Mixed QCD-EW corrections to NC-DY: numerical challenges

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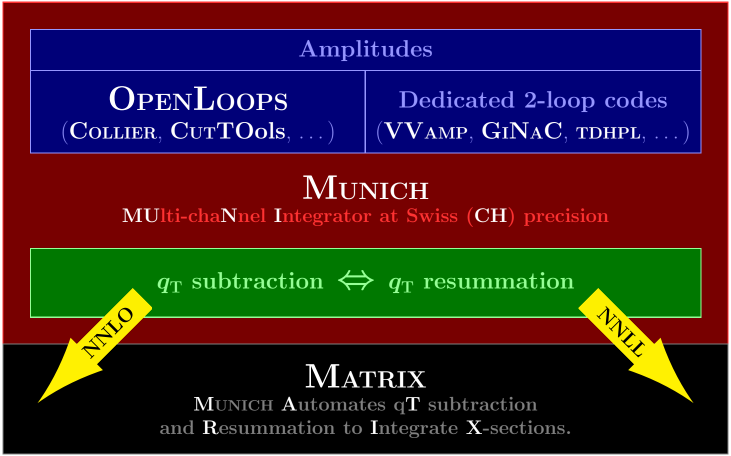
Outline

1. The MATRIX framework for precision calculations
   - Basic features of the MUNICH/MATRIX framework
   - Implementation of external amplitudes
   - The q_T subtraction method for higher-order corrections

2. Numerical challenges in the q_T subtraction method
   - Production of colourless final states at NNLO QCD accuracy
   - Extension to (associated) heavy-quark pair production at NNLO QCD accuracy
   - Extension to mixed NNLO QCD–EW corrections for charged massive colourless final states

The MATRIX framework for automated NNLO QCD calculations (and beyond)

[Grazzini, SK, Wiesemann (2018) + Rathlev; Buonocore, Devoto, Mazzitelli, Savoini, Yook, . . .]



available under `https://matrix.hepforge.org/`

MATRIX v1

- H, V, $\gamma\gamma$, Vγ, VV at NNLO QCD for all leptonic decay channels

MATRIX v2

- combination with NLO EW for all leptonic V and VV processes
- loop-induced *gg* channel at NLO QCD for neutral VV processes
- several technical improvements, e.g. tail enhancement runs

still not included: q_T resummation

but: **MATRIX+RADISH** available

➥ public interface to provide direct-space resummation (q_T, $p_{T,\text{jet}}$, combined) for all MATRIX processes

The MUNICH/MATRIX framework for automated NNLO calculations

MATRIX — MUNICH Automates qT-subtraction and Resummation to Integrate X-sections
[Grazzini, SK, Wiesemann (2018)]

- first public tool that performs NNLO QCD calculations for a large class of processes
- core of the framework: the C++ parton-level Monte Carlo generator
  MUNICH — MUlti-chaNnel Integrator at swiss (CH) precision [SK]
  - bookkeeping of partonic subprocesses for all contributions
  - fully automated dipole subtraction for NLO calculations (massive, QCD and EW)
  - general amplitude interface
  - highly efficient multi-channel Monte Carlo integration with several optimization features
  - simultaneous monitoring of slicing parameter and automated extrapolation
- PYTHON script to simplify the use of MATRIX
  - installation of MUNICH and all supplementary software
  - interactive shell steering all run phases without human intervention (grid-, pre-, main-run, summary)
  - organization of parallelized running on multicore machines and commonly used clusters: SLURM, HTCONDOR, LSF, etc.

