

Mixed QCD-EW corrections to NC-DY: numerical challenges

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Outline

1 The MATRIX framework for precision calculations

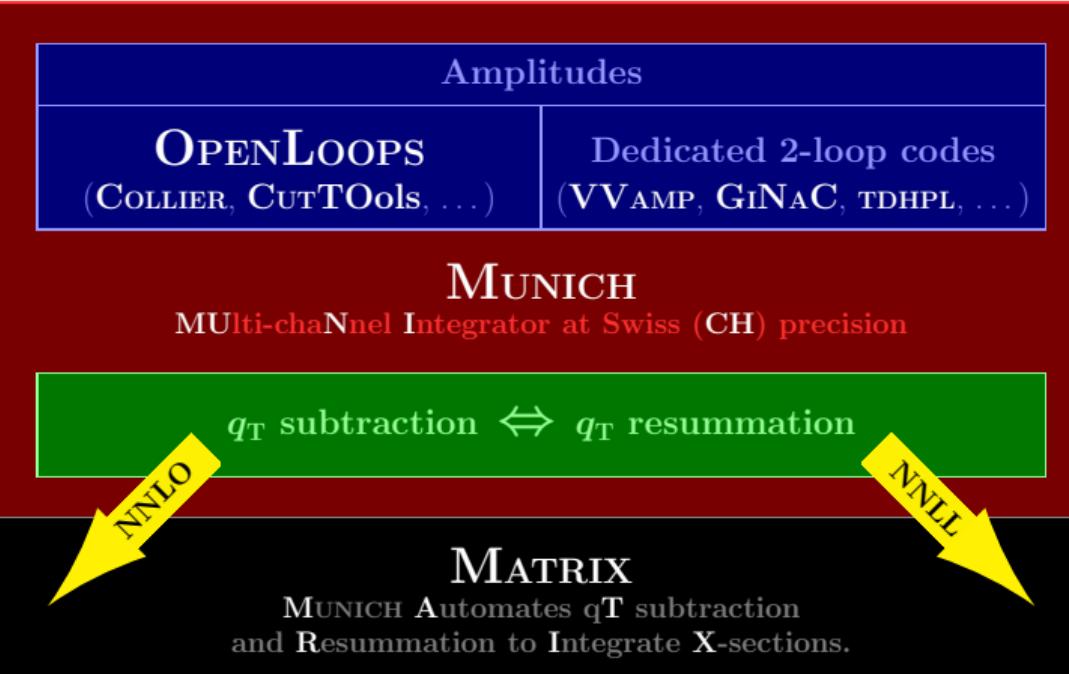
- Basic features of the MUNICH/MATRIX framework
- Implementation of external amplitudes
- The q_T subtraction method for higher-order corrections

2 Numerical challenges in the q_T subtraction method

- Production of colourless final states at NNLO QCD accuracy
- Extension to (associated) heavy-quark pair production at NNLO QCD accuracy
- Extension to mixed NNLO QCD-EW corrections for charged massive colourless final states

The MATRIX framework for automated NNLO QCD calculations (and beyond)

[Grazzini, SK, Wiesemann (2018) + Rathlev; Buonocore, Devoto, Mazzitelli, Savoini, Yook, ...]



available under <https://matrix.hepforge.org/>

MATRIX v1

- $H, V, \gamma\gamma, V\gamma, VV$ at NNLO QCD for all leptonic decay channels

MATRIX v2

- combination with NLO EW for all leptonic V and VV processes
- loop-induced gg channel at NLO QCD for neutral VV processes
- several technical improvements, e.g. tail enhancement runs

still not included: q_T resummation

but: **MATRIX+RADISH** available

- ➡ public interface to provide direct-space resummation ($q_T, p_{T,jet}$, combined) for all **MATRIX** processes

The MUNICH/MATRIX framework for automated NNLO calculations

MATRIX — MUNICH Automates qT-subtraction and Resummation to Integrate X-sections

[Grazzini, SK, Wiesemann (2018)]

- first public tool that performs NNLO QCD calculations for a large class of processes
- core of the framework: the C++ parton-level Monte Carlo generator

MUNICH — MUlti-chaNnel Integrator at swiss (CH) precision [SK]

- bookkeeping of partonic subprocesses for all contributions
 - fully automated dipole subtraction for NLO calculations (massive, QCD and EW)
 - general amplitude interface
 - highly efficient multi-channel Monte Carlo integration with several optimization features
 - simultaneous monitoring of slicing parameter and automated extrapolation
-
- PYTHON script to simplify the use of MATRIX
 - installation of MUNICH and all supplementary software
 - interactive shell steering all run phases without human intervention (grid-, pre-, main-run, summary)
 - organization of parallelized running on multicore machines and commonly used clusters:
SLURM, HTCONDOR, LSF, etc.

Performance features of the MUNICH phase space integrator

Issue of poorly populated regions

- sample case: high-energy tails
- standard phase space optimization samples points in bulk region

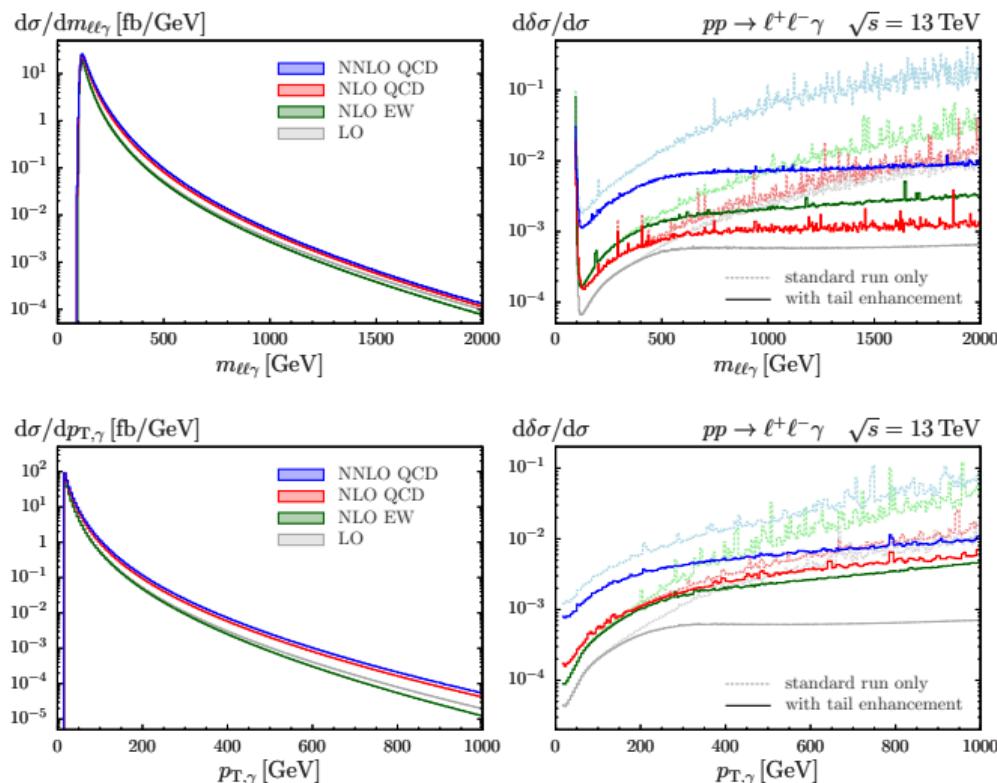
Solution in MUNICH integrator

- additional runs with optimization including a general bias factor
- sophisticated automated combination with results from standard runs

