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Parton Distribution Function for Gluon Condensate: One Loop Structure and "Zero Modes"

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Based on JHEP 12 (2021) 010 by A. Radyushkin and SZ

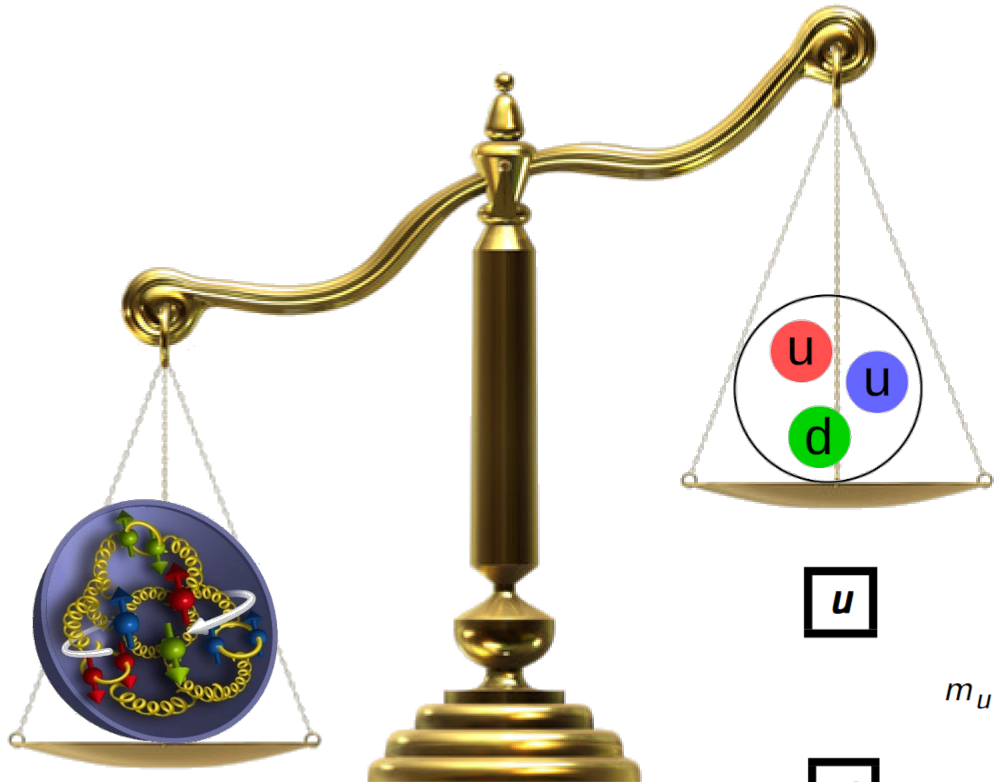
Outline

- Mass decomposition
  - Mass structure of nucleon
  - Sum rules, twist-4 PDFs
  - Light cone zero modes

- PDF for gluon condensate F(x)
  - One-loop structure of F(x)
  - Checking the gauge invariance
  - Zero modes contribution to F(x)

- Conclusion

Origin of proton mass



$\boxed{u}$

$$m_u = 2.16^{+0.49}_{-0.26}\ \text{MeV}$$

$\boxed{d}$

$$m_d = 4.67^{+0.48}_{-0.17}\ \text{MeV}$$

$\boxed{p}$

$$\text{Mass } m = 938.272081 \pm 0.000006\ \text{MeV}$$

Particle data group

Proton mass decomposition

$$M = \frac{\langle P| \int d^3x \, T^{00}(0, \mathbf{x})|P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle$$

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

Traceless

Trace

$$\overline{T}_q^{\mu\nu} = \bar{\psi}\gamma^{(\mu} i \overleftrightarrow{D}^{\nu)}\psi - \frac{1}{4} g^{\mu\nu}\overline{\psi} m \psi \, ,$$

