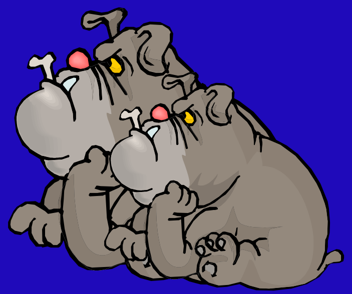
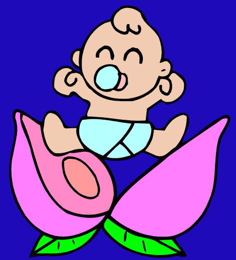
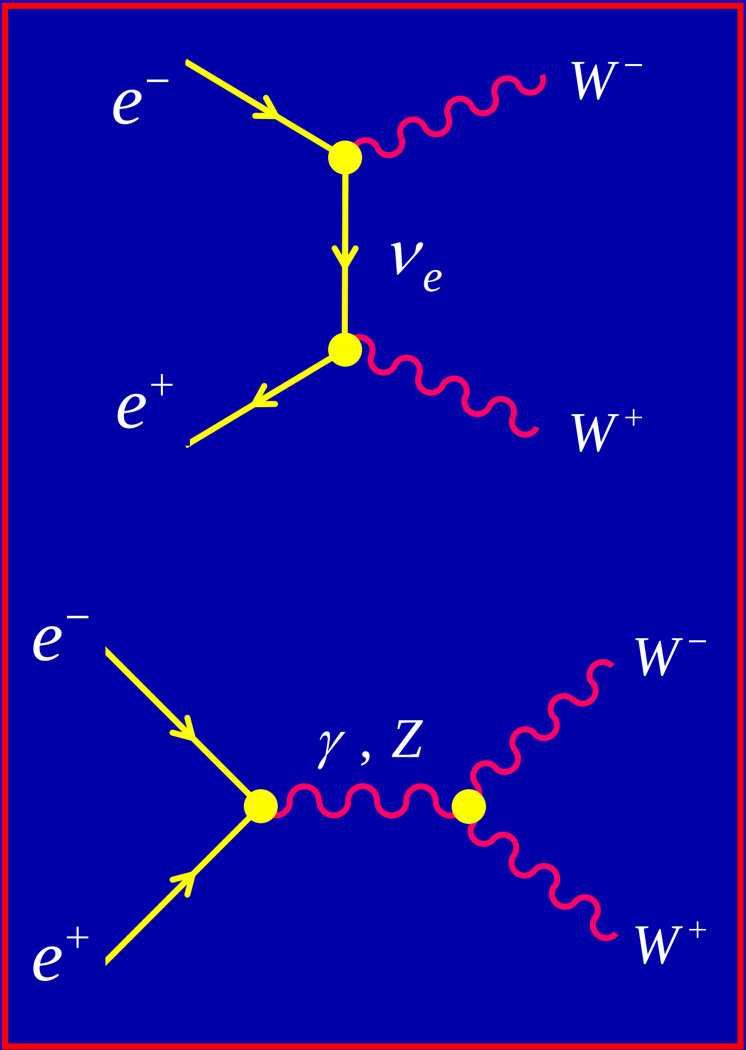
4. ASPECTS OF EW SSB

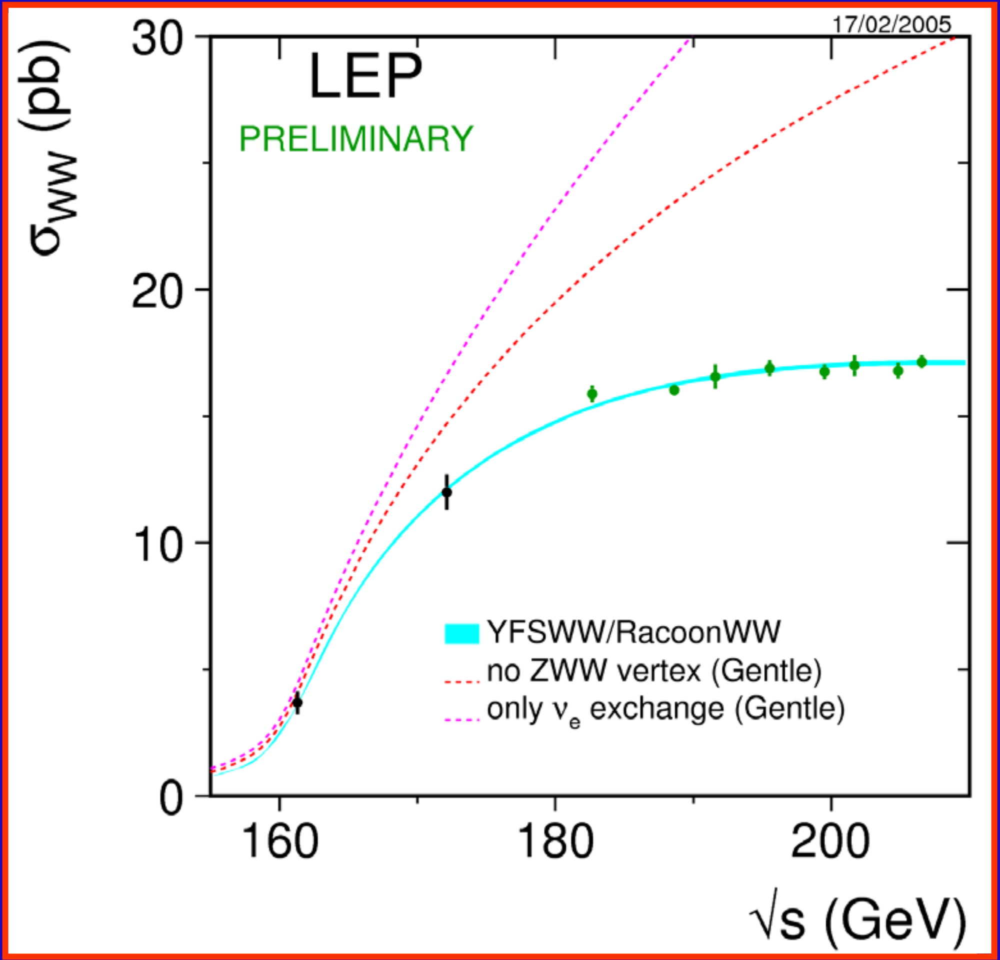


- Gauge Self-Interactions
- Longitudinal $W^{\pm}$ and Z
- Goldstone Dynamics
- Effective Higgs Potential
- Fermion Flavour



