Genuine weak corrections and τ lepton decays for phenomenology and Monte Carlo programs. From LEP to LHC, Belle2 and to FCC, ILC, CLIC

Z. Was*

* Institute of Nuclear Physics, Polish Academy of Sciences, Cracow, Poland

- **(1)** Genuine weak corrections, and libraries evolved over 4 decades. They were used with the KKMC and KORALZ and are archived in Comput.Phys.Commun. 260 (2021) 107734. Important for benchmarks.

- **(2)** Tau lepton decays and Belle 2, One day better hadronic currents from them...

- **(3)** Inject into $e^+e^- \to \tau^+\tau^- n\gamma$ (tau decayed) events, dark photon (dark(?) scalar), with the help of C, C++ variant photospp.

- **(4)** These are associated activities for presented earlier today KKMC developed first for LEP but now used for (i) LHC applications 2012.09298, (ii) future e^+e^- accelerators 2106.11802 and (iii) Belle 2, already installed in their basf2 software system.

- **(5)** In short, all these associated activities have independent life too. Web pages:

//tauolapp.web.cern.ch/tauolapp/ //photospp.web.cern.ch/photospp/ gitlab distr. not yet recommended outside Belle 2.

(1) Genuine weak: why LEP 1 pheno matters.

effective $\sin^2\theta_W^{eff}$: enumerate sensitivity of electroweak measurements.

●, $\sin^2\theta_W^{eff}$ ambiguity, SLD+LEP combined: $16\cdot 10^{-5}$ (Phys.Rep. 427 2006 257).

●, FCC-ee of distant future: expected ambiguity sizably smaller $0.6\cdot 10^{-5}$ (1905.05078).

●, for LHC it reads: (i) ATLAS: 0.23140 $\pm 21\cdot 10^{-5}$ (stat) $\pm 24\cdot 10^{-5}$ (PDF) $\pm 16\cdot 10^{-5}$ (syst) in total $\pm 36\cdot 10^{-5}$ (Eur.Phys.J.C 80 (2020) 831). (ii) CMS: 0.23101 $\pm 36\cdot 10^{-5}$ (stat) $\pm 18\cdot 10^{-5}$ (syst) $\pm 16\cdot 10^{-5}$ (theo) $\pm 31\cdot 10^{-5}$ (PDF) total $\pm 53\cdot 10^{-5}$ (Eur.Phys.J.C 78 (2018) 701). (iii) HL-LHC ATLAS prospects: $\pm 18\cdot 10^{-5}$ (total) and $\pm 15\cdot 10^{-5}$ (PDF) (ATL-PHYS-PUB-2018-037) (iv) HL-LHC CMS prospects: $\pm 12\cdot 10^{-5}$ (CMS-PAS-FTR-17-001).

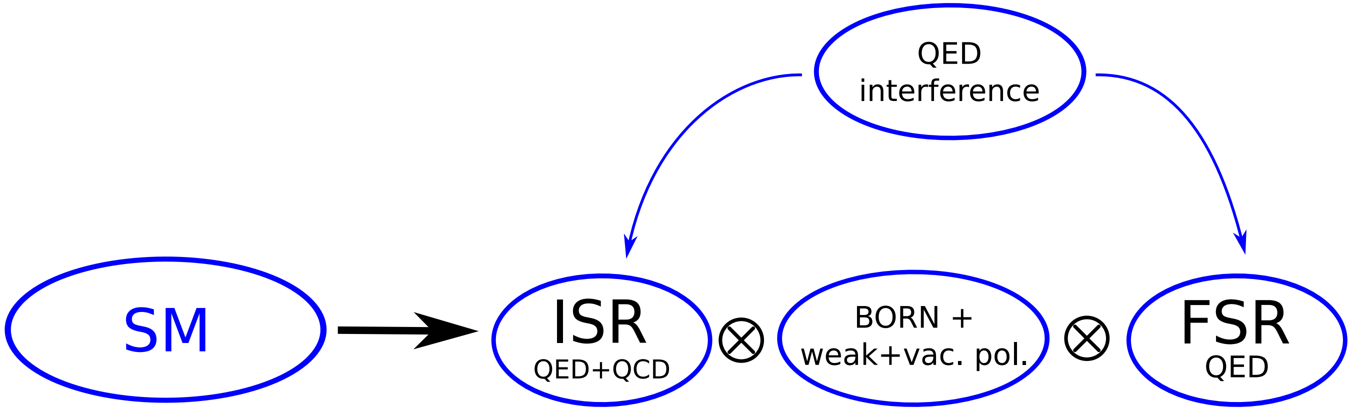
● One observes, that in all LHC cases, dominant uncertainties come from strong interactions: from parton distribution functions (PDF)s, configurations with additional jets and multi-loop corrections of QCD. That is where the bulk of effort is necessary. To ease the difficulties: eliminate or reduce complications due to electroweak effects. For LHC measurements, so far systematic error of $35\cdot 10^{-5}$ was achieved; expected to reach $18\cdot 10^{-5}$.

● It is comparable to SLD+LEP. LEP time EW sector theoretical predictions and programs should remain appropriate, if proper implementation into pp pheno. is prepared.

● LEP 1 software $\longrightarrow$ reference platform for efforts.

(1) Genuine weak: why LEP 1 pheno matters.

LEP 1: Production and decay of Z/γ^*



- Picture of applicability beyond LEP 1 e^+e^- Z-peak collisions.
- Genuine weak corrections were calculated at one loop level.
- Separated out QED corrections were treated at second order also exclusive exponentiation taken into account.
- Vaccum polarization corrections contributed to so called '**lineshape corrections**'. Up to 3 loop QCD contributions for quark loops were used. Also photon vaccum polarization $\Pi_{\gamma\gamma}(s)$, from dispersion relations and low energy $e^+e^- \to hadrons$ data.

(1) Genuine weak

Result of massive LEP 1 time effort: Genuine weak – QED QCD separation

$$
\begin{aligned}
ME_{Born+EW} = \quad \mathcal{N}\frac{\alpha}{s}\{ \quad & [\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u](q_e\cdot q_f)\Gamma_{V_{\Pi}}\chi_{\gamma}(s) + \big[\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}u\cdot(v_e\cdot v_f\cdot vv_{ef}) \\
& +\bar{u}\gamma^{\mu}v g_{\mu\nu}\bar{v}\gamma^{\nu}\gamma^{5}u\cdot(v_e\cdot a_f) + \bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{v}\gamma^{\nu}u\cdot(a_e\cdot v_f) \\
& +\bar{u}\gamma^{\mu}\gamma^{5}v g_{\mu\nu}\bar{v}\gamma^{\nu}\gamma^{5}u\cdot(a_e\cdot a_f)\big]\cdot Z_{V_{\Pi}}\cdot\chi_Z(s)\ \},
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
v_e &= (2\cdot T_3^e - 4\cdot q_e\cdot s_W^2\cdot K_e(s,t))/\Delta, \\
v_f &= (2\cdot T_3^f - 4\cdot q_f\cdot s_W^2\cdot K_f(s,t))/\Delta, \\
a_e &= (2\cdot T_3^e)/\Delta, \qquad s_W^2 = (1-c_W^2) = 1 - M_W^2/M_Z^2, \\
a_f &= (2\cdot T_3^f)/\Delta, \qquad \Delta = 4 s_W c_W, \\
\chi_Z(s) &= \frac{G_\mu\cdot M_z^2\cdot\Delta^2}{\sqrt{2}\cdot 8\pi\cdot\alpha}\cdot\frac{s}{s - M_Z^2 + i\cdot\Gamma_Z\cdot s/M_Z}, \\
\Gamma_{V_{\Pi}} &= \frac{1}{2-(1+\Pi_{\gamma\gamma}(s))}, \qquad Z_{V_{\Pi}} = \rho_{\ell f}(s,t), \qquad \chi_\gamma(s) = 1,
\end{aligned}
$$

(1) Genuine weak

$$vv_{ef} = \frac{1}{v_e \cdot v_f}[(2 \cdot T_3^e)(2 \cdot T_3^f) - 4 \cdot q_e \cdot s_W^2 \cdot K_f(s,t) - 4 \cdot q_f \cdot s_W^2 \cdot K_e(s,t)$$
$$+(4 \cdot q_e \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{ef}(s,t)] \frac{1}{\Delta^2}.$$

$u,\ v$ stand for fermion spinors and wave functions, $\mathcal{N}$ denotes normalization factor.

- Eq. (1) represent Improved Born of LEP. Its form-factors can be, **under some conditions**, used in the amplitudes of higher order QED or strong interactions directly.

- **Indeed** in KKMC, electroweak form-factors are installed into all spin amplitudes.

- This could be disastrous and damage gauge independence. Spinor techniques of Kleiss-Stirling are used for $e^+e^- \to l^+l^- n\gamma$ processes, with second order QED matrix element and coherent exclusive exponentiation. Nonphysical huge contributions proportional to $\sim 1/m_e^2$ could appear. This is not the case as contributions to Yennie Frautchi Suura β_0, β_1, ... terms, are calculated explicitly and gauge cancellations performed, before electroweak form-factors installation.

- This required hard work, effort may be not always justified. Convolutions of Improved Born with PDF's may be sufficent.

Where are the limits? Are there ways out, if PDF-style convolutions insufficient?

(1) Genuine weak

Target:

1. Multiloop EW+QCD calculations and MC available for **all processes** of interest.

2. Higher than second order terms included by resumations or validated negligible.

3. All in semi analytical programs and Monte Carlos

My aim is limited; exploit what is available from LEP software for today applications. For that purpose, first answer:

1. How to get Born kinematic for non-Born configurations.

2. How to simplify Improved Born so it can be used with QCD calculations/simulations. What precision loss simplifications bring to EW sector.

3. Estimations of parametric ambiguities, which higher (how high) orders are needed for the required precision target.

4. For the measurements interpretation $\sin^2\theta_W^{eff} = s_W^2 K_{e/f}(s,t)$ was used, $s = M_Z^2, t = -M_Z^2/2$. Sensitivity to EW couplings $\simeq$sensitivity to $\sin^2\theta_W^{eff}$. True also for ambiguities and errors.

(1) Genuine weak

LEP 1, typically: $3 \cdot 10^5$ – $1 \cdot 10^6$ event samples

A. Granularity of the detector was an issue: bare, (or dressed) leptons were used in predictions, (for dressed extra MC corrections were needed).

B. Semianalytical calculations used for phenomenology/fits.

C. Picture of logarithm power was used for preliminary estimation of required effects. Cut off induced logarithms were not dominating.

* Techniques:
  - d. Approach based on combination of semianalytical and Monte Carlo methods was established.
  - e. DIZET and ZFITTER were the semianalytical brands of that time.
  - f. Semianalytic calculation mean calculation where at most one (or two) integration are performed numerically. Usually for simplified cuts.
  - g. KKMC, KORALZ Monte Carlo programs used in evaluation of acceptance details.

