



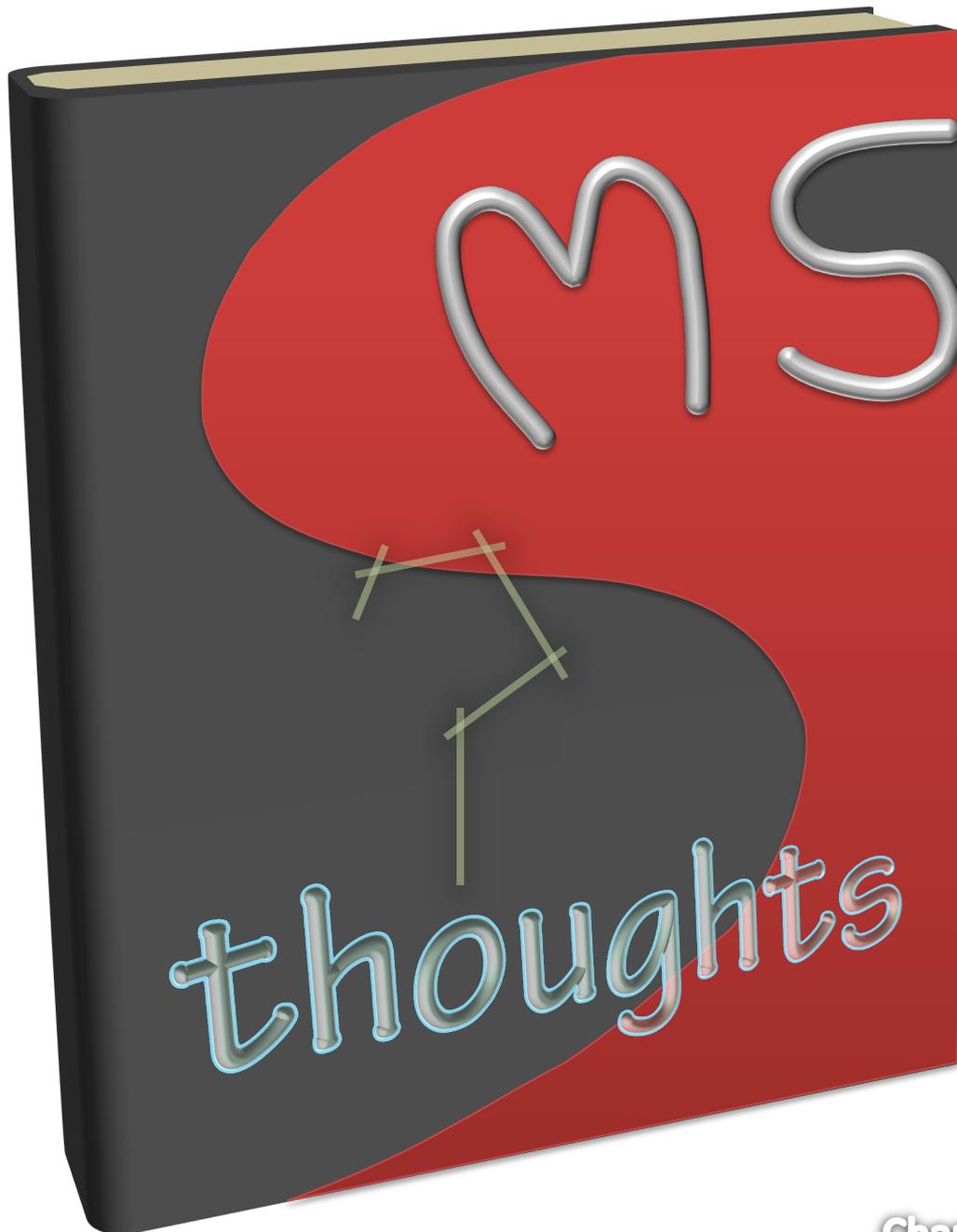
Author is not a statistician.
Author is not a software developer.
Use at your own risk.
May not compile in CMSSW X.Y.Z.
May be arbitrarily complicated.
Call 1-800-NO-SUPPORT for assistance.

> **Ctrl + C**

>

> **reboot**

—



J. Incandela
S. A. Koay
R. Rossin



P. Schuster
N. Toro



A **S**implified **M**odel **S**pectra
route towards physics intuition
and understanding of first
observations beyond SM.

- an experimentalist

5th November 2010. CERN
Characterization of new physics at the LHC, II

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1. Life at a hadron collider
2. Signal characterization, bottom-up
3. A hands-on procedure
4. Stages of understanding:
 - a) upper bound on a process
 - b) Composing multi-process models
 - c) Deducing new particle masses
5. Models: good, bad, better

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1. Life at a hadron collider

160 μb^{-1}	1.2 nb^{-1}	5.2 nb^{-1}	19 nb^{-1}	21 nb^{-1}	21 nb^{-1}	110 nb^{-1}	350 nb^{-1}	930 nb^{-1}	1.8 pb^{-1}	3.6 pb^{-1}	3.6 pb^{-1}	8.3 pb^{-1}	20 pb^{-1}	45 pb^{-1}
17/4	1/5	15/5	29/5	12/6	26/6	10/7	24/7	7/8	21/8	4/9	18/9	2/10	16/10	30/10

Signal characterization
Bottom-up

3. A hands-on procedure

4. Stages of understanding:

- Upper bound on a process
- Composing multi-process models
- Deducing new particle masses

5. Models: good, bad, better

Observation of an Excess above Standard Model Expectations in the Missing Transverse Energy and Jets Channel at $\sqrt{s} = ?\text{TeV}$

- Generic, “catch-all” searches:
 - Cuts and variables optimized for high signal significance; not always high efficiency.
 - Rarely a hunt for mass peaks.
- Interpretation likely to be non-obvious...
- ... but we want to do it anyway:
 - To gain confidence that the excess is not some strangely behaving background.
 - To have some idea where to look next.

– a SUSY analyst, 2010

Observation of an Excess above Standard Model Expectations in the Missing Transverse Energy and Jets Channel at $\sqrt{s} = ?\text{TeV}$

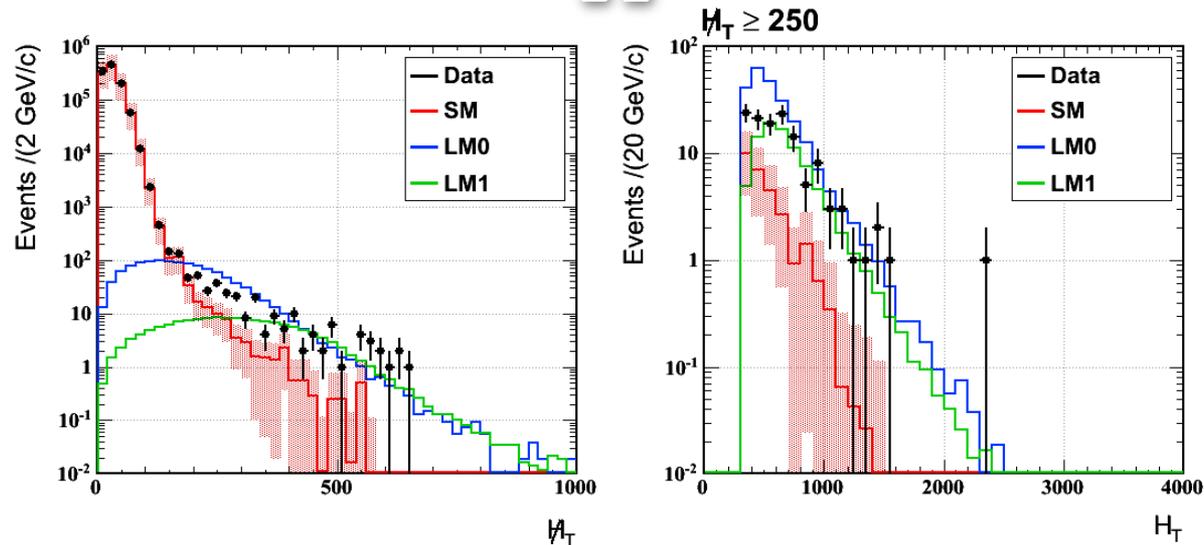


FIG 1. Distribution of H_T and H_T variables in data, overlaid on the expected Standard Model (SM) background. Two SUSY benchmark points LM0 and LM1 are also shown for comparison.

Example search that publishes two distributions.

Observation of an Excess above Standard Model Expectations in the Missing Transverse Energy and Jets Channel at $\sqrt{s} = ?\text{TeV}$

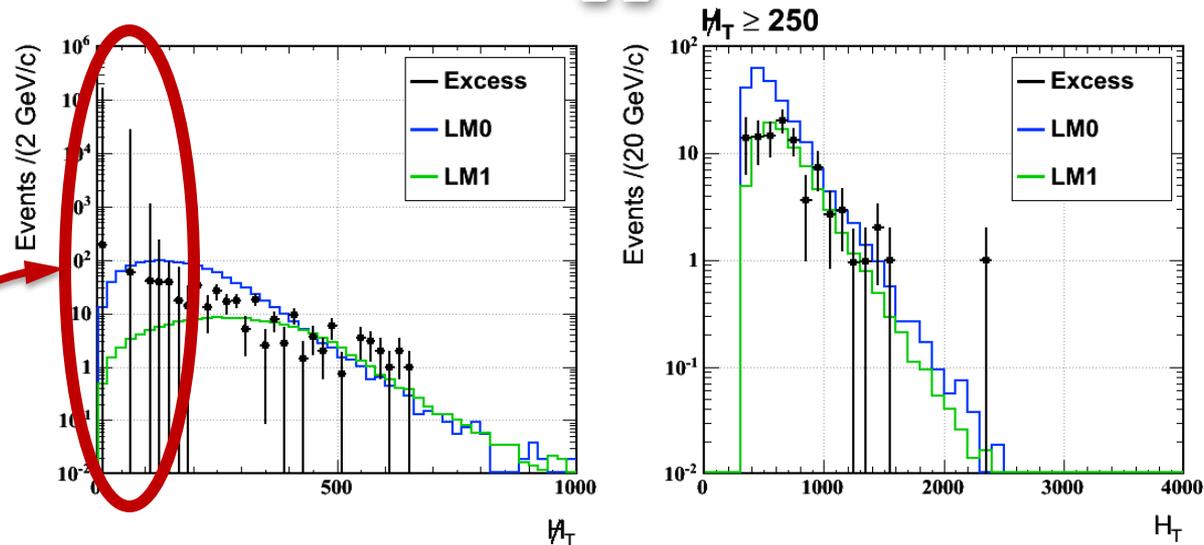


FIG 2. Excess in the M_T and H_T distributions in data, after subtraction of the expected SM background. Two SUSY benchmark points LM0 and LM1 are also shown for comparison.

Region unusable due to large background subtraction uncertainty

In the following discussion, assume working with background-subtracted excesses.

Observation of an Excess above Standard Model Expectations in the Missing Transverse Energy and Jets Channel at $\sqrt{s} = ?\text{TeV}$

What new particles?

*Masses?
Charges?
Spins?*

*Production
& decay
modes?*

*Identity of
WIMP?*

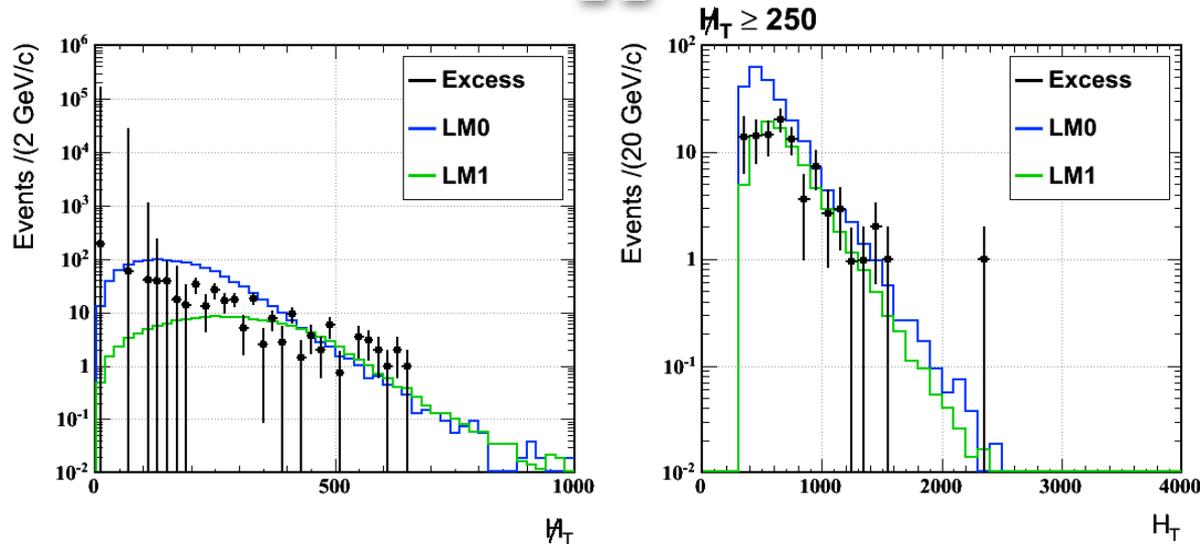


FIG 2. Excess in the M_T and H_T distributions in data, after subtraction of the expected SM background. Two SUSY benchmark points LM0 and LM1 are also shown for comparison.

