATA (REFITTING)
- COMPARE RESULTS $\Rightarrow$ STRONG CONSISTENCY CHECK

CONSISTENT INFORMATION PROCESSING II

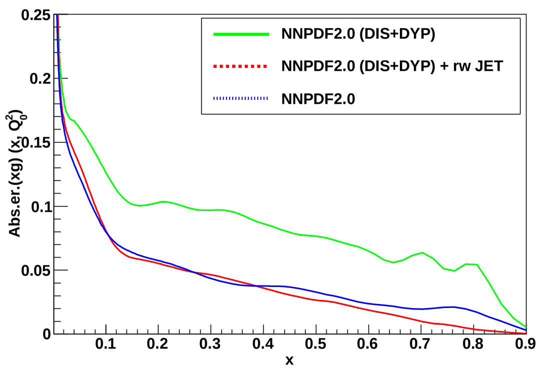
INCLUSION OF JET DATA: REWEIGHTING VS. REFITTING

NNPDF2.0DIS+DY VS. NNPDF2.0FULL

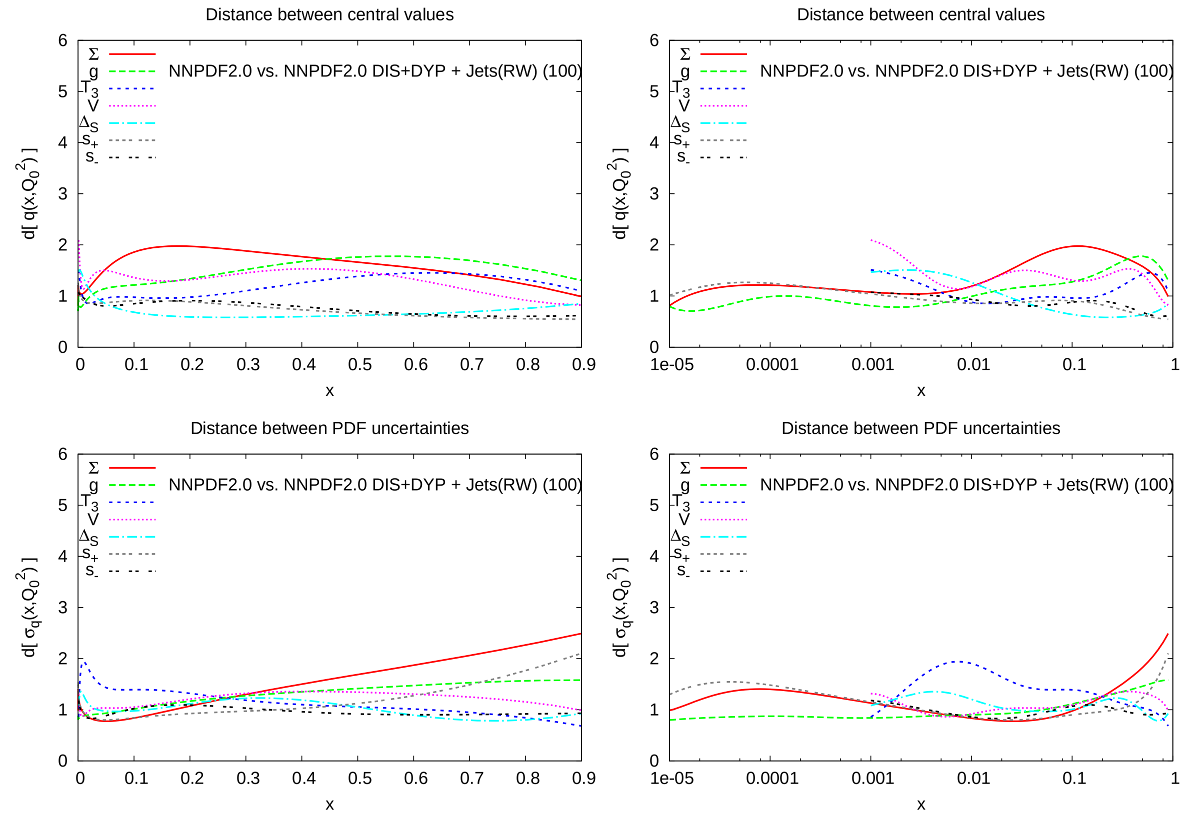
GLUON



GLUON UNCERTAINTY



DISTANCES

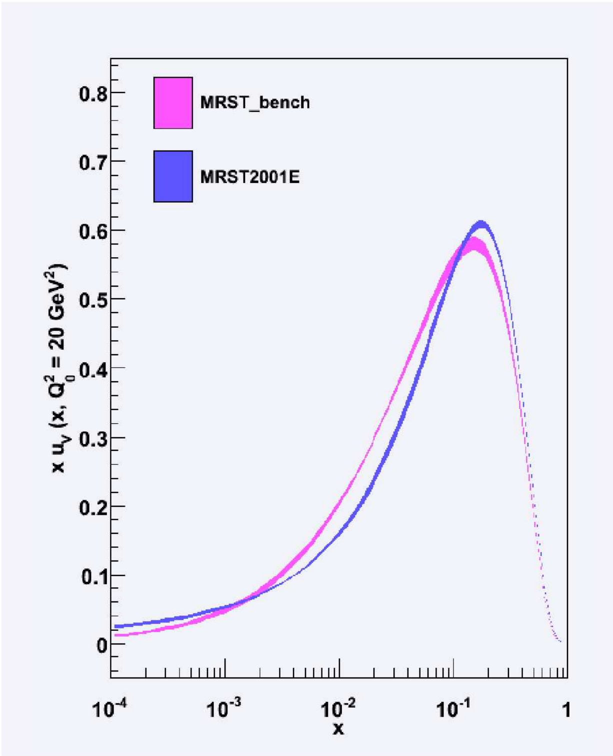


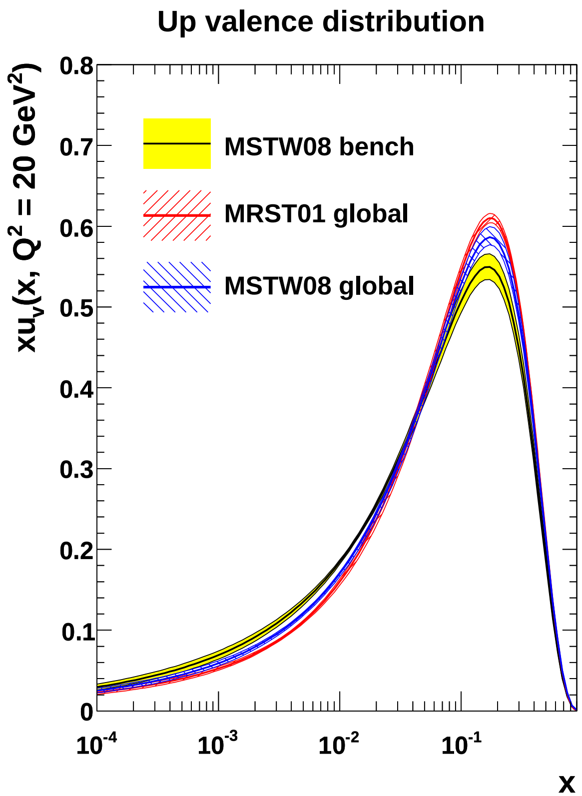
EXCELLENT CONSISTENCY!

DATA

THE PROBLEM OF BENCHMARK FITS... (HERALHC 2005-2008)

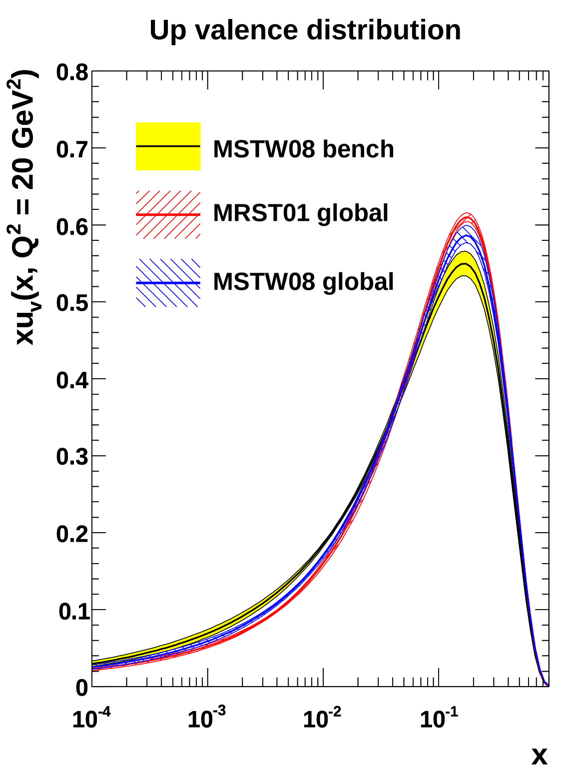
- PERFORM A MRST (MRSTBENCH) FIT TO A CONSISTENT SUBSET OF DATA, USE $\Delta\chi^2 = 1$
  $\Rightarrow$ RESULTS NOT CONSISTENT, UNCERTAINTY DOES NOT GROW AS DATASET DECREASES
- ...BUT MRST WAS DONE WITH TOLERANCE 50: REPEAT WITH DYNAMICAL TOLERANCE (MSTW08BENCH)
- IMPROVEMENT, BUT PROBLEM NOT SOLVED
  $\Rightarrow$ MUST TUNE PARAMETRIZATION AND STATISTICAL TREATMENT TO DATASET



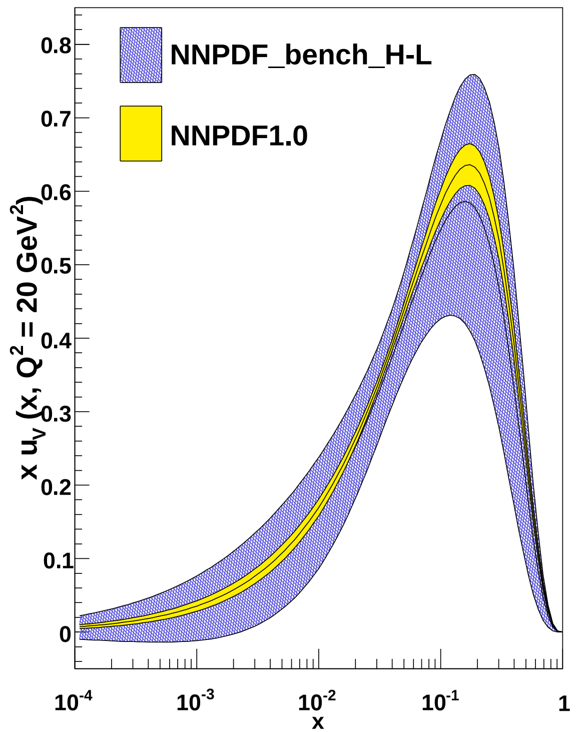


…AND THE NNPDF SOLUTION (HERALHC 2008)

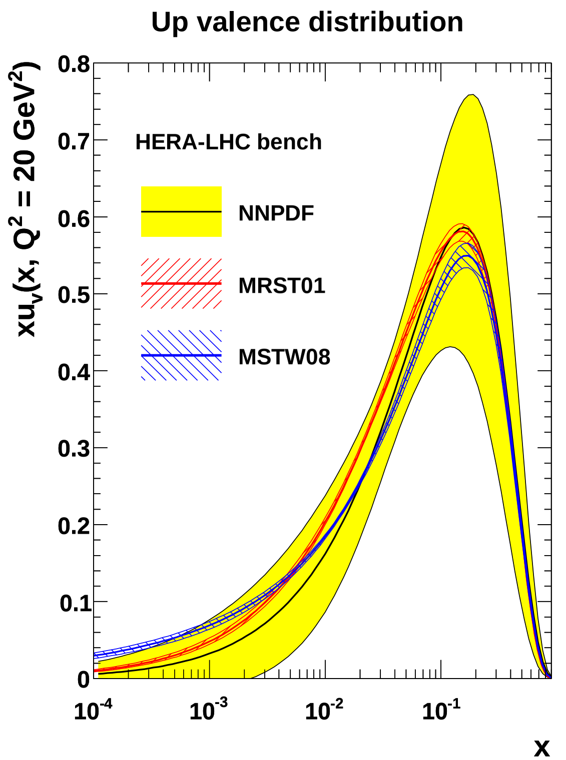
MRST/MSTW: BENCH VS REF



NNPDF: BENCH VS REF



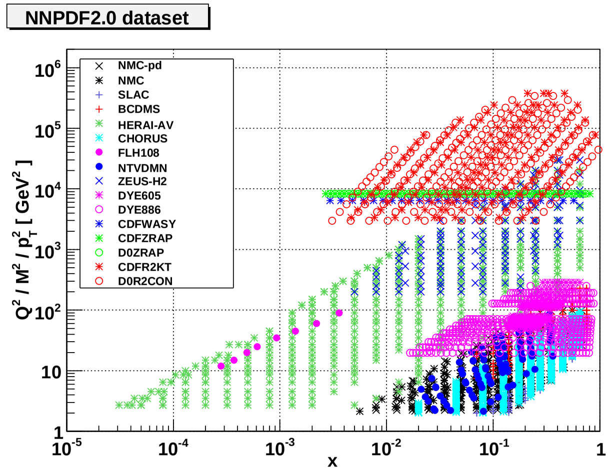
NNPDF BENCH VS MRST/MSTW BENCH



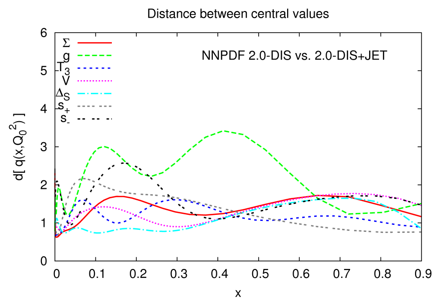
- SINGLE PARAMETRIZATION AND STAT. TREATMENT CAN ACCOMMODATE DIFFERENT DATASETS
- IMPACT OF DATA CAN BE STUDIED INDEPENDENT OF THEORETICAL FRAMEWORK