(1) Genuine weak

Keynotes

- LEP1: One loop genuine weak corrections, but up to 3 loops QCD effects and multiphoton corrections were necessary at the same time. Semianalytical calculations very useful, dressed leptons as well but it was not all.
- Luminosity: Highest precision: Theoretical predictions dependent on detector structure was a must. How (unresolved) photons affect detector response to leptons at the edge of acceptance or edge of detector pad.
- LEP2: Samples were tiny, solution for new problems such as gauge cancellation between s- and t-channel boson exchanges could be assured without worry of theoretical sophistications necessary at LEP 1.
- **Frame for predictions which was build at LEP times turned out to be robust. Useful over all Tevatron times.**
- **How it looks now?**

(1) Genuine weak

- Beyond LEP: no more Z peak only, t-channel diagrams etc
- Effort was needed, many aspects of Dima Bardin and his group work for Tevatron are important for that.
  His papers: Comput. Phys. Commun. 59 (1990) 303, Comput. Phys. Commun. 133 (2001) 229, Nucl. Phys.B175(1980) 435, Nucl. Phys. B197 (1982) 1 are quoted as late as in ATLAS-CONF-2018-037.
- I will adress what was needed for our approaches and tools.

TAUSPINNER algorithm for reweighting generated events I will use results from:T. Przedzinski, E. Richter-Was and Z. Was, “Documentation of TauSpinner algorithms – program for simulating spin effects in tau-lepton production at LHC,” arXiv:1802.05459 [hep-ph].

Effective Born How to improve using event weights sample to include genuine weak corrections; results from 2012.10997.

- Some other supplementary/supporting from other papers too.

(1) Genuine weak: strong effects factorization.

Parton level Born cross-section, for $q\bar{q} \to Z/\gamma^* \to \ell^+\ell^-$ can be expressed as:

$$\frac{d\sigma^{q\bar{q}}_{Born}}{d\cos\theta}(s,\cos\theta,p) = (1+\cos^2\theta)\,F_0(s)+2\cos\theta\,F_1(s)-p[(1+\cos^2\theta)\,F_2(s)+2\cos\theta\,F_3(s)] \tag{2}$$

p denotes polarization of the outgoing leptons, and form-factors read:

$$\begin{aligned}
F_0(s) &= \frac{\pi\alpha^2}{2s}[q_f^2 q_\ell^2 \cdot \chi_\gamma^2(s) + 2\cdot\chi_\gamma(s)Re\chi_Z(s)\, q_f q_\ell v_f v_\ell + |\chi_Z^2(s)|^2 (v_f^2+a_f^2)(v_\ell^2+a_\ell^2)], \\
F_1(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s)Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\; 2v_f a_f 2v_\ell a_\ell], \qquad (3 \\
F_2(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s)Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\; (v_f^2+a_f^2)2v_\ell a_\ell], \\
F_3(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s)Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\; (v_f^2+a_f^2)2v_\ell a_\ell],
\end{aligned}$$

$\cos\theta$ denotes angle between incoming quark and outgoing lepton in the rest frame of outgoing leptons. That is rather simple spherical harmonics of the second order.

(1) Genuine weak: strong effects factorization.

Mustraal frame

[18] F. A. Berends, R. Kleiss, and S. Jadach, *Comput. Phys. Commun.* **29** (1983) 185–200.

Mustraal: Monte Carlo for $e^+ e^- \to \mu^+ \mu^-$ (γ)

$$s = 2p_+ \cdot p_-, \quad t = 2p_+ \cdot q_+, \quad u = 2p_+ \cdot q_-$$

$$s' = 2q_+ \cdot q_-, \quad t' = 2p_- \cdot q_-, \quad u' = 2p_- \cdot q_+$$

$$\sigma_{\mathrm{hard}} = \int \mathrm{d}\tau \left(X_{\mathrm{i}} + X_{\mathrm{f}} + X_{\mathrm{int}} \right),$$

The explicit forms of the three terms in σ_{hard} read:

$$X_{\mathrm{i}} = \frac{Q^2 \alpha}{4\pi^2 s} \frac{1-\Delta}{k_+ k_-} s'^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t, u) + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t', u') \right], \qquad (3.4)$$

$$X_{\mathrm{f}} = \frac{Q'^2 \alpha}{4\pi^2 s} \frac{1-\Delta'}{k'_+ k'_-} s^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t', u) \right], \qquad (3.5)$$

$$X_{\mathrm{int}} = \frac{QQ'\alpha}{4\pi^2 s} W \frac{\alpha^2}{2ss'} \left[(u^2 + u'^2 + t^2 + t'^2) \tilde{f}(s, s') + \tfrac{1}{2}(u^2 + u'^2 - t^2 - t'^2) \tilde{g}(s, s') \right]$$
$$+ \frac{QQ'\alpha^3}{4\pi^2 s} \frac{(s-s') M \Gamma}{k_+ k_- k'_+ k'_-} \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma \left[\tilde{E}(s, s')(t^2 - t'^2) + \tilde{F}(s, s')(u^2 - u'^2) \right], \qquad (3.6)$$

Resulting optimal frame used to minimise higher order corrections from initial state radiation in $e^+e^- \to Z/\gamma^* \to \mu\,\mu$ for algorithms of genuine EW corrections implementation in LEP time Monte Carlo's like Koral Z.

Extending definition of Mustraal frame

- We extended this frame to pp -> $l^+ l^- j$ (j) case
  - reconstruct x_1, x_2 of incoming partons from final state kinematics (information on jets used)
  - assume the quark is following x_1 direction (equivalent to what done in CS frame)
  - calculate (θ_1, ϕ_1), (θ_2, ϕ_2) of two Born's, weight with probability calculated not using couplings

$$wt_1 = \frac{E_{p1}^2(1+\cos\theta_1^2)}{E_{p1}^2(1+\cos\theta_1^2)+E_{p2}^2(1+\cos\theta_2^2)}, \quad wt_2 = \frac{E_{p2}^2(1+\cos\theta_2^2)}{E_{p1}^2(1+\cos\theta_1^2)+E_{p2}^2(1+\cos\theta_2^2)}$$

3

(1) Genuine weak: strong effects factorization.

What Challenges come with jets and why solution justified:

- E. Mirkes and J. Ohnemus, "Angular distributions of Drell-Yan lepton pairs at the Tevatron: Order $\alpha - s^2$ corrections and Monte Carlo studies," Phys. Rev. D **51** (1995) 4891
- R. Kleiss, "Inherent Limitations in the Effective Beam Technique for Algorithmic Solutions to Radiative Corrections," Nucl. Phys. B **347**, 67 (1990).
- F. A. Berends, R. Kleiss and S. Jadach, "Monte Carlo Simulation of Radiative Corrections to the Processes e+ e- —> mu+ mu- and e+ e- —> anti-q q in the Z0 Region," Comput. Phys. Commun. **29**, 185 (1983).

Factorized Born level distribution preserved but choice of reference frame for lepton pair usually brings in all coefficients for second order spherical harmonics.

IMPORTANT: use one of the two Born kinematics $\rightarrow$ not average.

(1) Genuine weak: strong effects factorization.

In general, we get (with $\alpha_s^2 \sim 0.01$ corrections only) for the case when jets are present:

$$
\begin{aligned}
\frac{d\sigma}{dp_T^2 dY\, d\cos\theta d\phi} &= \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^2 dY}[(1+\cos^2\theta)+ \\
1/2\, A_0(1-3\cos^2\theta) &+ A_1\sin(2\theta)\cos\phi + 1/2\, A_2\sin^2\theta\cos(2\phi) + A_3\sin\theta\cos\phi \\
+A_4\cos\theta &+ A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\ \sin\phi]
\end{aligned}
$$

From Born formula this differ by rotation only.

Note: Collins-Soper and Mustraal frames differ by the reference frame rotation only.

For details see backup slides.

(1) Genuine weak: strong effects factorization.

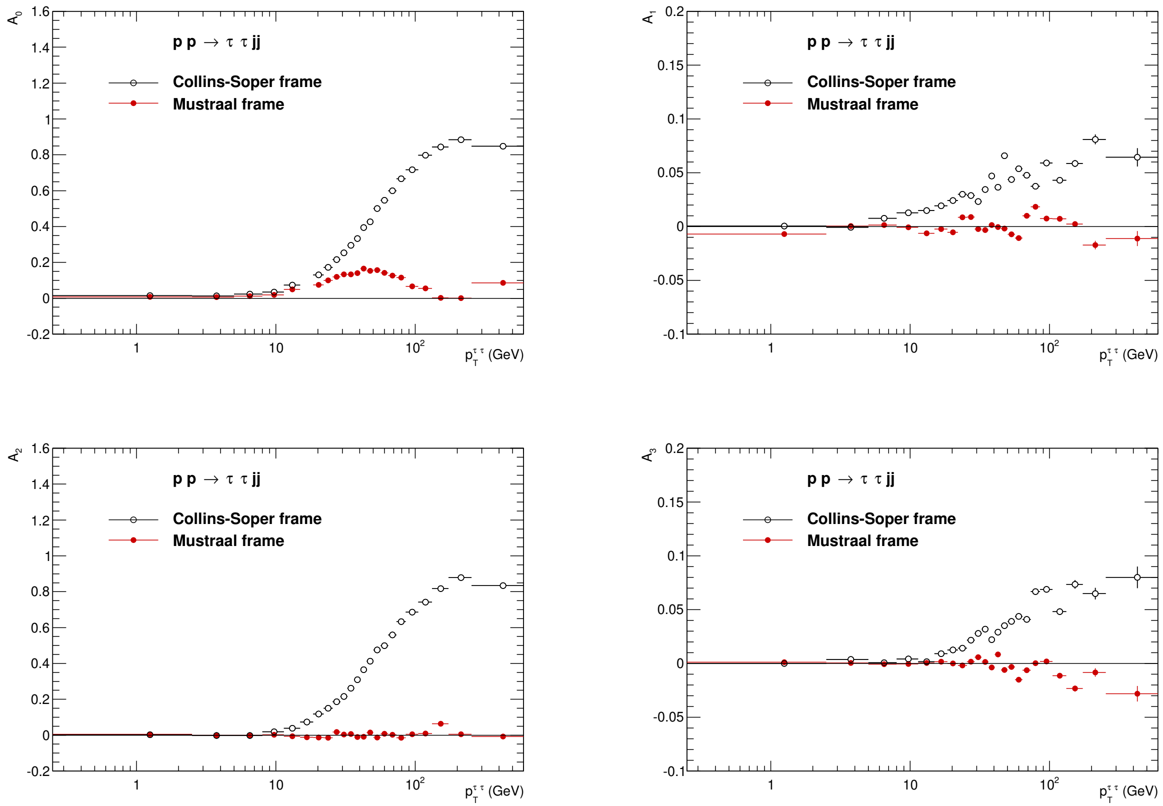


Figure 1: **arXiv:1605.05450**: The A_i coefficients of Eq. (4)) calculated in Collins-Soper (black) and in Mustraal (red) frames for $pp \to \tau\tau jj$ process generated with MadGraph. Details of initialization are given in the reference.

(1) Genuine weak: strong effects factorization.

