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MadGraph5_aMC@NLO for e^+e^- collisions

Based on: 1909.03886 (SF), 1911.12040 (Bertone, Cacciari, SF, Stagnitto) 2105.06688 (SF), and work in progress within MadGraph5_aMC@NLO (2108.10261, SF, Mattelaer, Zaro, Zhao)

ECFA 1^{st} topical meeting on generators, CERN, 9/11/2021

What is it

A single framework for the computation of:

A. Hard events (LHEF) at the NLO or LO, to be subsequently showered by either Pythia or Herwig [i.e. (N)LO+PS]

B. Infrared-safe observables at the NLO or LO [i.e. fixed order]

As in MadGraph* there is no pre-defined list of processes: all is generated/computed on the fly $\longleftarrow$ automation

The name is typically shortened as MG5_aMC

*MadGraph5_aMC@NLO has replaced MadGraph5 in 2014; the latter is obsolete

- Source code at:

  `https://launchpad.net/mg5amcnlo`

- Now on versions 2.9.6 (legacy) and 3.2.0 (that includes e^+e^- features). 3.3.0 is imminent (some new features + bug fixes)

- The code is routinely downloaded from the above web site by both LHC experiments and theorists. The site also features user support (questions, bug reports, . . .)

- Short-distance computations performed with a user-defined Lagrangian. Most models are included in the package (QCD+EW, MSSM, SMEFT, . . .)

The idea that underpins automation: the ability to carry out complex computations without necessarily having to understand any technical details

While automated codes have been employed predominantly in hadronic collisions, they *do* work for e^+e^- ones too

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With MG5_aMC, from 1405.0301 $\longrightarrow$