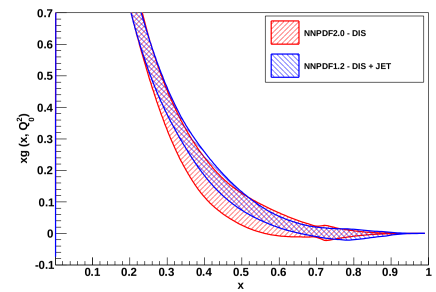
THE IMPACT OF NEW DATA:
ADDING JET DATA TO A DIS FIT

	DIS	DIS+JET	NNPDF2.0
χ^2_{tot}	1.20	1.18	1.21
NMC-pd	0.85	0.86	0.99
NMC	1.69	1.66	1.69
SLAC	1.37	1.31	1.34
BCDMS	1.26	1.27	1.27
HERAI	1.13	1.13	1.14
CHORUS	1.13	1.11	1.18
FLH108	1.51	1.49	1.49
NTVDMN	0.71	0.75	0.67
ZEUS-H2	1.50	1.49	1.51
CDFR2KT	**0.91**	**0.79**	0.80
D0R2CON	**1.00**	**0.93**	0.93
DYE605	7.32	10.35	0.88
DYE866	2.24	2.59	1.28
CDFWASY	13.06	14.13	1.85
CDFZRAP	3.12	3.31	2.02
D0ZRAP	0.65	0.68	0.47



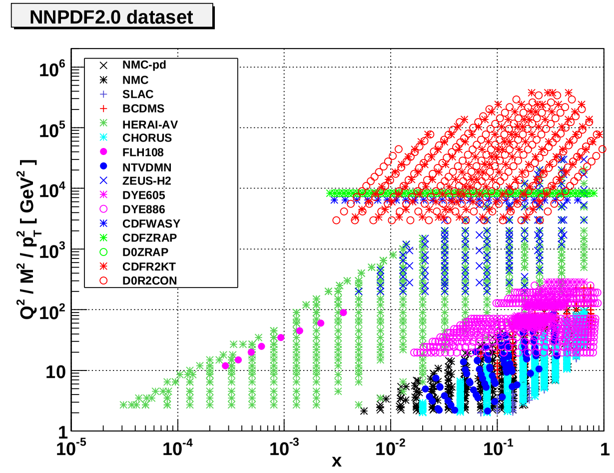
- HIGH E_T JET DATA WELL REPRODUCED EVEN WHEN NOT FITTED ⇒ LARGE x GLUON WELL DETERMINED BY SCALING VIOLATIONS!
- SIGNIFICANT IMPROVEMENT IN LARGE x GLUON ACCURACY
- OTHER PDFS UNCHANGED



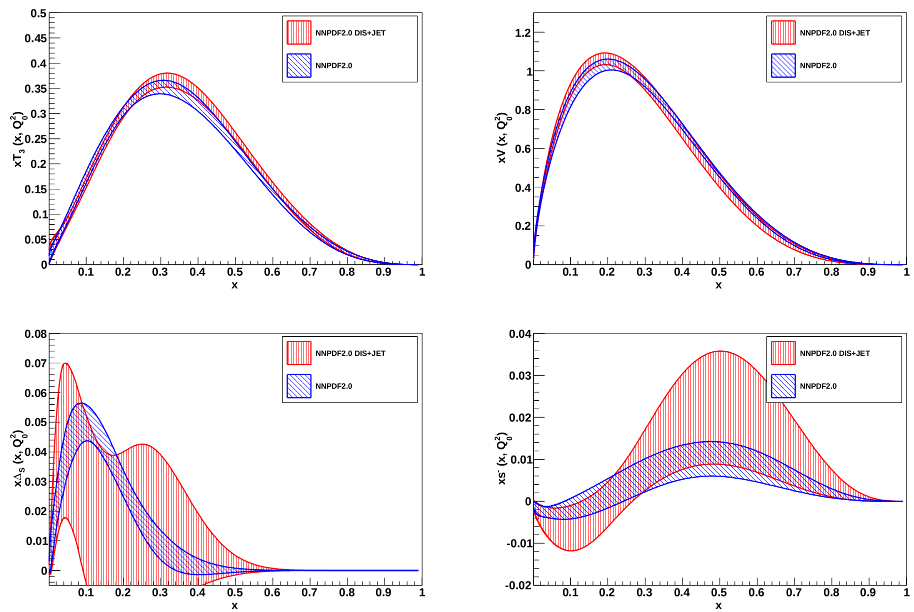


THE IMPACT OF NEW DATA:
ADDING DRELL-YAN (AND W, Z) TO DIS+JETS

	DIS	DIS+JET	NNPDF2.0
χ^2_{tot}	1.20	1.18	1.21
NMC-pd	0.85	0.86	0.99
NMC	1.69	1.66	1.69
SLAC	1.37	1.31	1.34
BCDMS	1.26	1.27	1.27
HERAI	1.13	1.13	1.14
CHORUS	1.13	1.11	1.18
FLH108	1.51	1.49	1.49
NTVDMN	0.71	0.75	0.67
ZEUS-H2	1.50	1.49	1.51
CDFR2KT	0.91	0.79	0.80
D0R2CON	1.00	0.93	0.93
DYE605	7.32	**10.35**	**0.88**
DYE866	2.24	**2.59**	**1.28**
CDFWASY	13.06	**14.13**	**1.85**
CDFZRAP	3.12	**3.31**	**2.02**
D0ZRAP	0.65	**0.68**	**0.47**



- VERY SUBSTANTIAL IMPROVEMENT IN FIT QUALITY WHEN DATA INCLUDED $\Rightarrow$ SOME PDF COMBINATIONS POORLY DETERMINED WITHOUT THESE DATA
- HUGE IMPROVEMENT IN SEA ASYM $\bar{u} - \bar{d}$ & STRANGENESS $s - \bar{s}$
- SIGNIFICANT IMPROVEMENT IN TOTAL VALENCE $(\sum_i (q_i - \bar{q}_i))$ & ISOTRIPLET $(u + \bar{u} - (d + \bar{d}))$

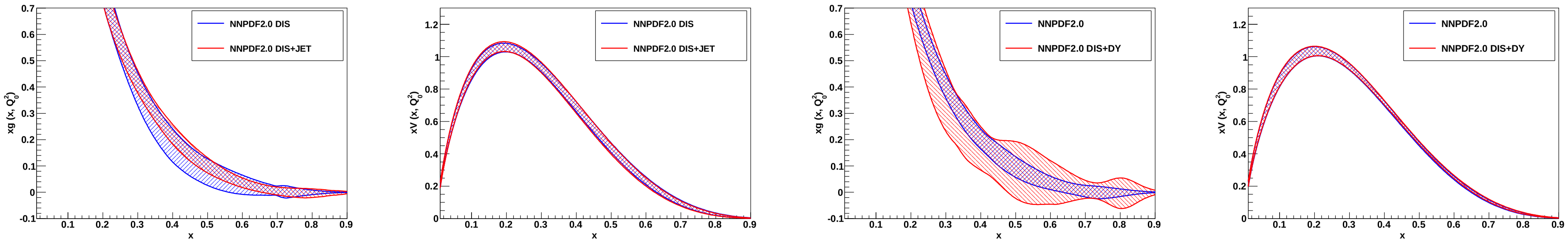


DATA COMPATIBILITY
DIS vs. HADRONIC DATA

A SENSITIVE TEST: IS THE IMPACT OF A DATASET INDEP. OF THE DATA IT IS ADDED TO?

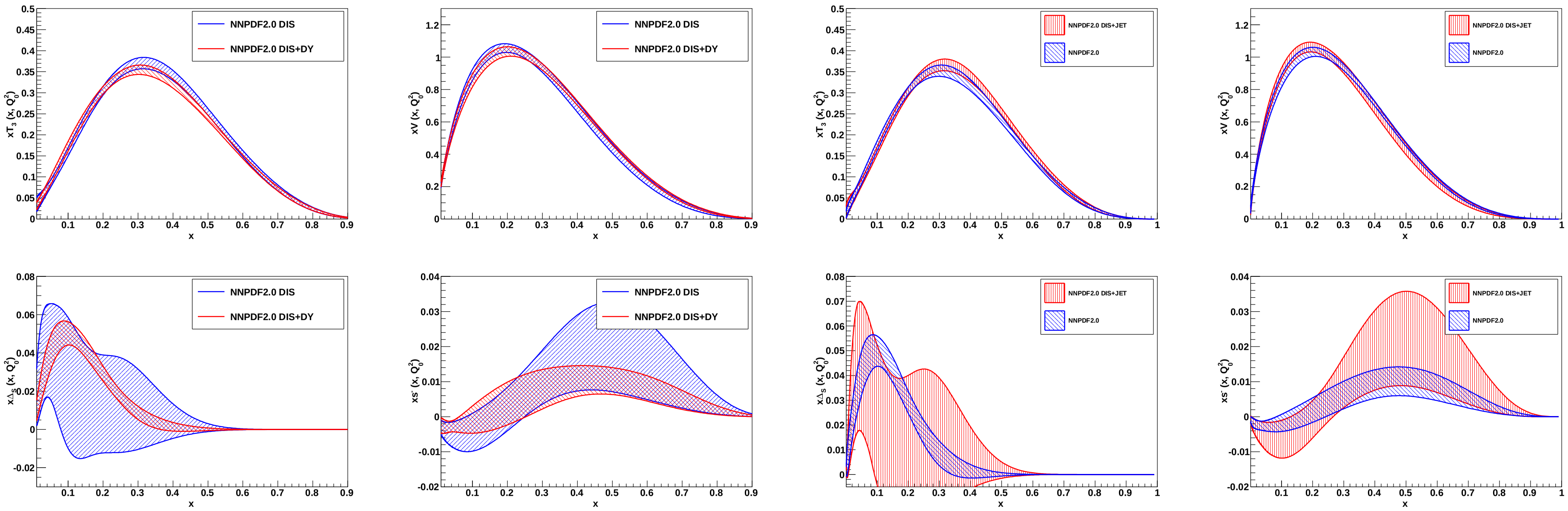
ADDING JET DATA. . .

. . . TO DIS DATA . . . TO DIS+DY DATA



ADDING DRELL-YAN DATA. . .

. . . TO DIS DATA . . . TO DIS+JET DATA

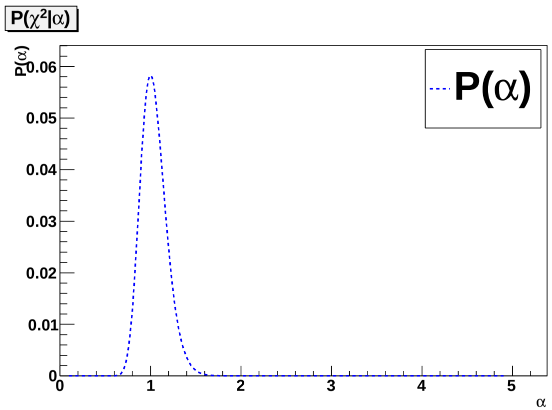


FITS “COMMUTE”

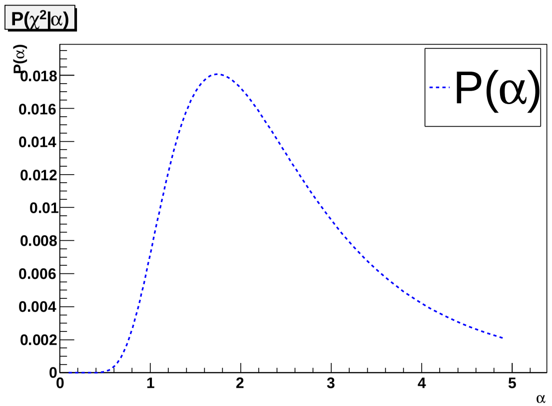
DATA COMPATIBILITY

- INCONSISTENT DATA $\Leftrightarrow$ UNDERESTIMATED UNCERTAINTIES
- RESCALE ALL UNCERTAINTIES IN A GIVEN EXPERIMENT BY SOME FACTOR α: $\chi^2_\alpha = \chi^2/\alpha$ (TOLERANCE)
- DETERMINE PROBABILITY DISTRIBUTION OF α VALUES BY BAYES' THEOREM $\Rightarrow$ REWEIGHTING: $\mathcal{P}(\alpha) = \frac{\mathcal{N}}{\alpha} \sum_{k=1}^{N} w_k w_k(\alpha)$.