Performance features of the MUNICH phase space integrator

Issue of poorly populated regions

- sample case: high-energy tails
- standard phase space optimization samples points in bulk region

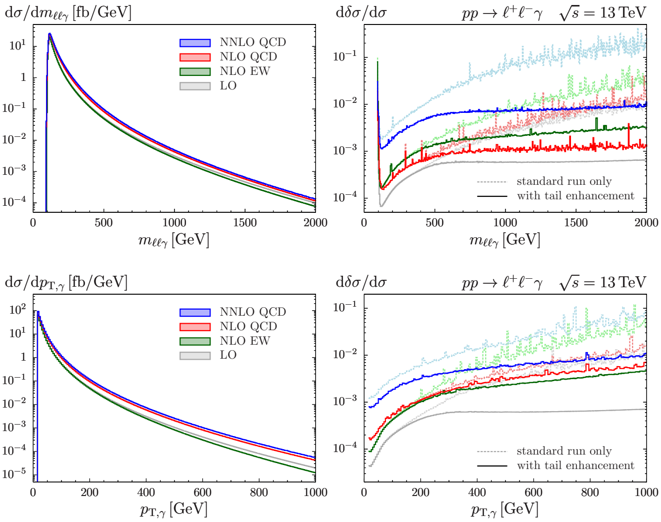
Solution in MUNICH integrator

- additional runs with optimization including a general bias factor
- sophisticated automated combination with results from standard runs

Significantly improved errors

- $\mathcal{O}(10)$ and better with doubled runtime
- simultaneous enhancement of observables

Good performance also for off-shell regions of intermediate resonances



Supplying MUNICH/MATRIX with 1-loop amplitudes

Process-independent interfaces to general automated amplitude generators

- OPENLOOPS [Cascioli, Maierhöfer, Pozzorini (2012); SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)] v2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)], written in FORTRAN
  - general code and process libraries
  - on-the-fly tensor reduction [Buccioni, Pozzorini, Zoller (2018)] with hybrid-precision stability system
  - scalar integrals from COLLIER [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2011)] or ONELOOP [van Hameren (2011)]
- RECOLA [Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)] v2 [Denner, Lang, Uccirati (2017)], written in FORTRAN
  - on-the-fly generation of amplitudes
  - tensor reduction and scalar integrals via COLLIER [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2003, 2006, 2011)]
  - different model files available, also for SMEFT and BSM applications
- modular structure of MUNICH allows other generators to be interfaced as well

Several dedicated interfaces developed in context of MATRIX applications

- loop×tree and loop×loop colour (and spin) correlators
- helicity amplitudes, colour-stripped amplitudes to construct 4-colour correlators
- imaginary parts of loop×tree amplitudes and correlators, helicity-flip amplitudes

Interfacing dedicated 2-loop amplitudes to MUNICH/MATRIX

- Higgs, Drell–Yan, **VH**, $\gamma\gamma$, **V**γ production
  - direct implementation of public analytic results, e.g. for Vγ [Gehrmann, Tandredi (2012)]
- **VV** production — qqVVAMP [Gehrmann, von Manteuffel, Tancredi (2015)] and ggVVAMP [von Manteuffel, Tancredi (2015)] libraries
  - C++ libraries using GINAC [Bauer, Frink, Kreckel (2002); Vollinga, Weinzierl (2005)] and CLN for arbitrary precision arithmetics
  - IBP approach, generated using MATHEMATICA, FORM [Vermaseren et al.], REDUZE2 [von Manteuffel, Studerus ('12)]
  - independent calculation of amplitudes in [Caola, Henn, Melnikov, Smirnov, Smirnov (2015; 2016)]
  - Higgs-mediated helicity amplitudes with full m_t dependence from [Harlander, Prausa, Usovitsch (2019; 2020)]
- $\gamma\gamma\gamma$ production — amplitudes from [Abreu, Page, Pascual, Sotnikov ('20)]
  - C++ library, generated by CARAVEL [Abreu et al. (2020)], applying PENTAGONFUNCTIONS++ [Chicherin, Sotnikov (2020)]
  - numerical unitarity and analytic reconstruction techniques [Ita (2015); Abreu et al. (2018; 2018; 2019; 2019)]
- **HH** production (full m_t dependence) — HHGRID library [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
  - PYTHON based numerical interpolation of amplitude grid
  - generated by 2-loop extension of GOSAM [Jones (2016)], REDUZE2 [von Manteuffel, Studerus ('12)], SECDEC3 [Borowka et al. (2015)]
- $\mathbf{Q\bar{Q}}$ production — amplitude grids from [Bärnreuther, Czakon, Fiedler (2014)]
  - FORTRAN routine for numerical interpolation of 2-dimensional grid, improved by expansions