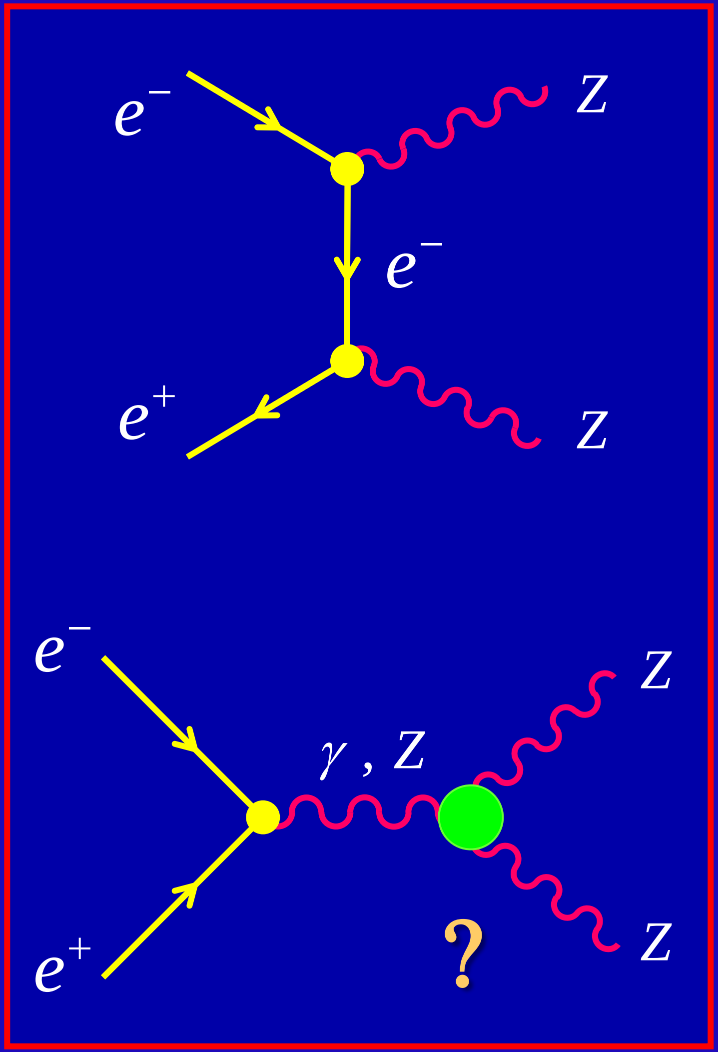
$$e^+ e^- \rightarrow W^+ W^-$$

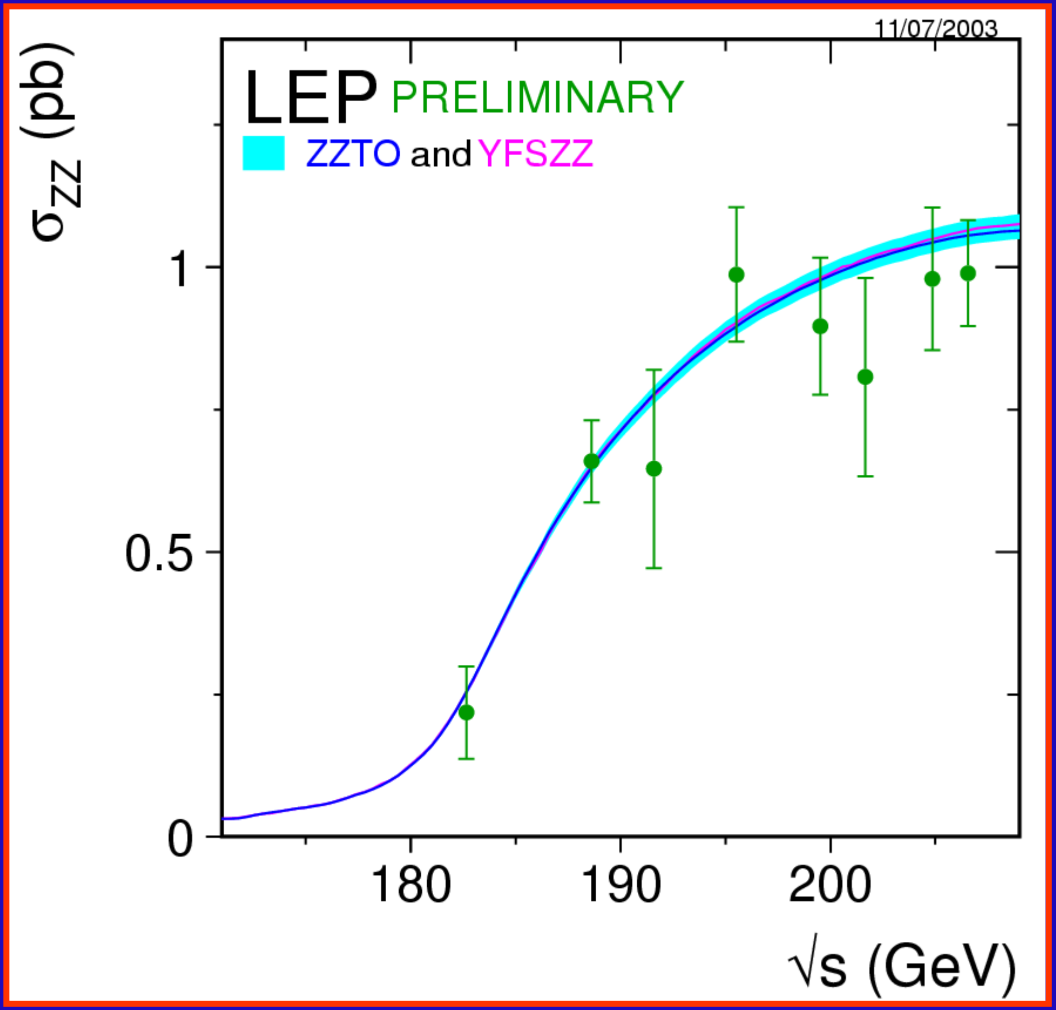




Evidence of Gauge Self-Interactions

$e^+ e^- \to Z\, Z$





No Evidence of γZZ or ZZZ couplings

Massive Spin-1 Boson: 3 Polarizations

- $k^\mu = (M,0,0,0)$: $\varepsilon_1^\mu = (0,1,0,0)$, $\varepsilon_2^\mu = (0,0,1,0)$, $\varepsilon_3^\mu = (0,0,0,1)$
- $k^\mu = (k^0,0,0,|\vec{k}|)$: $\varepsilon_1^\mu = (0,1,0,0)$, $\varepsilon_2^\mu = (0,0,1,0)$, $\varepsilon_3^\mu = \frac{1}{M}(|\vec{k}|,0,0,k^0)$

Transverse (ε_1, ε_2) — Longitudinal (ε_3)

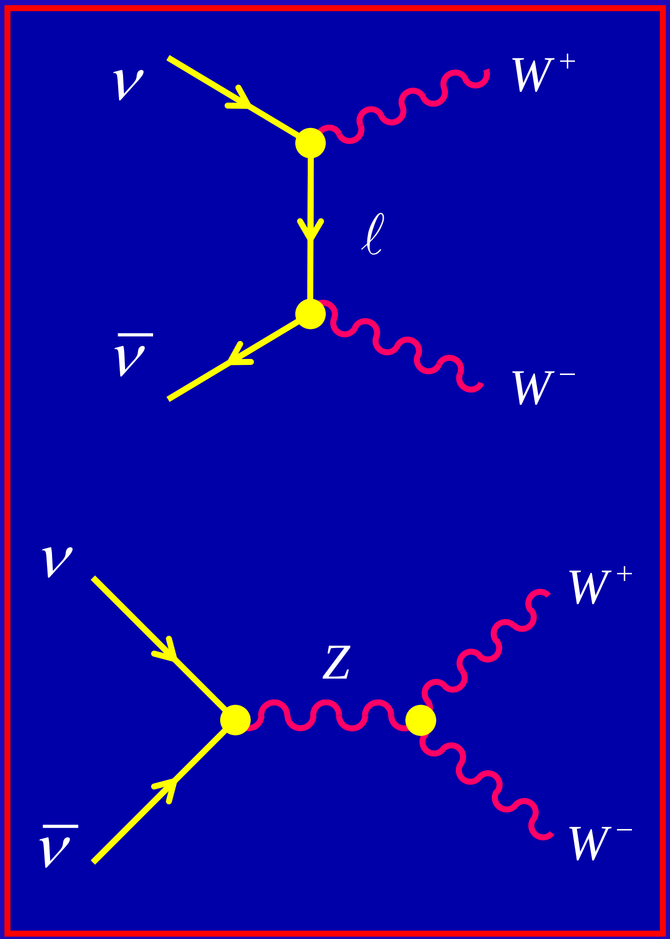
$$k_\mu\, \varepsilon_i^\mu(k) = 0 \quad , \quad \varepsilon_i^\mu(k)\, \varepsilon_{j\mu}(k) = -\delta_{ij} \quad , \quad \sum_{i=1}^{3} \varepsilon_i^\mu(k)\, \varepsilon_i^\nu(k) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}$$