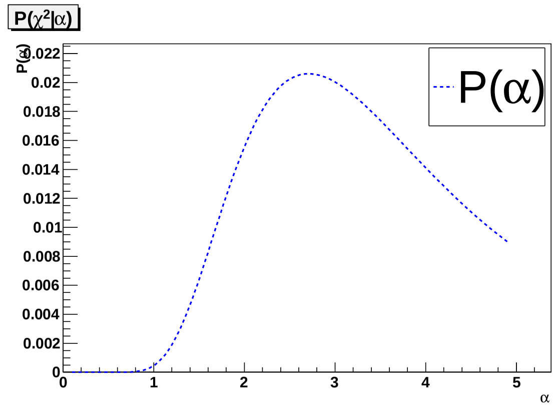
JETS



D0 e INCLUSIVE



D0 e HIGH E_T



- JETS: $\Rightarrow$ CONSISTENT DATA
- $W^\pm$ CHARGE ASYMMETRIES, D0 INCLUSIVE e DATA $\Rightarrow$ UNCERTAINTIES UNDERESTIMATED BY $\sim 30\%$ (PROB. PEAKS AT $\alpha \sim 1.7$)
- $W^\pm$ CHARGE ASYMMETRIES, D0 e DATA WITH $E_T > 35$ GEV $\Rightarrow$ INCONSISTENT DATA

APPLICATIONS

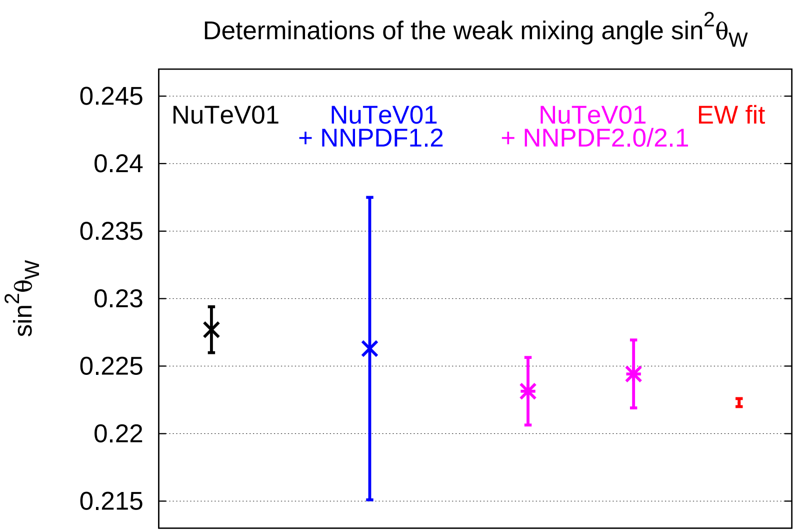
A DISCOVERY THAT WASN'T
THE NUTEV ANOMALY...

$$R_{\rm PW} \equiv \frac{\sigma(\nu\mathcal{N}\to\nu X)-\sigma(\bar\nu\mathcal{N}\to\bar\nu X)}{\sigma(\nu\mathcal{N}\to\ell X)-\sigma(\bar\nu\mathcal{N}\to\bar\ell X)} = \frac{1}{2}-\sin^2\theta_{\rm W}+\left(\frac{(U^- - D^-)+(C^- - S^-)}{\mathcal{Q}^-}\frac{1}{6}\left(3-7\sin^2\theta_{\rm W}\right)\right),$$

- PASCHOS-WOLFENSTEIN RATIO CAN BE MEASURED IN NEUTRINO DIS
- RESULT DEPENDS ON EW MIXING ANGLE, VALENCE ISOSPIN BREAKING (WITH ISOSINGLET TARGET), STRANGENESS VALENCE MOMENTUM ASYMMETRY
- STRANGENESS VALENCE MOMENTUM ASSUMED BY NUTEV TO VANISH ⇒ THREE σ DISCREPANCY WITH GLOBAL FIT

...IS GONE

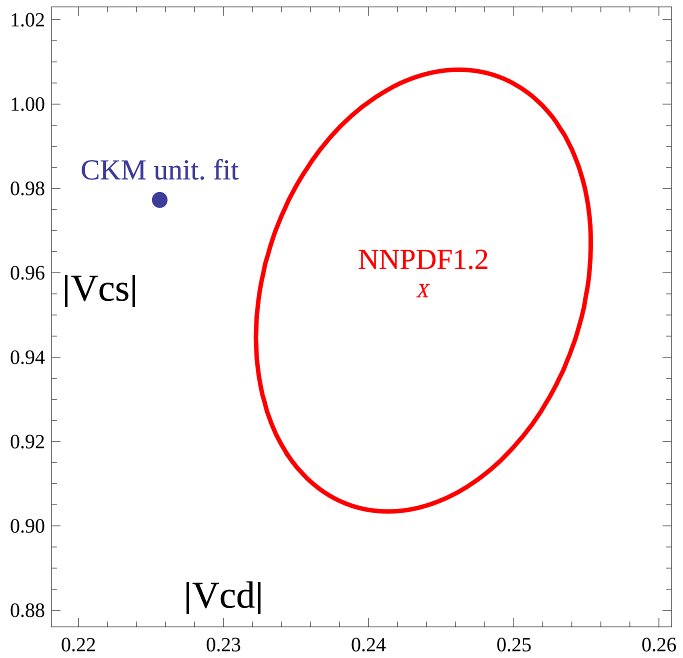
- NNPDF: s, $\bar s$ LIKE ANY OTHER PDF → 37 PARMS. EACH (CTEQ6.6: $s=\bar s$, TWO PARMS; MSTW08, S4S4, $\bar s$ TWO PARMS EACH)
- IF STRANGENESS UNCERTAINTY KEPT INTO ACCOUNT (DIS ONLY FIT: NNPDF1.2) ⇒ EFFECT LOSES STAT. SIGNIFICANCE
- IF HADRONIC DATA INCLUDED (NNPDF2.0 GLOBAL FIT) ⇒ STRANGENESS ASYMMETRY DETERMINED QUITE ACCURATELY → CORRECTED RESULT IN IMPRESSIVE AGREEMENT WITH SM GLOBAL FIT



(NNPDF2.1, 2010)

CKM MATRIX ELEMENTS

CAN BE DETERMINED WITH SURPRIZING ACCURACY



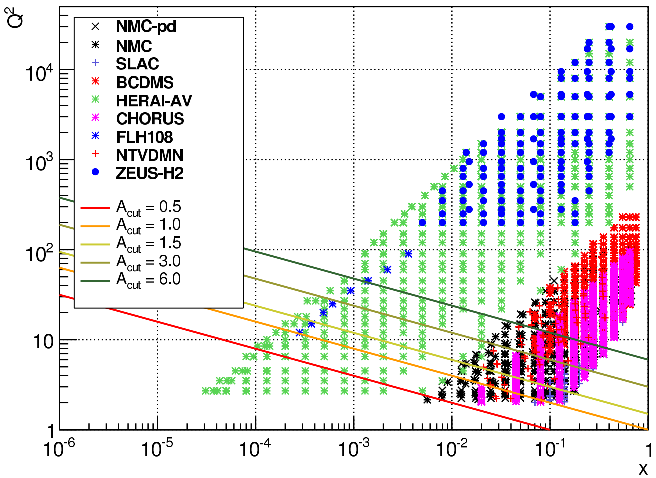
Analysis	Determination	$\vert V_{cs}\vert$
NNPDF1.2	Direct from global PDF analysis	0.96 ± 0.07^{tot}
CDHS	LO from $\nu N \to \mu^+\mu^- X$	≥ 0.59 (90% C.L.)
CCFR	NLO from $\nu N \to \mu^+\mu^- X$	≥ 0.74 (90% C.L.)
PDG08	Average from D decays	1.04 ± 0.06
Hocker	Average from $\nu N \to \mu^+\mu^- X$	1.04 ± 0.16
DELPHI	Direct from $W^+ \to c\bar{s}$ decays	$0.94^{+0.32}_{-0.26} \pm 0.13$
PDG08	CKM unitarity fit	0.97334 ± 0.00023

Analysis	Determination	$\vert V_{cd}\vert$
NNPDF1.2	Direct from global PDF analysis	0.244 ± 0.019^{tot}
CDHS	LO from $\nu N \to \mu^+\mu^- X$	0.24 ± 0.03
CCFR	NLO from $\nu N \to \mu^+\mu^- X$	$0.232^{+0.017}_{-0.019}$
PDG08	Average from $\nu N \to \mu^+\mu^- X$	0.230 ± 0.011
PDG08	Average from $D \to K/\pi l \nu$ decays	0.218 ± 0.023
PDG08	CKM unitarity fit	0.2256 ± 0.0010

- FIRST DETERMINATION OF CKM MATRIX ELEMENTS FROM DIS
- NNPDF1.2: ONLY DIS DATA
- MORE ACCURATE (V_{cs}) OR COMPETITIVE (V_{cd}) THAN OTHER DIRECT DETERMINATIONS

BEYOND DGLAP?

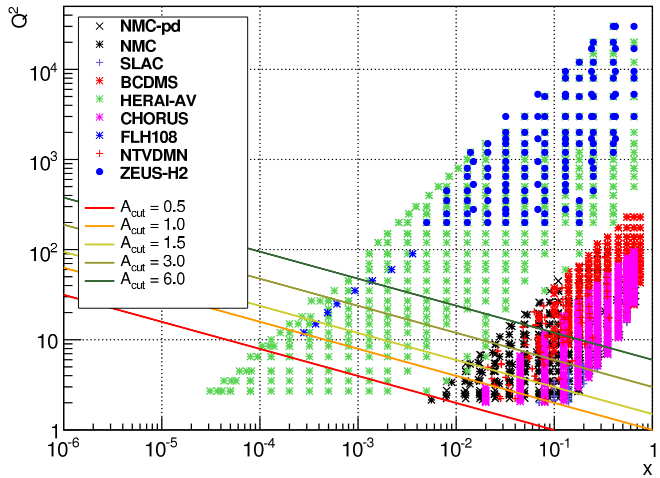
DISCOVERING A NEW QCD EFFECT IN HERA DATA



IDEA: (Gélis, 2008, ⇒ Caola, s.f. ,Rojo 2010)

- CUT OUT DATA IN THE "DANGEROUS" (SMALL x) REGION
- DETERMINE PDFS IN THE "SAFE" (LARGE x AND Q^2) REGION
- EVOLVE BACKWARDS AND COMPARE TO DATA

BEYOND DGLAP?
DISCOVERING A NEW QCD EFFECT IN HERA DATA

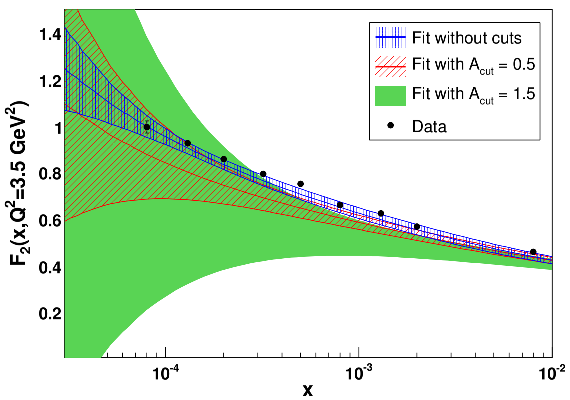


IDEA: (Gélis, 2008, $\Rightarrow$ Caola, s.f. ,Rojo 2010)

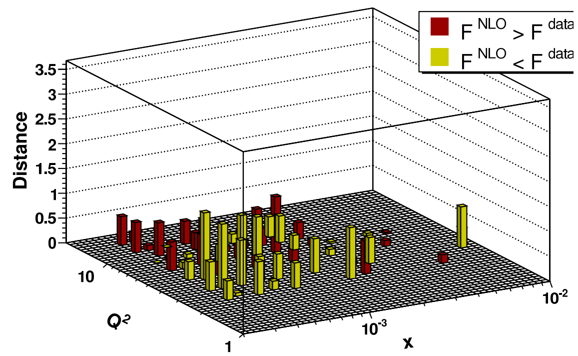
- CUT OUT DATA IN THE "DANGEROUS" (SMALL x) REGION
- DETERMINE PDFs IN THE "SAFE" (LARGE x AND Q^2) REGION
- EVOLVE BACKWARDS AND COMPARE TO DATA

