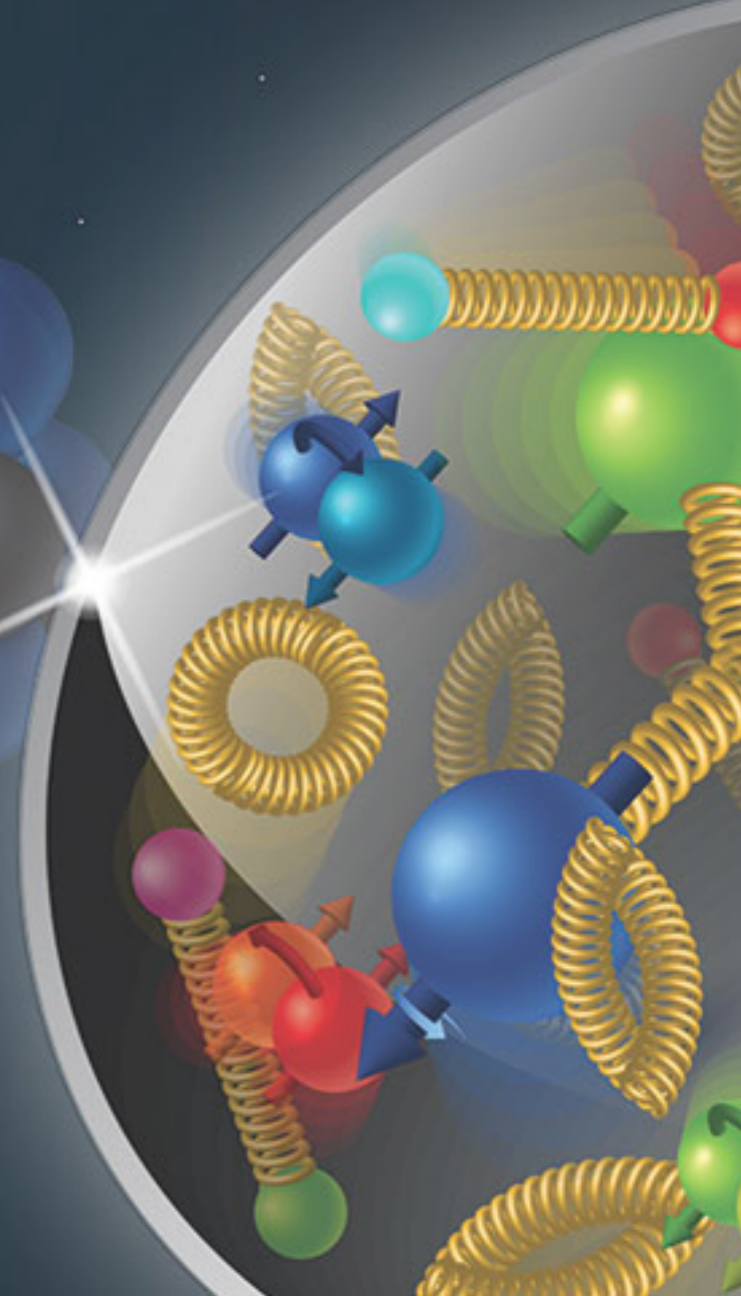
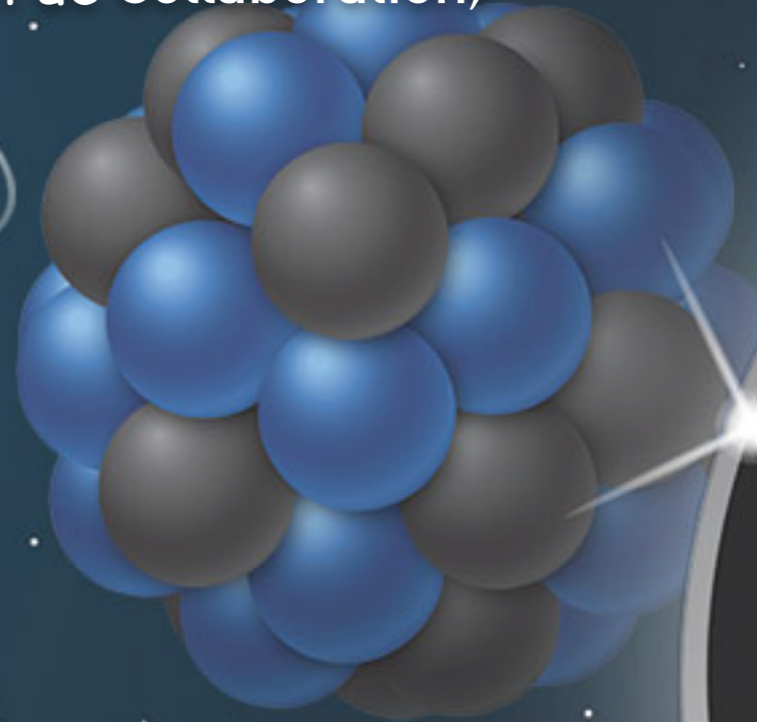
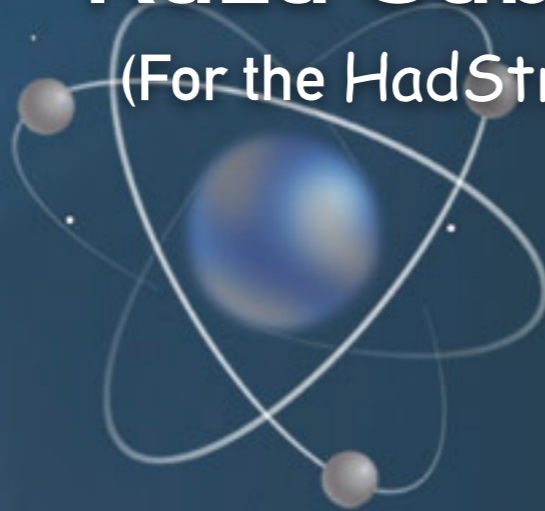


Gluon distribution in the nucleon from Lattice QCD

Raza Sabbir Sufian

(For the HadStruc Collaboration)



LaMET 2021 @ CNF



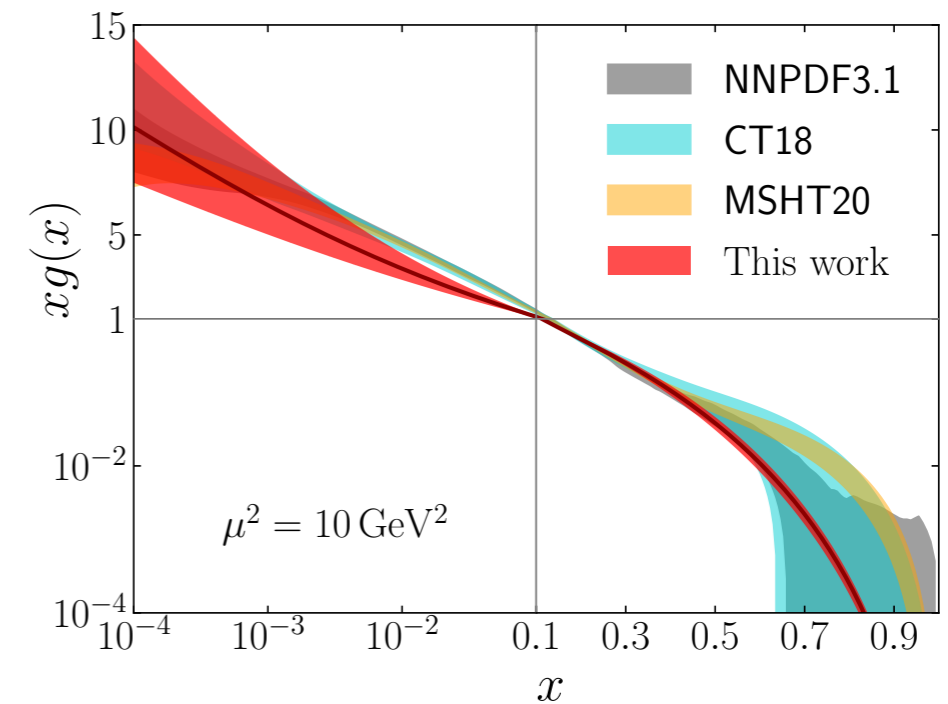
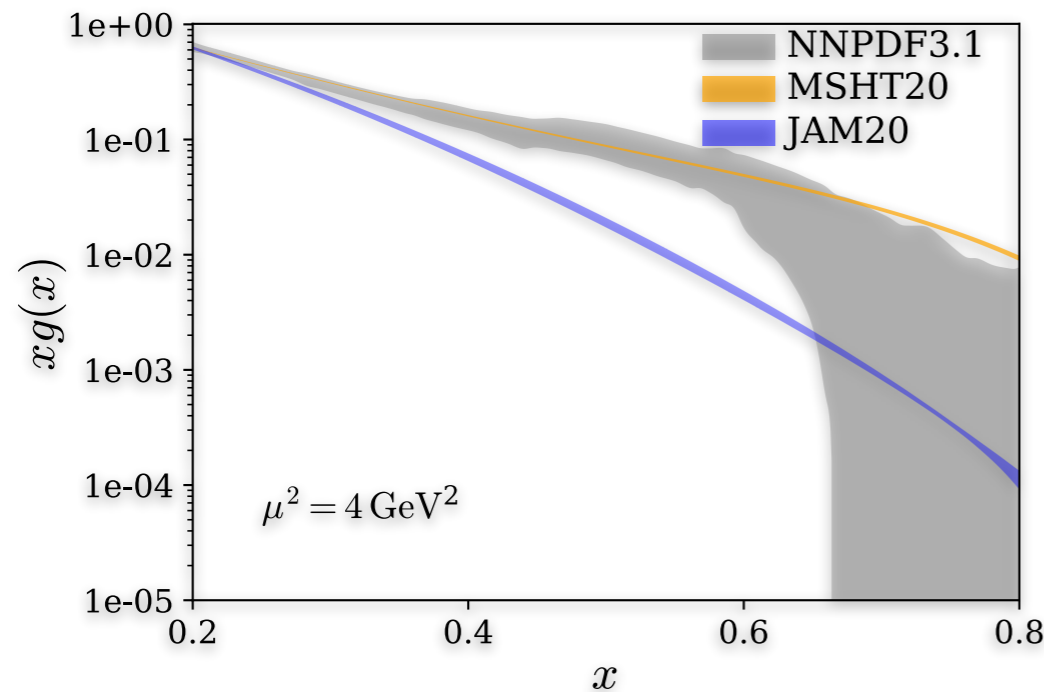
WILLIAM & MARY

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Jefferson Lab
Thomas Jefferson National Accelerator Facility

Gluon distributions and lattice QCD

- Gluon PDF is less explored in LQCD calculations and there is difference between PDF fits

