Disentangling Long and Short Distances in Momentum-Space TMDs

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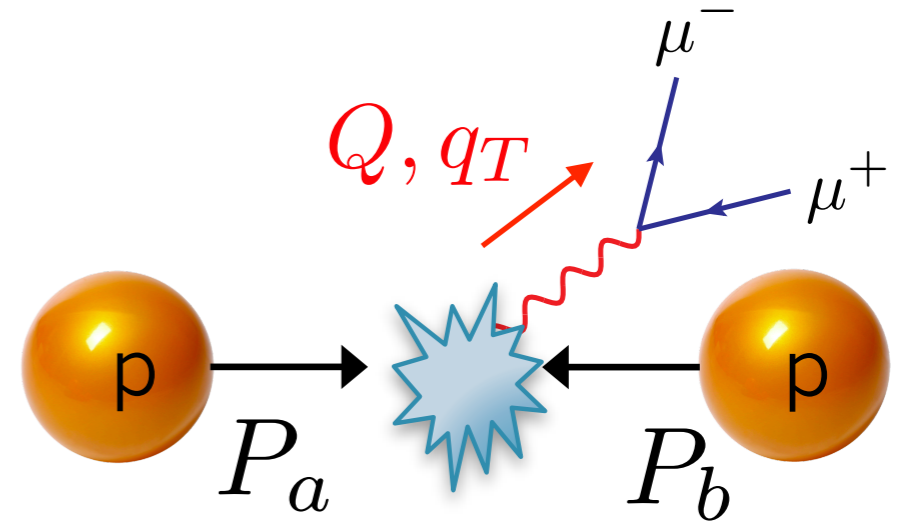*[to appear]*

LaMET 2021
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Massachusetts Institute of Technology

TMD Collaboration

TMDPDFs

- Factorization of Drell-Yan cross section:

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H(Q,\mu) \sum_i \int d^2\vec{b}_T\; e^{i\vec{q}_T\cdot\vec{b}_T}\, f_i(x_a, b_T, \mu, \zeta_a)\, f_{\bar{i}}(x_b, b_T, \mu, \zeta_b) \times \left[1 + \mathcal{O}\!\left(\frac{q_T^2}{Q^2}\right)\right]$$

Hard virtual corrections

Describe transverse momentum of the partons
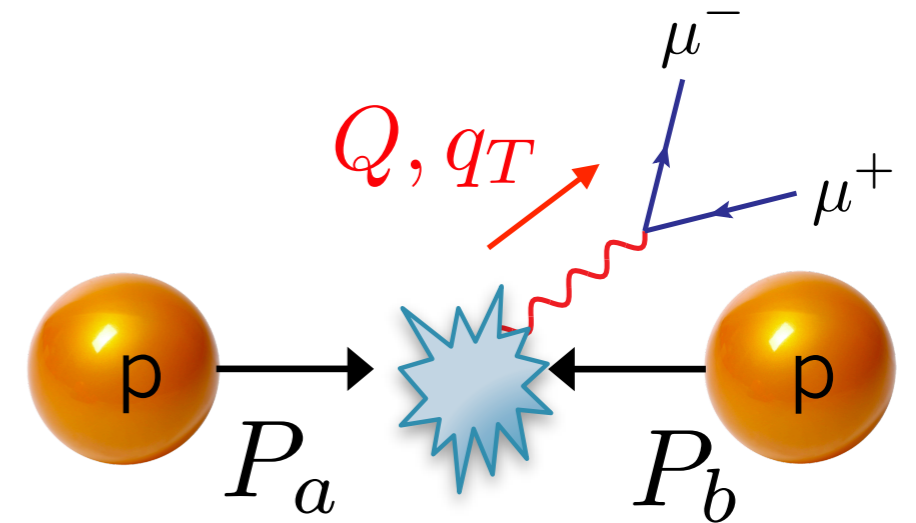
- Most easily written in position space

μ = Renormalization scale

ζ = Collins-Soper parameter
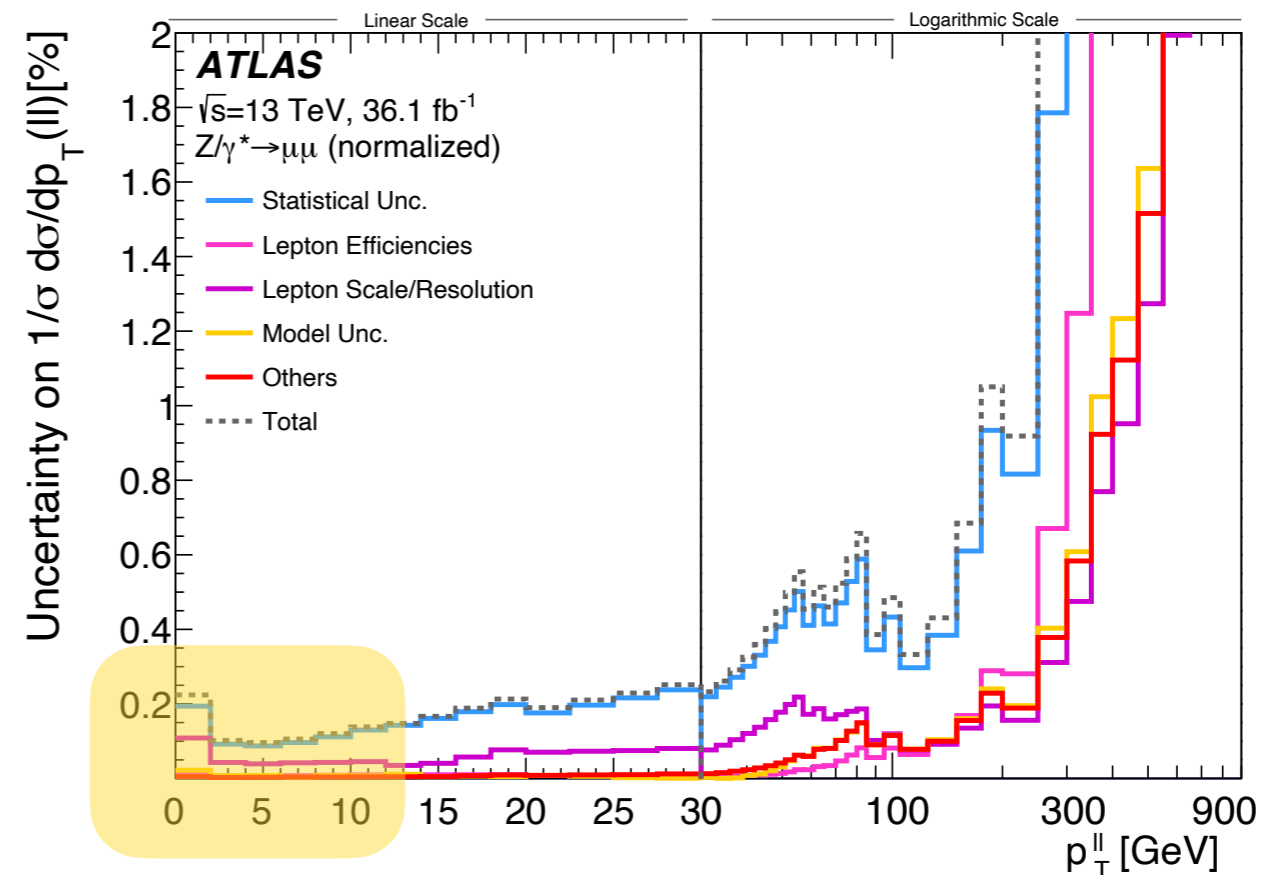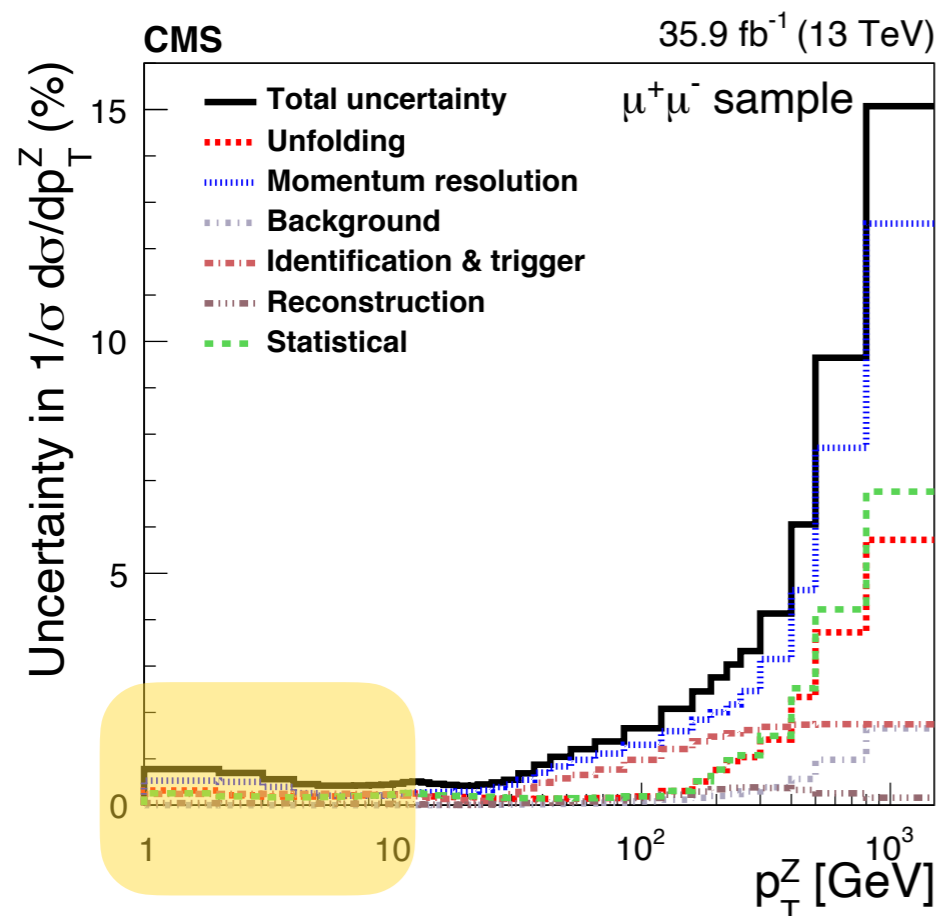
$\zeta_a\zeta_b = Q^4$

TMDPDFs

- Factorization of Drell-Yan cross section:

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H(Q,\mu)\sum_i \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_i(x_a, b_T, \mu, \zeta_a)\, f_{\bar{i}}(x_b, b_T, \mu, \zeta_b) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

- Measurements are done in momentum space!