- **Limit $M \to 0$ singular:** Only 2 polarizations for $M=0$
- **Longitudinal polarization grows with energy:**

$$\varepsilon_3^\mu(k) = \frac{k^\mu}{M} + O\left(\frac{M}{k^0}\right) \qquad (k^0 \gg M)$$

$$\nu\,\bar{\nu} \;\to\; W_L^+ W_L^-$$

$$\varepsilon_3^\mu(k) = \frac{k^\mu}{M_W} + O\left(\frac{M_W}{k^0}\right) \qquad \left(k^0 \gg M\right)$$



$$T_1 \simeq \frac{g^2}{2tM_W^2}\;\bar{v}_{\bar{\nu}}\,\not{k}_-\,\not{q}_l\,\not{k}_+\,u_\nu \simeq \frac{g^2}{2M_W^2}\;\bar{v}_{\bar{\nu}}\,\not{k}_+\,u_\nu$$

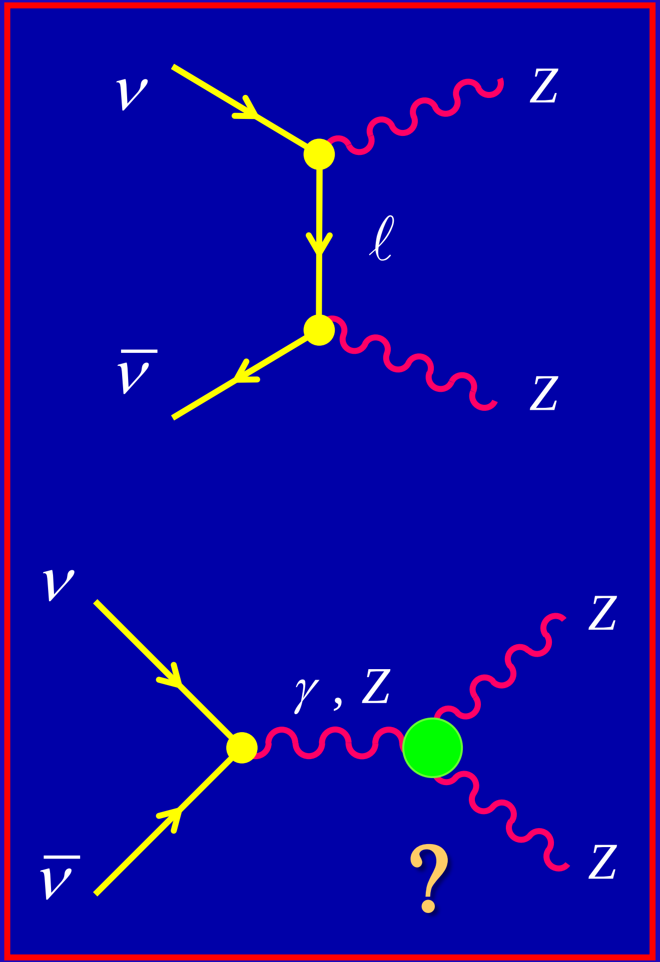
$$\sigma_1 \sim \frac{g^4}{M_W^4}\,s$$

$$T_2 \simeq -\frac{g^2}{2M_W^2}\;\bar{v}_{\bar{\nu}}\,\not{k}_+\,u_\nu$$

$T_1 + T_2 \simeq 0 \quad\to\quad$ **Well behaved** σ

The WWZ coupling restores a good high-energy behaviour

$\nu\,\bar{\nu} \to Z_L\,Z_L$

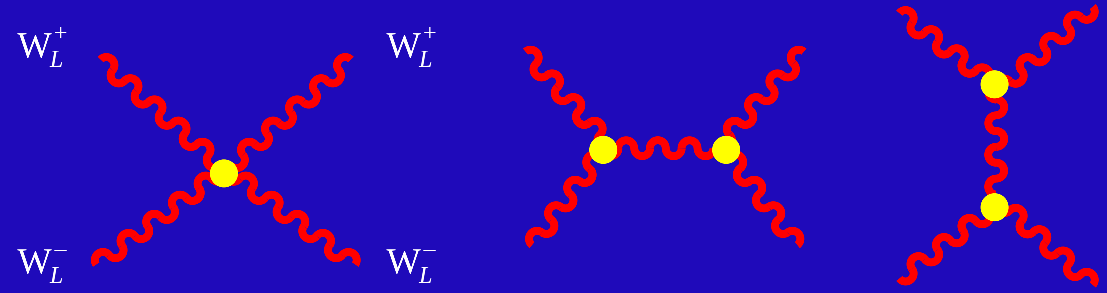


Bose Symmetry: (L=1)

$$T_1(\nu\bar{\nu} \to Z_{L1}Z_{L2}) + T_1(\nu\bar{\nu} \to Z_{L2}Z_{L1}) = 0$$

$$T_2(\nu\bar{\nu} \to Z_L Z_L) = 0$$

Longitudinal $W^+ W^-$ Scattering



$$\mathrm{T}\left(W_L^+ W_L^- \to W_L^+ W_L^-\right) = \frac{g^2}{4M_W^2}\,(s+t)$$

Badly behaved S- wave:

$$a_\ell = \frac{1}{32\pi}\int_{-1}^{+1} d\cos\theta \;\mathrm{T}(s,\theta)\; P_\ell(\cos\theta)$$

$$a_0\left(W_L^+ W_L^- \to W_L^+ W_L^-\right) = \frac{s}{32\,\pi\,\mathrm{v}^2}$$

Perturbative Unitarity:

$$|\operatorname{Im} a_\ell| = \left|a_\ell^2\right| \;\Rightarrow\; \left(\operatorname{Im} a_\ell - \frac{1}{2}\right)^2 + \left(\operatorname{Re} a_\ell\right)^2 = \frac{1}{4}$$

$$\sqrt{s} \le 2\sqrt{2\pi}\,\mathrm{v} = 1.2\ \mathrm{TeV}$$

Bosonic Degrees of Freedom

Massless $W^{\pm}, \; Z$

3 x 2 polarizations **= 6**

+

3 Goldstones $\vec{\theta}$

SSB ↓

Massive $W^{\pm}, \; Z$

3 x 3 polarizations **= 9**

SAME PHYSICS

CUSTODIAL SYMMETRY

$$\phi \equiv \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \quad \Rightarrow \quad \phi_C \equiv i\,\sigma_2\,\phi^* = \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix}$$

$$y_\phi = Q_\phi - T_3 = +\frac{1}{2}$$

$$y_{\phi_C} = Q_{\phi_C} - T_3 = -\frac{1}{2}$$

$$\Sigma \equiv (\phi, \phi_C) = \begin{pmatrix} \phi^{(+)} & \phi^{(0)*} \\ \phi^{(0)} & -\phi^{(-)} \end{pmatrix} \quad , \quad \mathbf{D}^\mu \Sigma \equiv \partial^\mu \Sigma + i g\, \mathbf{W}^\mu\, \Sigma + i\, g'\, \Sigma\, \frac{\sigma_3}{2} B^\mu$$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger\, \mathbf{D}^\mu \phi - \mu^2\, \phi^\dagger \phi - h\, (\phi^\dagger \phi)^2$$

$$= \frac{1}{2} \mathrm{Tr}\left[\left(\mathbf{D}^\mu \Sigma\right)^\dagger \mathbf{D}^\mu \Sigma \right] - \frac{h}{16} \left(\mathrm{Tr}\left[\Sigma^\dagger \Sigma\right] - \mathrm{v}^2 \right)^2 + \frac{h}{2} \mathrm{v}^4$$

