

Review on Linear Divergence

LaMET 2021

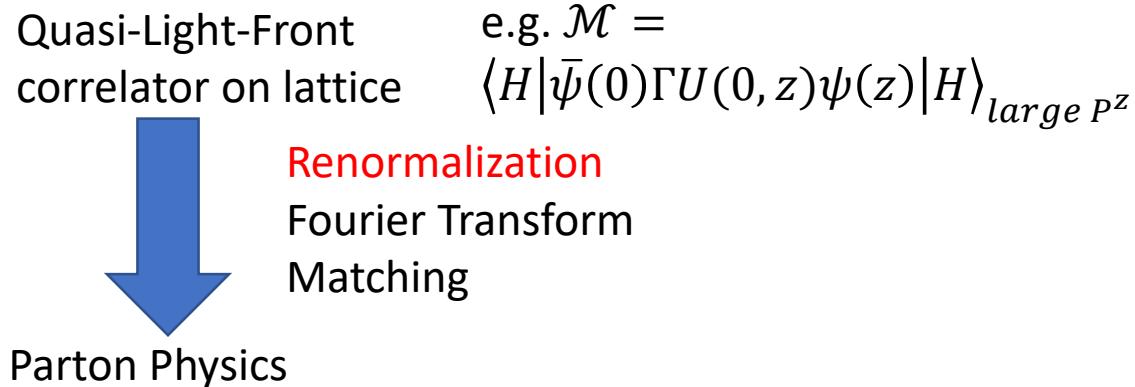
Yushan Su

University of Maryland

Background

X. Ji, Phys. Rev. Lett. 110, 262002 (2013)

Large Momentum Effective Theory (LaMET) proposes an effective way to get parton physics from lattice data:



Many nice works:

PDF (Parton Distribution Function)

C. Alexandrou et al. PRL (2018)

Y.-S. Liu et al.(Lattice Parton), PRD (2020)

T. Izubuchi et al. PRD (2019)

Z. Fan et al. PRD (2020)

GPD (Generalized Parton Distribution)

J.-W. Chen et al. NPB (2020)

C. Alexandrou et al. PoSLATTICE2019

DA (Distribution Amplitude)

J.-H. Zhang et al. PRD (2017)

R. Zhang et al. PRD (2020)

J.-W. Chen et al. arXiv:1803.04393

H.-W. Lin et al. PRL (2018)

J.-H. Zhang et al. PRD (2019)

Y. Chai et al., PRD (2020)

TMD (Transverse Momentum Dependent) Distributions

P. Shanahan et al. PRD (2020)

P. Shanahan et al. PRD (2020)

Q.-A. Zhang et al.(Lattice Parton), PRL (2020)

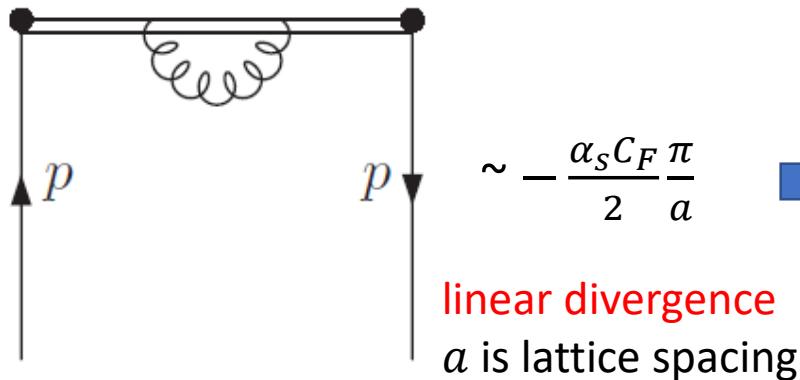
OutLine

- Why linear divergence is a problem?
- What is renormalization?
- Problem with RI/MOM Scheme
- Problem with Ratio Scheme
- Self-renormalization
- Hybrid-Renormalization

Why linear divergence is a problem?

$$\mathcal{M} = \langle H | \bar{\psi}(0) \Gamma \textcolor{red}{U}(0, z) \psi(z) | H \rangle_{large P^z}$$

Self-Energy of Wilson Link



Chen, Ji, Zhang, NPB (2017)

$$\sim -\frac{\alpha_s C_F}{2} \frac{\pi}{a}$$



$$\mathcal{M}(z, a) = \text{Exp} \left[-\frac{m_1}{a} z \right] f(z, a)$$

linear divergence factor

Ishikawa, Ma, Qiu, Yoshida,
arXiv:1609.02018

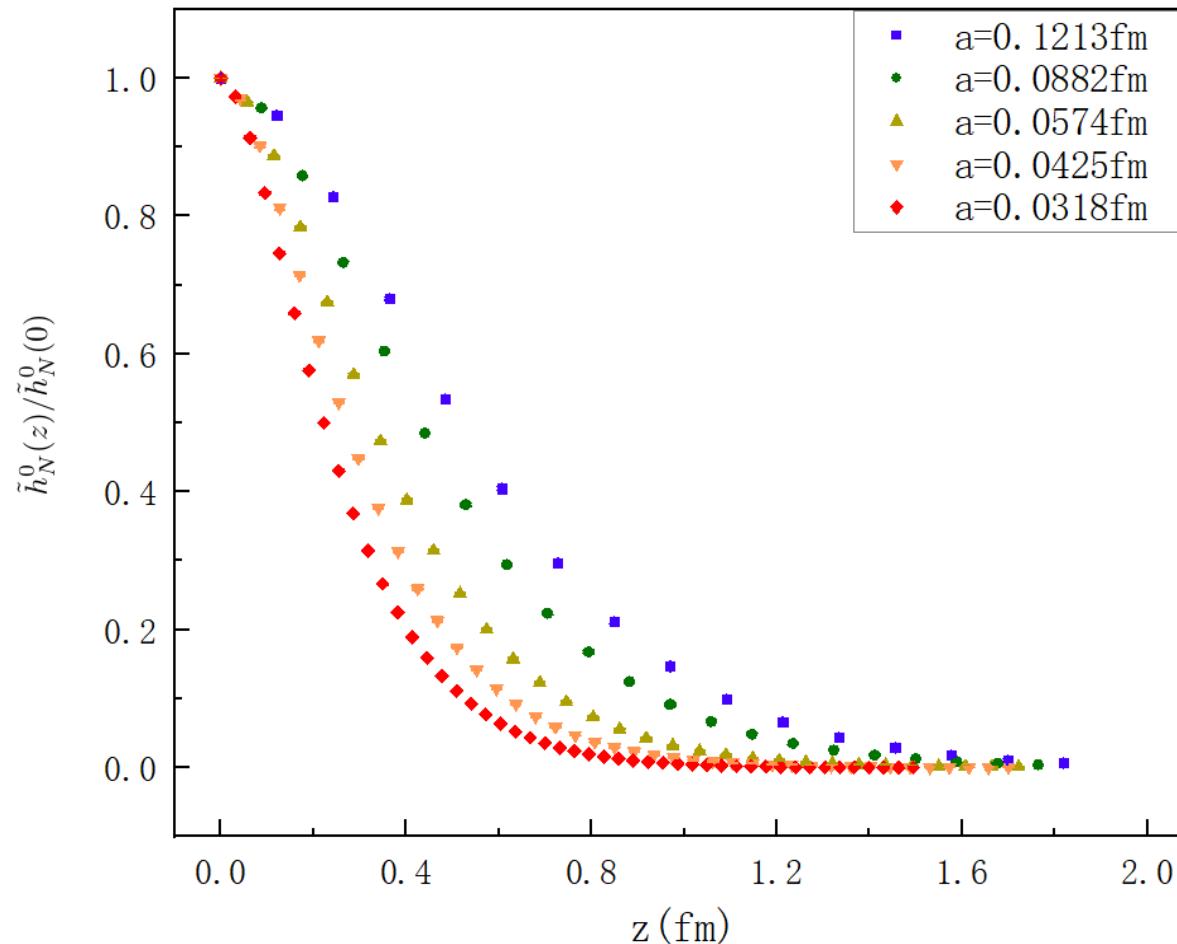
Ji, Zhang, Zhao, PRL (2018)