CMS: 1909.04133
ATLAS: 1912.02844

(Non)perturbative TMDPDFs

- Challenging to use the nonperturbative info that lattice provides

- Modeling TMDPDFs with both perturbative and nonperturbative parts is usually done by introducing $b*$:

$$f_i(x, b_T, \mu, \zeta) = f_{\text{pert}, i}(x, b^*(b_T), \mu, \zeta) \cdot f_{\text{NP}}(x, b_T, \zeta)$$

Calculated with expansion in $\alpha_s(1/b_T)$

- The perturbative part can be computed with an operator product expansion (OPE):

$$f_{\text{pert}, i}^{\text{TMD}}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{dz}{z} C_{ij}(\frac{x}{z}, b_T, \mu, \zeta) f_j^{\text{coll}}(z, \mu)$$

$$= f_i^{\text{coll}}(x, \mu) + \alpha_s C_{ij}^{(1)} \otimes f_j^{\text{coll}}(x, \mu) + \mathcal{O}(\alpha_s^2)$$
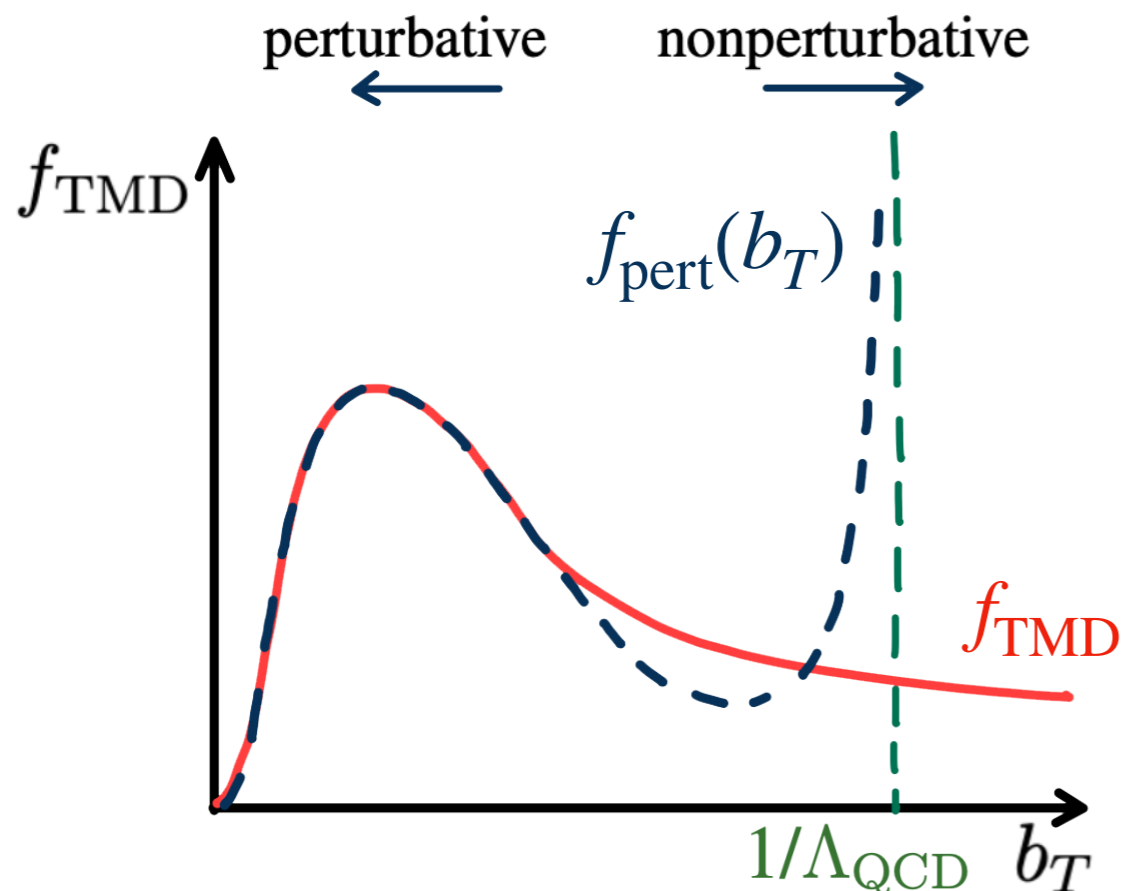
Modeling TMDPDFs

- Modeling TMDPDFs with both perturbative and nonperturbative parts is usually done by introducing $b*$:

$$f_i(x, b_T, \mu, \zeta) = f_{\text{pert}, i}(x, b^*(b_T), \mu, \zeta) \cdot f_{\text{NP}}(x, b_T, \zeta)$$

Calculated with expansion in $\alpha_s(1/b_T)$

Has to be $1 + \mathcal{O}(b_T^2)$

perturbative   nonperturbative

$f_{\text{pert}}(b_T)$

f_{TMD}

$1/\Lambda_{\text{QCD}}$ b_T

- $b^*(b_T)$ shields the Landau pole

- $b_T \ll 1/\Lambda_{\text{QCD}}$: $b^*(b_T) \to b_T$, $f_{\text{NP}} \to 1$

  f_{pert} dominates

- $b_T \gg 1/\Lambda_{\text{QCD}}$: $b^*(b_T) \to$ constant

  f_{NP} dominates

Modeling TMDPDFs

- Different models of f_{NP} are used for fitting to data

- $b^*(b_T)$ shields the Landau pole and is coupled to f_{NP}

$$f_{\mathrm{TMD}}(x, b_T, \mu, \zeta) = f_{\mathrm{pert}}(x, b_A^*(b_T), \mu, \zeta) \cdot f_{\mathrm{NP}}^A(x, b_T, \zeta)$$
$$= f_{\mathrm{pert}}(x, b_B^*(b_T), \mu, \zeta) \cdot f_{\mathrm{NP}}^B(x, b_T, \zeta)$$
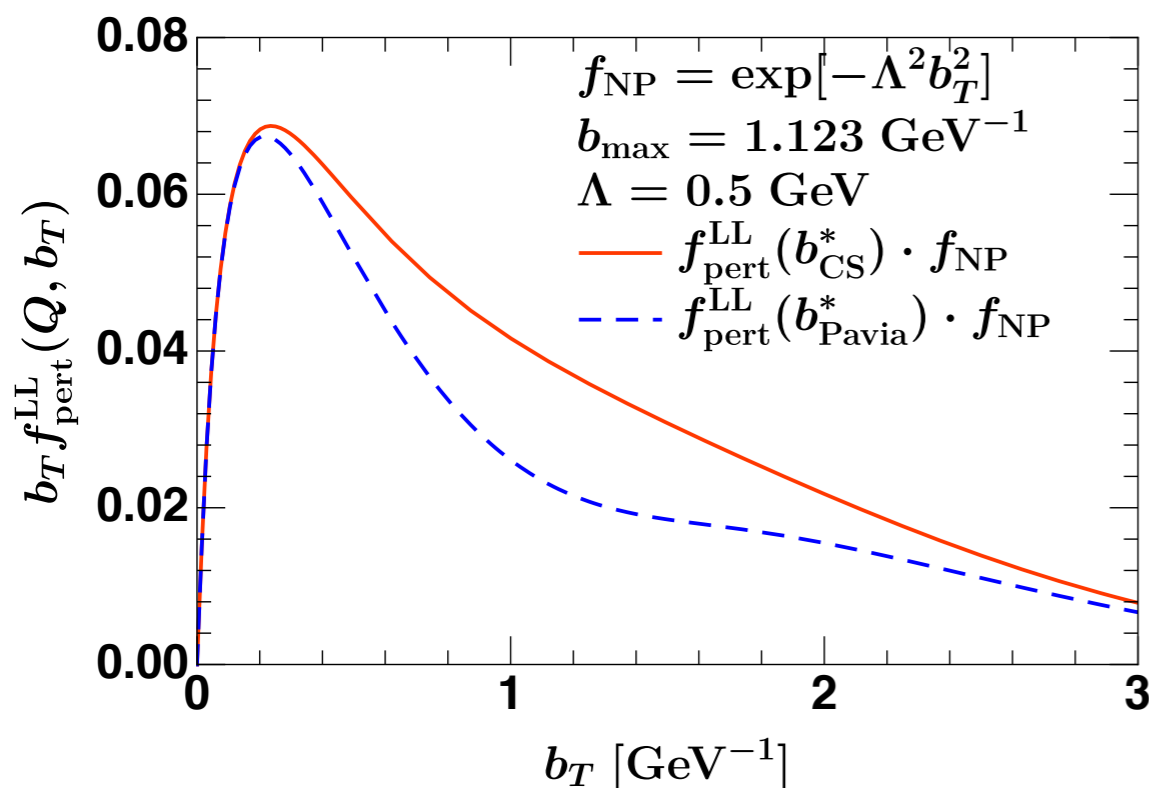
$$b_A^*(b_T) \neq b_B^*(b_T) \quad \Rightarrow \quad f_{\mathrm{NP}}^A(x, b_T) \text{ and } f_{\mathrm{NP}}^B(x, b_T) \text{ are not comparable!}$$

- The perturbative and nonperturbative effects are mixed up!