Invariant under global $SU(2)_L \otimes SU(2)_C \supset SU(2)_L \otimes U(1)_Y$

$$\Sigma \to \mathbf{U}_L \cdot \Sigma \cdot \mathbf{U}_C^\dagger \quad , \quad U_X \in SU(2)_X$$

Polar decomposition: $\Sigma(x) = \frac{1}{\sqrt{2}}\left[\mathrm{v} + H(x)\right] \mathbf{U}(\varphi)$, $\mathbf{U}(\varphi) = \exp\left\{\frac{i}{\mathrm{v}}\vec{\sigma}\vec{\varphi}\right\}$

Heavy Higgs $(h \gg 1)$

$$\mathcal{L}(\phi) = \frac{\mathrm{v}^2}{4}\,\mathrm{Tr}\left[\left(\mathbf{D}^\mu \mathbf{U}\right)^\dagger \mathbf{D}^\mu \mathbf{U}\right] + O\left(H/\mathrm{v}\right)$$

Same Lagrangian than **QCD pions:** $f_\pi \to \mathrm{v}$, $\pi^\pm, \pi^0 \to \varphi^\pm, \varphi^0 \to W_L^\pm, W_L^0$

Chiral Goldstone Bosons: $SU(2)_L \otimes SU(2)_C \to SU(2)_{L+C}$

Unitary Gauge $[\vec{\varphi}(x) = 0]$**:** $L(\phi) \doteq M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$

$$M_Z \cos\theta_W = M_W = \frac{1}{2}\,\mathrm{v}\,g$$

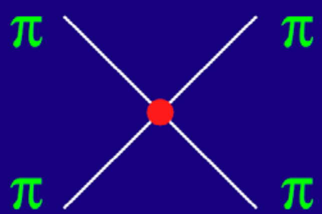
Universal Symmetry Relation

The $SU(2) \otimes SU(2) \to SU(2)$ Symmetry Determines the Interaction

$$\mathcal{L}_2 = \frac{f^2}{4}\,\mathrm{Tr}\left[\partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U}\right] = \partial_\mu \pi^- \partial^\mu \pi^+ + \frac{1}{2}\,\partial_\mu \pi^0 \partial^\mu \pi^0 + \cdots$$

$$+ \frac{1}{6f^2}\left\{\left(\pi^+ \overleftrightarrow{\partial}_\mu \pi^-\right)\left(\pi^+ \overleftrightarrow{\partial}^\mu \pi^-\right) + 2\left(\pi^0 \overleftrightarrow{\partial}_\mu \pi^+\right)\left(\pi^- \overleftrightarrow{\partial}^\mu \pi^0\right) + \cdots\right\}$$

$$+ O\left(\pi^6/f^4\right)$$



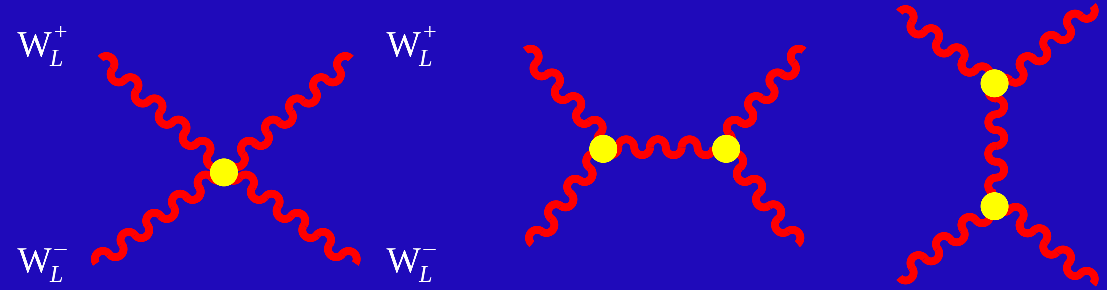
$$T\left(\pi^+\pi^- \to \pi^+\pi^-\right) = \frac{t+s}{f^2}$$

Pions $\quad \dfrac{1}{f^2} \quad \Longleftrightarrow \quad \dfrac{1}{v^2} = \dfrac{g^2}{4M_W^2} \quad$ **Longitudinal W's**

Strong Interaction $\qquad\qquad$ **EW Interaction**

- **Goldstone interactions vanish at zero momenta**
- **Non-linear representation:** $\mathbf{U}(\pi) = \exp\left\{\frac{i}{v}\,\vec{\sigma}\,\vec{\pi}\right\}$

Longitudinal $W^+ W^-$ Scattering



$$T\left(W_L^+ W_L^- \to W_L^+ W_L^-\right) = T\left(\varphi^+\varphi^- \to \varphi^+\varphi^-\right) = \frac{g^2}{4M_W^2}\,(s+t)$$

Equivalence Theorem

Badly behaved S- wave:

$$a_\ell = \frac{1}{32\pi}\int_{-1}^{+1} d\cos\theta\; T(s,\theta)\; P_\ell(\cos\theta)$$

$$a_0\left(W_L^+ W_L^- \to W_L^+ W_L^-\right) = \frac{s}{32\,\pi\,\mathrm{v}^2}$$

Perturbative Unitarity:

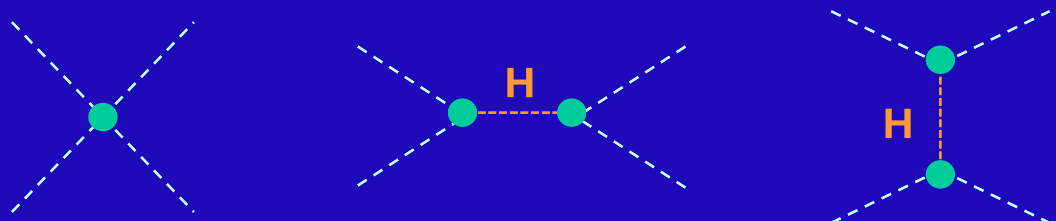
$$|\mathrm{Im}\, a_\ell| \le \left|a_\ell^2\right| \;\Rightarrow\; \left(\mathrm{Im}\, a_\ell - \frac{1}{2}\right)^2 + \left(\mathrm{Re}\, a_\ell\right)^2 \le \frac{1}{4}$$

$$\sqrt{s} \le 2\sqrt{2\pi}\,\mathrm{v} = 1.2\ \mathrm{TeV}$$

Higgs Exchange Restores Unitarity

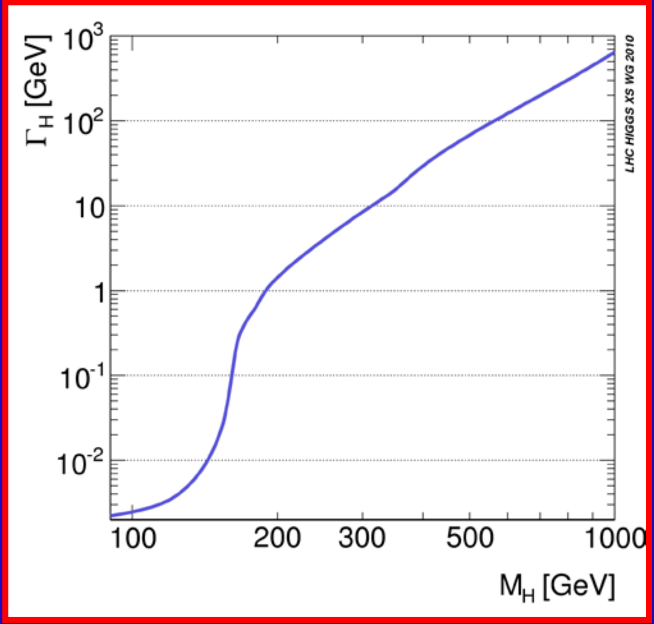
$$\mathcal{L}(\phi) = \frac{v^2}{4}\,\mathrm{Tr}\left[\left(\mathbf{D}^\mu \mathrm{U}\right)^\dagger \mathbf{D}^\mu \mathrm{U}\right]\left(1+\frac{H}{v}\right)^2$$

$$= \left(1+2\frac{H}{v}+\frac{H^2}{v^2}\right)\left\{\partial_\mu\varphi^+\partial^\mu\varphi^- + \frac{1}{2}\partial_\mu\varphi^0\partial^\mu\varphi^0 + \cdots\right\}$$