OLD HERA DATA

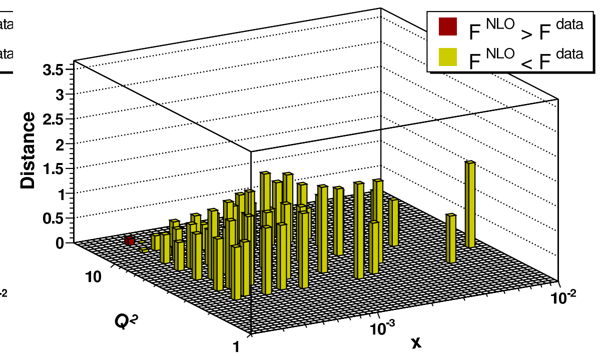
BACKWARD EV. VS DATA



DAT/TH DIST: NO CUT



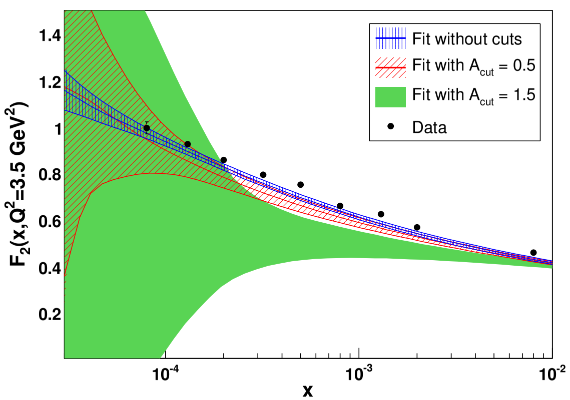
DAT/TH DIST: CUT



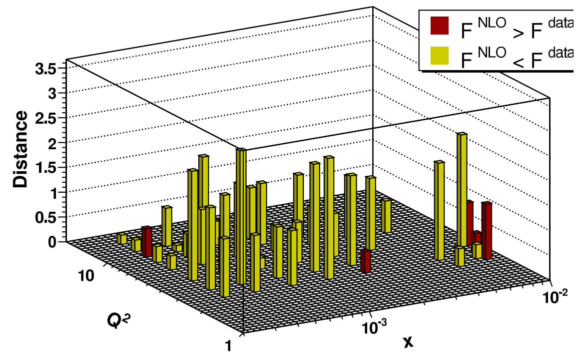
- BACKWARD EVOLVED FIT LIES SYSTEMATICALLY BELOW DATA
- DATA AT LOW x AND Q^2 SHOW LESS EVOLUTION THAN PREDICTED BY NLO DGLAP
- IF LOW x AND Q^2 DATA INCLUDED, THE FIT COMPENSATES READJUSTING PDFs

NEW COMBINED HERA DATA

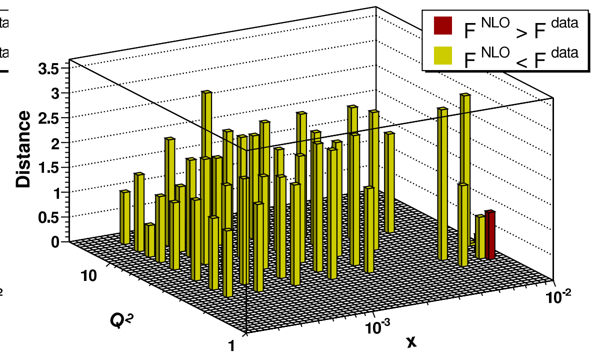
BACKWARD EV. VS DATA



DAT/TH DIST: NO CUT

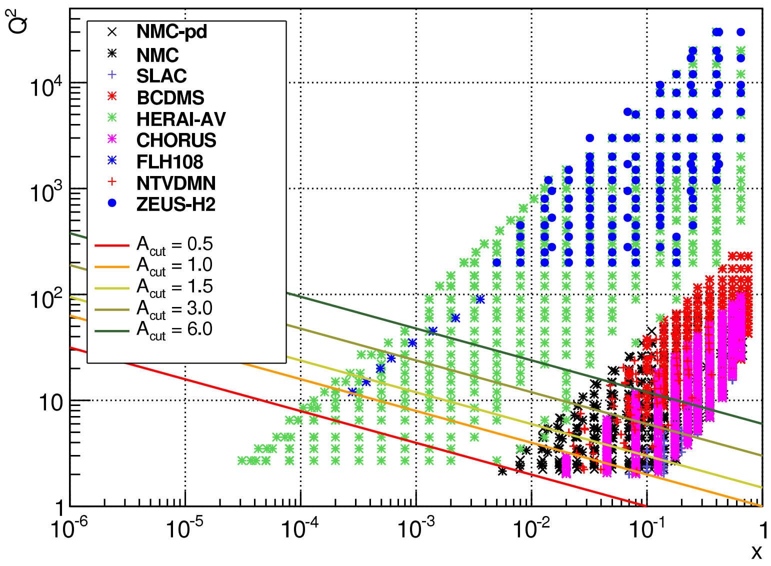


DAT/TH DIST: CUT

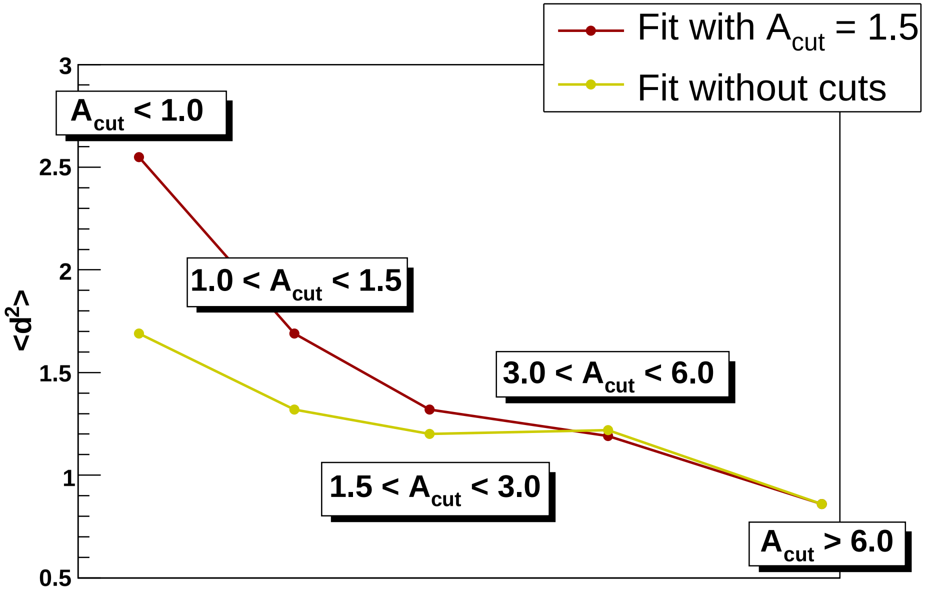


- DATA AT LOW x AND Q^2 SHOW LESS EVOLUTION THAN PREDICTED BY NLO DGLAP
- BACKWARD EVOLVED FIT LIES SYSTEMATICALLY BELOW DATA
- WITH MORE PRECISE DATA, THE FIT NO LONGER MANAGES TO COMPENSATE BY READJUSTING THE PDFs: EVEN FULL FIT LIES BELOW DATA

DETERIORATION IN FIT QUALITY:

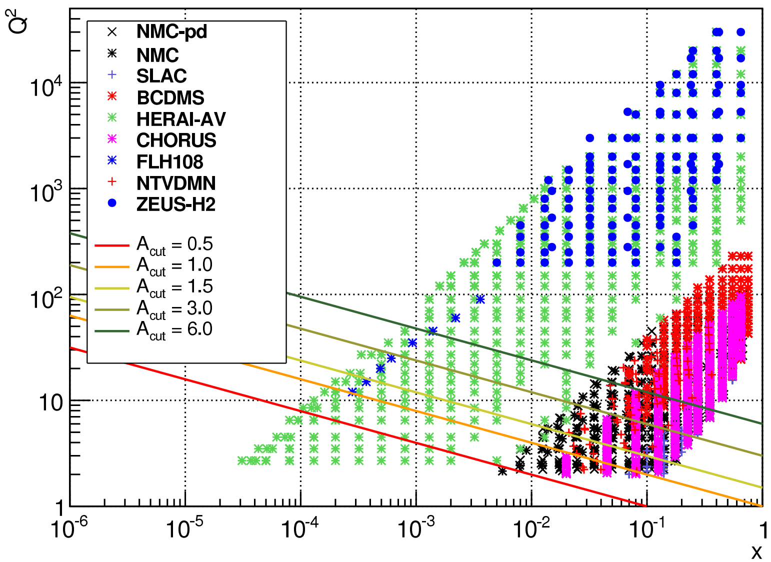


$\langle d^2 \rangle$ VS x SLICES

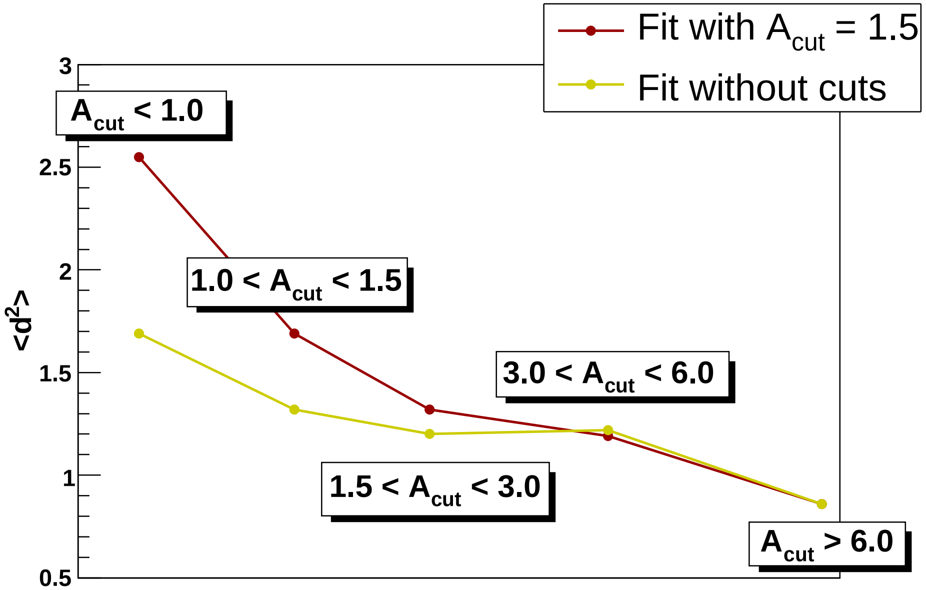


- QUALITY OF UNCUT FIT DETERIORATES IN LOW x REGIONS
- QUALITY OF CUT FIT INCREASINGLY POOR AS x DECREASES
- DISTANCE RISES DESPITE HUGE INCREASE IN UNCERTAINTY

DETERIORATION IN FIT QUALITY:

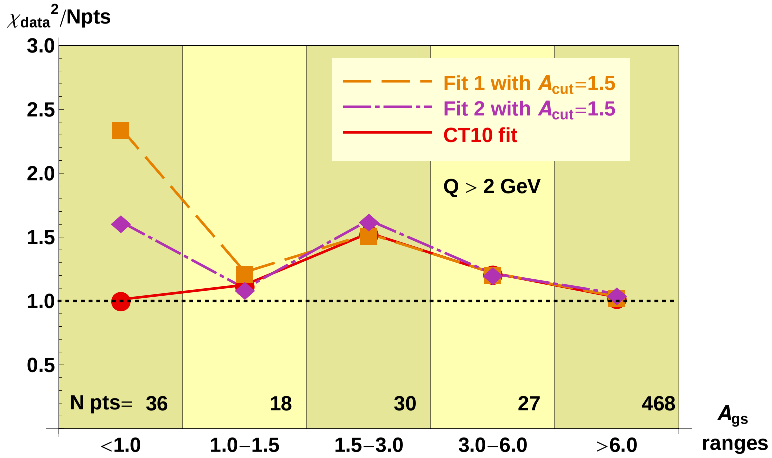


$\langle d^2 \rangle$ VS x SLICES



- QUALITY OF UNCUT FIT DETERIORATES IN LOW x REGIONS
- QUALITY OF CUT FIT INCREASINGLY POOR AS x DECREASES
- DISTANCE RISES DESPITE HUGE INCREASE IN UNCERTAINTY
- IN HESSIAN FIT (CTEQ) RESULTS DEPEND ON PARAMETRIZATION $\Rightarrow$ EVIDENCE INCONCLUSIVE