Why linear divergence is a problem?

$$\mathcal{M}(z, a) = \text{Exp} \left[-\frac{m_{-1}}{a} z \right] f(z, a)$$

linear divergence factor reduces
the data significantly at large z

bare nucleon
matrix element
in zero
momentum



What is renormalization?

$$\mathcal{M}_B(z, a) = Z_R(z, a, \mu) \mathcal{M}_R(z, \mu)$$

UV divergence factor

$$\mathcal{M}_R(z, \mu) = \frac{\mathcal{M}_B(z, a)}{Z_R(z, a, \mu)}$$

Purposes of renormalization:

1. Get rid of the UV divergence factor Z_R and take continuum limit ($a \rightarrow 0$)
2. Avoid introducing extra non-perturbative effect.
 Z_R should be IR free.

Problem with RI/MOM Scheme

RI/MOM is widely used in literature:

C. Alexandrou et al. NPB (2017)

J. Green et al. PRL (2018)

J.-W. Chen et al. PRD (2018)

I. Stewart and Y.Z., PRD (2018)

Large Momentum Hadron state matrix element

$$\mathcal{M}(z, a, P_z) = \langle H | \bar{\psi}(0) \gamma^t U(0, z) \psi(z) | H \rangle_{large P_z} \sim \text{Exp} \left[-\frac{m_{-1,h}}{a} z \right] \mathcal{M}_R(z, P_z)$$

RI/MOM factor, quark state matrix element

$$Z_{RI}(z, a) = \frac{1}{4N_c} \text{Tr} [\gamma_t \langle q | \bar{\psi}(0) \gamma^t U(0, z) \psi(z) | q \rangle] \Big|_{p^2 = -\mu_R^2, p_z = p_t = 0} \sim \text{Exp} \left[-\frac{m_{-1,q}}{a} z \right] Z_{RI,R}(z)$$

Renormalized matrix element

$$\mathcal{M}_{RI}(z, a, P_z) = \frac{\mathcal{M}(z, a, P_z)}{Z_{RI}(z, a)} \sim \frac{\text{Exp} \left[-\frac{m_{-1,h}}{a} z \right] \mathcal{M}_R(z, P_z)}{\text{Exp} \left[-\frac{m_{-1,q}}{a} z \right]} Z_{RI,R}(z)$$

Large Euclidean momentum,
perturbative state

Extra non-perturbative
effect if z is large

Residual linear divergence in RI/MOM renormalization

Kuan Zhang et al. PRD (2021)

Problem with Ratio Scheme

A. V. Radyushkin, PRD (2017)

K. Orginos et al. PRD (2017)

T. Izubuchi et al. PRD (2018)

Large Momentum Hadron state matrix element

$$\mathcal{M}(z, a, P_z) = \langle H | \bar{\psi}(0) \gamma^t U(0, z) \psi(z) | H \rangle_{large \ P_z} \sim \text{Exp} \left[-\frac{m_{-1}}{a} z \right] \mathcal{M}_R(z, P_z)$$

Zero Momentum Hadron state matrix element

$$\mathcal{M}(z, a, P_z = 0) = \langle H | \bar{\psi}(0) \gamma^t U(0, z) \psi(z) | H \rangle_{P_z=0} \sim \text{Exp} \left[-\frac{m_{-1}}{a} z \right] \mathcal{M}_R(z, P_z = 0)$$

Non-perturbative state

Renormalized matrix element

$$\mathcal{M}_{ratio}(z, a) = \frac{\mathcal{M}(z, a, P_z)}{\mathcal{M}(z, a, P_z = 0)} \sim \frac{\text{Exp} \left[-\frac{m_{-1}}{a} z \right] \mathcal{M}_R(z, P_z)}{\text{Exp} \left[-\frac{m_{-1}}{a} z \right] \mathcal{M}_R(z, P_z = 0)} = \frac{\mathcal{M}_R(z, P_z)}{\mathcal{M}_R(z, P_z = 0)}$$

Extra non-perturbative effect caused by non-perturbative state and large z

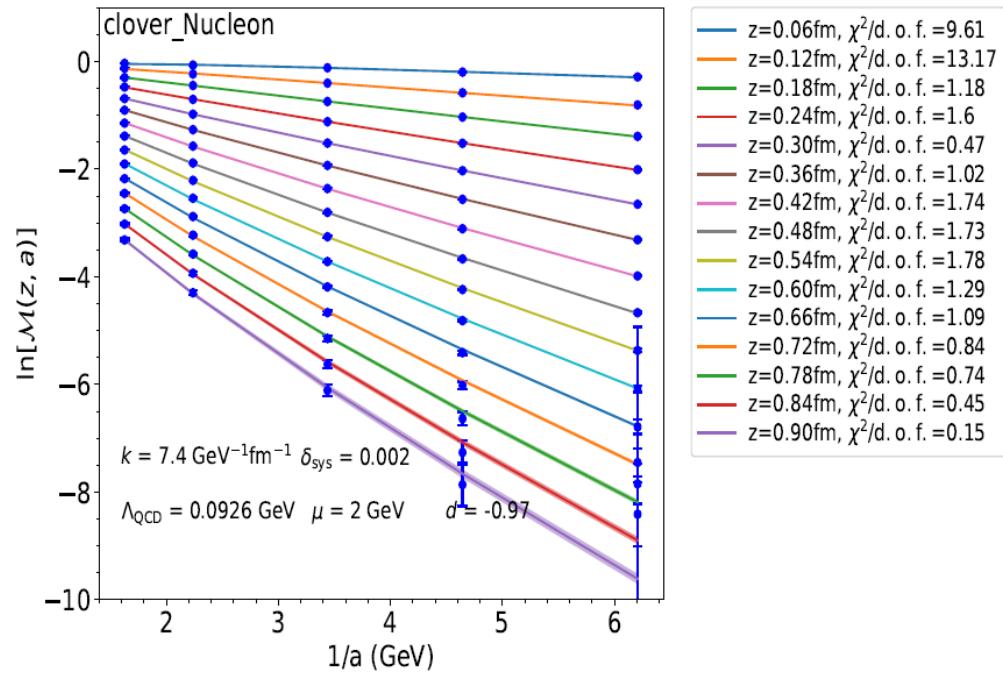
Self-renormalization

How to disentangle the UV divergence factor from lattice matrix elements?