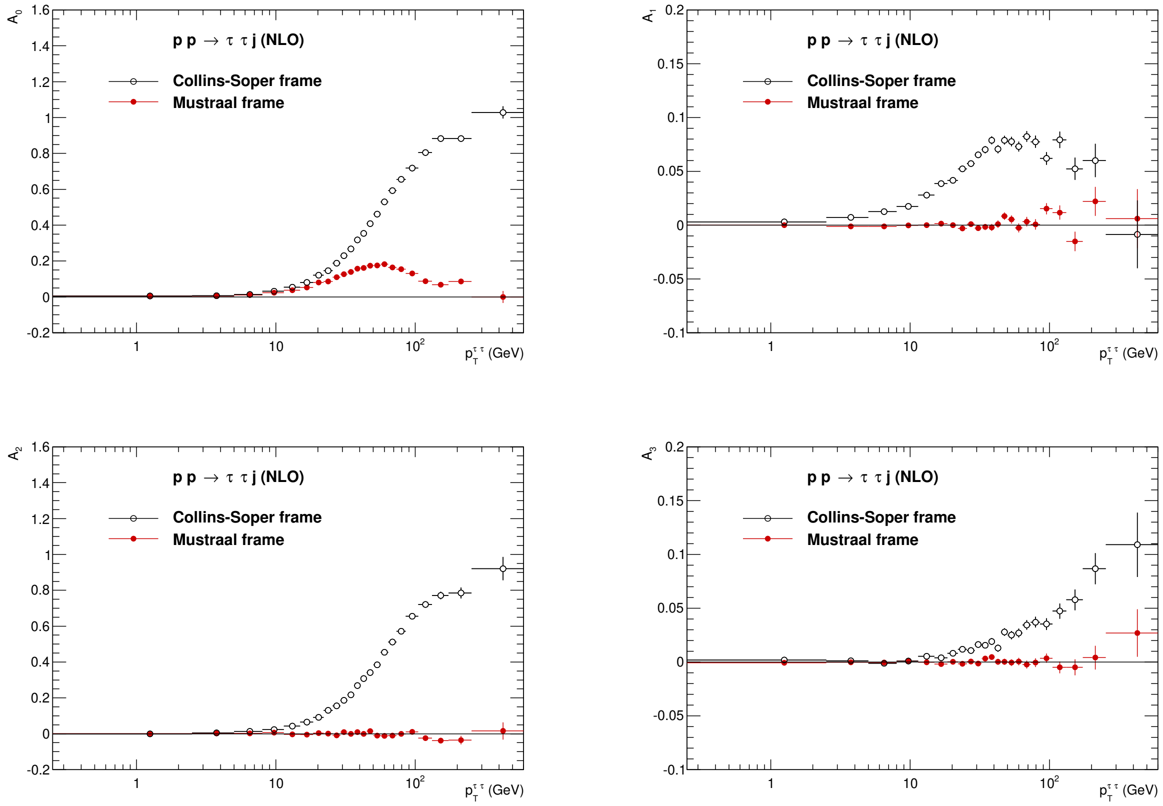


Figure 2: **arXiv:1605.05450**: The A_i coefficients of Eq. (4)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp \to \tau\tau j$ (NLO) process generated with `Powheg+MiNLO`. Details of initialization are given in the reference.

(1) Genuine weak: strong effects factorization.

- The choice of Mustraal frame is result of careful study of single photon (gluon emission amplitude)
- In Ref of 1982 it was shown, that differential distribution is a sum of two born-like distributions convoluted with emission factors.
- This is a consequence of Lorentz group representation and that is why it generalizes to the case of double gluon or even double parton emissions.
- Presence of jets effects mainly orientation of frames.
- That is why use of electroweak Borns is justified.
- Figures demonstrate how proper choice of frames can turn high p_T events into electroweak Born kinematic.
- Recall papers by Mirkes and Kleiss.
- **Now electroweak sector, it could kill Effective Born picture too.**

(1) Genuine weak: no damage to Born.

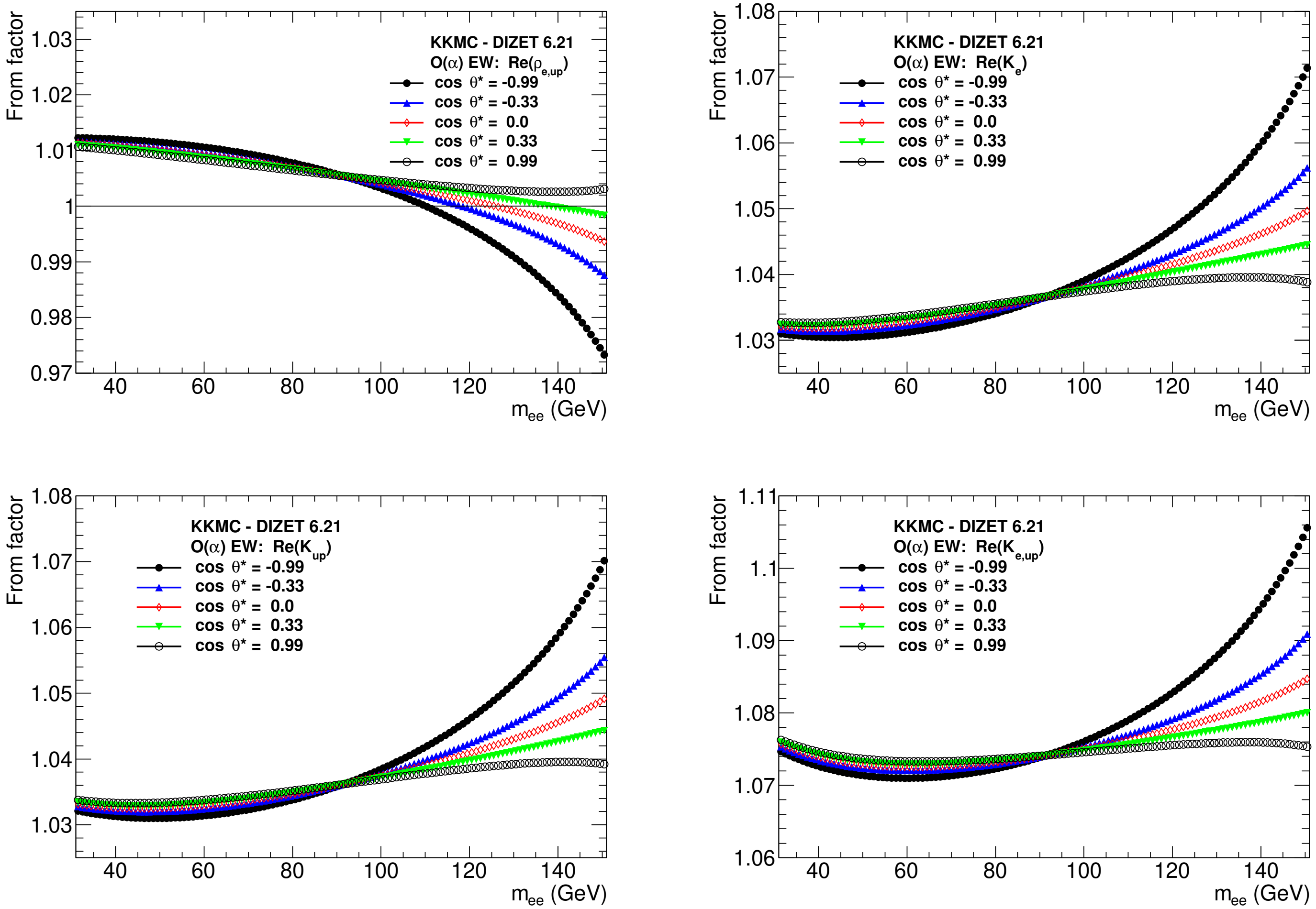


Figure 3: Real part of $\rho_{e,up}$, K_e, K_{up} and $K_{e,up}$ EW form factors as a function of m_{ee} for few values of $\cos\theta^*$ and u-type quark flavour. Note that close to the Z peak angular dependence is minimal. For lower virtualities photon exchange dominates. At Z-peak electroweak effects do not damage picture of spherical harmonics: θ^* dependence is mild. At higher virtualities reweighting may be the only option, Effective Born insuffiicent.

(1) Genuine weak: no damage to Born.

Observations

- Formfactors distort, but in numerically not so significant manner, the lepton angular distributions from second order spherical harmonics.
- Limitation for histogram level. Re-weights at fully differential level!
- We can explore Mustraal frames for reweighting algorithms, which can then be used to install better genuine weak effects into 'any' MC sample, provided in its generation known (constant) couplings of Z bosons were used.
- Then reweighting is dfferential at least for kinematic of first jet.
- To Effective Born $\rightarrow$ no form-factors, For direct implementation into non-electroweak savy simulations.
- Now numerical results for $v0, v1, v2$ variants of Effective Born, no formfactors but of increasing number of constants:

(1) Genuine weak: $\sin^2\theta_W^{eff}$ and Effective Born limitations

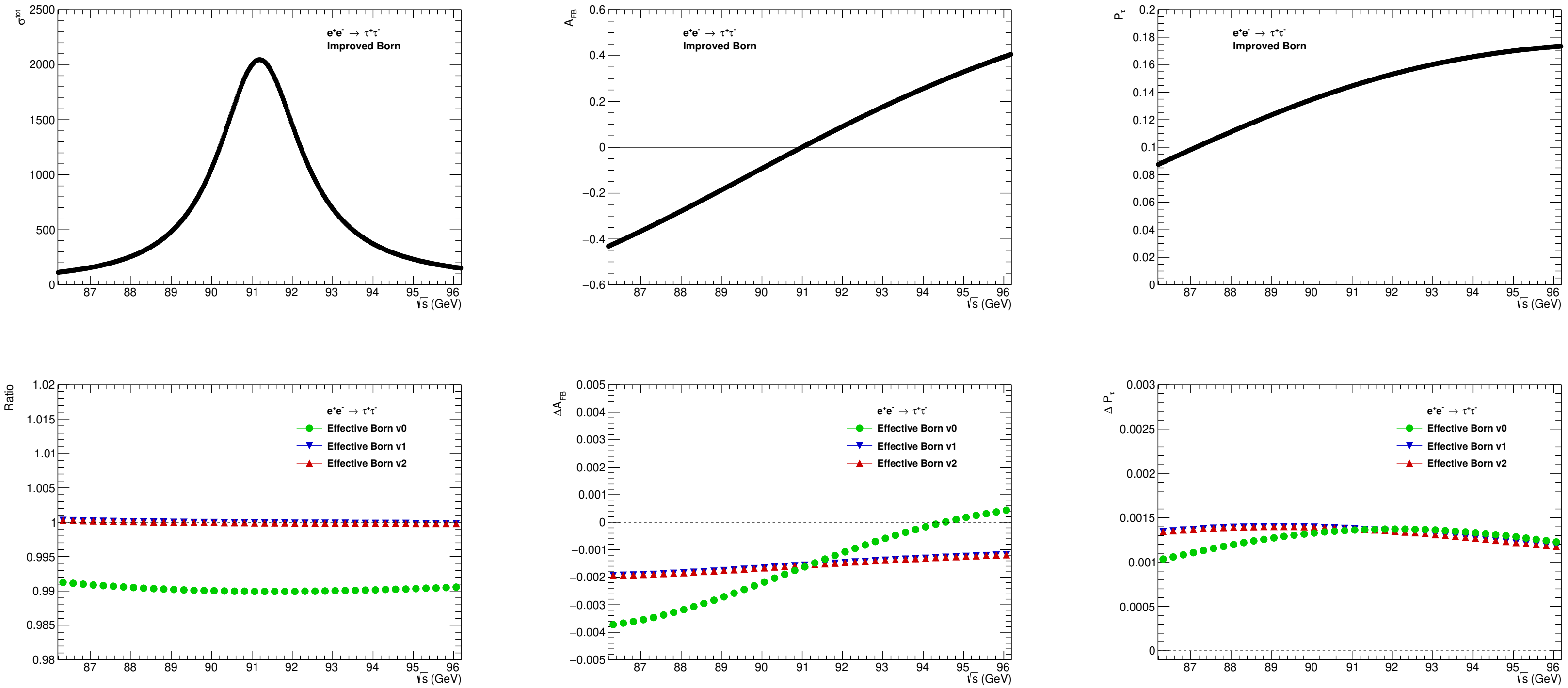


Figure 4: Left side plots, Improved Born in the vicinity of the Z peak: σ^{tot} (left), A_{FB} (middle) and P_τ (right). Bottom plots enumerate, with ratios or differences the effects of Effective Born simplifications with respect to Improved Born. Green points: Effective Born $\mathtt{v0}$, Blue triangles: Effective Born $\mathtt{v1}$ Red (rotated) triangles: Effective Born $\mathtt{v2}$.

(1) Genuine weak: $\sin^2\theta_W^{eff}$ and Effective Born limitations

The forward-backward asymmetry A_{FB} (top central plot) for the $pp \to Z/\gamma^* \to l^+l^-$ process, lepton pair virtuality range 60 to 150 GeV, it is of interest for EW effects. Shape and size of the corrections depend on whether box exchange diagrams are included in the Improved Born form-factors. The A_{FB} distribution, as generated (EW LO) is superimposed with EW corrected result, the two cases are practically indistinguishable.

Below the plot with three lines, for the difference $\Delta A_{FB} = A_{FB} - A_{FB}^{ref}$. As reference A_{FB}^{ref}, the three versions of the Effective Born are used: (i) EW LO $\alpha(0)$, (ii) $\mathtt{v0}$ and (iii) $\mathtt{v2}$, while for A_{FB} Improved Born always is used. The EW corrections for A_{FB}^{ref} of EW LO Born with $\alpha(0)$ scheme, integrated around the Z-pole, necessary to reproduce Improved Born result can reach -0.03514. The Effective Born $\mathtt{v0}$ reproduces Improved Born up to ΔA_{FB} of about -0.0004, while the Effective Born $\mathtt{v2}$ up to -0.0002. The $\mathtt{v2}$ variant is again better by a factor of two than $\mathtt{v0}$.

Similar plots for cross section (left side) and τ polarization (right side) are also presented.

These results point to limitation **at about** $20 \cdot 10^{-5}$ for $\sin^2\theta_W^{eff}(M_Z)$ ambiguity Effective Born parametrization that is real constants. Even if $\alpha(M_Z)$ and $\rho_{lf}(M_Z)$ is used $\mathtt{v2}$.

(1) Genuine weak: $\sin^2\theta_W^{eff}$ and Effective Born limitations

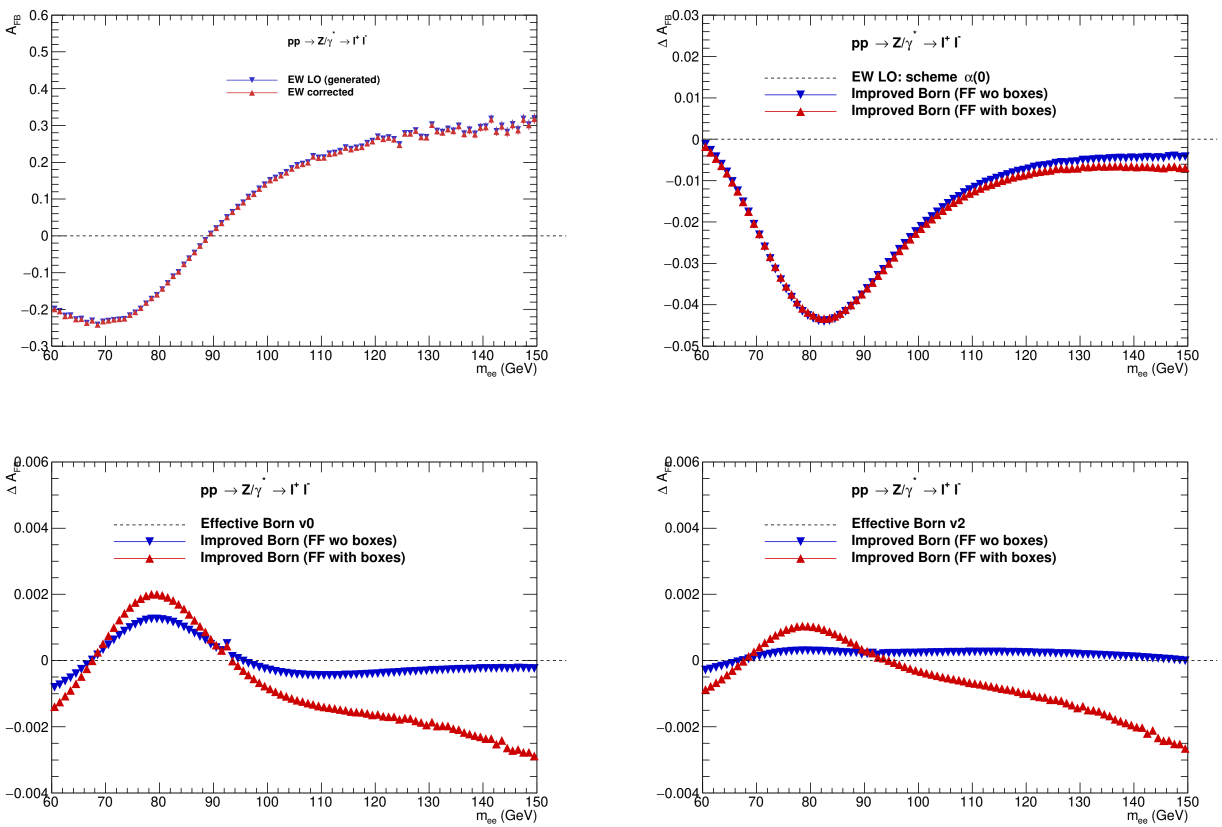


Figure 5: The A_{FB} generated by Powheg+MiNLO (blue triangles) and with re-weighting for all EW corrections (red triangles); the two are barely distinguishable. Following plots $\Delta A_{FB} = A_{FB} - A_{FB}^{ref}$: Improved Born (EW boxes on/off) minus Effective Born for (i) EW LO $\alpha(0)$ scheme, (ii) Effective Born v0, (iii) Effective Born v2.

(1) Genuine weak: parametric (corrections) uncertainty

Table 1: The `DIZET` predictions with improved treatment of two-loop corrections. Other flags set for defaults.

Parameter	AMT4= 4	AMT4 = 8	Δ
$\alpha(M_Z^2)$	0.0077549256113	0.0077549256002	
$1/\alpha(M_Z^2)$	128.95030206	128.95030224	
M_W (GeV)	80.361846	80.358936	- 2.9 MeV
Δr	0.03640338	0.03640338	
Δr_{rem}	0.01167960	0.01167960	
s_W^2	0.22333971	0.22340108	+ 0.00006
$sin^2\theta_W^{eff\ lepton}(M_Z^2)$	0.23157938	0.23149900	-0.00008
$sin^2\theta_W^{eff\ up-quark}(M_Z^2)$	0.23147290	0.23139248	-0.00008
$sin^2\theta_W^{eff\ down-quark}(M_Z^2)$	0.23134590	0.23126543	-0.00008

(1) Genuine weak: parametric (corrections) uncertainty

Table 2: The `DIZET 6.45` predictions: $\Delta/2$–selected parameters shifts, due to $\pm$ 0.0001 variation from $\Delta\alpha_h^{(5)}(M_Z^2)$=0.0275762 of Ref. 1711.06089.

Parameter	$\Delta\alpha_h^{(5)}$ - 0.0001	$\Delta\alpha_h^{(5)}$	$\Delta\alpha_h^{(5)}$ + 0.0001	$\Delta/2$
$\alpha(M_Z^2)$	0.0077541016	0.0077549256	0.0077557498	
$1/\alpha(M_Z^2)$	128.96400565	128.95030224	128.93659846	
M_W (GeV)	80.360747	80.358936	80.357124	1.8 MeV
Δr	0.03629414	0.03640338	0.03651261	
Δr_{rem}	0.01167983	0.01167960	0.01167938	
s_W^2	0.22336607	0.22340108	0.22343610	0.000035
$\sin^2\theta_W^{eff\ lepton}(M_Z^2)$	0.23146409	0.23149900	0.23153392	0.000035
$\sin^2\theta_W^{eff\ up-q}(M_Z^2)$	0.23135758	0.23139248	0.23142737	0.000035
$\sin^2\theta_W^{eff\ down-q}(M_Z^2)$	0.23123057	0.23126543	0.23130029	0.000035

Summary

1. **Radiative corrections, their separation into parts was essential for the phenomenology picture enabling verification that Standard Model is Quantum Field theory describing essentially all phenomena we observe.**
2. Matching Lagrangian approach, gauge invariance with the need of resummation of some corrections to higher orders (other not) was an important task.
3. Dispersion relations were used for low energy $\Pi_{\gamma\gamma}(s)$. This makes analytic continuation to introduce Z or W width perilous, beyond one loop. Mind parametric ambiguity e.g. $\alpha_{QED}(M_Z)$ measurements.
4. This effort was completed for e^+e^- colisons and $\sqrt{s} \simeq M_Z$ first but later extended to larger energies and $p\bar{p}$, pp collisons as well.
5. I have recalled some arguments, results of electroweak and strong interaction calculations, why it can work.
6. Numerical results were presented:
   (a) on parametric ambiguities,
   (b) on precision limits for $\sin^2\theta_W^{eff}$, effctive Born applicability.