Process		Syntax	Cross section (pb)			
Heavy quarks and jets			LO 1 TeV		NLO 1 TeV	
i.1	$e^+e^- \to jj$	`e+ e- > j j`	$6.223 \pm 0.005 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$6.389 \pm 0.013 \cdot 10^{-1}$	$^{+0.2\%}_{-0.2\%}$
i.2	$e^+e^- \to jjj$	`e+ e- > j j j`	$3.401 \pm 0.002 \cdot 10^{-1}$	$^{+9.6\%}_{-8.0\%}$	$3.166 \pm 0.019 \cdot 10^{-1}$	$^{+0.2\%}_{-2.1\%}$
i.3	$e^+e^- \to jjjj$	`e+ e- > j j j j`	$1.047 \pm 0.001 \cdot 10^{-1}$	$^{+20.0\%}_{-15.3\%}$	$1.090 \pm 0.006 \cdot 10^{-1}$	$^{+0.0\%}_{-2.8\%}$
i.4	$e^+e^- \to jjjjj$	`e+ e- > j j j j j`	$2.211 \pm 0.006 \cdot 10^{-2}$	$^{+31.4\%}_{-22.0\%}$	$2.771 \pm 0.021 \cdot 10^{-2}$	$^{+4.4\%}_{-8.6\%}$
i.5	$e^+e^- \to t\bar{t}$	`e+ e- > t t~`	$1.662 \pm 0.002 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$1.745 \pm 0.006 \cdot 10^{-1}$	$^{+0.4\%}_{-0.4\%}$
i.6	$e^+e^- \to t\bar{t}j$	`e+ e- > t t~ j`	$4.813 \pm 0.005 \cdot 10^{-2}$	$^{+9.3\%}_{-7.8\%}$	$5.276 \pm 0.022 \cdot 10^{-2}$	$^{+1.3\%}_{-2.1\%}$
i.7*	$e^+e^- \to t\bar{t}jj$	`e+ e- > t t~ j j`	$8.614 \pm 0.009 \cdot 10^{-3}$	$^{+19.4\%}_{-15.0\%}$	$1.094 \pm 0.005 \cdot 10^{-2}$	$^{+5.0\%}_{-6.3\%}$
i.8*	$e^+e^- \to t\bar{t}jjj$	`e+ e- > t t~ j j j`	$1.044 \pm 0.002 \cdot 10^{-3}$	$^{+30.5\%}_{-21.6\%}$	$1.546 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-11.6\%}$
i.9*	$e^+e^- \to t\bar{t}t\bar{t}$	`e+ e- > t t~ t t~`	$6.456 \pm 0.016 \cdot 10^{-7}$	$^{+19.1\%}_{-14.8\%}$	$1.221 \pm 0.005 \cdot 10^{-6}$	$^{+13.2\%}_{-11.2\%}$
i.10*	$e^+e^- \to t\bar{t}t\bar{t}j$	`e+ e- > t t~ t t~ j`	$2.719 \pm 0.005 \cdot 10^{-8}$	$^{+29.9\%}_{-21.3\%}$	$5.338 \pm 0.027 \cdot 10^{-8}$	$^{+18.3\%}_{-15.4\%}$
i.11	$e^+e^- \to b\bar{b}$ (4f)	`e+ e- > b b~`	$9.198 \pm 0.004 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$9.282 \pm 0.031 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$
i.12	$e^+e^- \to b\bar{b}j$ (4f)	`e+ e- > b b~ j`	$5.029 \pm 0.003 \cdot 10^{-2}$	$^{+9.5\%}_{-8.0\%}$	$4.826 \pm 0.026 \cdot 10^{-2}$	$^{+0.5\%}_{-2.5\%}$
i.13*	$e^+e^- \to b\bar{b}jj$ (4f)	`e+ e- > b b~ j j`	$1.621 \pm 0.001 \cdot 10^{-2}$	$^{+20.0\%}_{-15.3\%}$	$1.817 \pm 0.009 \cdot 10^{-2}$	$^{+0.0\%}_{-3.1\%}$
i.14*	$e^+e^- \to b\bar{b}jjj$ (4f)	`e+ e- > b b~ j j j`	$3.641 \pm 0.009 \cdot 10^{-3}$	$^{+31.4\%}_{-22.1\%}$	$4.936 \pm 0.038 \cdot 10^{-3}$	$^{+4.8\%}_{-8.9\%}$
i.15*	$e^+e^- \to b\bar{b}b\bar{b}$ (4f)	`e+ e- > b b~ b b~`	$1.644 \pm 0.003 \cdot 10^{-4}$	$^{+19.9\%}_{-15.3\%}$	$3.601 \pm 0.017 \cdot 10^{-4}$	$^{+15.2\%}_{-12.5\%}$
i.16*	$e^+e^- \to b\bar{b}b\bar{b}j$ (4f)	`e+ e- > b b~ b b~ j`	$7.660 \pm 0.022 \cdot 10^{-5}$	$^{+31.3\%}_{-22.0\%}$	$1.537 \pm 0.011 \cdot 10^{-4}$	$^{+17.9\%}_{-15.3\%}$
i.17*	$e^+e^- \to t\bar{t}b\bar{b}$ (4f)	`e+ e- > t t~ b b~`	$1.819 \pm 0.003 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$2.923 \pm 0.011 \cdot 10^{-4}$	$^{+9.2\%}_{-8.9\%}$
i.18*	$e^+e^- \to t\bar{t}b\bar{b}j$ (4f)	`e+ e- > t t~ b b~ j`	$4.045 \pm 0.011 \cdot 10^{-5}$	$^{+30.5\%}_{-21.6\%}$	$7.049 \pm 0.052 \cdot 10^{-5}$	$^{+13.7\%}_{-13.1\%}$