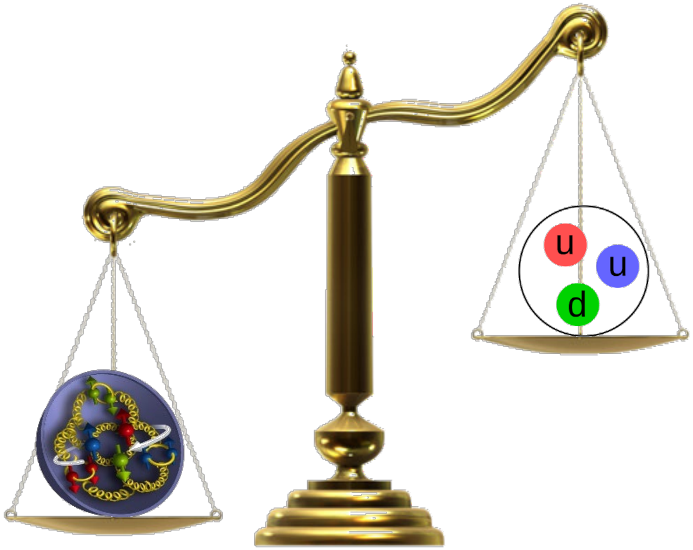
$$\hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu}\left[(1+\gamma_m)\overline{\psi} m \psi + \frac{\beta(g)}{2g} F^{\mu\nu}F_{\mu\nu}\right]$$

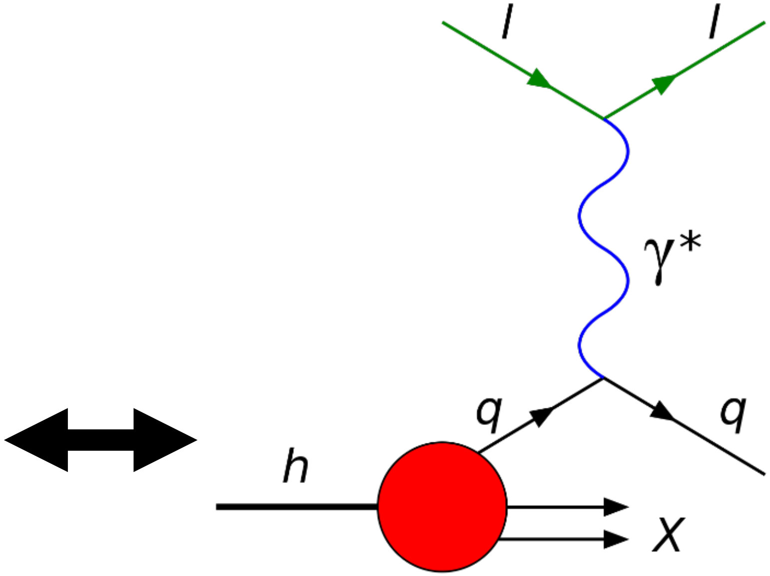
$$\overline{T}_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha} \, ,$$

$$\boxed{M_p = M_q + M_g + M_m + M_a}$$

X.Ji,1995

Linking to DIS





Mass decomposition in rest frame

- Mass decomposition (rest frame) Ji,1995

$$M = M^q_{\rm kin} + M^g_{\rm kin} + M_m + \boxed{M_a},$$

$$M_m \sim \langle P|\bar{\psi}\psi|P\rangle$$

2nd moments of twist-2 PDFs

1st moment of e(x)

- Twist-4 PDF F(x) for gluon condensate Y. Hatta, Y. Zhao 2020

$$F(x) = \frac{P^+}{2M^2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P|F_{\mu\nu}(0)W[0,z]F^{\mu\nu}(z^-)|P\rangle$$

- Sum rule for M_a

$$\int dx F(x) \sim M_a.$$

Light front proton mass sum rule

- Proton mass decomposition (Infinite momentum frame)

$$M = 2\langle P| \left[\left(\overline{T}_q^{+-} + \hat{T}_m^{+-} \right) + \overline{T}_g^{+-} + \hat{T}_a^{+-} \right] |P\rangle$$
$$= M_q^{\mathrm{LF}} + M_g^{\mathrm{LF}} + M_a^{\mathrm{LF}}$$

$$M_q^{\mathrm{LF}} = (a+b)M/2$$
$$M_g^{\mathrm{LF}} = (1-a)M/2$$
$$M_a^{\mathrm{LF}} = (1-b)M/2,$$

$$a = \sum_q \int dx\, x q(x) \qquad b = \sum_q \sigma_q (m_q/M)$$

- Chiral limit:
  - a/2 from quark kinematic energy
  - (1-a)/2 from gluon kinematic energy
  - ½ from trace anomaly