Idea of the q_T subtraction method for (N)NLO cross sections

Consider the production of a **colourless final state F** via $\mathbf{q\bar{q} \to F}$ or $\mathbf{gg \to F}$: $\quad \mathrm{d}\sigma_{\mathrm{F}}^{\mathrm{(N)NLO}}\Big|_{q_T \neq 0} = \mathrm{d}\sigma_{\mathrm{F+jet}}^{\mathrm{(N)LO}}$

where q_T refers to the transverse momentum of the colourless system F [Catani, Grazzini (2007)]

- $\mathrm{d}\sigma_{\mathrm{F}}^{\mathrm{(N)NLO}}\Big|_{q_T \neq 0}$ is singular for $q_T \to 0$
  - ➥ limiting behaviour known from transverse-momentum resummation [Bozzi, Catani, de Florian, Grazzini (2006)]
- Define a universal counterterm $\mathbf{\Sigma}$ with the complementary $q_T \to 0$ behaviour [Bozzi, Catani, de Florian, Grazzini (2006)]

  $\mathrm{d}\sigma^{\mathrm{CT}} = \Sigma(q_T/q) \otimes \mathrm{d}\sigma^{\mathrm{LO}}$ where q is the invariant mass of the colourless system F
- Add the $q_T = 0$ piece with the hard–virtual coefficient $\mathcal{H}_{\mathrm{F}}$, which contains the 1-(2-)loop amplitudes at (N)NLO and compensates for the subtraction of $\mathbf{\Sigma}$ [Catani, Cieri, de Florian, Ferrera, Grazzini (2013)]

➥ **Master formula for (N)NLO cross section in q_T subtraction method**

$$\mathrm{d}\sigma_{\mathrm{F}}^{\mathrm{(N)NLO}} = \mathcal{H}_{\mathrm{F}}^{\mathrm{(N)NLO}} \otimes \mathrm{d}\sigma^{\mathrm{LO}} + \left[\mathrm{d}\sigma_{\mathrm{F+jet}}^{\mathrm{(N)LO}} - \Sigma^{\mathrm{(N)NLO}} \otimes \mathrm{d}\sigma^{\mathrm{LO}}\right]_{\mathrm{cut}_{q_T} \to 0}$$

- all ingredients known for extension to N^{3}LO [Luo, Yang, Zhu, Zhu (2019; 2020), Ebert, Mistlberger, Vita (2020), Cieri, Chen, Gehrmann, Glover, Huss (2019), Camarda, Cieri, Ferrera ('21), Chen, Gehrmann, Glover, Huss, Yang, Zhu (2021)]

Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — sample case $pp \to \gamma\gamma + X$

Result for $r_{\text{cut}} \to 0$ via extrapolation

- automated and simultaneous scan over reasonable range of r_{cut} values
- quadratic least-χ^2 fit with variable range

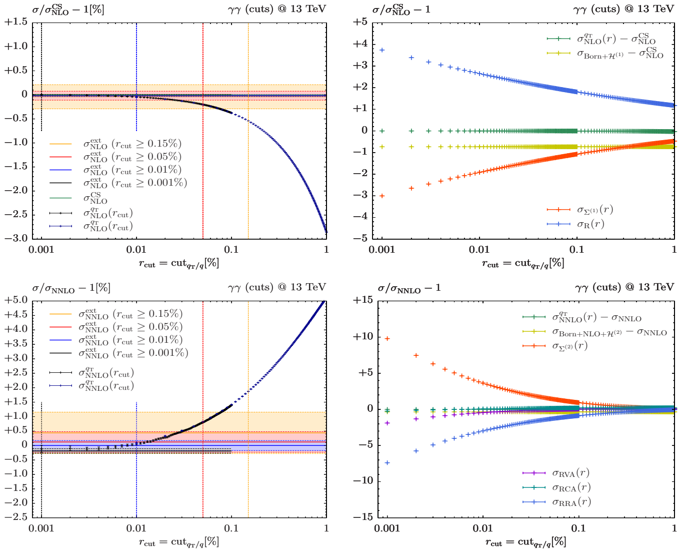
$$\sigma_{\text{(N)NLO}}(r_{\text{cut}}) = A r_{\text{cut}}^2 + B r_{\text{cut}} + \sigma_{\text{(N)NLO}}$$

