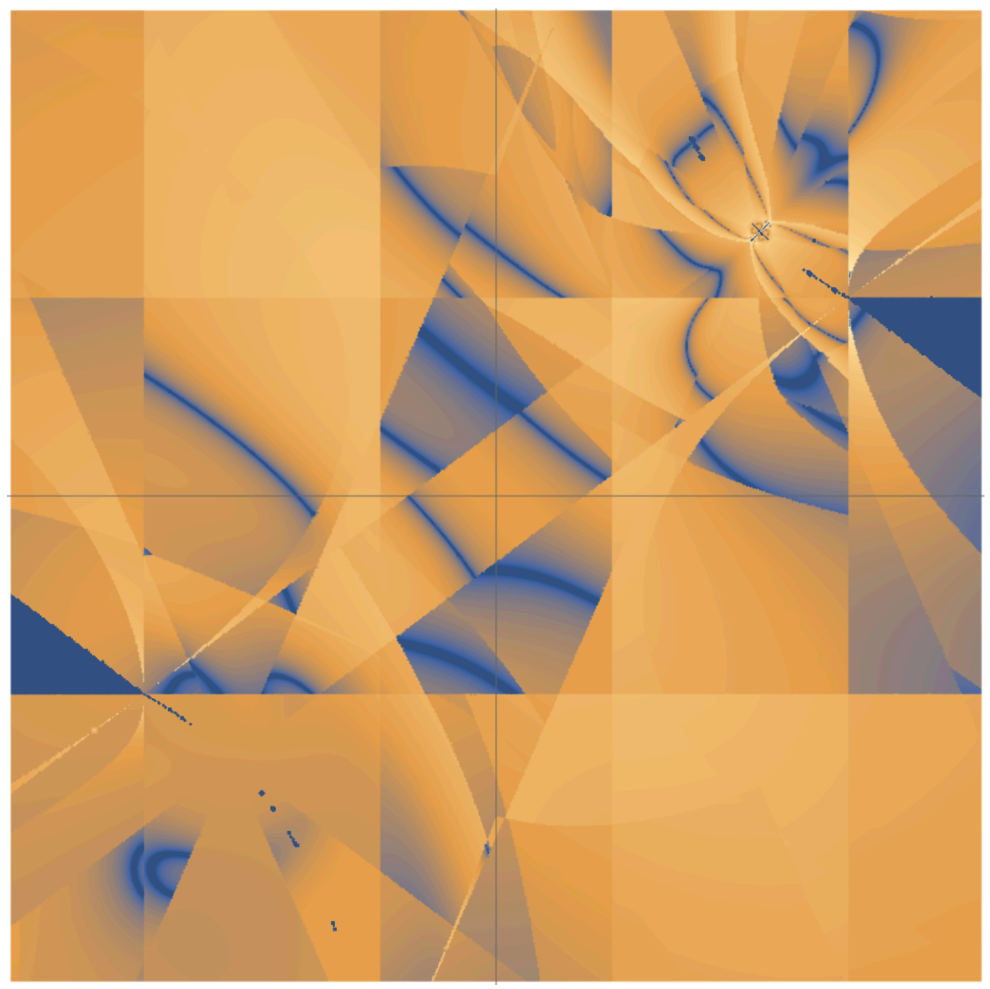
Local Unitarity

Zeno Capatti
ETH Zürich



Pictorial representation of the $e^+e^- \to 2j$ @ NLO *differential x-section*

2010.01068: in collaboration with V. Hirschi, A. Pelloni and B. Ruijl
1906.06138, 1912.09291, 2009.05509: in collaboration with V. Hirschi, D. Kermanschah, A. Pelloni and B. Ruijl

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Introduction

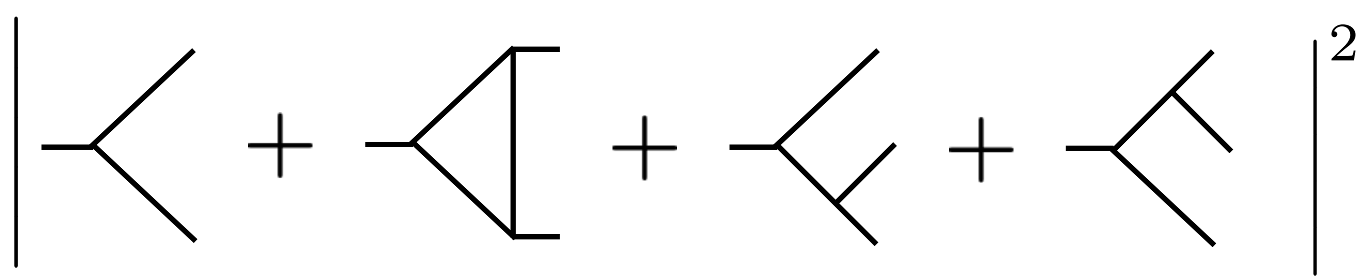
Local Unitarity: framing the problem

A cross-section admits a perturbative expansion when $\alpha < 1$

$$\sigma = \sum_{L=1}^{\infty} \alpha^L \sigma^{(L)}$$

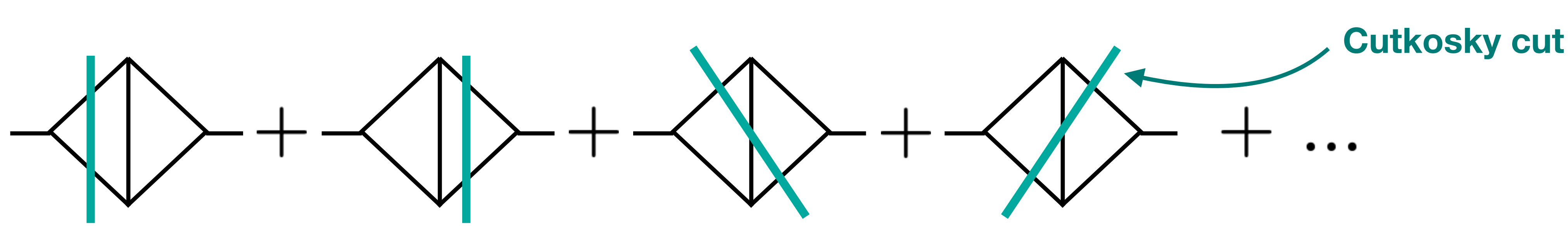
The coefficients of the power series can be obtained by "squaring the S-matrix"

$$\sigma \approx |S|^2 =$$



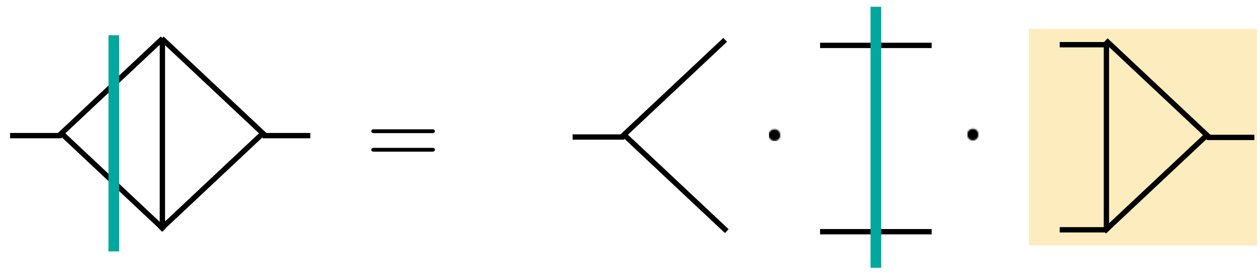
The coefficients can be represented as a sum of interference diagrams

$$\sigma^{(2)} =$$

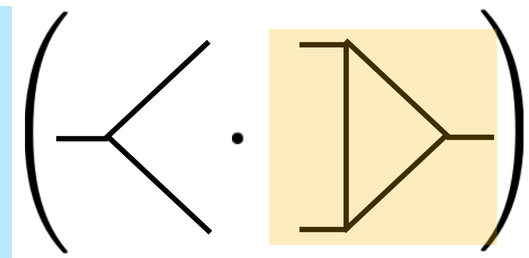


Each side of Cutkosky cut corresponds to a diagram building up the S-matrix

Interference diagrams themselves can be represented as integrals of amplitudes

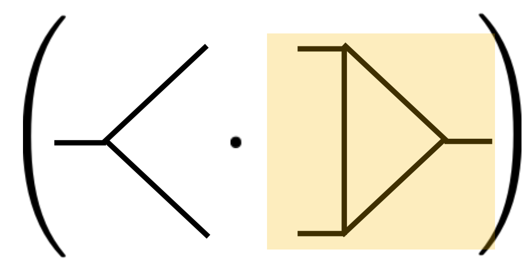


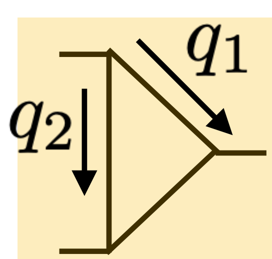
$$= \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \delta^{(+)}(p_1^2)\, \delta^{(+)}(p_2^2)\, \delta^4(p_1 + p_2 - q)$$



Phase space integral

$$= \mathrm{i} \int \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3} \delta(|\vec{p}| + |\vec{p} - \vec{q}| - Q^0)$$





$$= \int \frac{\mathrm{d}^4 k}{D_1 D_2 D_3}$$

Loop integral

Problem: both types of integrals are divergent!

- Collinear divergences $q_1 // q_2$
- Soft divergences $q_1 = 0$

} **Non-integrable**

- Thresholds } **Integrable**

Loop integrals $\mathrm{d}^4 k$

Infrared singularities

Final state singularities (FSS)

Initial state singularities (ISS)

Integrable singularities

Loops

LTD/cLTD/TOPT
Causal flow

KLN theorem

$\mathrm{d}^3 \vec{p}$ **Phase space integrals**

Infrared singularities

Final state radiation (FSR)

Initial state radiation (ISR)

Trees

This subdivision **hides an inherent simplicity**

Integrals

ISS + ISR

Integrable singularities

Trees

IR singularities appear in **separate pieces** of the computation of LHC observables, but **not in the final result** (IR-safety)

Forcing IR-safety to be realised **locally** loosens the distinction between phase space and loop integrals

Our objective

Computing cross-sections fully numerically by locally combining real and virtual contributions

That is: Find a representation of perturbative cross-sections in the form

$$\sigma = \sum_{L=1}^{\infty} \alpha^L \int \mathrm{d}\Pi_L \sigma_{\mathrm{d}}^{(L)}$$

where $\sigma_{\mathrm{d}}^{(L)}$ is an **integrable** function, can be **MonteCarlo** integrated.

This can be achieved with no subtraction and dimensional regularisation

Using robustness of MonteCarlo methods to automate fixed order corrections

Objective: $$\sigma = \sum_{L=1}^{\infty} \alpha^L \int \mathrm{d}\Pi_L \sigma_{\mathrm{d}}^{(L)}$$

Monte Carlo methods are fine, but how do we construct $\sigma_{\mathrm{d}}^{(L)}$?