X.Ji,2020

Twist-4 PDFs and proton mass

- Twist-4 PDFs

$$Q_q(x,\mu^2) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P|\overline{\psi}_q(0) W(0,\lambda n)(\gamma^+ i \overleftrightarrow{D}^- + \gamma^- i \overleftrightarrow{D}^+)\psi_q(\lambda n)|P\rangle ,$$

$$E(x,\mu^2) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P|[F^{+-}(0) W(0,\lambda n) F^{+-}(\lambda n)|P\rangle ,$$

$$B(x,\mu^2) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P|F^{12}(0) W(0,\lambda n) F^{12}(\lambda n)|P\rangle ,$$

$$A(x,\mu^2) = \sum_i \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P|F^{+i}(0) W(0,\lambda n) F^{-i}(\lambda n) + F^{-i}(0) W(0,\lambda n) F^{+i}(\lambda n)|P\rangle$$

- Sum rules

$$M_q^{\mathrm{LF}} = \int_{-1}^{1} dx \sum_q Q_q(x,\mu^2)) ,$$

$$M_g^{\mathrm{LF}} = \int_{-1}^{1} dx [B(x) + E(x)] ,$$

$$M_a^{\mathrm{LF}} = \int_{-1}^{1} dx \frac{\beta(g)}{2g} (B(x,\mu^2) - E(x,\mu^2) - A(x,\mu^2))$$

X.Ji,2020

Zero modes and normal components

- Normal components
  - x>0
  - probed in experiments (e.g.,DIS)
  - Calculated in models
- Zero mode components
  - x=0
  - Cannot be probed experimentally
  - Long range LF correlations

X.Ji,2020

- Zero modes may lead to δ(x) contributions to high-twist PDFs (e.g., e(x)) Burkardt,Koike,2001; Aslan,Burkardt, 2019
- Mass may receive contributions from zero modes through twist-4 PDFs

$$M = \delta M_0 + M_n$$

- Mass will not be calculated correctly if the zero mode contributions are ignored
- Does F(x) (or E(x), B(x), A(x), etc) have δ(x) contributions?

F(x) regularized by offshellness

- It was proposed that F(x) has zero modes contribution.

$$F(x) = F_{\text{reg}}(x) + \delta(x)\mathcal{C}.$$

- One-loop computation with gluon target (light cone gauge)

$$F(x) = \delta(1-x) + \frac{\alpha_s N_c}{2\pi}\left[2 - x - \frac{2}{[x]_+} + \left(-\frac{3}{2} + \frac{\beta_0}{2N_c}\right)\delta(1-x)\right]\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)$$
$$+ \frac{\alpha_s N_c}{2\pi}\left[-x + (x-2)\ln x(1-x) + \frac{2\ln(1-x)}{x} + \left(\frac{11}{9} - \frac{5n_f}{9N_c}\right)\delta(1-x)\right].$$

where $$\frac{1}{[x]_+} \equiv \frac{1}{x^{1+\epsilon}} - \delta(x)\int_0^1 \frac{dx'}{x'^{1+\epsilon}}.$$ Y. Hatta, Y. Zhao 2020

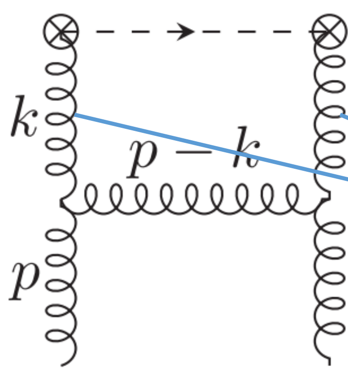
- Mandelstam-Leibbrandt prescription Leibbrandt, RMP,1987