- error estimate based on combination of statistical error and variation of r_{cut} range

➥ **Significant $r_{\text{cut}} \to 0$ dependence for processes involving isolated photons** (similar between NLO and NNLO QCD)

➥ good agreement of extrapolated results within errors for different start values

- $r_{\text{cut}} \geq 0.15\%$
- $r_{\text{cut}} \geq 0.05\%$
- $r_{\text{cut}} \geq 0.01\%$
- $r_{\text{cut}} \geq 0.001\%$



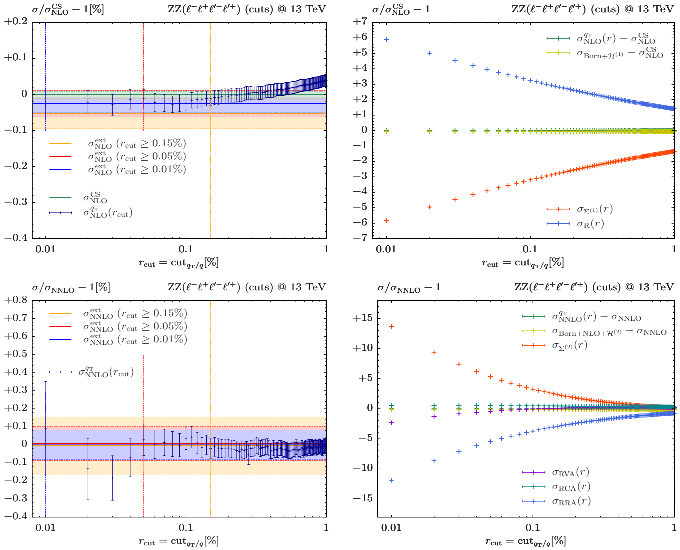
Investigation of $r_{\rm cut} = {\rm cut}_{q_T/q}$ dependence — sample case $pp \to \ell^-\ell^+\ell'^-\ell'^+ + X$

Result for $r_{\rm cut} \to 0$ via extrapolation

- same procedure for all processes

$$\sigma_{\rm (N)NLO}(r_{\rm cut}) = A r_{\rm cut}^2 + B r_{\rm cut} + \sigma_{\rm (N)NLO}$$

- **No significant $r_{\rm cut} \to 0$ dependence for processes without isolated photons** (similar between NLO and NNLO QCD)
  - Important exception: symmetric cut configurations (e.g. standard Drell–Yan setup)
- good agreement of extrapolated results within errors for different start values
  - $r_{\rm cut} \geq 0.15\%$
  - $r_{\rm cut} \geq 0.05\%$
  - $r_{\rm cut} \geq 0.01\%$
- larger cancellation between contributions (factor of ≈ 15 at $r_{\rm cut} = 0.01\%$)



Investigation of $r_{cut} = \mathrm{cut}_{q_T/q}$ dependence — Drell–Yan processes

$pp \to Z$

- fully inclusive

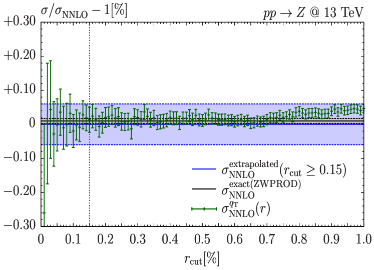
$pp \to \ell^+ \nu_\ell$

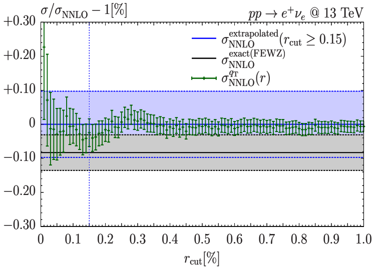
- $p_{T,\ell^+} > 25$ GeV $\quad \eta_{\ell^+} < 2.47$
- $p_{T,\text{miss}} > 20$ GeV

