# The `BabaYaga` event generator: overview and future prospects

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# Outline

- References & motivations
- Theoretical formulation: QED PS and NLO matching
- Results
- Future developments
- Conclusions

# Relevant papers

- **BabaYaga** core references:

| | |
|---|---|
| Barzè et al., Eur. Phys. J. C **71** (2011) 1680 | BabaYaga with dark photon |
| Balossini et al., Phys. Lett. **663** (2008) 209 | BabaYaga@NLO for $e^+e^- \to \gamma\gamma$ |
| Balossini et al., Nucl. Phys. **B758** (2006) 227 | BabaYaga@NLO for Bhabha |
| C.M.C.C. et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 | BabaYaga@NLO |
| C.M.C.C., Phys. Lett. B **520** (2001) 16 | improved PS BabaYaga |
| C.M.C.C. et al., Nucl. Phys. B **584** (2000) 459 | BabaYaga |

- Related work:
  - S. Actis et al.
    "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
    Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
  - C.M.C.C. et al., JHEP **1107** (2011) 126
    NNLO massive pair corrections

# Motivations: reference processes for luminometry

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

- Reference processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ **Large-angle QED processes** $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to get typical precision at the $0.1\%$ level

↪ QED radiative corrections are mandatory

↦ `BabaYaga` was developed for high-precision simulation of QED processes at flavour factories (up to $\sqrt{s} \simeq 10$ GeV), primarily for luminosity determination

↦ At energies for which the code was developed for, only QED corrections are important

# The event generator `BabaYaga`/`BabaYaga@NLO`

- Theoretical accuracy at $0.1\%$ (or slightly better) for integrated cross sections for luminosity monitoring
- Based on an *in-house* implementation of a **QED Parton Shower**, ***consistently matched with exact QED NLO matrix elements***
  - ↪ An arbitrary number of (extra) photons can be generated
- Used by KLOE/KLOE2, Babar, BesIII, Novosibirsk experiments, Belle
- The same QED PS & (EWK) NLO matching framework successfully applied also to Drell-Yan processes (`HORACE`[1]) and $H \to 4\ell$ (`Hto4l`[2])

[1] CMCC *et al.*, JHEP 0612 (2006) 016, JHEP 0710 (2007) 109

[2] S. Boselli *et al.*, JHEP 1506 (2015) 023, JHEP 01 (2018) 096

- B.t.w., also `Hto4l` might be of interest to this meeting

  It's a code for Higgs decay into 4 charged leptons including one-loop EW corrections and multiple photon radiation or dim6 operators in EFT

# QED Parton Shower

- ⋆ Common methods used to account for multiple photon corrections are analytical collinear QED Structure Functions (SF), YFS exponentiation, QED Parton Shower (PS)

- The QED PS is an exact MC solution of the QED DGLAP equation for the non-singlet electron SF $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

- The PS solution can be cast into the form

$$D(x, Q^2) \; = \; \Pi(Q^2, \epsilon) \sum_{n=0}^{\infty} \; \frac{1}{n!} \int \delta(x - x_1 \cdots x_n) \; \prod_{i=0}^{n} \left[ \frac{\alpha}{2\pi} P(x_i) \; L \; dx_i \right]$$

  → $\Pi(Q^2, \epsilon) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$,
  $L \equiv \ln Q^2/m^2$ collinear log, $\epsilon$ soft/hard separator and $Q^2$ virtuality scale

  → the kinematics of the photon emissions can be recovered → exclusive photons generation

- The accuracy is improved by matching leading-log PS with exact NLO matrix elements (NLOPS)

  ⇝ theoretical error starts then at $\mathcal{O}(\alpha^2)$ (NNLO)

# Matching NLO and PS in BabaYaga@NLO

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual ($SV$) corrections and hard-bremsstrahlung ($H$) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma_{PS}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma_{PS}^{\alpha} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma_{PS}^{SV}(\varepsilon) + d\sigma_{PS}^{H}(\varepsilon)$
- $d\sigma_{\mathrm{NLO}}^{\alpha} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_1|^2 d\Phi_3 \equiv d\sigma_{\mathrm{NLO}}^{SV}(\varepsilon) + d\sigma_{\mathrm{NLO}}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

$$d\sigma_{\mathrm{matched}}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$$