“Self Renormalization”: determine the perturbative renormalization factor $Z(z, a)_R$ through fitting the data

$$\mathcal{M}(z, a) = Z(z, a)_R \mathcal{M}_R(z)$$

$$Z(z, a)_R \sim \text{Exp} \left[-\frac{m_{-1}}{a} z \right]$$



Fitting functions are determined by perturbation theory:

$$\ln(\mathcal{M}) = \frac{kz}{a \ln(a \Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a \Lambda_{QCD})} \right]$$

Requirements on data:

1. multi lattice spacings a ; 2. data with high precision

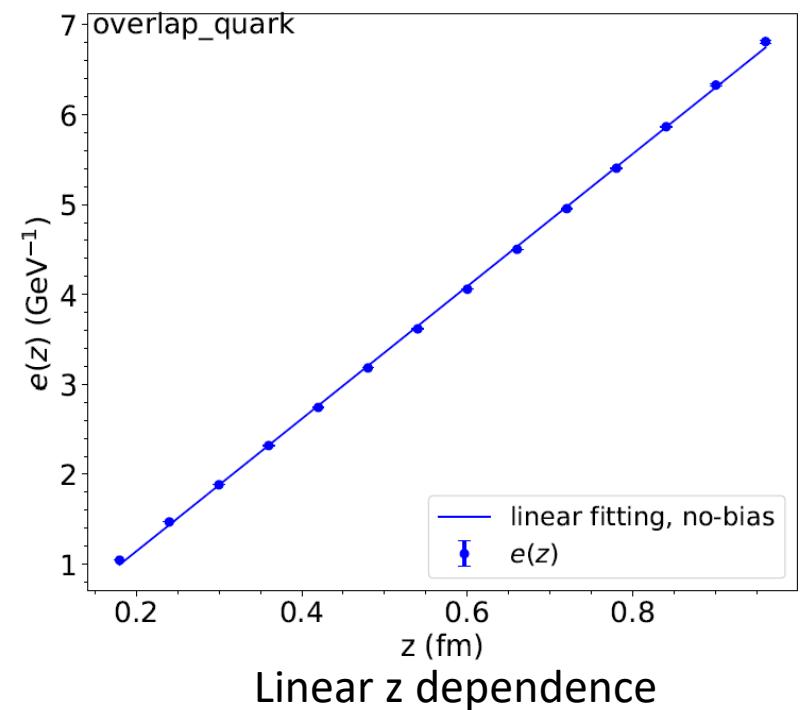
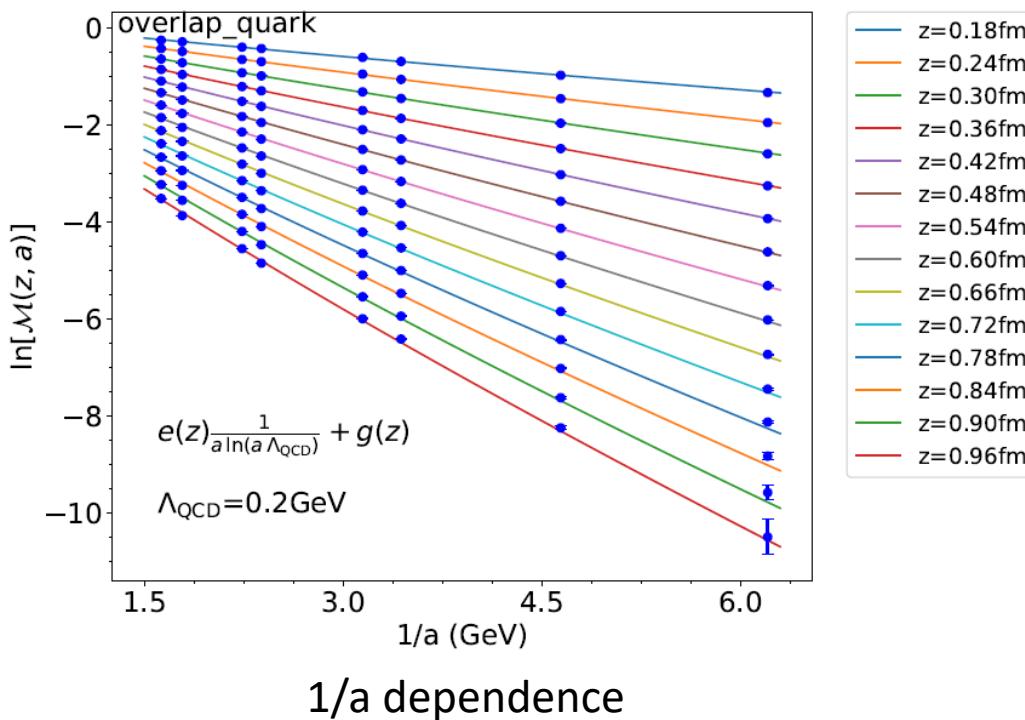
Self-renormalization

Check whether lattice data have the linear divergence factor

$$\mathcal{M}(z, a) = \text{Exp} \left[-\frac{m_{-1}}{a} z \right] f(z, a)$$

We have tested that linear divergence factors exist in RI/MOM factor, zero momentum hadron matrix element, Wilson Loop...

e.g. for RI/MOM factor:

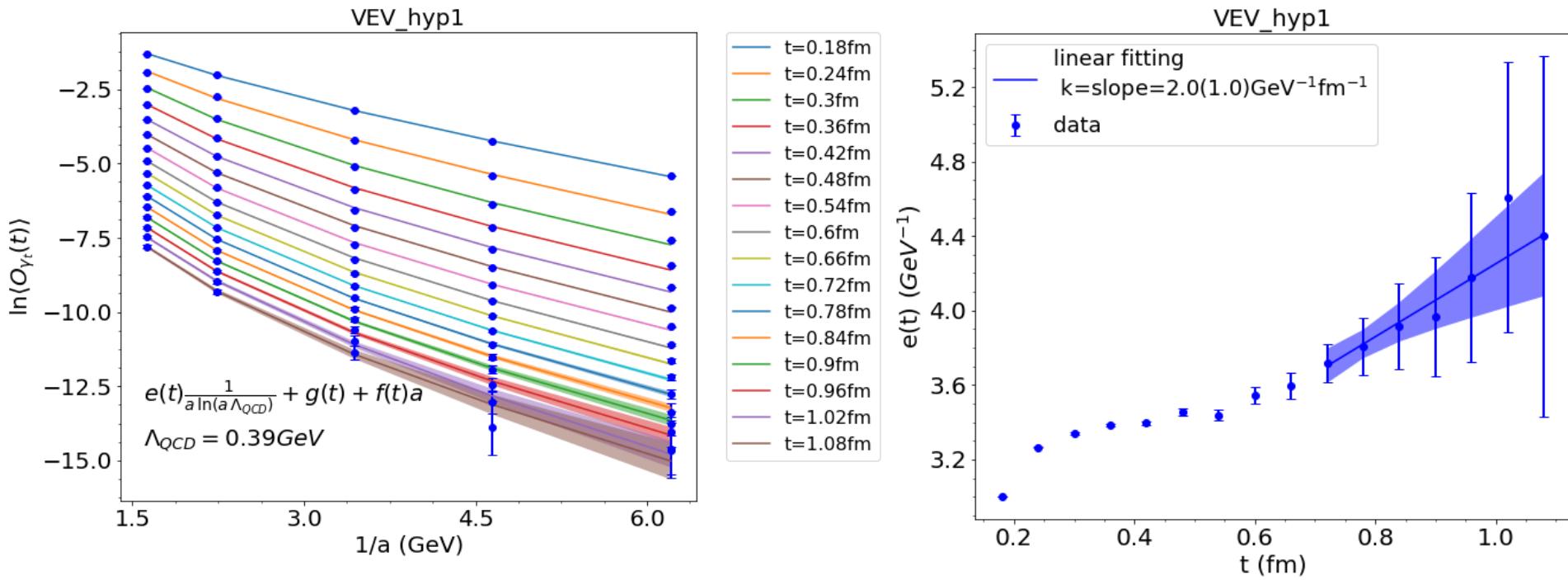


Self-renormalization

Check whether lattice data have the linear divergence factor

$$\mathcal{M}(z, a) = \text{Exp} \left[-\frac{m_{-1}}{a} z \right] f(z, a)$$

Linear divergence factor doesn't exist in the vacuum expectation value (VEV) of quasi PDF operator: $\langle \bar{\psi}(0)\gamma^t U(0, t)\psi(t) \rangle$