*BSM group
structure?*

*SUSY?
UED?
LHT?
(your
favorite
TOE)?*

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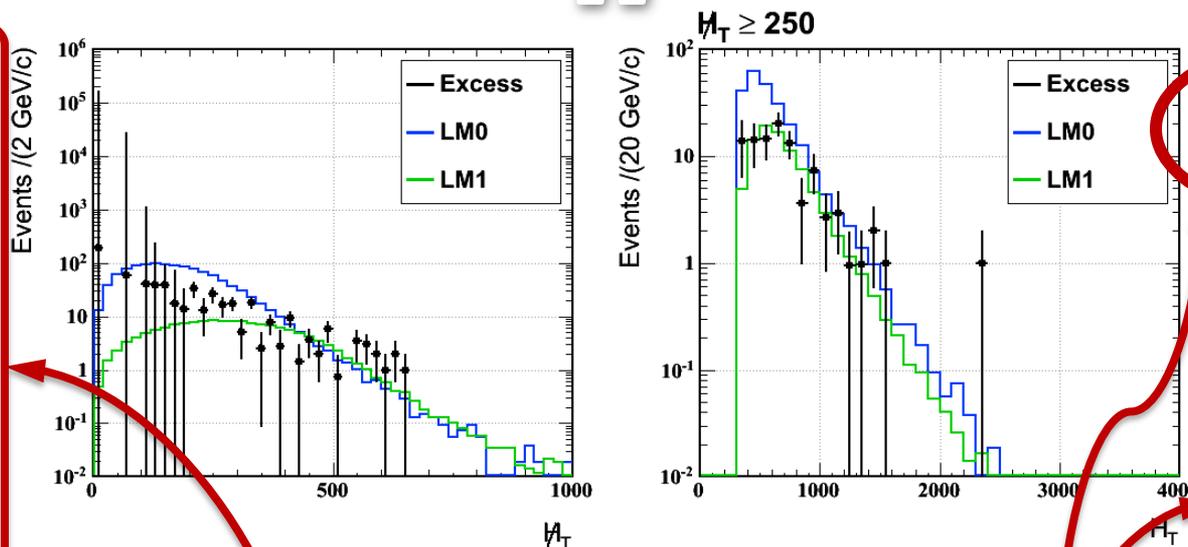


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BSM group
structure?

SUSY?
UED?
LHT?
(your
favorite
TOE)?

$\mu^+\mu^-$ machine: Direct measurement of many quantities.

pp machine: Mostly *implicit* measurement via sensitive variables...
... but details will be hard to resolve early on.

Holy grail (a.k.a.
far, far away)

Observation of an Excess above Standard Model Expectations in the Missing Transverse Energy and Jets Channel at $\sqrt{s} = ?\text{TeV}$

The dominant backgrounds to this search—QCD multi-jet, top pair, and W boson production—are estimated using data-driven techniques.

- *Variable-specific in many cases (e.g. matrix method), for simplicity, expediency, ...*
- *May not be able to predict shapes, only counts above particular set(s) of cuts.*

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- *Variable-specific in many cases (e.g. matrix method), for simplicity, expediency, ...*
- *May not be able to predict shapes, only counts above particular set(s) of cuts.*

- *Hard to imagine being able to characterize signal using only a count in a single bin.*
- *The following discussion assumes that a well-established excess of reasonably many events ($\gg 1$) is available in the form of one or more distributions.*
- *In general, expect a limited set of variables and regions in which signal can be tested.*

Observation of an Excess above Standard Model Expectations in the Missing Transverse Energy and Jets Channel at $\sqrt{s} = ?\text{TeV}$

The dominant backgrounds to this search—QCD multi-jet, top pair, and W boson production—are estimated using data-driven techniques.

- Variable-specific in many cases (e.g. matrix method), for simplicity, expediency, ...
- May not be able to predict shapes, only counts above particular set(s) of cuts.

Expect a limited set of variables and regions in which signal can be tested.

Comparisons of the data-driven predictions to Monte Carlo simulated expectations are shown in Figs. 3 to 5.

- “ Let’s just use MC backgrounds, and take a very large uncertainty. ”
- The proper thing to do still requires validation of MC vs. data in unexplored \sqrt{s} territory.
 - Otherwise, what is a “very large” theoretical uncertainty? Scale variations? NLO? NNLO?

And this is assuming we’ve pinned down all aspects of detector modeling!

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1. *Life at a hadron collider*
2. *Signal characterization, bottom-up*

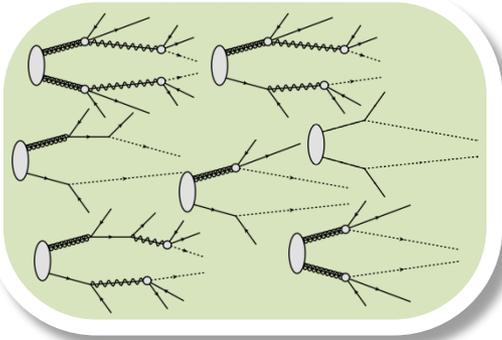
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3. *A hands-on procedure*
4. *Stages of understanding:*
 - a) *Upper bound on a process*
 - b) *Composing multi-process models*
 - c) *Deducing new particle masses*
5. *Models: good, bad, better*

Signal characterization, top down

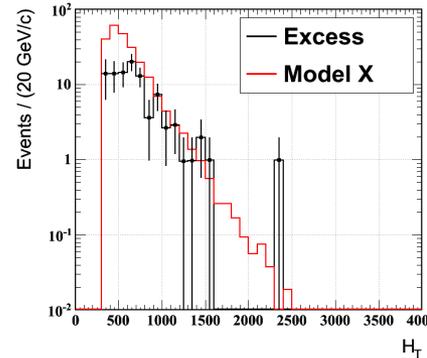
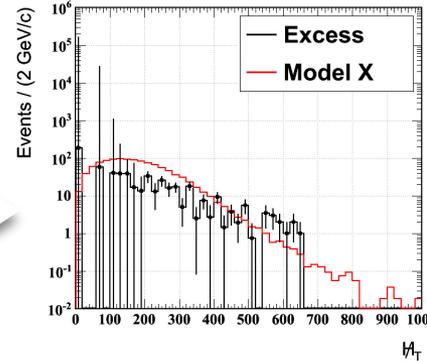
“Signal” \equiv background-subtracted data

Model X



Defined by:

- Multiple processes implied by particle spectrum and decay channels.
- Cross section from matrix element calculation.



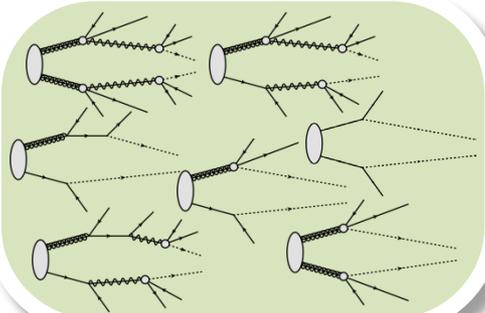
Compare distributions with a histogram compatibility test (χ^2 , log-likelihood, ...); or more sophisticated statistical techniques.

“Distance”
III
degree of compatibility
(χ^2 probability, ...)

Signal characterization, top down

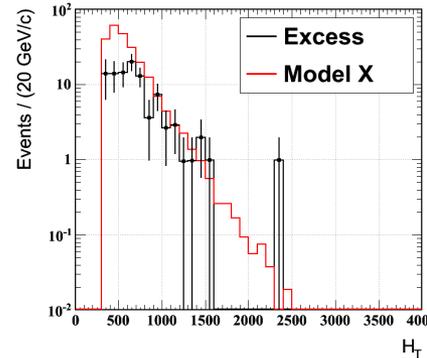
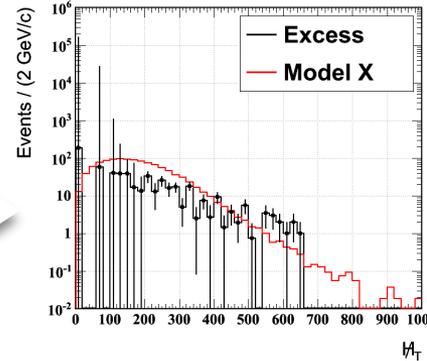
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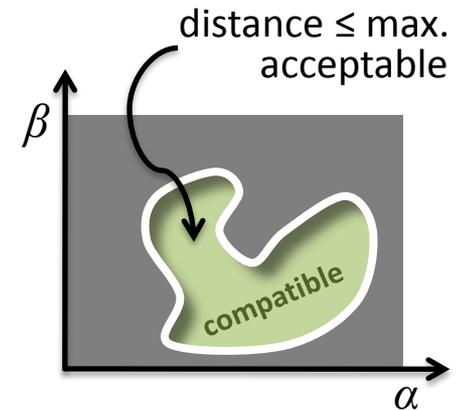
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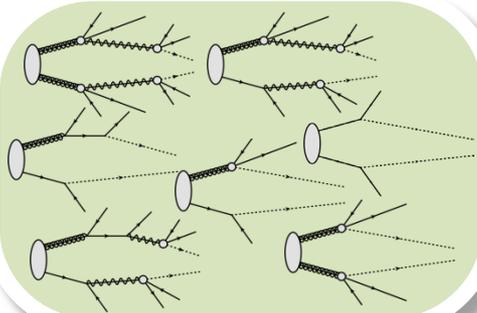
(theory with 2 parameters)

Typically a point in parameter space of some larger theory

Signal characterization, bottom up

“Signal” \equiv background-subtracted data

Model X



Defined by:

- Multiple processes implied by particle spectrum and decay channels.
- Cross section from matrix element calculation.

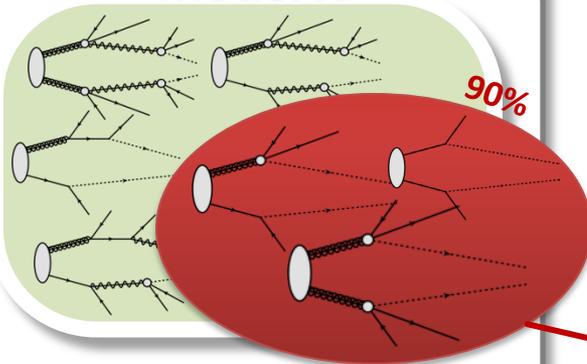
- The bottom up approach differs in how one selects/evolves the models to test.
- Statistics treatment of whether a model is feasible (or not) remains the same.

Rather than checking all models in an $O(100)$ -dimensional TOE parameter space, the idea is to progressively use information from data to narrow down possibilities:

Signal characterization, bottom up

“Signal” \equiv background-subtracted data

Model X



Defined by:

- Multiple processes implied by particle spectrum and decay channels.
- Cross section from matrix element calculation.

- The bottom up approach differs in how one selects/evolves the models to test.
- Statistics treatment of whether a model is feasible (or not) remains the same.

Rather than checking all models in an $O(100)$ -dimensional TOE parameter space, the idea is to progressively use information from data to narrow down possibilities:

- Condense indistinguishable processes into representative classes.
 - Regions can differ only by change in fractions of each topology.
- \Rightarrow Fit for fractions in data.

- Typically the most constraining quantity.
 - Depends strongly on quantities with little observable impact—spins of produced particles, species and masses of internal propagators.
- \Rightarrow Extract directly from data.