Want method to be **generic (scattering process and perturbative order) and competitive**, yield new results in **reasonable time** with **limited resources**

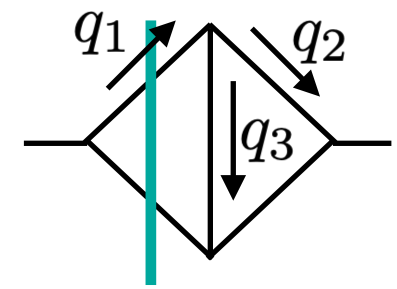
Studying this we learn about

- The singular structure of amplitudes in momentum space
- The singular structure of phase space integrals

Conceptual shift: from amplitudes+phase space integrals to interference diagrams

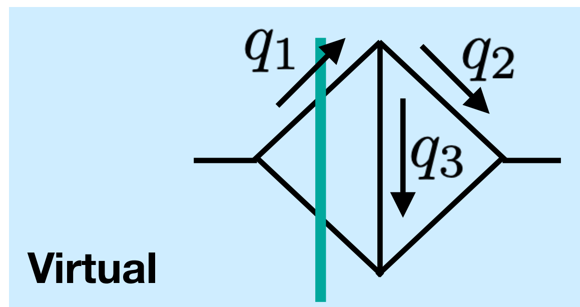
Real and virtual contributions

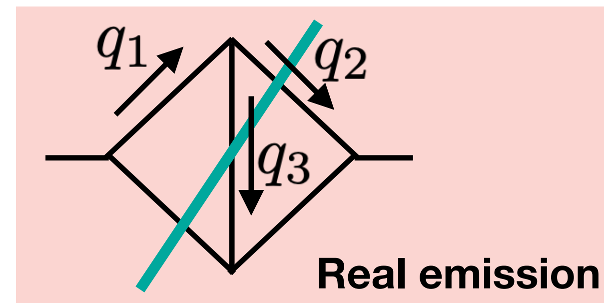
Interference diagram may have a collinear singularity, e.g.



$$\begin{cases} q_2 = x q_1 \\ q_3 = (1-x) q_1 \end{cases}$$

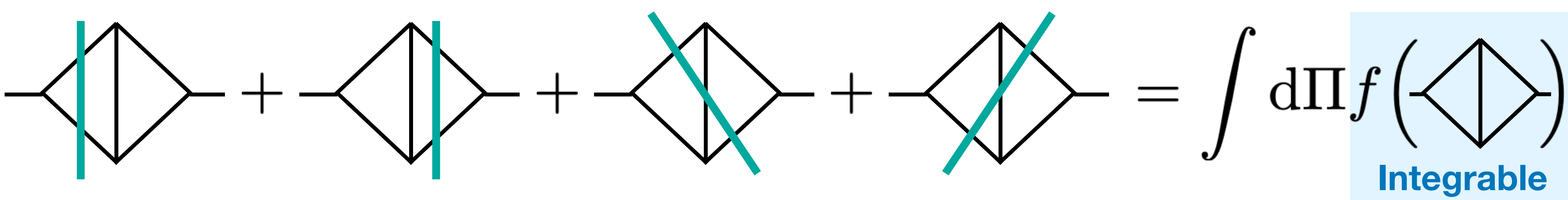
This sum of diagrams is finite in this collinear limit (KLN theorem)



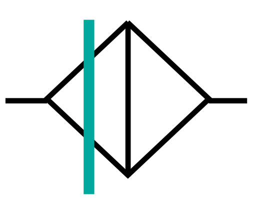


$(+\, c.c)$

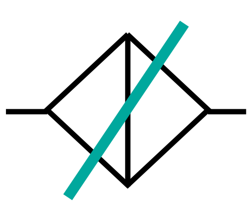
Sum over all the Cutkosky cuts of the double triangle is finite in any IR limit



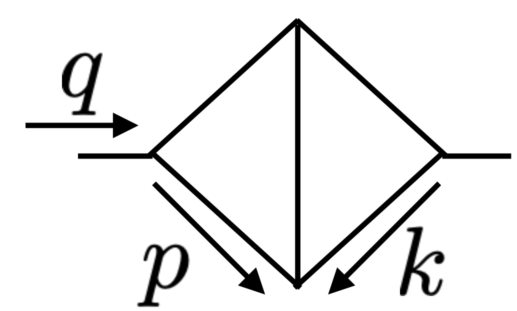
Problem: there is a difference in dimensionality between phase space and loop integrals



$$\frac{\mathrm{d}^3\vec{p}}{2|\vec{p}|}\mathrm{d}^4k\,\delta(|\vec{p}|+|\vec{p}-\vec{q}|-Q_0)$$



$$\frac{\mathrm{d}^3\vec{p}}{2|\vec{p}|}\frac{\mathrm{d}^3\vec{k}}{2|\vec{k}|}\delta(|\vec{p}|+|\vec{p}-\vec{k}|+|\vec{k}-\vec{q}|-Q_0)$$



Loop-Tree Duality

Loop Tree Duality

The **LTD representation** allows for explicit integration of the energy components using residue theorem

$$\int \left[\prod_{m=1}^{M} \mathrm{d}^4 k_m\right] \frac{N}{\prod_i D_i} = \int \left[\prod_{m=1}^{M} \mathrm{d}^3 \vec{k}_m\right] f_{\mathrm{ltd}}$$

With this result, both loop and phase space integrals are defined over 3D space

Catani, Gleisberg, Krauss, Rodrigo, Winter
arXiv: 0804.3170 (2008)

Bierenbaum, Catani, Draggiotis, Rodrigo
arXiv: 1007.0194 (2010)

Runkel, Ször, Vesga, Weinzierl
arXiv: 1902.02135 (2019)

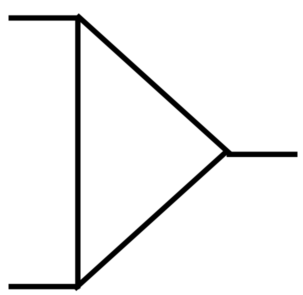
Verdugo, Driencout-Mangin, et al.
arXiv: 2001.03564 (2020)

ZC, Hirschi, Kermanshah, Ruijl
arXiv: 1906.06138 (2019)

ZC, Hirschi, Kermanshah, Pelloni, Ruijl
arXiv: 2009.05509 (2020)

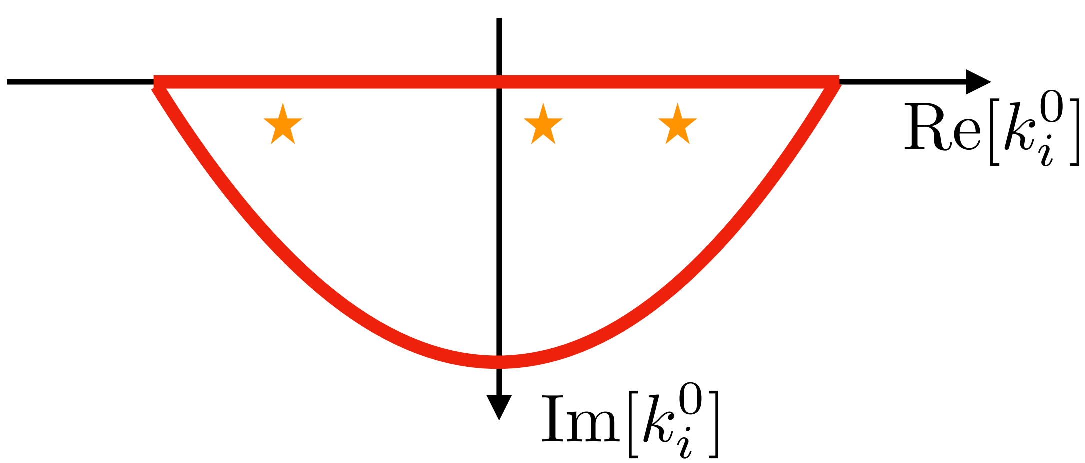
Automation of LTD and cLTD (arbitrary loops, topologies, numerators)

Example:



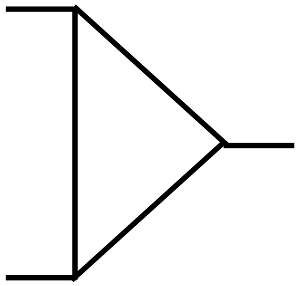
$$= \int \mathrm{d}^4 k \frac{N}{(k^2 - i\epsilon)((k-p_1)^2 - i\epsilon)((k+p_2)^2 - i\epsilon)}$$

Analytically continue energy component. Close the contour in the lower complex plane. Poles are:



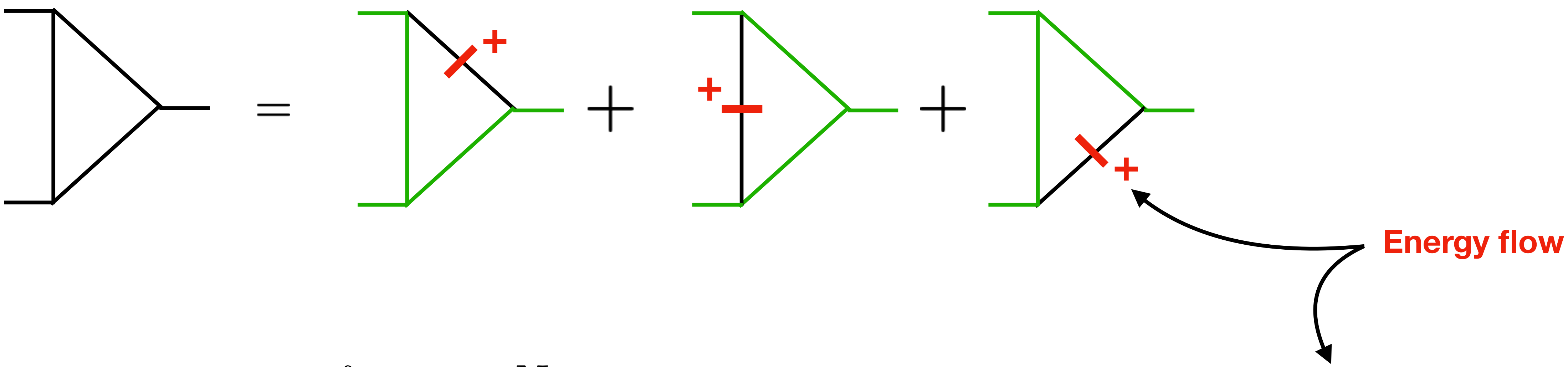
$$\begin{cases} k^0 = \sqrt{|\vec{k}|^2 - i\epsilon} & \mathrm{Im}[k^0] < 0 \quad 1 \\ k^0 = p_1^0 + \sqrt{|\vec{k} - \vec{p}_1|^2 - i\epsilon} & \mathrm{Im}[k^0] < 0 \quad 2 \\ k^0 = -p_2^0 + \sqrt{|\vec{k} + \vec{p}_2|^2 - i\epsilon} & \mathrm{Im}[k^0] < 0 \quad 3 \end{cases}$$

Then using residue theorem



$$= \int \mathrm{d}^3\vec{k} \left[\mathrm{Res}_1\left[\frac{N}{D_1 D_2 D_3}\right] + \mathrm{Res}_2\left[\frac{N}{D_1 D_2 D_3}\right] + \mathrm{Res}_3\left[\frac{N}{D_1 D_2 D_3}\right]\right]$$

Residues can be represented as cuts:

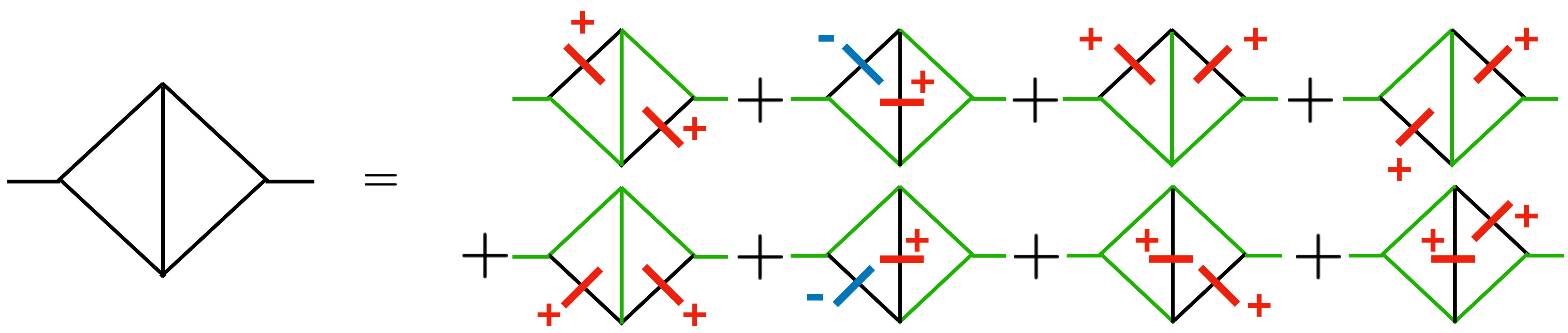


$$= \int \mathrm{d}^4 k \frac{N}{D_1 D_2 D_3}\left(D_1 \delta^{(+)}(D_1) + D_2 \delta^{(+)}(D_2) + D_3 \delta^{(+)}(D_3)\right)$$

Delete the cut edges, obtain spanning trees ⟶ **Tree processes** **(virtual/real particles)**

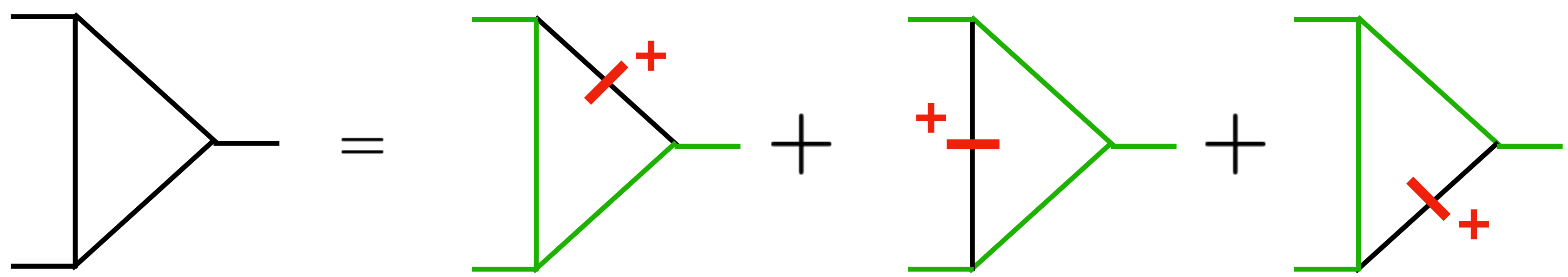
One loop is easy! The technicalities start to appear at higher loops

For example, applying LTD to a double triangle



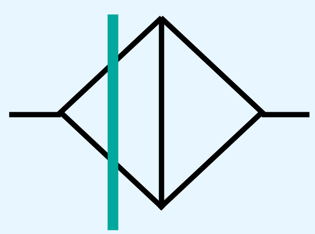
Interplay of momentum conservation and epsilon prescription is key to obtain the energy flow

Going back to the triangle

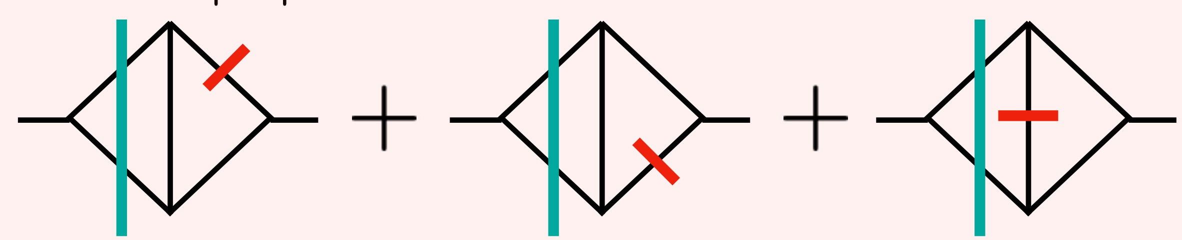


Finally, we can apply LTD for our original purpose

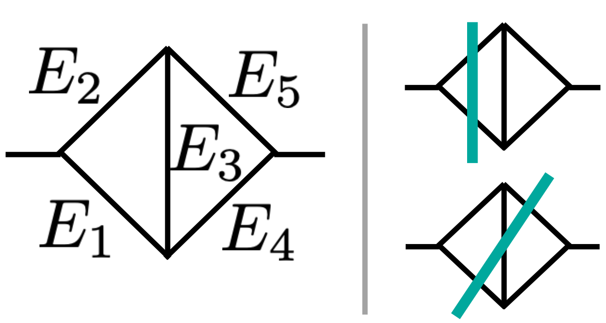
$$\frac{\mathrm{d}^3\vec{p}}{2|\vec{p}|}\mathrm{d}^4k\,\delta(|\vec{p}|+|\vec{p}-\vec{q}|-Q_0) \rightarrow \frac{\mathrm{d}^3\vec{p}}{2|\vec{p}|}\frac{\mathrm{d}^3\vec{k}}{2|\vec{k}|}\delta(|\vec{p}|+|\vec{p}-\vec{q}|-Q_0)$$



$=$



Applying LTD to the interference diagrams, we can bring them under the same integral sign

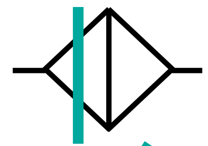


$$= \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\,\left(\delta(E_1+E_2-Q_0)f_{\mathrm{virt}} + \delta(E_1+E_3+E_5-Q_0)f_{\mathrm{real}}\right)$$

The causal flow

Causal flow

The measure now differs only in the **delta enforcing on shell energy conservation**

 $\sim \delta(E_1 + E_2 - Q_0)$

 $\sim \delta(E_1 + E_3 + E_5 - Q_0)$

Find a variable to solve both deltas. Here the first energy works, in general there is not a unique energy that allows that.

Side-step phase space mapping problems

Solution: introduce a fictitious variable in which to solve the delta

$$\delta(|\vec{k}| - Q_0) \underset{\vec{k} \to t\vec{k}}{\longrightarrow} \delta(t|\vec{k}| - Q_0) \longrightarrow t = \frac{Q_0}{|\vec{k}|}$$

Soper,
arXiv: 9804454 (1998)

Soper,
arXiv: 0102031 (2001 @ RADCOR)

ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

General FSR cancellations
For N to M NkLO processes

A toy example:

$$\int \mathrm{d}^3\vec{k}\, \delta(|\vec{k}| - Q_0) f(\vec{k})$$

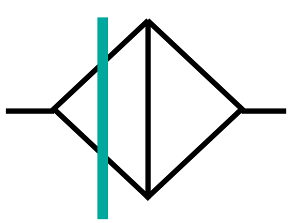
$$= \int \mathrm{d}^3\vec{k} \int \mathrm{d}t\, h(t)\, \delta(|\vec{k}| - Q_0) f(\vec{k}) \qquad \text{using} \qquad 1 = \int \mathrm{d}t\, h(t)$$

$$= \int \mathrm{d}^3\vec{k} \int \mathrm{d}t\, t^3\, h(t)\, \delta(t|\vec{k}| - Q_0) f(t\vec{k}) \qquad \text{using} \qquad \vec{k} \to t\vec{k}$$

$$= \int \mathrm{d}^3\vec{k}\, \frac{Q_0^3}{|\vec{k}|^4}\, h(Q_0/|\vec{k}|)\, f(Q_0\vec{k}/|\vec{k}|) \qquad \text{with} \qquad t^\star = Q_0/|\vec{k}|$$

Solve delta in scaling variable. Phase space has same dimensionality

Then



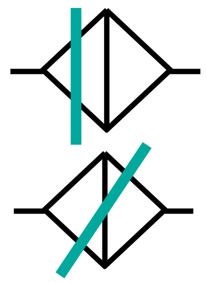
$$= \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\delta(E_1 + E_2 - Q_0) f_{\text{virt}} = \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\, g_{\text{v}}(t_{\text{v}}^\star)$$

where $t_{\text{v}}^\star = t_{\text{v}}^\star(\vec{k},\vec{p}) = \dfrac{Q_0}{E_1 + E_2}$

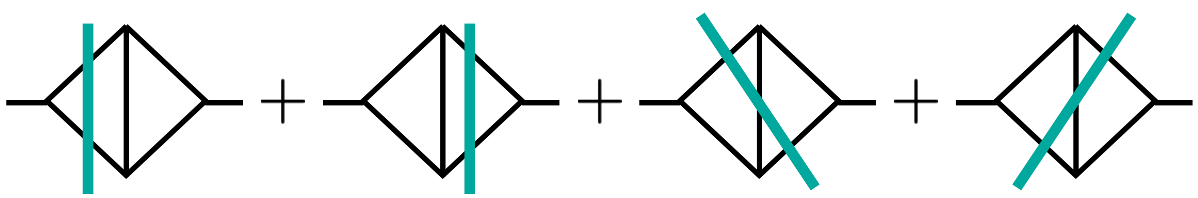
$$(\vec{p},\vec{k}) \to \vec{\phi}(t,(\vec{p},\vec{k})) \qquad \begin{cases} \partial_t \vec{\phi} = \vec{\kappa} \circ \vec{\phi} \\ \vec{\phi}(0,(\vec{k},\vec{l})) = (\vec{k},\vec{l}) \end{cases}$$

Why “causal flow”?

Apply same procedure to real…

Then:  $= \int \mathrm{d}^3\vec{p}\,\mathrm{d}^3\vec{k}\,(g_{\mathrm{v}}(t_{\mathrm{v}}^{\star}) + g_{\mathrm{r}}(t_{\mathrm{r}}^{\star}))$

We have aligned the measure!



$$= \sum_{i=1}^{4} \int \mathrm{d}\Pi_i f_i \;=\; \int \mathrm{d}\Pi \sum_{i=1}^{4} g_i$$

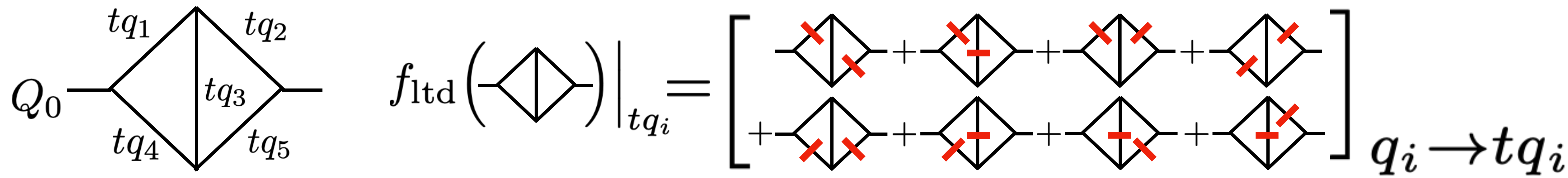
It turns out that doing so also achieves IR-finiteness at **the local level (causal flow)**

- We constructed our local representation of differential cross-sections
- The two main ingredients (LTD and the causal flow) can be generically applied at any order.