Violation of Linear z dependence

Self-renormalization

Linear divergence parameters

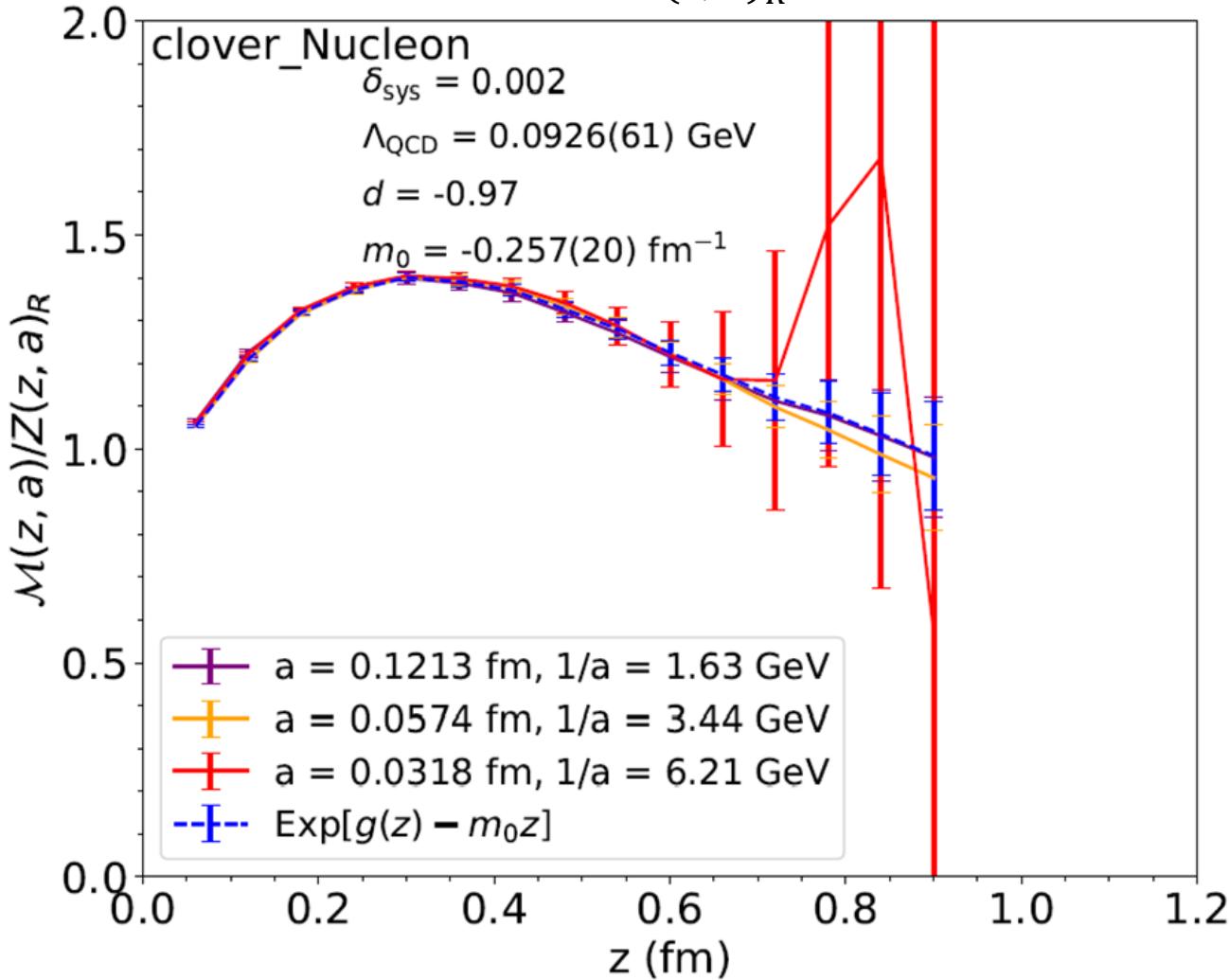
| | Cases | k | $\Lambda_{\text{QCD}}(\text{GeV})$ |
|---------------------------|----------------|---------------------------|------------------------------------|
| RI/MOM factor | overlap quark | 0.5521(07) | 0.39 |
| | clover quark | 0.6328(05) | 0.39 |
| P=0 hadron matrix element | overlap pion | 0.5191(14) | 0.39 |
| | clover pion | 0.5178(18) | 0.39 |
| | clover nucleon | 0.5139(54) | 0.39 |
| | Wilson Loop | 0.4921(02) | 0.39 |
| | VEV | 0.39(21) | 0.39 |
| | Wilson Link | 0.5402(04), 0.7099(09) | 0.39 |

Fitted by $\ln \mathcal{M}(z, a) = \frac{e(z)}{a \ln[a \Lambda_{\text{QCD}}]} + g(z) + f_{1,2}(z)a$,
 k is the slope of $e(z)$

Self-renormalization

Continuum Limit

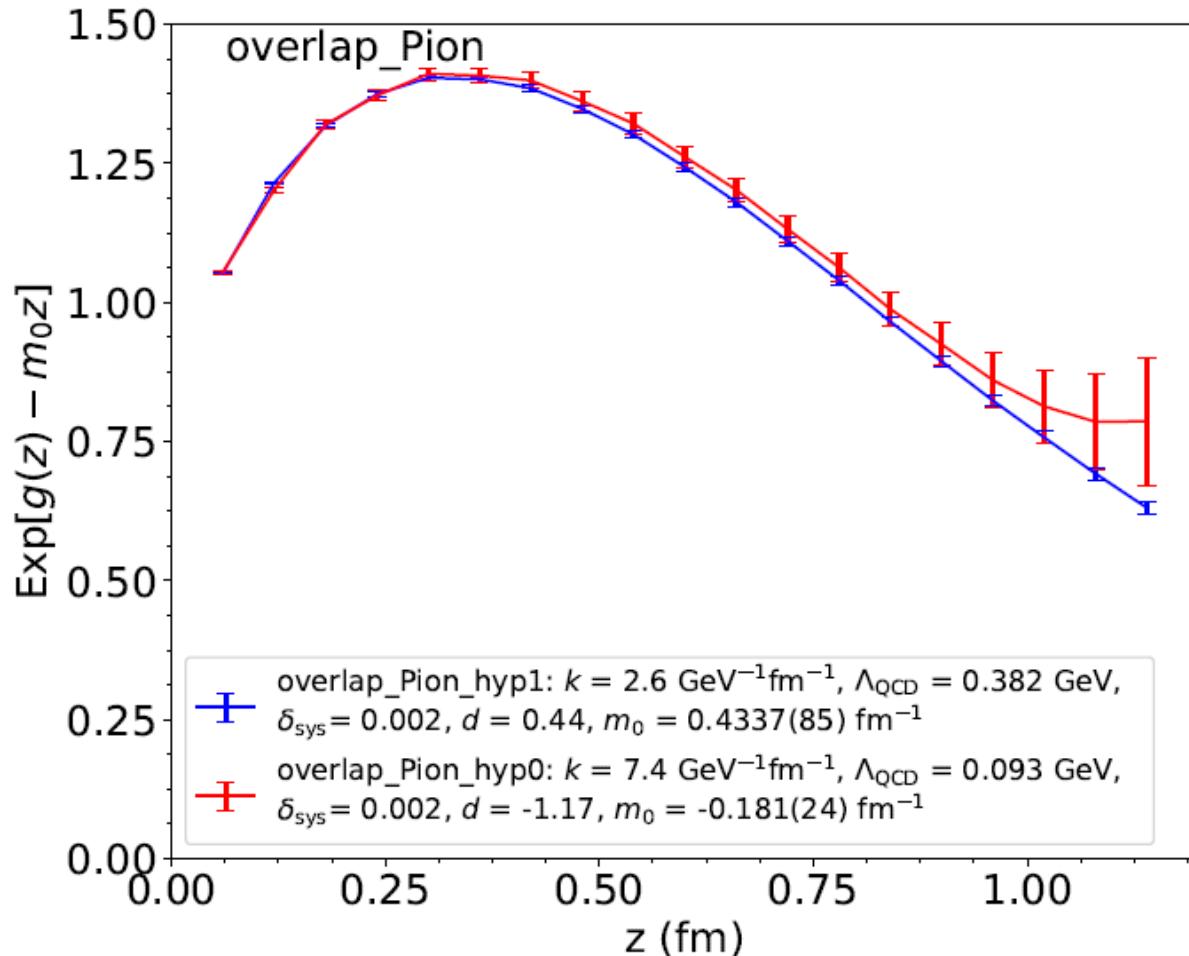
The renormalized matrix element is: $\mathcal{M}_R(z) = \frac{\mathcal{M}(z, a)}{Z(z, a)_R}$