Computation with offshell gluon

- External gluon is offshell ($p^2<0$), which may break the gauge symmetry

- No $1/(1-x)_+$ from bremsstrahlung

- "Zero mode contributions" appear as $1/x$, instead of $\delta(x)$



$$g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{\boxed{k \cdot n}}$$

Checking the gauge invariance

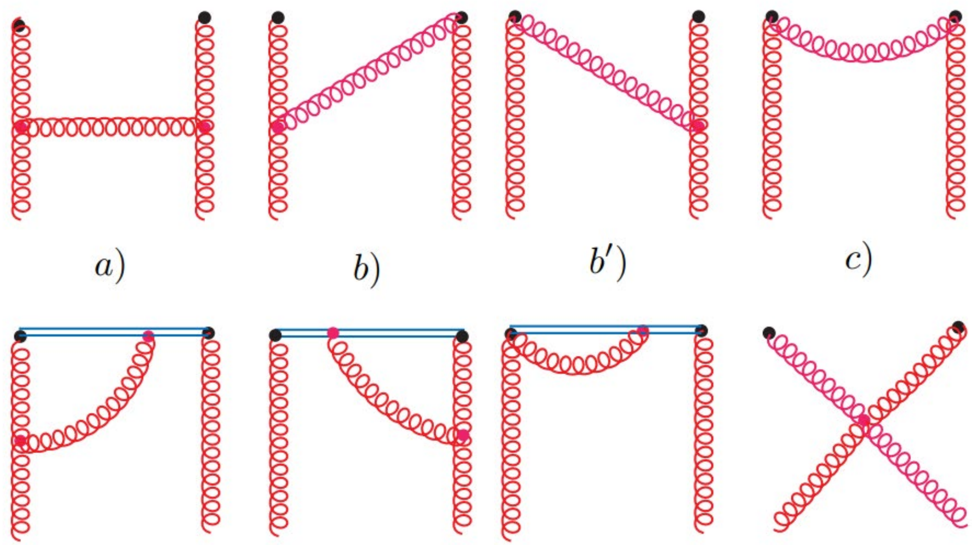
- Tree-level matrix element

$$\frac{p^+}{N_g}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\left\langle g\left(p,\epsilon_2^*\right)\left|G^a_{\mu\nu}(0)W[0,z]G_a^{\mu\nu}\left(z^-\right)\right|g\left(p,\epsilon_1\right)\right\rangle^{(0)}$$

$$=2\left(p^2\epsilon_1\cdot\epsilon_2^*-p\cdot\epsilon_1\,p\cdot\epsilon_2^*\right)\delta(1-x)+\{x\to -x\}$$

N_c^2-1

- One-loop "real" corrections



a) *b)* *b′)* *c)* *d)* *d′)* *e)* *f)*

Checking the gauge invariance

- Checking in light cone and Feynman gauges

$$F^{\mathrm{LC}}(x,p^2)\Big|_{(a+b+b')} = -\frac{\alpha_s}{2\pi}C_A\left\{\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{-p^2x(1-x)}\right]\left(\frac{2}{x}-2+x\right)+x\right\}\theta(0<x<1)$$
$$+\,(x\to -x)\ .$$

$$F^{\mathrm{F}}(x)\Big|_{(a+b+b'+d+d')} = \frac{\alpha_s}{4\pi}C_A\left\{\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{-p^2x(1-x)}\right]\left(\frac{2}{1-x}+1-x\right)-\frac{1+2x^2}{x}\right\}$$
$$\times\,\theta(0<x<1)+(x\to -x)\ .$$

Off shell vs on shell

- In QCD, gluon operator may get mixed with

  Gauge invariant operators

  Gauge variant operators

  Equation of motion operators,…

- If the external gluon is off shell, the gauge variant contribution will get mixed in

- Extremely dangerous to use off-shell gluon for F(x) because:
  - M^2 F(x) ~ p^2(…)
  - The violation of gauge invariance is also related to p^2

Onshell forward matrix element

- If $p^2 = 0,\ p\cdot\epsilon_1 = p\cdot\epsilon_2^* = 0$

$$\frac{p^+}{N_g}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\left\langle g\left(p,\epsilon_2^*\right)\left|G^a_{\mu\nu}(0)W[0,z]G_a^{\mu\nu}\left(z^-\right)\right|g\left(p,\epsilon_1\right)\right\rangle^{(0)}$$

$$=2\left(p^2\epsilon_1\cdot\epsilon_2^* - p\cdot\epsilon_1\, p\cdot\epsilon_2^*\right)\delta(1-x)+\{x\to -x\}$$

  The tree-level matrix element is zero

- One-loop is also zero. No useful information can be extracted

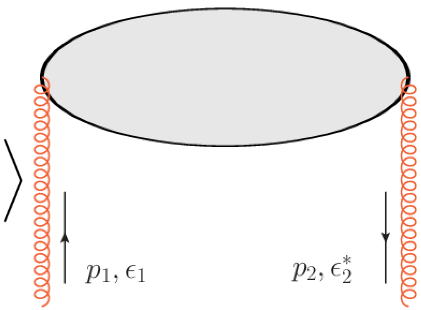
- However, we can start with nonforward kinematics

Generalized parton distributions

- Double distribution (GPD) for gluon condensate

$$F(x, p_1, p_2; \epsilon_1, \epsilon_2)$$

$$= \frac{P^+}{N_g} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle g(p_2, \epsilon_2) \middle| G^a_{\mu\nu}\left(-\frac{z^-}{2}\right) W\left[-\frac{z}{2}, \frac{z}{2}\right] G^{\mu\nu}_a\left(\frac{z^-}{2}\right) \middle| g(p_1, \epsilon_1) \right\rangle$$