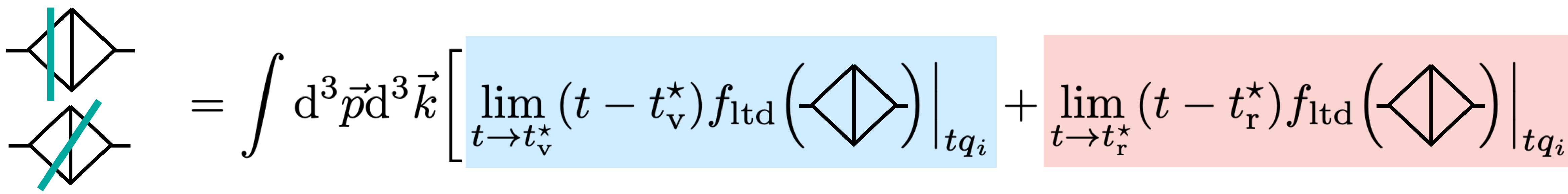
Showing IR finiteness we have to recast this expression is a slightly different but illuminating way

Local Unitarity representation

The LTD representation of the double triangle with rescaled momenta is

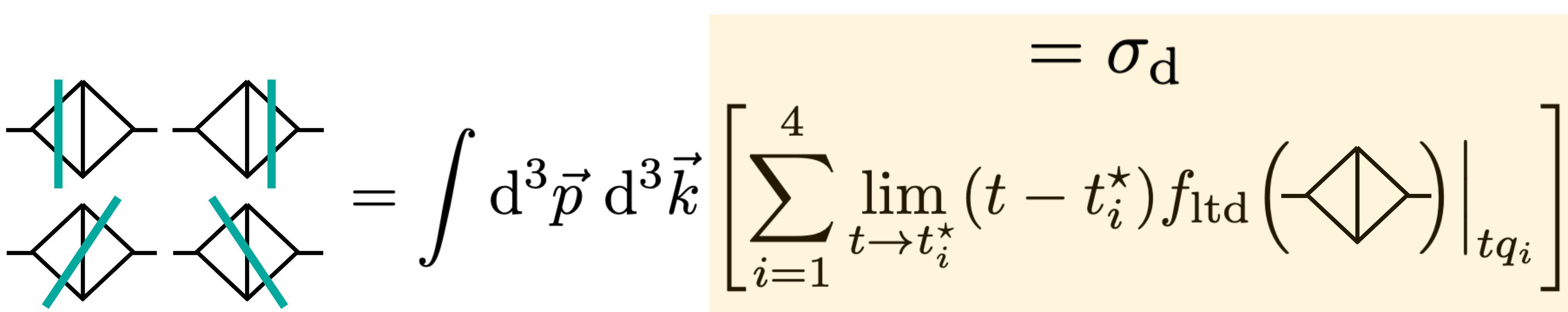


Then



$g_{\rm v}\,,\ \ g_{\rm r}$ can be written as different limits of the same function!

Solving delta in the scaling variable $\Rightarrow$ 1d residue theorem along the line $\gamma(t) = (t\vec{k}, t\vec{p})$

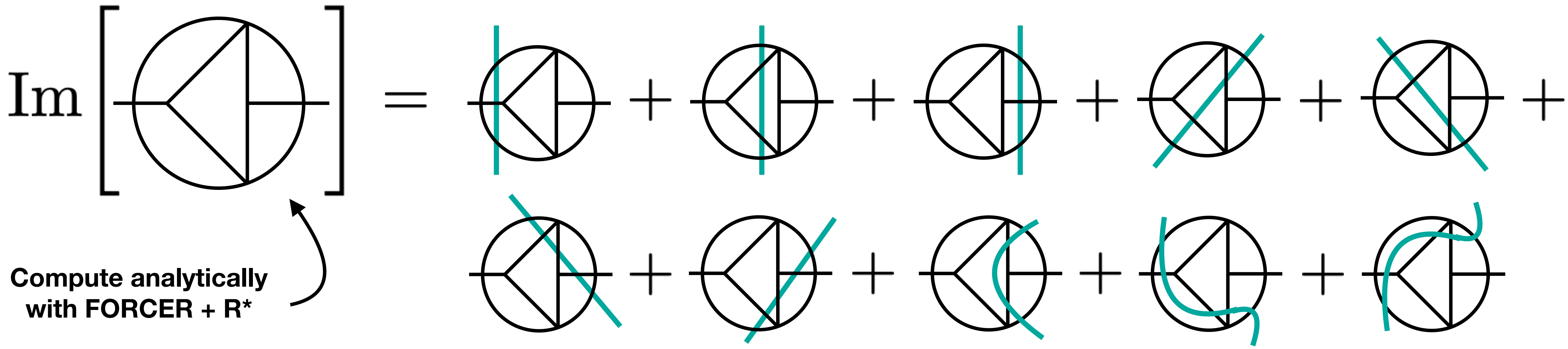


LU representation

Cutkosky, but at the local level!

Local IR cancellations: 5-loop example

We proved cancellations rigorously for FSR singularities. Here we use an example

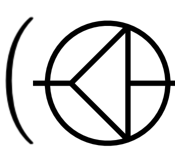


Compute analytically with FORCER + R*

B. Ruijl, T. Ueda, J. Vermaseren
arXiv: 1704.06650 (2017)

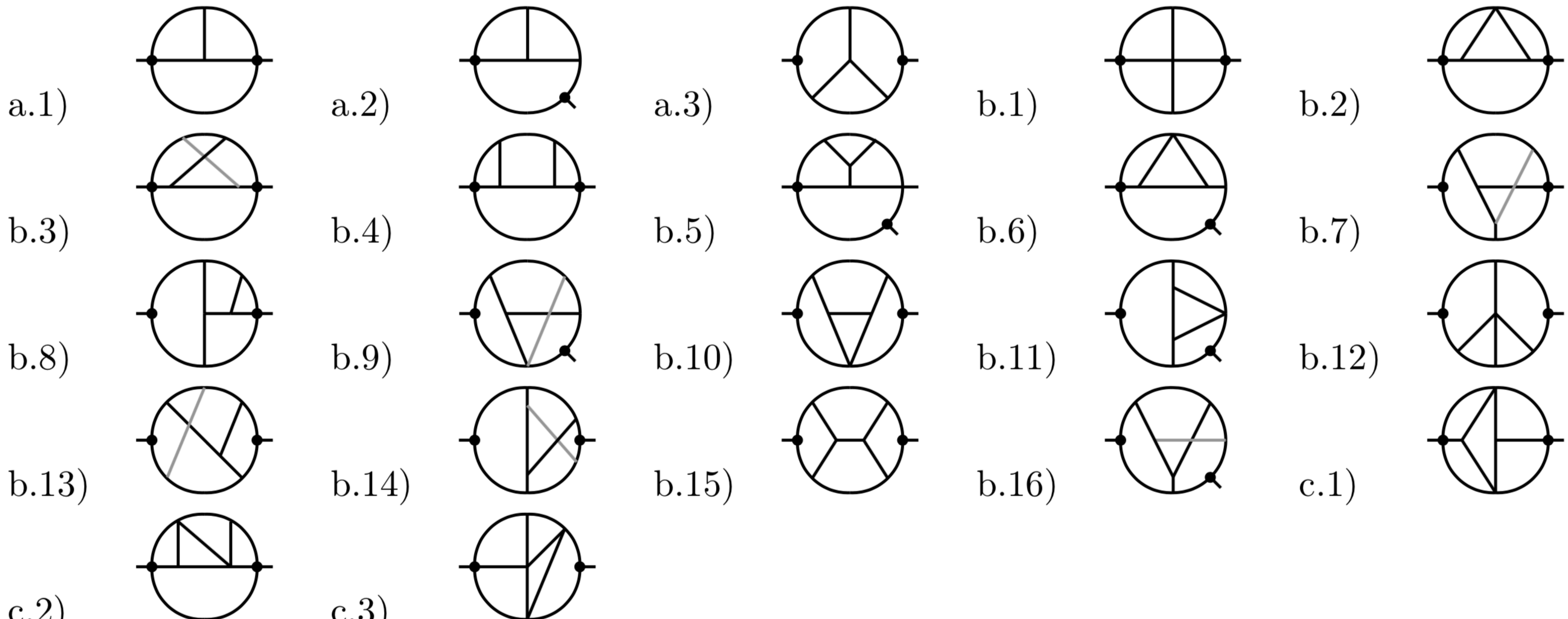
F. Herzog, B. Ruijl
arXiv: 1703.03776 (2017)

$$= \int \left[\prod_{j=1}^{5} \mathrm{d}^3 \vec{k}_j \right] \sum_{i=1}^{10} \lim_{t \to t_i} (t - t_i) f_{\mathrm{ltd}} \left(\cdot \right)$$



Monte Carlo Integration

$\mathtt{N_p}$ $[10^6]$	t/p $[\mu s]$ min	avg	$\mathtt{N_{ch}}$	FORCER $[\mathrm{GeV}^2]$	αLOOP $[\mathrm{GeV}^2]$	exp.	Δ $[\sigma]$	Δ $[\%]$
Inclusive cross-section per supergraph								
1	1100	49000	128	1.66419	1.6691(79)	-9	0.62	0.0029

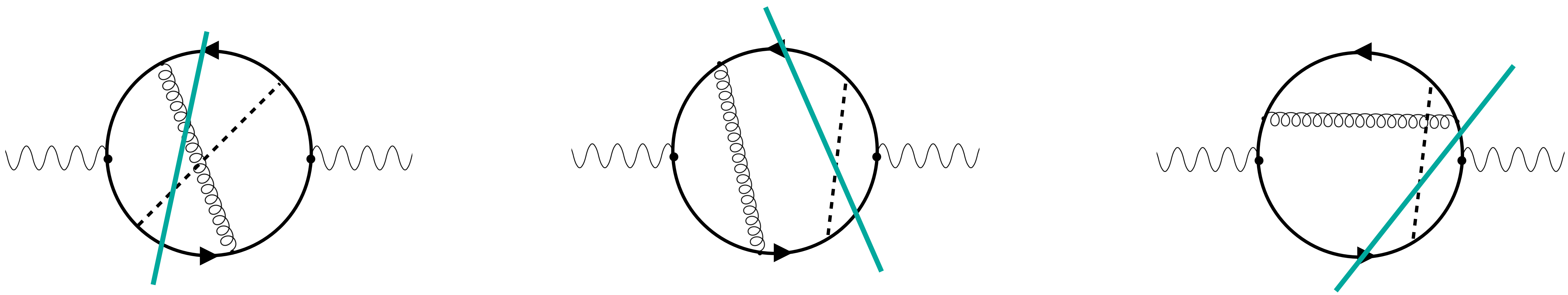


We did the same for all 3-4-5 loop two-point functions that are finite in scalar theory

Inclusive $e^+e^- \to t\bar{t}H$ @ NLO

Same procedure is applied to physical case.