Self-renormalization

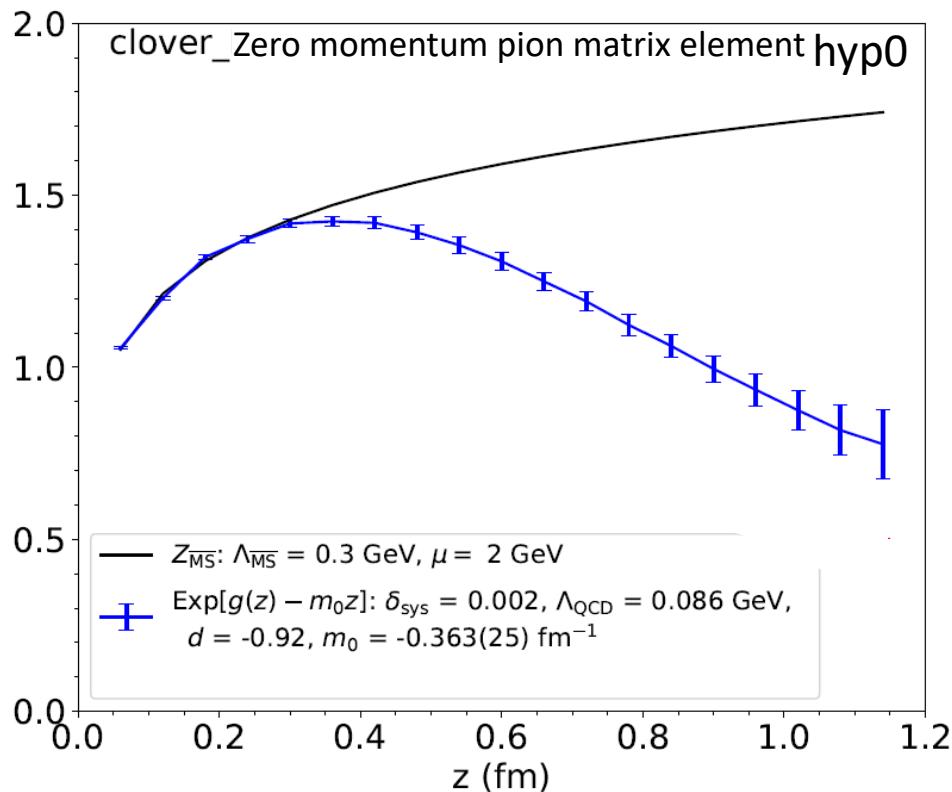
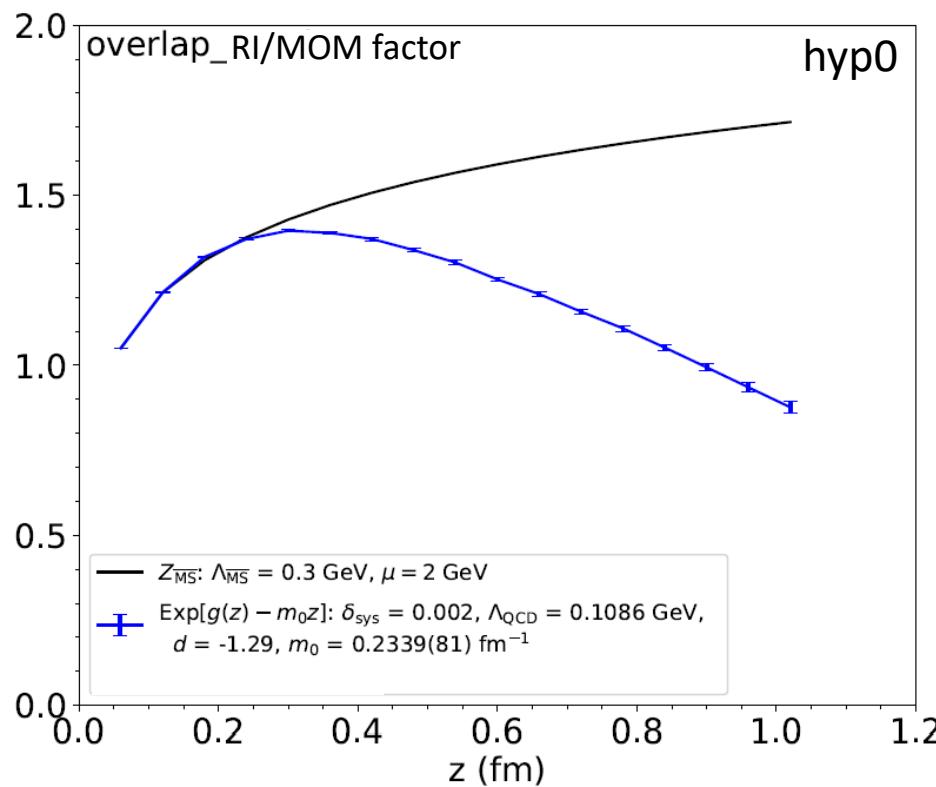
Can we apply the perturbative renormalization (unsmeared theory) to the link smearing cases?

$$\ln(\mathcal{M}) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$



Non-perturbative effect at large z

Compare renormalized lattice matrix elements (blue) with perturbation theory (black)



Large extra non-perturbative effect at large distance in RI/MOM and Ratio scheme, which requires a new renormalization method after RI/MOM and Ratio scheme.

Hybrid Renormalization Method

The renormalized matrix element in Hybrid scheme:

$$\mathcal{M}_R(z, Pz) = \frac{\mathcal{M}(z, a, Pz)}{\mathcal{M}(z, a, Pz = 0)} \theta(z_S - |z|) + \frac{\mathcal{M}(z, a, Pz)}{Z(z, a)_R} Z_{\text{hybrid}}(z_S) \theta(|z| - z_S)$$



Short Distance: Ratio Scheme,
divided by zero momentum
matrix element.