Modeling TMDPDFs

- $b*$ prescriptions makes different f_{NP} not comparable

- For example, take the same $f_{\mathrm{NP}}(b_T) = e^{-(0.5\mathrm{GeV}\ b_T)^2}$,

  use either $b^*_{\mathrm{CS}}(b_T)$ or $b^*_{\mathrm{Pavia}}(b_T)$:

$$b^*_{\mathrm{CS}}(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{\max})^2}} = b_T\left(1 + \mathcal{O}(b_T^2)\right)$$

Collins+Soper (1982)

$$b^*_{\mathrm{Pavia}}(b_T) = b_{\max}\left(1 - \exp(-\frac{b_T^4}{b_{\max}^4})\right)^{\frac{1}{4}} = b_T\left(1 + \mathcal{O}(b_T^4)\right)$$

Pavia: 1703.10157

In-figure text:
$f_{\mathrm{NP}} = \exp[-\Lambda^2 b_T^2]$
$b_{\max} = 1.123\ \mathrm{GeV}^{-1}$
$\Lambda = 0.5\ \mathrm{GeV}$
— $f^{\mathrm{LL}}_{\mathrm{pert}}(b^*_{\mathrm{CS}}) \cdot f_{\mathrm{NP}}$
-- $f^{\mathrm{LL}}_{\mathrm{pert}}(b^*_{\mathrm{Pavia}}) \cdot f_{\mathrm{NP}}$

Axis labels: $b_T f^{\mathrm{LL}}_{\mathrm{pert}}(Q, b_T)$ (vertical), $b_T\ [\mathrm{GeV}^{-1}]$ (horizontal)

- **Goal:** extract nonperturbative physics without $b*$ contamination

Momentum Space

- Measurements are in q_T space: Fourier transform

$$\frac{d\sigma}{dq_T} = 2\pi q_T \int_0^\infty \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \sigma(b_T)$$

$$= q_T \int_0^\infty db_T \, b_T \int_0^{2\pi} \frac{d\phi}{2\pi} \, e^{iq_T b_T \cos\phi} \sigma(b_T) = q_T \int_0^\infty db_T \, b_T \, J_0(q_T b_T) \sigma(b_T)$$

q_T spectrum

- For perturbative q_T, integral still includes nonperturbative b_T !

- Intuition: perturbative q_T should be dominated
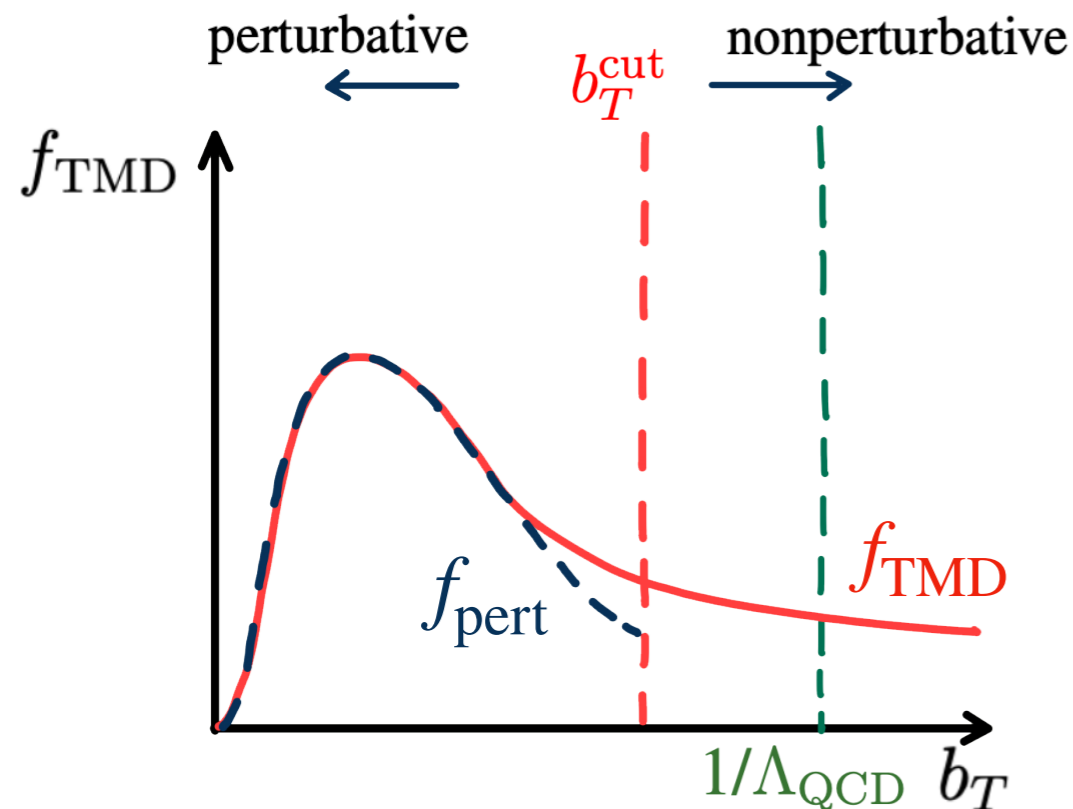 by perturbative $b_T \sim 1/q_T$

Momentum Space

- Intuition: perturbative q_T should be dominated by perturbative b_T

- Goal: make this intuition manifest

- Solution: introducing b_T^{cut}

Can use perturbative OPE   Nonperturbative physics

$$S[f](q_T) \equiv q_T \int_0^\infty db_T \, b_T J_0(q_T b_T) f(b_T) = S_<[f](q_T) + S_>[f](q_T)$$

full spectrum

perturbative b_T^{cut} nonperturbative

f_{TMD}

f_{pert} f_{TMD}

$1/\Lambda_{\text{QCD}}$ b_T

$$S_<[f](q_T) \equiv q_T \int_0^{b_T^{\text{cut}}} db_T \, b_T J_0(q_T b_T) f(b_T),$$

$$S_>[f](q_T) \equiv q_T \int_{b_T^{\text{cut}}}^\infty db_T \, b_T J_0(q_T b_T) f(b_T)$$

Truncated Functionals

- Want to approximate $S[f]$ using perturbative $b_T \leq b_T^{\text{cut}}$

- Can use $S_<[f]$, but need to systematically account for $S_>[f]$

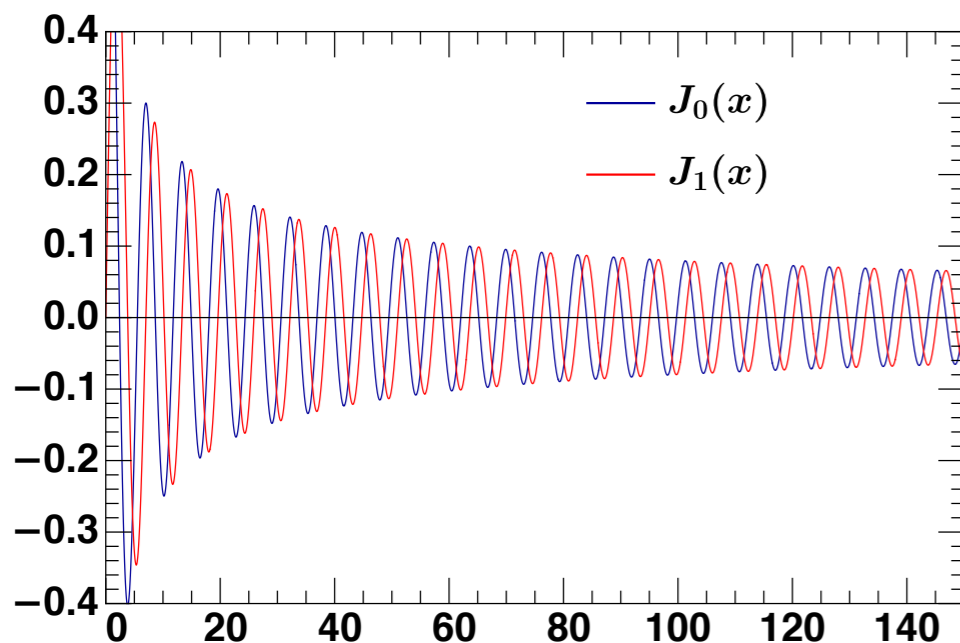$$S_>[f](q_T, b_T^{\text{cut}}) = q_T \int_{b_T^{\text{cut}}}^{\infty} db_T \ b_T J_0(q_T b_T) f(b_T)$$

$$= -b_T^{\text{cut}} \underbrace{J_1(q_T b_T^{\text{cut}}) f(b_T^{\text{cut}})} - \int_{b_T^{\text{cut}}}^{\infty} db_T \ b_T J_1(q_T b_T) \underbrace{f'(b_T)}_{< b_T^{-\rho-1}}$$

Assumption:

a) $f(b_T \to \infty) < b_T^{-\rho}, \ \rho > \dfrac{1}{2}$

b) $f(b_T)$ differentiable at b_T^{cut}

asymptotic form $\quad\searrow\quad q_T \gg 1/b_T^{\text{cut}} > \Lambda_{\text{QCD}}$

$$= \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} \cos\left(q_T b_T^{\text{cut}} + \frac{\pi}{4}\right) f(b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} q_T)^{-\frac{3}{2}}]$$

$$J_0(x \to \infty) = \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4}) + \mathcal{O}(x^{-\frac{3}{2}})$$

$$J_1(x \to \infty) = -\sqrt{\frac{2}{\pi x}} \cos(x + \frac{\pi}{4}) + \mathcal{O}(x^{-\frac{3}{2}})$$

Truncated Functionals

- Define a systematic series to approximate $S[f]$ using $b_T \leq b_T^{\text{cut}}$

$$S^{(0)}[f](q_T) \equiv S_<[f](q_T) = q_T \int_0^{b_T^{\text{cut}}} db_T \, b_T J_0(q_T b_T) f(b_T)$$

- Define $S^{(1)}[f]$ to include leading boundary contribution from $S_>[f]$

$$S^{(1)}[f](q_T) \equiv S^{(0)}[f] + \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} \cos\left(q_T b_T^{\text{cut}} + \frac{\pi}{4}\right) f(b_T^{\text{cut}})$$ ← First correction!

$$\boxed{S[f](q_T) = S^{(1)}[f](q_T, b_T^{\text{cut}}) + \frac{1}{q_T}\mathcal{O}[(b_T^{\text{cut}} q_T)^{-\frac{1}{2}}]}$$