This has many forward-scattering diagrams and Cutkosky cuts, e.g.



	name	multiplicity	neval	real	real_err	eval_time
0	SG_QG0	2.000000e+00	5850000	6.689035e-05	9.240544e-08	0 days 00:15:35.553646000
1	SG_QG2	2.000000e+00	2080000	2.349607e-05	4.541978e-08	0 days 00:00:41.443805000
2	SG_QG6	1.000000e+00	2080000	-8.346356e-05	9.293410e-08	0 days 00:00:53.087342000
			...			
14	SG_QG46	2.000000e+00	2080000	3.534058e-05	3.903003e-08	0 days 00:00:59.076110000
15	SG_QG47	2.000000e+00	2080000	-1.618672e-06	1.686635e-09	0 days 00:00:09.248204000

15 forward-scattering diagrams

O(50) interference diagrams

```
Pure NLO correction:
MG res:         -1.38400e-04 +/- 1.4e-07
aL real res:    -1.38320e-04 +/- 5.9e-07
|(MG-aL)/MG|:    5.75e-04
```

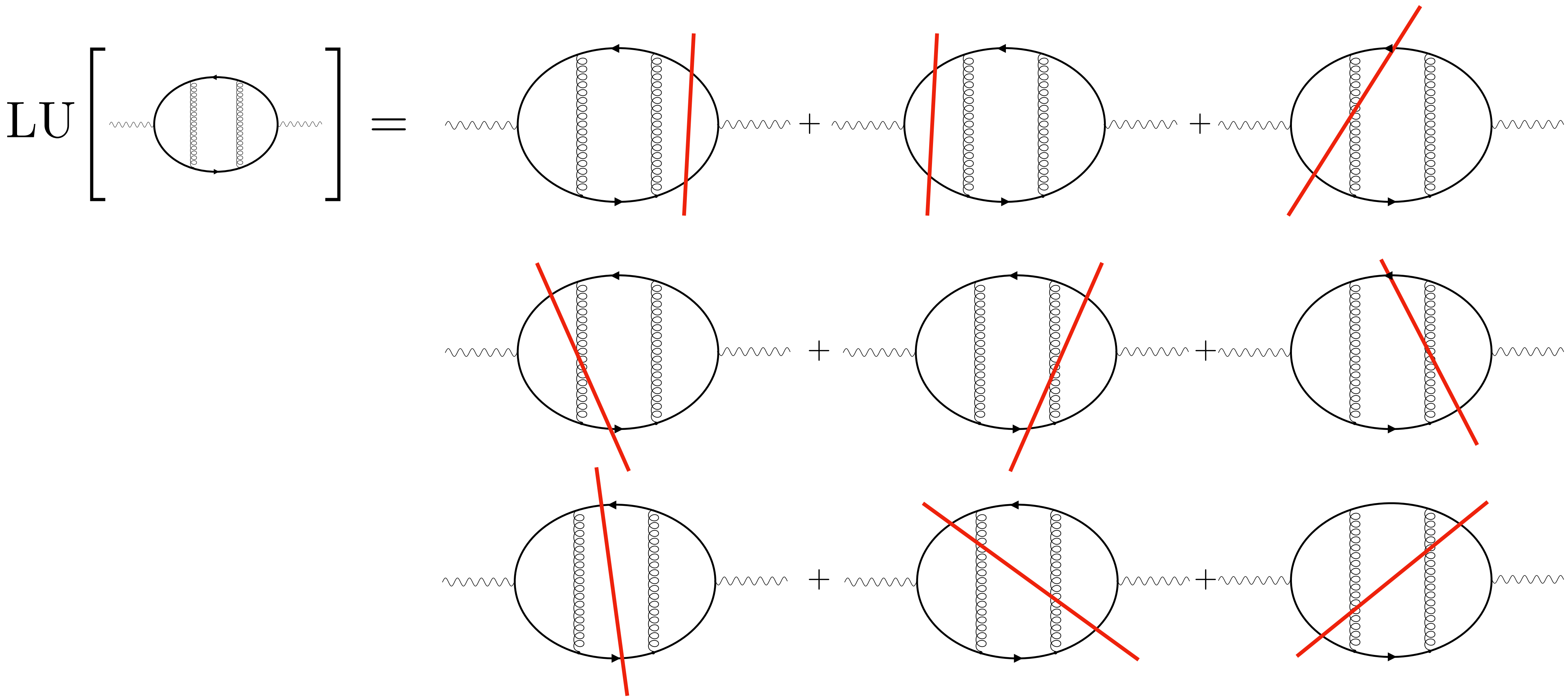
Matches benchmark
From MG5_aMC@NLO

Only dim. reg.
for UV counter-terms

No IR counter-terms

Alwall, Frederix, Frixione, Hirschi, Maltoni
arXiv: 1405.0301 (2014)

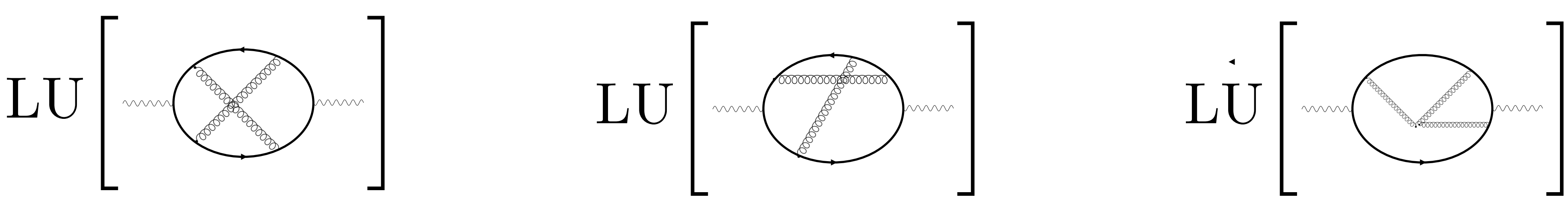
$e^+e^- \to t\bar{t}$ @ NNLO



Very complicated singular structure, regulated without counter-terms!

```
| SG_QG3 | -0.000685226 +- 1.86e-06 (0.27%)  | chi2=0.875|mwi=1.04|n=19057.0M(11%)|n0=73.4M(0.39%)|p=11%
```

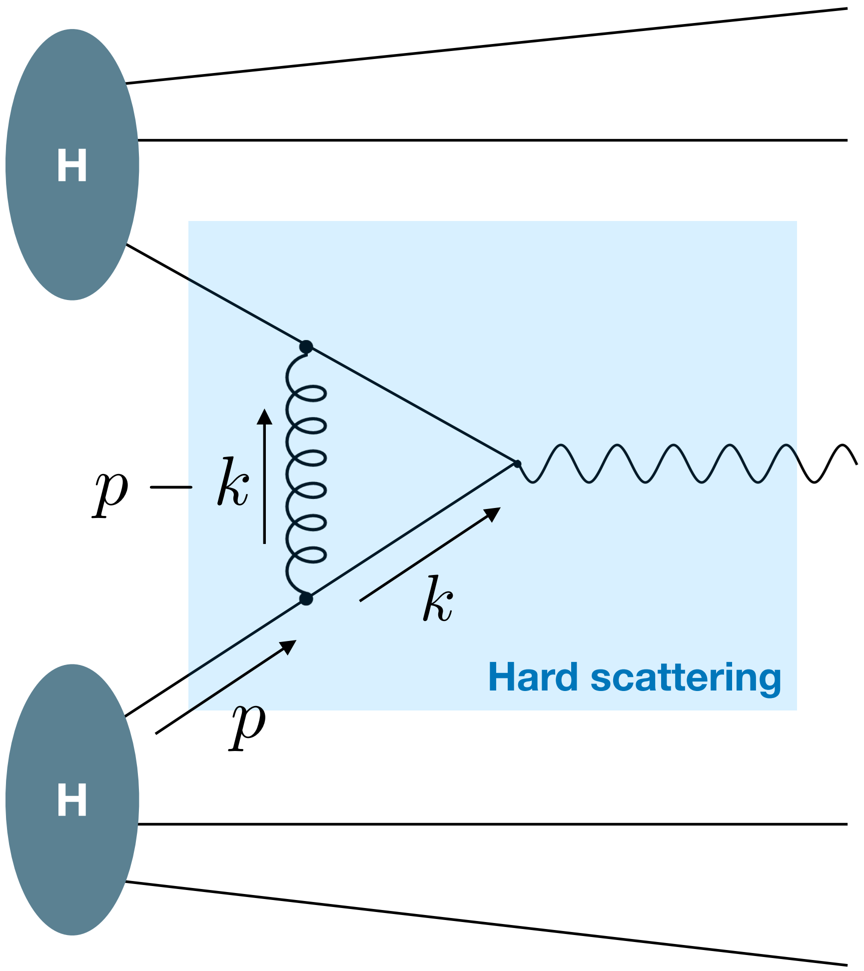
Same with



Initial State Singularities

Initial State Singularities

Scattering of hadrons is treated in the parton model



Assumptions:

- Interactions between partons inside the same hadron are negligible
- Scattering occurs between one Parton from one hadron and one Parton from the other

These assumptions lead to singularities that are incurable from KLN

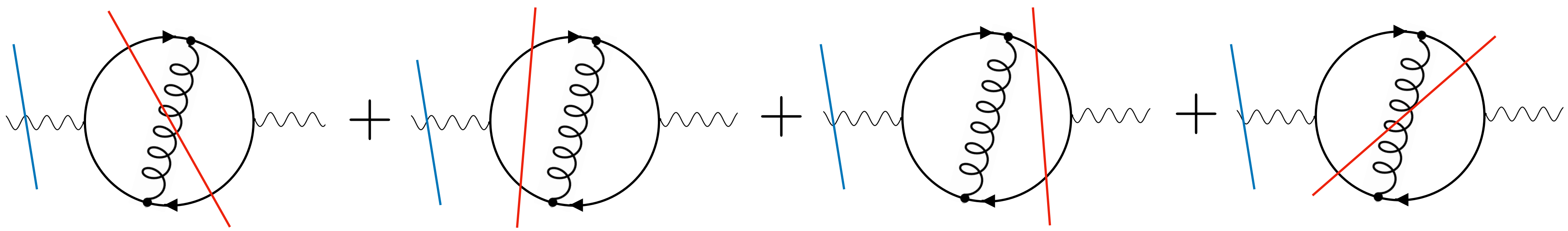
$$k = xp$$

The current paradigm uses PDF **renormalisation** + **resummation** to eliminate them

Idea

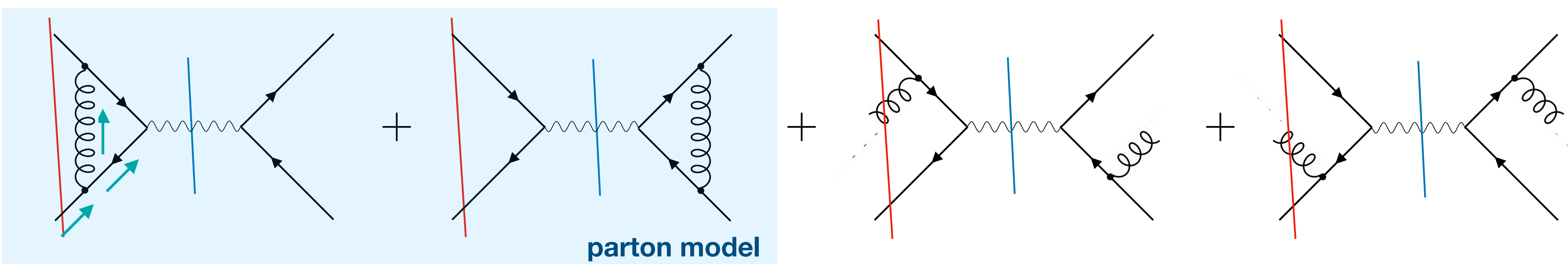
Instead of using the Parton model, we take inspiration from KLN

For $e^+e^- \to 2j$ @ NLO we used



(include all degenerate configurations, higher final-state multiplicities)

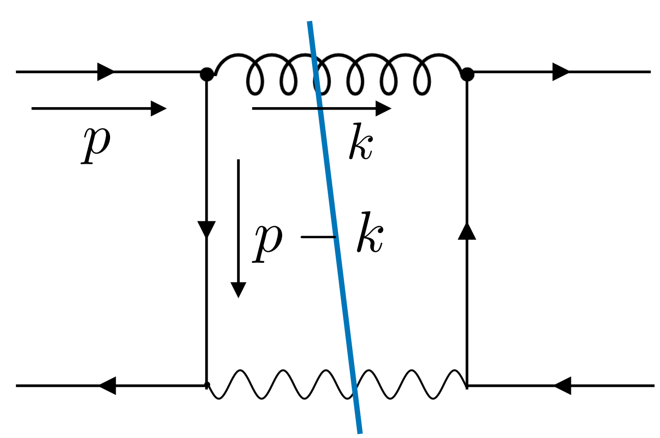
Flip it, and obtain the answer for Drell-Yan, $2\,\mathrm{j} \to e^+e^-$



The initial state singularity is now absent!