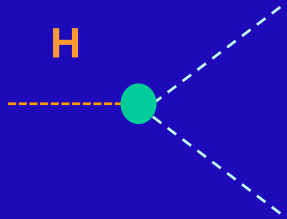
$$\mathrm{T}\left(\varphi^+\varphi^- \to \varphi^+\varphi^-\right) = \frac{1}{v^2}\left\{s+t-\frac{s^2}{s-M_H^2}-\frac{t^2}{t-M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s-M_H^2}-\frac{t}{t-M_H^2}\right\}$$

Higgs Decay Width



$$\mathcal{L}(\phi) = \frac{v^2}{4} \mathrm{Tr}\left[\left(\mathbf{D}^\mu \mathbf{U}\right)^\dagger \mathbf{D}^\mu \mathbf{U}\right] \left(1+\frac{H}{v}\right)^2$$

$$= \left(1+2\frac{H}{v}+\frac{H^2}{v^2}\right) \left\{\partial_\mu\varphi^+\partial^\mu\varphi^- + \frac{1}{2}\partial_\mu\varphi^0\partial^\mu\varphi^0 + \cdots\right\}$$



$$T\left(H\to\varphi^+\varphi^-\right) = \frac{2}{v}\, k_+\cdot k_- = \frac{M_H^2}{v}\left\{1-\frac{2M_W^2}{M_H^2}\right\}$$

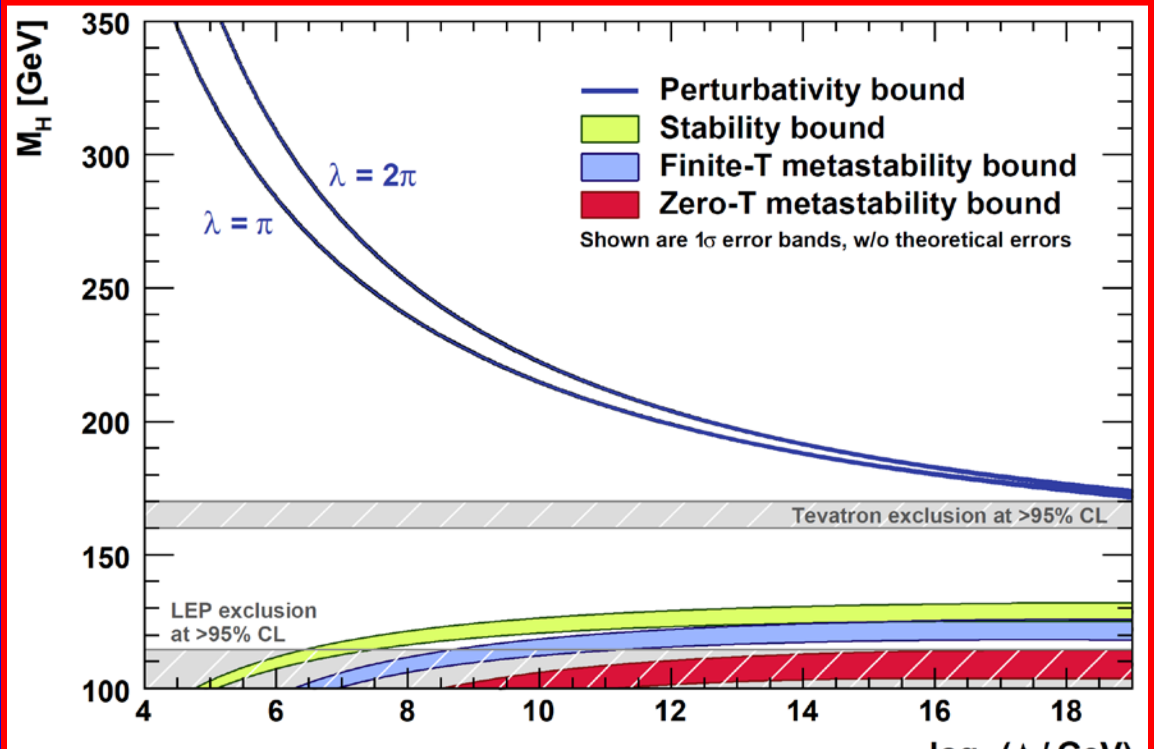
$$\Gamma\left(H\to W_L^+ W_L^-\right) \simeq \Gamma\left(H\to\varphi^+\varphi^-\right) \simeq \frac{M_H^3}{16\pi v^2}$$

Longitudinal W's generate a fast growing of Γ_H with M_H

Effective Higgs Potential (2-loop RGE)