Truncated Functionals

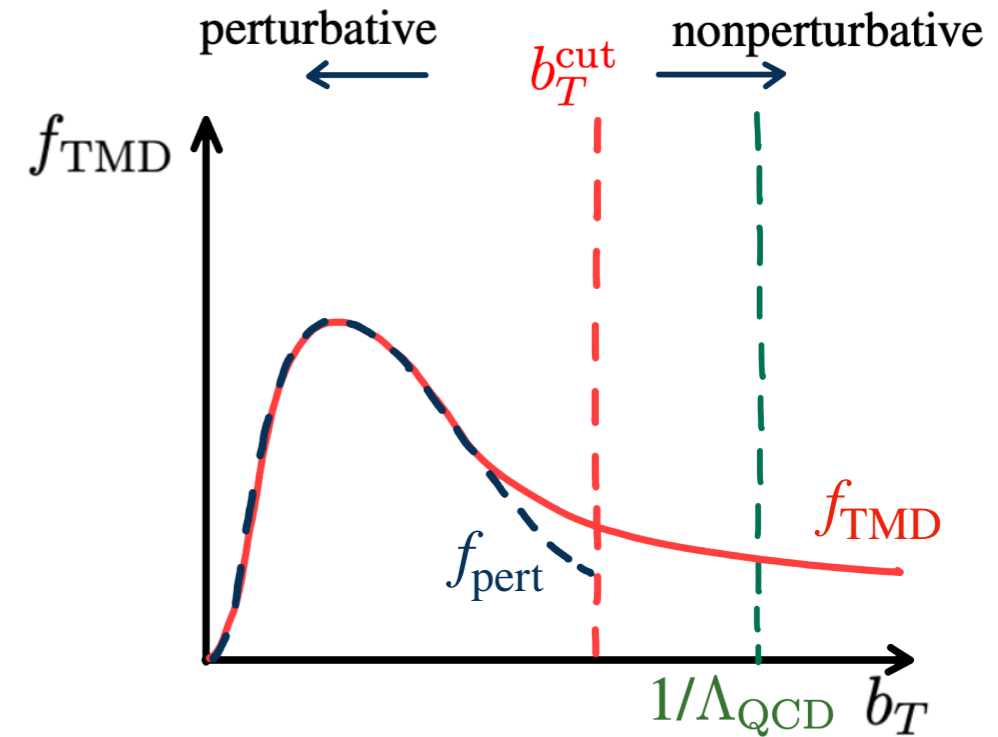- Systematically add on power corrections

so $S^{(n)}[f] \to S[f]$

$$S^{(0)}[f](q_T, b_T^{\mathrm{cut}}) = \int_0^{b_T^{\mathrm{cut}}} db_T\, b_T J_0(q_T b_T) f(b_T)$$

$$S^{(1)}[f](q_T, b_T^{\mathrm{cut}}) = S^{(0)}[f] + \sqrt{\frac{2b_T^{\mathrm{cut}}}{\pi q_T}} f(b_T^{\mathrm{cut}}) \cdot \cos\left(b_T^{\mathrm{cut}} q_T + \frac{\pi}{4}\right)$$

$$S^{(2)}[f](q_T, b_T^{\mathrm{cut}}) = S^{(1)}[f] - \sqrt{\frac{2b_T^{\mathrm{cut}}}{\pi q_T}} \left(\frac{3\, f(b_T^{\mathrm{cut}})}{8\, b_T^{\mathrm{cut}} q_T} + \frac{f'(b_T^{\mathrm{cut}})}{q_T}\right) \cdot \cos\left(b_T^{\mathrm{cut}} q_T - \frac{\pi}{4}\right)$$

$$S^{(3)}[f](q_T, b_T^{\mathrm{cut}}) = S^{(2)}[f] + \sqrt{\frac{2b_T^{\mathrm{cut}}}{\pi q_T}} \left(\frac{15\, f(b_T^{\mathrm{cut}})}{128\, b_T^{\mathrm{cut}\,2} q_T^2} - \frac{7\, f'(b_T^{\mathrm{cut}})}{8\, b_T^{\mathrm{cut}} q_T^2} - \frac{f''(b_T^{\mathrm{cut}})}{q_T^2}\right) \cdot \cos\left(b_T^{\mathrm{cut}} q_T + \frac{\pi}{4}\right)$$
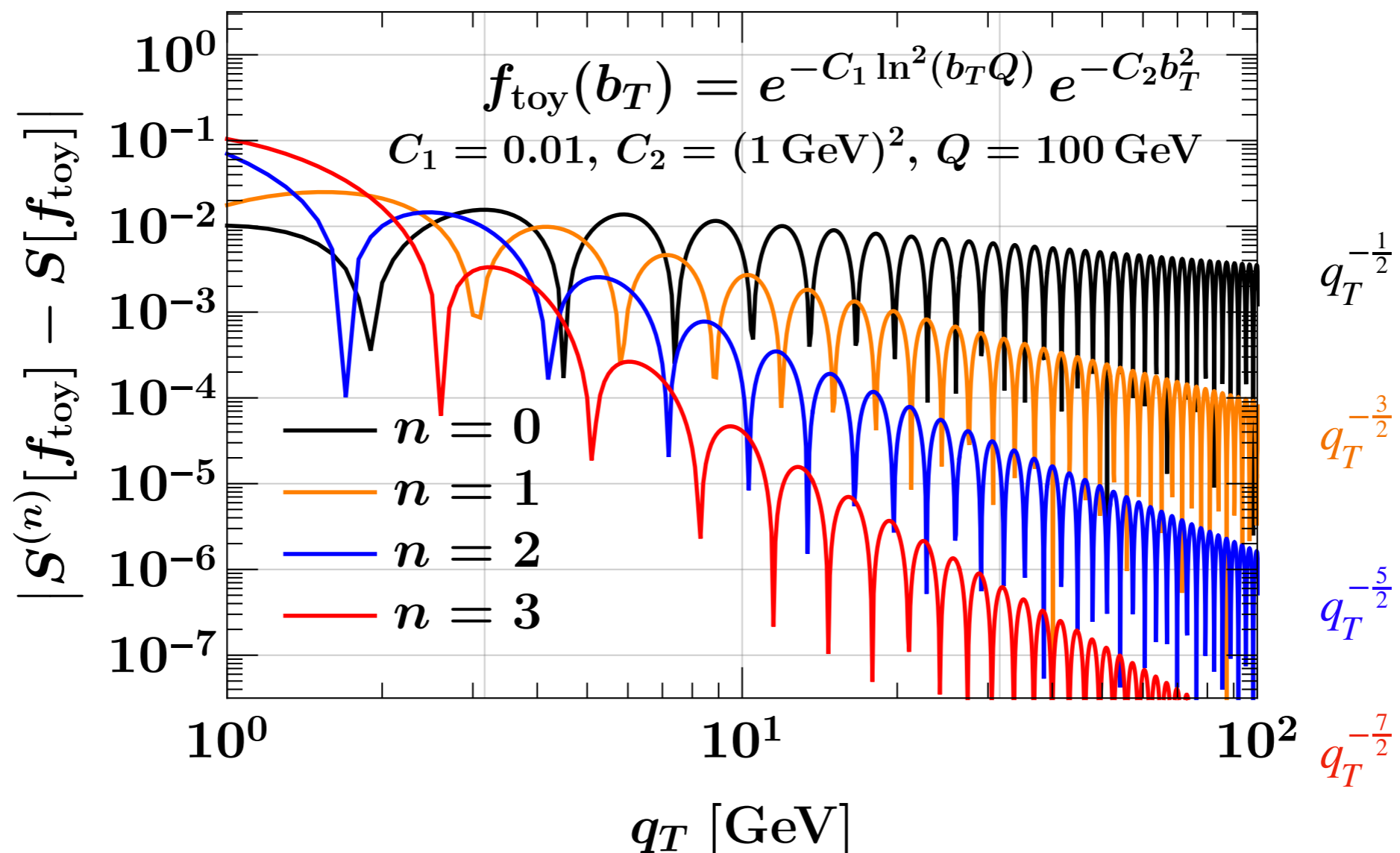
$$\boxed{S[f](q_T) = S^{(n)}[f] + \frac{1}{q_T} \mathcal{O}\left[(b_T^{\mathrm{cut}} q_T)^{-n+\frac{1}{2}}\right]}$$