Significantly improved errors

- $\mathcal{O}(10)$ and better with doubled runtime
- simultaneous enhancement of observables

Good performance also for off-shell regions of intermediate resonances



Supplying MUNICH/MATRIX with 1-loop amplitudes

Process-independent interfaces to general automated amplitude generators

- **OPENLOOP^S** [Cascioli, Maierhöfer, Pozzorini (2012); SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)], written in **FORTRAN**
 - general code and process libraries
 - on-the-fly tensor reduction [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)] with hybrid-precision stability system
 - scalar integrals from **COLLIER** [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2011)] or **ONELOOP** [van Hameren (2011)]
- **RECOLA** [Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)], written in **FORTRAN**
 - on-the-fly generation of amplitudes
 - tensor reduction and scalar integrals via **COLLIER** [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2003, 2006, 2011)]
 - different model files available, also for SMEFT and BSM applications
- modular structure of **MUNICH** allows other generators to be interfaced as well

Several dedicated interfaces developed in context of **MATRIX** applications

- loop×tree and loop×loop colour (and spin) correlators
- helicity amplitudes, colour-stripped amplitudes to construct 4-colour correlators
- imaginary parts of loop×tree amplitudes and correlators, helicity-flip amplitudes

Interfacing dedicated 2-loop amplitudes to MUNICH/MATRIX

- Higgs, Drell–Yan, **VH**, $\gamma\gamma$, $V\gamma$ production
 - direct implementation of public analytic results, e.g. for $V\gamma$ [Gehrmann, Tandredi (2012)]
- **VV** production — **qqVVAMP** [Gehrmann, von Manteuffel, Tancredi (2015)] and **ggVVAMP** [von Manteuffel, Tancredi (2015)] libraries
 - C++ libraries using **GINAC** [Bauer, Frink, Kreckel (2002); Vollinga, Weinzierl (2005)] and **CLN** for arbitrary precision arithmetics
 - IBP approach, generated using **MATHEMATICA**, **FORM** [Vermaseren et al.], **REDUCE2** [von Manteuffel, Studerus ('12)]
 - independent calculation of amplitudes in [Caola, Henn, Melnikov, Smirnov, Smirnov (2015; 2016)]
 - Higgs-mediated helicity amplitudes with full m_t dependence from [Harlander, Prausa, Usovitsch (2019; 2020)]
- $\gamma\gamma\gamma$ production — amplitudes from [Abreu, Page, Pascual, Sotnikov ('20)]
 - C++ library, generated by **CARAVEL** [Abreu et al. (2020)], applying **PENTAGONFUNCTIONS++** [Chicherin, Sotnikov (2020)]
 - numerical unitarity and analytic reconstruction techniques [Ita (2015); Abreu et al. (2018; 2018; 2019; 2019)]
- **HH** production (full m_t dependence) — **HHGRID** library [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
 - PYTHON based numerical interpolation of amplitude grid
 - generated by 2-loop extension of **GoSAM** [Jones (2016)], **REDUCE2** [von Manteuffel, Studerus ('12)], **SECDEC3** [Borowka et al. (2015)]
- **Q \bar{Q}** production — amplitude grids from [Bärnreuther, Czakon, Fiedler (2014)]
 - FORTRAN routine for numerical interpolation of 2-dimensional grid, improved by expansions

Idea of the q_T subtraction method for (N)NLO cross sections

Consider the production of a **colourless final state F** via $q\bar{q} \rightarrow F$ or $gg \rightarrow F$: $d\sigma_F^{(N)\text{NLO}} \Big|_{q_T \neq 0} = d\sigma_{F+\text{jet}}^{(N)\text{LO}}$
where q_T refers to the transverse momentum of the colourless system F [Catani, Grazzini (2007)]

- $d\sigma_F^{(N)\text{NLO}} \Big|_{q_T \neq 0}$ is singular for $q_T \rightarrow 0$
 - limiting behaviour known from transverse-momentum resummation [Bozzi, Catani, de Florian, Grazzini (2006)]
- Define a universal counterterm Σ with the complementary $q_T \rightarrow 0$ behaviour [Bozzi, Catani, de Florian, Grazzini (2006)]

$$d\sigma^{\text{CT}} = \Sigma(q_T/q) \otimes d\sigma^{\text{LO}}$$
 where q is the invariant mass of the colourless system F
- Add the $q_T = 0$ piece with the hard-virtual coefficient \mathcal{H}_F , which contains the 1-(2-)loop amplitudes at (N)NLO and compensates for the subtraction of Σ [Catani, Cieri, de Florian, Ferrera, Grazzini (2013)]
 - **Master formula for (N)NLO cross section in q_T subtraction method**

$$d\sigma_F^{(N)\text{NLO}} = \mathcal{H}_F^{(N)\text{NLO}} \otimes d\sigma^{\text{LO}} + \left[d\sigma_{F+\text{jet}}^{(N)\text{LO}} - \Sigma^{(N)\text{NLO}} \otimes d\sigma^{\text{LO}} \right]_{\text{cut } q_T \rightarrow 0}$$

- all ingredients known for extension to $N^3\text{LO}$ [Luo, Yang, Zhu, Zhu (2019; 2020), Ebert, Mistlberger, Vita (2020), Cieri, Chen, Gehrmann, Glover, Huss (2019), Camarda, Cieri, Ferrera ('21), Chen, Gehrmann, Glover, Huss, Yang, Zhu (2021)]

Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — sample case $\text{pp} \rightarrow \gamma\gamma + X$

Result for $r_{\text{cut}} \rightarrow 0$ via extrapolation

- automated and simultaneous scan over reasonable range of r_{cut} values
- quadratic least- χ^2 fit with variable range