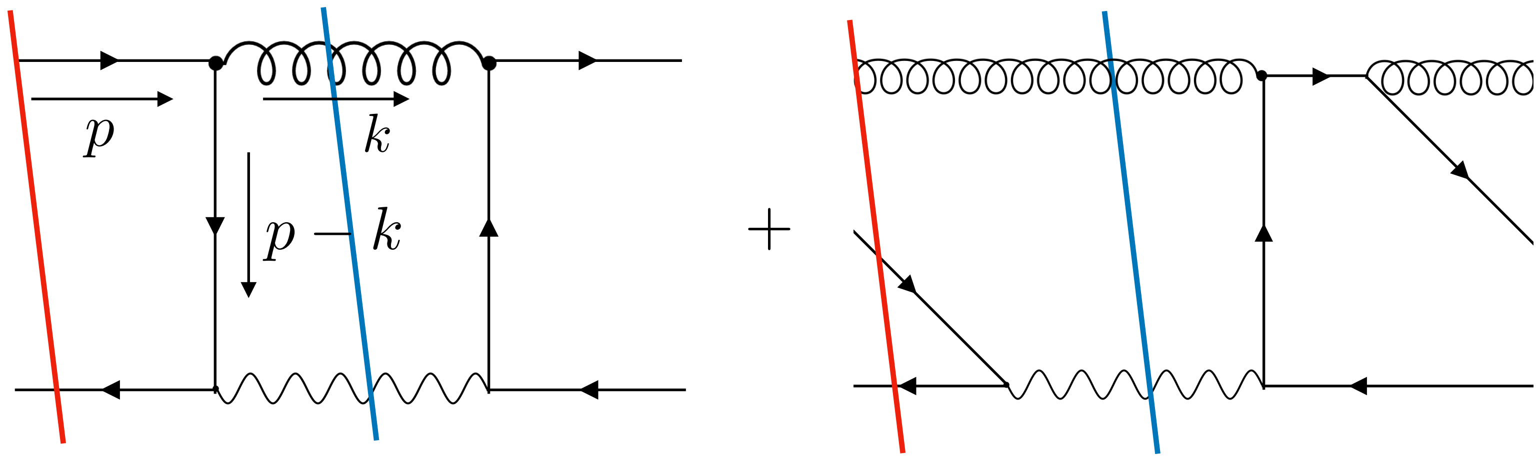
Include degenerate initial states $\longrightarrow$ **Higher multiplicity initial states**

What about this diagram?



Also has collinear singularity at $k = xp$

In this case, the cancelling partner is



Higher multiplicity initial states, but also **disconnected**! **Free travelling gluon!**

The sum of these two diagrams is finite everywhere in phase space

Not a new idea!

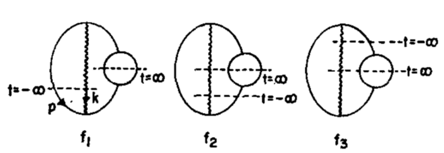
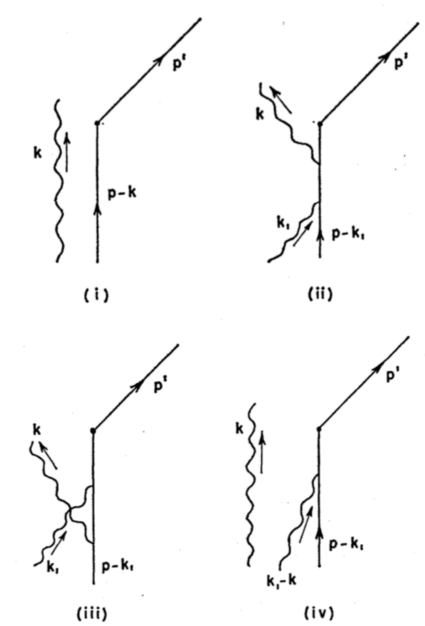


FIG. 15. Additional double cut diagrams which are introduced to take account of the degeneracy of the initial state.

T. Kinoshita
"Mass singularities of Feynman amplitudes"
(1962)

The methods described here cannot be directly applied to processes with definite numbers of hadrons in the initial state. However, it may be that for some purposes a high energy hadron behaves like a jet. In this case, the differential cross section for jet production in high energy hadronic collisions could also be calculated in QCD by ordinary perturbation theory.

G. Sterman, S. Weinberg,
"Jets from Quantum Chromodynamics"
(1977)

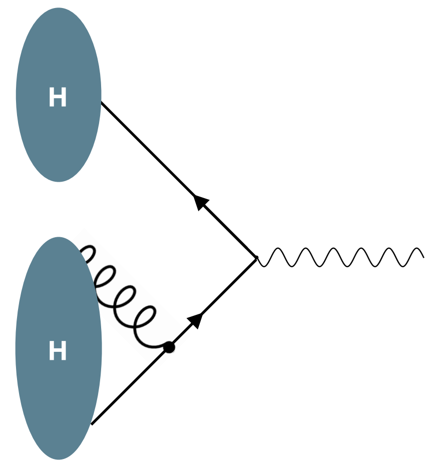


T.D. Lee, M. Nauenberg
"Degenerate systems and mass singularities"
(1963)

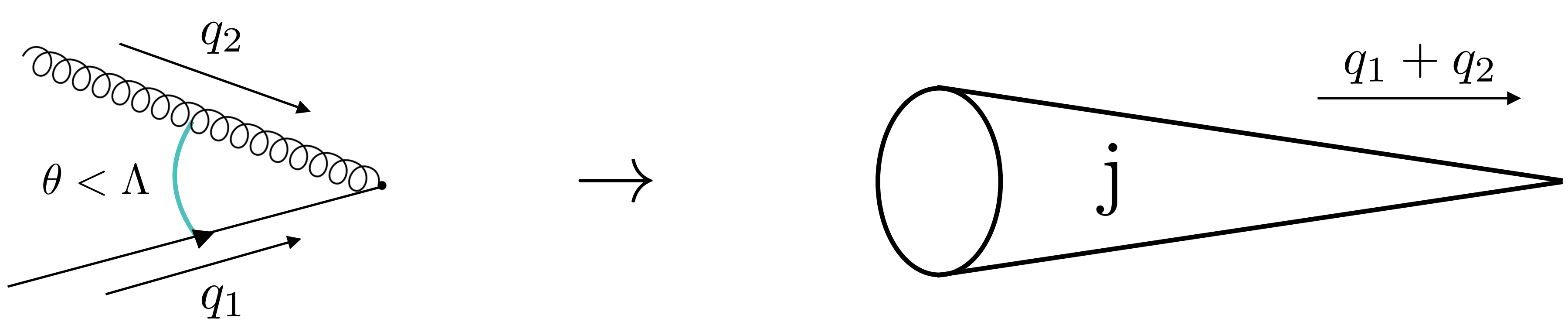
Also look at Frye, Hannesdottir, Paul, Schwartz, Yan
arXiv:1810.10022 (2019)

Multiple initial state partons

This argument suggests that, in order to maintain IR-finiteness, one requires more than two initial state partons



and that the multiple partons should be clustered into **two jet-like objects** that resemble high energy hadrons



After clustering, we get two jets with momenta

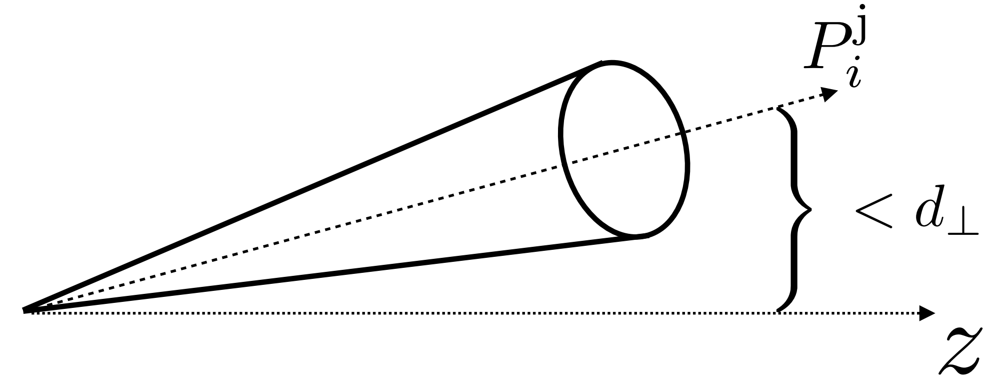
$$P_1^{\mathrm{j}} \qquad\qquad P_2^{\mathrm{j}}$$

Cluster initial states analogously to final states: symmetry initial-final state

The use of a jet algorithm naturally comes with two scales

- One measuring the allowed phase space for the **total momentum** of the jet

$$(P_i^{\mathrm{j}})_\perp < d_\perp$$



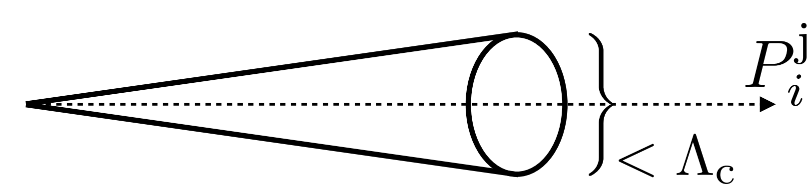
If the scale is zero, the jet lies **exactly** on the z axis

If $d_\perp = 0$ the two jets are exactly back-to-back. This is equivalent to the parton's model

$$p_1 = (x_1\sqrt{s}, 0, 0, x_1\sqrt{s}), \quad p_2 = (x_2\sqrt{s}, 0, 0, -x_2\sqrt{s})$$

- One measuring the maximum angular separation between two partons in a jet

$$(P_{ij})_\perp < \Lambda_{\mathrm{c}}$$

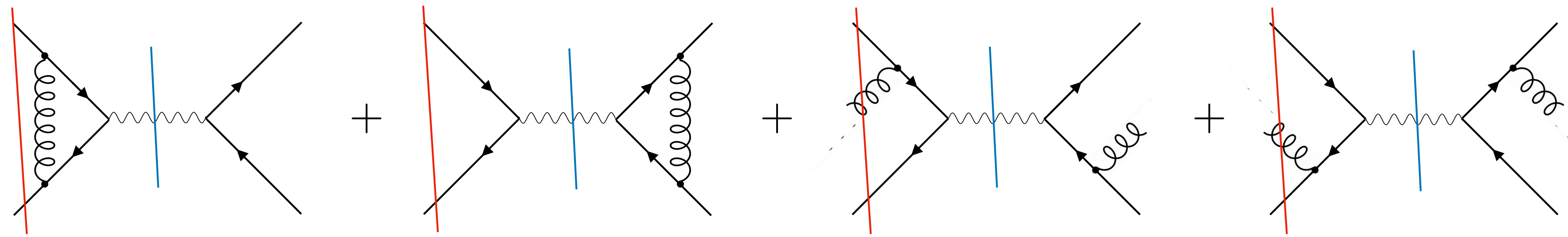


The smaller this scale, the more **collinear** the partons are

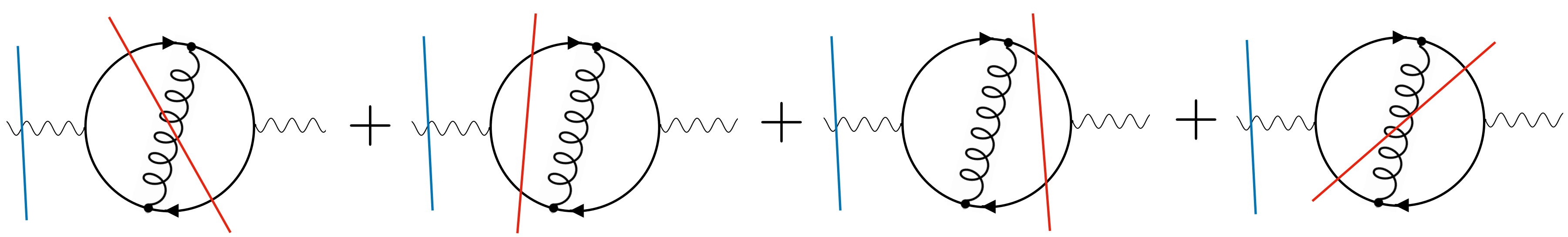
The more collinear the partons, the more divergent the observable