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4. Stages of understanding:

a) Upper bound on a process

b) Composing multi-process models

c) Deducing new particle masses

5. Models: good, bad, better

But hasn't this all already been said and done?

Bard: Interpreting New Frontier Energy Collider Physics
([arXiv:hep-ph/0602101v1](https://arxiv.org/abs/hep-ph/0602101v1))

MUSiC – An Automated Scan for Deviations between
Data and Monte Carlo Simulation
(<http://cms-physics.web.cern.ch/cms-physics/public/EXO-08-005-pas.pdf>)

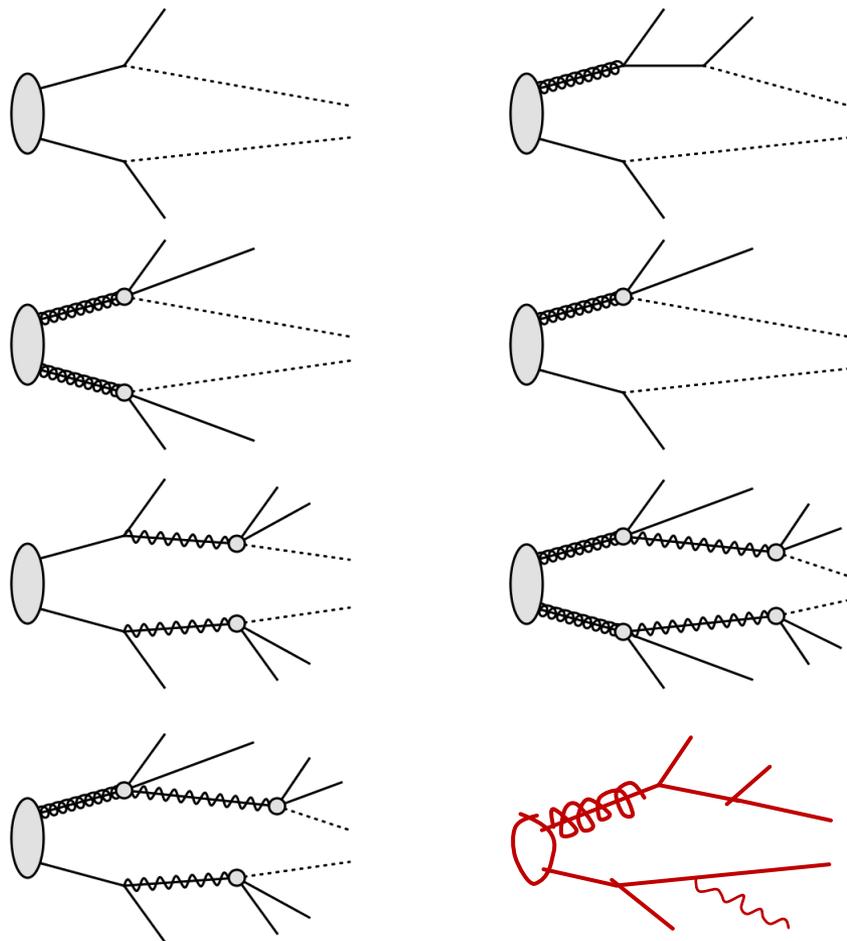
MARMOSET: The Path from LHC Data to the New
Standard Model via On-Shell Effective Theories
([arXiv:hep-ph/0703088v1](https://arxiv.org/abs/hep-ph/0703088v1))

*The writing and people
who initiated this work.*

*The strategy shown
here follows in this
spirit; think of the
choice as a matter of
taste (or ignorance).*

An “informed search” through BSM space

SKETCH



SUSY

etc.

UED

LHT

Start from a “comprehensive” list of SMS topologies.

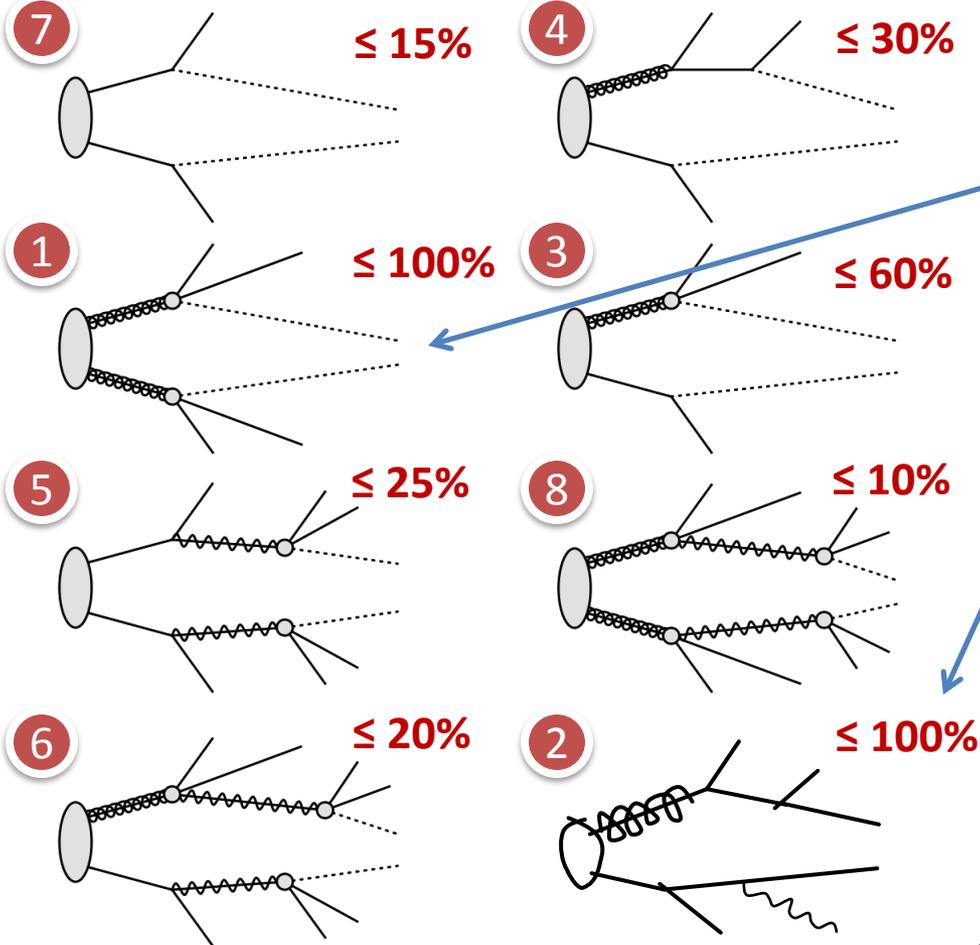
Most of these will come from the sets proposed a priori by theorists.

Some may be inspired during the course of investigation.

We focus on a SUSY-inspired example; the treatment of other theories is analogous. Some topologies can anyway be interpreted in the language of several theories.

An “informed search” through BSM space

SKETCH



SUSY

etc.

UED

LHT

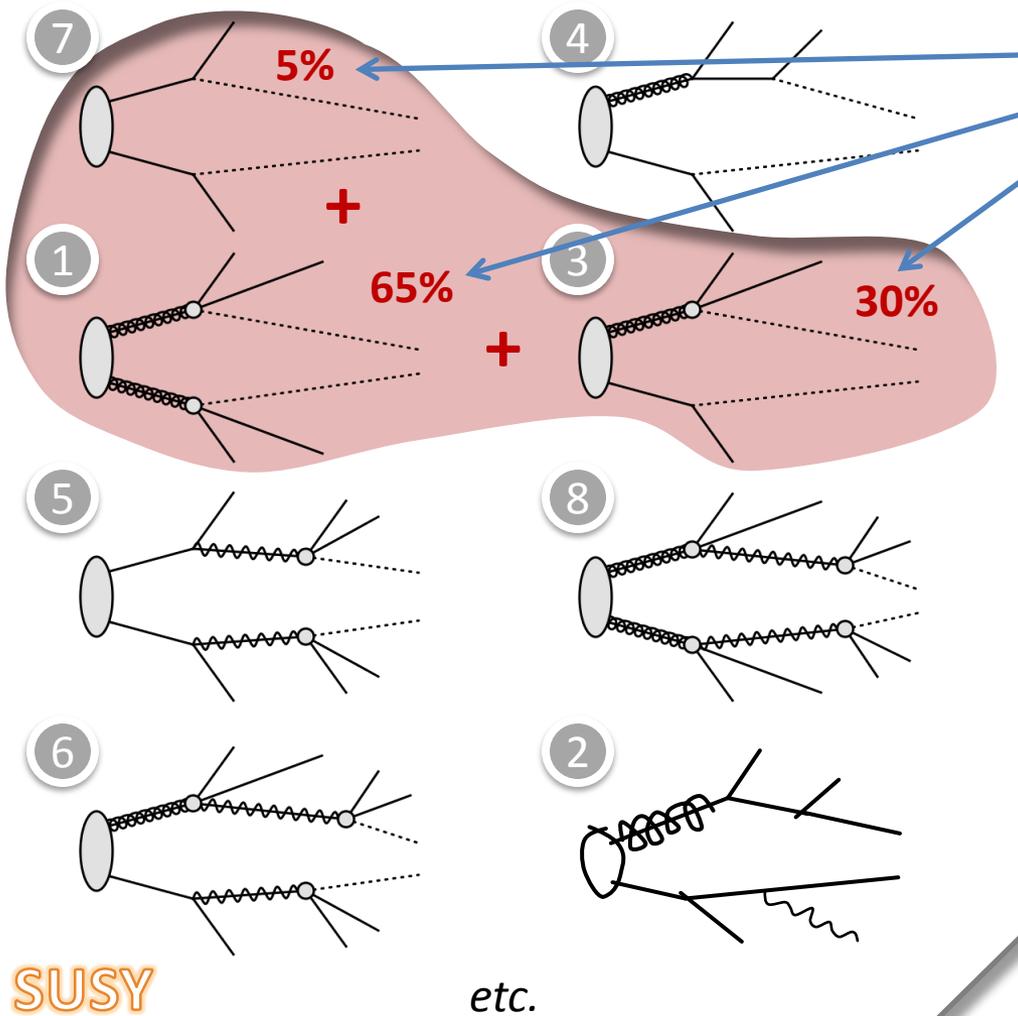
Signal could be these (i.e. models composed predominantly of a single process).

Compare each topology to data, and extract the maximum allowable fraction of signal that it can account for.

Ranking topologies in decreasing order of this maximum fraction gives us a priority list for what to check (albeit not strictly necessary).

An “informed search” through BSM space

SKETCH



Also test if signal is compatible with multi-process models:

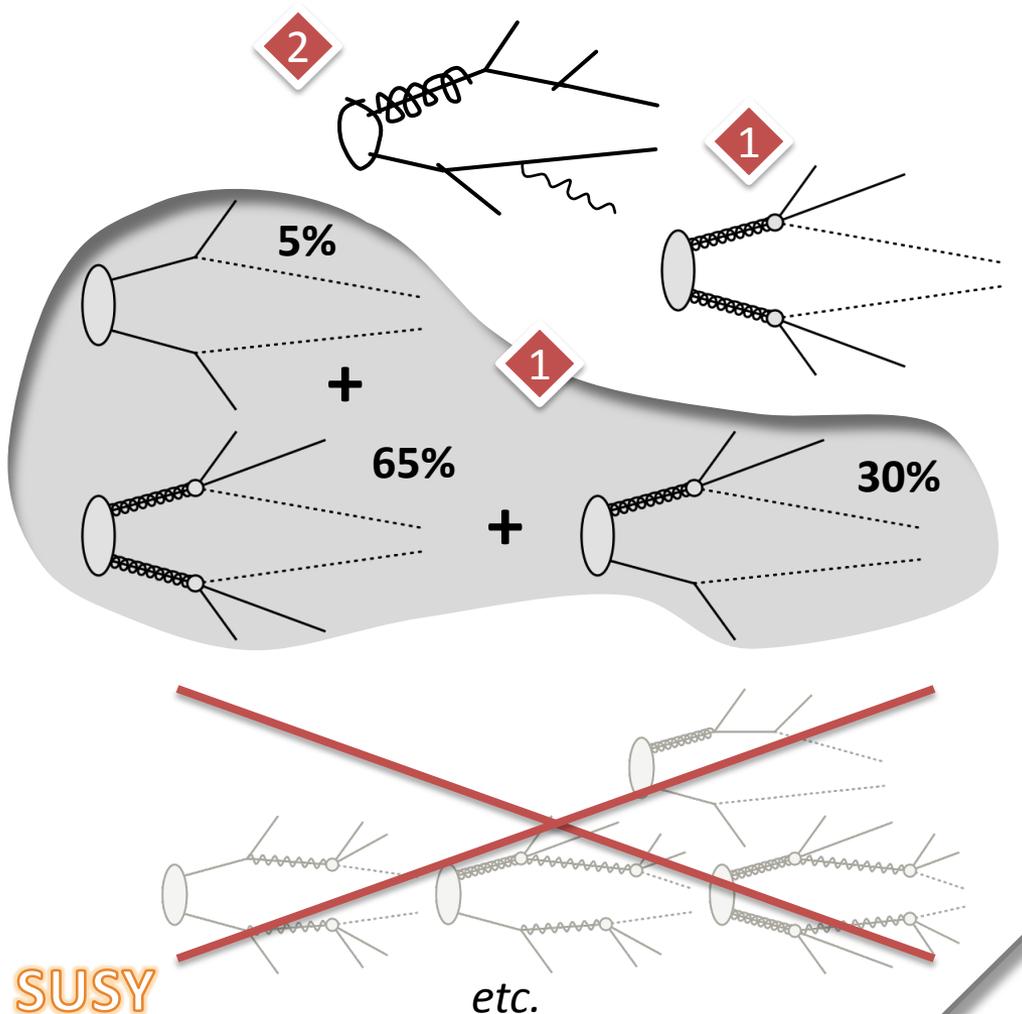
- Simplified Model Sets in fact provide physically “combinable” collections of processes.
- Of course signal could as well be composed of two or more SMS’s – something for future iterations (and so on *ad infinitum*).