- Parameterization

$$F\left(x, p_1, p_2; \epsilon_1, \epsilon_2\right) = \epsilon_1{}^{\mu} \epsilon_2^{*\,\nu} \left[\left(2\, q_\mu q_\nu - q^2 g_{\mu\nu}\right) F_1(x, \xi, q^2) + g_{\mu\nu} F_2(x, \xi, q^2)\right]$$
$$\equiv \Pi(q, \epsilon_1, \epsilon_2) F_1(x, \xi, q^2) + \epsilon_1 \cdot \epsilon_2^* F_2(x, \xi, q^2) \ .$$

$$\xi = \frac{p_1^+ - p_2^+}{p_1^+ + p_2^+}$$

$$\boxed{\begin{array}{c} p_1^2 = 0,\ p_2^2 = 0,\ p_1 \cdot \epsilon_1 = 0,\ p_2 \cdot \epsilon_2^* = 0 \\ \hline p_1 + p_2 = 2P,\ p_2 - p_1 = q \end{array}}$$

- At tree level: $F_1 = \delta(1-x) + \delta(1+x)$, $F_2 = 0$

F(x) at one loop: x≠±1

- GPDs at x≠ ±1

$$F_1^{(1)}(x,\xi,q^2)\,|_{x\neq\pm1}=\frac{\alpha_s}{\pi}C_A\frac{1}{1-x}\left\{\left(\frac{1}{\epsilon_{\rm UV}}+\ln\frac{\mu_{\rm UV}^2(1-\xi^2)}{-q^2(1-x)^2}\right)\theta(\xi<x<1)\right.$$
$$\left.+\frac{1}{2}\left(\frac{1}{\epsilon_{\rm UV}}+\ln\frac{\mu_{\rm UV}^2(\xi-x)(1+\xi)^2}{-q^2(1-x)^2(x+\xi)}\right)\theta(-\xi<x<\xi)\right\}+\{x\to -x\}\ .$$

$$F_2^{(1)}(x,\xi,q^2)=\frac{\alpha_s}{\pi}C_A q^2\left\{\frac{1-x}{1-\xi^2}\theta(\xi<x\leq1)-\frac{1-\xi}{4\xi(1+\xi)}\theta(-\xi<x<\xi)\right\}$$
$$+\{x\to -x\}\ .$$

- IR poles and scales cancel Y.S.Liu, SZ et al 2019;Ji,Schäfer,Xiong,Zhang 2015
- ξ → 0 limit

$$F_1^{(1)}(x,q^2)\Big|_{x\neq\pm1}=\frac{\alpha_s}{\pi}C_A\left\{\frac{\theta(0\leq x<1)}{1-x}\left(\frac{1}{\epsilon_{\rm UV}}+\ln\frac{\mu_{\rm UV}^2}{-q^2(1-x)^2}\right)\right\}$$
$$+\{x\to -x\}\ .$$

$$F_2^{(1)}(x,q^2)=\frac{\alpha_s}{\pi}C_A q^2\left\{(1-x)\theta(0\leq x\leq1)-\frac{1}{2}\delta(x)\right\}+\{x\to -x\}$$

- When $q^2 \rightarrow 0$, F_2 contribution disappears

Checking the "real corrections"

- The results are obtained in both light-cone and Feynman gauges
- We also calculated $\xi=0$ case from the beginning and got the same result as taking $\xi \to 0$ limit in GPD
- Nontrivial check. No diagram-by-diagram correspondence. Only the total results coincide

F(x) at one loop: x=±1

Plus prescription of "real correction"

"Sudakov term"

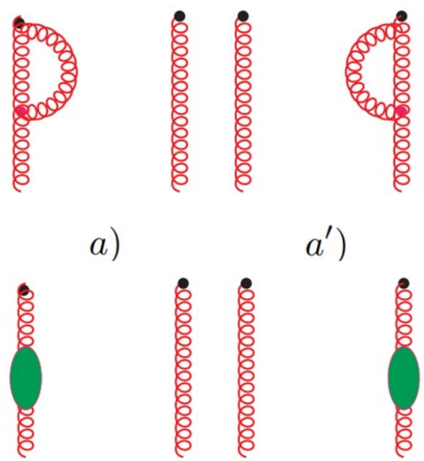
$$\frac{\alpha_s}{2\pi}C_A\delta(1-x)\left(-\frac{1}{\epsilon_{\rm IR}^2}+\frac{1}{\epsilon_{\rm IR}}\ln\frac{-q^2}{\mu_{\rm IR}^2}-\frac{1}{2}\ln^2\frac{-q^2}{\mu_{\rm IR}^2}+\frac{\pi^2}{12}\right)\Pi(q,\epsilon_1,\epsilon_2)\ .$$