Power Correction of Functionals

- Toy function $\quad f = \exp[-C_1 \ln^2(b_T Q)] \exp[-C_2 b_T^2]$

- Errors of truncated functionals follow expected power law

$$S[f](q_T) = S^{(n)}[f] + \frac{1}{q_T}\mathcal{O}[(b_T^{\mathrm{cut}} q_T)^{-n+\frac{1}{2}}]$$
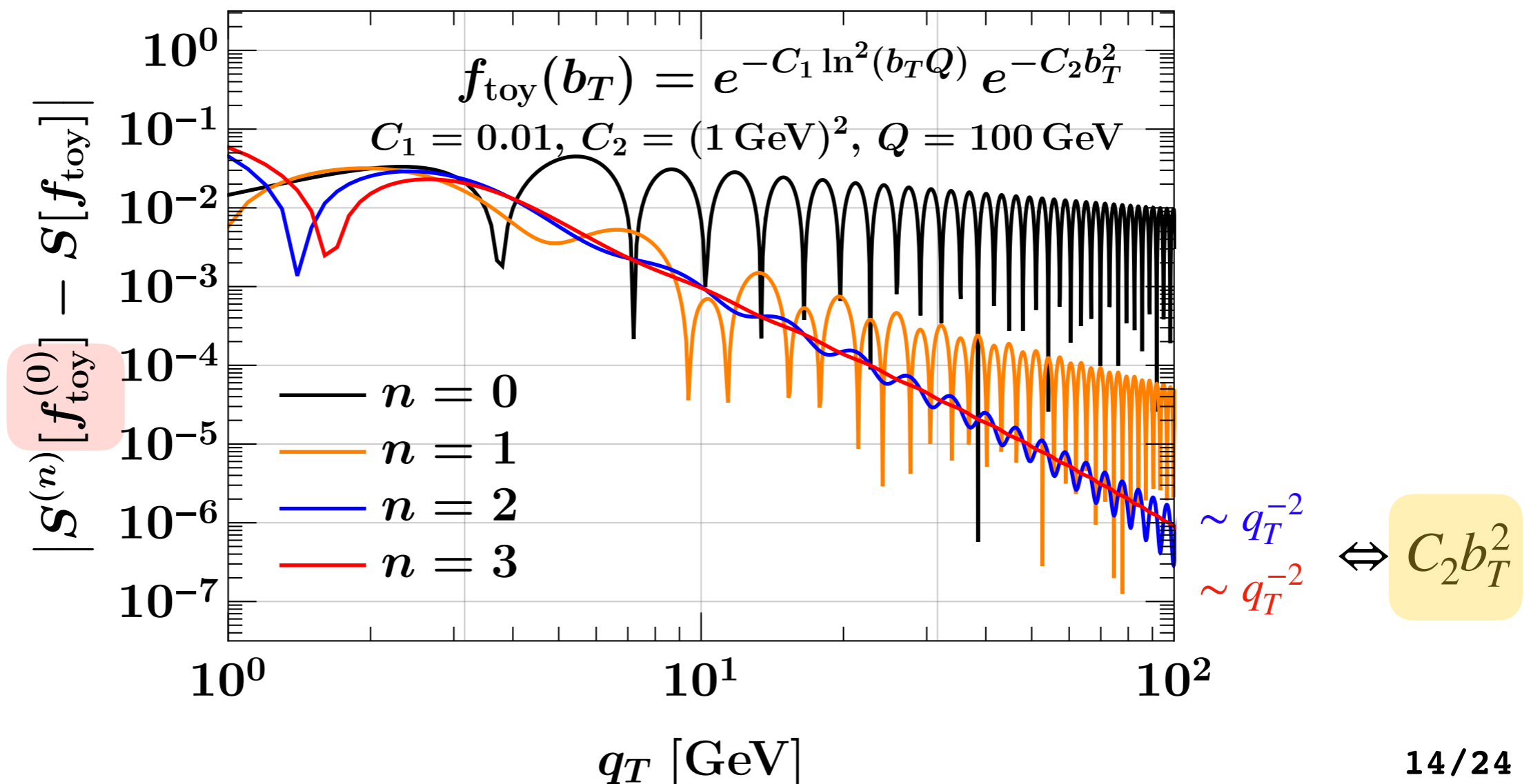
Power Correction of Functionals

- Power expand toy function and use only "perturbative" input $f^{(0)}$

$$f = \underbrace{\exp[-C_1 \ln^2(b_T Q)]}_{f^{(0)}}(1 - C_2 b_T^2 + \mathcal{O}(b_T^4))$$

- "Errors" of truncated functionals identify missing quadratic term

$$f_{\text{toy}}(b_T) = e^{-C_1 \ln^2(b_T Q)} e^{-C_2 b_T^2}$$

$$C_1 = 0.01, \ C_2 = (1\,\text{GeV})^2, \ Q = 100\,\text{GeV}$$

$\left| S^{(n)}[f_{\text{toy}}^{(0)}] - S[f_{\text{toy}}] \right|$

- $n = 0$
- $n = 1$
- $n = 2$
- $n = 3$

$\sim q_T^{-2}$
$\sim q_T^{-2}$ $\Leftrightarrow C_2 b_T^2$

q_T [GeV]

Cumulative Functionals

- We are often interested in the cumulative distribution:

$$\int_{|k_T| \leq k_T^{\mathrm{cut}}} d^2\vec{k}_T \, f(k_T) = \int_{|k_T| \leq k_T^{\mathrm{cut}}} d^2\vec{k}_T \int \frac{d^2 b_T}{(2\pi)^2} \, e^{+i\vec{k}_T \cdot \vec{b}_T} f(b_T)$$

$$= \int_0^{k_T^{\mathrm{cut}}} dk_T \, k_T \int_0^\infty db_T \, b_T \, J_0(b_T k_T) f(b_T)$$

$$= \underbrace{k_T^{\mathrm{cut}} \int_0^\infty db_T \, J_1(b_T k_T^{\mathrm{cut}}) \, f(b_T)}_{K[f](k_T^{\mathrm{cut}})}$$

- Approximate using perturbative region:

$$K^{(0)}[f](k_T^{\mathrm{cut}}, b_T^{\mathrm{cut}}) = k_T^{\mathrm{cut}} \int_0^{b_T^{\mathrm{cut}}} db_T \, J_1(b_T k_T^{\mathrm{cut}}) \, f(b_T)$$

Cumulative Functionals

- Systematically add on power corrections so $K^{(n)}[f] \to K[f]$

$$K^{(0)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = k_T^{\text{cut}} \int_0^{b_T^{\text{cut}}} db_T \, J_1(b_T k_T^{\text{cut}}) \, f(b_T)$$

$$K^{(1)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = K^{(0)}[f] + f(b_T^{\text{cut}}) \cdot \frac{\cos(b_T^{\text{cut}} k_T^{\text{cut}} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\text{cut}} k_T^{\text{cut}})^{1/2}}$$

$$K^{(2)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = K^{(1)}[f] - \left(\frac{f(b_T^{\text{cut}})}{8 \, b_T^{\text{cut}} k_T^{\text{cut}}} - \frac{f'(b_T^{\text{cut}})}{k_T^{\text{cut}}} \right) \cdot \frac{\cos(b_T^{\text{cut}} k_T^{\text{cut}} + \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\text{cut}} k_T^{\text{cut}})^{1/2}}$$
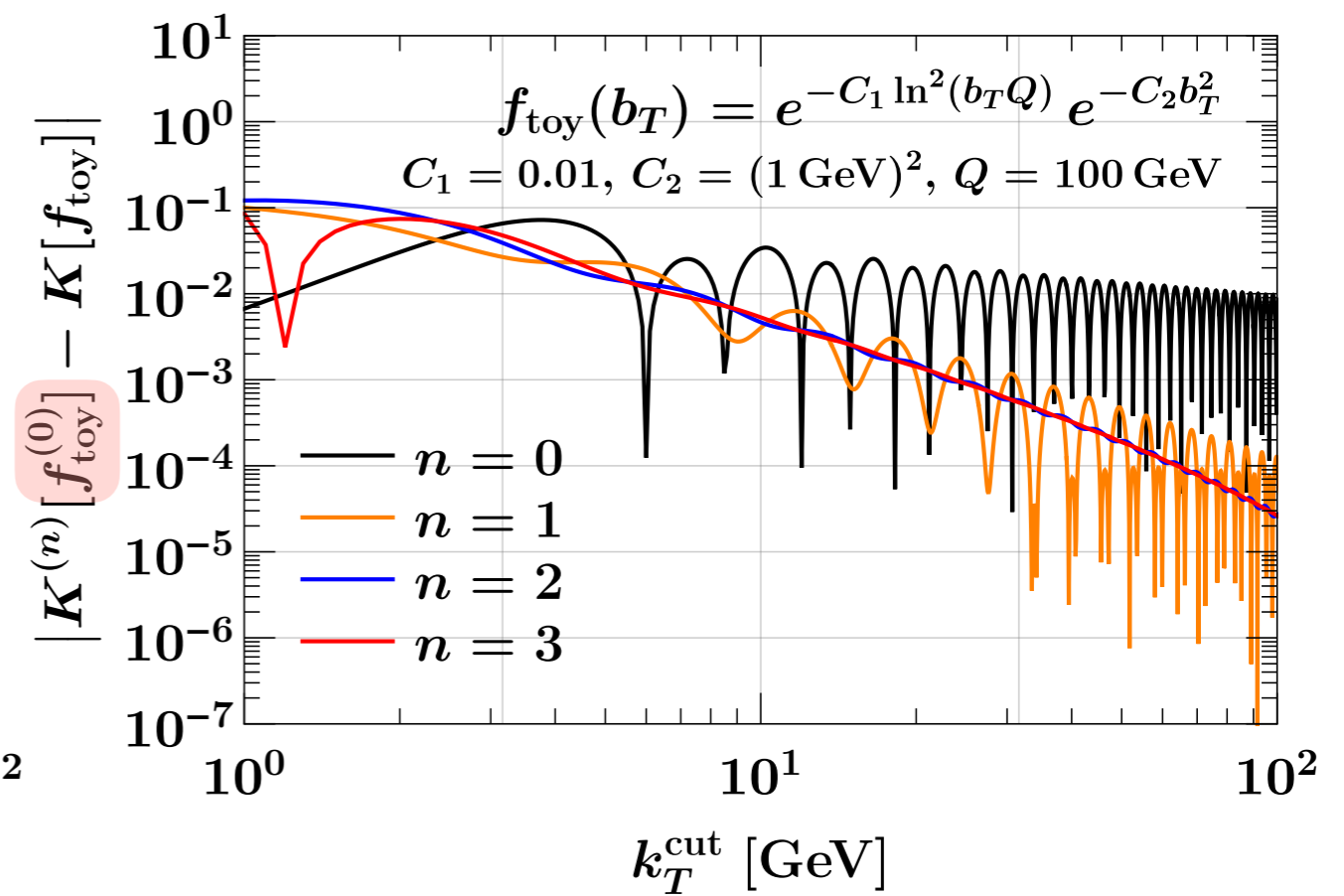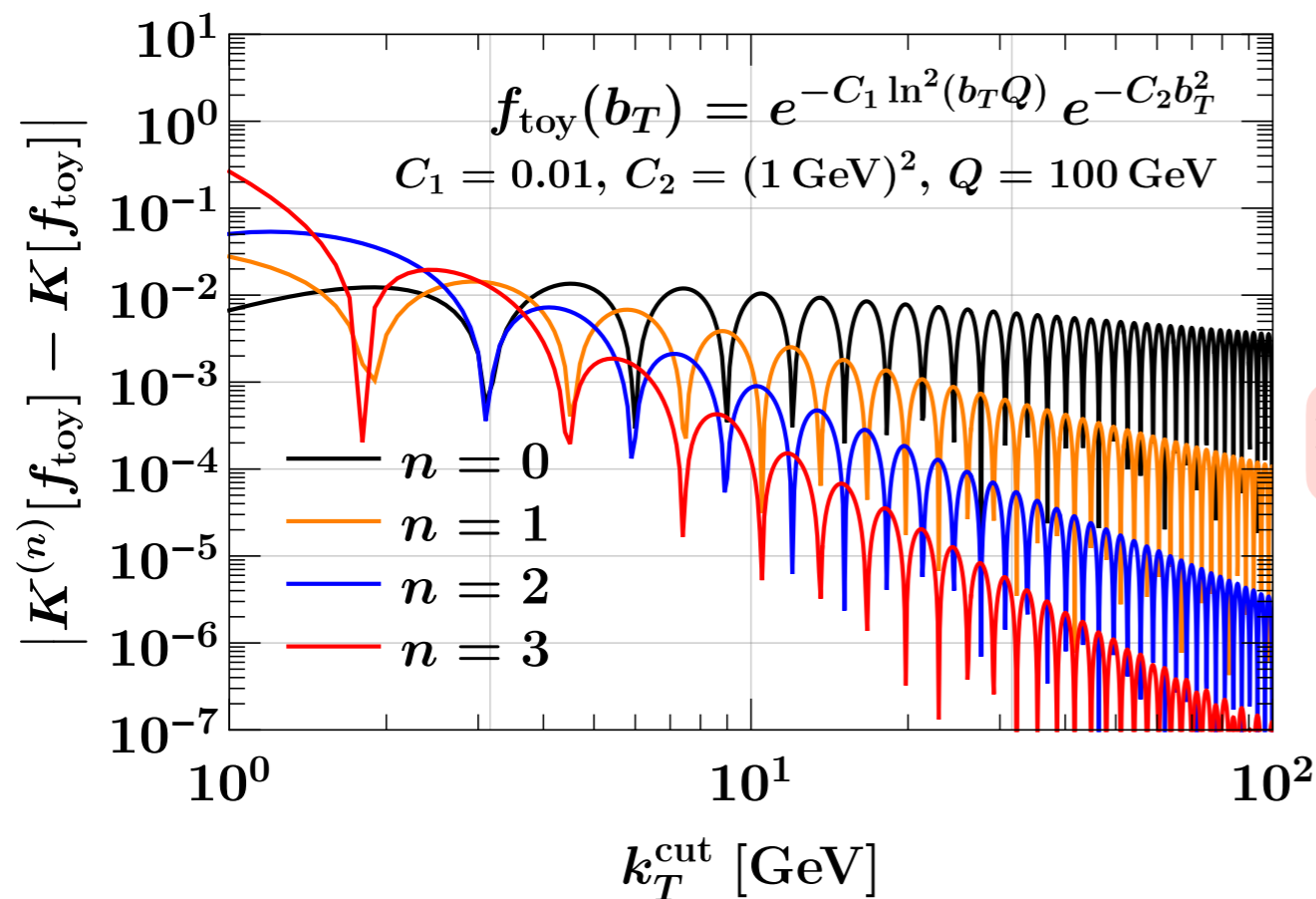
$$K^{(3)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) = K^{(2)}[f] - \left(\frac{9 f(b_T^{\text{cut}})}{128 \, b_T^{\text{cut}\,2} k_T^{\text{cut}\,2}} - \frac{5 f'(b_T^{\text{cut}})}{8 \, b_T^{\text{cut}} k_T^{\text{cut}\,2}} + \frac{f''(b_T^{\text{cut}})}{k_T^{\text{cut}\,2}} \right) \cdot \frac{\cos(b_T^{\text{cut}} k_T^{\text{cut}} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\text{cut}} k_T^{\text{cut}})^{1/2}}$$