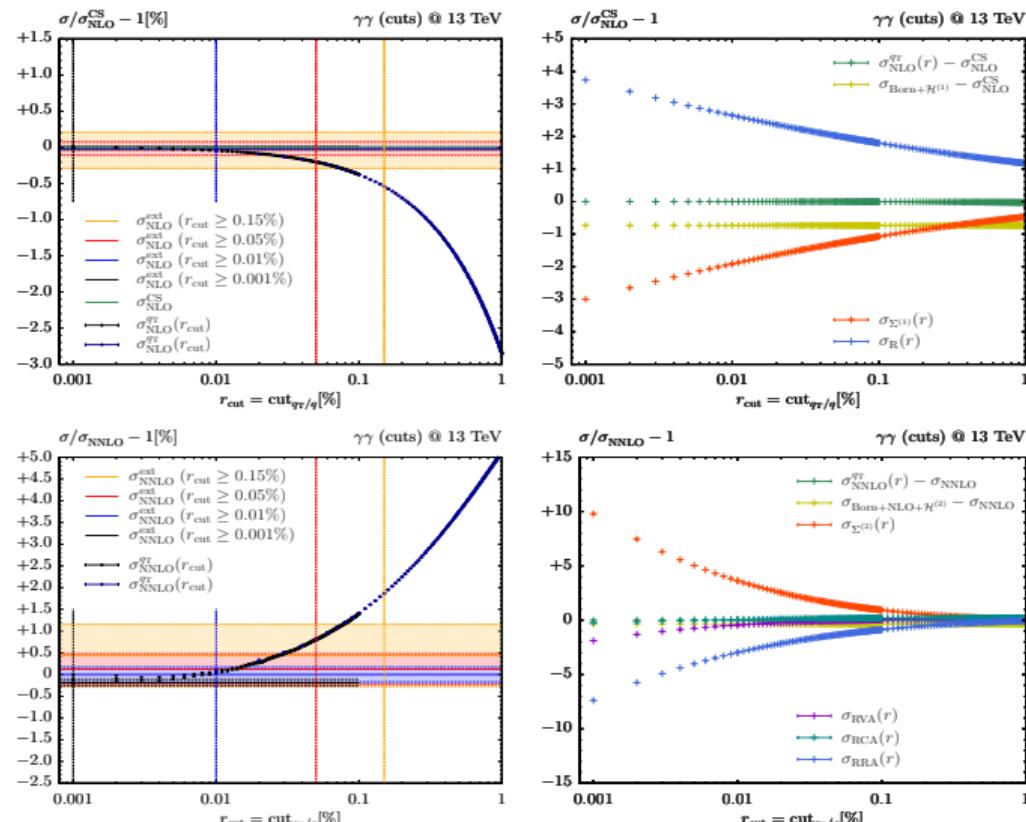
$$\sigma_{(N)\text{NLO}}(r_{\text{cut}}) = Ar_{\text{cut}}^2 + Br_{\text{cut}} + \sigma_{(N)\text{NLO}}$$

- error estimate based on combination of statistical error and variation of r_{cut} range

→ Significant $r_{\text{cut}} \rightarrow 0$ dependence for processes involving isolated photons
(similar between NLO and NNLO QCD)

→ good agreement of extrapolated results within errors for different start values

- $r_{\text{cut}} \geq 0.15\%$
- $r_{\text{cut}} \geq 0.05\%$
- $r_{\text{cut}} \geq 0.01\%$
- $r_{\text{cut}} \geq 0.001\%$



Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — sample case $\text{pp} \rightarrow \ell^-\ell^+\ell^-\ell^+ + X$

Result for $r_{\text{cut}} \rightarrow 0$ via extrapolation

- same procedure for all processes

$$\sigma_{(N)\text{NLO}}(r_{\text{cut}}) = Ar_{\text{cut}}^2 + Br_{\text{cut}} + \sigma_{(N)\text{NLO}}$$

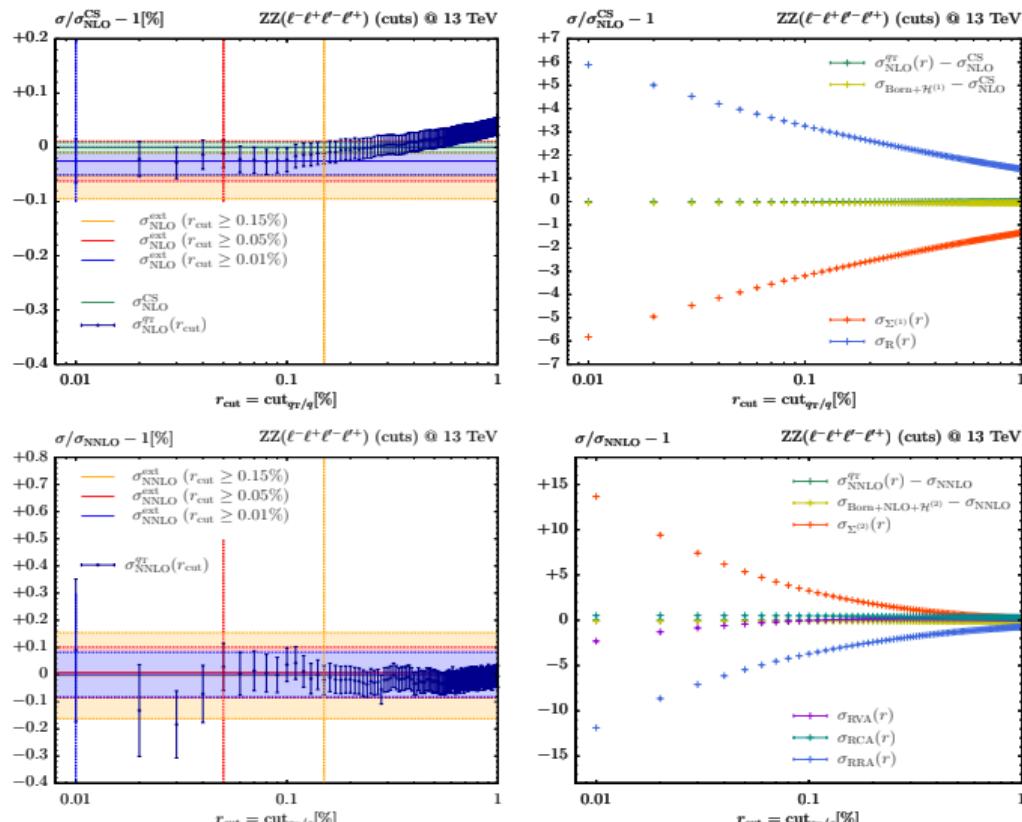
→ **No significant $r_{\text{cut}} \rightarrow 0$ dependence for processes without isolated photons**
 (similar between NLO and NNLO QCD)

- Important exception:
 symmetric cut configurations
 (e.g. standard Drell–Yan setup)

→ good agreement of extrapolated results
 within errors for different start values

- $r_{\text{cut}} \geq 0.15\%$
- $r_{\text{cut}} \geq 0.05\%$
- $r_{\text{cut}} \geq 0.01\%$

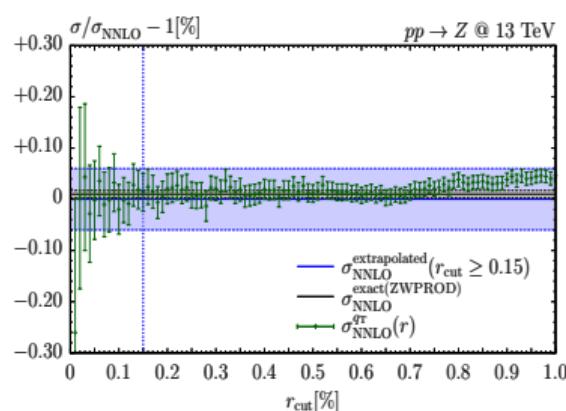
→ larger cancellation between contributions
 (factor of ≈ 15 at $r_{\text{cut}} = 0.01\%$)



Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — Drell-Yan processes

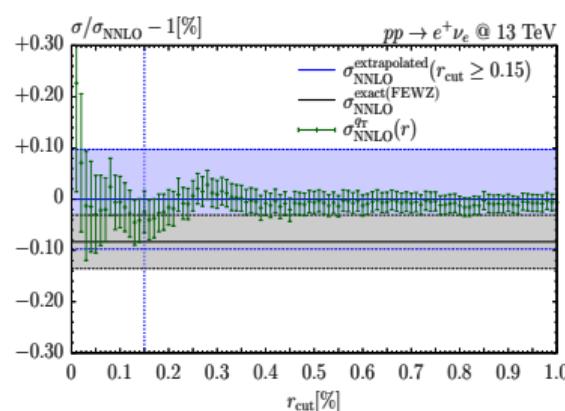
$pp \rightarrow Z$

- fully inclusive



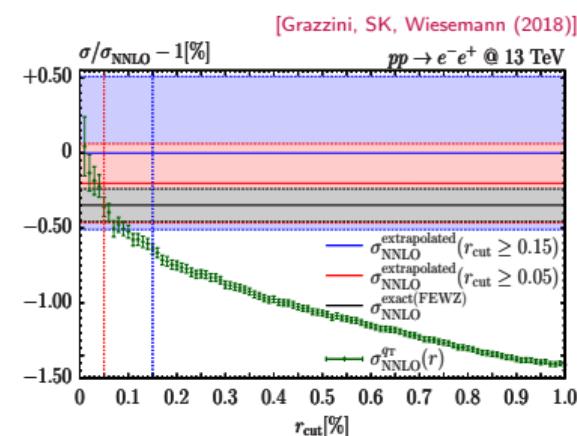
$pp \rightarrow \ell^+ \nu_\ell$

- $p_{T,\ell^+} > 25 \text{ GeV}$ $\eta_{\ell^+} < 2.47$
- $p_{T,\text{miss}} > 20 \text{ GeV}$



$pp \rightarrow \ell^- \ell^+$

- $p_{T,\ell^\pm} > 25 \text{ GeV}$ $\eta_{\ell^\pm} < 2.47$
- $66 \text{ GeV} < m_{\ell^+\ell^-} < 116 \text{ GeV}$



➡ no phase space restrictions

➡ no significant dependence on r_{cut}

➡ asymmetric cuts on two individual leptons (ℓ^+ and ν_ℓ)

➡ no significant dependence on r_{cut}

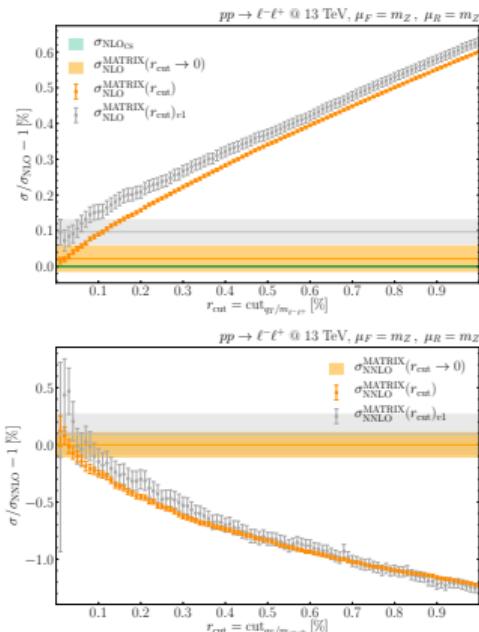
➡ symmetric cuts on both leptons (either on ℓ^+/ℓ^- or on ℓ_1/ℓ_2)

➡ large power corrections in r_{cut}

Dependence on r_{cut} in different cut scenarios for the NC Drell–Yan process

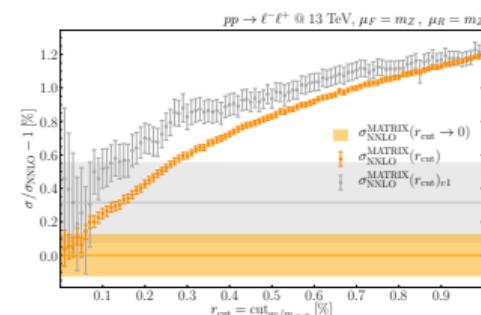
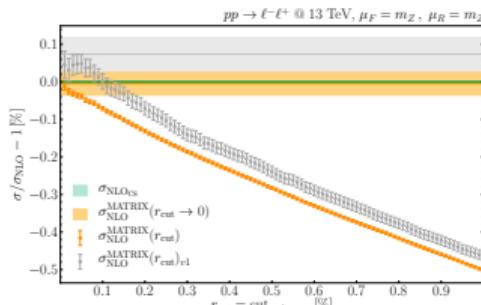
Symmetric cuts

- $p_{T,\ell^\pm} > 25 \text{ GeV}$



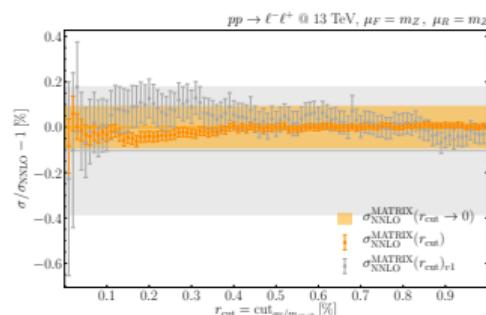
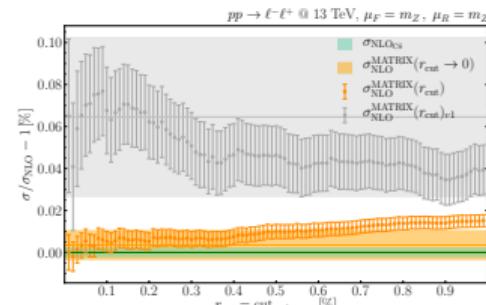
Asymmetric cuts on ℓ_1 and ℓ_2

- $p_{T,\ell_1} > 25 \text{ GeV} \quad p_{T,\ell_2} > 20 \text{ GeV}$



Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{T,\ell^+} > 25 \text{ GeV} \quad p_{T,\ell^-} > 20 \text{ GeV}$



➡ large power corrections in r_{cut}

➡ large power corrections in r_{cut}

➡ no significant dependence on r_{cut}

Production of heavy coloured particles at NNLO QCD accuracy

Extension of q_T subtraction method to production of heavy coloured particles (e.g. top-quark pairs)

$$d\sigma_{\text{NNLO}}^{t\bar{t}} = \mathcal{H}_{\text{NNLO}}^{t\bar{t}} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{\text{NLO}}^{t\bar{t}+\text{jet}} - d\sigma_{\text{NNLO}}^{t\bar{t}, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

- counterterm accounts for IR behaviour of real contribution, including soft singularities related to emissions from final-state quarks [Catani, Grazzini, Torre (2014), Ferroglio, Neubert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
- $\mathcal{H}_{\text{NNLO}}^{t\bar{t}}$ contains remainder of integrated final-state soft singularities
[Catani, Devoto, Grazzini, Mazzitelli (to appear), Angeles-Martinez, Czakon, Sapeta (2018)]
- massive NLO subtraction required for real-emission part, e.g. massive dipole subtraction
[Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]