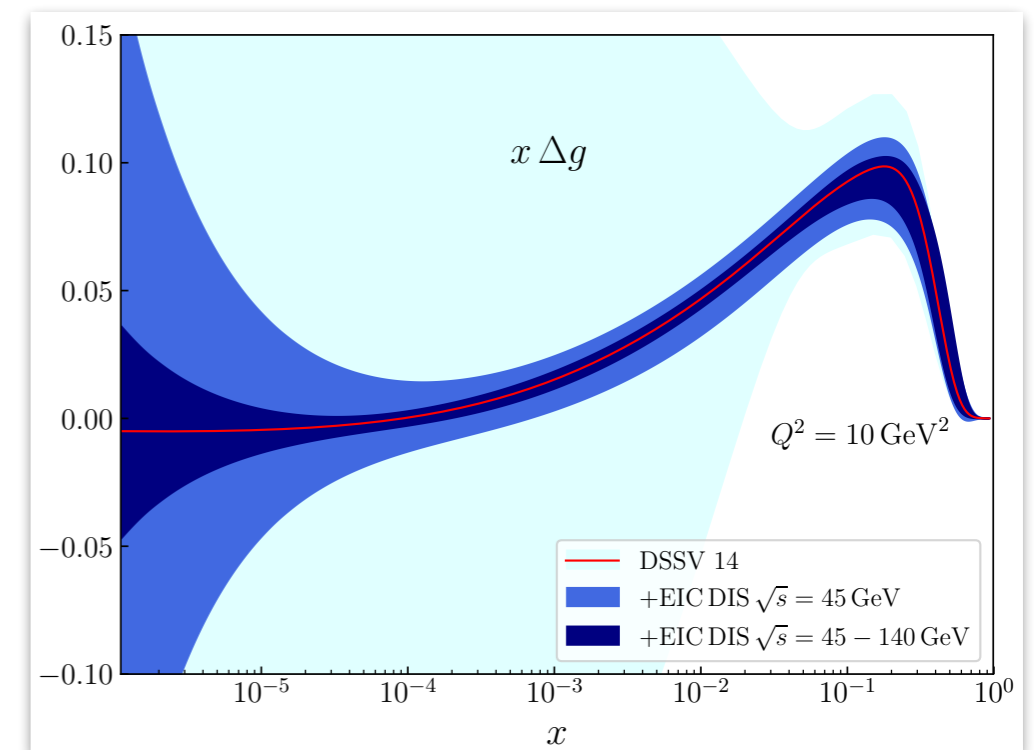


de Teramond, Dosch, Liu, RSS, Brodsky, Deur
PRD 2021

- Origin of proton spin : A BIG question

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma(\mu) + \Delta G(\mu) + L_{Q+G}(\mu)$$

- Gluon contribution to proton spin is not well-constrained from experiment



Gluon distribution in Pseudo-PDF approach

- On the lattice, calculate **spatial** correlation in **coordinate space**

$$M_{\mu\alpha;\lambda\beta}(z, p) \equiv \langle p | G_{\mu\alpha}(z) [z, 0] G_{\lambda\beta}(0) | p \rangle \quad \text{X. Ji [PRL 2013]}$$

- “Pseudo-PDF” formalism: Based on coordinate-space factorization

Radyushkin [PLB 2017]

- Renormalization: $\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}} \right) / \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0, z=0}} \right)$



Reduced Ioffe-time distribution

Radyushkin [PLB 2017]
Orginos, et al [PRD 2017]
Joo, et al [JHEP 2019]

- Ioffe time, $\nu = p_z z$ (convention from Braun, et al [PRD 1995])

Gluon distribution in Pseudo-PDF approach

- To determine unpolarized gluon distribution

$$M_{0i;i0} = \langle p | G_{0i}(z) [z, 0] G_{i0}(0) | p \rangle = 2 p_0^2 \mathcal{M}_{pp} + 2 \mathcal{M}_{gg}$$

$$M_{ji;ij} = -2 \mathcal{M}_{gg} \quad \boxed{i, j \rightarrow x, y}$$

$$M_{0i;i0} + M_{ji;ij} = 2 p_0^2 \mathcal{M}_{pp}$$

► Combination is multiplicatively renormalizable

Balitsky, et al [PLB 2020]

► Also see

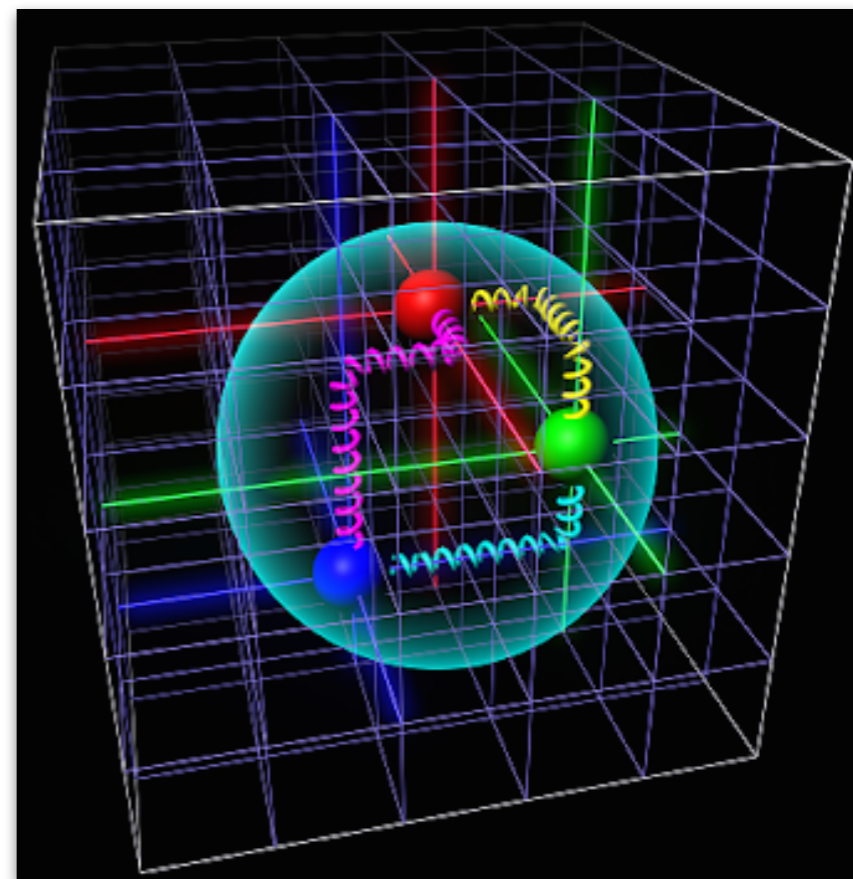
Zhang, et al [PRL 2019] & Li, et al [PRL 2019]

- After renormalization and perturbative matching

$$\mathcal{M}_{pp}(\nu, z^2) \rightarrow \mathcal{I}_g(\nu, \mu^2) = \int_0^1 dx \cos(x\nu) x g(x, \mu^2)$$

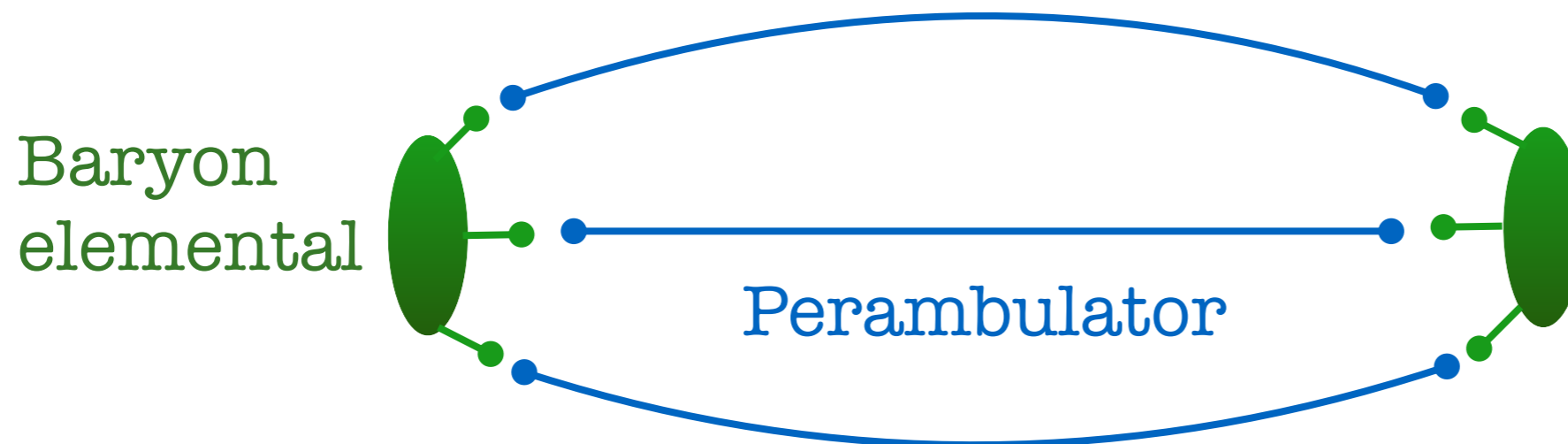
Lattice QCD calculation

- 2+1 flavor clover Wilson fermions
 - Lattice size, $L \times T = 32^3 \times 64$
 - Lattice spacing, $a \approx 0.094$ fm
 - Pion mass, $m_\pi = 358$ MeV
 - 349 configurations
- ↓
- **1899 configurations**



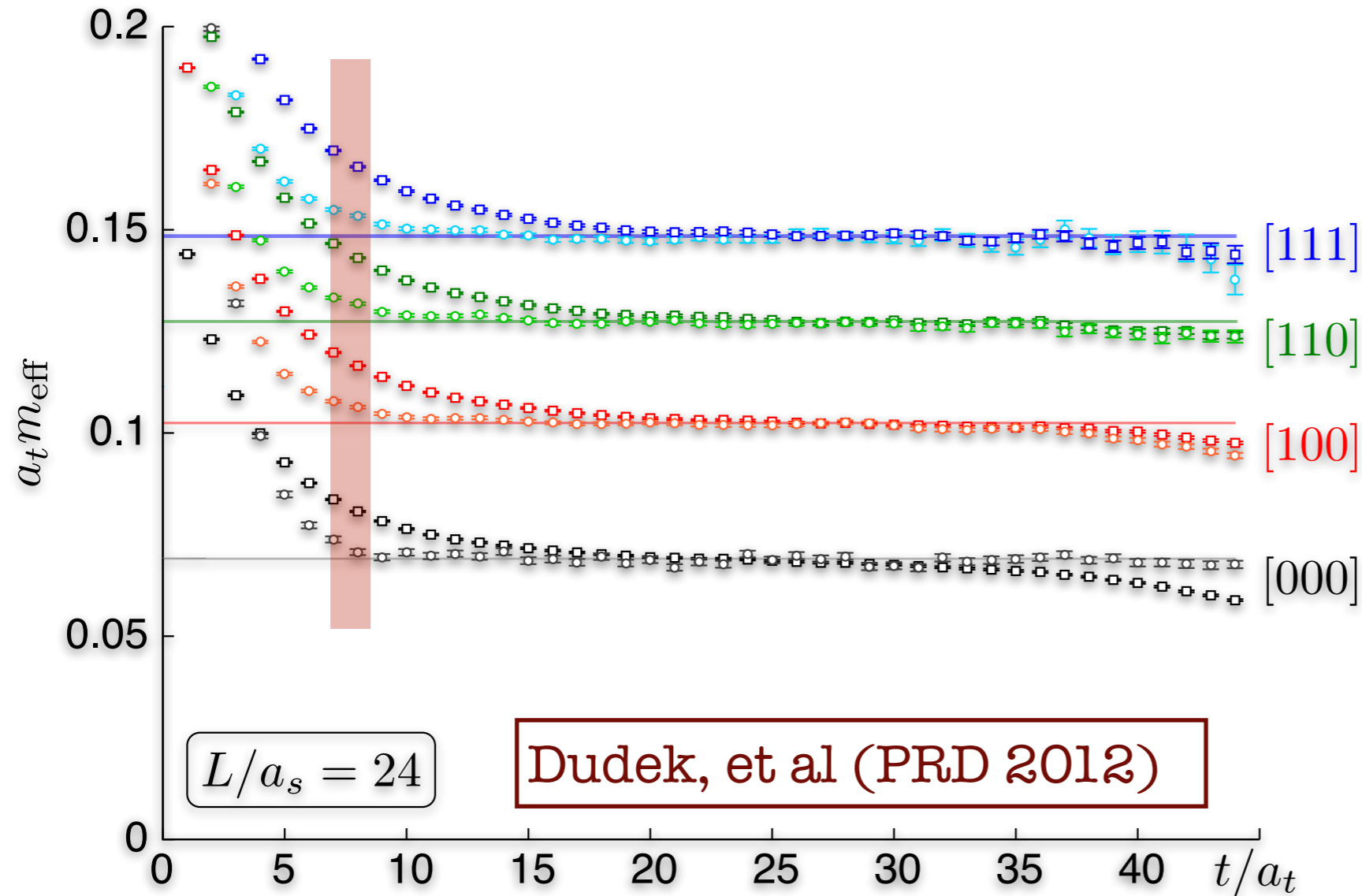
Special features: optimized operators and nucleon correlator