Summary

1. Results and arrangements for numerical (semi analytical or Monte Carlo) studies of genuine (non QED) electroweak effects for high enery physics observables presented.
2. Formalism developed over the years of LEP can be useful for LHC and future applications with minor upgrades.
3. Concepts of: Effective Born, Improved Born, electroweak form factors survive.
4. Even if underlying approximations may be not suffcent for high precision phenomenology they can be helpful to estimate which effects need to be then taken into account.
5. For today, solutions can be helpful to improve electroweak content of simulations with tools targetting difficulties of strong interactions.
6. I have collected (not presented) some slides on τ decays and on possibility to imprint dark photons (or non SM scalars) into events simulated with KKMC.
7. **Thanks for the opportunity to talk on EW! Topic (2) and (3) follow**

(2) Tau lepton decays, prepared for Belle 2

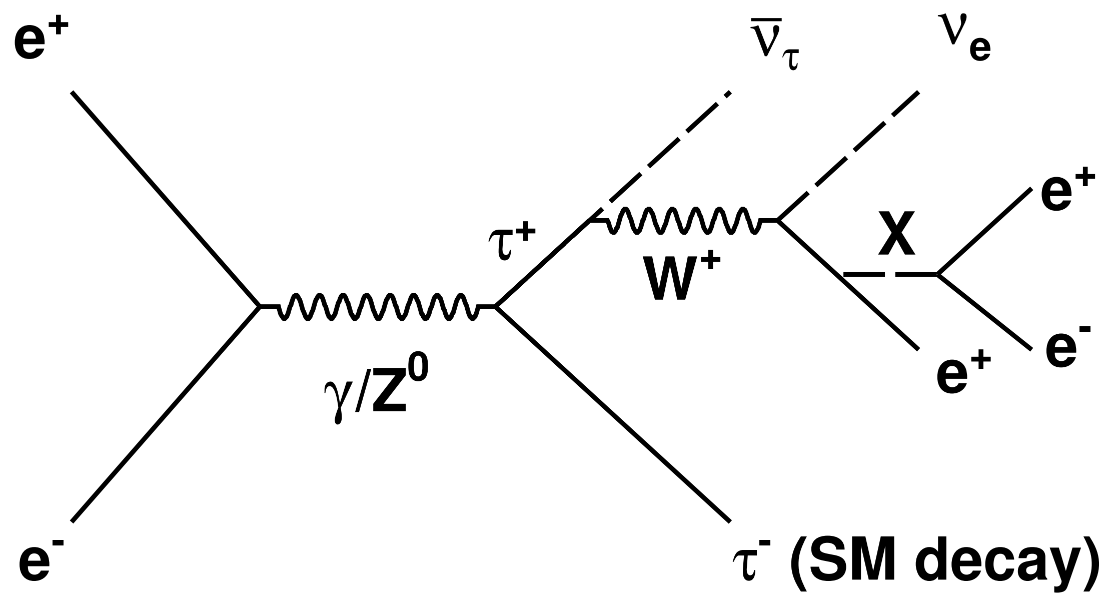
`basf2` software of Belle II, new τ decay channels prepared for `tauola`:

1. Total number of decay channels: 278
2. 2 body neutrinoless non SM decays: 58
3. 3 body neutrinoless non SM decays: 46
4. Number of generic SM decay channels : 92 [initialized with PDG2020 Branching Fractions]. For matrix elements choices from older versions of parametrisations were taken, but ...
   (a) ...for high precision data, new parametrisations will be needed.
   (b) Theoretical uncertainty of models is worse than quality of data.
5. Decay modes with l^+l^- pair from SM photons or from Dark photon (or NP scalar) with mass $\in [50, 1500]$ MeV (ME cross-validated with `MadGraph`)
   - $\tau^- \to \nu_\tau \bar{\nu}_\ell \ell^- \ell^+ \ell^-$
   - $\tau^- \to \nu_\tau \pi^- \ell^+ \ell^-$

Hopefully, one day new Belle2 τ decay parametrizations will be for everybody.

(3) Dark photons or new physcics scalars

... in tauola:



Once we could rely on tests for SM pairs, we could include safely $X \to e^- e^+$ too.

Feynman diagram for $e^- e^+ \to \tau^- \ (SM \ decay) \ \tau^+ (\to \bar{\nu}_\tau \nu_e e^+ X (\to e^- e^+))$ process with X emitted in τ^+ decay.

(3) Dark photons or new physcics scalars

... in τ lepton pair production

1. `photospp` (in C, C++) MC for radiative corrections in decays and `tauola` (still F77), MC for tau lepton decays have new presamples for phase-space which are useful for narrow resonances (e.g. decaying to lepton pairs).

2. Approximations in matrix elements, but not in phase space:

   (a) Tests with semianalytical calculations

   (b) Tests with matrix element exact phase space, event samples...

   (c) ...from `KORALW` for FSR QED pair emission in Z decay,

   (d) ... from`MadGraph` for $\tau\tau$ scalar($\rightarrow$ lepton pair) sample (solution like for ML used for Higgs parity observables Phys.Rev.D 103 (2021), 036003).

3. Tests evolved into presented now physics extensions, that is why dark photon, dark scalar implementation was relatively easy.

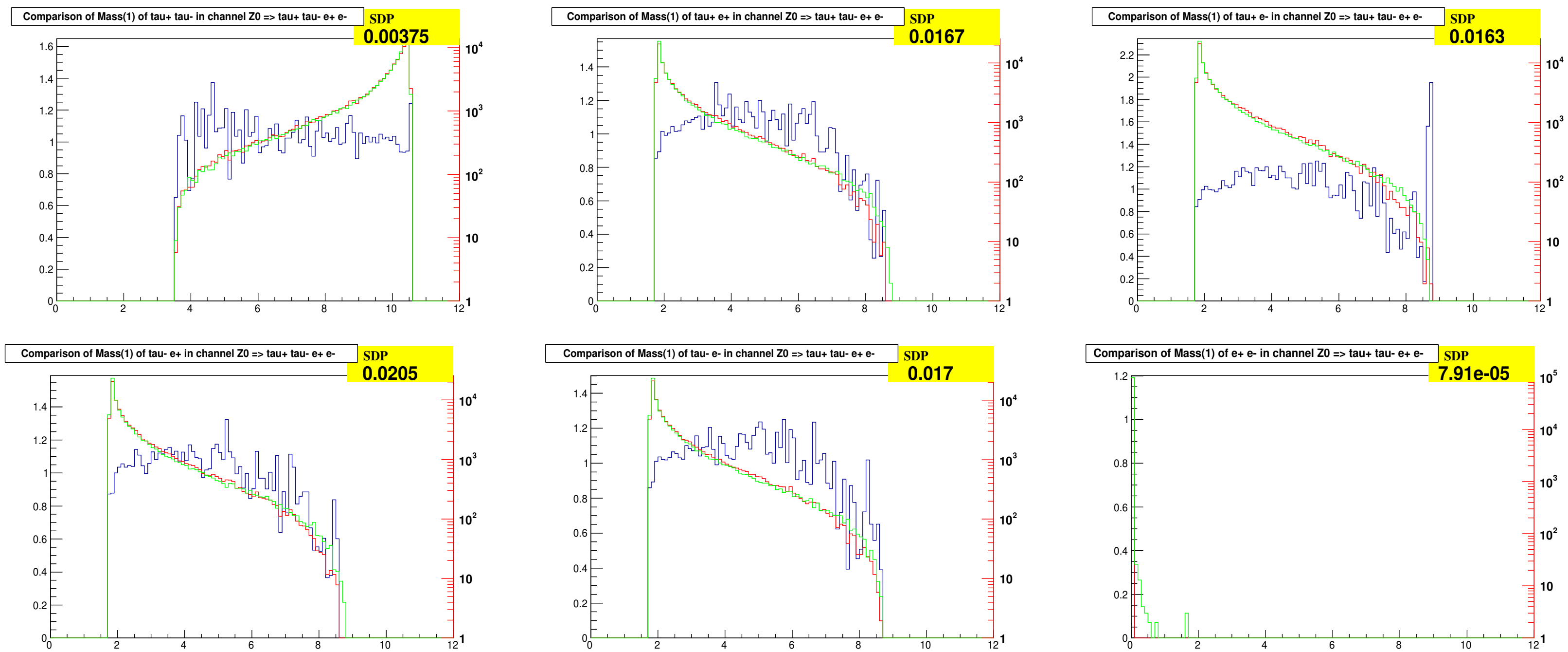
(3) Dark photons or new physcics scalars

$$e^-e^+ \to \tau^-\tau^+\phi_{\mathrm{Dark\ Scalar}}\ (50\ MeV)\ (\to e^-e^+)$$

Red lines: 50K events from `MadGraph` (MG5)

Green lines: 100K events from `KKMC+photos`

First check: ISR bremsstrahlung effects turned off in both generators



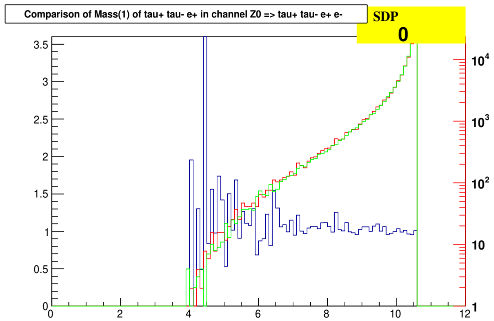
(3) Dark photons or new physcics scalars

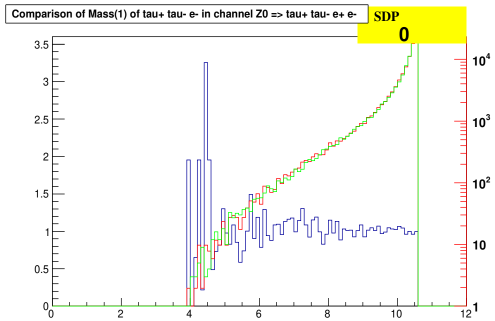
$$e^- e^+ \to \tau^- \tau^+ \phi_{\text{Dark Scalar}}\,(50\;MeV)\;(\to e^- e^+)$$

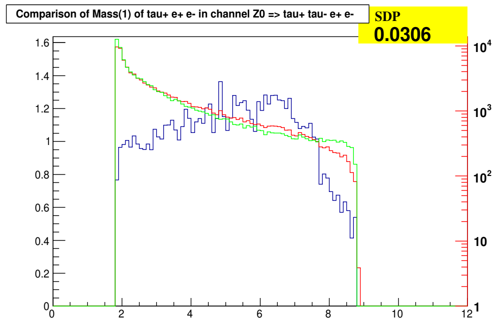
Red lines: 50K events from `MadGraph` (MG5)

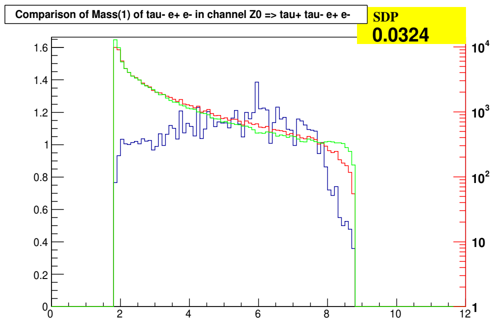
Green lines: 100K events from `KKMC+photos`