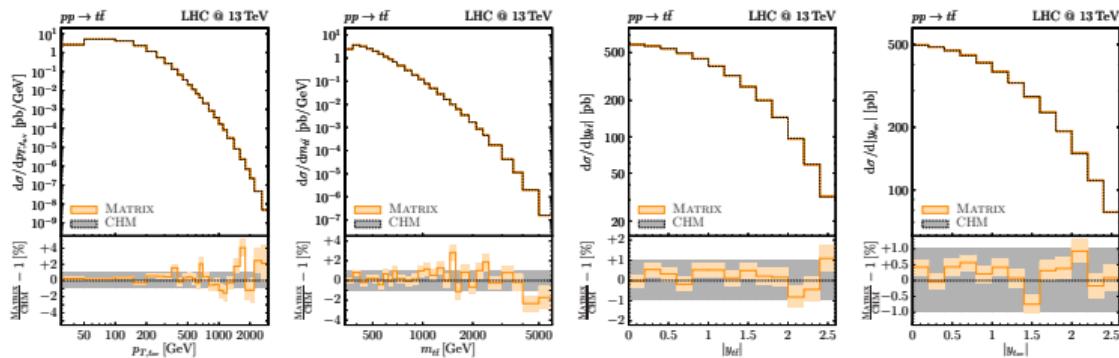
Associated heavy-quark pair production ($t\bar{t}H$, $t\bar{t}V$, ...) with identical singularity structure

- numerical solutions required for evaluation of the soft function due to more involved kinematics
 - ➡ no back-to-back configuration of heavy quarks
- proof-of-principle calculation for non-diagonal channels in $t\bar{t}H$ [Catani, Fabre, SK, Grazzini (2021)]
- two-loop amplitudes as the bottleneck for any beyond $2 \rightarrow 2$ NNLO calculations

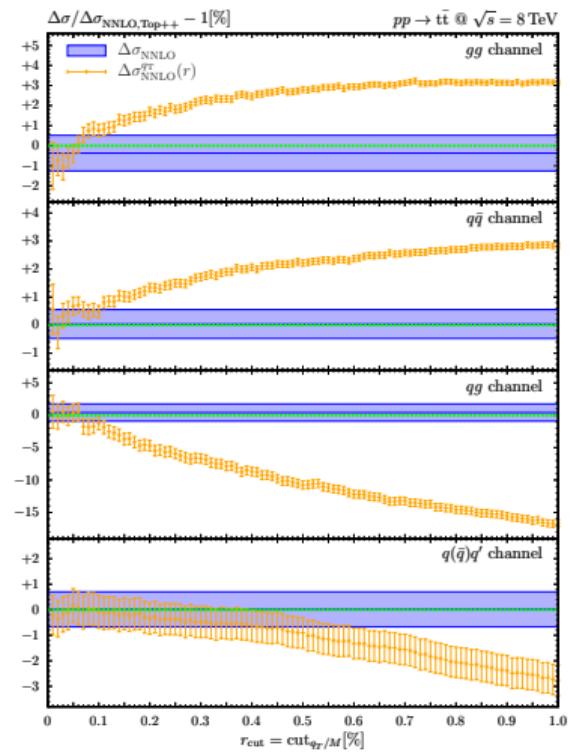
Top-quark pair production at NNLO QCD accuracy

First MATRIX calculation for colourful final states at NNLO QCD

- 2-loop amplitudes from numerical result [Bärnreuther, Czakon, Fiedler (2014)]
- slicing parameter dependence under good numerical control; investigation after splitting into partonic channels
 - full agreement with TOP++ [Czakon, Mitov (2014)]
- successful validation also on the level of differential distributions
 [Catani, Devoto, Grazzini, SK, Mazzitelli (2019)]
 - (comparison against results from [Czakon, Heymes, Mitov (2017)])



[Catani, Devoto, Grazzini, SK, Mazzitelli, Sargsyan (2019)]



Mixed NNLO QCD-EW calculation for production of massive charged particles

Extension of q_T subtraction method to mixed QCD-EW corrections of $\mathcal{O}(\alpha_s^m \alpha^n)$

$$d\sigma_{(m,n)}^{\ell\ell/\ell\nu} = \mathcal{H}_{(m,n)}^{\ell\ell/\ell\nu} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{(m,n)}^{\ell\ell/\ell\nu, \text{R}} - d\sigma_{(m,n)}^{\ell\ell/\ell\nu, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

- $m = 1(2)$ and $n = 0$: (N)NLO QCD corrections
- $m = 0$ and $n = 1$: NLO EW corrections
- $m = 1$ and $n = 1$: mixed NNLO QCD-EW corrections

(limitation: no massless jets (for $m \geq 1$) and no massless charged particles (for $n \geq 1$) allowed at LO)

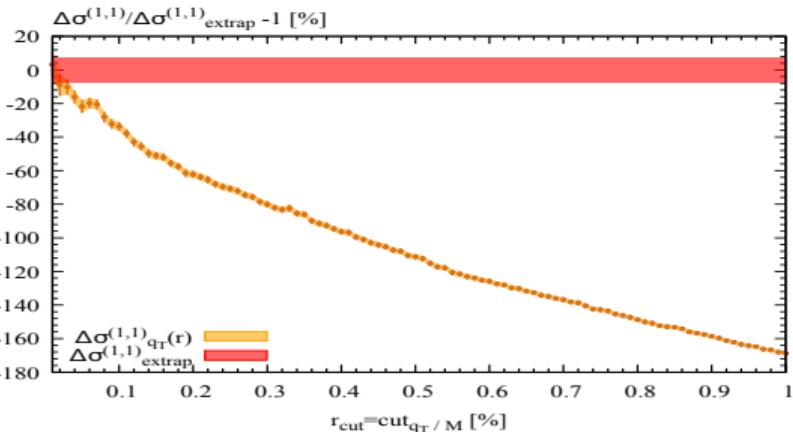
Strategy to cancel IR singularities in mixed QCD-EW corrections

- abelianisation procedure, starting from heavy-quark pair production at NNLO QCD
 - ➡ for neutral final states, abelianisation of standard q_T subtraction method is sufficient
(mixed QCD-QED corrections on $pp \rightarrow Z$ [De Florian, Der, Fabre (2018)], $pp \rightarrow \nu\bar{\nu}$ [Cieri, De Florian, Der, Mazzitelli (2020)])
- colourless final state ($\ell\ell/\ell\nu$) results in soft final-state singularities of pure QED origin
 - ➡ much simpler IR structure than in heavy-quark pair production at NNLO QCD
- finite charged-lepton mass required to regularize collinear final-state singularities

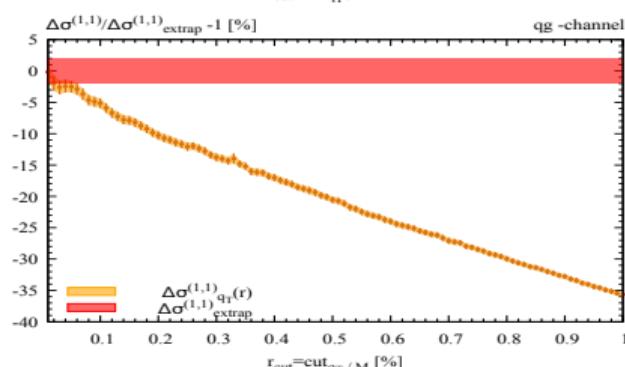
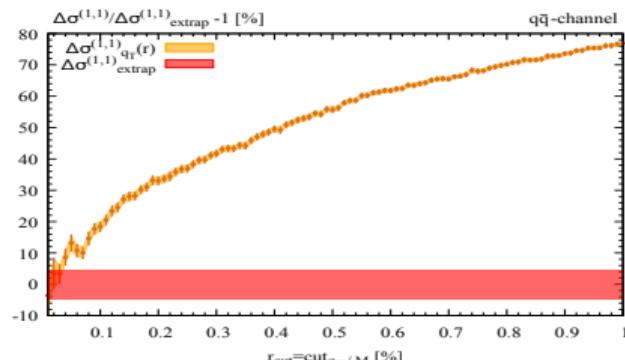
Dependence on r_{cut} of the mixed NNLO QCD-EW corrections for NC Drell-Yan