UV term

$$\frac{\alpha_s}{2\pi}C_A\delta(1-x)\frac{5}{2}\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}+\ln\frac{\mu_{\rm UV}^2}{\mu_{\rm IR}^2}\right)\Pi(q,\epsilon_1,\epsilon_2)$$

$$-\delta(1-x)\frac{\alpha_s}{\pi}C_A\frac{3}{4}\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}+\ln\frac{\mu_{\rm UV}^2}{\mu_{\rm IR}^2}\right)\Pi(q,\epsilon_1,\epsilon_2)$$
$$+\{x\to -x\}\ .$$

Self-energy-like contributions



$a)$ $a')$ $b)$ $b')$

$$\delta(1-x)\frac{\alpha_s}{\pi}\left(\frac{5}{12}C_A-\frac{1}{3}T_Fn_f\right)\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}+\ln\frac{\mu_{\rm UV}^2}{\mu_{\rm IR}^2}\right)\Pi(q,\epsilon_1,\epsilon_2)$$
$$+\{x\to -x\}\ .$$

$$\beta_0=\frac{11}{3}C_A-\frac{4}{3}T_Fn_f$$

$$\boxed{\delta(1-x)\frac{\alpha_s}{4\pi}\beta_0\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}+\ln\frac{\mu_{\rm UV}^2}{\mu_{\rm IR}^2}\right)\Pi(q,\epsilon_1,\epsilon_2)+\{x\to -x\}}$$

Evolution

- $G^2\beta(\alpha_s)/\alpha_s$ is related to trace anomaly

- $g_s^2\, G^2$ is independent of UV scale

  Kluberg-Stern, Zuber, 1975; Nielsen, 1977

  Anomalous dimension of G^2 should be proportional to β

- The evolution equation of F(x) (gg-part)

$$F(x,\mu_1^2) = F(x,\mu_2^2) + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_1^2}{\mu_2^2}\right) \int_x^1 \frac{dz}{z} P_{gg}^F(z)\, F(x/z,\mu_2^2)$$

$$P_{gg}^F(z) = \frac{\beta_0}{2}\delta(1-z) + C_A\left[\frac{2}{1-z}\right]_+$$

LaMET and zero mode δ(x)

- Zero modes contribution may be studied with LaMET

X.Ji,2020

- Zero mode contribution in e(x)

$$e^{(1a)}_{(s)}(x)\Big|_{m_q} = \frac{\alpha_s C_F}{2\pi}\delta(x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2_{\rm UV}}{m_q^2} - 1\right),$$

$$\mathcal{P}_{\rm UV} = \frac{1}{\epsilon_{\rm UV}} + \ln 4\pi - \gamma_E$$

- Zero mode contribution in $\tilde{e}(x)$

$$e^{(1a)}_{\rm Q(s)}(x)\Big|_{m_q} = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{x} & x>1 \\ \frac{p^3}{\sqrt{x^2p_3^2+m_q^2}} & -1<x<1 \\ -\frac{1}{x} & x<-1\,, \end{cases}$$

$$\frac{\theta(1-x)\,\theta(1+x)}{\sqrt{x^2+\eta^2}} = \delta(x)\left(\ln\frac{4}{\eta^2}\right) + \mathrm{R}_0(|x|) + \mathcal{O}(\eta^2)$$

S.Bhattacharya et al, 2020

See also V.Braun,Y.Ji, A.Vladimirov,2021 and Yao Ji's talk tomorrow

- Evaluate F(x) with LaMET, Pseudo-PDF, …

Summary

- Twist-4 gluon PDF F(x) is important in understanding the mass structure, gluon condensate, …
- A gauge invariant calculation disagrees the existence of δ(x) for F(x) at one loop; but δ(x) exists in GPD $F_2(x,0,q^2)$ when $q^2 \neq 0$
- Gauge invariance should be carefully examined in a gauge theory calculation

Thank you!

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