χ^2 VS x SLICES FOR CT10



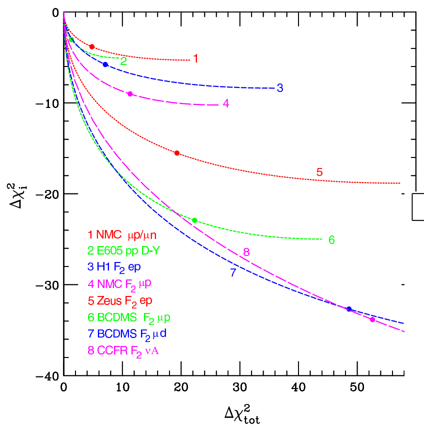
PDF UNCERTAINTIES

THE TOLERANCE PROBLEM

MSTW/CTEQ: ONE σ IS DEFINED UP TO A "TOLERANCE"

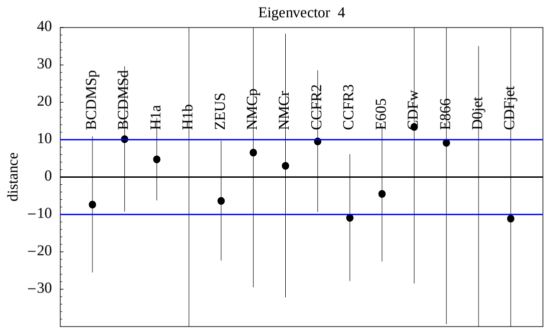
- STANDARD $\Delta\chi^2 = 1$ BANDS TOO NARROW $\Rightarrow$ LARGE DISCREPANCIES FOR INDIVIDUAL EXPERIMENTS
- TOLERANCE $\Rightarrow$ ENVELOPE OF UNCERTAINTIES OF EXPERIMENTS
- DYNAMICAL $\Rightarrow$ SEPARATELY DETERMINED FOR EACH HESSIAN EIGENVECTOR

MINIMUM χ_i^2 VS GLOBAL χ^2

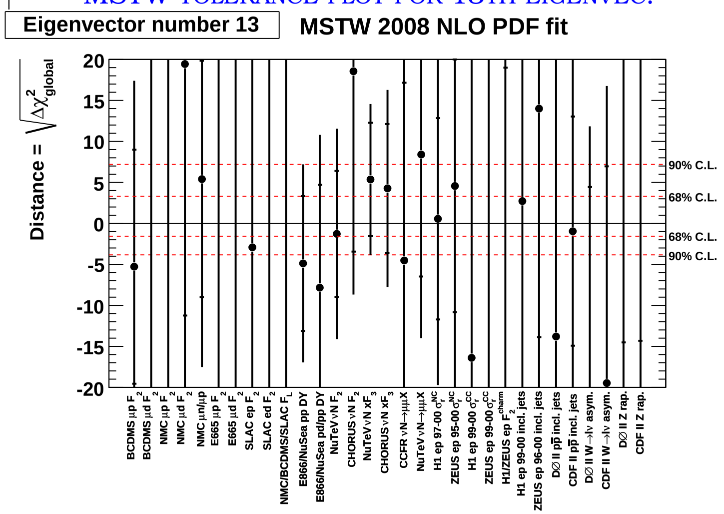


Collins, Pumplin 2001

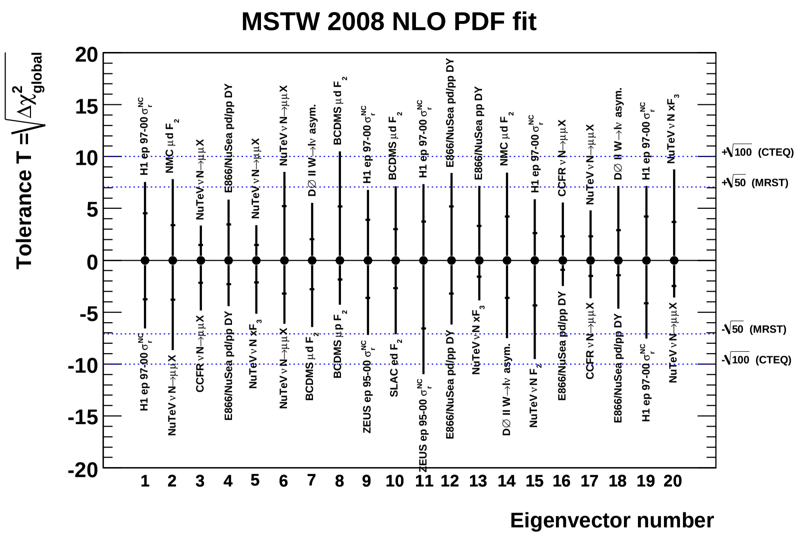
CTEQ TOLERANCE PLOT FOR 4TH EIGENVEC.



MSTW TOLERANCE PLOT FOR 13TH EIGENVEC.



GLOBAL MSTW TOLERANCE

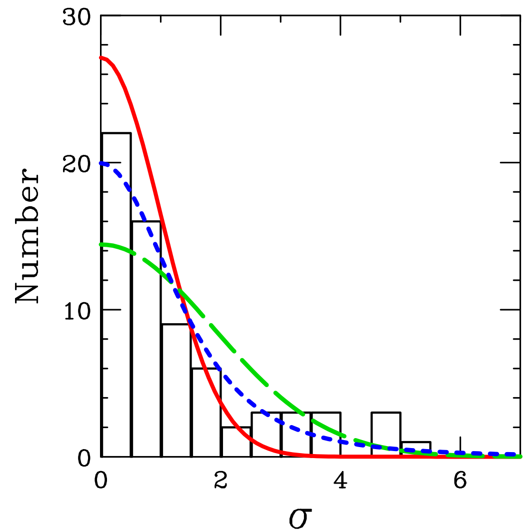


WHERE IS THE UNCERTAINTY COMING FROM?

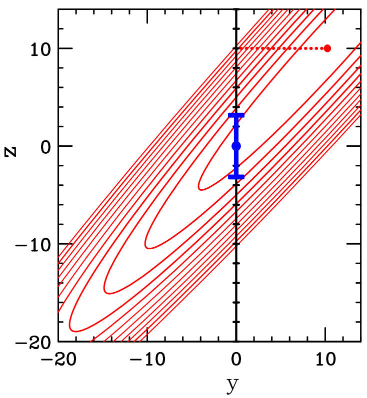
WHY DOES ONE NEED LARGE TOLERANCES?

DATA INCOMPATIBILITY (Pumplin, 2009)

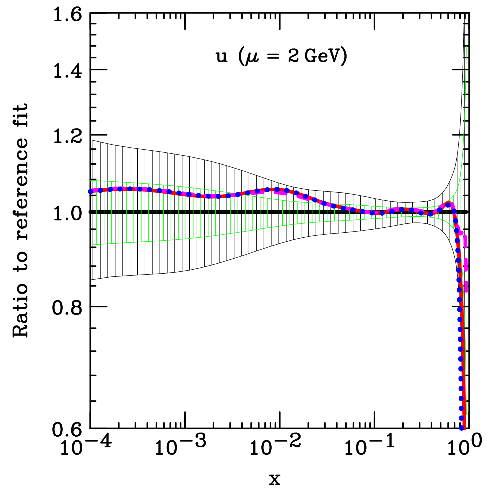
- CAN "REDIAGONALIZE": DIAGONALIZE SIMULTANEOUSLY χ^2 FOR TOTAL AND i–TH EXPT $\Rightarrow$ COMPATIBILITY OF EACH EXPT WITH GLOBAL FIT
- STUDY DISTRIBUTION OF DISCREPANCIES
- APPROX. GAUSSIAN WITH UNCERTAINTIES RESCALED BY $2 \Rightarrow$ $\Delta\chi^2 \sim 10$ FOR 90%C.L.



FUNCTIONAL BIAS (Pumplin, 2010)



- IF PARM. NOT GENERAL ENOUGH, GLOBAL MIN. IS NOT TRUE MIN.
- ONE-σ VARIATION ABOUT FAKE MIN CORRESP. TO LARGE χ^2 VARIATION
- USE OF CHEBYSHEV POLYNOMIALS SUGGESTS "MOST GENERAL" PARM. COMPATIBLE WITH $\Delta\chi^2 = 100$ RANGE OF CT10 PARM.



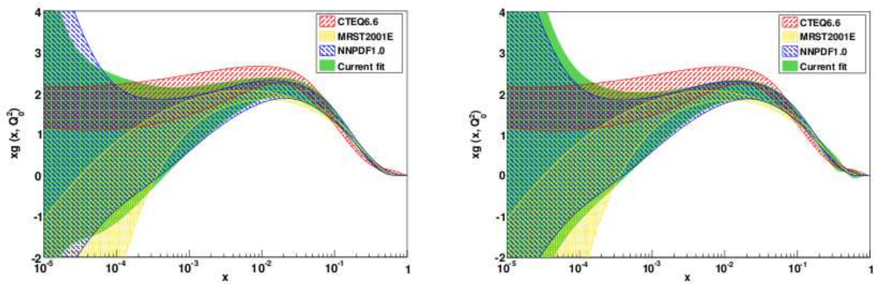
WHERE IS THE UNCERTAINTY COMING FROM?

FIT TO REPLICAS VS RANDOM SUBSET OF CENTRAL VAL.S

	REPLICAS	CENTRAL V.
χ^2	1.32	1.32
$\langle\chi^2\rangle_{\rm rep}$	2.79 ± 0.24	1.65 ± 0.20
$\langle\sigma^{\rm dat}\rangle$	0.039	0.035

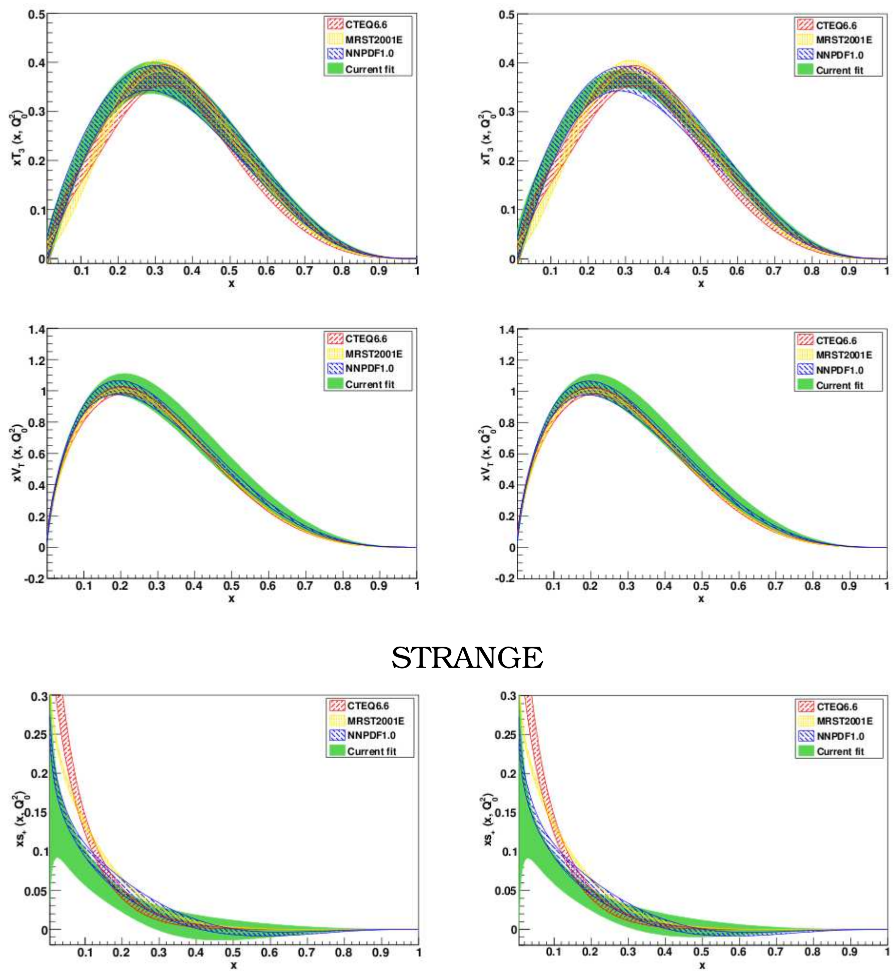
GLUE

replicas c. vals.



LIGHT QUARKS

STRANGE



- QUALITY OF FIT &PDFs UNCHANGED
- REDUCTION OF $\langle\chi^2\rangle_{\rm rep}$ BY FACTOR $\sim 2 \Rightarrow$ FLUCTUATIONS ABOUT TRUE VALUE HALVED
- UNCERTAINTY ON DATA ONLY REDUCED BY 1.1 $\Rightarrow$ EXPT. UNCERTAINTIES UNDERESTIMATED OR UNDERLYING INCOMPRESSIBLE UNCERTAINTY

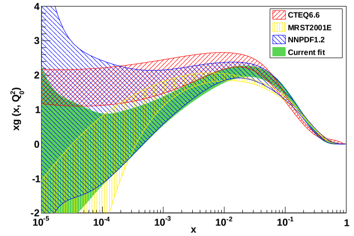
WHERE IS THE UNCERTAINTY COMING FROM?