Eliminate discretization effect
at short distance.

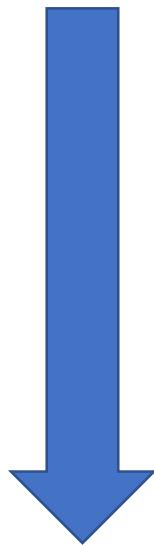


Long Distance: divided by the
renormalization factor $Z(z, a)_R$,
which **only contains UV effect**.
So it will not introduce extra non-
perturbative effect (IR effect).
 $Z(z, a)_R$ can be extracted by **self-**
renormalization method.

e.g. Isovector unpolarized proton PDF @ CLS H102 $a=0.0854$ fm

Lattice Matrix element

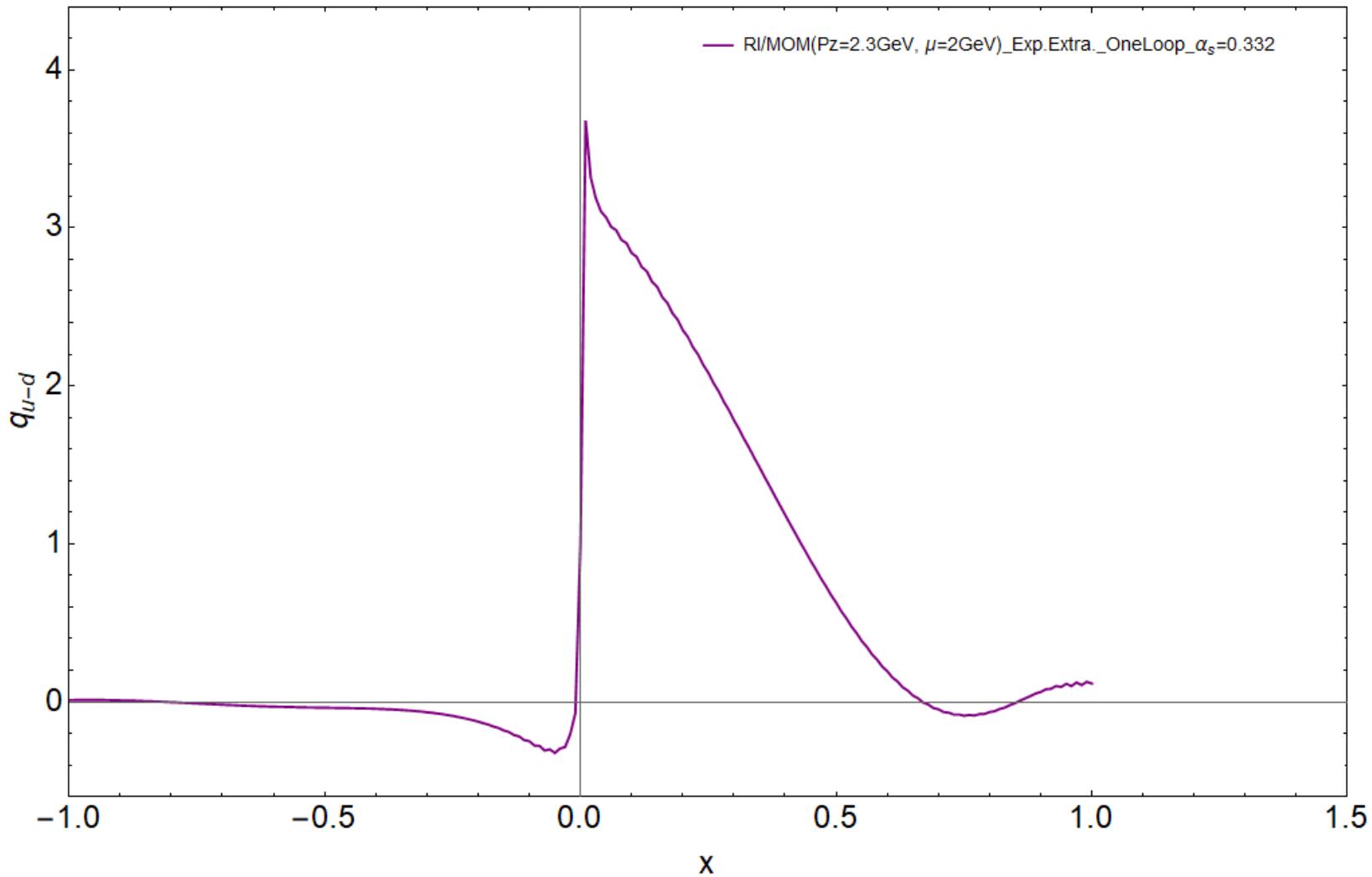
$$\mathcal{M}(z, a, P_z) = \langle \text{proton} | \bar{u}(0) \gamma^t U(0, z) u(z) - \bar{d}(0) \gamma^t U(0, z) d(z) | \text{proton} \rangle_{\text{large } P_z}$$



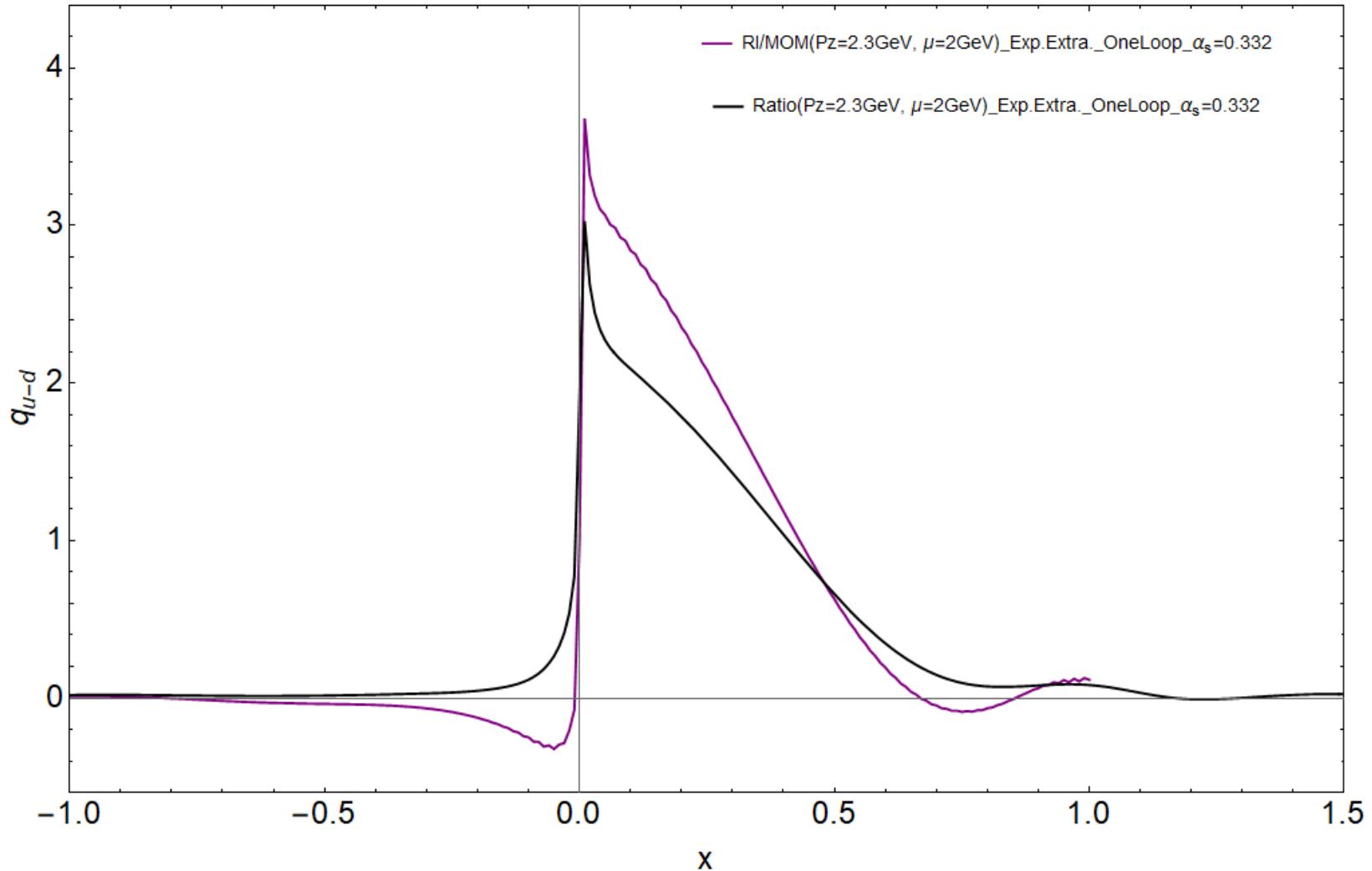
Renormalization
Fourier Transform
Matching