UED

LHT

An “informed search” through BSM space

SKETCH



Perform a “final” ranking of all the tested models:

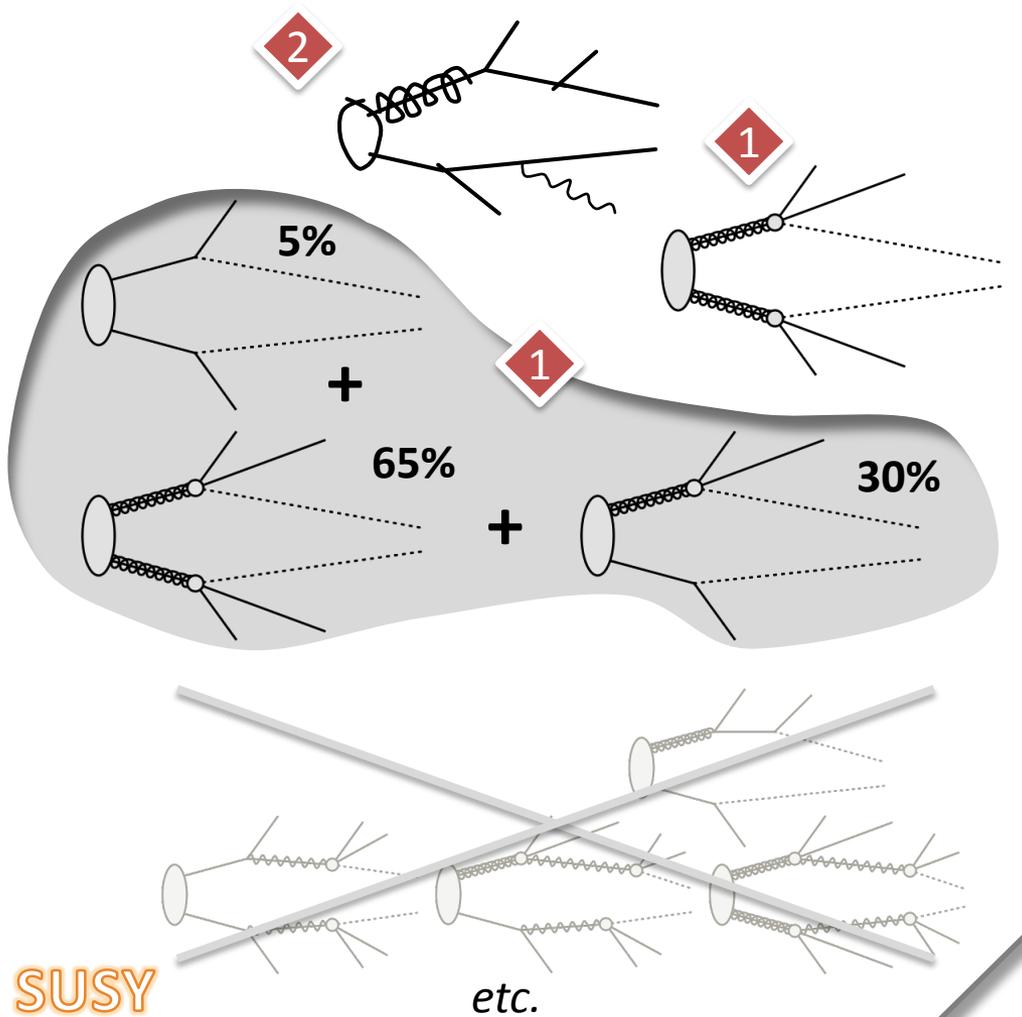
- Models that are not compatible with signal are eliminated.
- Remaining viable models are ordered by decreasing goodness-of-fit to signal.

UED

LHT

An “informed search” through BSM space

SKETCH



*Why bother to rank?
Because then an
experimentalist could
pick a top few off this
list to start looking.*

Perform a “final” ranking
of all the tested models:

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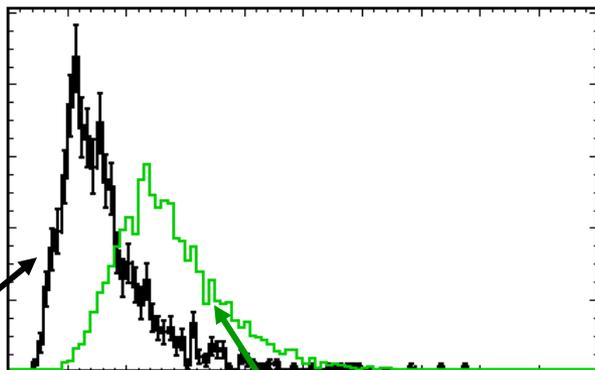
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5. Models: good, bad, better

Some terminology

Template = 1.0 × Signal

*Template scaled to
number of signal events*

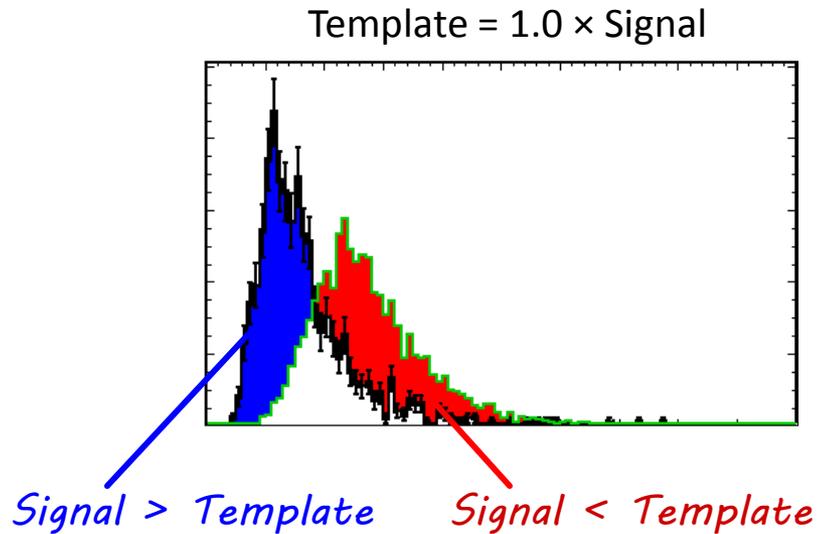


*Signal
(≡ background
subtracted data)*

*Template (from a
particular SMS
model hypothesis)*

- Our staple tool is the simple histogram comparison.
 - Assume χ^2 test, to be concrete.

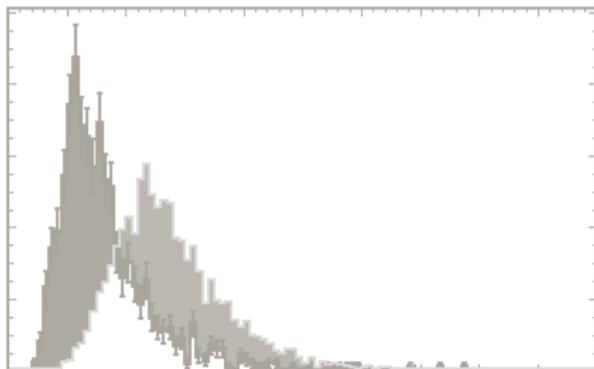
Upper bound on a process X



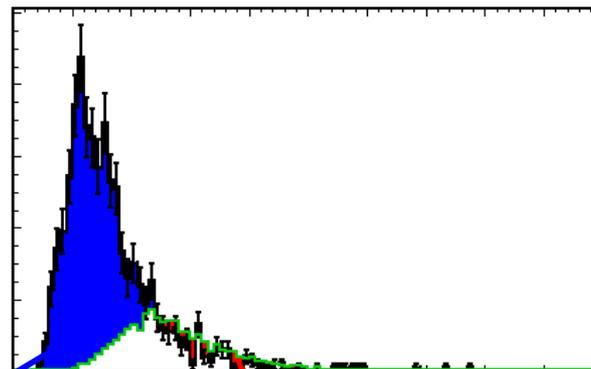
- For this SMS process, a shape comparison of this distribution indicates that it is not compatible with the observed signal.
- However the above does **not** mean that process X cannot be present:
- It only means that it cannot be the **only** source of the signal.

Upper bound on process X

Template = $1.0 \times \text{Signal}$



Template = $0.3 \times \text{Signal}$

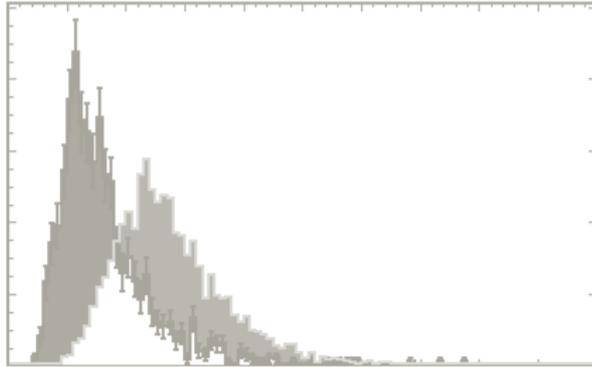


*This part
could be due
to other
process(es)*

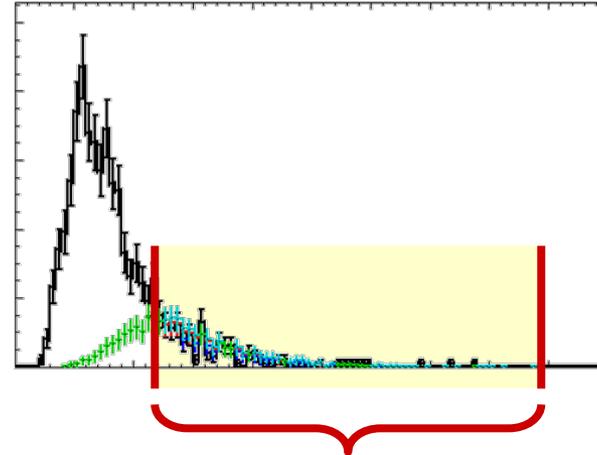
*The maximum fractional contribution of
process X is what can manage to fit
under the entire distribution...
... with a $\sim 2\sigma$ excess allowed in case there
was a downwards Poisson fluctuation.*

Upper bound on process X

Template = $1.0 \times$ Signal



Template = $0.3 \times$ Signal

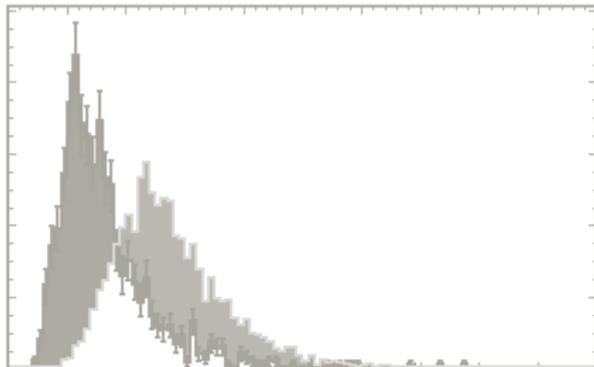


Procedure

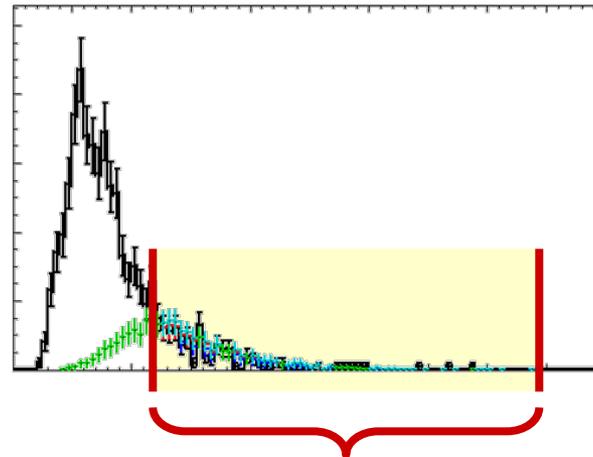
Compute a “constrained distance” using only those bins where template $>$ signal

Upper bound on process X

Template = $1.0 \times$ Signal



Template = $0.3 \times$ Signal



Procedure

Compute a “constrained distance” using only those bins where template > signal

- This upper bound must be checked as a function of the new particle masses.
- Provides a crude first pass to see which processes can be the dominant source of signal, versus those that cannot be present except in conjunction with some other process(es).