Λ_{c} **is the equivalent of the factorisation scale!** $\approx \log(\Lambda_{\mathrm{c}})$

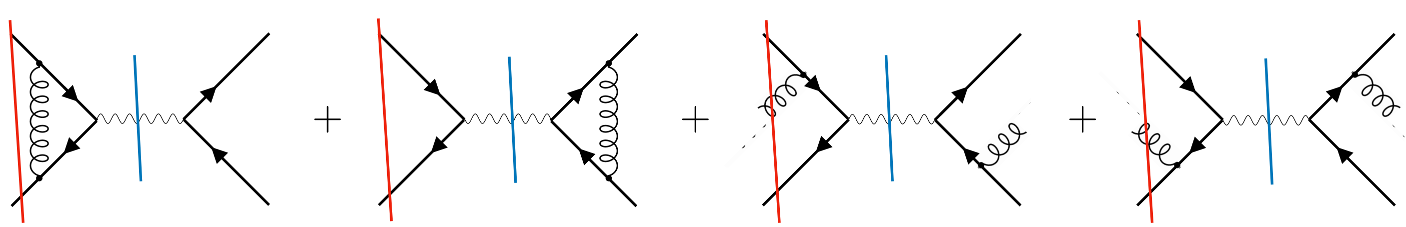
But how do we compute all this?



Up to redefinition of observables, this is equivalent to



So we can use **Local Unitarity**!



$$= \int \mathrm{d}\Pi \, f_{\mathrm{LU}}$$

Where the integrand is locally finite! We can Monte Carlo integrate it.

What can we do?

- Take the limit $d_\perp \to 0$ **analytically** and obtain **exact back to back jets**

$$P_i^{\text{j}} = ((P_i^{\text{j}})^0, 0, 0, (P_i^{\text{j}})^3)$$

This allows us to define **Bjorken variables**

$$x_1 = \frac{(P_1^{\text{j}})^0 + (P_1^{\text{j}})^3}{2} \qquad x_2 = \frac{(P_2^{\text{j}})^0 - (P_2^{\text{j}})^3}{2}$$

For a 2 to N diagram this reproduces the parton model

- Bin the distribution in the Bjorken variables $\longrightarrow$ **Fit PDFs!**

  (not in MS bar)

- Vary the factorisation scale Λ_c and interpolate the dependence on the factorisation scale

Numerical resummation Banfi, Salam, Zanderighi, arXiv:0407286 (2004)

Hadronic cross-sections

We started with a very generic formalism for scattering

$$\sigma(HH \to X + n\,\mathrm{j}) = \sum_m \int \left[\prod_{i=1}^{m} \mathrm{d}^3\vec{p}_i\right] f(p_1, \ldots, p_m) \; \frac{\mathrm{d}^m\sigma}{\mathrm{d}p_1 \ldots \mathrm{d}p_m}(p_1, \ldots, p_m \to X + n\mathrm{j})$$

Sum over number of initial state partons

Integration over initial state partons momenta

Weight

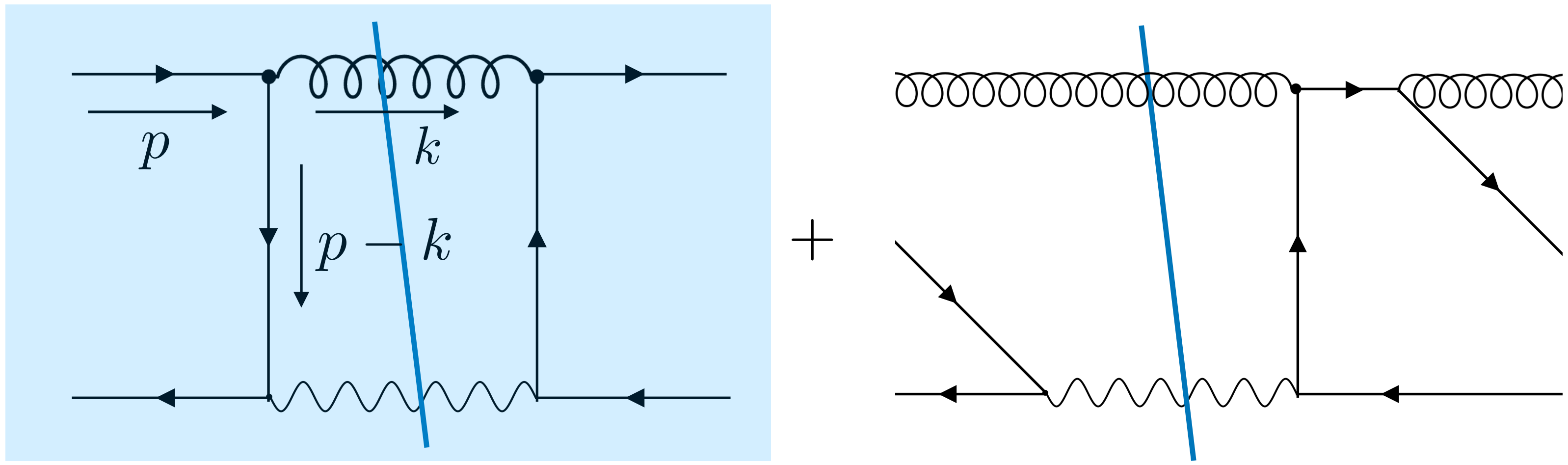
Cross-sections for m initial state partons

In the end, we arrive to

$$\sigma(HH \to X + n\,\mathrm{j}) = \int \mathrm{d}x_1 \mathrm{d}x_2 \; f(x_1, \Lambda_\mathrm{c}) \; f(x_2, \Lambda_\mathrm{c}) \; \frac{\mathrm{d}^2\sigma_p}{\mathrm{d}x_1 \mathrm{d}x_2}(2\,\mathrm{j} \to X + n\,\mathrm{j}, \Lambda_\mathrm{c})$$

In a sense, we get the renormalised result to begin with!

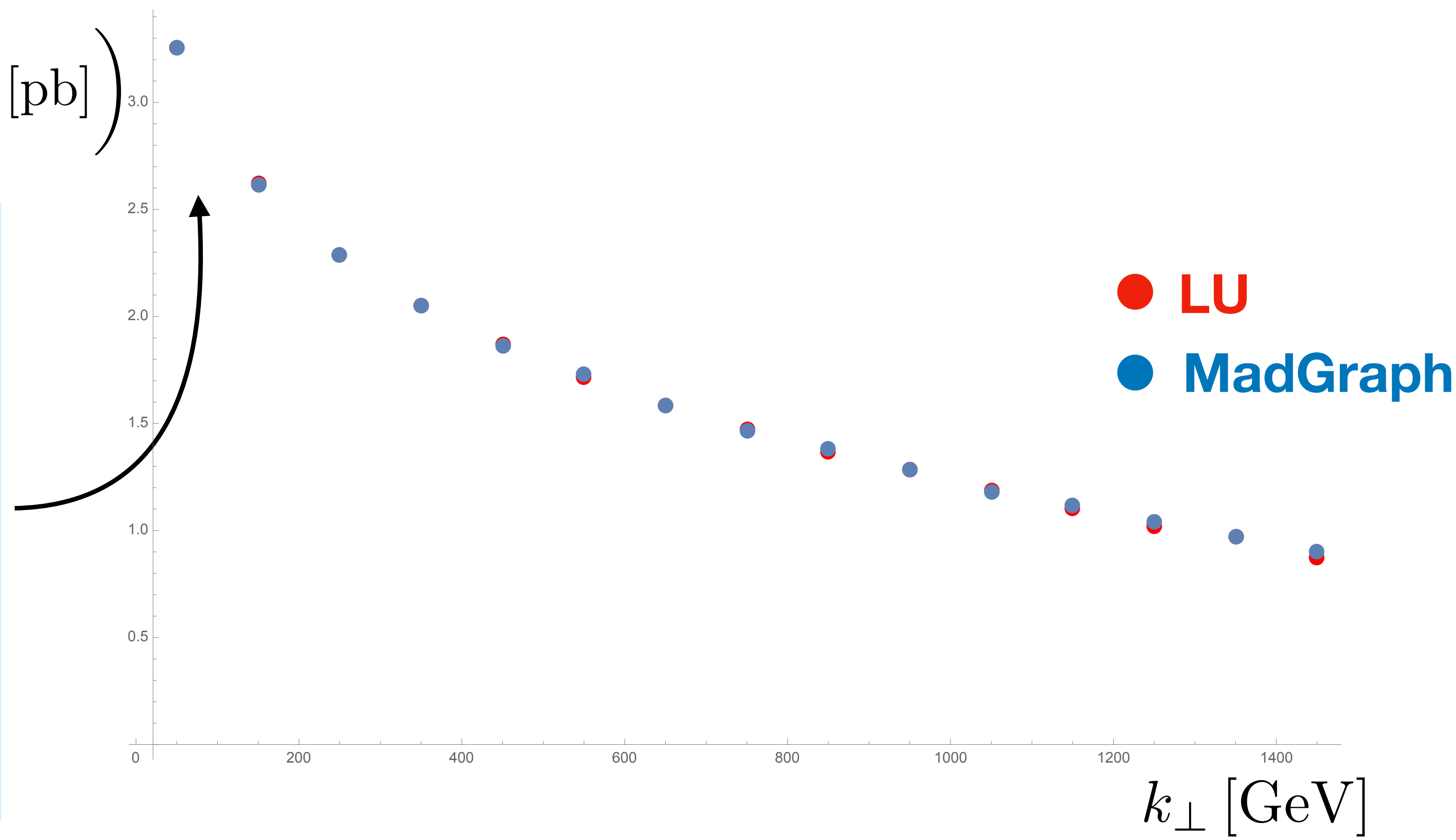
Let us look specifically at:



We integrate the “usual” diagram and check that it matches with traditional computations

And bin the **transverse momentum distribution of the gluon (jet)**

$$\log_{10}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp}\,[\mathrm{pb}]\right)$$