- Nucleon correlation function using “Distillation” Peardon, et al [PRD 2009]
- Distillation : low rank approximation of gauge covariant smearing kernel - increases operator state overlaps onto low-lying modes
- Elementals encode the choice of nucleon operator [7-9 operators]
- Perambulators encode quark propagation



Features of this calculation

● Basis of operators (positive/negative parity, hybrid, higher spin)



had spec

● Gluonic operator using “Wilson flow”

M. Luscher, JHEP 2010

- Flow of gauge field, $B_\mu(\tau, x_\mu)$ so that $B_\mu|_{\tau=0} = A_\mu$
- Diffusion length in x is $\sqrt{8\tau}$ ($\tau \sim a^2$)

Extraction of matrix elements

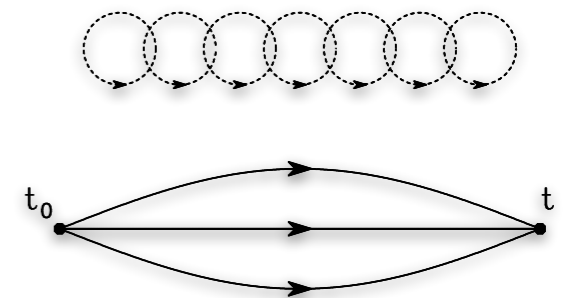
- Correlation matrix analysis using variational technique
 - ▶ System of generalized eigenvalue equations for correlation matrix
 - ▶ Orthogonality conditions on the eigenvectors of different states

Difficult to distinguish degenerate states by their time-dependence alone

- Use summed generalized eigenvalue problem (sGEVP)

J. Bulava, et al, JHEP 2012

- ▶ $C \exp(-\Delta E t / 2)$ (GEVP)
- ▶ $D t \exp(-\Delta E t)$ (sGEVP)

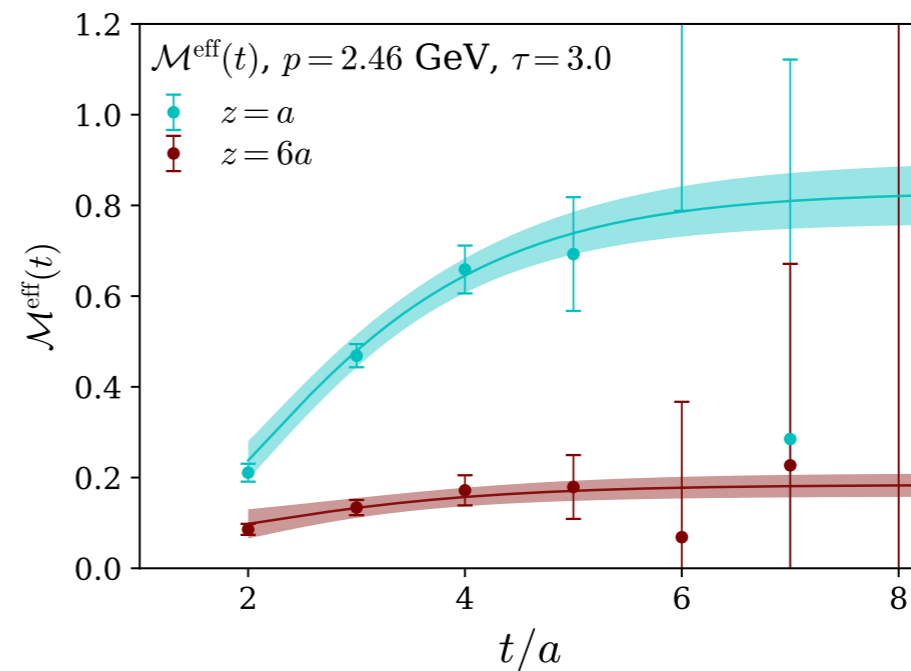
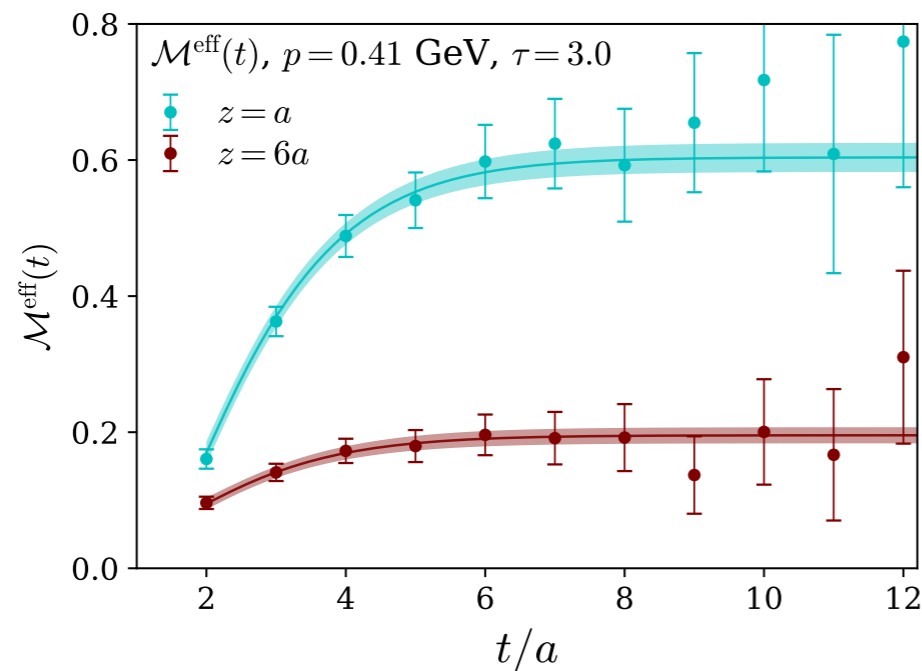
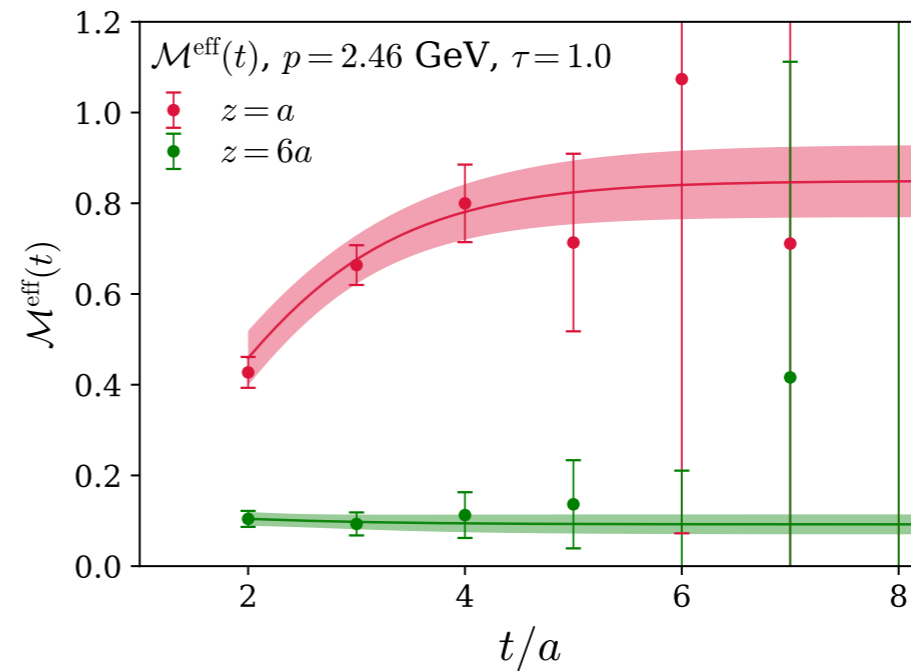
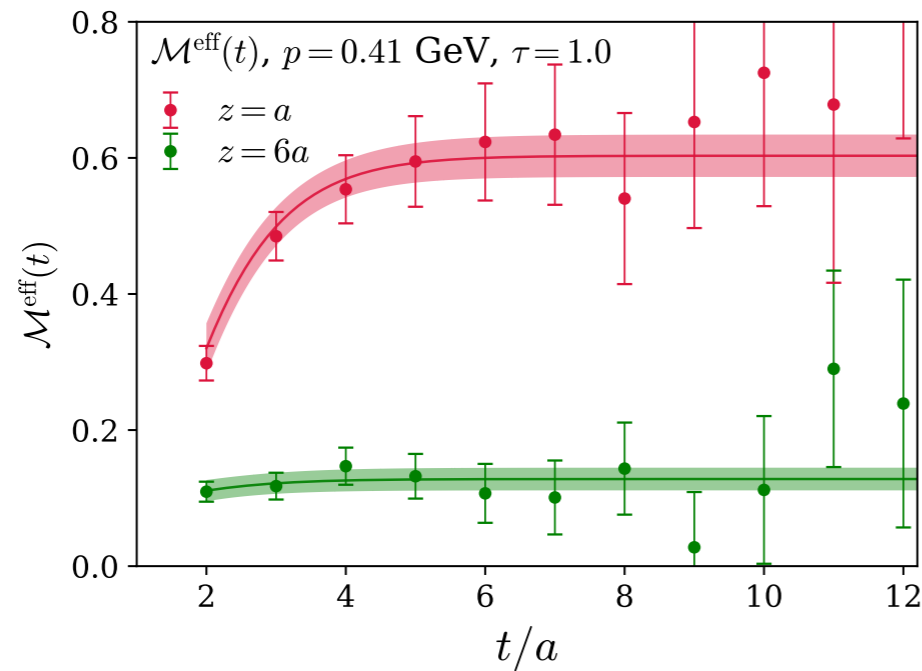


$$\mathcal{M}_{nn}^{\text{eff}}(t, t_0) = \mathcal{M}_{nn} + \mathcal{O}(\Delta E_{N+1,n} t \exp(-\Delta E_{N+1,n} t))$$