$$V \approx \frac{1}{4}\lambda\ H^4$$

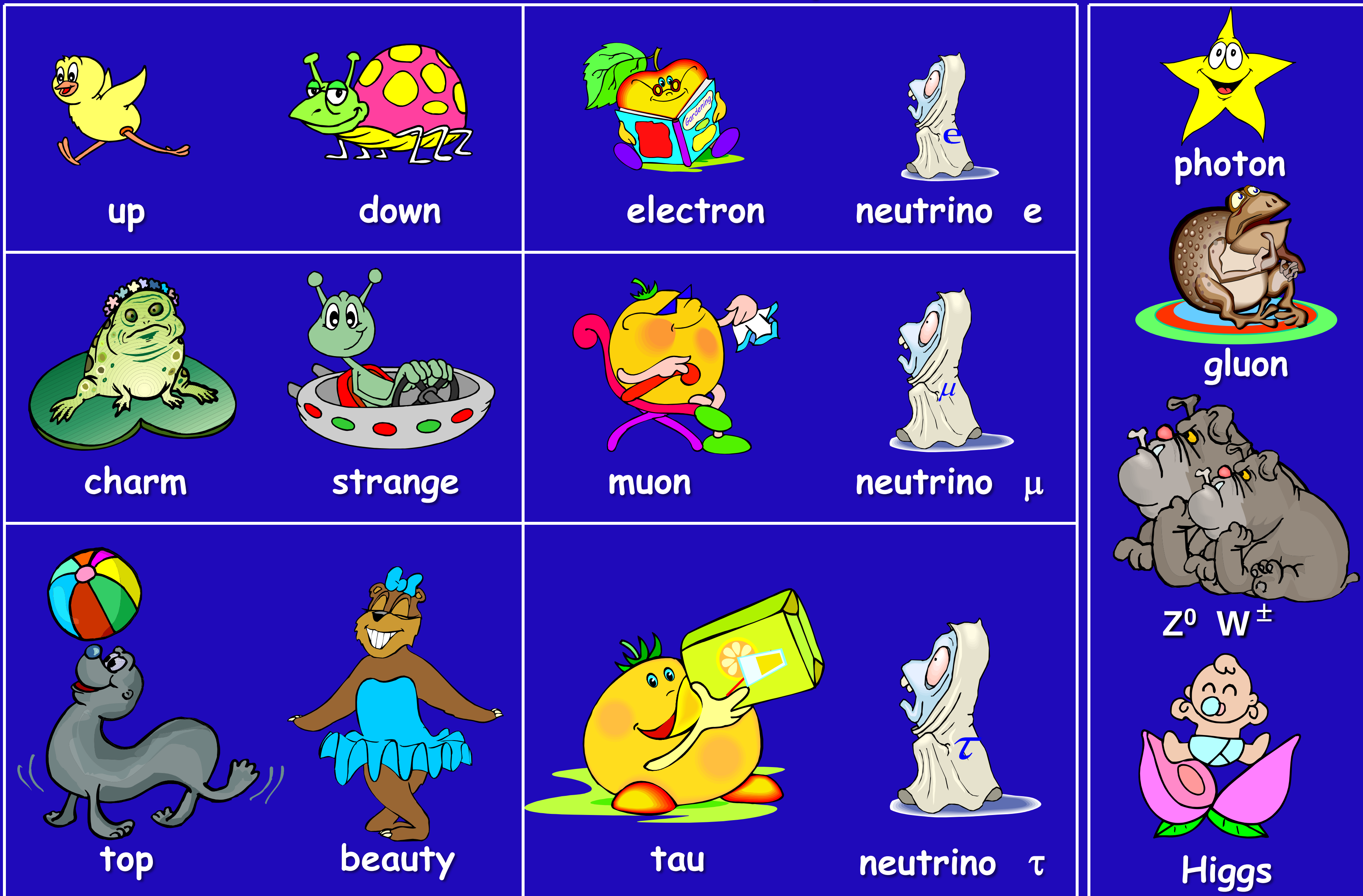
$$\lambda(\Lambda) = \frac{M_H^2}{2v^2}\{1+\delta_H(\Lambda)\}$$



Ellis et al

- $M_H \to \infty$: λ non perturbative
- $M_H \to 0$: $\lambda < 0$ possible ➡ unstable vacuum

Quarks Leptons Bosons



FERMION MASSESS

Scalar – Fermion Couplings allowed by Gauge Symmetry

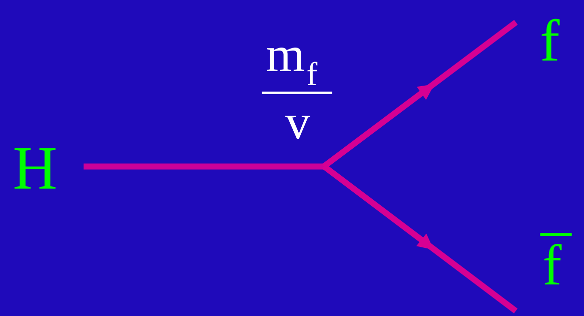
$$\mathcal{L}_Y = -(\bar{q}_u , \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] - (\bar{\nu}_l , \bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ m_{q_d} \; \bar{q}_d \, q_d + m_{q_u} \; \bar{q}_u \, q_u + m_l \; \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d} \; , \; m_{q_u} \; , \; m_l \right] = \left[c^{(d)} \; , \; c^{(u)} \; , \; c^{(l)} \right] \frac{v}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \frac{m_f}{v}$

FERMION GENERATIONS

$N_G = 3$ **Identical Copies** **Masses are the only difference**

$$Q=0 \qquad \begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix} \qquad Q=+2/3$$
$$Q=-1 \qquad\qquad\qquad\qquad Q=-1/3$$

$$(j = 1, \cdots, N_G)$$

WHY ?

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d \,,\, \mathbf{M}'_u \,,\, \mathbf{M}'_l \right]_{jk} = \left[c^{(d)}_{jk} \,,\, c^{(u)}_{jk} \,,\, c^{(l)}_{jk} \right] \frac{v}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathrm{S}_u \cdot \mathbf{U}_u$$
$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$
$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$
$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$

$$\mathcal{L}_Y = -\left(1+\frac{H}{\mathrm{v}}\right) \left\{ \bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l \right\}$$

$$\mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t) \quad ; \quad \mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad ; \quad \mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates
$\neq$
Weak Eigenstates

$$\bar{\mathrm{f}}'_L\, \mathrm{f}'_L = \bar{\mathrm{f}}_L\, \mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R\, \mathrm{f}'_R = \bar{\mathrm{f}}_R\, \mathrm{f}_R \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{NC}} = \mathcal{L}_{\mathrm{NC}}$$

$$\bar{\mathrm{u}}'_L\, \mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{CC}} \neq \mathcal{L}_{\mathrm{CC}}$$

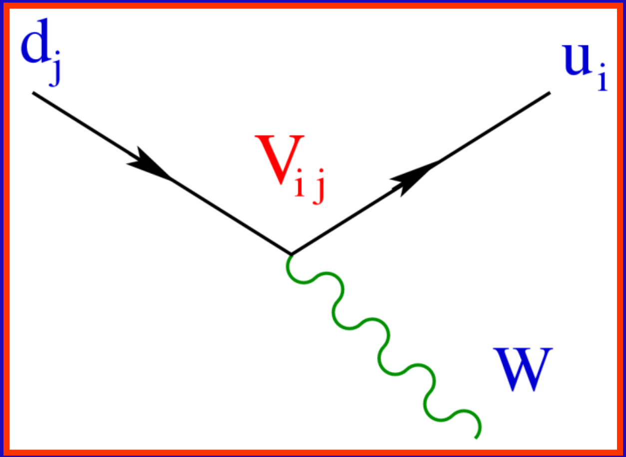
QUARK MIXING

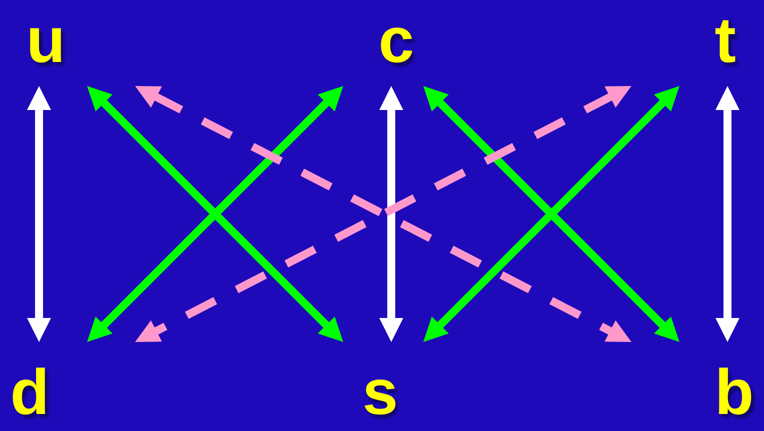
$$\mathcal{L}_{NC}^{Z} = -\frac{e}{2\sin\theta_W\cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu [v_f - a_f\gamma_5] f$$

Flavour Conserving Neutral Currents (GIM)

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$

Flavour Changing Charged Currents





QUARK MIXING MATRIX: $\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**
- $2\,N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i}\, u_i \quad ; \quad d_j \to e^{i\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)}\, \mathbf{V}_{ij}$$