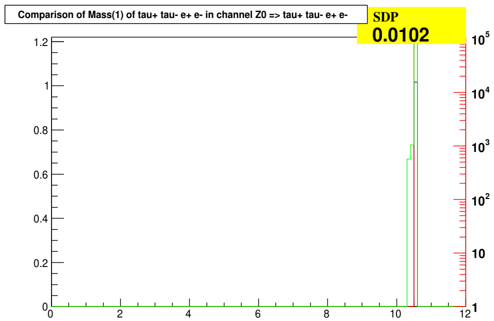
First check: ISR bremsstrahlung effects turned off in both generators



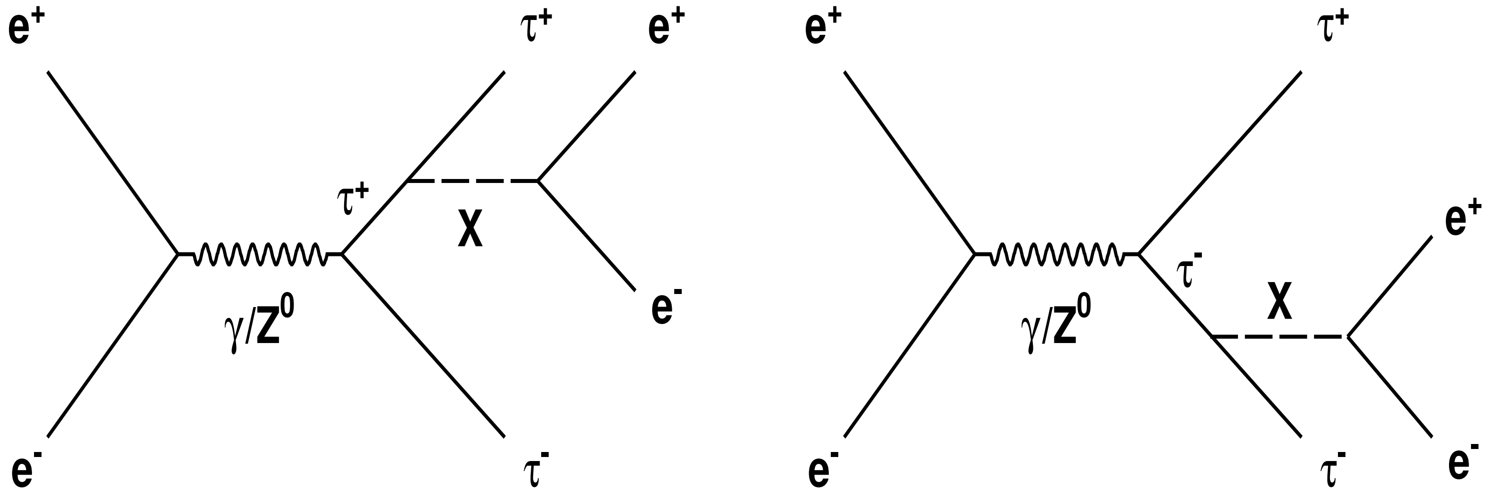






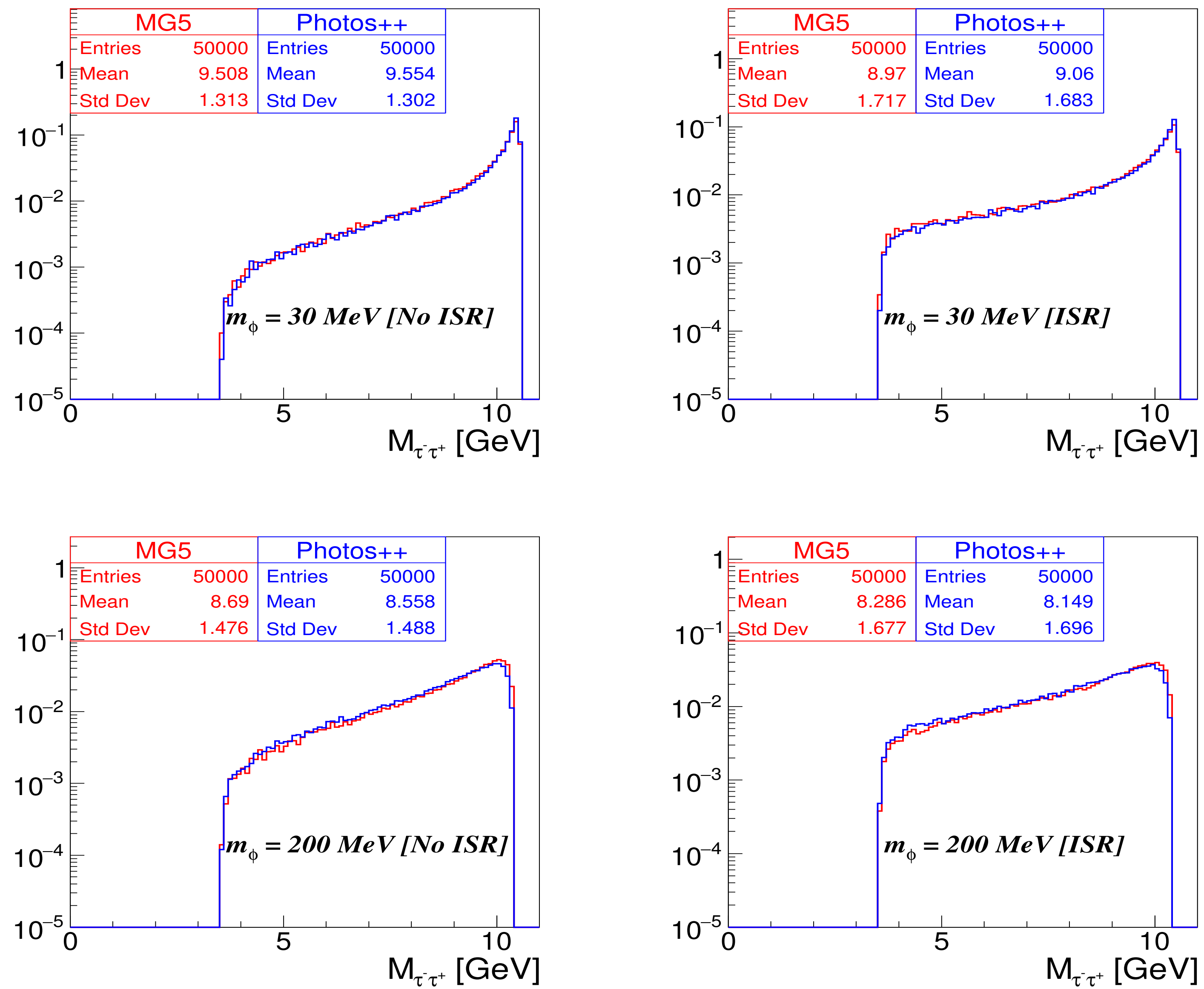


(3) Dark photons or new physcics scalars

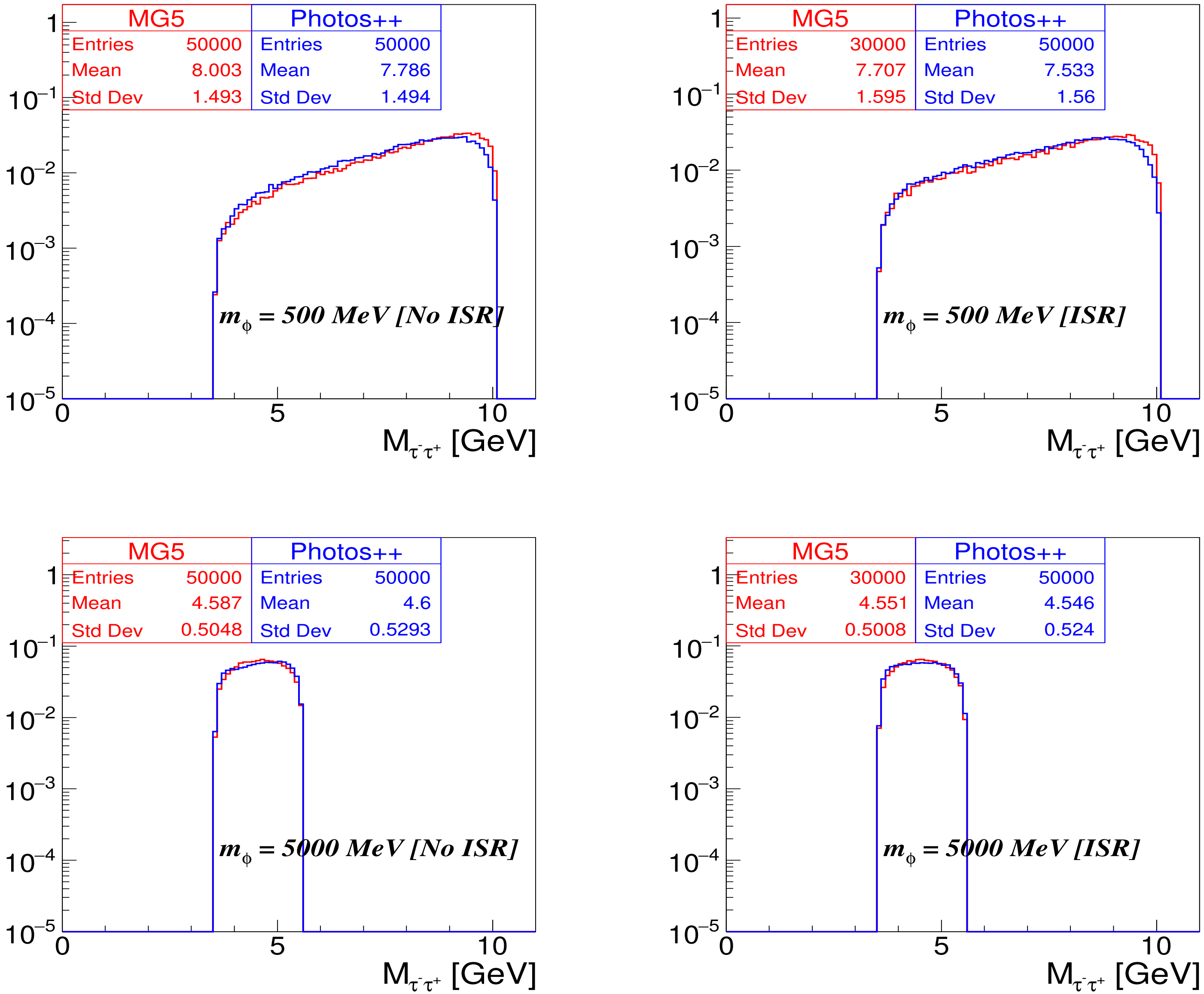


Dominant Feynman diagrams for $e^- e^+ \to \tau^- \tau^+ X (\to e^- e^+)$ as installed in `MadGraph`. For`photospp` dominant part, which can be used together with bremsstrahlung is used. It is an approximation, but validated. Note that spin state of τ flips if X is scalar. That need to be taken into account (done within basf2 version of `KKMC` for $\tau - \tau$ spin correlations.