Isovector unpolarized proton PDF

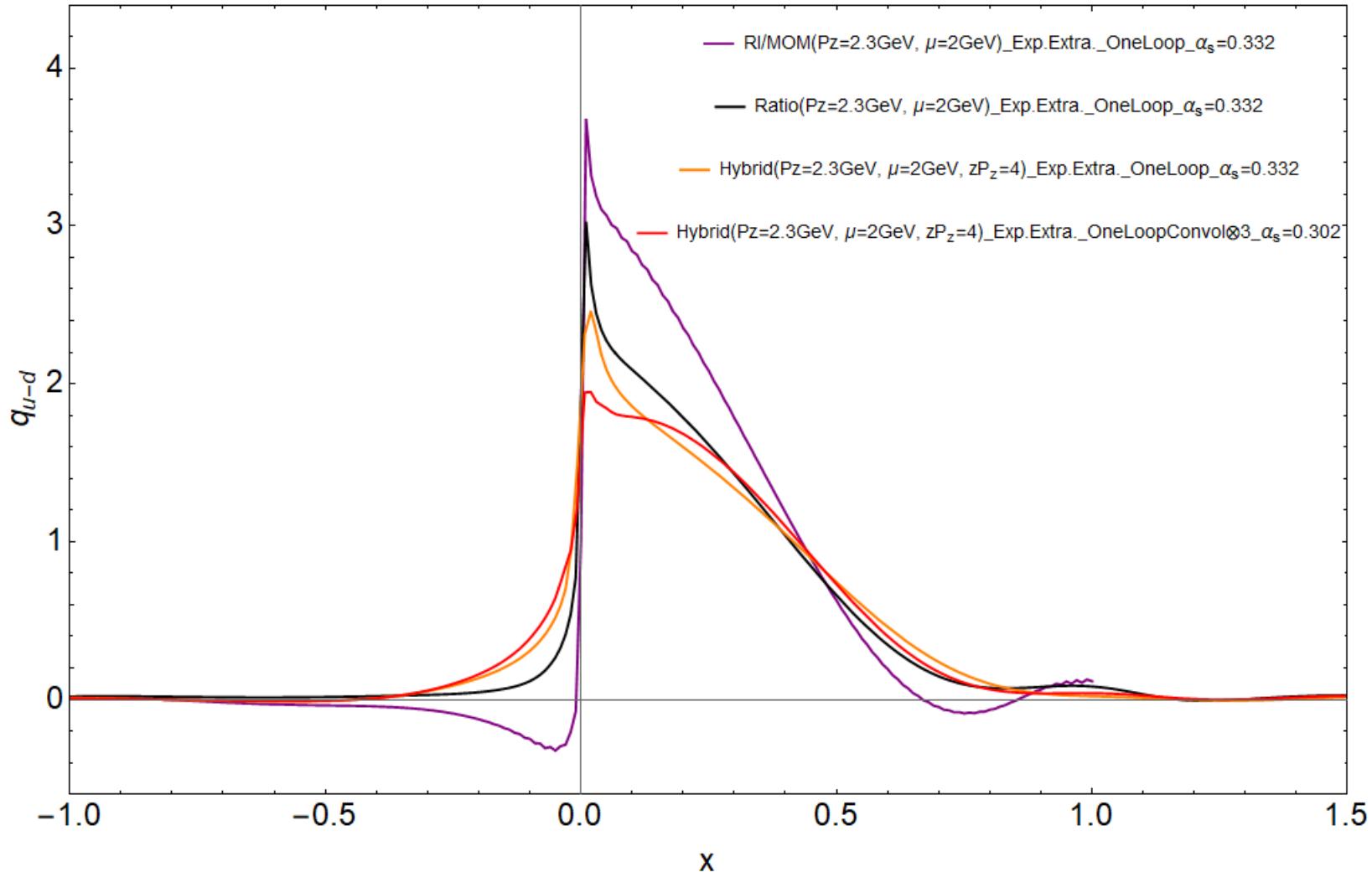
Isovector unpolarized proton PDF @ CLS H102 $\alpha=0.0854$ fm