Lattice QCD matrix elements

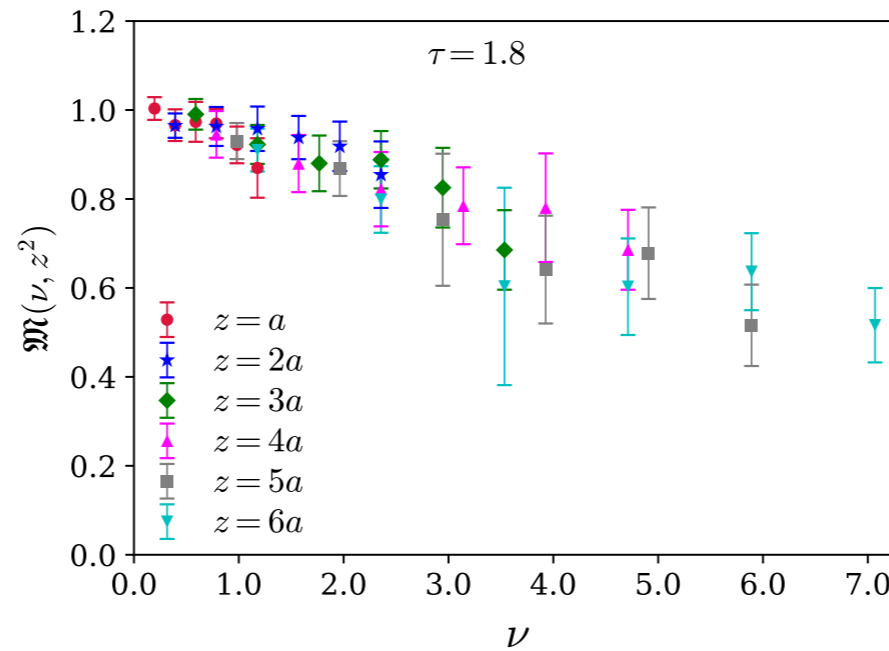
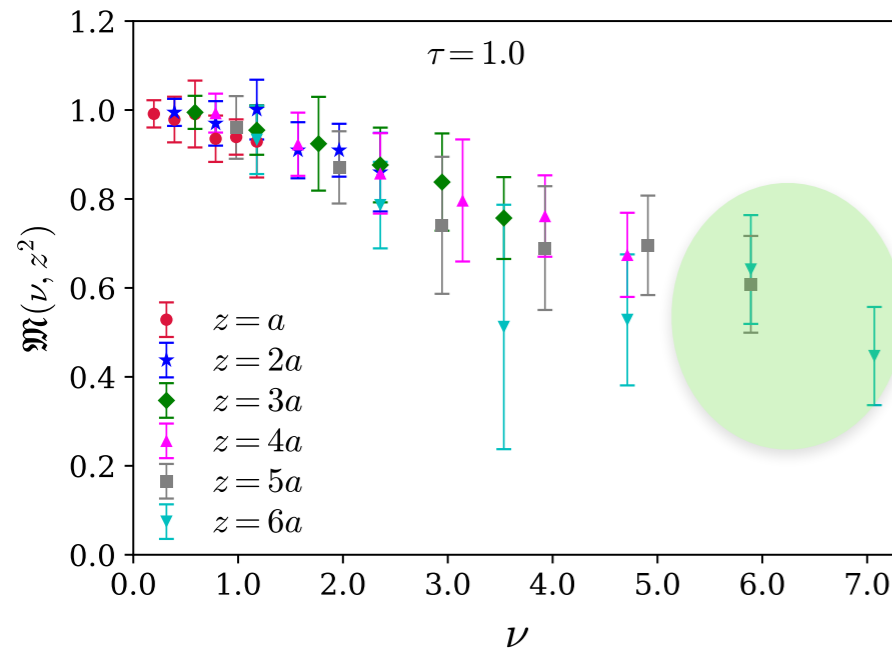
Simultaneous correlated fit to matrix elements for all
(fixed momentum & gradient flow)

$$\mathcal{M}^{\text{eff}}(t)_i = A_i + B_i t \exp(-\Delta E t)$$



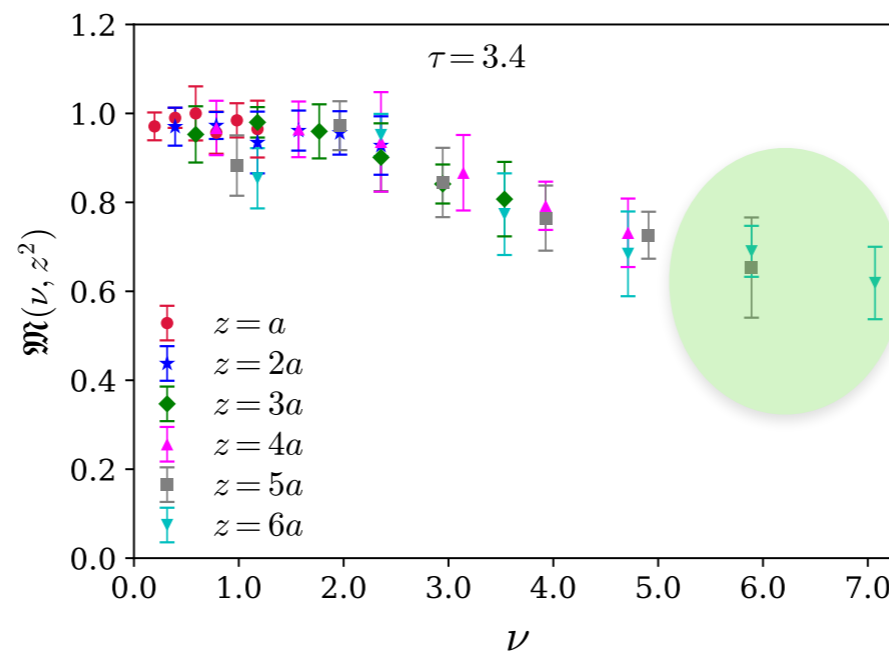
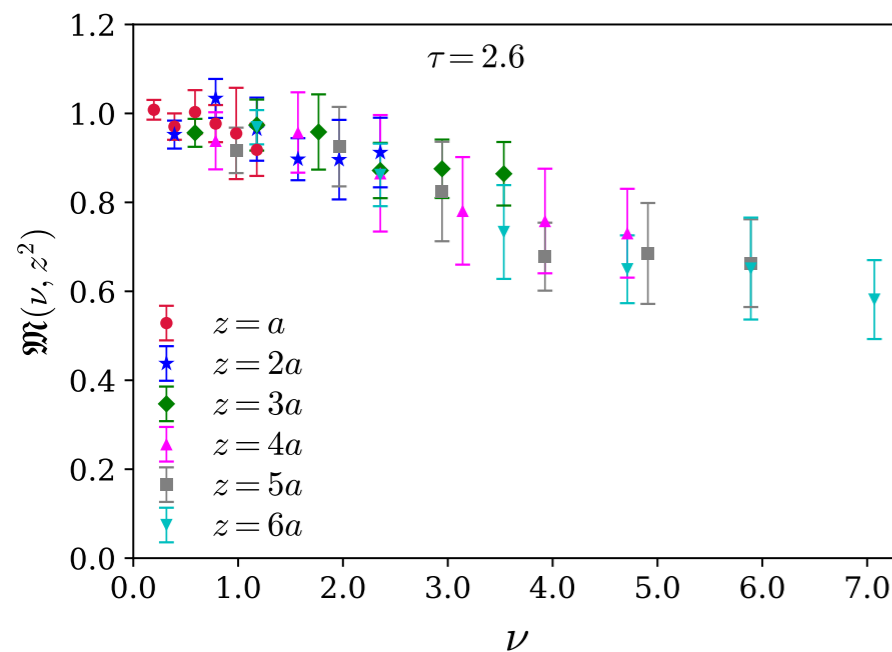
Lattice QCD rITD as a function of flow time

● Reduced ITD: $\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}} \right) / \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0, z=0}} \right)$