$e^-e^+ \to \tau^-\tau^+\phi_{\text{Dark Scalar}}(\to e^-e^+)$



$e^-e^+ \to \tau^-\tau^+\phi_{\text{Dark Scalar}}(\to \mu^-\mu^+)$



$e^-e^+ \to \tau^-\tau^+\phi_{\rm Dark\ Scalar}(\to \mu^-\mu^+)$

1. This work was quick, because of previous steps and installation into `photospp` possibility to generate lepton pairs:
   (a) produced through virtual photon Comput.Phys.Commun. 199 (2016) 86 1912.11376
   (b) produced through narrow vector resonance (like dark photon) 1912.11376 footnote 9
2. These changes were introduced first to `tauola`,
3. Both programs have similar phase space parametrisations which is exact.

(1) Genuine weak; backup slides.

- Lessons from LEP: with the improved statistical precision, more and more sophistication for theory is needed.
- Not only in the establised sectors such as:
  - QED,
  - genuine weak,
  - strong interaction,
  - detector granularity details,
  - beam properties (energy spread etc.),
- ... but also in the ways how to combine them for final predictions.
- The t-channel mean not only pressure on gauge cancellation constraints but also enhance importance of detector details, cells geometry and QED brem. etc.
- All this was necessary to understand at LEP. Conditions for LHC work, more complex.

(1) Genuine weak; backup slides.

Questions of genuine weak:

1. Improved Born of LEP I and electroweak Form factors
2. Why this could have been useful at LEP 1, but challenging later
3. Nonetheless approach is in use all these years since and form reference basis.
4. Even if one does not like (or find insufficent) this so colled 1- (called sometimes 1.5-) loop level picture, it is required, because of wealth LEP and Tevatron measurements.
5. Effective $\sin^2\theta_W$ and effective Born were in use.
6. Why this old e^+e^- collision picture, may be used for LHC applications how merge strong interactions into the picture?
7. Can LEP 1 pheno tools evaluate for observables size of some higher order effects?

$\mathcal{D}$erived up to quite high precision level, reliable basis.

- Separation of amplitudes into parts such as QED/QCD ISR, QED FSR: no loss of precision. See eg. *The standard model in the making: Precision study of the electroweak interactions,* **Dmitri Yu. Bardin**, (Dubna, JINR) , G. Passarino, Oxford, UK: Clarendon (1999).

- Beyond LEP 1, e.g. what to do with emissions from intermediate (e.g. W's): Leading pole approximation as in U. Baur NLO calculation for $W\gamma$ (anomalous) coupling (Phys. Rev. D 47 (1993) 4889), separation of QED FSR from the rest of EW effects. Nearly direct consequence of formula:
$$\frac{1}{\big((P+k)^2-M_W^2\big)\big(P^2-M_W^2\big)} = \left(\frac{1}{P^2-M_W^2} - \frac{1}{(P+k)^2-M_W^2}\right)\frac{1}{2Pk}$$

- **Confront field theory with the data. Precision regime; first missing terms $\sim \frac{\alpha_{QED}^2}{\pi^2}$. All necessary resummations included. Long work on precision required effort on strong interactions.**

(1) Genuine weak; backup slides.

From D. Bardin Comput.Phys.Commun. 133 (2001) 229

$$\rho_{ef} = 1+\frac{g^2}{16\pi^2}\Bigg\{-\Delta\rho^F_z+\mathcal{D}^F_z(s)+\frac{5}{3}B^F_0(-s;M_W,M_W)-\frac{9}{4}\frac{c^2_W}{s^2_W}\ln c^2_W-6$$
$$+\frac{5}{8}c^2_W\left(1+c^2_W\right)+\frac{1}{4c^2_W}\left(3v^2_e+a^2_e+3v^2_f+a^2_f\right)\mathcal{F}_Z(s)+\mathcal{F}^0_W(s)+\hat{\mathcal{F}}_W(s)$$
$$-\frac{r_t}{4}\left[B^F_0(-s;M_W,M_W)+1\right]-c^2_W(R_Z-1)\,s\hat{\mathcal{B}}^d_{WW}(s,t)\Bigg\}, \qquad (A.4.80)$$

$$\kappa_e = 1+\frac{g^2}{16\pi^2}\Bigg\{-\frac{c^2_W}{s^2_W}\Delta\rho^F-\Pi^F_{Z\gamma}(s)-\frac{1}{6}B^F_0(-s;M_W,M_W)-\frac{1}{9}-\frac{v_e\sigma_e}{2c^2_W}\mathcal{F}_Z(s)$$
$$-\hat{\mathcal{F}}^0_W(s)+(R_Z-1)\left[\frac{|Q_f|}{2}\left(1-4|Q_f|s^2_W\right)\mathcal{F}_Z(s)+c^2_W\Big[\hat{\mathcal{F}}_{W_n}(s)\right.$$
$$\left.-|Q_{f'}|\mathcal{F}_{W_a}(s)+s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\right]\Bigg\}, \qquad (A.4.81)$$

$$\kappa_f = 1+\frac{g^2}{16\pi^2}\Bigg\{-\frac{c^2_W}{s^2_W}\Delta\rho^F-\Pi^F_{Z\gamma}(s)-\frac{1}{6}B^F_0(-s;M_W,M_W)-\frac{1}{9}-\frac{v_f\sigma_f}{2c^2_W}\mathcal{F}_Z(s)$$
$$-\hat{\mathcal{F}}_W(s)+(R_Z-1)\left[\frac{|Q_e|}{2}\left(1-4|Q_e|s^2_W\right)\mathcal{F}_Z(s)+c^2_W\Big[\hat{\mathcal{F}}^0_{W_n}(s)\right.$$
$$\left.-|Q_{e'}|\mathcal{F}_{W_a}(s)+s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\right]-\frac{r_t}{4}\left[B^F_0(-s;M_W,M_W)+1\right]\Bigg\}, \qquad (A.4.82)$$

interference

$$\kappa_{ef} = 1+\frac{g^2}{16\pi^2}\Bigg\{-2\frac{c^2_W}{s^2_W}\Delta\rho^F-2\Pi^F_{Z\gamma}(s)-\frac{1}{3}B^F_0(-s;M_W,M_W)-\frac{2}{9}$$
$$-\frac{1}{4c^2_W}\left[\frac{\delta^2_e+\delta^2_f}{s^2_W}(R_W-1)+3v^2_e+a^2_e+3v^2_f+a^2_f\right]\mathcal{F}_Z(s)$$
$$-\hat{\mathcal{F}}^0_W(s)-\hat{\mathcal{F}}_W(s)-\frac{r_t}{4}\left[B^F_0(-s;M_W,M_W)+1\right]$$
$$+c^2_W(R_Z-1)\left[\frac{2}{3}-\hat{\Pi}^{\text{bos},F}_{\gamma\gamma}(s)+s\hat{\mathcal{B}}^d_{WW}(s,t)\right]\Bigg\}. \qquad (A.4.83)$$

BOX

Fermionic loops in γ propagator

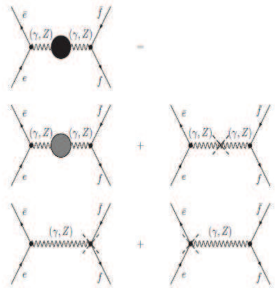


Figure A.11. Bosonic self-energies and bosonic counter-terms for $e\bar{e} \to (Z,\gamma) \to f\bar{f}$

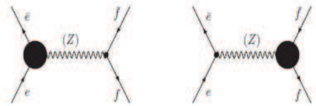


Figure A.10. Electron (a) and final fermion (b) vertices in $e\bar{e} \to (Z) \to f\bar{f}$

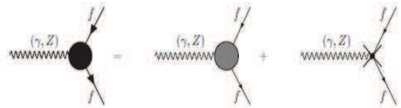


Figure A.6. Off-shell $Zf\bar{f}$ and $\gamma f\bar{f}$ vertices

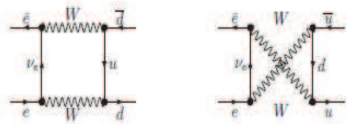


Figure A.7. The WW boxes

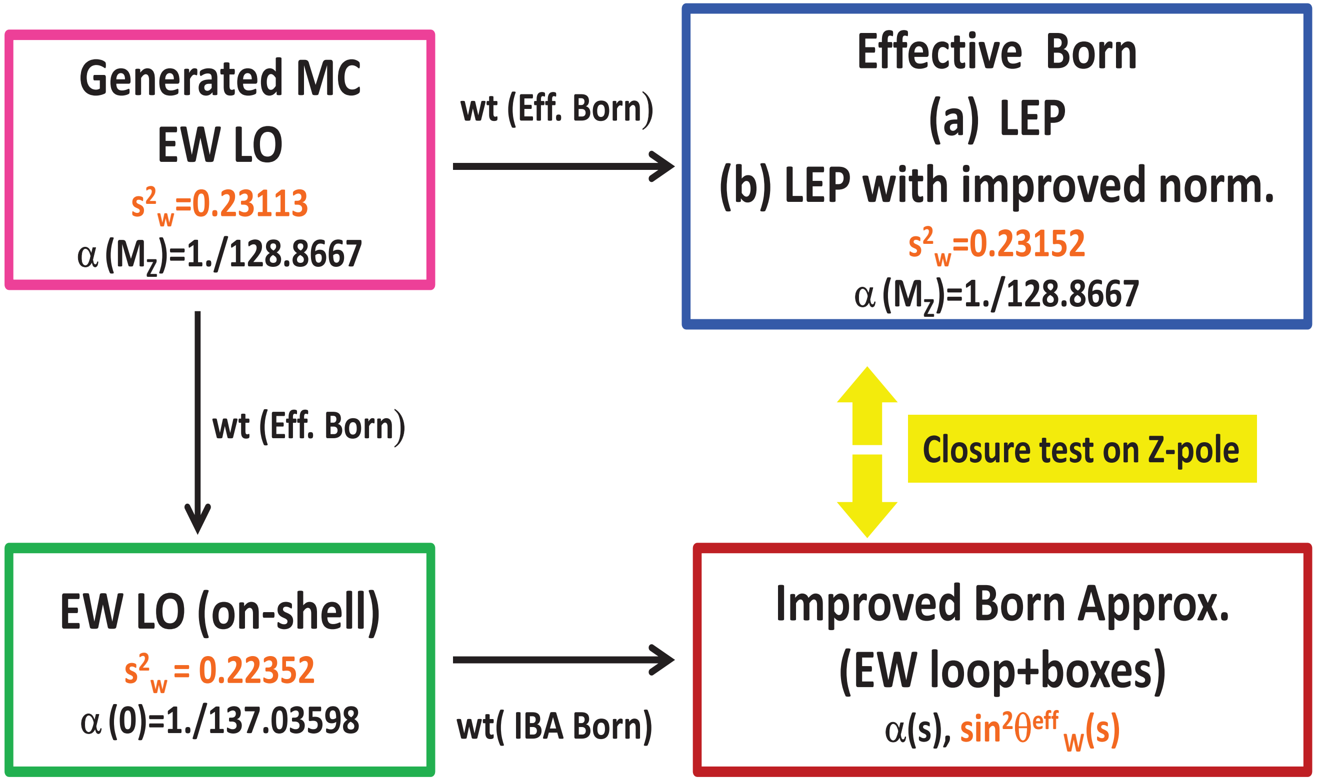
etc. etc.