Isovector unpolarized proton PDF @ CLS H102 $\alpha=0.0854$ fm

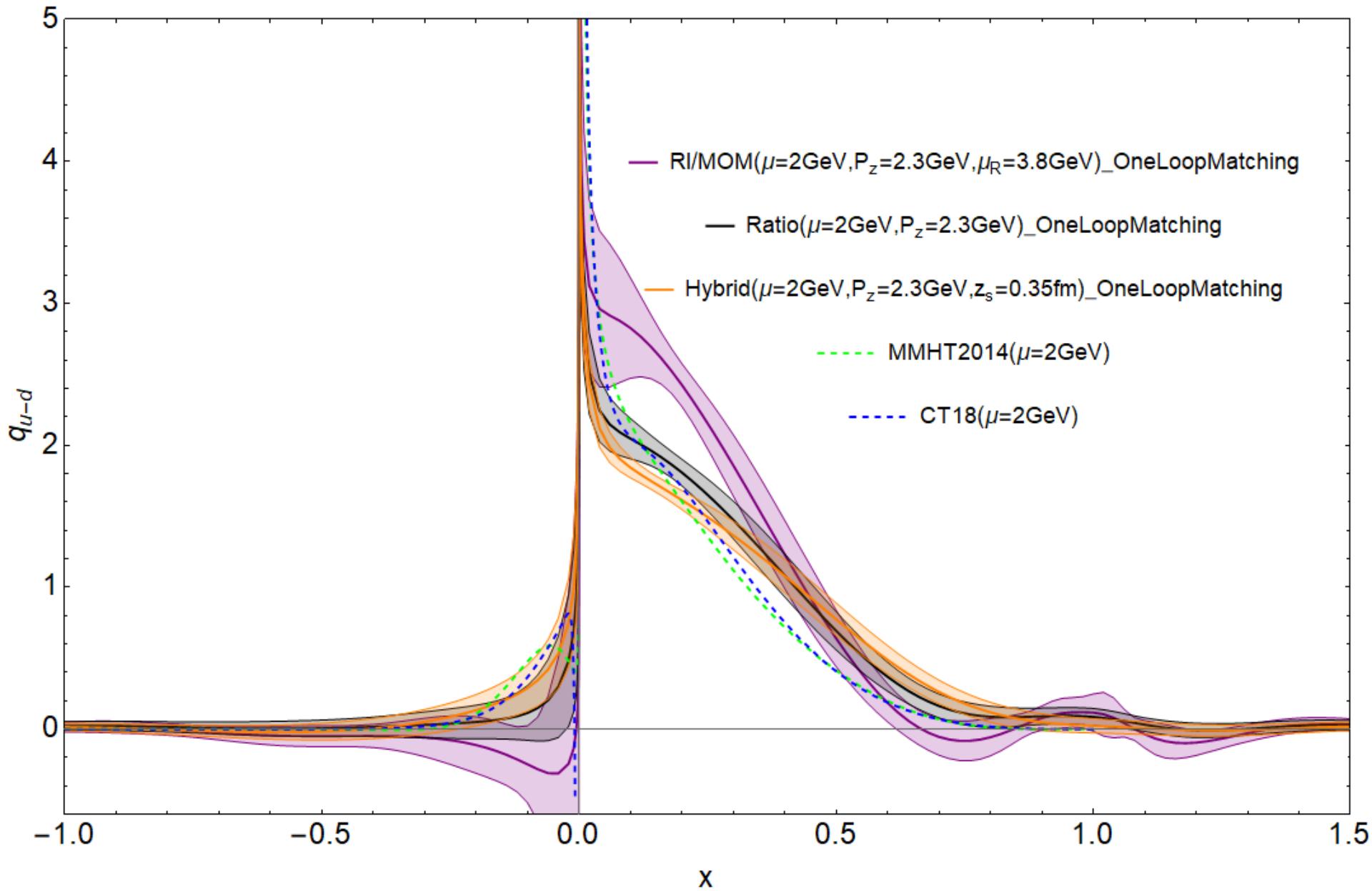


Isovector unpolarized proton PDF @ CLS H102 $\alpha=0.0854$ fm



The large discrepancy between
different renormalization schemes

Isovector unpolarized proton PDF @ CLS H102 $\alpha=0.0854$ fm

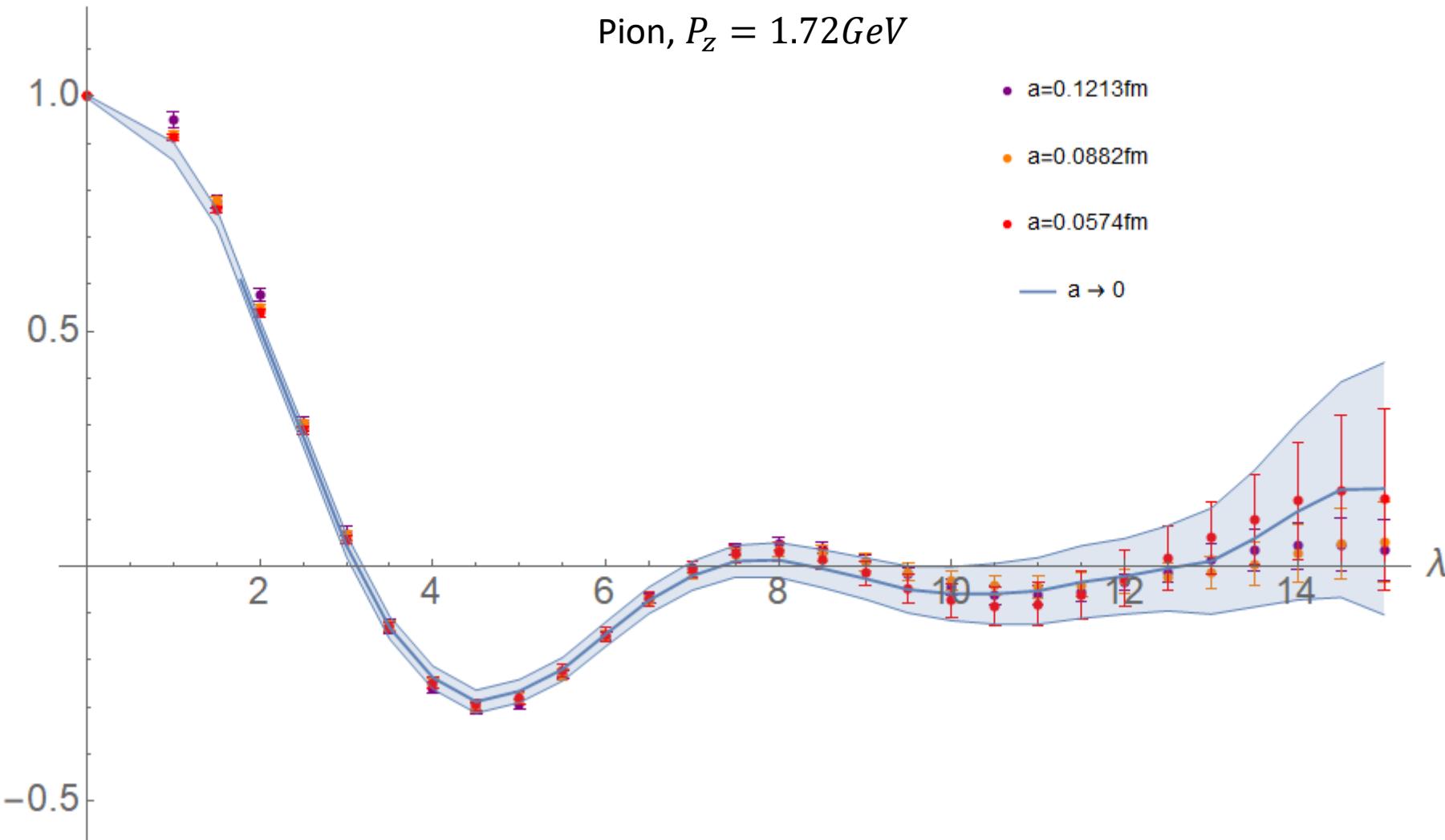


Continuum Limit with Hybrid Renormalization

Re[quasiDAcorr_{Hybrid}]

Hybrid Renormalized quasi DA matrix element

Pion, $P_z = 1.72\text{GeV}$

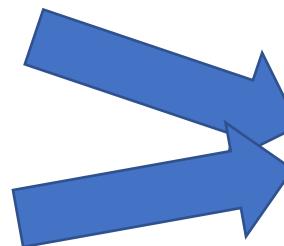


Lattice Parton Collaboration. "Pion and kaon distribution amplitudes..."
Jinchen He will give this talk tomorrow.

Conclusion

Purposes of renormalization:

1. Achieve nice continuum limit
2. Avoid introducing extra non-perturbative effect



Self Renormalization and
Hybrid Renormalization

The renormalized matrix element in Hybrid scheme

$$\mathcal{M}_R(z, P_z) = \frac{\mathcal{M}(z, a, P_z)}{\mathcal{M}(z, a, P_z = 0)} \theta(z_s - |z|) + \frac{\mathcal{M}(z, a, P_z)}{Z(z, a)_R} Z_{\text{hybrid}}(z_s) \theta(|z| - z_s)$$

extracted from lattice data based on
self renormalization