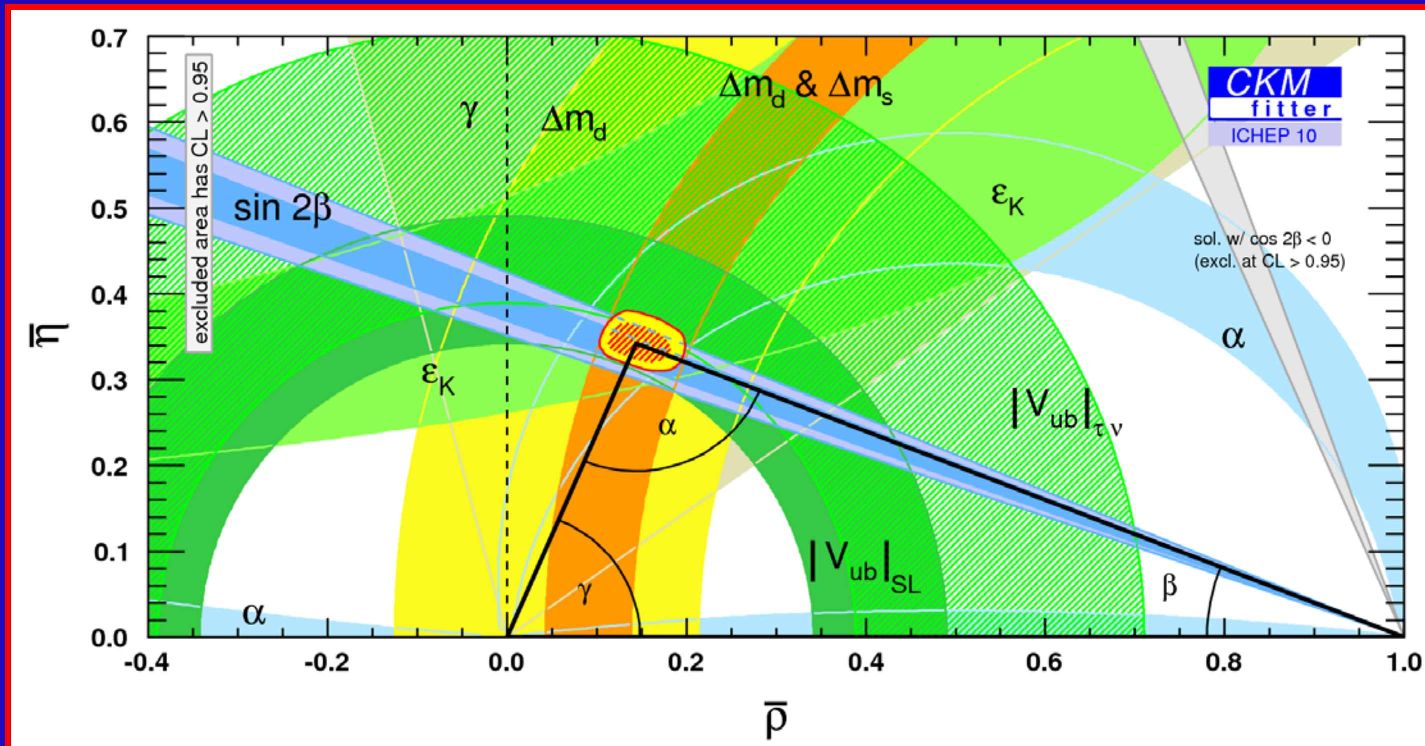
$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

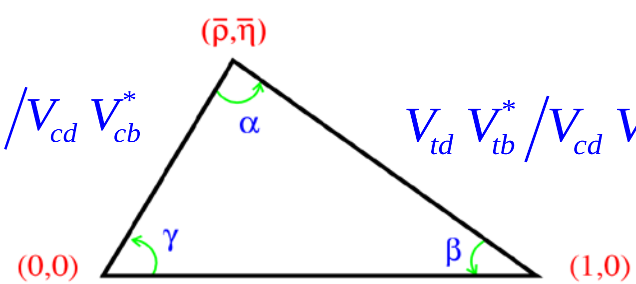
$\frac{1}{2} N_G \left(N_G - 1\right)$ **Moduli** ; $\frac{1}{2} (N_G - 1)\,(N_G - 2)$ **phases**

$N_G = 3$: **3 angles, 1 phase** **(CKM)** $\Rightarrow$ $\not{CP}$

$$V_{ud}\, V_{ub}^{*} + V_{cd}\, V_{cb}^{*} + V_{td}\, V_{tb}^{*} = 0$$



$V_{ud}\, V_{ub}^{*} / V_{cd}\, V_{cb}^{*}$



$V_{td}\, V_{tb}^{*} / V_{cd}\, V_{cb}^{*}$

UT*fit*

$$\bar{\eta} \equiv \eta \left(1 - \frac{1}{2}\lambda^2\right) = 0.358 \pm 0.012$$

$$\bar{\rho} \equiv \rho \left(1 - \frac{1}{2}\lambda^2\right) = 0.132 \pm 0.022$$

$$\alpha = 87.8 \pm 3.0^\circ \;;\; \beta = 22.42 \pm 0.74^\circ \;;\; \gamma = 69.8 \pm 3.0^\circ$$

Standard Model Parameters

QCD: $\alpha_s\,(M_Z)$ **1**

EW Gauge / Scalar Sector: **4**

$$g\,,\,g'\,,\,\mu^2\,,\,h \quad \Leftrightarrow \quad \alpha\,,\,\theta_W\,,\,M_W\,,\,M_H \quad \Leftrightarrow \quad \alpha\,,\,G_F\,,\,M_Z\,,\,M_H$$

Yukawa Sector: **13**

$$m_e\ ,\ m_\mu\ ,\ m_\tau$$

$$m_d\ ,\ m_s\ ,\ m_b$$

$$m_u\ ,\ m_c\ ,\ m_t$$

$$\theta_1\,,\,\theta_2\,,\,\theta_3\,,\,\delta$$

➡ **18 Free Parameters** **(+ Neutrino Masses / Mixings ?)**

TOO MANY !

THE STANDARD THEORY OF FUNDAMENTAL INTERACTIONS

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak + Strong Forces

- Gauge Symmetry → Dynamics
- 3 Gauge Parameters: $\alpha_s(M_Z^2)$, α , θ_W
- All Known Experimental Facts Explained
- Problem with Mass Scales → SSB, Higgs, Yukawas, Mixings

- 15 Additional Parameters
- Why 3 Families ?
- Why Left ≠ Right ?
- Why $m_t > M_Z$?
- Does the Higgs Exist ?
- Flavour Mixing
- $\mathcal{CP}$ Violation
- Neutrino Masses / Oscillations


WANTED
Higgs
GREAT REWARD
STOCKHOLM

Backup Slides

Effective EW Goldstone Lagrangian

$$\mathcal{L}_{\rm Eff} = \frac{v^2}{4}\,\mathrm{Tr}\left[\left(\mathbf{D}^\mu \mathbf{U}\right)^\dagger \mathbf{D}^\mu \mathbf{U}\right] + \mathcal{L}_{\rm EW}^{(4)} + \cdots \quad , \qquad \mathcal{L}_{\rm EW}^{(4)} = \sum_{i=0}^{14} a_i\, O_i$$

$$O_0 = \frac{v^2}{4}\,\langle T V_\mu\rangle^2,$$

$$O_1 = i\frac{gg'}{2}\,B_{\mu\nu}\,\langle T\widehat{W}^{\mu\nu}\rangle, \qquad O_2 = -i\frac{g'}{2}\,B_{\mu\nu}\,\langle T\,[V^\mu, V^\nu]\rangle,$$

$$O_3 = -g\,\langle \widehat{W}_{\mu\nu}\,[V^\mu, V^\nu]\rangle, \qquad O_4 = \langle V_\mu V_\nu\rangle\,\langle V^\mu V^\nu\rangle,$$