Process		Syntax	Cross section (pb)			
Top quarks +bosons			LO 1 TeV		NLO 1 TeV	
j.1	$e^+e^- \to t\bar{t}H$	`e+ e- > t t~ h`	$2.018 \pm 0.003 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$1.911 \pm 0.006 \cdot 10^{-3}$	$^{+0.4\%}_{-0.5\%}$
j.2*	$e^+e^- \to t\bar{t}Hj$	`e+ e- > t t~ h j`	$2.533 \pm 0.003 \cdot 10^{-4}$	$^{+9.2\%}_{-7.8\%}$	$2.658 \pm 0.009 \cdot 10^{-4}$	$^{+0.5\%}_{-1.5\%}$
j.3*	$e^+e^- \to t\bar{t}Hjj$	`e+ e- > t t~ h j j`	$2.663 \pm 0.004 \cdot 10^{-5}$	$^{+19.3\%}_{-14.9\%}$	$3.278 \pm 0.017 \cdot 10^{-5}$	$^{+4.0\%}_{-5.7\%}$
j.4*	$e^+e^- \to t\bar{t}\gamma$	`e+ e- > t t~ a`	$1.270 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$1.335 \pm 0.004 \cdot 10^{-2}$	$^{+0.5\%}_{-0.4\%}$
j.5*	$e^+e^- \to t\bar{t}\gamma j$	`e+ e- > t t~ a j`	$2.355 \pm 0.002 \cdot 10^{-3}$	$^{+9.3\%}_{-7.9\%}$	$2.617 \pm 0.010 \cdot 10^{-3}$	$^{+1.6\%}_{-2.4\%}$
j.6*	$e^+e^- \to t\bar{t}\gamma jj$	`e+ e- > t t~ a j j`	$3.103 \pm 0.005 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$4.002 \pm 0.021 \cdot 10^{-4}$	$^{+5.4\%}_{-6.6\%}$
j.7*	$e^+e^- \to t\bar{t}Z$	`e+ e- > t t~ z`	$4.642 \pm 0.006 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$4.949 \pm 0.014 \cdot 10^{-3}$	$^{+0.6\%}_{-0.5\%}$
j.8*	$e^+e^- \to t\bar{t}Zj$	`e+ e- > t t~ z j`	$6.059 \pm 0.006 \cdot 10^{-4}$	$^{+9.3\%}_{-7.8\%}$	$6.940 \pm 0.028 \cdot 10^{-4}$	$^{+2.0\%}_{-2.6\%}$
j.9*	$e^+e^- \to t\bar{t}Zjj$	`e+ e- > t t~ z j j`	$6.351 \pm 0.028 \cdot 10^{-5}$	$^{+19.4\%}_{-15.0\%}$	$8.439 \pm 0.051 \cdot 10^{-5}$	$^{+5.8\%}_{-6.8\%}$
j.10*	$e^+e^- \to t\bar{t}W^{\pm}jj$	`e+ e- > t t~ wpm j j`	$2.400 \pm 0.004 \cdot 10^{-7}$	$^{+19.3\%}_{-14.9\%}$	$3.723 \pm 0.012 \cdot 10^{-7}$	$^{+9.6\%}_{-9.1\%}$
j.11*	$e^+e^- \to t\bar{t}HZ$	`e+ e- > t t~ h z`	$3.600 \pm 0.006 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$3.579 \pm 0.013 \cdot 10^{-5}$	$^{+0.1\%}_{-0.0\%}$
j.12*	$e^+e^- \to t\bar{t}\gamma Z$	`e+ e- > t t~ a z`	$2.212 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$2.364 \pm 0.006 \cdot 10^{-4}$	$^{+0.6\%}_{-0.5\%}$
j.13*	$e^+e^- \to t\bar{t}\gamma H$	`e+ e- > t t~ a h`	$9.756 \pm 0.016 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$9.423 \pm 0.032 \cdot 10^{-5}$	$^{+0.3\%}_{-0.4\%}$
j.14*	$e^+e^- \to t\bar{t}\gamma\gamma$	`e+ e- > t t~ a a`	$3.650 \pm 0.008 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$3.833 \pm 0.013 \cdot 10^{-4}$	$^{+0.4\%}_{-0.4\%}$
j.15*	$e^+e^- \to t\bar{t}ZZ$	`e+ e- > t t~ z z`	$3.788 \pm 0.004 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$4.007 \pm 0.013 \cdot 10^{-5}$	$^{+0.5\%}_{-0.5\%}$
j.16*	$e^+e^- \to t\bar{t}HH$	`e+ e- > t t~ h h`	$1.358 \pm 0.001 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$1.206 \pm 0.003 \cdot 10^{-5}$	$^{+0.9\%}_{-1.1\%}$
j.17*	$e^+e^- \to t\bar{t}W^+W^-$	`e+ e- > t t~ w+ w-`	$1.372 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$1.540 \pm 0.006 \cdot 10^{-4}$	$^{+1.0\%}_{-0.9\%}$

So are we done?