Symmetric-cut scenario

$$p_{T,\ell^\pm} > 25 \text{ GeV} \quad y_{\ell^\pm} < 2.5 \quad m_{\ell\ell} > 50 \text{ GeV}$$



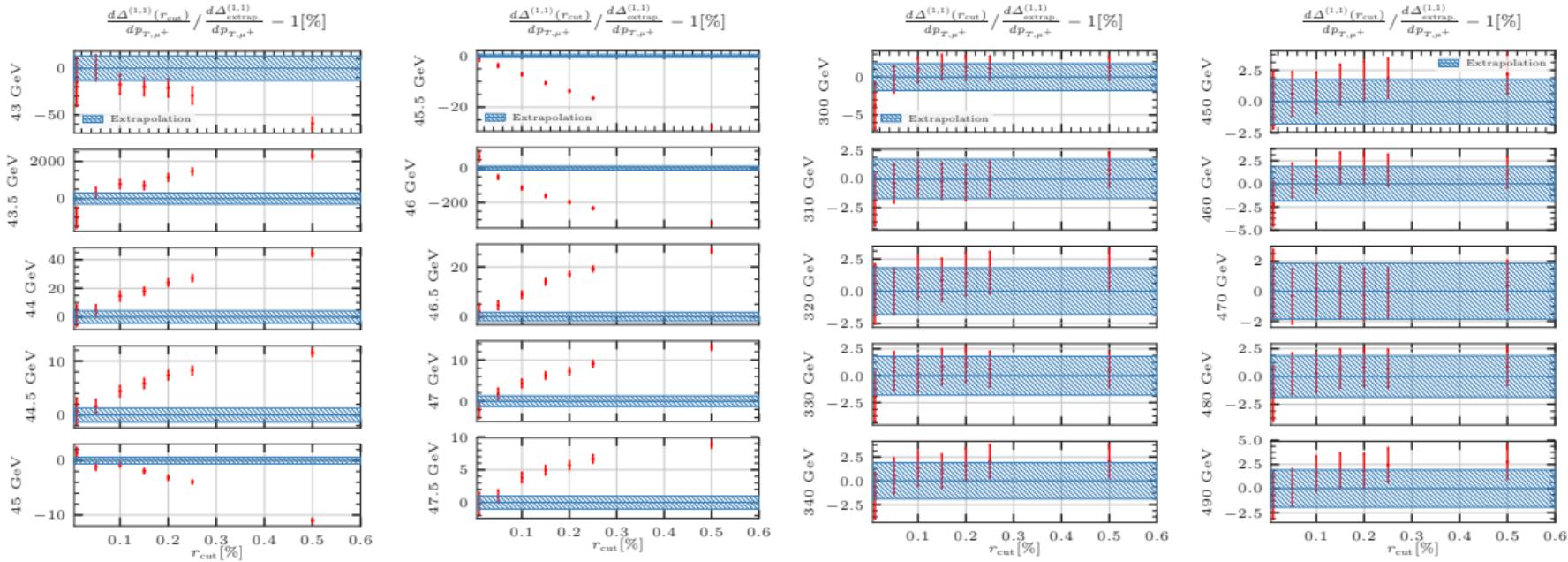
Splitting into partonic channels



- **large power corrections in r_{cut} for mixed corrections**
 - explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
 - better than permille precision at inclusive level

Binwise r_{cut} dependence of the mixed NNLO QCD-EW corrections for NC Drell-Yan

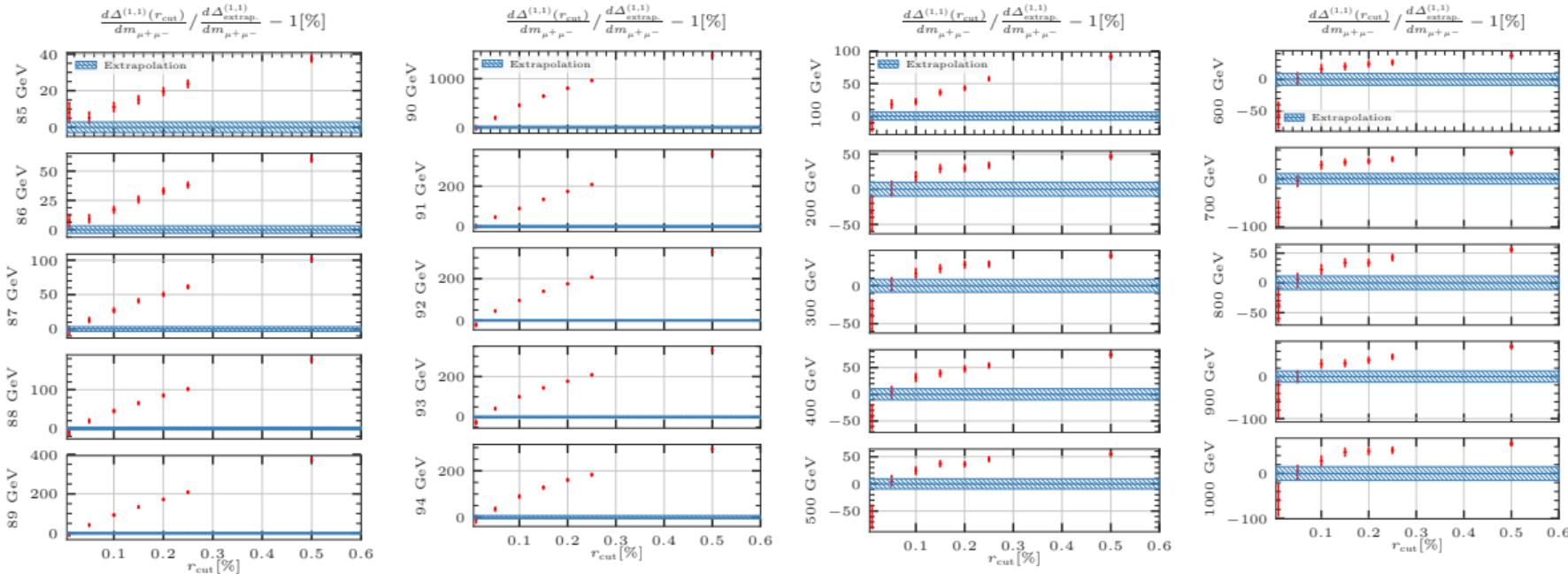
Differential distribution in p_{T,μ^+} : peak (left panels) and tail (right panels) regions