CENTRAL VALUES: VARYING PARTITION VS FIXED PARTITION

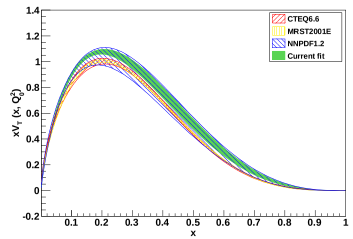
	REPLICAS	CENTRAL VALUE	FIXED PARTITION
χ^2	1.32	1.32	$\sim$1.3
$\langle\chi^2\rangle_{\rm rep}$	2.79 ± 0.24	1.65 ± 0.20	$\sim 1.6 \pm 0.2$
$\langle\sigma^{\rm dat}\rangle$	0.039	0.035	$\sim$0.03

fixed partition results obtained averaging over 5 different choices of partition (100 replicas each); more partitions needed for accurate results

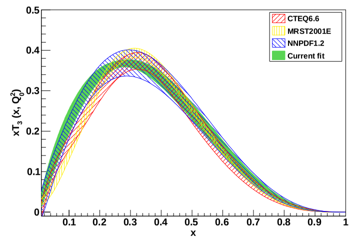
GLUE



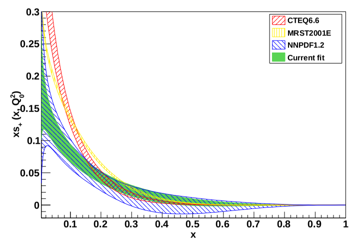
VALENCE



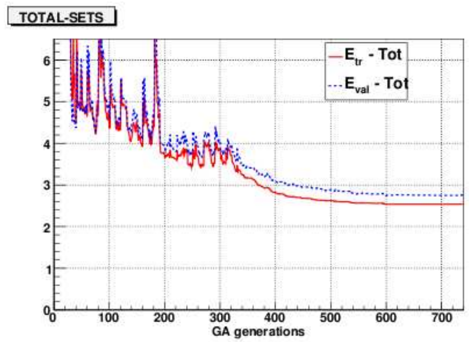
TRIPLET

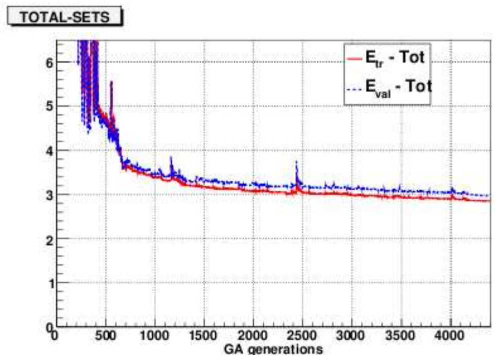


STRANGE



- QUALITY OF FIT UNCHANGED
- $\langle\chi^2\rangle_{\rm rep}$ UNCHANGED $\Rightarrow$ CENTRAL FIT UNCHANGED
- UNCERTAINTY ON PREDICTION (I.E. ON PDFS) REDUCED



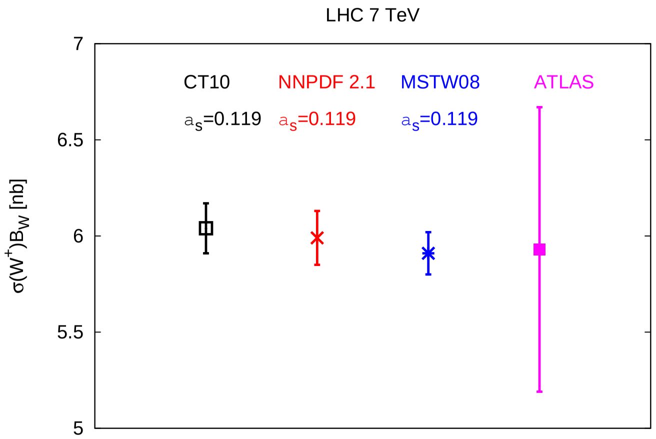


FUNCTIONAL UNCERTAINTY

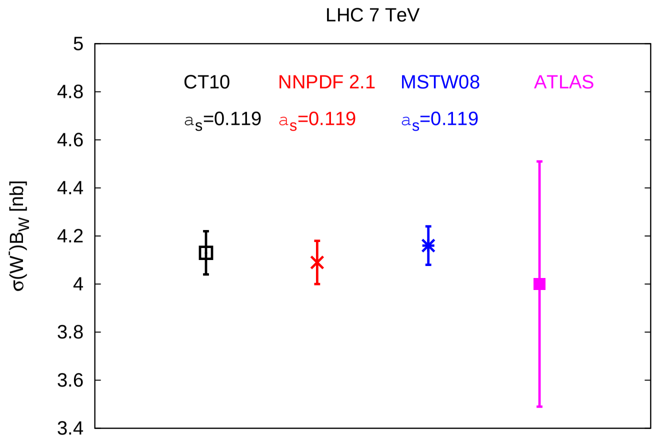
- MORE THAN HALF OF UNCERTAINTY DUE TO "FUNCTIONAL FORM": $\langle\sigma^{\rm dat}\rangle =\sim 0.03$ SMALLER FOR HERA DATA
- REMAINING UNCERTAINTY ROUGHLY SCALES WITH DATA UNCERTAINTY: $\langle\sigma^{\rm dat}\rangle =\sim 0.005$ CENT.; $\langle\sigma^{\rm dat}\rangle =\sim 0.009$ REP.

CONCLUSION

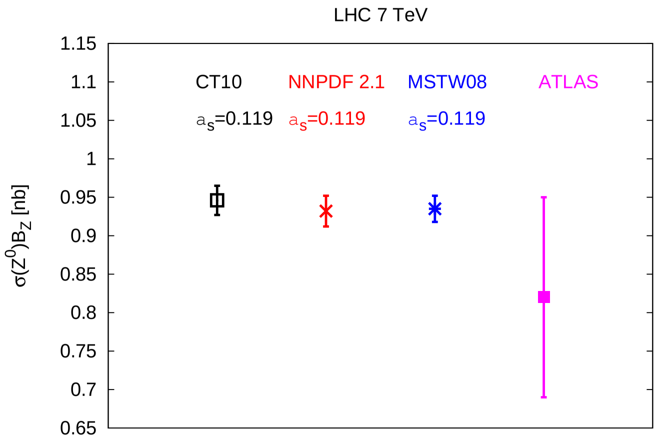
W^+



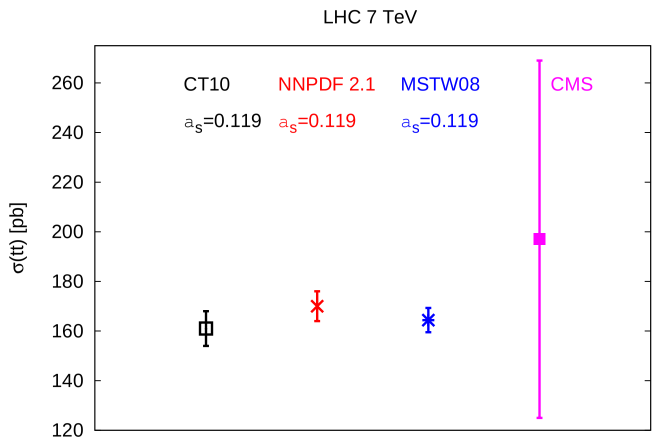
W^-



Z



TOP



PDFs ARE NOW PRECISION PHYSICS

WE ARE READY FOR DISCOVERY PHYSICS

EXTRAS

Monte Carlo data generation

- BCDMS+ NMC proton & deuteron F_2 data (full correlated systematics available), taken at 4 beam energies

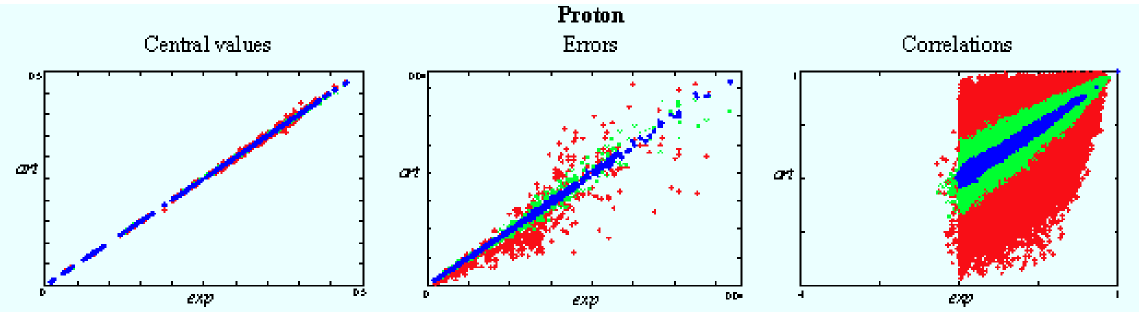
- on top of stat. errors, 4 systematics + 1 normalization (NMC) or 6 systematics + 1 absolute & 2 relative normalizations (BCDMS), with various forms of correlation (full, or for each target, or for each beam energy)

Generate data according to a multigaussian distribution

$$F_i^{(art)\,(k)} =$$

$$(1 + r_5^{(k)}\,\sigma_N)\sqrt{1 + r_{i,6}^{(k)}\,\sigma_{N_t}}\sqrt{1 + r_{i,7}^{(k)}\,\sigma_{N_b}}\left[F_i^{(exp)} + \frac{r_{i,1}^{(k)}\,f_b + r_{i,2}^{(k)}\,f_{i,s} + r_{i,3}^{(k)}\,f_{i,r}}{100}F_i^{(exp)} + r_{i,s}^{(k)}\,\sigma_s^i\right]$$

r univariate gaussian random nos., one $r_{i,s}$ for each data, but single $r_{i,j}$ for all correlated data



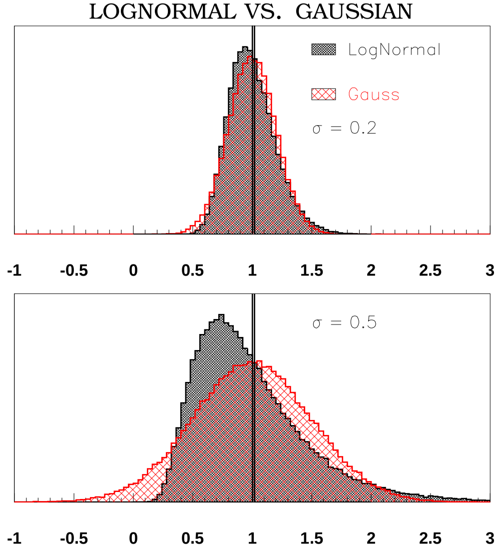
scatter plot art. vs. exp. for 10 (red) 100 (green) and 1000 (blue) replicas

need 1000 replicas to reproduce correlations to percent accuracy

PARAMETRIZATION UNCERTAINTIES?

NONGAUSSIAN BEHAVIOUR?

LOGNORMAL VS. GAUSSIAN

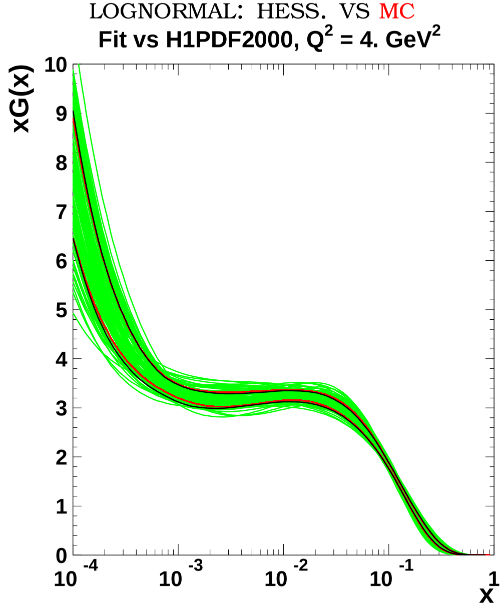


THE HERALHC BENCHMARK

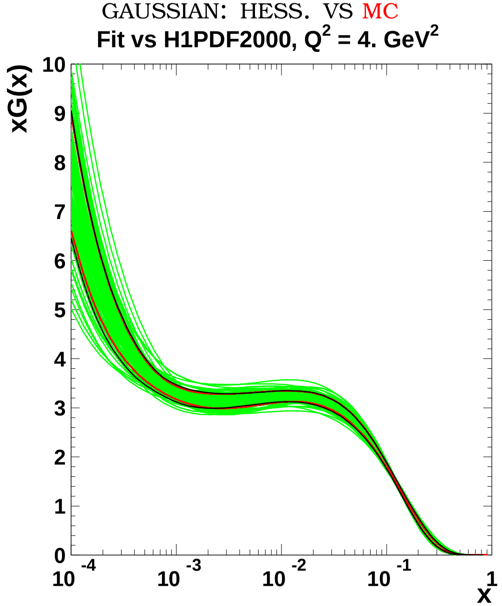
(Feltesse, Glazov, Radescu + NNPDF 2008)

- TRY EXPERIMENTAL SYSTEMATICS GIVEN BY EITHER GAUSSIAN OR LOGNORMAL DISTRIBUTION
- REPEAT (BENCHMARK) HERAPDF, WITH MONTECARLO LOGNORMAL OR GAUSSIAN, IN EITHER CASE DETERMINE UNCERTAINTY EITHER WITH HESSIAN OR MONTECARLO

LOGNORMAL: HESS. VS MC



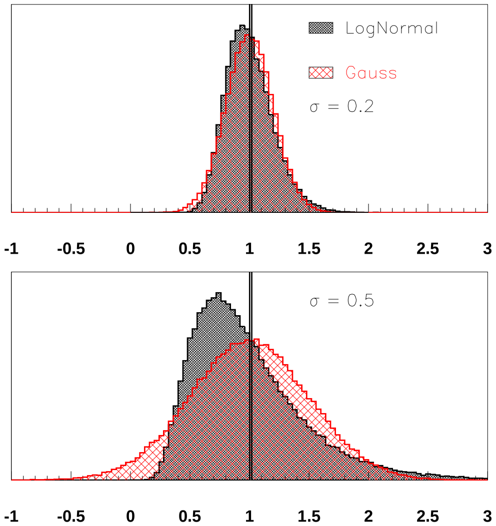
GAUSSIAN: HESS. VS MC



- NO DIFFERENCE BETWEEN LOGNORMAL, GAUSSIAN, MC, HESSIAN

PARAMETRIZATION UNCERTAINTIES?
NONGAUSSIAN BEHAVIOUR?