$$O_5 = \langle V_\mu V^\mu\rangle^2, \qquad O_6 = \langle V_\mu V_\nu\rangle\,\langle T V^\mu\rangle\,\langle T V^\nu\rangle,$$

$$O_7 = \langle V_\mu V^\mu\rangle\,\langle T V_\nu\rangle^2, \qquad O_8 = \frac{g^2}{4}\,\langle T\widehat{W}_{\mu\nu}\rangle^2,$$

$$O_9 = -\frac{g}{2}\,\langle T\widehat{W}_{\mu\nu}\rangle\,\langle T\,[V^\mu, V^\nu]\rangle, \qquad O_{10} = \{\langle T V_\mu\rangle\,\langle T V_\nu\rangle\}^2,$$

$$O_{11} = \langle (D_\mu V^\mu)^2\rangle, \qquad O_{12} = \langle T D_\mu D_\nu V^\nu\rangle\,\langle T V^\mu\rangle,$$

$$O_{13} = \frac{1}{2}\,\langle T D_\mu V_\nu\rangle^2, \qquad O_{14} = -ig\varepsilon^{\mu\nu\rho\sigma}\,\langle \widehat{W}_{\mu\nu} V_\rho\rangle\,\langle T V_\sigma\rangle.$$

$$\widehat{W}_{\mu\nu} \equiv \frac{i}{g}\left[\left(\partial_\mu - ig\widehat{W}_\mu\right), \left(\partial_\nu - ig\widehat{W}_\nu\right)\right] = \frac{\vec{\tau}}{2}\,\vec{W}_{\mu\nu}$$

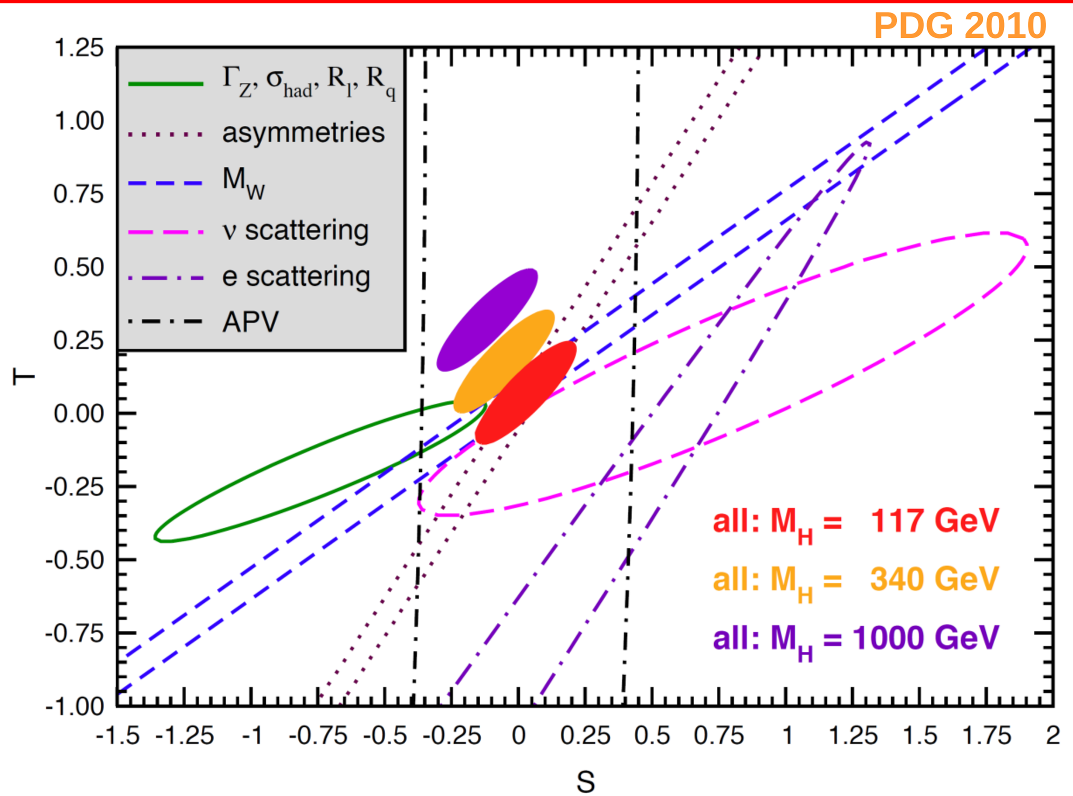
$$\widehat{B}_{\mu\nu} \equiv \partial_\mu \widehat{B}_\nu - \partial_\nu \widehat{B}_\mu = \frac{\tau_3}{2}\,B_{\mu\nu}\,.$$

$$T \equiv U\,\tau^3\,U^\dagger, \quad V_\mu \equiv D_\mu U\,U^\dagger, \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - ig\left[\widehat{W}_\mu, V_\nu\right]$$

$$\frac{\alpha(M_Z)}{4\sin^2\theta_W}(\mathbf{S}+\mathbf{U}) = \qquad \Delta r \doteq -2\,\frac{\cos^2\theta_W}{\sin^2\theta_W}\,a_0 + \left(1-\frac{\cos^2\theta_W}{\sin^2\theta_W}\right) g^2\,(a_8+a_{13}) - 2g^2\,(a_1+a_{13})\,,$$

$$\alpha(M_Z)\,\mathbf{T} = \qquad \Delta\rho \doteq 2a_0,$$

$$\Delta k \doteq \frac{2\cos^2\theta_W\,a_0 + g^2\,(a_1+a_{13})}{\sin^2\theta_W - \cos^2\theta_W}\,.$$



Non-Decoupling

$\mathbf{Log(M_H)}$ **corrections**

Custodial Symmetry

Electroweak chiral coefficients, in units of $1/(16\pi^2)$, for different limits of the SM.

	$M_H \to \infty$ [172,196,197]	$M_{t',b'} \to \infty$ [193]	$M_t \to \infty$ [194]
a_0	$-\frac{3}{4} g'^2 \left[\log(M_H/\mu) - \frac{5}{12}\right]$	0	$\frac{3}{2}\frac{M_t^2}{v^2}$
a_1	$-\frac{1}{6}\log(M_H/\mu) + \frac{5}{72}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{1}{4}$
a_2	$-\frac{1}{12}\log(M_H/\mu) + \frac{17}{144}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{3}{4}$
a_3	$\frac{1}{12}\log(M_H/\mu) - \frac{17}{144}$	$\frac{1}{2}$	$\frac{3}{8}$
a_4	$\frac{1}{6}\log(M_H/\mu) - \frac{17}{72}$	$\frac{1}{4}$	$\log(M_t/\mu) - \frac{5}{6}$
a_5	$\frac{2\pi^2 v^2}{M_H^2} + \frac{1}{12}\log(M_H/\mu) - \frac{79}{72} + \frac{9\pi}{16\sqrt{3}}$	$-\frac{1}{8}$	$-\log(M_t/\mu) + \frac{23}{24}$
a_6	0	0	$-\log(M_t/\mu) + \frac{23}{24}$
a_7	0	0	$\log(M_t/\mu) - \frac{23}{24}$
a_8	0	0	$\log(M_t/\mu) - \frac{7}{12}$
a_9	0	0	$\log(M_t/\mu) - \frac{23}{24}$
a_{10}	0	0	$-\frac{1}{64}$
a_{11}	—	$-\frac{1}{2}$	$-\frac{1}{2}$
a_{12}	—	0	$-\frac{1}{8}$
a_{13}	—	0	$-\frac{1}{4}$
a_{14}	0	0	$\frac{3}{8}$

Hard $\mathbf{m_t^2}$ **corrections**