$p = 2.46 \text{ GeV}$

Before ratio, SNR = 6.5

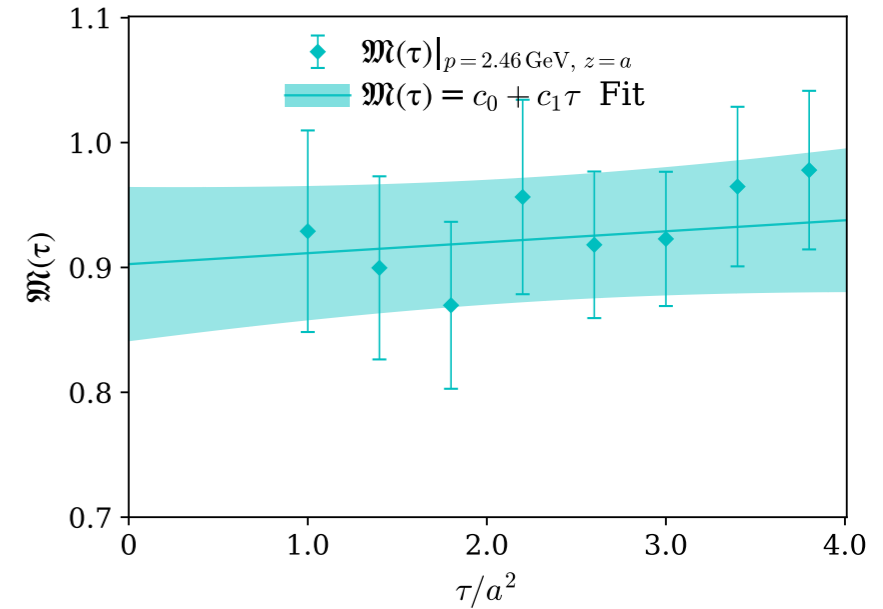
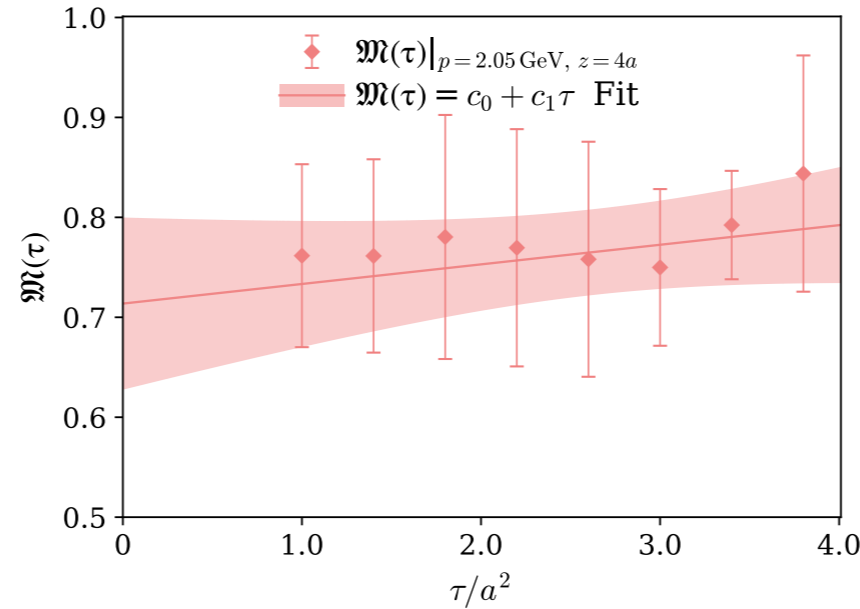
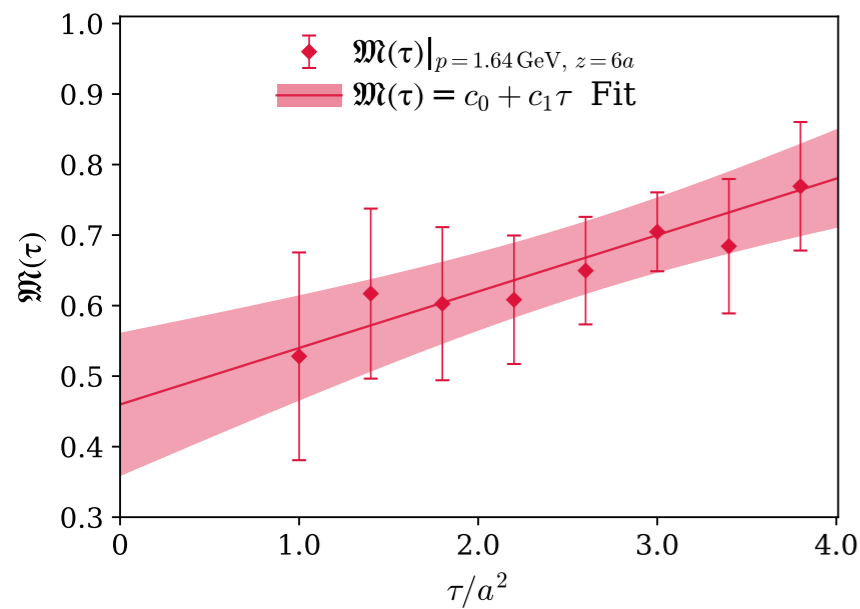
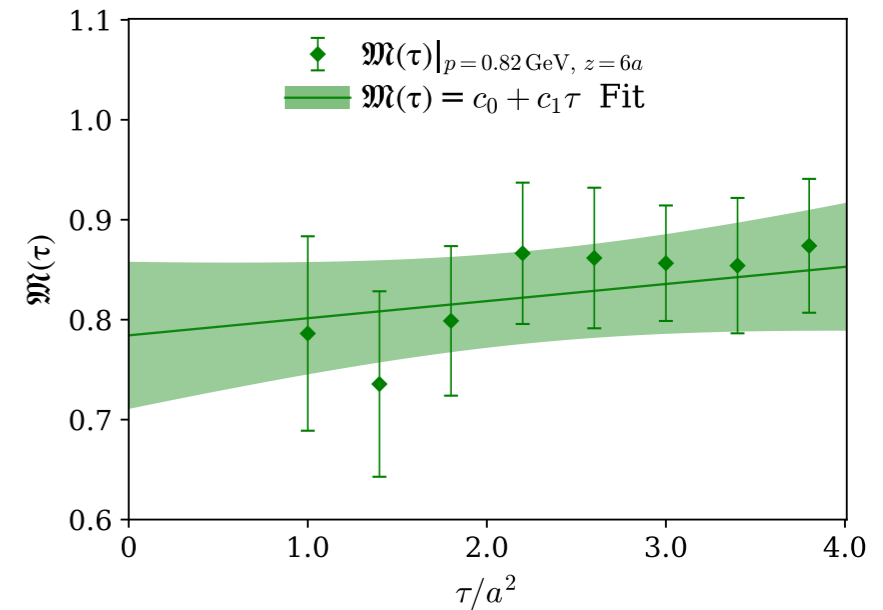
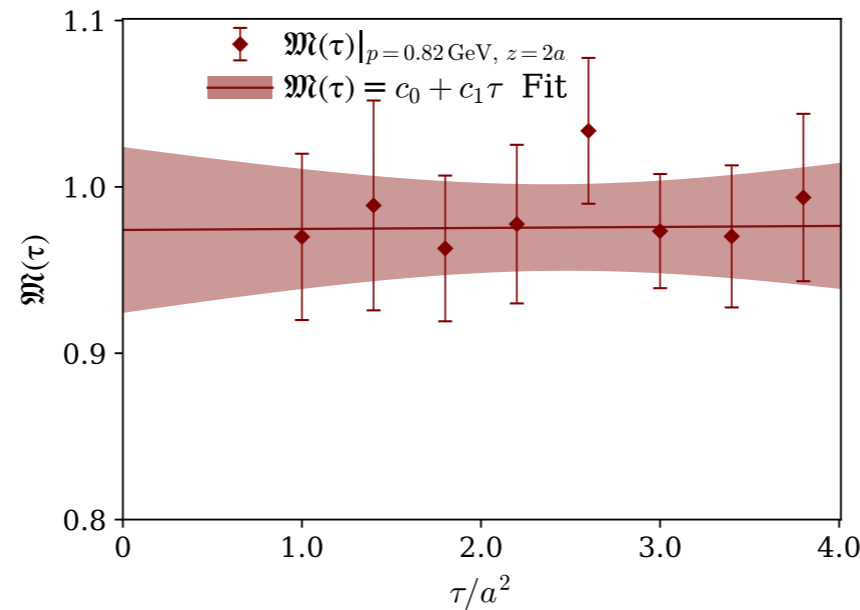
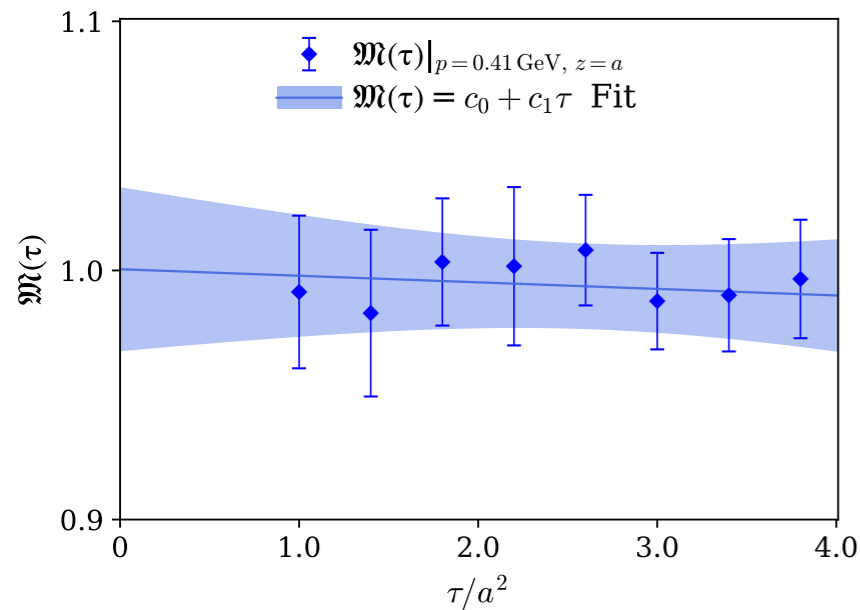


After ratio, SNR = 12.1

● Flow time dependence is minimized in the double ratio

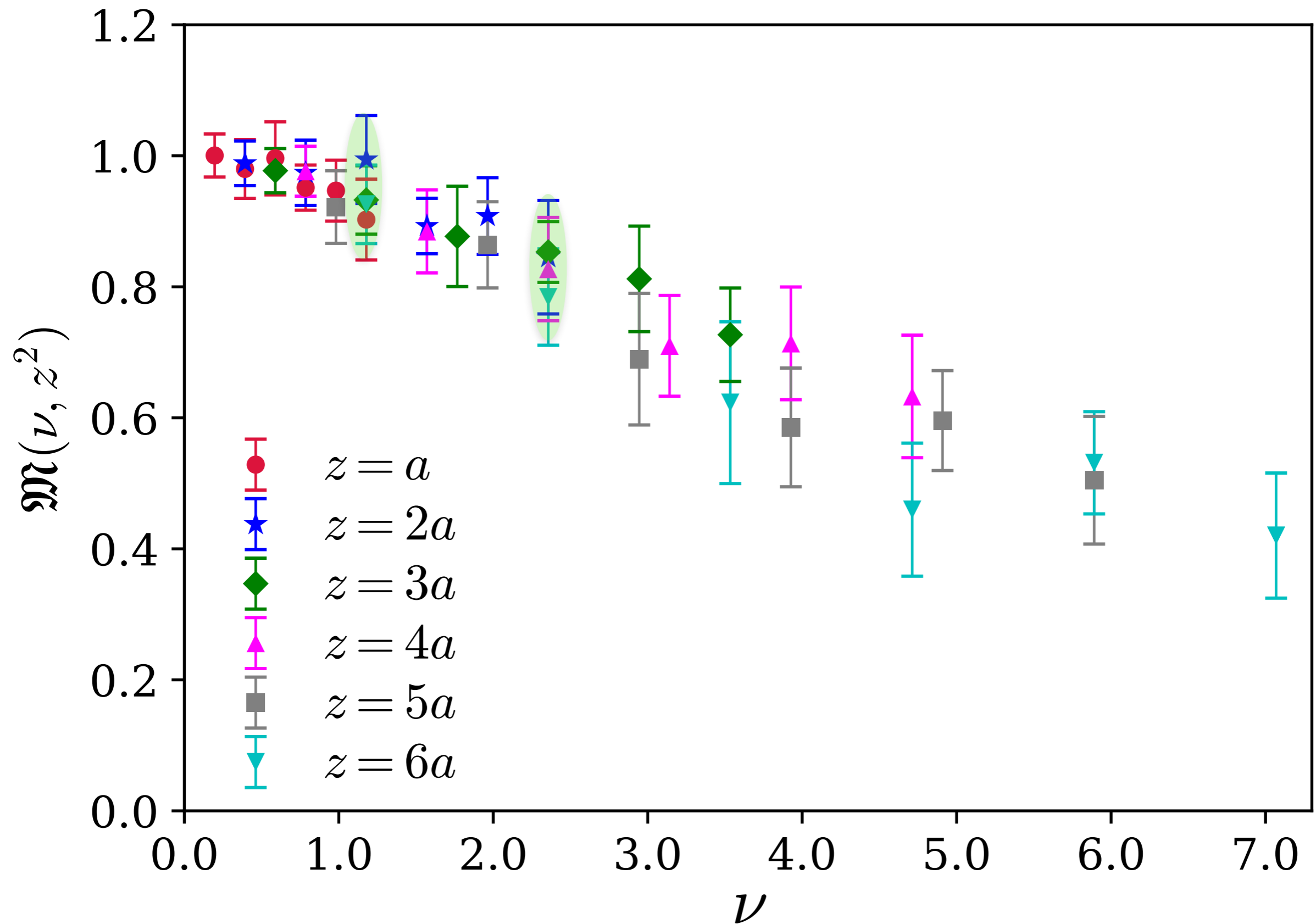
Zero flow time extrapolation of rITD (examples)

● For fixed p & z , fit forms: $A + B\tau$, $A + B\tau + C\tau^2$, etc.