LOGNORMAL VS. GAUSSIAN



THE HERALHC BENCHMARK
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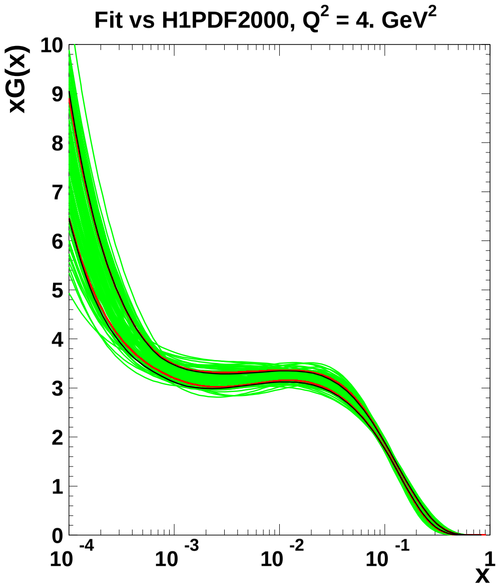
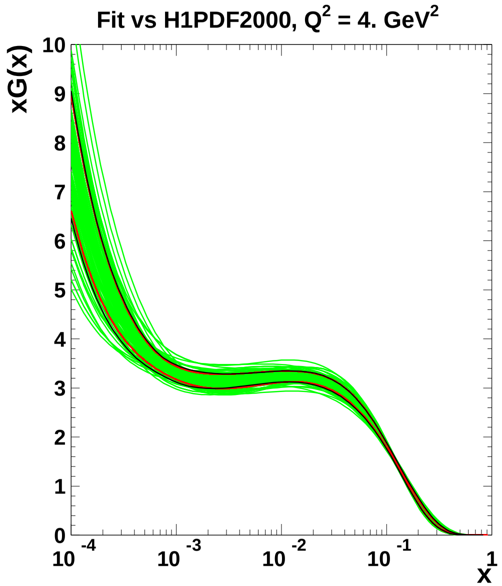
- REPEAT (BENCHMARK) HERAPDF, WITH MONTECARLO LOGNORMAL OR GAUSSIAN, IN EITHER CASE DETERMINE UNCERTAINTY EITHER WITH HESSIAN OR MONTECARLO
- COMPARE TO NNPDF FIT TO SAME DATA

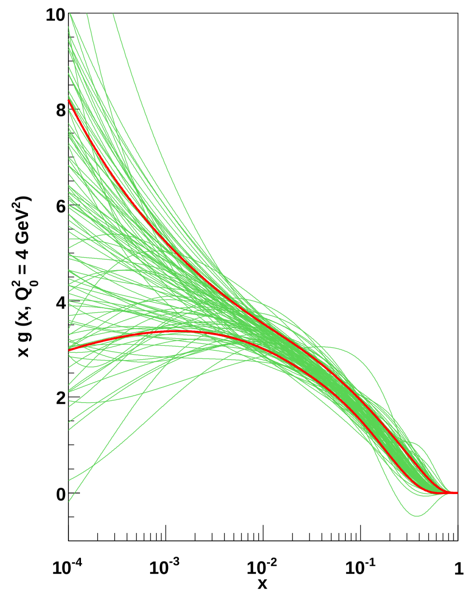
LOGNORMAL: HESS. VS MC



GAUSSIAN: HESS. VS MC



NNPDF

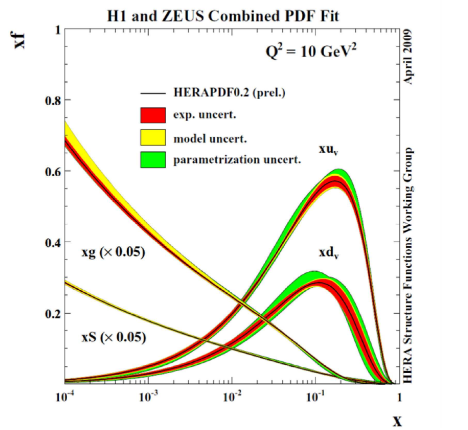


- NO DIFFERENCE BETWEEN LOGNORMAL, GAUSSIAN, MC, HESSIAN
- SIZABLE DIFFERENCE WR TO FLEXIBLE NNPDF PARAMETRIZATION

PARAMETRIZATION UNCERTAINTIES?

EXPLORING THE SPACE OF PARAMETERS: HESSIAN APPROACH

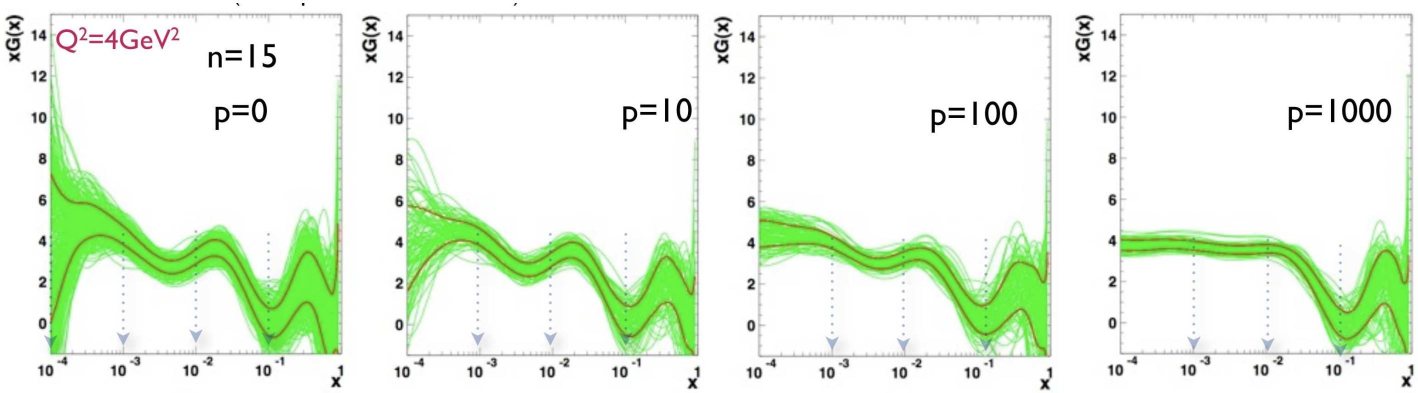
- IN HESSIAN APPROACH CAN VARY THE FUNCTIONAL FORM, ASSUMPTIONS, STARTING SCALE
- DONE IN THE HERAPDF1.0 FIT: VARIATION OF STRANGENESS FRACTION, LARGE x BEHAVIOUR, HIGHER ORDER POLYNOMIAL TERMS
- NO TOLERANCE ($\Delta\chi^2 = 1$), UNCERTAINTY DOUBLED



- Red: experimental uncertainties
- Yellow: model uncertainties
- Green: pdf parametrization uncertainties

ORTHOGONAL POLYNOMIALS

- OLD IDEA (PARISI, SOURLAS, 1978; ZOMER 1996): EXPAND PDFS OVER BASIS OF ORTHOGONAL POLYNOMIALS
- GLAZOV, RADESCU, 2009: COUPLED TO MONTE CARLO METHOD
- LENGTH PENALTY TO STABILIZE THE FIT



(Glazov, Radescu, 2009)

NORMALIZATION UNCERTAINTIES

- NORMALIZATION UNCERTAINTIES IN COVARIANCE MATRIX
  $(\mathrm{cov}_{t_0})_{IJ} = \sigma_{I,n}\sigma_{J,n}F_I F_J$
  $\Rightarrow$ MAXIMUM-LIKELIHOOD RESULT BIASED (d'Agostini, 1994)

- "PENALTY TRICK": RESCALE BY λ & ADD $\frac{(\lambda-1)^2}{\sigma_n}^2$ TO χ^2
  SOMETIMES ALSO HIGHER ORDER POWERS
  $\Rightarrow$ ALSO BIASED, THOUGH BIAS DOES NOT GROW WITH N_{dat}

- BIAS DUE TO χ^2 NOT QUADRATIC IN MEASURED QUANTITY
  (NONGAUSSIAN AND IMPROPERLY NORMALIZED LIKELIHOOD)

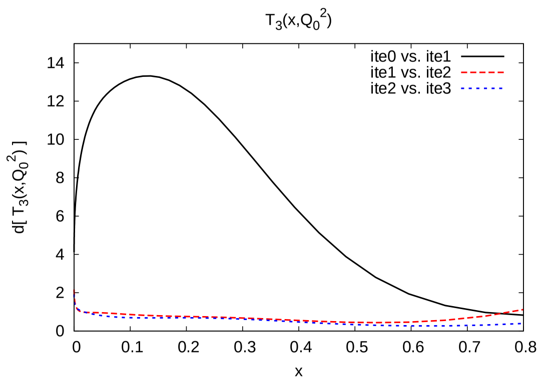
THE t_0 METHOD (R.D. Ball et al., 2010)

- NORMALIZATION UNCERTAINTIES IN COVARIANCE MATRIX, BUT COMPUTED AS FUNCTION OF RESULT OF PREVIOUS FIT $F_I^{(0)}$: $(\mathrm{cov}_{t_0})_{IJ} = \sigma_{I,n}\sigma_{J,n}F_I^{(0)}F_J^{(0)}$

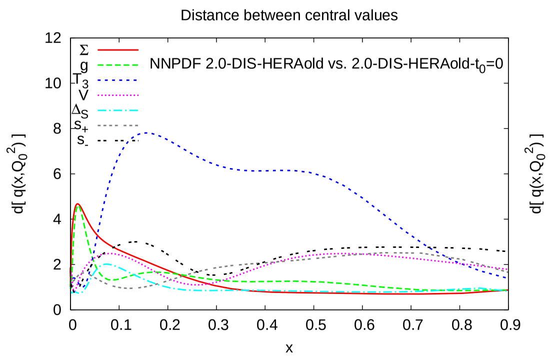
- ITERATE UNTIL CONVERGENCE

DISTANCE BETWEEN ITERATIONS

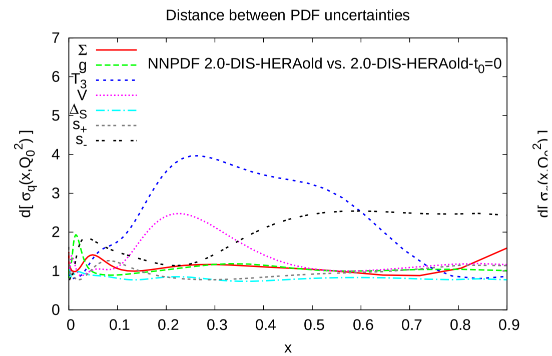
(TRIPLET)



t_0: IMPACT ON CENTRAL VALS



t_0: IMPACT ON UNCERTAINTIES



PREPROCESSING

- PDFs PARAMETRIZED AS $f(x,Q_0^2) = A_f\,(1-x)^{m_f}\,x^{-n_f}\mathrm{N}N_f(x)$, WITH $\mathrm{N}N_f(x)$ A NEURAL NETWORK
- PREFACTOR NECESSARY IF ONE WANTS $f(x)$ UNBOUNDED IN $0,1$ BECAUSE OUTPUT OF NEURAL NET BOUNDED
- WITH ARBITRARY LONG TRAINING, ANY ADMISSIBLE BEHAVIOUR CAN BE LEARNT, BUT TRAINING VERY LONG IF NN HAS TO OFFSET SHARP RISE OR DROP
- EXPONENTS m_f, n_f CHOSEN A RANDOMN WITH FLAT DISTRIBUTION IN WIDE RANGE
- RANGE DETERMINED BY WIDENING UNTIL FIT QUALITY DETERIORATES
- TEST FOR UNIFORM FIT QUALITY BY DETERMINING CORRELATION BETWEEN χ^2 AND PREPROCESSING EXPONENTS IN GIVEN RANGE