Singularity at zero transverse momentum

$$k_\perp \to 0$$

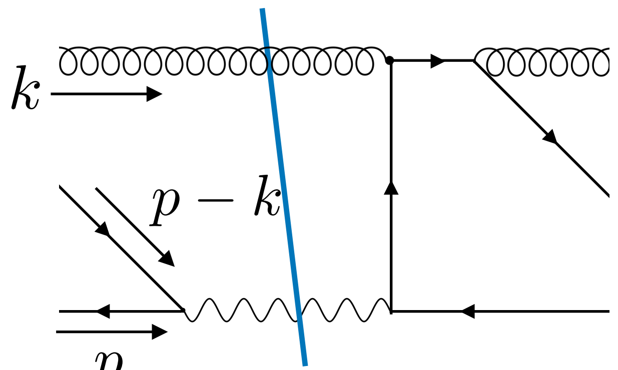
Since

$$p_\perp = 0$$

This implies

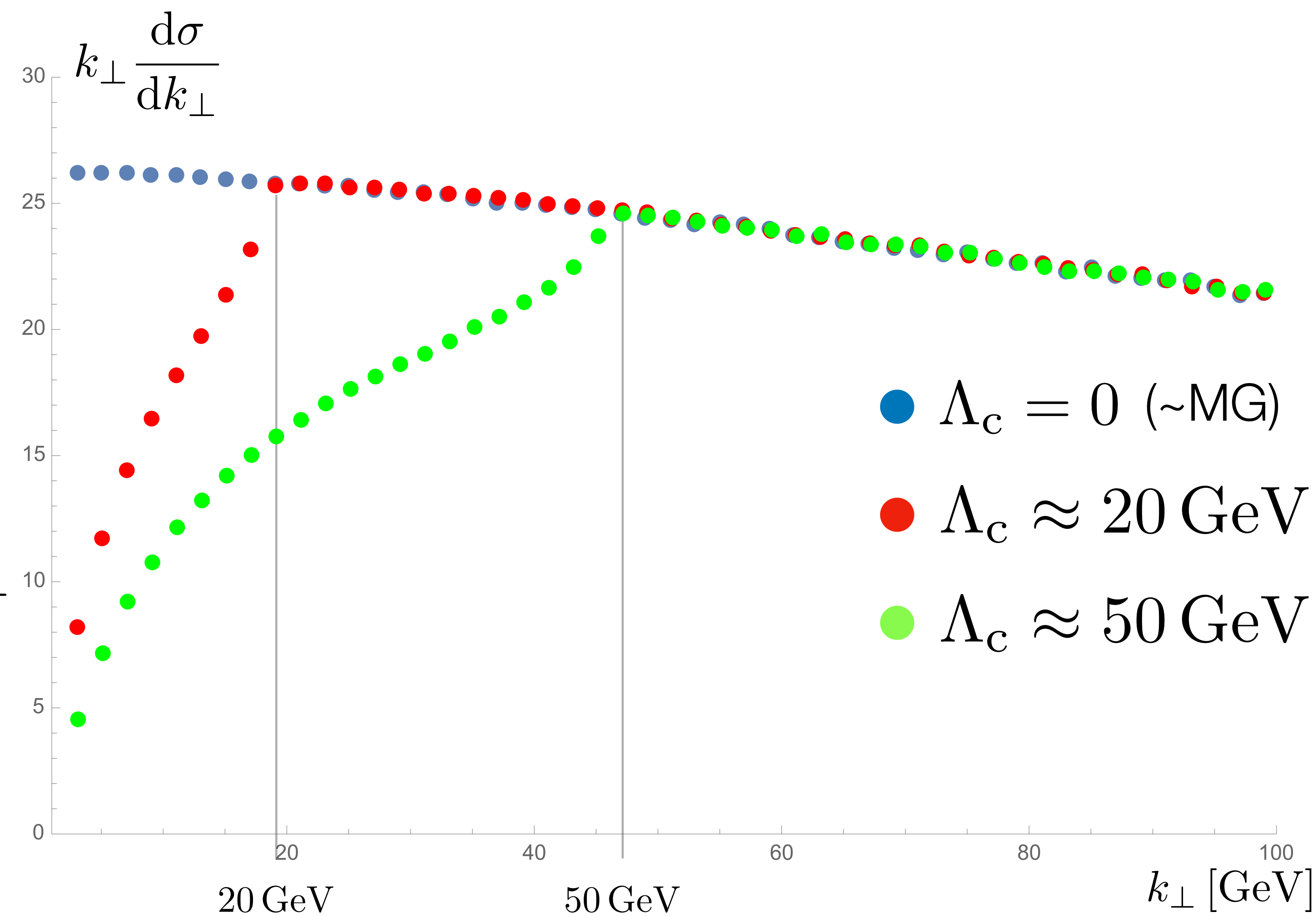
$$k = xp$$

Now include the other graph



Distribution is finite for any $k_\perp$

We have to choose Λ_c



First observation: for $\Lambda_c > 50\,\mathrm{GeV}$ the distribution does not change anymore

$(p-k)_\perp,\, p_\perp < m_Z \quad k_\perp < m_Z$ due to energy/momentum conservation

Highest separation of two partons in a jet is of **order of Z mass**

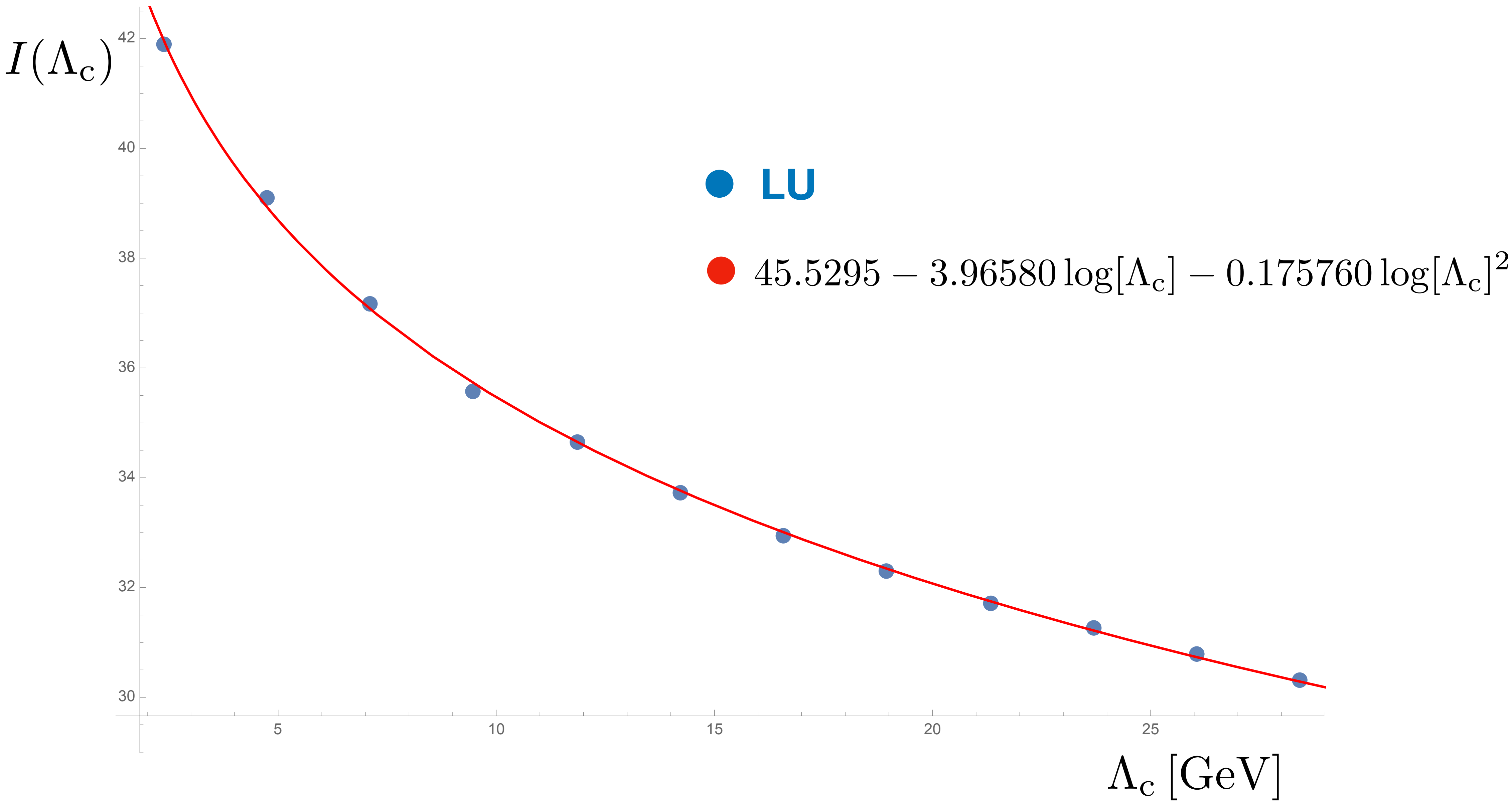
Factorisation scale has natural range choice associated to process scale

Scale dependence

Consider now the integral

$$I(\Lambda_c) = \int_0^{100} \mathrm{d}k_\perp \frac{\mathrm{d}\sigma}{\mathrm{d}k_\perp}(\Lambda_c)$$

as a function of the factorisation scale



Local Unitarity is a fully **local** and **generic** paradigm

- **IR singularities** regulated by realising **KLN** locally
- **Thresholds** regulated using **local deformation**
- **Local UV** and **renormalisation fully automated**

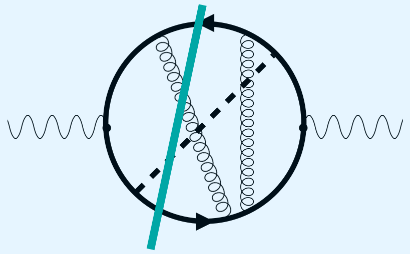
It forces to **unify** different aspects of fixed-order computations

- **Phase-space/Loop integrals**
- **Initial State Singularities/Final State Singularities**

It is particularly suitable to numerical integration!

- Take advantage of the **robustness** of **MonteCarlo methods**
- **Full automation underway**
- **Evaluation speed** and **convergence** constitute **important challenges**

Future application: $e^+e^- \to t\bar{t}H$ $\mathrm{N^2LO}$ **cross-section**



Selected reads

LTD/cLTD

Catani, Gleisberg, Krauss, Rodrigo, Winter
arXiv: 0804.3170 (2008)

Bierenbaum, Catani, Draggiotis, Rodrigo
arXiv: 1007.0194 (2010)

Runkel, Ször, Vesga, Weinzierl
arXiv: 1902.02135 (2019)

ZC, Hirschi, Kermanshah, Ruijl
arXiv: 1906.06138 (2019)

ZC, Hirschi, Kermanshah, Pelloni, Ruijl
arXiv: 2009.05509 (2020)

Verdugo, Hernandez-Pinto, Rodrigo, Sborlini et al.
arXiv: 2010.12971 (2020)

KLN for ISS

Kinoshita, "Mass singularities of Feynman amplitudes" (1962)

Lee, Nauenberg, "Degenerate systems and mass singularities" (1963)

Khalil, Abdullah, Horowitz,
arXiv:1701.00763 (2017)

Frye, Hannesdottir, Paul, Schwartz, Yan
arXiv:1810.10022 (2019)

Local Unitarity

Soper,
arXiv: 9804454 (1998)

Soper,
Beowulf (pages.uoregon.edu/soper/beowulf/)

ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

Contour Deformation

Gong, Nagy, Soper
arXiv:0812.3686 (2009)

Becker, Reuschle, Weinzierl
arXiv:1010.4187 (2010)

Becker, Götz, Reuschle, Schwan, Weinzierl
arXiv:1111.1733 (2011)

Buchta, Chachamis, Draggiotis, Rodrigo
arXiv:1510.00187 (2017)

ZC, Hirschi, Kermanschah, Pelloni, Ruijl
arXiv: 1510.00187 (2019)

Subtraction of loop integrals

Becker, Reuschle, Weinzierl
arXiv:1010.4187 (2010)

Anastasiou, Sterman
arXiv:1812.03753 (2018)

Yao Ma
arXiv:1910.11304 (2019)

Anastasiou, Haindl, et al.
arXiv:2008.12293 (2020)