Ioffe time distribution in the zero flow time limit

● Unpolarized gluon pseudo-ITD



Some phenomenology

Fit NNPDF3.1 gluon PDF using ansatz

$$xg^+(x) = x^\alpha [A(1-x)^{4+\beta} + B(1-x)^{5+\beta}] \times (1 + \gamma\sqrt{x} + \delta x)$$

$$xg^-(x) = x^\alpha [A(1-x)^{6+\beta} + B(1-x)^{7+\beta}] \times (1 + \gamma\sqrt{x} + \delta x)$$

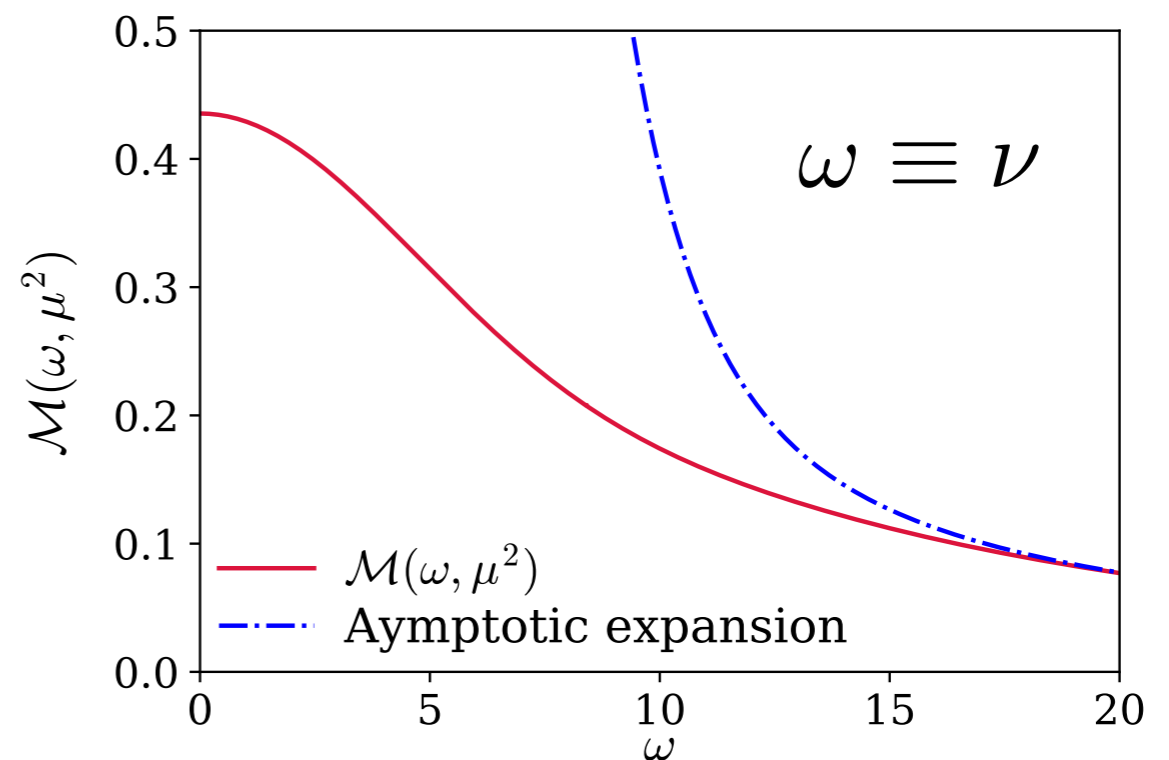
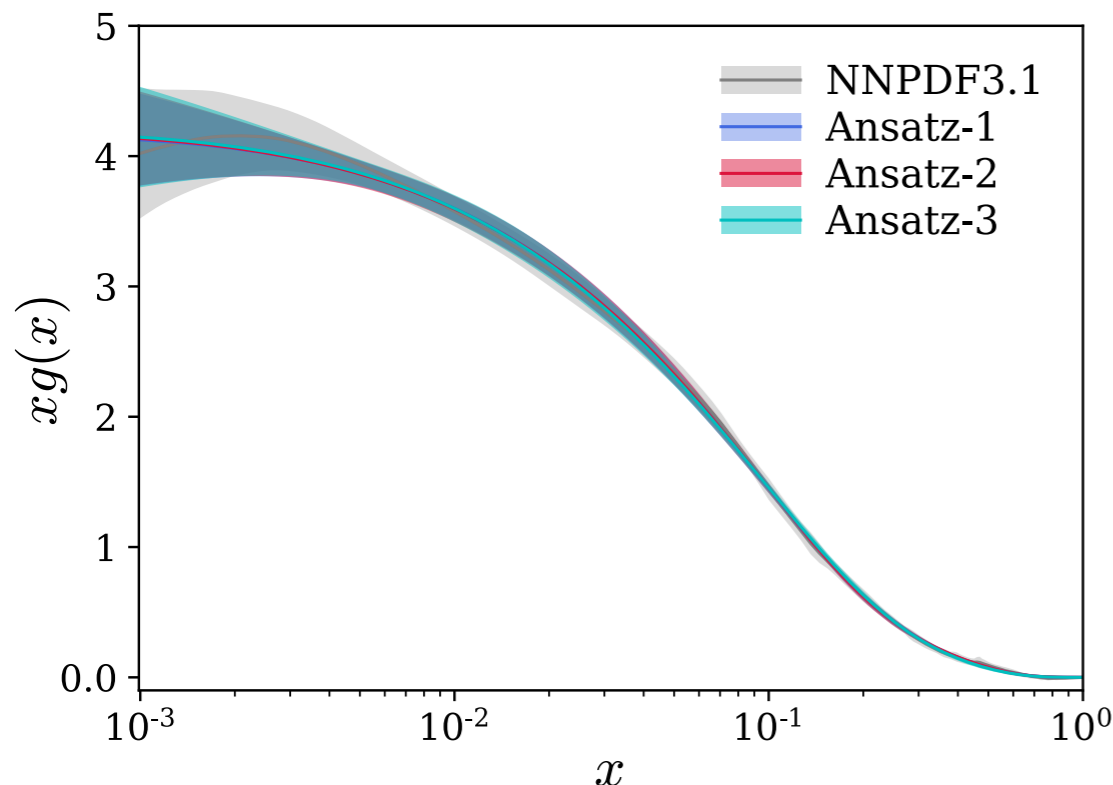
$$xg(x) \equiv xg^+(x) + xg^-(x)$$

$$x\Delta g(x) \equiv xg^+(x) - xg^-(x)$$

Brodsky, Burkardt, Schmidt [NPB 95]

RSS, Liu, Paul
PRD 2021

Asymptotic form: $\mathcal{M}(\omega, \mu^2) = A \left[\left(C_R(\alpha, 4 + \beta; \omega) \right. \right.$
 $\left. + \gamma C_R(\alpha + 1/2, 4 + \beta; \omega) + \delta C_R(\alpha + 1, 4 + \beta; \omega) \right)$
 $\left. + (\beta \rightarrow \beta + 2) \right] + B \left[\beta \rightarrow \beta + 1 \right] + \mathcal{O}(1/\omega^{a+R+1})$



From pseudo-distribution to light-cone distribution

PDF fits

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx \mathcal{K}(x\nu, \mu^2 z^2) \frac{x^\alpha (1-x)^\beta}{B(\alpha+1, \beta+1)}$$

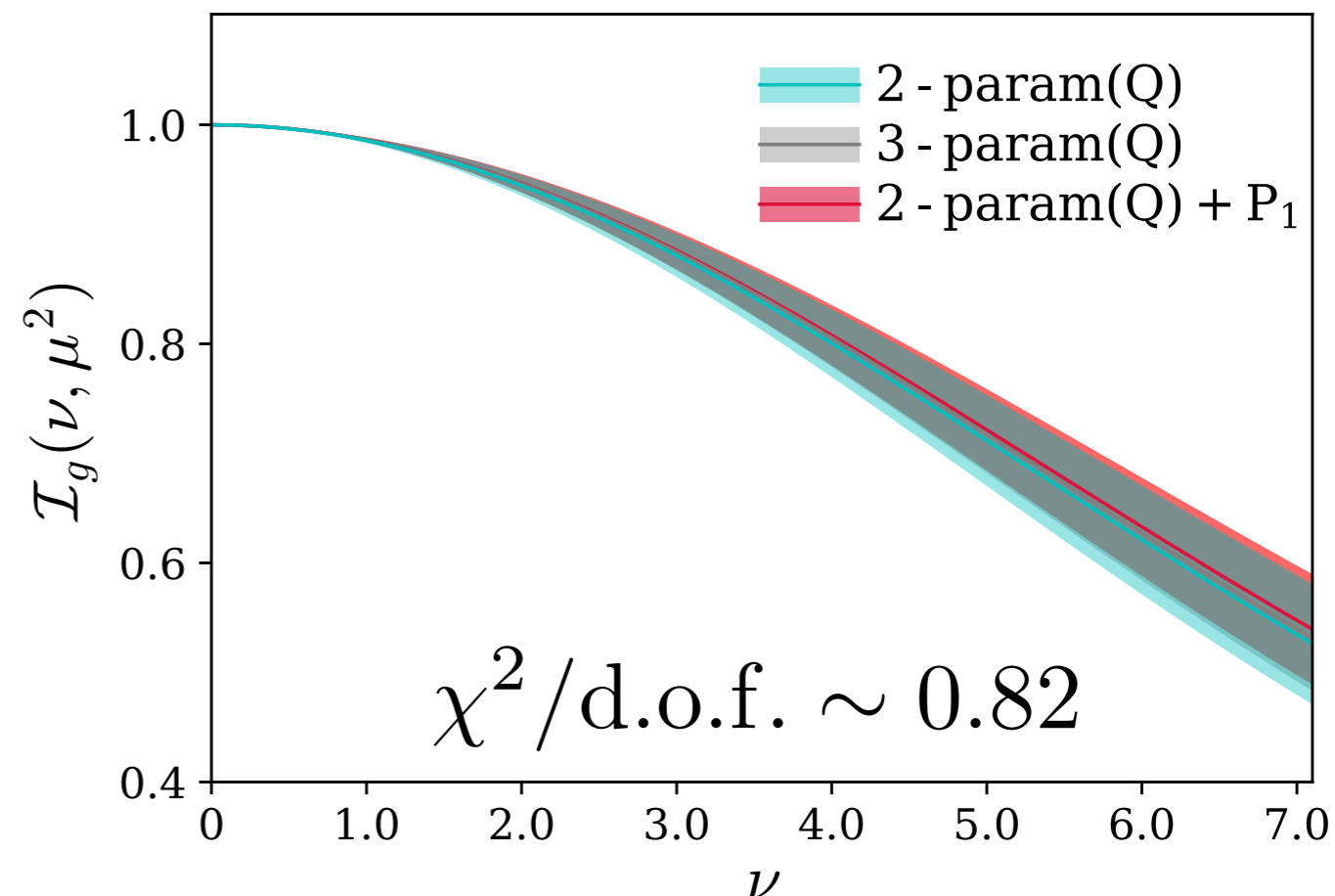
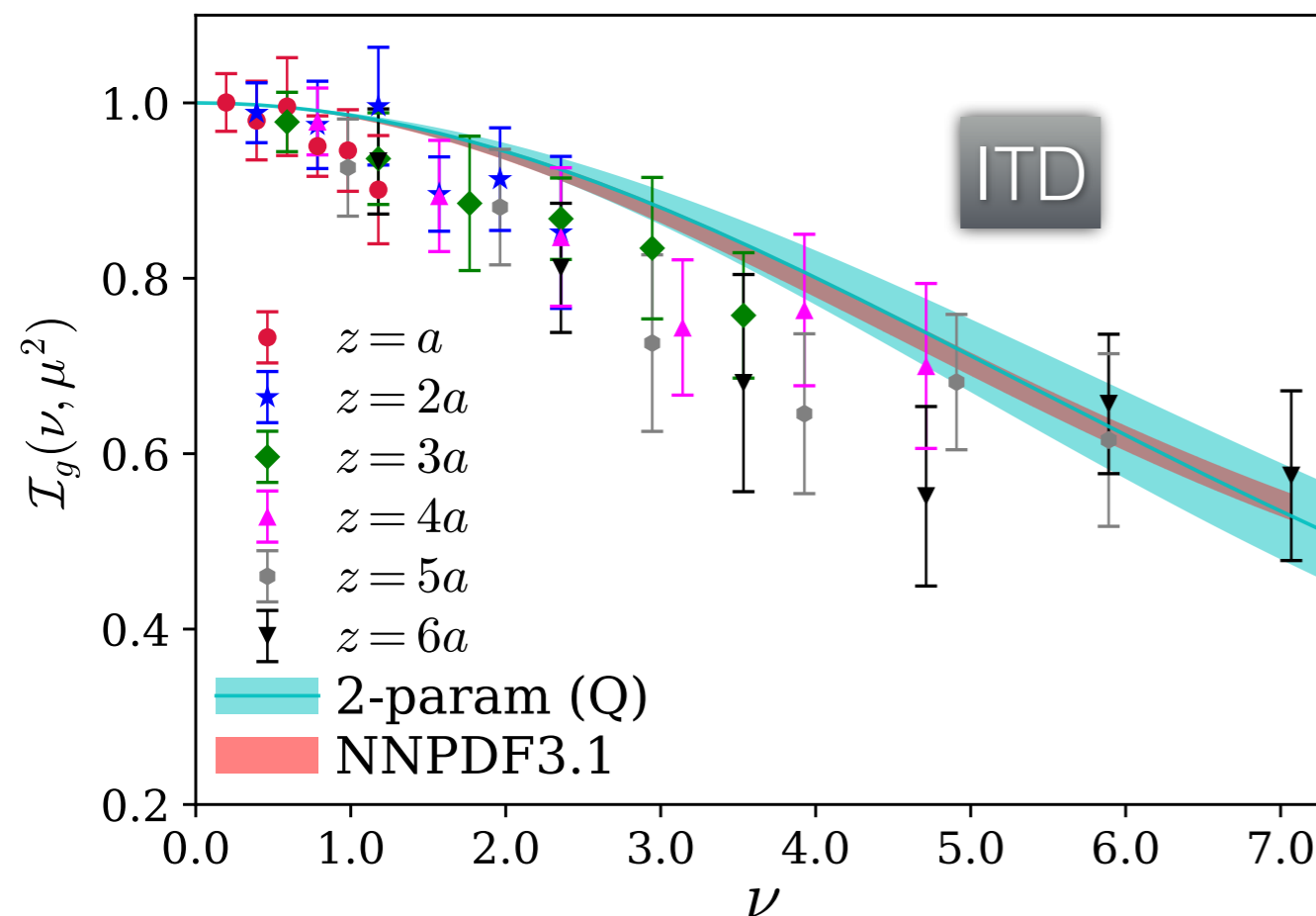
+ correction to 2-parameter form