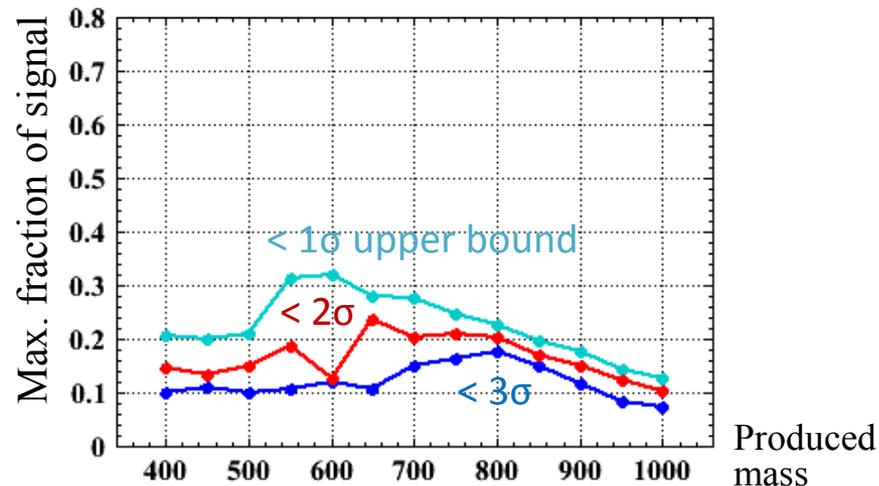


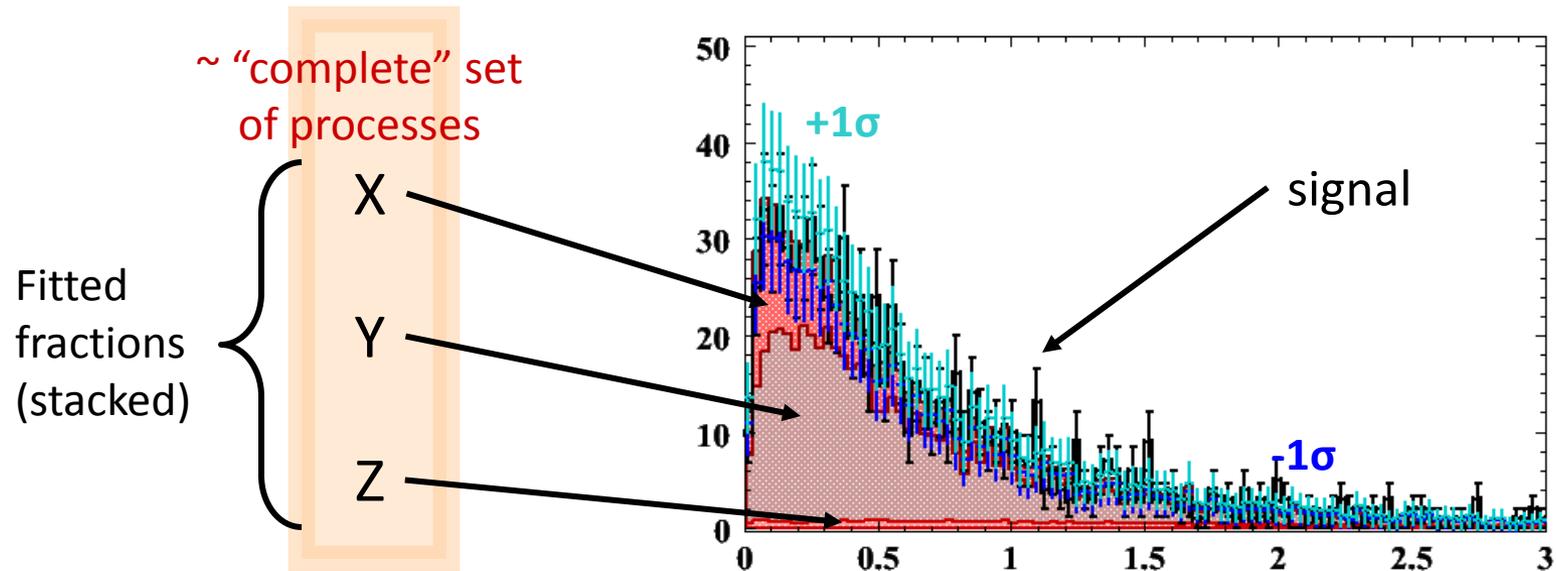
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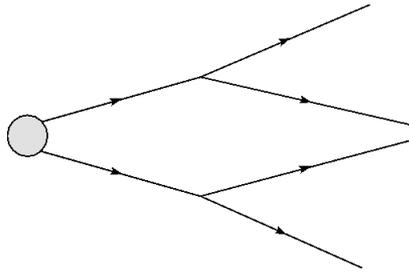
7. Models: good, bad, better

Fitting for fractions of processes in a SMS

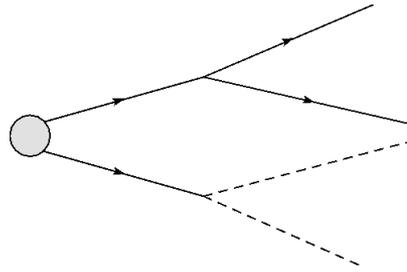


- To compose a viable BSM model, the processes must sum up to be compatible with the entire signal in all distributions.
- Standard template-fitting techniques (e.g. **TFractionFitter** @ [ROOT](#)) can be used to extract the best composition of processes to explain signal.
 - These methods typically implicitly or explicitly constrain the total yield (\equiv crosssection \times search cut efficiency) of the model to be equal to the number of observed signal events.
 - The cross-section deduced in this way can provide clues towards interpreting the SMS in terms of full theories (SUSY, UED, LTH, ...).

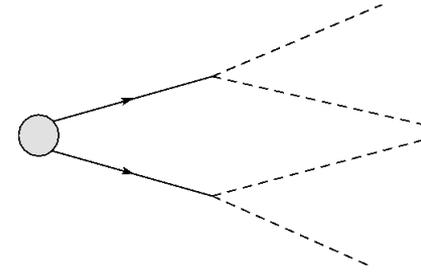
Fitting for fractions of processes in a SMS



$$\sim \text{BR}(\rightarrow \text{solid lines})$$



$$\sim \text{BR}(\rightarrow \text{solid lines}) \text{BR}(\rightarrow \text{dashed lines})$$



$$\sim [\text{BR}(\rightarrow \text{dashed lines})]^2$$

- The main subtlety here is if the fractions of the processes are constrained by some model relationship, e.g. a single BSM particle with exactly two decay modes $\rightarrow \text{solid lines}$ and $\rightarrow \text{dashed lines}$.

- Unitarity requires

$$\text{BR}(\rightarrow \text{solid lines}) = 1 - \text{BR}(\rightarrow \text{dashed lines})$$

which is a constraint that must be used to eliminate one degree of freedom in the otherwise 3-template fraction fit problem illustrated here.

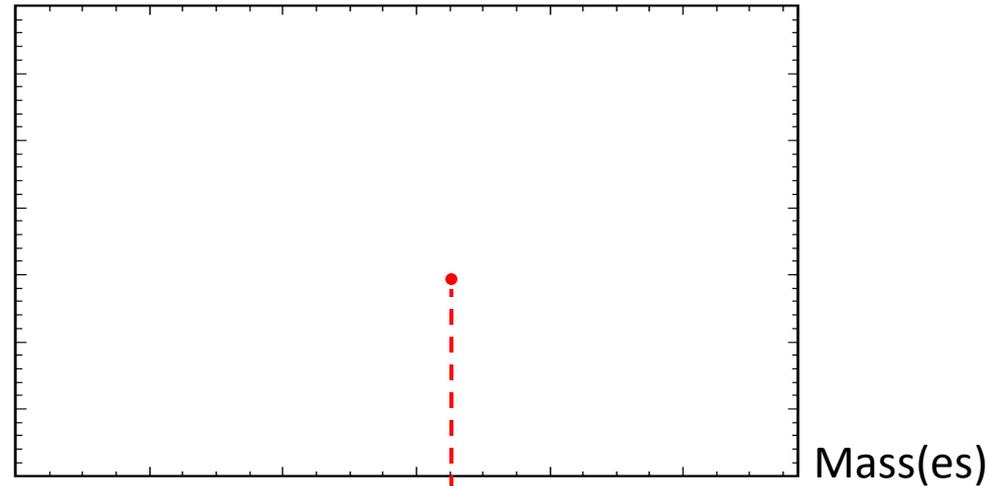
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Best-fit masses for each SMS model

So far:



At each set of masses that we generate MC for:

mass(1) = 80 GeV
mass(2) = 100 GeV
...

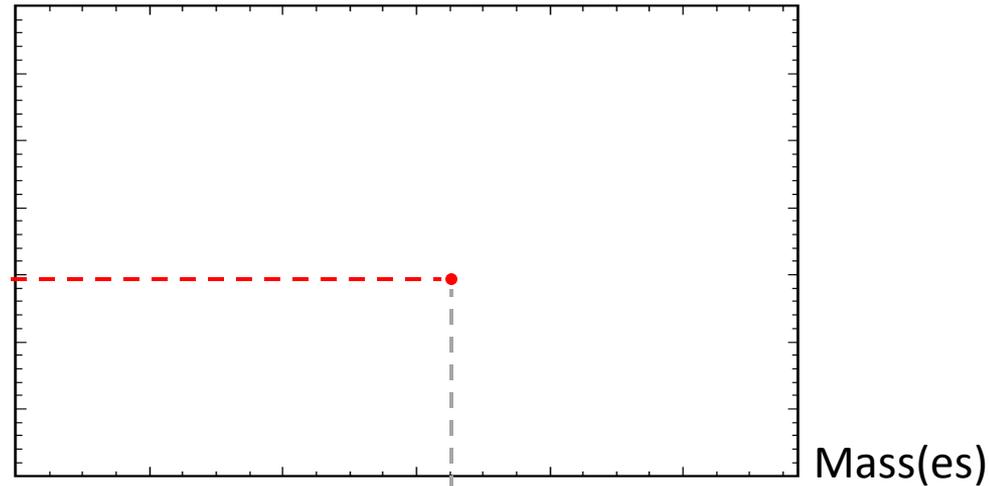
Perform a fit for other parameters (σ , BR)

(a cartoon)

Best-fit masses for each SMS model

So far:

“Height” of mass point
≡ Goodness of best fit
(χ^2 probability, ...)