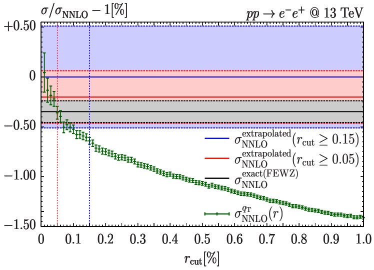
$pp \to \ell^- \ell^+$

- $p_{T,\ell^\pm} > 25$ GeV $\quad \eta_{\ell^\pm} < 2.47$
- $66\,\text{GeV} < m_{\ell^+\ell^-} < 116$ GeV

[Grazzini, SK, Wiesemann (2018)]







- no phase space restrictions
- no significant dependence on r_{cut}

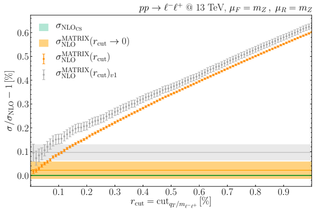
- asymmetric cuts on two individual leptons (ℓ^+ and ν_ℓ)
- no significant dependence on r_{cut}

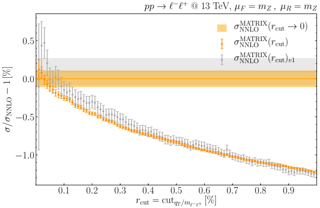
- symmetric cuts on both leptons (either on ℓ^+/ℓ^- or on ℓ_1/ℓ_2)
- large power corrections in r_{cut}

Dependence on r_{cut} in different cut scenarios for the NC Drell–Yan process

Symmetric cuts

- $p_{\mathrm{T},\ell^\pm} > 25$ GeV

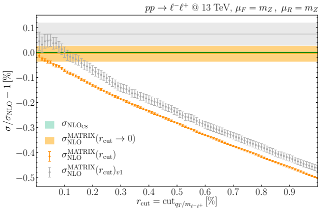


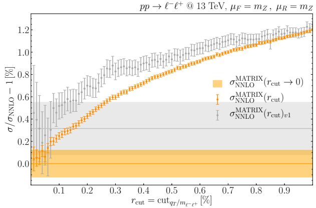


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ_1 and ℓ_2

- $p_{\mathrm{T},\ell_1} > 25$ GeV $p_{\mathrm{T},\ell_2} > 20$ GeV

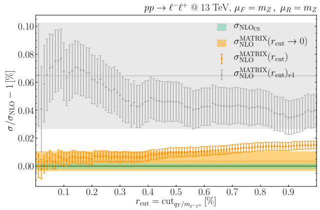


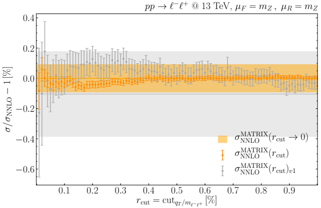


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{\mathrm{T},\ell^+} > 25$ GeV $p_{\mathrm{T},\ell^-} > 20$ GeV





➥ no significant dependence on r_{cut}

Production of heavy coloured particles at NNLO QCD accuracy

Extension of q_T subtraction method to production of heavy coloured particles (e.g. top-quark pairs)

$$d\sigma^{t\bar{t}}_{\mathrm{NNLO}} = \mathcal{H}^{t\bar{t}}_{\mathrm{NNLO}} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{NLO}} - d\sigma^{t\bar{t},\mathrm{CT}}_{\mathrm{NNLO}} \right]_{r_{\mathrm{cut}} \to 0}$$

- counterterm accounts for IR behaviour of real contribution, including soft singularities related to emissions from final-state quarks [Catani, Grazzini, Torre (2014), Ferroglia, Neubert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
- $\mathcal{H}^{t\bar{t}}_{\mathrm{NNLO}}$ contains remainder of integrated final-state soft singularities [Catani, Devoto, Grazzini, Mazzitelli (to appear), Angeles-Martinez, Czakon, Sapeta (2018)]
- massive NLO subtraction required for real-emission part, e.g. massive dipole subtraction [Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]