**$d\Phi_n$ is the exact phase space for $n$ final-state particles**

(2 fermions + an arbitrary number of photons)

***Any approximation is confined into matrix elements***

# Matching NLO and PS in `BabaYaga@NLO`

- $F_{SV}$ and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\mathrm{NLO}}$
- resummation of higher orders LL (PS) contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles ($F$'s correction factors are calculated and applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \otimes$ [leading-logs]

G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \simeq \begin{cases} 5\times10^{-4} & \text{at 1 GeV} \\ 1\times10^{-3} & \text{at 1 TeV} \end{cases}$$

# Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

| | | | |
|---|---|---|---|
| LO | $\alpha^0$ | | |
| NLO | $\alpha L$ | $\alpha$ | |
| NNLO | $\frac{1}{2}\alpha^2 L^2$ | $\frac{1}{2}\alpha^2 L$ | $\frac{1}{2}\alpha^2$ |
| h.o. | $\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$ | $\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$ | $\cdots$ |

Blue: Leading-Log PS, SF

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| h.o. | $\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$ | $\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$ | $\cdots$ |

Red: matched PS, SF + NLO

# Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

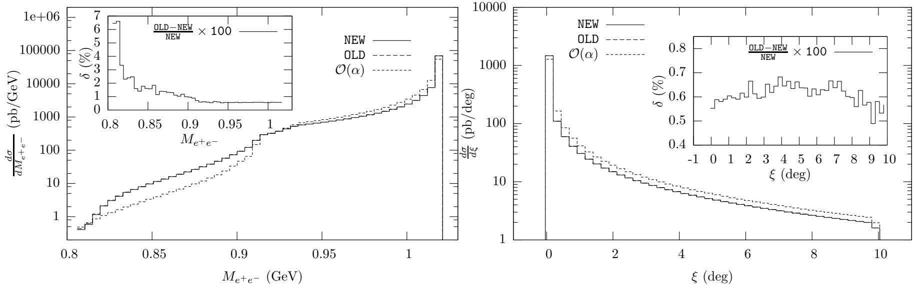
| | | | |
|---|---|---|---|
| LO | $90\%$ | | |
| NLO | $10\%$ | $0.5\%$ | |
| NNLO | $0.5\%$ | $0.05\%$ | $0.01\%$ |
| h.o. | $0.01\%$ | $\cdots$ | $\cdots$ |

Typically at flavour factories (on integrated Bhabha $\sigma$)

# Results with `BabaYaga@NLO`

from Balossini et al., Nucl. Phys. **B758** (2006) 227

↦ $M_{e^+e^-}$ invariant mass and acollinearity distributions, at KLOE



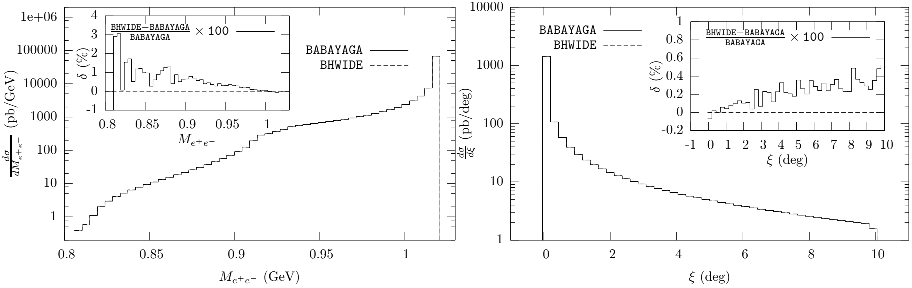
`OLD` → pure PS    `NEW` → matched PS with NLO (NLOPS)    $\mathcal{O}(\alpha)$ → exact NLO

⇝ both exact QED NLO and higher orders resummation are essential for high-precision simulations