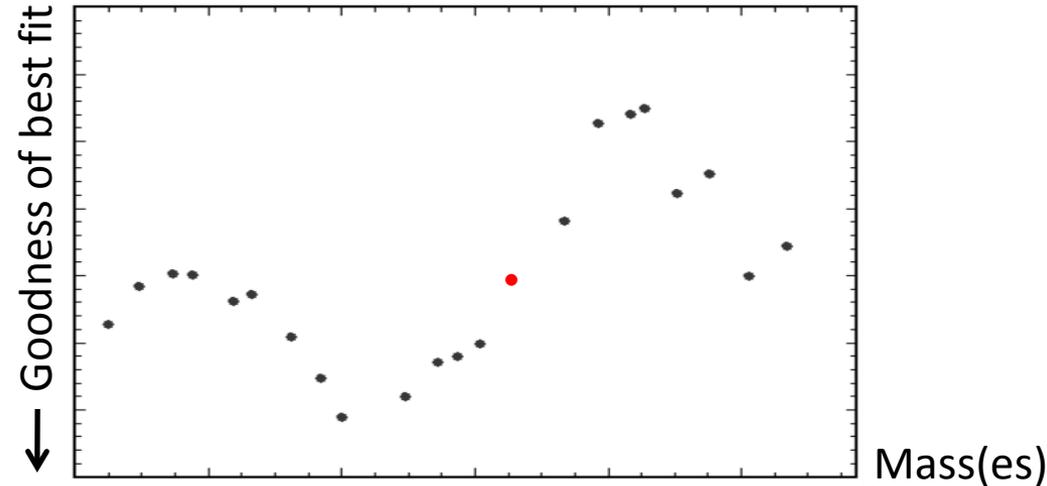
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Perform a fit for other
parameters (σ , BR)

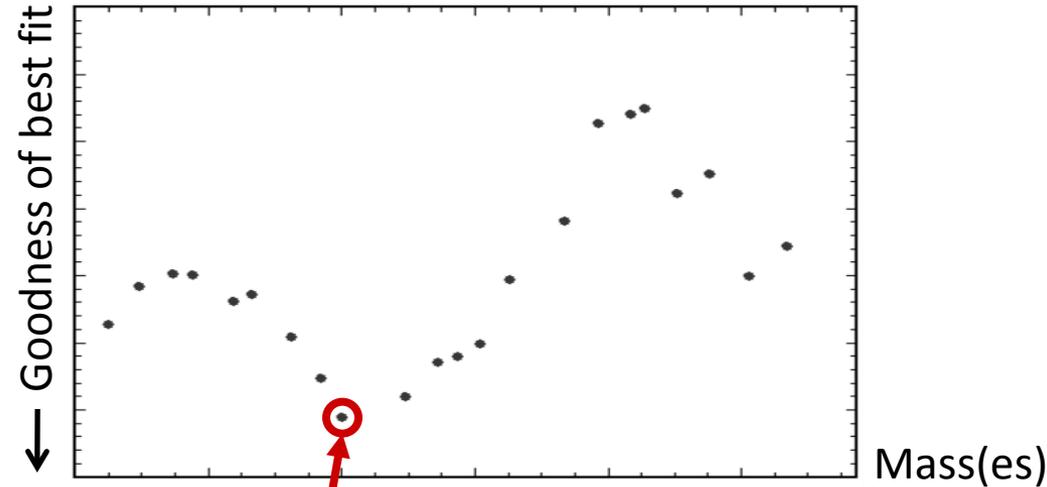
(a cartoon)

Best-fit masses for each SMS model



- Mass fitting is technically different (and more challenging) due to the computational burden of generating MC events for each mass hypothesis.
- One is likely restricted to a finite, discrete grid of mass points, and cannot directly use a minimization package to locate the “true” minimum.

Best-fit masses for each SMS model



Procedure 1

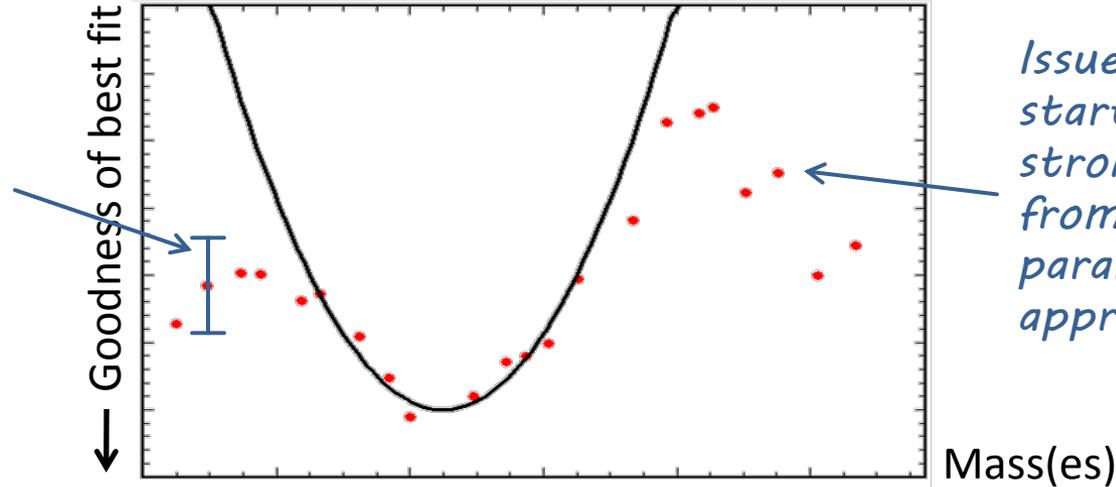
Pick the best generated point.

Not bad. But can do better by using information from neighboring points.

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Best-fit masses for each SMS model

Issue #1: What "error bars" can we assign to a goodness of fit?



Issue #2: Points start to diverge strongly away from the parabolic approximation.

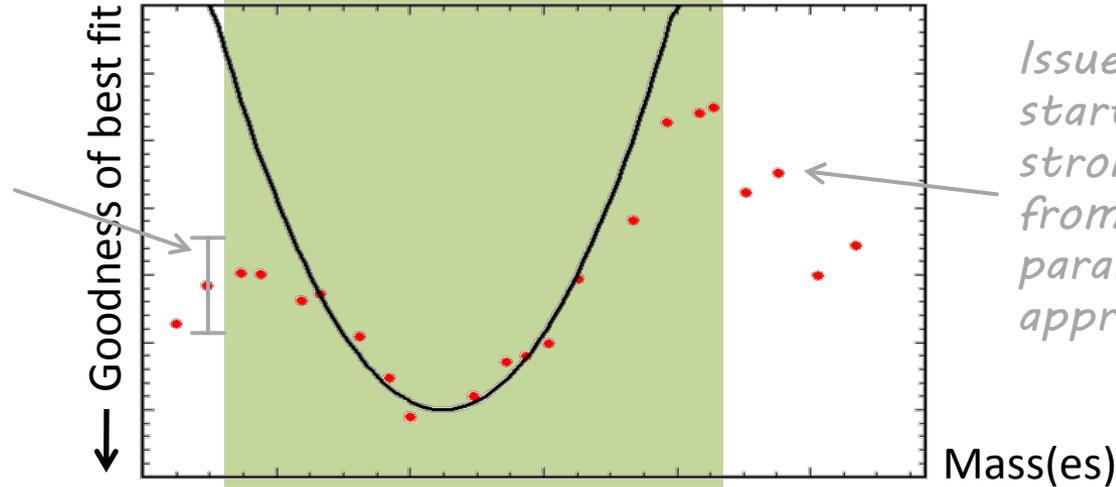
Procedure 2

Fit for parabolic expansion around minimum.

Issue #3: Can have multiple minima.

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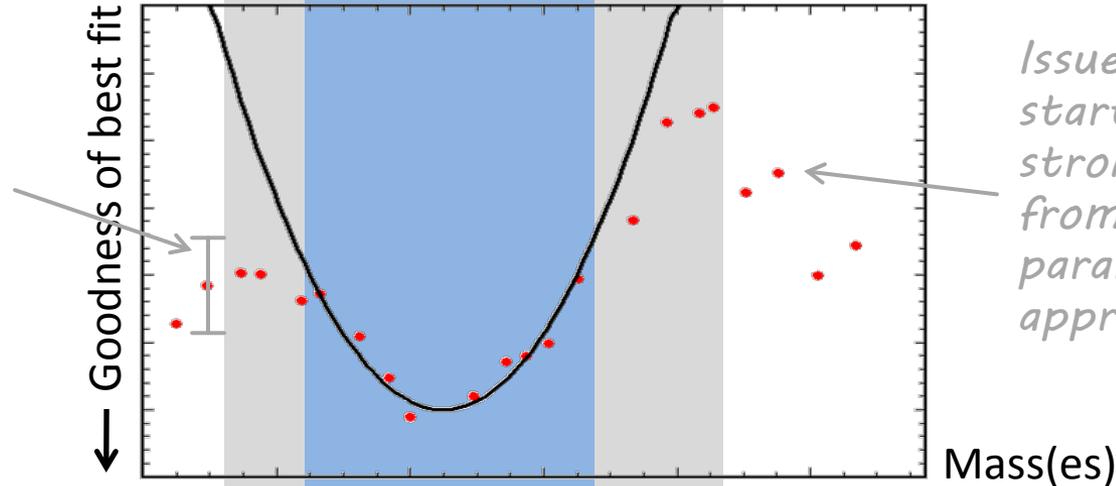
An "obvious" solution (at least for a 1D problem):

- Group points into regions containing exactly one minimum.

-

Best-fit masses for each SMS model

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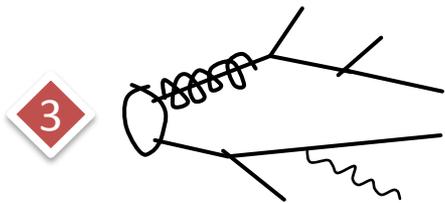
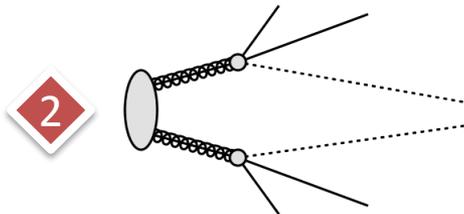
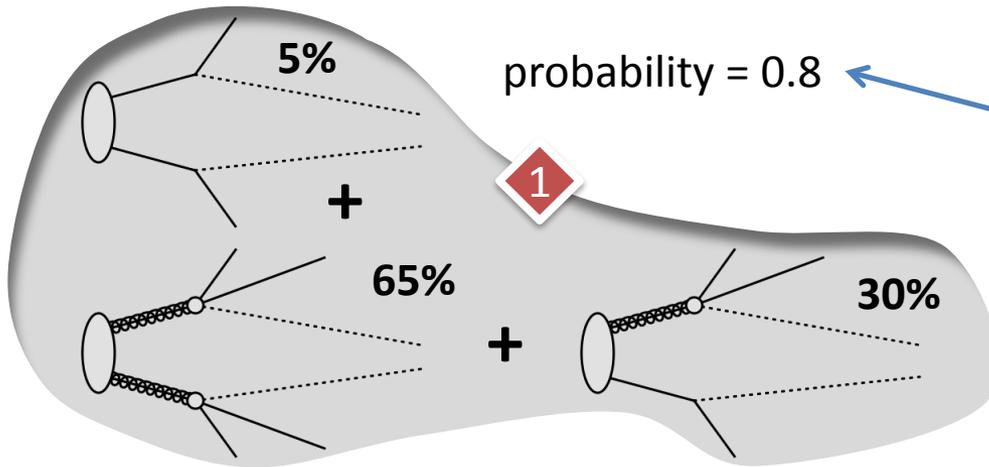
- Group points into regions containing exactly one minimum.
- Use only those points that are reasonably within the desired approximation in the parabola fit.