$$\boxed{K[f](k_T^{\text{cut}}) = K^{(n)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} k_T^{\text{cut}})^{-n-\frac{1}{2}}]}$$

Power Correction of $K^{(n)}$

- Same toy function $\quad f = \exp[-C_1 \ln^2(b_T Q)] \exp[-C_2 b_T^2]$

- Errors of truncated functionals follow expected power law

$$K[f](k_T^{\text{cut}}) = K^{(n)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} k_T^{\text{cut}})^{-n-\frac{1}{2}}]$$
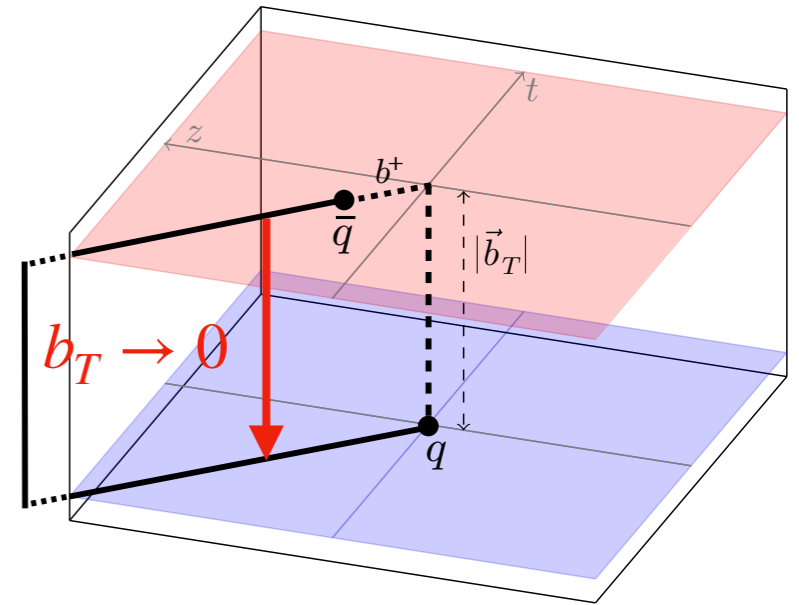
Apply Cumulative Functionals

- What's the normalization of the TMDPDFs?

$$\int d^2\vec{k}_T \, f^{\mathrm{TMD}}(x, k_T, \mu, \zeta) \stackrel{?}{=} f^{\mathrm{coll}}(x, \mu)$$

naively yes :)

- Renormalization breaks the naive expectation

$$\mu \frac{d}{d\mu} \int d^2\vec{k}_T \, f^{\mathrm{TMD}}(x, k_T, \mu, \zeta) \neq \mu \frac{d}{d\mu} f^{\mathrm{coll}}(x, \mu)$$

renormalization says no :(

Apply Cumulative Functionals

- We can systematically compute the normalization of the TMDPDF

- Cut off the integral in UV with k_T^{cut}

$$\int_{|k_T| \leq k_T^{\text{cut}}} d^2 \vec{k}_T \, f^{\text{TMD}}(k_T) = K[f^{\text{TMD}}](k_T^{\text{cut}})$$

- Apply cumulative functional $K^{(3)}$ to compute the normalization!

$$K[f^{\text{TMD}}](k_T^{\text{cut}}) = K^{(3)}[f^{\text{TMD}}](k_T^{\text{cut}}, b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} k_T^{\text{cut}})^{-\frac{7}{2}}]$$

Recall: we have perturbative knowledge of f^{TMD} from the OPE

$$f_{\text{pert},i}^{\text{TMD}} = \sum_j \int \frac{dz}{z} C_{ij}(\frac{x}{z}, b_T, \mu, \zeta) f_j^{\text{coll}}(z, \mu)$$

- Include N3LL evolution from boundary condition at $\zeta_0 \sim 1/b_T^2$ to overall ζ