+ discretization error

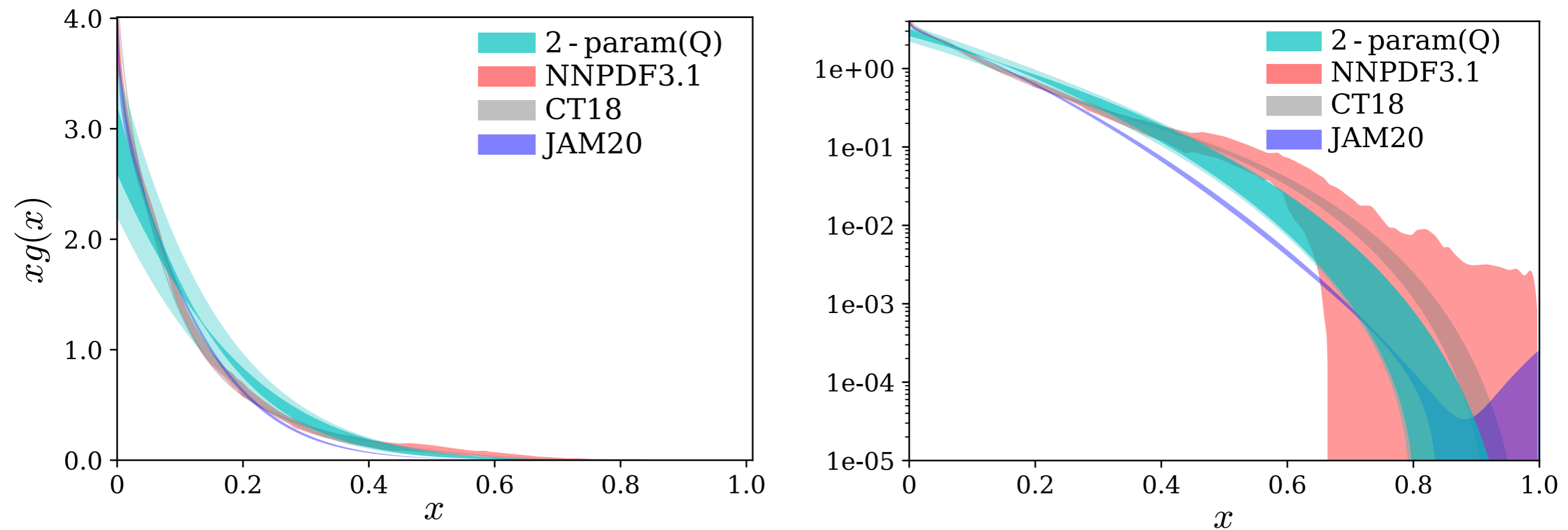
Karpie, et al [arXiv:2105.13313]

Perturbative matching
Balitsky, et al [PLB 2020]

*Talk by Nikhil Karthik
@ LaMET2021*



Determination of unpolarized gluon distribution



PHYSICAL REVIEW D **104**, 094516 (2021)



Unpolarized gluon distribution in the nucleon from lattice quantum chromodynamics

Tanjib Khan¹, Raza Sabbir Sufian^{1,2}, Joseph Karpie,³ Christopher J. Monahan,^{1,2} Colin Egerer,^{1,2} Bálint Joó,⁴ Wayne Morris,^{5,2} Kostas Orginos,^{1,2} Anatoly Radyushkin,^{5,2} David G. Richards,² Eloy Romero,² and Savvas Zafeiropoulos⁶

(On behalf of the *HadStruc Collaboration*)

Towards determining gluon helicity distribution

- For gluon helicity distribution

$$\Delta\mathcal{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p,s | G_{\mu\alpha}(z)[z,0] \tilde{G}_{\lambda\beta}(0) | p,s \rangle$$

- To determine **polarized** gluon Ioffe-time distribution

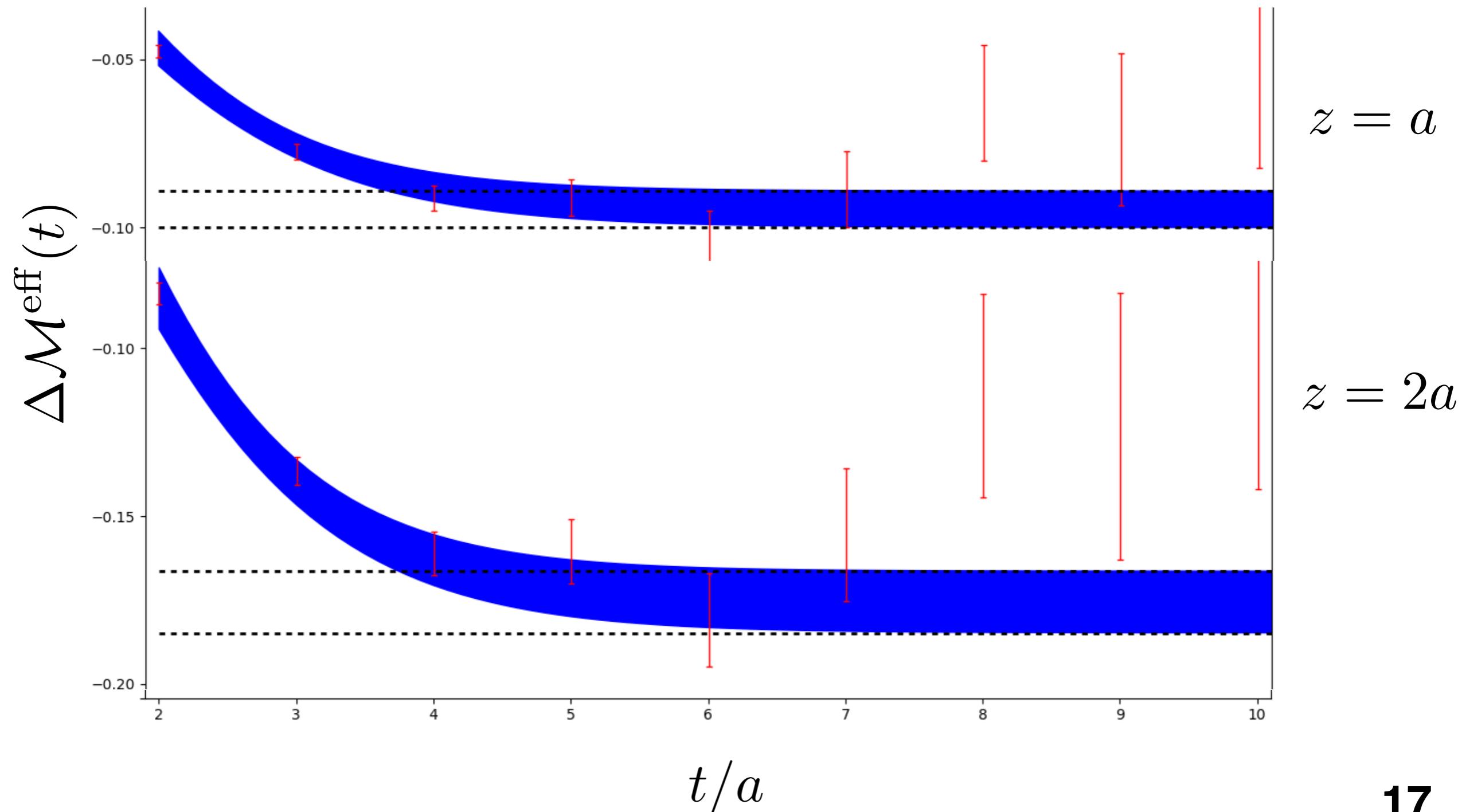
$$\text{Lattice} \quad \Delta\mathcal{M}_{0i;0i} + \Delta\mathcal{M}_{ij;ij} = -2p_0p_z \left[\Delta\mathcal{M}_{ps} - (1 + m_N^2/p_z^2) \nu \Delta\mathcal{M}_{pp} \right]$$

$$[\Delta\mathcal{M}_{ps} - \nu \Delta\mathcal{M}_{pp}](\nu, \mu^2) = \int_0^1 dx \, x \Delta g(x) \sin(x\nu)$$