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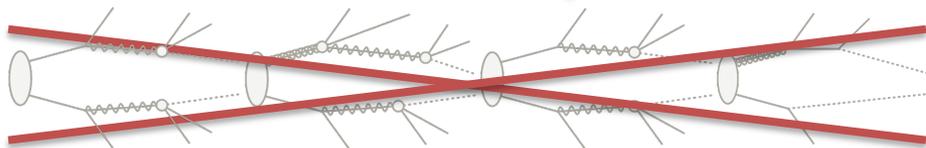
1. *Life at a hadron collider*
2. *Signal characterization, bottom-up*
3. *A hands-on procedure*
4. *Stages of understanding:*
 - a) *Upper bound on a process*
 - b) *Composing multi-process models*
 - c) *Deducing new particle masses*
5. *Models: good, bad, better*

160 μb^{-1}	1.2 nb^{-1}	5.2 nb^{-1}	19 nb^{-1}	21 nb^{-1}	21 nb^{-1}	110 nb^{-1}	350 nb^{-1}	930 nb^{-1}	1.8 pb^{-1}	3.6 pb^{-1}	3.6 pb^{-1}	8.3 pb^{-1}	20 pb^{-1}	45 pb^{-1}
17/4	1/5	15/5	29/5	12/6	26/6	10/7	24/7	7/8	21/8	4/9	18/9	2/10	16/10	30/10

Ranking models by compatibility with signal



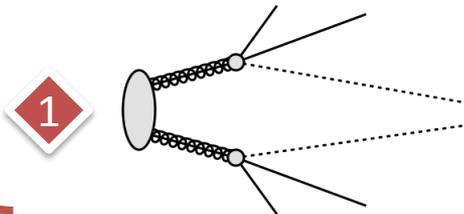
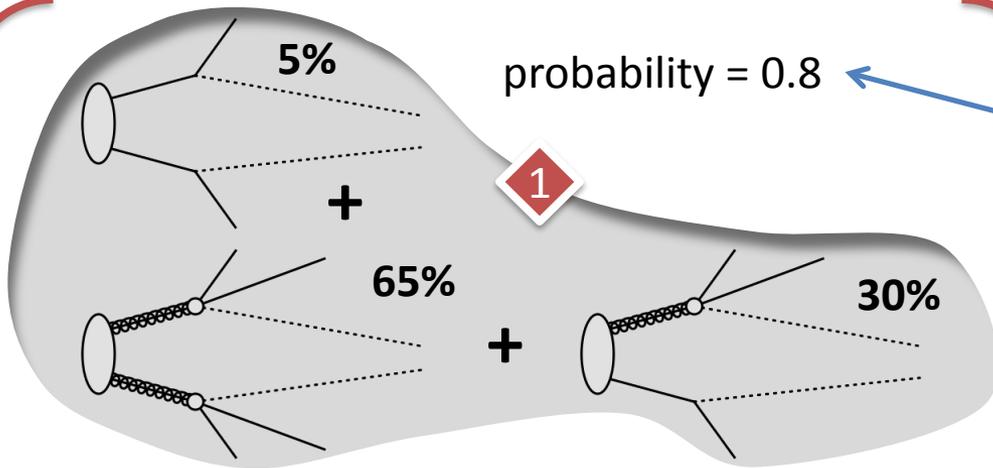
▼ probability $< 10^{-3}$



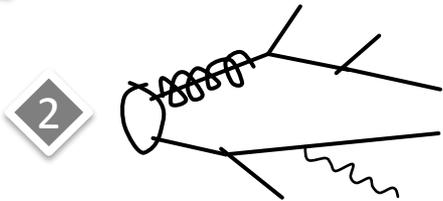
Increasing the number of free parameters (e.g. processes) always increases goodness of fit, but this may be a spurious effect.

All models with “low” probability are discarded as incompatible.

Ranking models by compatibility with signal

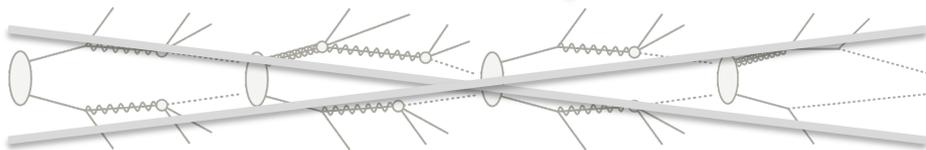


probability = 0.7



probability = 0.5

▼ probability $< 10^{-3}$



Increasing the number of free parameters (e.g. processes) always increases goodness of fit, but this may be a spurious effect.

Instead need to check if the two viable models are actually mutually compatible (within data uncertainties); if so they should be "tied" in rank.

All models with "low" probability are discarded as incompatible.

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1. Life at a hadron collider

jets + jets + ... + jets
= takes many pb^{-1} to
resolve details
= first pass will be
macroscopic

2. Signal characterization,
bottom-up

More flexible than top-
down tests for specific
models; quickly map
out signal possibilities

3. A hands-on procedure

4. Stages of understanding:

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b) Composing multi-process models

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Unsophisticated
techniques using
histogram
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Complementary
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Rank models by degree of
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... But can our searches predict all of these distributions?

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Need variables that can discriminate topologies...
... But can our searches predict all of these distributions?

300 recorded/s
 4×10^6 events/s
→ 99.99% events discarded:

Signal in some new, powerful corner of phase space need not be sitting around waiting to be analyzed at our convenience.

1. Life at a hadron collider

2. Signal characterization, bottom-up

3. A hands-on procedure

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Unsophisticated techniques using histogram comparison

Complementary to direct measurement (mass peaks)

⇒ Plan on planning ahead!

Rank models by degree of compatibility with observation

Chapter 1

It was a dark and stormy night. A team of physicists (armed with tall flasks of coffee) were watching with utmost care the monitors at _____, when all of a sudden...

a.k.a. The story is just beginning. 😊

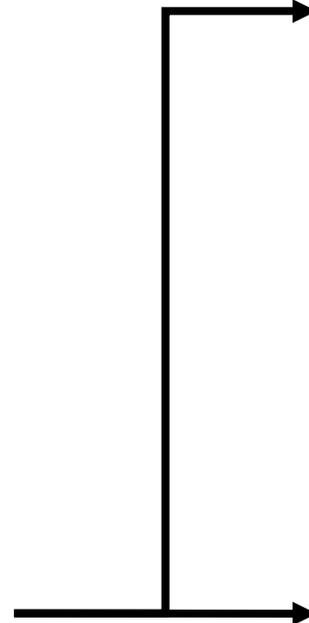


A problem encountered even for very simple pseudo-experiments.



BUT...

- How do we know what region falls within the parabolic approximation?
- Are we assured of there being exactly one minimum?



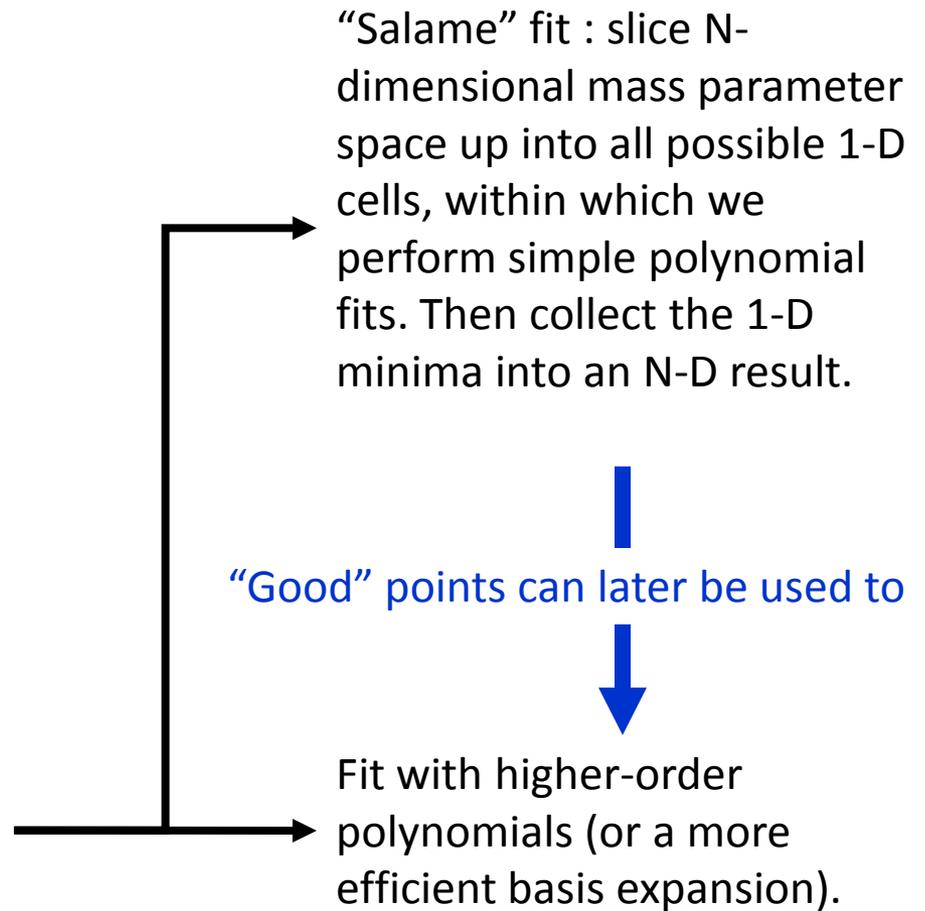
“Salame” fit : slice N-dimensional mass parameter space up into all possible 1-D cells, within which we perform simple polynomial fits. Then collect the 1-D minima into an N-D result.

Fit with higher-order polynomials (or a more efficient basis expansion).

Goodness-of-fit:

- Compare mass hypotheses to locate most likely spectrum

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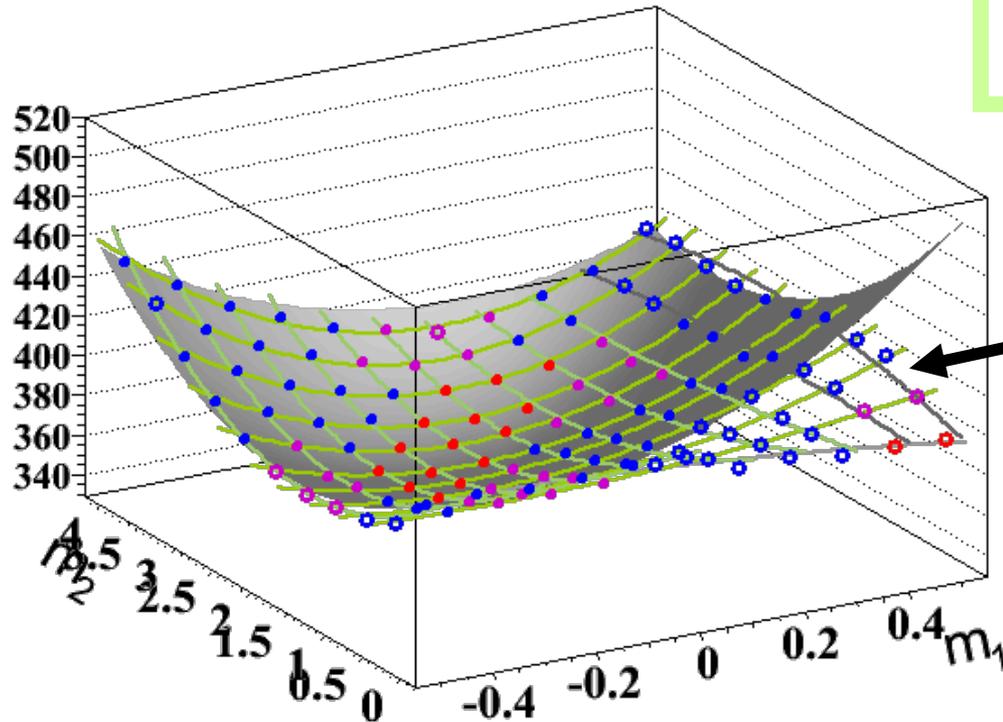
Goodness-of-fit:

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e.g. Particular cell in a 3-mass parameter space

"Salame" fit

$m_1 = x, m_2 = y, m_3 = 5$: 1D fits



The (mostly) green mesh are the 1-D slice fits to the shape function:

$$a + b x + c x^2$$

If a slice is convex ($c < 0$), it likely does not belong to an N-D *minimum* — an expansion around a minimum must have positive 2nd order derivatives.



Omit these points from N-D fit

BUT...