- OPE matching coefficients up to two loops

- As implemented by `SCETlib`

Impact of Evolution Effects

$$\int^{k_T^{\mathrm{cut}}} \mathrm{d}^2\vec{k}_T \, f_d(x, \vec{k}_T, \mu, \zeta)/f_d(x,\mu) - 1$$
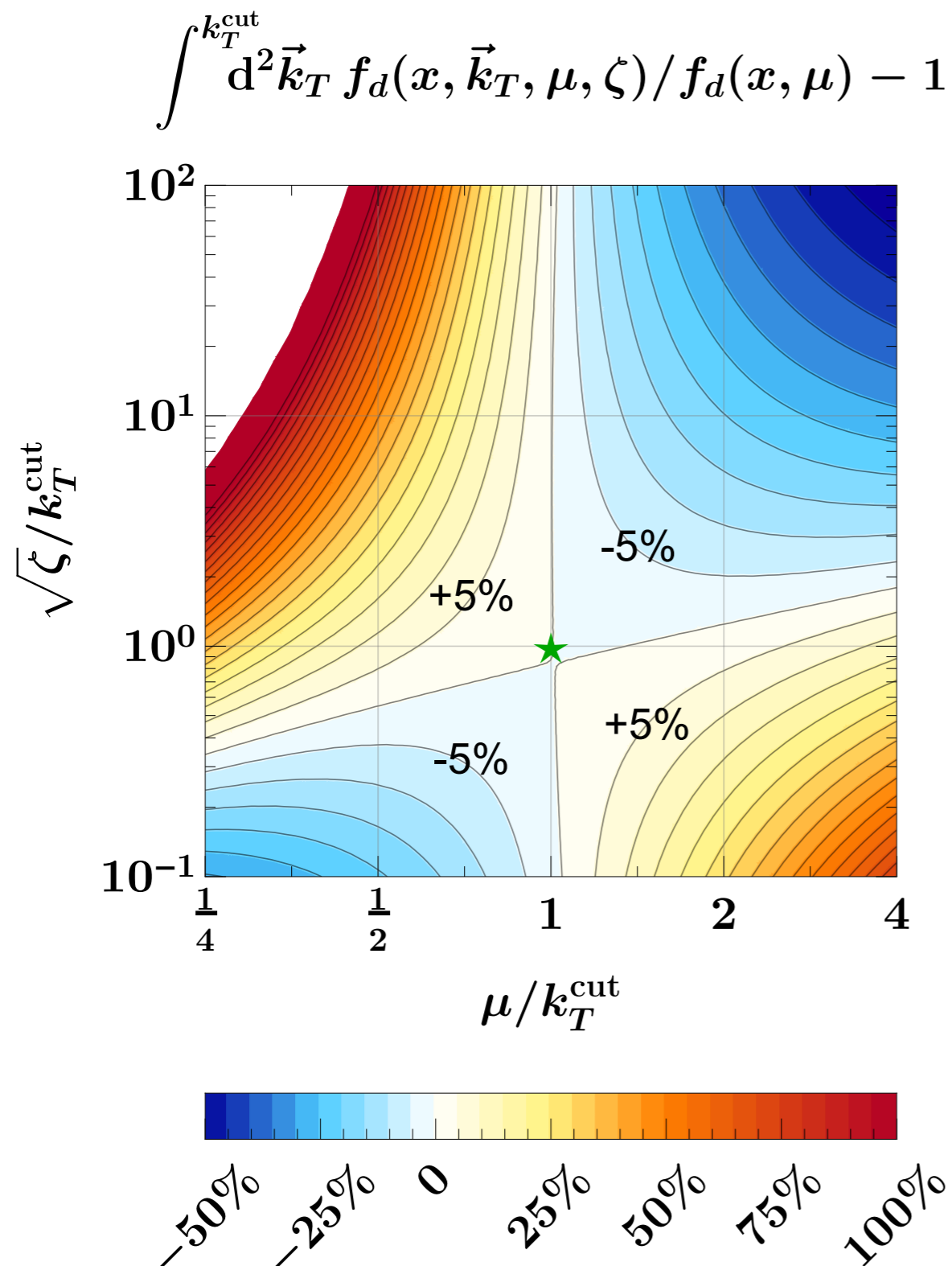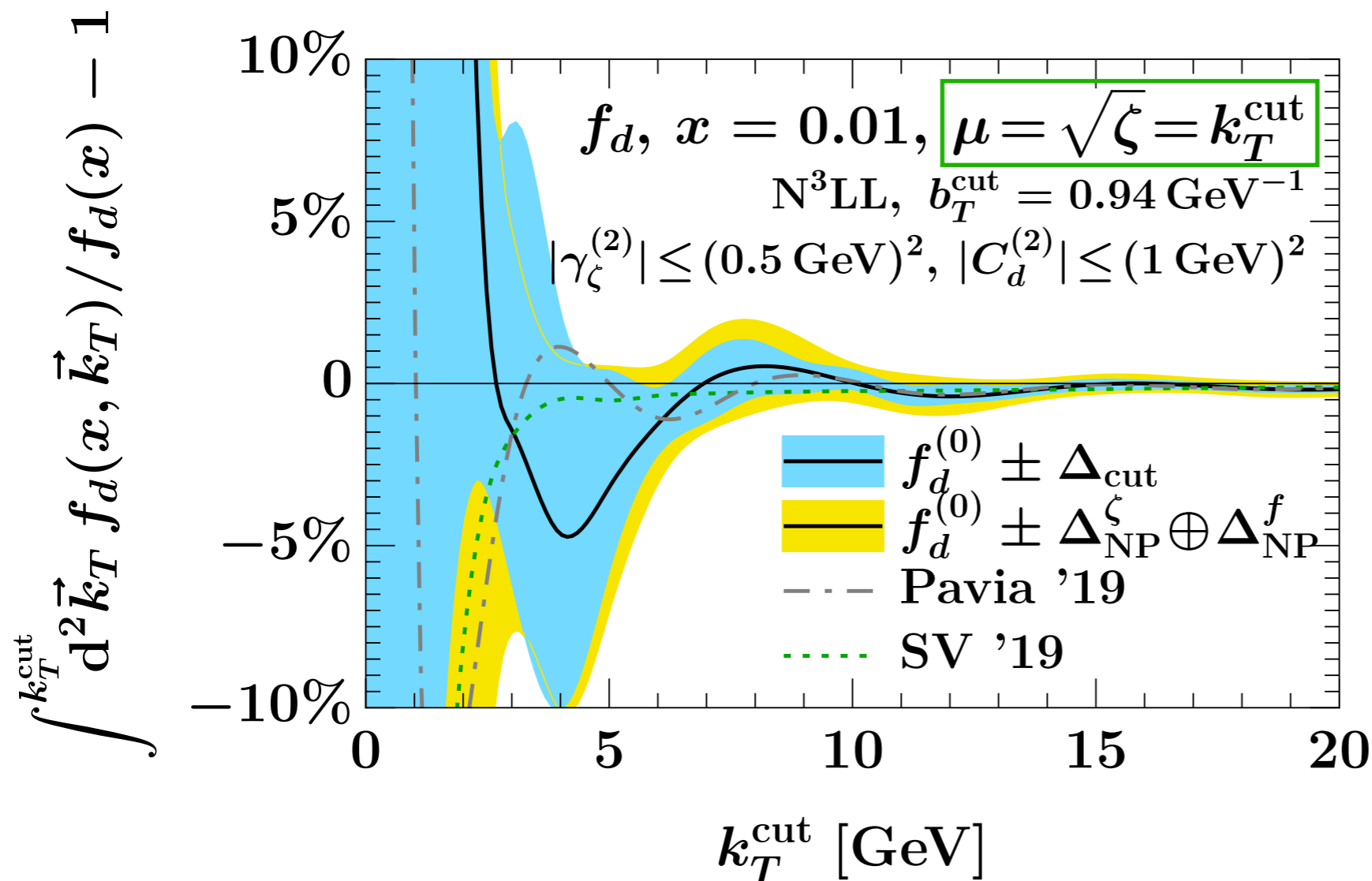

- $K^{(3)}[f^{\mathrm{TMD}}]$ allows us to model-independently access the evolution effects

- Intuitive expectation is robust in the vicinity of $\mu = \sqrt{\zeta} = k_T^{\mathrm{cut}}$

- For $\mu = k_T^{\mathrm{cut}}$, the ζ evolution is negligible

- Sizable corrections from evolution away from these regions, due to the cusp anomalous dimension

$$\int^{k_T^{\mathrm{cut}}} d^2\vec{k}_T \, f^{\mathrm{TMD}}(x, k_T, \mu = k_T^{\mathrm{cut}}, \zeta) \stackrel{\star}{=} f^{\mathrm{coll}}(x, \mu = k_T^{\mathrm{cut}})$$

Impact of Nonperturbative Effects

$$f_{\text{TMD}}(x, b_T, \mu, \zeta) = f_{\text{TMD}}^{(0)}(x, b_T, \mu, \zeta_0) \left[1 + \left(C^{(2)} - \frac{1}{2}L_\zeta \gamma_\zeta^{(2)}\right) b_T^2\right] + \mathcal{O}(b_T^4)$$

boundary condition

evolution kernel

$f_d, \ x = 0.01, \ \boxed{\mu = \sqrt{\zeta} = k_T^{\text{cut}}}$

$\text{N}^3\text{LL}, \ b_T^{\text{cut}} = 0.94 \, \text{GeV}^{-1}$

$|\gamma_\zeta^{(2)}| \leq (0.5 \, \text{GeV})^2, \ |C_d^{(2)}| \leq (1 \, \text{GeV})^2$

$\int^{k_T^{\text{cut}}} \mathrm{d}^2\vec{k}_T \, f_d(x, \vec{k}_T)/f_d(x) - 1$

$k_T^{\text{cut}} \, [\text{GeV}]$

Legend:
- $f_d^{(0)} \pm \Delta_{\text{cut}}$
- $f_d^{(0)} \pm \Delta_{\text{NP}}^\zeta \oplus \Delta_{\text{NP}}^f$
- Pavia '19
- SV '19

- Δ_{cut} from varying b_T^{cut}
- Small corrections for large k_T^{cut}
- Significant NP effects for small k_T^{cut}
- Agree with SV and Pavia global fits which use $b*$