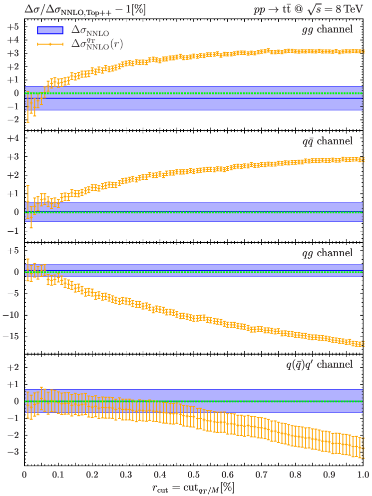
Associated heavy-quark pair production ($t\bar{t}$H, $t\bar{t}$V, . . .) with identical singularity structure

- numerical solutions for required for evaluation of the soft function due to more involved kinematics
  - ➥ no back-to-back configuration of heavy quarks
- proof-of-principle calculation for non-diagonal channels in $t\bar{t}\mathrm{H}$ [Catani, Fabre, SK, Grazzini (2021)]
- two-loop amplitudes as the bottleneck for any beyond $2 \to 2$ NNLO calculations

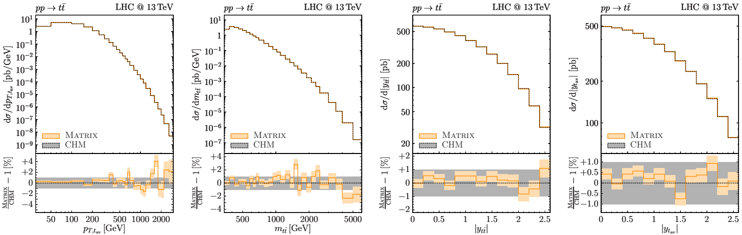
Top-quark pair production at NNLO QCD accuracy

First MATRIX calculation for colourful final states at NNLO QCD

[Catani, Devoto, Grazzini, SK, Mazzitelli, Sargsyan (2019)]



- 2-loop amplitudes from numerical result [Bärnreuther, Czakon, Fiedler (2014)]
- slicing parameter dependence under good numerical control; investigation after splitting into partonic channels
  - ➥ full agreement with TOP++ [Czakon, Mitov (2014)]
- successful validation also on the level of differential distributions [Catani, Devoto, Grazzini, SK, Mazzitelli (2019)] (comparison against results from [Czakon, Heymes, Mitov (2017)])



Mixed NNLO QCD–EW calculation for production of massive charged particles

Extension of q_T subtraction method to mixed QCD–EW corrections of $\mathcal{O}(\alpha_s^m \alpha^n)$

$$d\sigma_{(m,n)}^{\ell\ell/\ell\nu} = \mathcal{H}_{(m,n)}^{\ell\ell/\ell\nu} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma_{(m,n)}^{\ell\ell/\ell\nu,\mathrm{R}} - d\sigma_{(m,n)}^{\ell\ell/\ell\nu,\mathrm{CT}} \right]_{r_{\mathrm{cut}} \to 0}$$

- $m = 1(2)$ and $n = 0$: (N)NLO QCD corrections
- $m = 0$ and $n = 1$: NLO EW corrections
- $m = 1$ and $n = 1$: mixed NNLO QCD–EW corrections

(limitation: no massless jets (for $m \geq 1$) and no massless charged particles (for $n \geq 1$) allowed at LO)

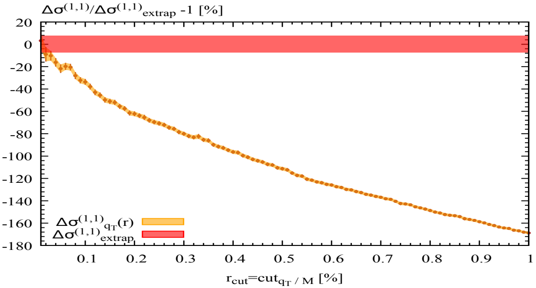
Strategy to cancel IR singularities in mixed QCD–EW corrections