- large r_{cut} dependence in particular around the peak of the distribution, and typically precision of $\lesssim 3\%$ on the relative mixed QCD-EW corrections (artificially large where corrections are basically zero)

Binwise r_{cut} dependence of the mixed NNLO QCD-EW corrections for NC Drell-Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



- quite large r_{cut} dependence throughout, and lower numerical precision of $\lesssim 10\%$ on the relative mixed QCD-EW corrections (but still permille-level precision at the level of cross sections)

The Matrix framework for automated NNLO QCD calculations (and beyond)

MATRIX v1
→ $\gamma^* V, \gamma\gamma V, VV$ at NNLO-QCD
for all leptonic decay channels

MATRIX v2
→ combination with NLO EW for all leptons V and VV processes
→ higher-order corrections at NLO EW for neutral VV processes
→ several technical improvements,
e.g. fast parallel matrix multiplication
still not included: ℓ^2 subtraction, ℓ^2 renormalization

MATRIX v3
→ $\gamma^* VV$ at NNLO-QCD available
→ public interface to provide direct-space renormalization (at ℓ^2 and combined) for all MATRIX processes

available under <https://matrix.hepforge.org>

03

Interfacing dedicated 2-loop amplitudes to MUSCH/MATRIX

- Highly-Drell-Yan, VH, VV, V γ , V γ production
- direct subtraction of 2-loop results, e.g. for $V\gamma$ [11]
- **VH**: $\gamma^* VV$ — $\gamma^* VV$ (matrix), $\gamma^* VV$ (matrix) and $\gamma^* VV$ (matrix) libraries
- **VV**: VV library using **GALORE** (matrix), VV (matrix) and VV (matrix) libraries
- **HF**: approach generated using **MATHEMATICA**, **FORM** (matrix) libraries [12]
- **indirect calculation of amplitudes at NNLO**: ℓ^2 subtraction of 2-loop contributions from 3-loop contributions [13]
- **V γ production — amplitudes from** $\gamma^* VV$ [14]
- **C++ library generated by** **CoCoA** [15, 16], applying **PENTAGONFACTORY** [17] + **pentagon** [18, 19]
- numerical unitarity and analysis/reconstruction techniques [18, 19]
- **NN production (full or dependence)** — **matrix** library
- **Process**: full or dependence of amplitudes grid generated by 2-loop extension of **GoLoR** [20], **Resum2** [21] (matrix), **Resum3** [22] (matrix), **SetC3** [23] (matrix) and **SetC4** [24]
- **Q δ production — amplitude grids from** **matrix** [25]
- **Fortran** routine for numerical integration of grid, imposed by expansions

07

Investigation of ϵ_{cut} = $cut_{q\bar{q}1q_2}$ dependence — Drell-Yan processes

pp → Z
fully inclusive:
 $\bullet p_T > 25$ GeV, $|y| < 2.47$
 $\bullet p_T > 25$ GeV, $y_2 < 2.47$
 $\bullet p_T > 25$ GeV, $y_2 < 2.47$, $y_3 < 2.47$
 $\bullet p_T > 25$ GeV, $y_2 < 2.47$, $y_3 < 2.47$, $y_4 < 2.47$

→ no phase space restrictions
→ asymmetric cuts on two individual leptons (ℓ^1/ℓ^2 and ℓ^3/ℓ^4)
→ symmetric cuts on both leptons (either on ℓ^1/ℓ^2 or on ℓ^3/ℓ^4)
→ ϵ_{cut} → large power corrections

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Mixed NNLO QCD-EW calculation for production of massive charged particles

Extension of ℓ^2 subtraction method to mixed QCD-EW corrections of $\mathcal{O}(1/\alpha_s^n)$

$$\Delta m_{\text{EW}}^{(n)} = \mathcal{M}_{\text{EW}}^{(n)} \otimes (\Delta m_{\text{QCD}}^{(n)} - \Delta m_{\text{EW}}^{(n)})$$

→ $n=1, 2$ → mixed QCD-EW corrections
→ $n=0, 1, 2$ → NNLO QCD corrections
→ $n=1$ and $n=2$ → mixed NNLO QCD-EW corrections
(initialization: no massive jets (for $n=2$) and no massless charged particles (for $n \geq 1$) allowed at LO)

Strategy to cancel IR singularities in mixed QCD-EW corrections

→ subtraction procedure, starting from heavy-quark pair production at NNLO QCD
→ for neutral final states, subtraction of standard ℓ^2 subtraction method is sufficient [mixed QCD-QED] (11) → results in full cancellation of IR singularities [mixed QCD-QED]
→ cancellation of IR singularities in mixed QCD-EW corrections at NNLO QCD
→ much simpler IR structure than in heavy-quark pair production at NNLO QCD
→ false charged-lepton mass required to regularize singularities

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The MUSCH/MATRIX framework for automated NNLO calculations

MATRIX v1
→ $\gamma^* V, \gamma\gamma V, VV$ at NNLO-QCD

MATRIX v2
→ combination with NLO EW for all leptons V and VV processes
→ higher-order corrections at NLO EW for neutral VV processes
→ several technical improvements, e.g. fast parallel matrix multiplication

MATRIX v3
→ $\gamma^* VV$ at NNLO-QCD available
→ public interface to provide direct-space renormalization (at ℓ^2 and combined) for all MATRIX processes

available under <https://matrix.hepforge.org>

04

Idea of the ℓ^2 subtraction method for (N)NLO cross sections

Consider the production of a colourless final state F via $\gamma\ell^1$ or $\ell^1\ell^2$ or $\gamma\gamma$ or $\gamma\gamma$ → Δm_{NNLO} → Δm_{NNLO}
where $\gamma\ell^1$ refers to the transverse momentum of the colourless system F [paper [here](#)]

→ Δm_{NNLO} is singular for $\ell^1 = 0$
→ singling behavior lesson from transverse-momentum renormalization

→ Define a universal counterterm Σ with the same symmetry as $\ell^1 = 0$ behavior [paper [here](#)]:

$$\Delta m_{\text{NNLO}} = \Sigma(\ell^1, \ell^2, \ell^3, \ell^4) + \Delta m_{\text{NLO}} + \Delta m_{\text{EW}}$$

→ Add the $\ell^1 = 0$ limit with the soft-cut variable ϵ_{cut} which satisfies the 3-2-loop amplitudes at NNLO and compensates for the subtraction Σ [paper [here](#)]

→ Master formula for (NNLO) cross section in ℓ^2 subtraction method

$$\Delta m_{\text{NNLO}} = \Delta m_{\text{NLO}} + \Delta m_{\text{EW}} + [\Delta m_{\text{NNLO}} - \Delta m_{\text{NLO}} - \Delta m_{\text{EW}}]_{\ell^1 = 0}$$

→ all ingredients known for extension to N ℓ^2

08

Dependence on ϵ_{cut} in different cut scenarios for the NC Drell-Yan process

Symmetric cuts:
 $\bullet p_T > 25$ GeV

Asymmetric cuts on ℓ_1 and ℓ_2 :
 $\bullet p_{T,\ell_1} > 25$ GeV, $p_{T,\ell_2} > 20$ GeV
 $\bullet p_{T,\ell_1} > 25$ GeV, $p_{T,\ell_2} > 15$ GeV