Not quite. In those results:

- NLO was in α_s, not α
- No beamstrahlung
- No description of all-order electron-mass factorisable effects
  (which implies collider energy $\equiv$ collision energy)

Consider the production of a system X at an e^+e^- collider:

$$e^+(P_{e^+}) + e^-(P_{e^-}) \longrightarrow X$$

Its cross section is written as follows:

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl=e^+e^-\gamma} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

Here:

- $d\Sigma_{e^+e^-}$: the collider-level cross section
- $d\sigma_{kl}$: the particle-level cross section
- $\mathcal{B}_{kl}(y_+, y_-)$: describes beam dynamics (including beamstrahlung)
- e^+, e^- on the lhs: the beams
- e^+, e^-, γ on the rhs: the particles

The particle-level cross section $d\sigma$ embeds all that is not beam dynamics

It is perturbatively computable, but plagued by $\log(m/E)$ terms to all orders. Fortunately, the dominant classes of these are factorisable:

$$d\sigma\left(\log(m/E), m/E\right) = \mathcal{K}\left(\log(m/E)\right) \otimes d\hat{\sigma}\left(m/E\right)$$

The idea is to compute $d\hat{\sigma}$ to some fixed order in perturbation theory, and $\mathcal{K}$ to all orders (so that logs are resummed)

The definitions of $\mathcal{K}$ and of the convolution $(\otimes)$ determine unambiguously how the logs are resummed. Historically (LEP), simulations have been predominantly done by adopting the YFS formalism

Therefore, two things to be done:

1. Compute $d\hat{\sigma}$

2. Compute $\mathcal{K}$ to all orders within a definite convolution scheme

Therefore, two things to be done:

1. Compute $d\hat{\sigma}$

   As shown before, MG5_aMC automated LO and NLO (in α_S) results.

   A further major progress has been achieved in 1804.10017: full automation of NLO computations in α (as well as for any combination $\alpha_S^k \alpha^p$).