- abelianisation procedure, starting from heavy-quark pair production at NNLO QCD
  - ➥ for neutral final states, abelianisation of standard q_T subtraction method is sufficient (mixed QCD–QED corrections on $pp \to \mathrm{Z}$ [De Florian, Der, Fabre (2018)], $pp \to \nu\bar{\nu}$ [Cieri, De Florian, Der, Mazzitelli (2020)])
- colourless final state ($\ell\ell/\ell\nu$) results in soft final-state singularities of pure QED origin
  - ➥ much simpler IR structure than in heavy-quark pair production at NNLO QCD
- finite charged-lepton mass required to regularize collinear final-state singularities

Dependence on r_{cut} of the mixed NNLO QCD–EW corrections for NC Drell–Yan

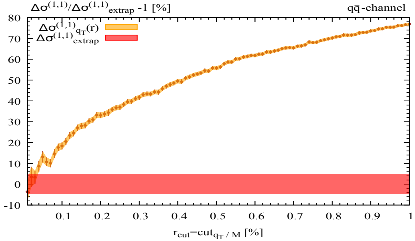
Symmetric-cut scenario

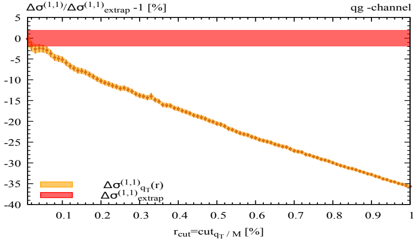
$p_{T,\ell^\pm} > 25$ GeV $y_{\ell^\pm} < 2.5$ $m_{\ell\ell} > 50$ GeV



- **large power corrections in r_{cut} for mixed corrections**
  - explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
  - better than permille precision at inclusive level