PREPROCESSING EXPONENTS: RANGE AND CORRELATION

PDF	$[m_{\min}, m_{\max}]$	$[n_{\min}, n_{\max}]$	$r\left[\chi^2, m\right]$	$r\left[\chi^2, n\right]$
$\Sigma(x,Q_0^2)$	$[2.55, 3.45]$	$[1.05, 1.35]$	-0.018	0.131
$g(x,Q_0^2)$	$[1.05, 1.35]$	$[1.05, 1.35]$	-0.002	0.050
$T_3(x,Q_0^2)$	$[2.55, 3.45]$	$[0, 0.5]$	-0.023	-0.130
$V_T(x,Q_0^2)$	$[2.55, 3.45]$	$[0, 0.5]$	0.003	-0.068
$\Delta_S(x,Q_0^2)$	$[12, 14]$	$[-0.95, -0.65]$	0.000	-0.069
$s^+(x,Q_0^2)$	$[2.55, 3.45]$	$[1.05, 1.35]$	0.021	-0.055
$s^-(x,Q_0^2)$	$[2.55, 3.45]$	$[0, 0.5]$	-0.027	-0.015

THE “HESSIAN MONTE CARLO”

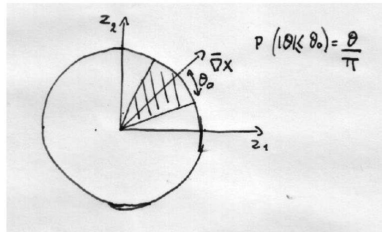
Q: IF ONE PICKS REPLICAS AT RANDOM ON THE ONE-SIGMA CONTOUR WHAT IS THE CHANCE OF “FILLING” THE ENVELOPE?

A: DETERMINE THE PROBABILITY FOR AT LEAST ONE REPLICA TO BE WITHIN ANGLE θ OF DIRECTION $\vec{\nabla} X$ OF MAX

TWO PARAMETERS: ONE REPLICA WITH $\theta < \theta_0 \Rightarrow P(2,1:\theta_0) = \frac{\theta_0}{\pi}$

PROBABILITY OF MAX(ENVELOPE)=$\sigma_X \cos\theta_0$

$\Rightarrow$ ALL n REPLICAS HAVE $\theta > \theta_0 \Rightarrow P(2,n;\theta_0) = \left(1 - \frac{\theta_0}{\pi}\right)^n$

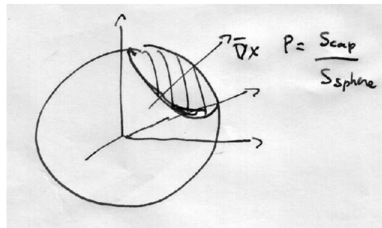


d PARAMETERS: ONE REPLICA WITH $\theta < \theta_0$

$$\Rightarrow P(d,1:\theta_0) = \frac{\Gamma\left(\frac{d}{2}\right)}{(d-1)\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)}\theta_0^{d-1}(1+O(\theta_0)) \approx \frac{\theta_0^{d-1}}{\sqrt{2\pi d}}$$

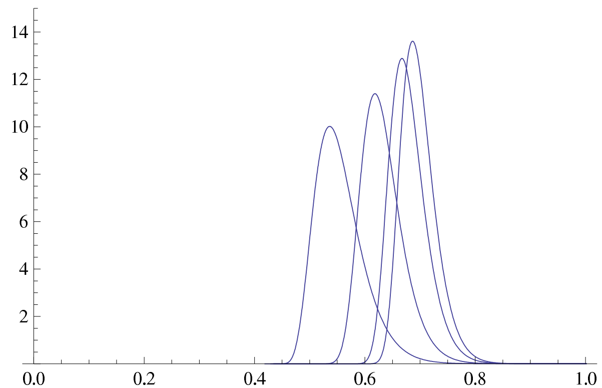
PROBABILITY OF MAX(ENVELOPE)=$\sigma_X \cos\theta_0$

$$\Rightarrow P(d,n;\theta_0) = \left(1 - \frac{\theta_0^{d-1}}{\sqrt{2\pi d}}\right)^n$$



PROBABILITY FOR THE WIDTH OF THE ENVELOPE TO BE SMALLER BY A FACTOR R THAN THE STANDARD DEVIATION σ_X PLOTTED VS R FOR

$d = 23$ PARAMETERS AND $n = 10,\ 100,\ 500,\ 1000$ REPLICAS



MONTE CARLO ERROR ESTIMATES

PARAMETER SPACE: NOT ADVISABLE

OBSERVABLE X DEPENDS ON PARAMETERS $\vec{z}$
VARIANCE: $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$
AVERAGES: $\langle X \rangle = \int d^d z X(\vec{z}) P(\vec{z})$, WITH
$P(\vec{z}) \Rightarrow$ PROBABILITY DISTN. OF PARAMETER VALUES
& INTEGRAL PERFORMED BY MONTE CARLO SAMPLING
HOW MANY REPLICAS DOES ONE NEED? THREE BINS PER PARM $\Rightarrow 3^d$ BINS
FOR 23 PARMS., NEED $> 10^{11}$ REPLICAS

DATA SPACE

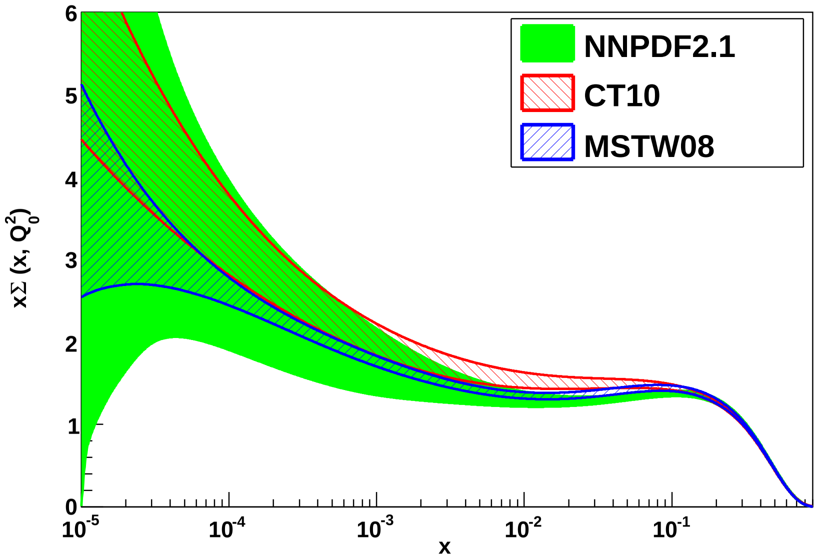
DIAGONALIZATION: CHOOSE PARM z_1 ALONG $\vec{\nabla} X$
ALL OTHER PARMS $\Rightarrow$ FLAT DIRECTIONS
AVERAGES: $\langle X \rangle = \int dz_1 X(\vec{z}) P(z_1)$
HOW MANY REPLICAS DOES ONE NEED? ONE–DIMENSIONAL AVERAGE OF n REPLICAS
CONVERGES TO TRUE AVERAGE WITH STANDARD DEV. $\frac{\sigma}{\sqrt{n}}$
10 REPLICAS ENOUGH FOR $\frac{\sigma}{3}$ ACCURACY
Q: HOW IS IT DONE IN PRACTICE?
A: CHOOSE REPLICAS OF THE DATA, DISTRIBUTED AS THE DATA

NNPDF2.1 PDFs

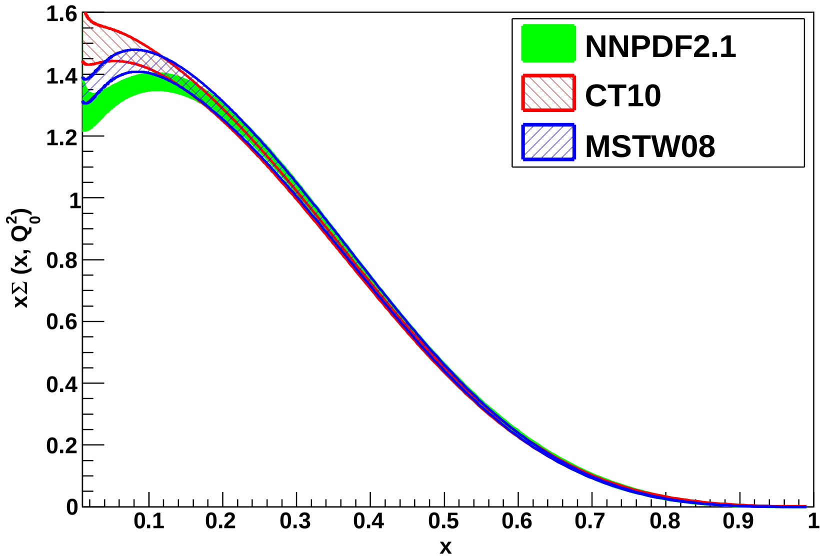
COMPARED TO OTHER GLOBAL PDF SETS (MSTW08, CT10)

SINGLET SECTOR

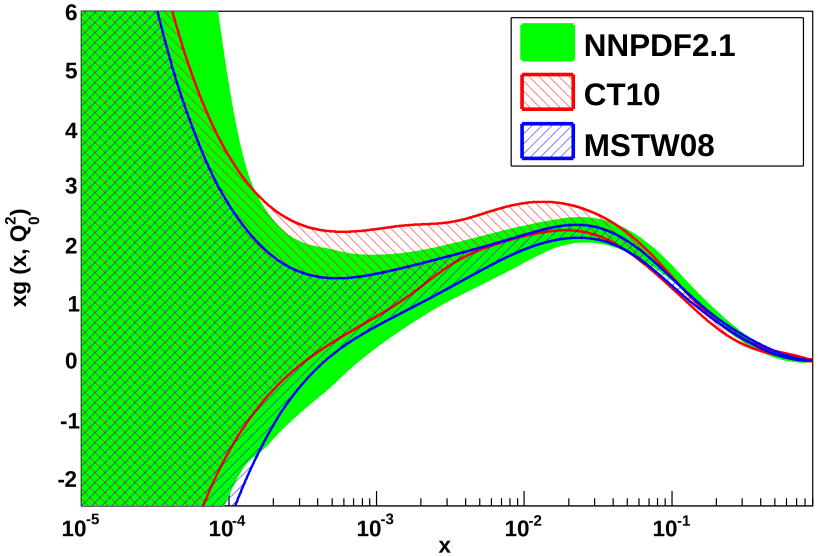
SINGLET



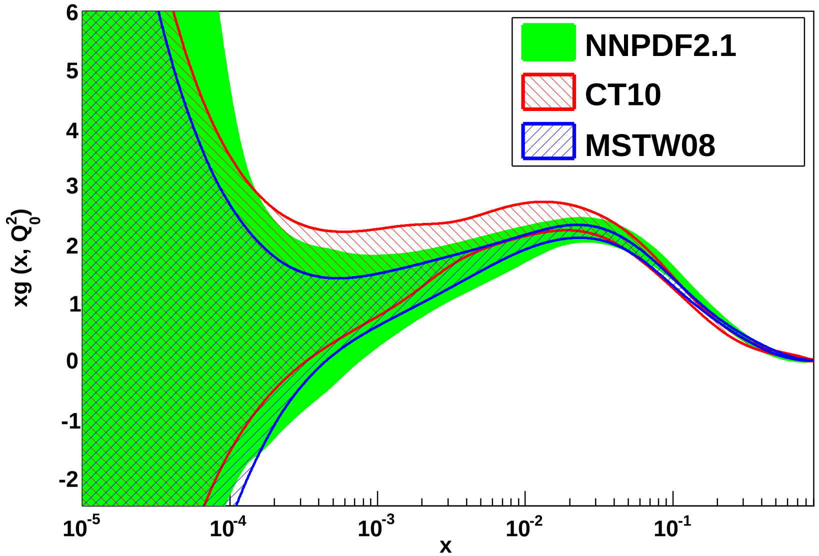
SINGLET



GLUON



GLUON



NNPDF2.1 PDFs

COMPARED TO OTHER GLOBAL PDF SETS (MSTW08, CT10)

NONSINGLET SECTOR

TOTAL VALENCE ISOSPIN TRIPLET