   NLO+PS results still restricted to QCD showers

   $\longrightarrow$

Process	Syntax	Cross section (in pb)		Correction (in %)
		LO	NLO	
$pp \to e^+\nu_e$	`p p > e+ ve QCD=0 QED=2 [QED]`	$5.2498 \pm 0.0005 \cdot 10^3$	$5.2113 \pm 0.0006 \cdot 10^3$	-0.73 ± 0.01
$pp \to e^+\nu_e j$	`p p > e+ ve j QCD=1 QED=2 [QED]`	$9.1468 \pm 0.0012 \cdot 10^2$	$9.0449 \pm 0.0014 \cdot 10^2$	-1.11 ± 0.02
$pp \to e^+\nu_e jj$	`p p > e+ ve j j QCD=2 QED=2 [QED]`	$3.1562 \pm 0.0003 \cdot 10^2$	$3.0985 \pm 0.0005 \cdot 10^2$	-1.83 ± 0.02
$pp \to e^+e^-$	`p p > e+ e- QCD=0 QED=2 [QED]`	$7.5367 \pm 0.0008 \cdot 10^2$	$7.4997 \pm 0.0010 \cdot 10^2$	-0.49 ± 0.02
$pp \to e^+e^- j$	`p p > e+ e- j QCD=1 QED=2 [QED]`	$1.5059 \pm 0.0001 \cdot 10^2$	$1.4909 \pm 0.0002 \cdot 10^2$	-1.00 ± 0.02
$pp \to e^+e^- jj$	`p p > e+ e- j j QCD=2 QED=2 [QED]`	$5.1424 \pm 0.0004 \cdot 10^1$	$5.0410 \pm 0.0007 \cdot 10^1$	-1.97 ± 0.02
$pp \to e^+e^-\mu^+\mu^-$	`p p > e+ e- mu+ mu- QCD=0 QED=4 [QED]`	$1.2750 \pm 0.0000 \cdot 10^{-2}$	$1.2083 \pm 0.0001 \cdot 10^{-2}$	-5.23 ± 0.01
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	`p p > e+ ve mu- vm~ QCD=0 QED=4 [QED]`	$5.1144 \pm 0.0007 \cdot 10^{-1}$	$5.3019 \pm 0.0009 \cdot 10^{-1}$	$+3.67 \pm 0.02$
$pp \to He^+\nu_e$	`p p > h e+ ve QCD=0 QED=3 [QED]`	$6.7643 \pm 0.0001 \cdot 10^{-2}$	$6.4914 \pm 0.0012 \cdot 10^{-2}$	-4.03 ± 0.02
$pp \to He^+e^-$	`p p > h e+ e- QCD=0 QED=3 [QED]`	$1.4554 \pm 0.0001 \cdot 10^{-2}$	$1.3700 \pm 0.0002 \cdot 10^{-2}$	-5.87 ± 0.02
$pp \to Hjj$	`p p > h j j QCD=0 QED=3 [QED]`	$2.8268 \pm 0.0002 \cdot 10^0$	$2.7075 \pm 0.0003 \cdot 10^0$	-4.22 ± 0.01
$pp \to W^+W^-W^+$	`p p > w+ w- w+ QCD=0 QED=3 [QED]`	$8.2874 \pm 0.0004 \cdot 10^{-2}$	$8.8017 \pm 0.0012 \cdot 10^{-2}$	$+6.21 \pm 0.02$
$pp \to ZZW^+$	`p p > z z w+ QCD=0 QED=3 [QED]`	$1.9874 \pm 0.0001 \cdot 10^{-2}$	$2.0189 \pm 0.0003 \cdot 10^{-2}$	$+1.58 \pm 0.02$
$pp \to ZZZ$	`p p > z z z QCD=0 QED=3 [QED]`	$1.0761 \pm 0.0001 \cdot 10^{-2}$	$0.9741 \pm 0.0001 \cdot 10^{-2}$	-9.47 ± 0.02
$pp \to HZZ$	`p p > h z z QCD=0 QED=3 [QED]`	$2.1005 \pm 0.0003 \cdot 10^{-3}$	$1.9155 \pm 0.0003 \cdot 10^{-3}$	-8.81 ± 0.02
$pp \to HZW^+$	`p p > h z w+ QCD=0 QED=3 [QED]`	$2.4408 \pm 0.0000 \cdot 10^{-3}$	$2.4809 \pm 0.0005 \cdot 10^{-3}$	$+1.64 \pm 0.02$
$pp \to HHW^+$	`p p > h h w+ QCD=0 QED=3 [QED]`	$2.7827 \pm 0.0001 \cdot 10^{-4}$	$2.4259 \pm 0.0027 \cdot 10^{-4}$	-12.82 ± 0.10
$pp \to HHZ$	`p p > h h z QCD=0 QED=3 [QED]`	$2.6914 \pm 0.0003 \cdot 10^{-4}$	$2.3926 \pm 0.0003 \cdot 10^{-4}$	-11.10 ± 0.02
$pp \to t\bar{t}W^+$	`p p > t t~ w+ QCD=2 QED=1 [QED]`	$2.4119 \pm 0.0003 \cdot 10^{-1}$	$2.3025 \pm 0.0003 \cdot 10^{-1}$	-4.54 ± 0.02
$pp \to t\bar{t}Z$	`p p > t t~ z QCD=2 QED=1 [QED]`	$5.0456 \pm 0.0006 \cdot 10^{-1}$	$5.0033 \pm 0.0007 \cdot 10^{-1}$	-0.84 ± 0.02
$pp \to t\bar{t}H$	`p p > t t~ h QCD=2 QED=1 [QED]`	$3.4480 \pm 0.0004 \cdot 10^{-1}$	$3.5102 \pm 0.0005 \cdot 10^{-1}$	$+1.81 \pm 0.02$
$pp \to t\bar{t}j$	`p p > t t j QCD=3 QED=0 [QED]`	$3.0277 \pm 0.0003 \cdot 10^2$	$2.9683 \pm 0.0004 \cdot 10^2$	-1.96 ± 0.02
$pp \to jjj$	`p p > j j j QCD=3 QED=0 [QED]`	$7.9639 \pm 0.0010 \cdot 10^6$	$7.9472 \pm 0.0011 \cdot 10^6$	-0.21 ± 0.02
$pp \to tj$	`p p > t j QCD=0 QED=2 [QED]`	$1.0613 \pm 0.0001 \cdot 10^2$	$1.0539 \pm 0.0001 \cdot 10^2$	-0.70 ± 0.02

From 1804.10017; this is NLO in α; e^+e^- results can be obtained as easily as these ones, provided a definite scheme for item 2. above has been chosen (as is now the case)

Therefore, two things to be done:

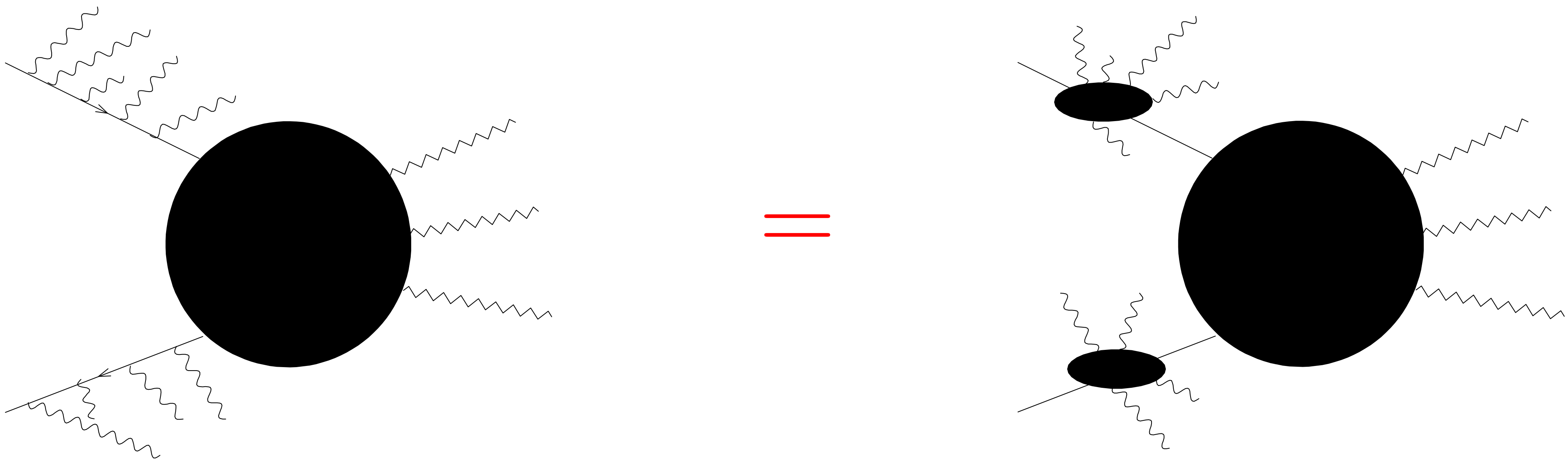
1. Compute $d\hat{\sigma}$

2. Compute $\mathcal{K}$ to all orders within a definite convolution scheme

   We adopt a collinear-factorisation approach. Comparisons with YFS-based predictions will help assess theoretical systematics in a comprehensive way

   (I'll concentrate here on ISR. Analogous formulae hold for FSR)

Collinear factorisation



$$d\sigma = \mathrm{PDF} \star \mathrm{PDF} \star d\hat{\sigma}$$

PDFs collect (universal) small-angle dynamics

$$d\sigma_{kl}(p_k,p_l) = \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_-\, \Gamma_{i/k}(z_+,\mu^2,m^2)\, \Gamma_{j/l}(z_-,\mu^2,m^2)$$
$$\times\, d\hat{\sigma}_{ij}(z_+p_k, z_-p_l, \mu^2) + \mathcal{O}\left(\left(\frac{m^2}{s}\right)^p\right)$$

where one calculates Γ and $d\hat{\sigma}$ to predict $d\sigma$

- $k\,, l = e^+\,, e^-\,, \gamma$ on the lhs: the particles that emerge from beamstrahlung
- $i\,, j = e^+\,, e^-\,, \gamma$ on the rhs: the partons
- $d\sigma_{kl}$: the particle-level (ie observable) cross section
- $d\hat{\sigma}_{ij}$: the subtracted parton-level cross section.
  Generally with $m = 0$ $\Longrightarrow$ power-suppressed terms in $d\sigma$ discarded
- $\Gamma_{i/k}$: the PDF of parton i inside particle k
- μ: the hard scale, $m^2 \ll \mu^2 \sim s$

As I have said, parton-level cross section computations are highly automated, and can now be carried out at the NLO in both α and α_S with MG5_aMC

Conversely, until recently PDFs were only available at the LO+LL, which is insufficient in the context of NLO (in α) simulations

$\Longrightarrow$

z-space LO+LL PDFs $(\alpha \log(E/m))^k$:

$\sim$ 1992

- $0 \leq k \leq \infty$ for $z \simeq 1$ (Gribov, Lipatov)
- $0 \leq k \leq 3$ for $z < 1$ (Skrzypek, Jadach; Cacciari, Deandrea, Montagna, Nicrosini; Skrzypek)
- matching between these two regimes

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- matching between these two regimes

z-space NLO+NLL PDFs $(\alpha \log(E/m))^k + \alpha\,(\alpha \log(E/m))^{k-1}$:

$\longrightarrow$ 1909.03886, 1911.12040, 2105.06688

- $0 \le k \le \infty$ for $z \simeq 1$
- $0 \le k \le 3$ for $z < 1 \quad \Longleftrightarrow \quad \mathcal{O}(\alpha^3)$
- matching between these two regimes
- for e^+, e^-, and γ
- both numerical and analytical

Main tool: the solution of PDFs evolution equations

In summary:

- The computations of both LO- and NLO-accurate (in both α_S and α) short-distance cross sections are fully automated in MG5_aMC
- QED PDFs of matching accuracy (NLL) are now also available

Embed the latter, plus beamstrahlung simulation, in the former, to obtain physical predictions at e^+e^- colliders for arbitrary processes

$\longrightarrow$ 2108.10261, and work in progress

Note: the above deals with *QED* ISR effects by means of an explicit convolution between PDFs and cross sections

Alternatively, one can simulate such effects by means of parton showers as is normally done in (N)LO+PS calculations

We have not (yet) implemented the latter in e^+e^- collisions, because:

- Non-trivial technical matching issues, due to the functional forms of the PDFs
- A genuine physics problem: no current parton shower can handle the NLL $e^\pm$ PDFs

Conversely, *QCD* (N)LO+PS calculations can be performed

Beamstrahlung in MG5_aMC (2108.10261)

General representation of the beamstrahlung function:

$$\mathcal{B}_{kl}(y_+, y_-) \approx \sum_{n=1}^{N} b_{n,kl}^{(e^+)}(y_+)\, b_{n,kl}^{(e^-)}(y_-)$$

In practice (so far):

$$N = 4\,, \quad k = e^+\,, \quad l = e^-\,,$$

and (separation of variables):

$$\begin{aligned}
\mathcal{B}_{e^+e^-}(y_+, y_-) \quad &= \quad \hat{f}_{11}\, \delta(1-y_+)\, \delta(1-y_-) \\
&+ \quad (1-y_+)^{\kappa_+} f_{01}(y_+)\, \delta(1-y_-) \\
&+ \quad \delta(1-y_-)\, (1-y_-)^{\kappa_-} f_{10}(y_-) \\
&+ \quad (1-y_+)^{\kappa_+} f_{00+}(y_+)\, (1-y_-)^{\kappa_-} f_{00-}(y_-)
\end{aligned}$$

The idea:

- Choose functional forms for the $f_\alpha(y_\pm)$ functions
  (possibly collider-specific: see 2108.10261 for examples)
- For any given collider, run GuineaPig with very high statistics
- Fit $f_\alpha(y_\pm)$ on GuineaPig results
  In 2108.10261 we have considered FCC-ee, CEPC, ILC, CLIC

Obviously process independent, i.e. to be done once and for all

Also, combine beamstrahlung and ISR, by exploting the separation of variables to write the collider-level cross section as:

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{n=1}^{N} \sum_{ijkl} \int dx_+ dx_- \, \phi^{(e^+)}_{i/k,n,kl}(x_+, \mu^2, m^2) \, \phi^{(e^-)}_{j/l,n,kl}(x_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(x_+ P_{e^+}, x_- P_{e^-}, \mu^2)$$

with:

$$\phi^{(e^\pm)}_{i/k,n,kl}(x, \mu^2, m^2) = \int dy \, dz \, \delta(x - yz) \, b^{(e^\pm)}_{n,kl}(y) \, \Gamma_{i/k}(z, \mu^2, m^2)$$

This is also process independent, i.e. to be done once and for all. MG5_aMC will include a code that performs this convolution

Take-home message

- MadGraph5_aMC@NLO now includes fully-realistic e^+e^- features: NLO corrections in both α and α_S, beamstrahlung, QED ISR effects by means of NLL-accurate PDFs, QCD NLO+PS simulations

- Current *public* version (3.2.0) still restricted to LL+LO in α
(again: no restrictions on QCD corrections)

- Bear in mind the possibility of performing short-distance computations with a user-defined Lagrangian (e.g. BSM, EFTs)

- A single code handles all type of collisions: features developed for hadronic collisions have been/will be ported to e^+e^-
[e.g.: work has started (2106.10279, 2106.12631) for the usage with GPUs and vector CPUs $\longrightarrow$ see A. Valassi's talk]