Flow chart

for EW

reweighting

tests



Backup

Collins-Soper: the polar θ and azimuthal ϕ angles are constructed in lepton pair rest-frame. Since the Z-boson has usually a transverse momentum, the directions of initial protons are not collinear. The polar axis (z-axis) is bisecting the angle between the momentum of one of the proton and inverse of the momentum of the other one. The sign of the z-axis is defined by the sign of the lepton-pair momentum with respect to z-axis in the laboratory frame. The y-axis is defined as the normal vector to the plane spanned by the two incoming proton momenta.

Mustraal:

- **Definition below is for reference. It is important that every event may contribute with one of two configurations, defined either with the help of first or second beam (reconstructed parton) as seen in the rest frame of lepton pair. The final choice is made with probability independent of any couplings or PDFs.**
- We start from the following information, which turns out to be sufficient: (i) The 4-momenta and charges of outgoing leptons τ_1, τ_2. (ii) The sum of 4-momenta of all outgoing partons.
- The orientation of incoming beams b_1, b_2 is fixed as follows: b_1 is chosen to be always along positive z-axis of the laboratory frame and b_2 is anti-parallel to z axis. The information on incoming partons of p_1, p_2 is not taken from the event record. It is recalculated from kinematics of outgoing particles and knowledge of the center of mass energy of colliding protons. In this convention the energy fractions x_1 and x_2 of p_1, p_2 carried by colliding partons, define also the 3-momenta which are along b_1, b_2 respectively.
- The flavour of incoming partons (quark or antiquark) is attributed as follows: incoming parton of larger x_1 (x_2) is assumed to be the quark. This is equivalent to choice that the quark follow direction of the outgoing $\ell\ell$ system, similarly as it is defined for the Collins-Soper frame. This choice is necessary to fix sign of $\cos\theta_{1,2}$ defined later.
- The 4-vectors of incoming partons and outgoing leptons are boosted into lepton-pair rest frame.
- To fix orientation of the event we use versor $\hat{x}_{lab}$ of the laboratory reference frame. It is boosted into

lepton-pair rest frame as well. It will be used in definition of azimuthal angle ϕ, which has to extend over the range $(0, 2\pi)$.

- We first calculate $\cos\theta_1$ (and $\cos\theta_2$) of the angle between the outgoing lepton and incoming quark (outgoing anti-lepton and incoming anti-quark) directions.

$$\cos\ \theta_1 = \frac{\vec{\tau_1} \cdot \vec{p_1}}{|\vec{\tau_1}||\vec{p_1}|}, \qquad \cos\ \theta_2 = \frac{\vec{\tau_2} \cdot \vec{p_2}}{|\vec{\tau_1}||\vec{p_2}|} \tag{4}$$

- The azimuthal angles ϕ_1 and ϕ_2 corresponding to θ_1 and θ_2 are defined as follows. We first define $e\vec{y_{1,2}}$ versors and with their help later $\phi_{1,2}$ as:

$$\vec{e_y} = \frac{x\vec{_{lab}} \times \vec{p_2}}{|\vec{e_y}|}, \quad \vec{e_x} = \frac{\vec{e_y} \times \vec{p_2}}{|\vec{e_x}|}$$

$$\begin{aligned}
\cos\ \phi_1 &= \frac{\vec{e_x} \cdot \vec{\tau_1}}{\sqrt{(\vec{e_x} \cdot \vec{\tau_1})^2 + (\vec{e_y} \cdot \vec{\tau_1})^2}} \\
\sin\ \phi_1 &= \frac{\vec{e_y} \cdot \vec{\tau_1}}{\sqrt{(\vec{e_x} \cdot \vec{\tau_1})^2 + (\vec{e_y} \cdot \vec{\tau_1})^2}}
\end{aligned} \tag{5}$$

and similarly for ϕ_2:

$$\vec{e_y} = \frac{x\vec{_{lab}} \times \vec{p_1}}{|\vec{e_y}|}, \quad \vec{e_x} = \frac{\vec{e_y} \times \vec{p_1}}{|\vec{e_x}|}$$

$$\cos\ \phi_2 = \frac{\vec{e_x} \cdot \vec{\tau_2}}{\sqrt{(\vec{e_x} \cdot \vec{\tau_2})^2 + (\vec{e_y} \cdot \vec{\tau_2})^2}}$$
$$\sin\ \phi_2 = \frac{\vec{e_y} \cdot \vec{\tau_2}}{\sqrt{(\vec{e_x} \cdot \vec{\tau_2})^2 + (\vec{e_y} \cdot \vec{\tau_2})^2}}. \qquad (6)$$

- Each event contributes with two Born-like kinematics configurations $\theta_1\phi_1$, ($\theta_2\phi_2$), respectively with wt_1 (and wt_2) weights; $wt_1 + wt_2 = 1$ where

$$wt_1 = \frac{E_{p1}^2(1+\cos^2\theta_1)}{E_{p1}^2(1+\cos^2\theta_1) + E_{p2}^2(1+\cos^2\theta_2)},$$
$$wt_2 = \frac{E_{p2}^2(1+\cos^2\theta_2)}{E_{p1}^2(1+\cos^2\theta_1) + E_{p2}^2(1+\cos^2\theta_2)}. \qquad (7)$$

In the calculation of the weight, incoming partons energies E_{p1}, E_{p2} in the rest frame of lepton pair are used, but not their couplings or flavours. That is also why, instead of $\sigma_B(s, \cos\theta)$ the simplification $(1 + \cos^2\theta)$ is used in Eq. (7).

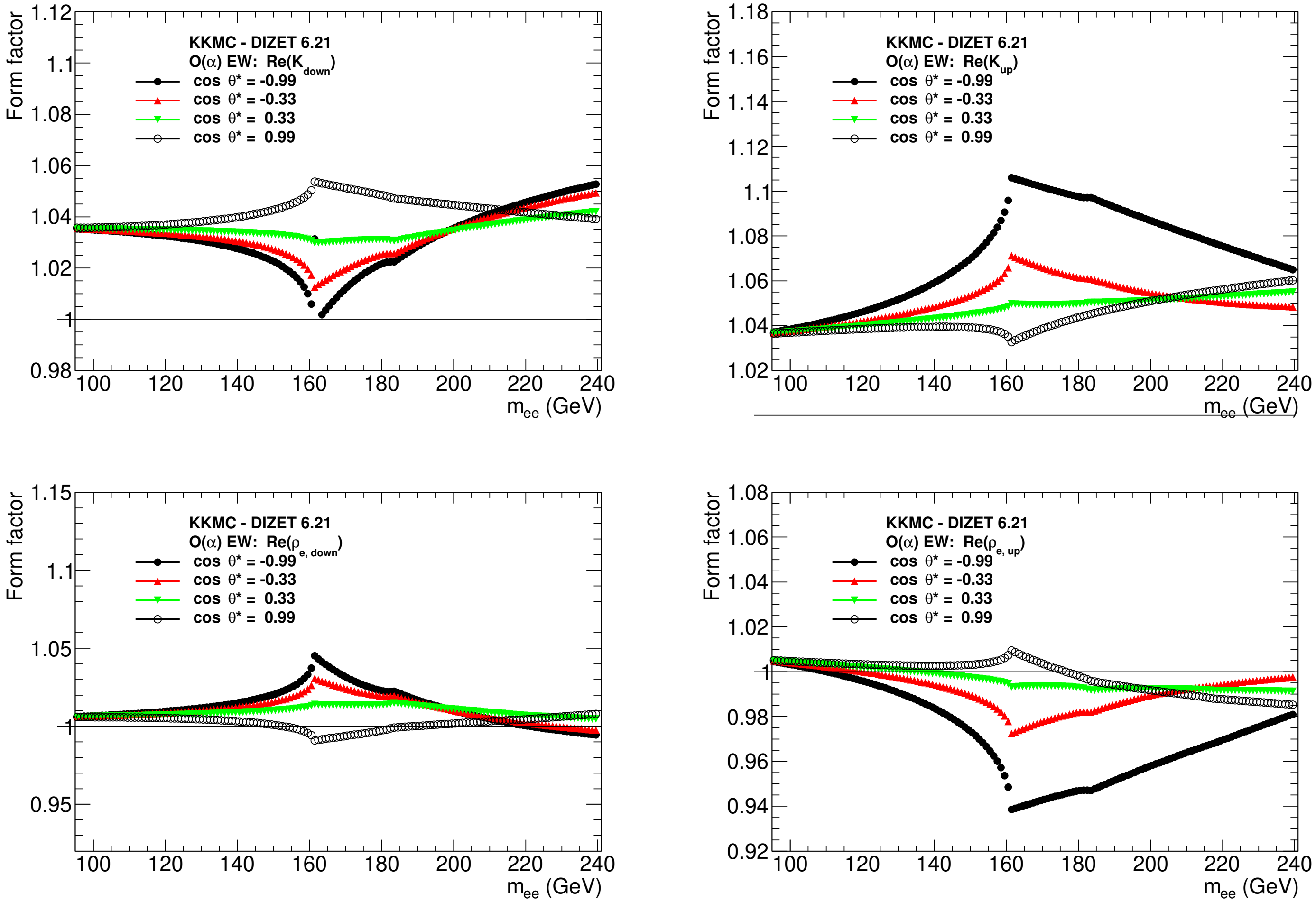


Figure 6: Real part of K_{down}, K_{up}, $\rho_{e,down}$ and $\rho_{e,up}$ as a function of m_{ee} for few values of $\cos\theta^*$. Note the WW and ZZ threshold effects which exhibits as discontinuity. At higher virtualities θ^* dependence is sizable, there electroweak effects distort picture of spherical harmonics. Important for LHC reweighting, important for KKMC.

Table 3: The DIZET predictions for two different parametrisations of $\Delta\alpha_h^{(5)}(M_Z^2)$. Other flags set for defaults.

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2)$ = 0.0280398 (param. Jegerlehner 1995)	$\Delta\alpha_h^{(5)}(M_Z^2)$ = 0.0275762 (param. Jegerlehner 2017)	Δ
$\alpha(M_Z^2)$	0.0077587482	0.0077549256	
$1/\alpha(M_Z^2)$	128.88676996	128.95030224	
M_W (GeV)	80.350538	80.358936	+8.4 MeV
Δr	0.03690873	0.03640338	
Δr_{rem}	0.01168001	0.01167960	
s_W^2	0.22356339	0.22340108	- 0.00016
$sin^2\theta_W^{eff\ lepton}(M_Z^2)$	0.23166087	0.23149900	- 0.00023
$sin^2\theta_W^{eff\ up-quark}(M_Z^2)$	0.23155425	0.23139248	- 0.00016
$sin^2\theta_W^{eff\ down-quark}(M_Z^2)$	0.23142705	0.23126543	- 0.00016