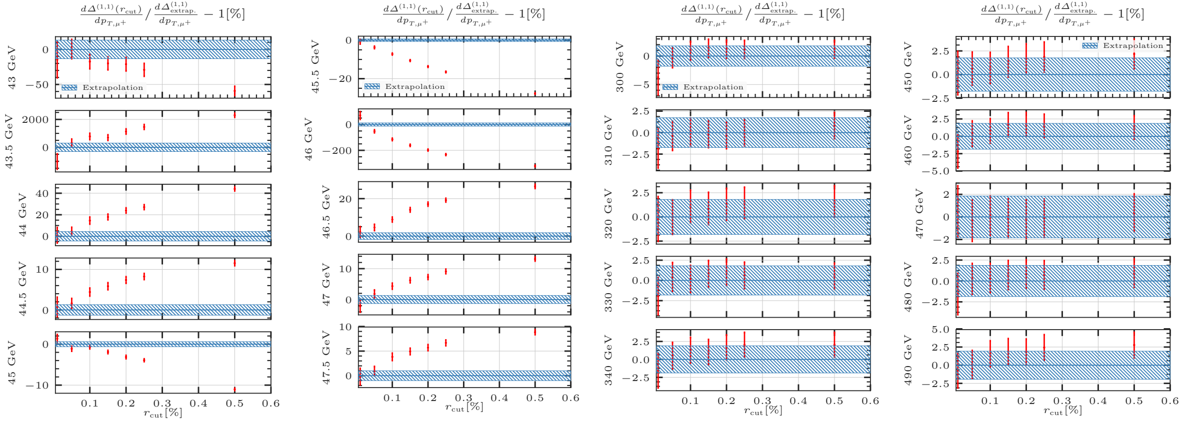
Splitting into partonic channels





Binwise r_{cut} dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

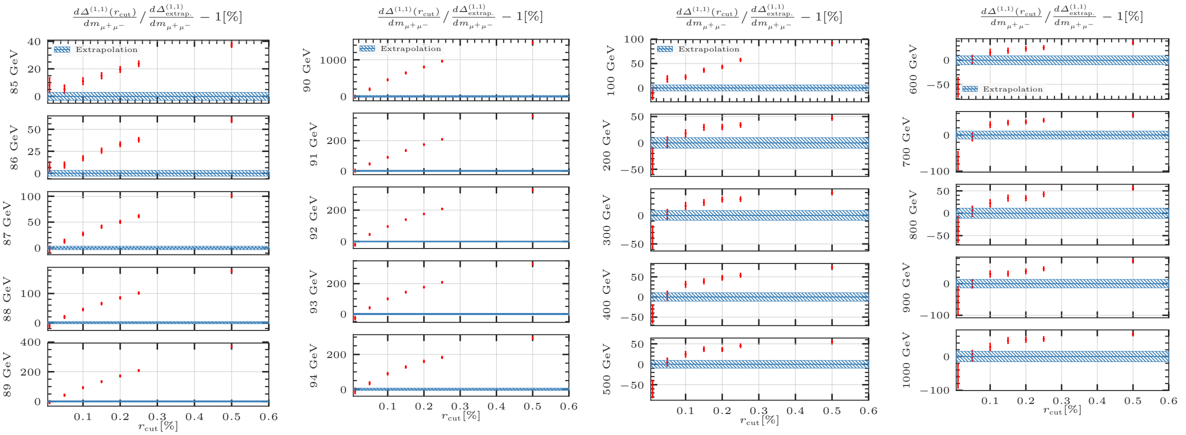
Differential distribution in p_{T,μ^+}: peak (left panels) and tail (right panels) regions



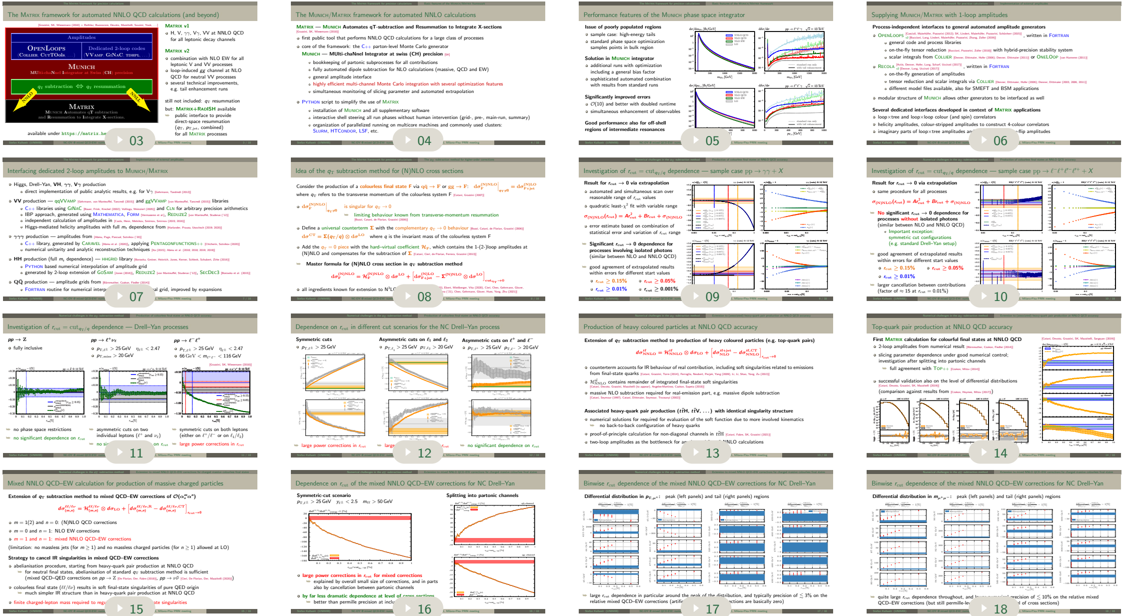
➥ large r_{cut} dependence in particular around the peak of the distribution, and typically precision of $\lesssim 3\%$ on the relative mixed QCD–EW corrections (artificially large where corrections are basically zero)

Binwise $r_{\rm cut}$ dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



➥ quite large $r_{\rm cut}$ dependence throughout, and lower numerical precision of $\lesssim 10\%$ on the relative mixed QCD–EW corrections (but still permille-level precision at the level of cross sections)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