# Estimating the theoretical accuracy

from S. Actis et al. Eur. Phys. J. C **66** (2010) 585

- It is extremely important to compare independent calculations/implementations/codes, in order to
  - ↦ asses the technical precision, spot bugs (with the same “theory ingredients”)
  - ↦ estimate theoretical errors when including partial/incomplete higher-order corrections
- E.g. comparison `BabaYaga@NLO` vs. `BHWIDE` (Jadach, Płaczek, Ward) at KLOE

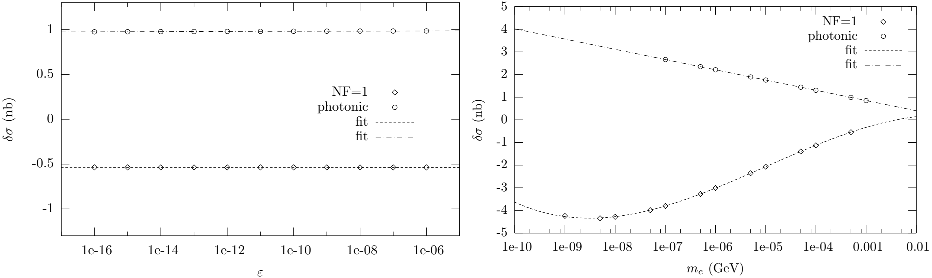


# Estimating the theoretical accuracy by measuring missing NNLO

Using realistic cuts for luminosity at KLOE

The `BabaYaga@NLO` NLOPS master formula can be expanded up to NNLO and consistently compared to exact results

- e.g., vs the class of exact photonic soft+virtual QED NNLO corrections, function of the soft photon cut-off $\varepsilon$ and $m_e$



★ differences are infrared safe and $\delta\sigma(\text{photonic})/\sigma^{\text{LO}} \propto \alpha^2 L$, as expected

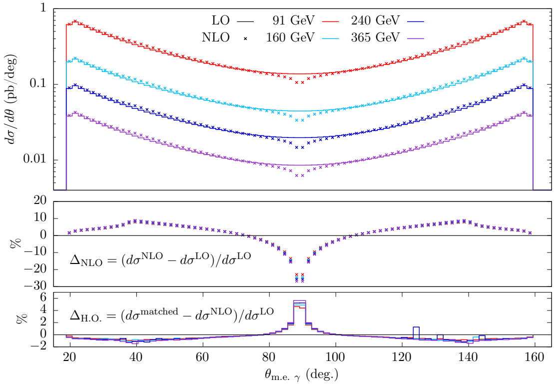
- Numerically, for various selection criteria at the $\Phi$ and $B$ factories

$$\sigma_{\text{SV}}^{\text{NNLO}}(\text{photonic}) - \sigma_{\text{SV}}^{\text{NNLO}}(\texttt{BabaYaga@NLO}) < 0.02\% \times \sigma^{\text{LO}}$$

# `BabaYaga@NLO` at FCC energies: $e^+e^- \to \gamma\gamma$

CMCC et al., Phys. Lett. B **798** (2019), 134976

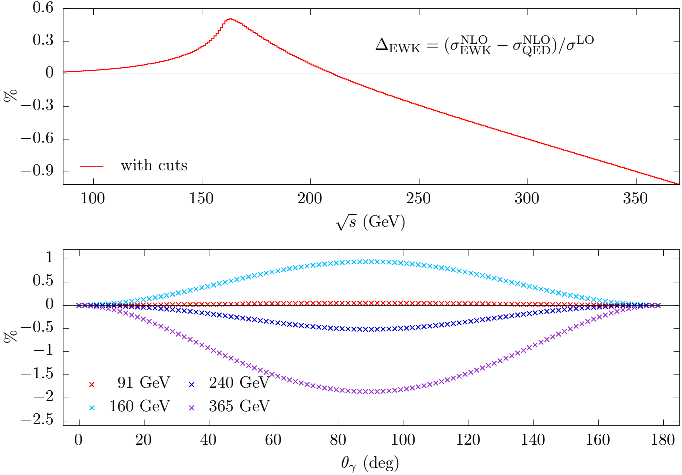
- We explored the possibility to use $e^+e^- \to \gamma\gamma$ at large angle as luminosity monitor
- at LO, it's a pure QED process at any energy
- at NLO, weak corrections enter but no fermionic-loops yet
  (chiefly no hadronic loops, source of theorerical uncertainty for Bhabha already at NLO)
- a $\mathcal{O}(10^{-4})$ theoretical accuracy is likely achievable