SV: 1912.06532
Pavia: 1912.07550

Resummation Orders

- Perturbative uncertainty estimated by scale variations *2006.11382*
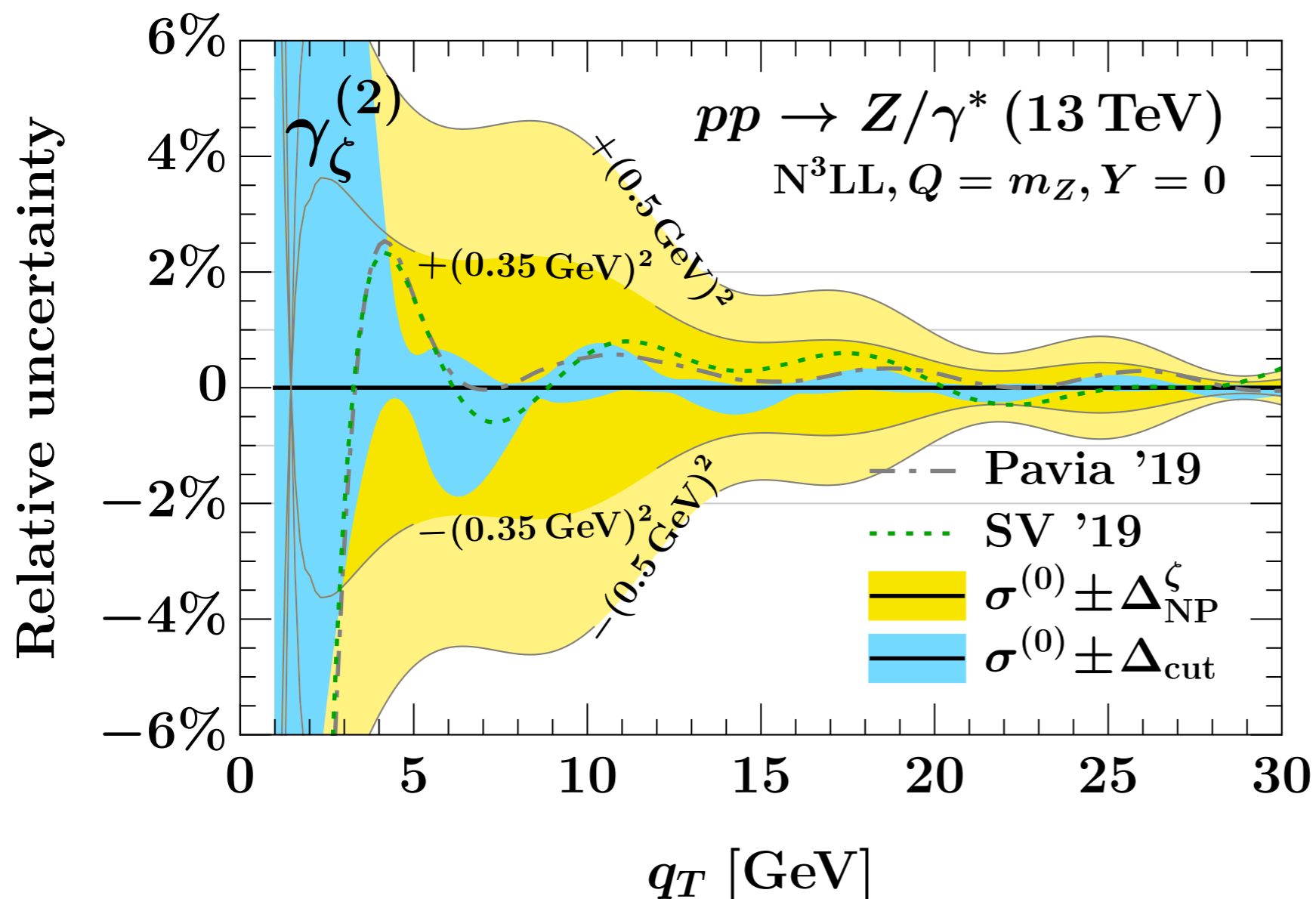
- Increasing orders show convergence
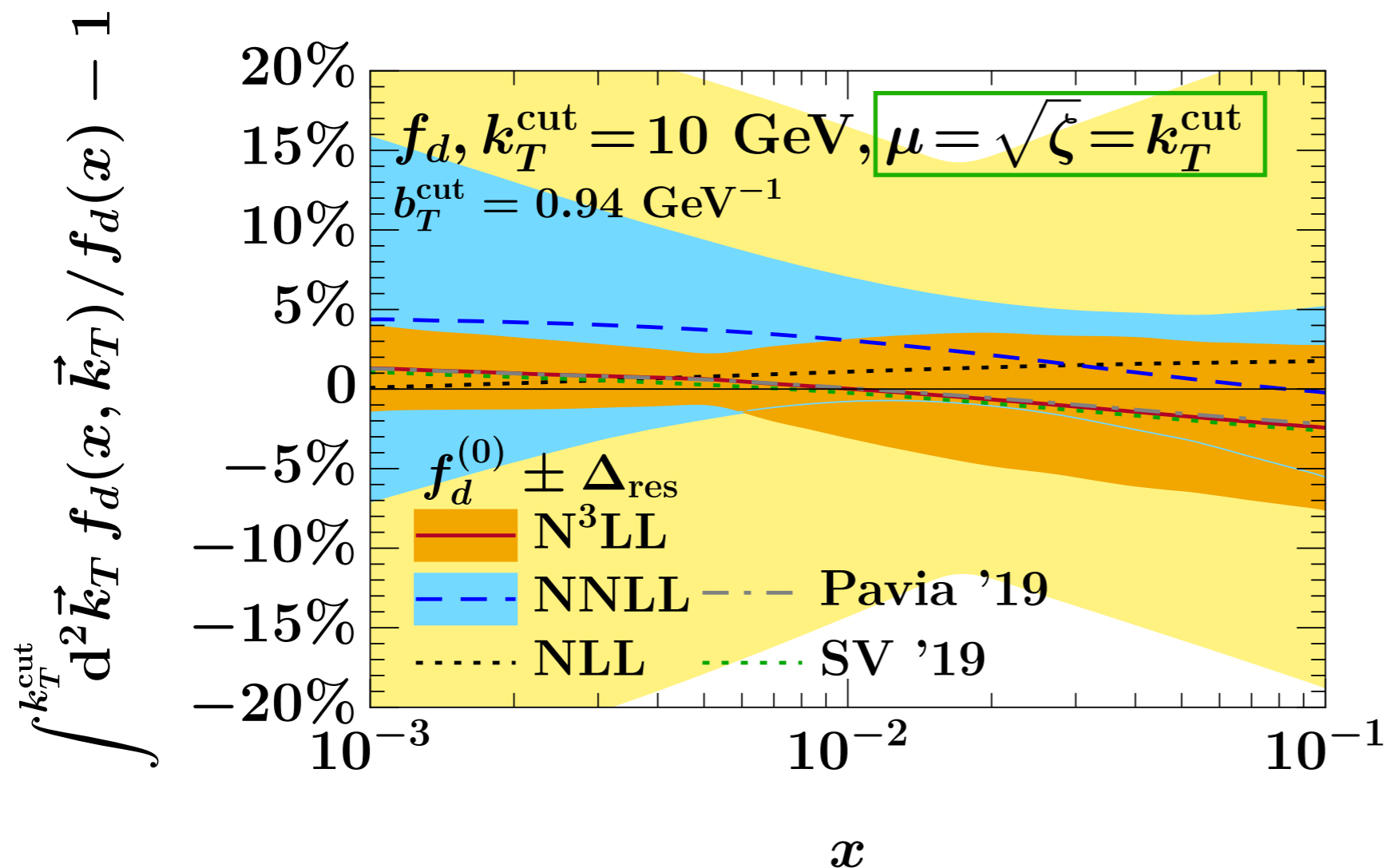
- Resummation uncertainty as a function of x while keeping k_T^{cut} fixed

- Central value can differ from zero ($\pm$ 2%)

- Agree with SV and Pavia global fits

Momentum-Space Spectrum

- Apply same procedures to q_T cross section

$$\sigma(b_T) = H(Q, b_T) f_{\text{TMD}}(x_a, b_T) f_{\text{TMD}}(x_b, b_T) \exp[-L_\zeta \gamma_\zeta]$$
$$= \sigma^{(0)}(b_T) \left[1 + \left(\tilde{C}^{(2)} - L_\zeta \gamma_\zeta^{(2)} \right) b_T^2 + \mathcal{O}(b_T^4) \right]$$

- The impact of $\gamma_\zeta^{(2)}$ is linear (the values chosen are representative and can be rescaled)

- Important to determine $\gamma_\zeta^{(2)}$ nonperturbatively from first principles!
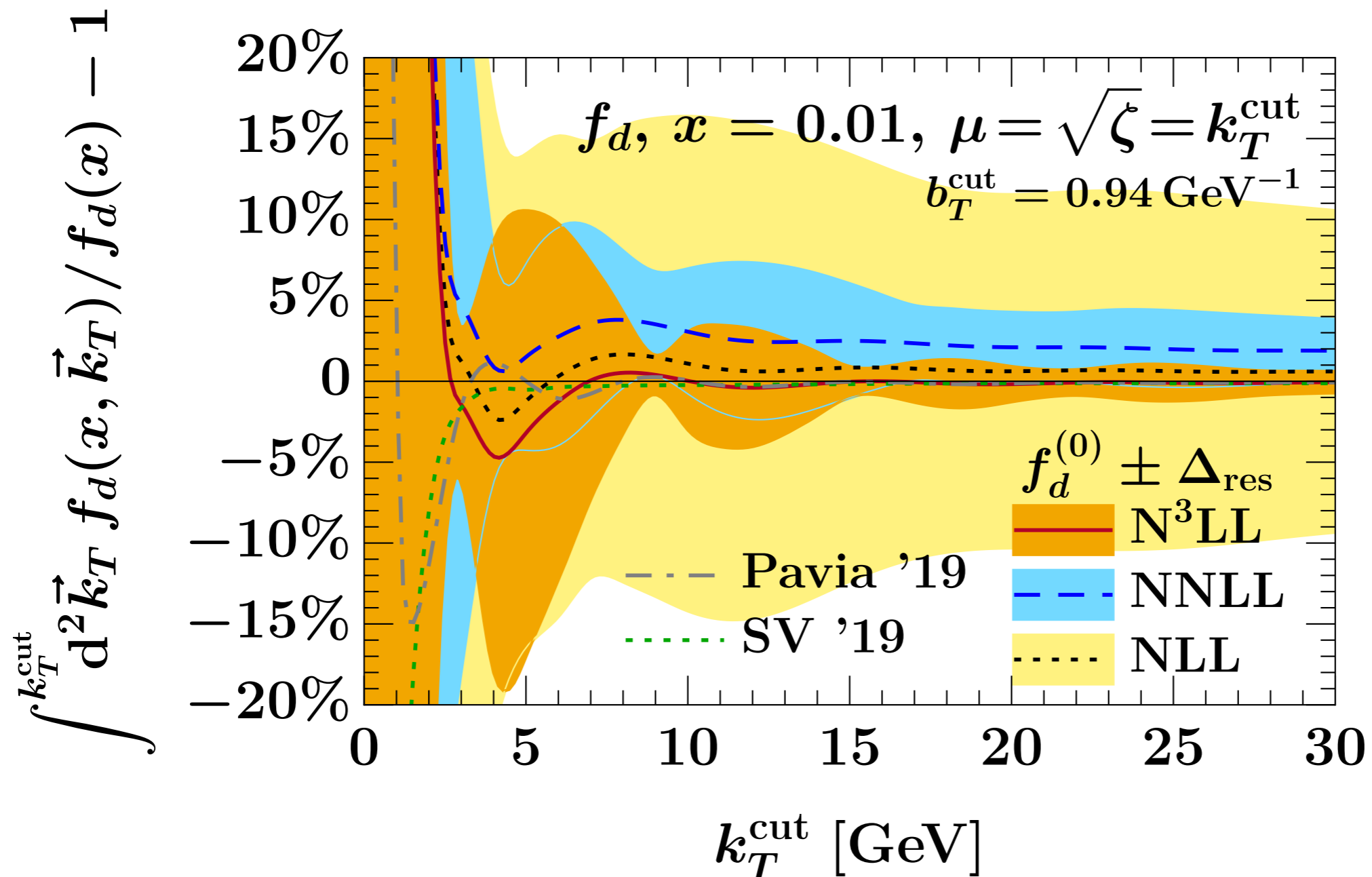
Conclusions

- Perturbative and nonperturbative physics in TMDPDFs are usually hard to disentangle because of $b*$ prescriptions

- Truncated functionals provide a model-independent and systematically improvable method to map perturbative results from position to momentum space

- Demonstrated that integrating the unpolarized TMDPDF over $[0, k_T^{\mathrm{cut}}]$ gives the collinear PDF to the percent level (when renormalization scale $\mu = k_T^{\mathrm{cut}}$)

- Developed model-independent method to assess the impact of non-perturbative effects (OPE coefficients) in momentum space

Thank you!!!

Back-up Slides

Resummation Orders

- Resummation orders: as a function of k_T^{cut}

More on NP effects