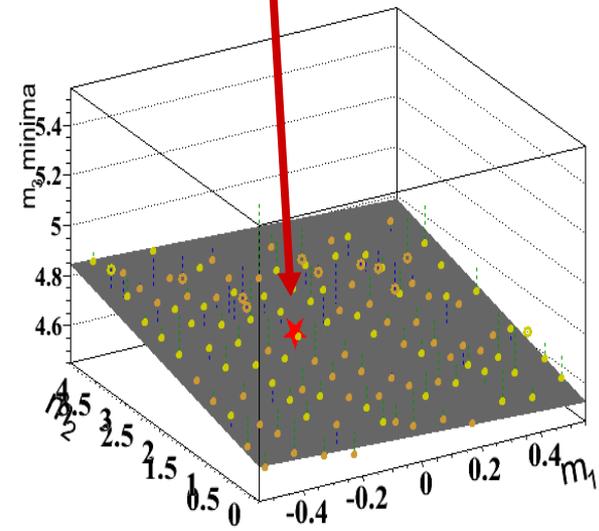
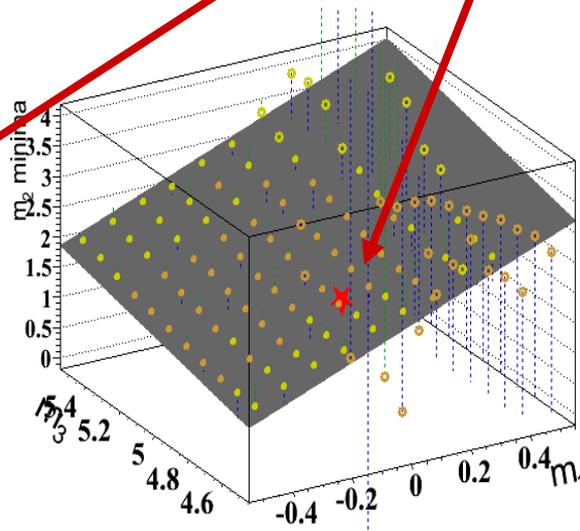
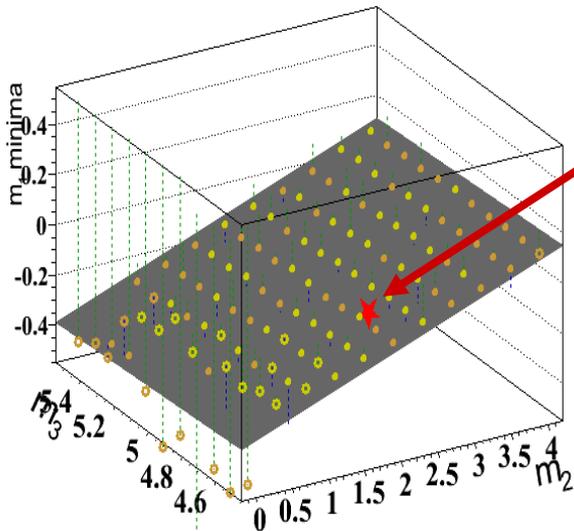
- How do we know what region falls within the parabolic approximation?
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The 1-D minima form N "planes" that are close to the principal axes of the N-D parabola around the global minimum.

The intersection of all these planes are a none-too-shabby estimator for the global minimum.



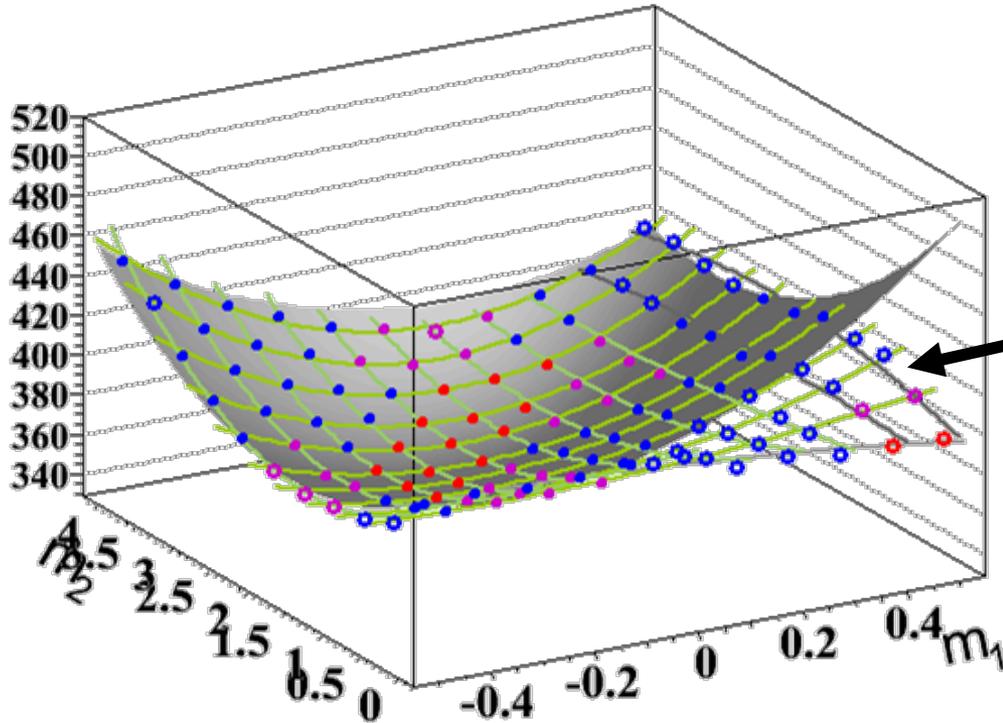
BUT...

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For a better estimator, we can also use the 1-D fits to more correctly seed the N-D fit.



If a slice is convex ($c < 0$), it likely does not belong to an N-D

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Omit these points from N-D fit

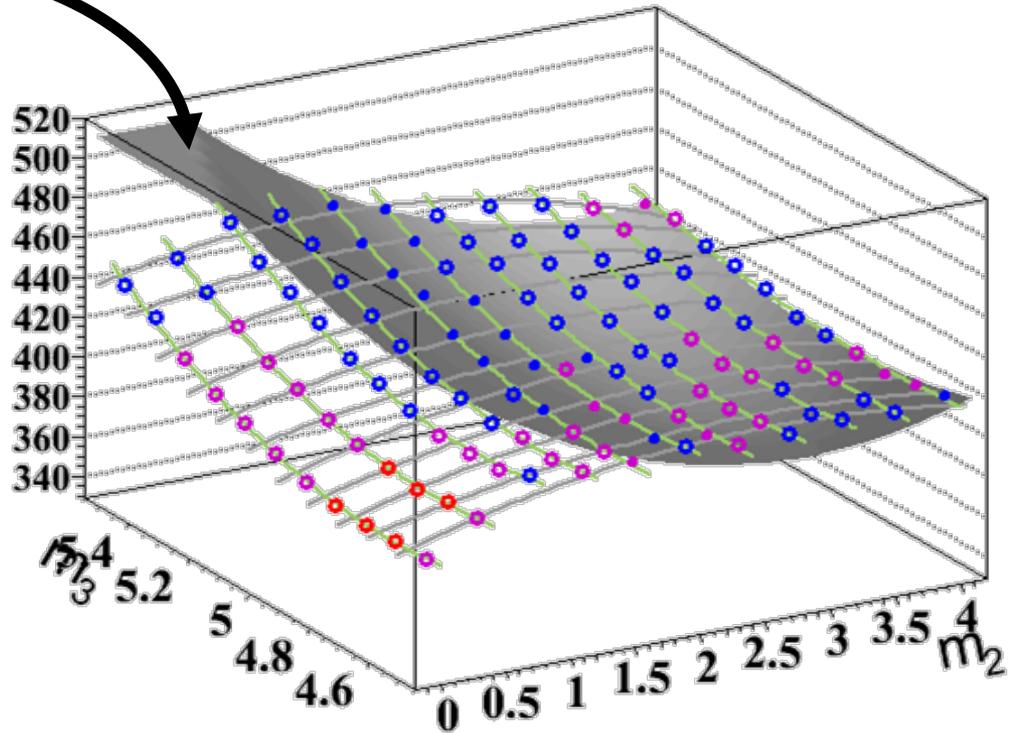
BUT...

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Goodness-of-fit:

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The N-D fit (gray surface) successfully ignores points that can't reasonably lie within the region where the parabolic expansion is valid (empty circles)



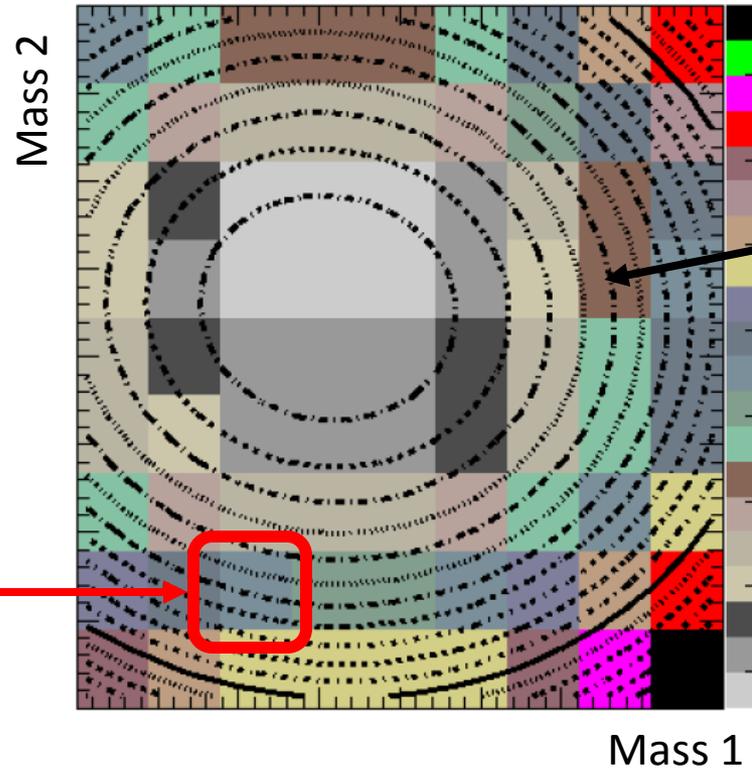
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Goodness-of-fit:

- Compare mass hypotheses to locate most likely spectrum

For a specific set of mass hypotheses (at this point of the mass grid): record how well we can fit the three processes to explain signal.



Contours of parabolic fit interpolating between points in (possibly coarse) grid

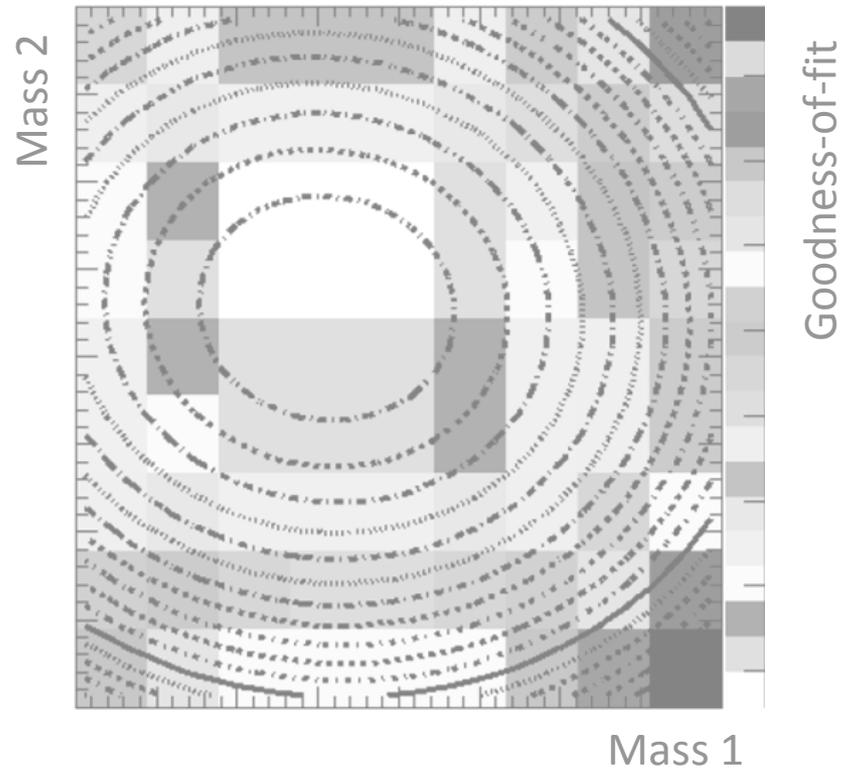
Refine grid after roughly locating minimum

For each particular model:

The goodness-of-fit (for the various processes) as we vary the mass parameters can be used to locate the most probable mass spectrum.

Goodness-of-fit:

- Compare mass hypotheses to locate most likely spectrum



BUT...

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