# $e^+e^- \to \gamma\gamma$ at FCC energies

CMCC et al., Phys. Lett. B **798** (2019), 134976

- purely weak corrections at NLO



using **`Recola`**

S. Actis et al., JHEP 04:037, 2013 & CPC 214:140–173, 2017

# Needed improvements for the future & technicalities

- In order to be used at higher energies, the full set of EWK corrections must be implemented and included for all the processes

  $\rightsquigarrow$ In principle, a doable task, the same framework already implemented in **`HORACE`** with QED PS/EWK NLO matching (for Drell-Yan)
- Going from NLOPS to NNLOPS accuracy, by matching to NNLO

- The code is written in `Fortran` (77!)
- It is currently availabe upon request from the authors (although an out-dated website exists)

  $\rightsquigarrow$ We plan to make it public on a `GitHub/GitLab` repository
- Events format at the moment is not standardized, we followed users' requests

  $\rightsquigarrow$ A LHE interface is however implemented for $e^+e^- \to \gamma\gamma$
- All the calculations are performed for unpolarized cross sections

  $\rightsquigarrow$ In principle, polarization can be implemented
- Neither beamsstrahlung nor crossing angle included yet, but in principle not an issue

  $\rightsquigarrow$ Gaussian smearing of beam momenta is already implemented
- CPU performance:

  depends on generator level cuts, included RCs, requiring weighted or unweighted events.

  $\rightsquigarrow$ E.g., for Bhabha at NLOPS at KLOE, $10^7$ weighted events in 45s on my laptop (single core), with relative MC error on cross section of $\simeq 3 \cdot 10^{-4}$

# Conclusions

- The `BabaYaga@NLO` event generator is a “special-purpose” Monte Carlo event generator for high-precision simulation of **Bhabha**, $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \gamma\gamma$ processes with multiple photon emission at NLOPS accuracy in QED
- As of now, it is tailored for flavour factories, thus only QED corrections are included
- Only $e^+e^- \to \gamma\gamma$ has been “ported” to FCC energies
- Large room for improvements under consideration:
  - $\rightsquigarrow$ including full set of EWK corrections
  - $\rightsquigarrow$ match QED PS at NNLO
  - $\rightsquigarrow$ standardize event storage
  - $\rightsquigarrow$ make the code public on a `git`-like repository

# SPARES

# Error budget for Bhabha luminometry at flavour factories

main conclusion of the Luminosity Section of Eur. Phys. J. C **66** (2010) 585

- Putting the sources of uncertainties (in large-angle Bhabha) all together:

| Source of error (%) | $\Phi-$factories | $\sqrt{s}=3.5$ GeV | $B-$factories |
|---|---|---|---|
| $\vert\delta^{\rm err}_{\rm VP}\vert$ [Jegerlehner] | 0.00 | 0.01 | 0.03 |
| $\vert\delta^{\rm err}_{\rm VP}\vert$ [HMNT] | 0.02 | 0.01 | 0.02 |
| $\vert\delta^{\rm err}_{{\rm SV},\alpha^2}\vert$ | 0.02 | 0.02 | 0.02 |
| $\vert\delta^{\rm err}_{{\rm HH},\alpha^2}\vert$ | 0.00 | 0.00 | 0.00 |
| $\vert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\vert$ | 0.05 | 0.05 | 0.05 |
| $\vert\delta^{\rm err}_{\rm pairs}\vert$ | 0.03 | 0.016 | 0.03 |
| $\vert\delta^{\rm err}_{\rm total}\vert$ linearly | 0.12 | 0.1 | 0.13 |
| $\vert\delta^{\rm err}_{\rm total}\vert$ in quadrature | 0.07 | 0.06 | 0.06 |

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements at the 0.1% level.
It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP

- For the experiments on top of and closely around the narrow resonances ($J/\psi$, $\Upsilon$,...), the accuracy quickly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ parameterizations and/or their intrinsic error