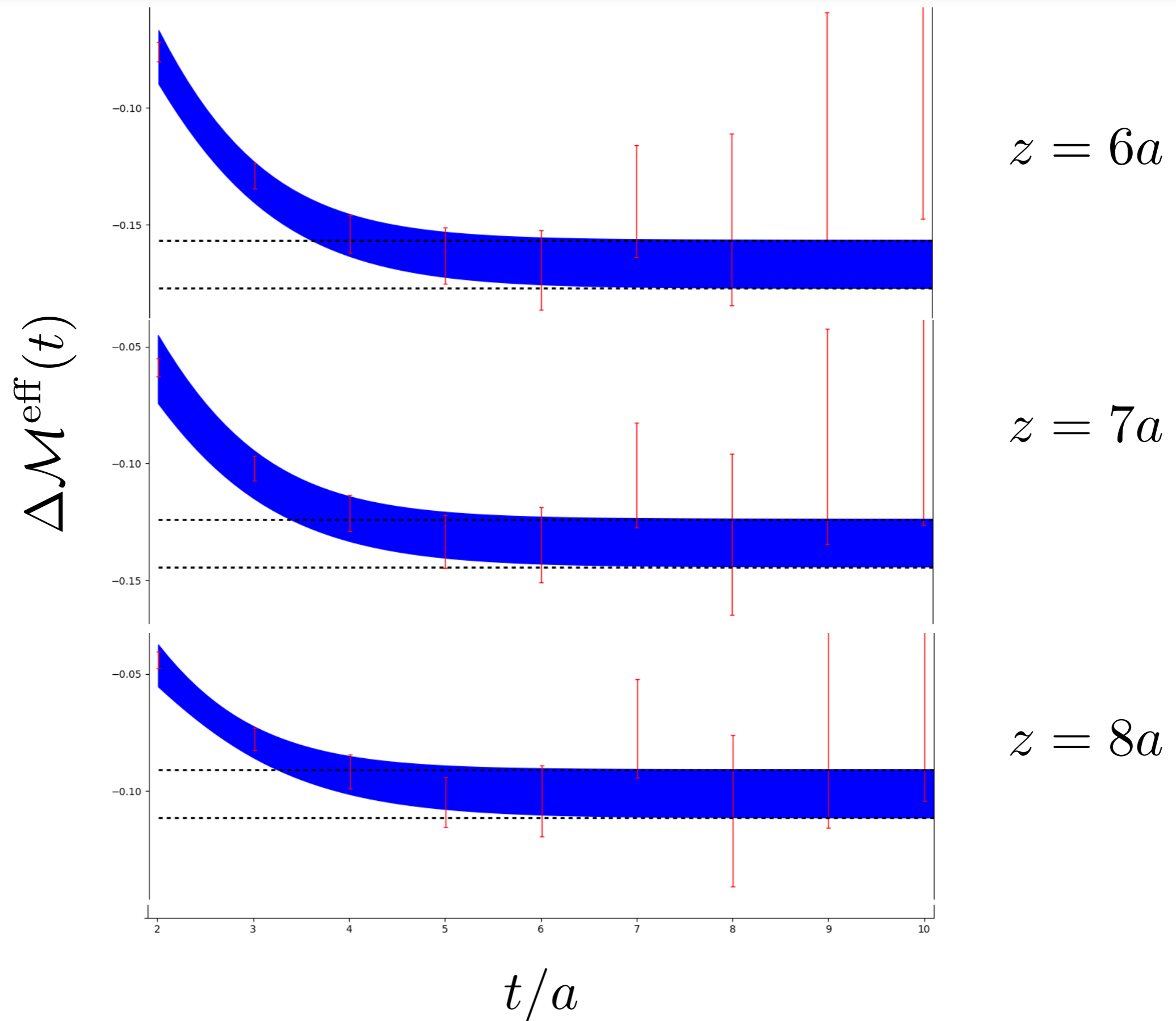
Lattice QCD matrix elements with 1899 configs

- Simultaneous correlated fit to matrix elements for all z
($p = 0.41 \text{ GeV}$ & gradient flow $\tau = 2.2$)

$$\Delta\mathcal{M}^{\text{eff}}(t)_i = A_i + B_i t \exp(-\Delta E t)$$



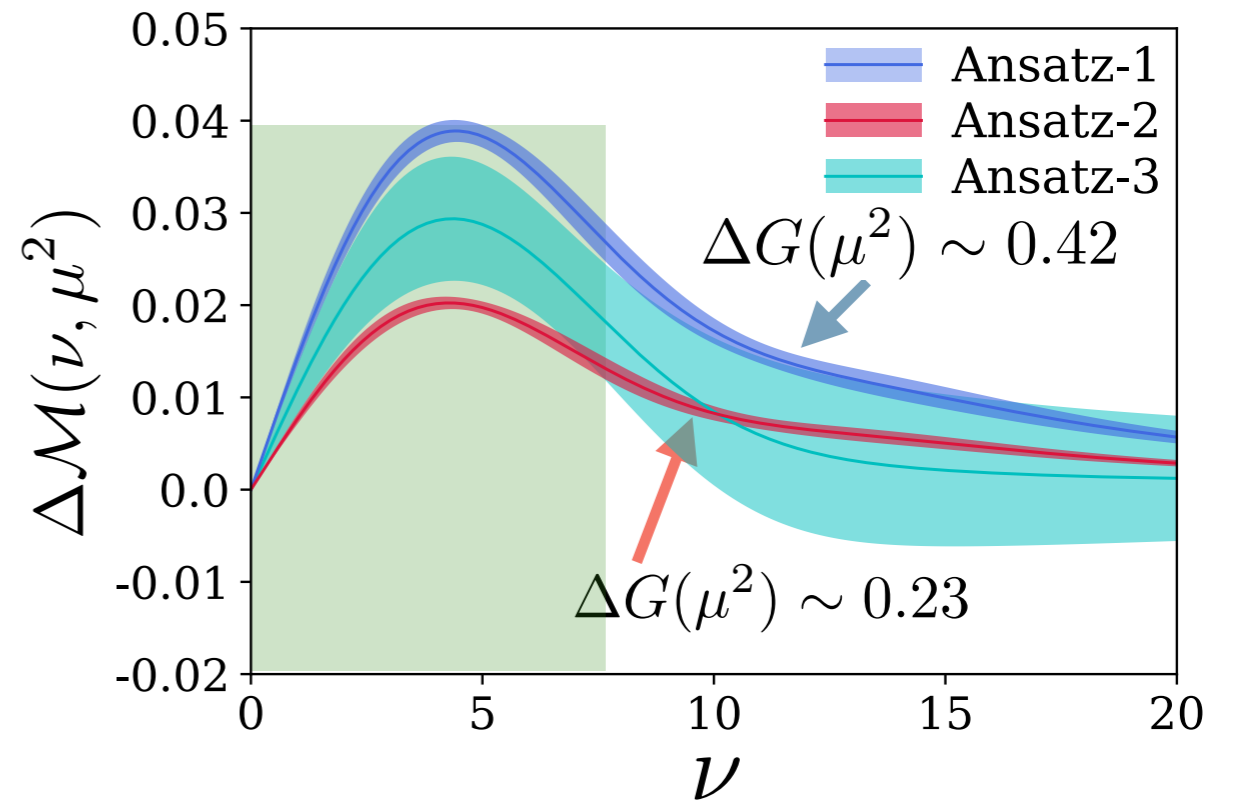
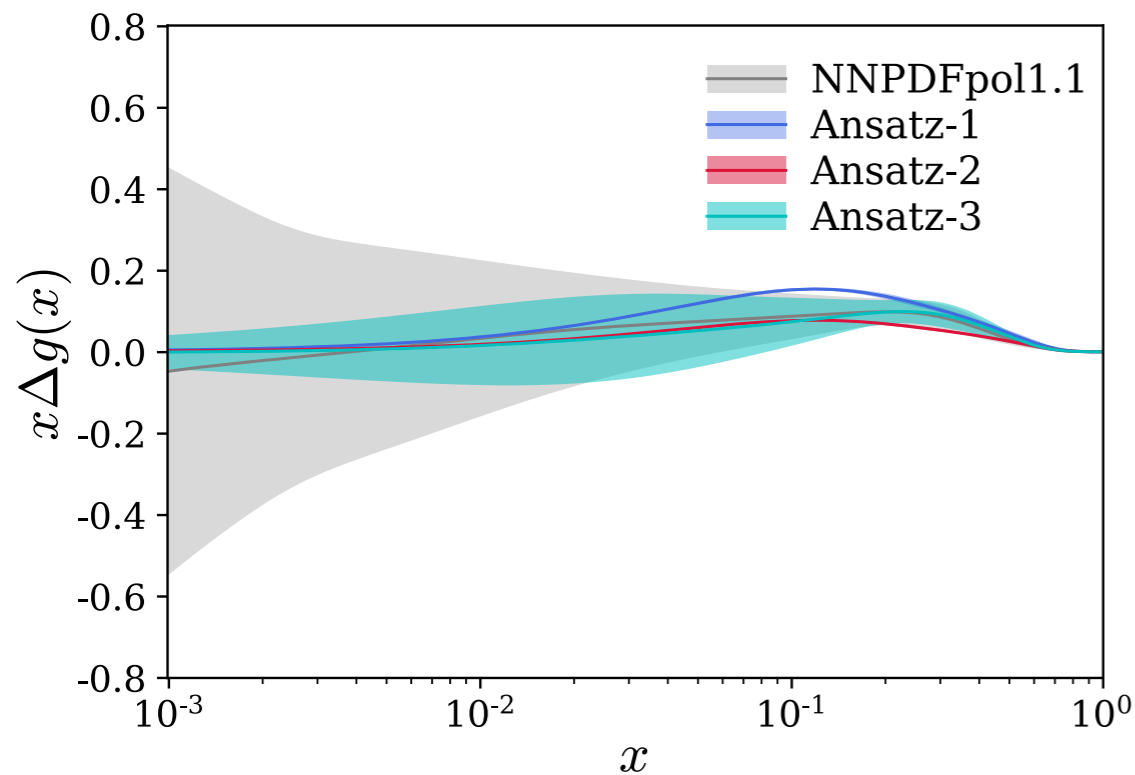
Lattice QCD matrix elements with 1899 configs



Prospect of Lattice QCD on gluon helicity distribution

● Gluon helicity from light cone Ioff-time distribution

$$\Delta G(\mu^2) = \int_0^\infty d\nu \Delta \mathcal{M}_{\text{light-cone}}(\nu, \mu^2)$$



RSS, Liu, Paul
PRD 2021

● LQCD determination of polarized gluon ITD, even at small Ioffe-time window can have important impact

Summary & Outlook

- In progress: unpolarized gluon distribution with 5 times stats
- Future consideration: mixing of quark-gluon operator
- Calculation of gluon helicity ITD
- Challenge: many systematics to be understood

Thank you!