Asymmetric cuts on ℓ^1 and ℓ^2 :
 $\bullet p_{T,\ell^1} > 25$ GeV, $p_{T,\ell^2} > 20$ GeV
 $\bullet p_{T,\ell^1} > 25$ GeV, $p_{T,\ell^2} > 15$ GeV

→ large power corrections in ϵ_{cut}
→ large power corrections in ϵ_{cut}
→ no significant dependence on ϵ_{cut}

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Dependence on ϵ_{cut} of the mixed NNLO QCD-EW corrections for NC Drell-Yan

Symmetric cut scenario:
 $\bullet p_{T,\ell^1} > 25$ GeV, $\ell^1 < 2.5$, $m_{\ell^1\ell^2} > 50$ GeV

Splitting into partonic channels:
 $\bullet p_{T,\ell^1} > 25$ GeV, $\ell^1 < 2.5$, $m_{\ell^1\ell^2} > 50$ GeV

→ large power corrections in ϵ_{cut} for mixed corrections
→ explained by overall small size of corrections, and is partly also by cancellation between partonic channels

→ no massless dependence at level of cross sections
→ better than previous precision at level

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Performance features of the MUSCH phase space integrator

Issue of poorly populated regions

- sample size: high-energy tails
- standard phase space optimization samples points in bulk regions

Solution in MUSCH integrator

- additional noise with optimization
- smaller noise with standard noise
- sophisticated automated combination with results from standard noise

Significantly improved errors

- O(10) and better with doubled runtime
- simultaneous enhancement of observables

Good performance also for off-shell regions of intermediate resources

05

Investigation of ϵ_{cut} = $cut_{q\bar{q}1q_2}$ dependence — sample case pp → $\gamma\gamma + X$

Result for $\epsilon_{cut} \rightarrow 0$ via extrapolation

- small ϵ_{cut} → large statistical error in measurable range of ϵ_{cut} values
- quadratic linear fit with variable range Δm_{EW} → $\Delta m_{\text{EW}} = \Delta m_{\text{NLO}} + \Delta m_{\text{EW}} + \Delta m_{\text{NNLO}}$
- over estimate based on combination of statistical error and variation of ϵ_{cut} range

→ Significant ϵ_{cut} dependence for processes involving isolated photons (similar between NLO and NNLO QCD)

- good agreement of extrapolated photon ratios within errors for different start values
- $\epsilon_{cut} \geq 0.05\%$ → $\Delta m_{\text{EW}} \geq 0.05\%$
- $\epsilon_{cut} \geq 0.01\%$ → $\Delta m_{\text{EW}} \geq 0.001\%$

09

Production of heavy coloured particles at NNLO QCD accuracy

Extension of ℓ^2 subtraction method to production of heavy coloured particles (e.g. top-quark pair)

$$\Delta m_{\text{EW}} = \Delta m_{\text{EW}} \otimes (\Delta m_{\text{QCD}} - \Delta m_{\text{EW}})_{\text{soft}}$$

→ cancellation accounts for IR behavior of real contribution, including soft singularities related to emissions from final-state quarks [paper [here](#)]

→ Δm_{EW} contains remainder of integrated final-state soft singularities [paper [here](#)]

→ massive NNLO subtraction required for non-resonant part, e.g. massive dipole subtraction [paper [here](#)]

Associated heavy quark pair production ($t\bar{t}, b\bar{b}$) → with identical kinematics structure

- numerical solution required for evaluation of the soft function due to more involved kinematics
- no back-to-back configuration of heavy quarks
- proof-of-principle calculation for one-diagonal channel in **ttH** [paper [here](#)]
- two-loop amplitudes as the bottleneck for an automated NNLO calculation

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Bimass ϵ_{cut} dependence of the mixed NNLO QCD-EW corrections for NC Drell-Yan

Differential distribution in $m_{\ell^1\ell^2}^{-1}$ peak (left panels) and tail (right panels) regions

→ large ϵ_{cut} dependence in particular around the peak of the distribution, and typically precision of $\lesssim 2\%$ on the relative mixed QCD-EW corrections (with ϵ_{cut} resolution of $\lesssim 10\%$, the relative mixed QCD-EW corrections (but still precisely))

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Supplying MUSCH/MATRIX with 1-loop amplitudes

Process-independent interface to generate automated amplitude generators

- **Diagrams**: **Diagrams** (matrix), **Diagrams** (matrix), **Diagrams** (matrix), written in **FORTRAN**
- **General code and process libraries**
 - on-the-fly reduction (**Matrix**, **Matrix**, **Matrix**), with hybrid precision stability system
 - scalar integrals from **CoCoA** (**Matrix**, **Matrix**, **Matrix**) or **OneLoop** (**Matrix**, **Matrix**)
- **RECOLLA**: **RECOLLA** (matrix), **RECOLLA** (matrix), **RECOLLA** (matrix), written in **FORTRAN**
 - on-the-fly generation of amplitudes
 - tensor integral calculations, via **CoCoA** (**Matrix**, **Matrix**, **Matrix**)
 - different model files available, also for **SMETT** and **BSM** applications
- modular structure of **MUSCH** allows other generators to be interfaced as well

Several dedicated interfaces developed in context of MUSCH applications

- **Top-quark pair production**
 - velocity amplitudes, colour-striped amplitudes to construct 4-color correlators
 - imaginary parts of loop-tree amplitudes as
 - flip amplitudes

06

Investigation of ϵ_{cut} = $cut_{q\bar{q}1q_2}$ dependence — sample case pp → $t^+ t^- \ell^+ \ell^- + X$

Result for $\epsilon_{cut} \rightarrow 0$ via extrapolation

- same procedure for all processes
- Δm_{EW} → $\Delta m_{\text{EW}} = \Delta m_{\text{NLO}} + \Delta m_{\text{EW}} + \Delta m_{\text{NNLO}}$
- no significant ϵ_{cut} dependence for processes involving isolated photons (similar between NLO and NNLO QCD)
- important exception: top quark pair production (top quarks are gluon-gluon fusion)
- good agreement of extrapolated results within errors for different start values
- $\epsilon_{cut} \geq 0.05\%$ → $\Delta m_{\text{EW}} \geq 0.05\%$
- $\epsilon_{cut} \geq 0.01\%$ → $\Delta m_{\text{EW}} \geq 0.001\%$
- larger cancellation between contributions (factor of ≈ 15 at $\epsilon_{cut} = 0.05\%$)

10

Top-quark pair production at NNLO QCD accuracy

Fast **Matrix** calculator for colored final states at NNLO QCD

→ 2-loop amplitudes from numerical results

→ closing parameter dependence under good numerical control, investigation after splitting into partonic channels

- full agreement with **TOP** [1]: $\epsilon_{cut} \geq 0.05\%$

→ numerical validation also on the level of differential distributions (comparison against results from **SMETT** [1])

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Bimass ϵ_{cut} dependence of the mixed NNLO QCD-EW corrections for NC Drell-Yan

Differential distribution in $m_{\ell^1\ell^2}^{-1}$ peak (left panels) and tail (right panels) regions

→ quite large ϵ_{cut} dependence throughout, and $\lesssim 2\%$ on the relative mixed QCD-EW corrections (but still precisely))

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