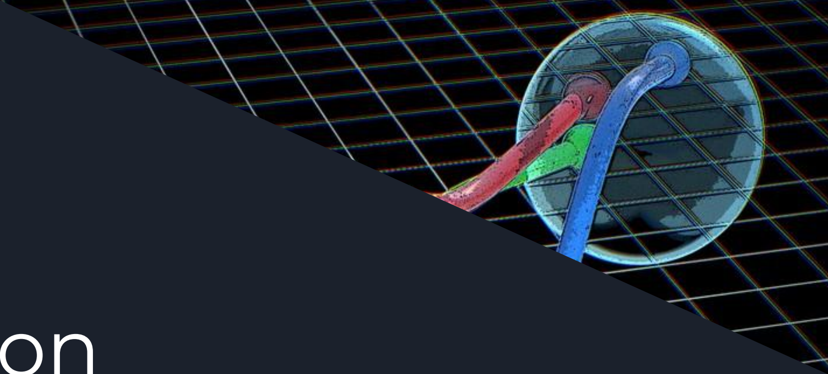

Towards High-Precision Nucleon Parton Distributions via Distillation

JHEP 11 (2021) 148; arXiv: 2107.05199 [hep-lat]

LaMET 2021

December 7th, 2021

CENTER for NUCLEAR FEMTOGRAPHY

Jefferson Lab

Colin Egerer
(For the HadStruc Collaboration)

Matrix Elements of Non-Local Parton Bilinears

<u>Focus of this talk:</u>
A matrix element of a distinct character

$$M^{\alpha}\left(p,z\right)=\langle h\left(p\right)|\,\overline{\psi}\left(z\right)\gamma^{\alpha}\Phi_{\hat{z}}^{(f)}\left(\{z,0\}\right)\psi\left(0\right)|h\left(p\right)\rangle$$

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Unpolarized leading-twist PDF defined in terms of $k^-, \mathbf{k}_\perp$ integrated parton correlator

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle h(p)|\,\overline{\psi}\left(\tfrac{z}{2}\right)\gamma^+\Phi^{(f)}_{\hat{z}^-}\left(\{\tfrac{z}{2},-\tfrac{z}{2}\}\right)\psi\left(-\tfrac{z}{2}\right)|h(p)\rangle$$

$$p^\alpha = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right)$$

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- universality/constrained by global analyses

$$F_i\left(x,Q^2\right) = \sum_{a=q,\bar{q},g} f_{a/h}\left(x,\mu^2\right)\otimes H_i^a\left(x,\frac{Q^2}{\mu^2},\alpha_s\left(\mu^2\right)\right) + h.t.$$

J. Collins, D. Soper, G. Sterman, Adv. Ser. Direct. High Energy Phys. 5, 1 (1989)

- complement 3D imaging efforts

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A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025

A frame amenable to Lattice QCD

$$p^\alpha = (E, \mathbf{0}_\perp, p_z)$$
$$z^\alpha = (0, \mathbf{0}_\perp, z_3) \quad \alpha = 0$$

Ioffe-time Pseudo-distribution (pseudo-ITD)

$$\mathcal{M}(p_z z_3, z_3^2) = \int_{-1}^{1} dx\ e^{i\nu x}\mathcal{P}(x, z_3^2)$$

Generalization of light-cone PDFs onto space-like intervals w/ Lorentz covariant parton momentum fraction

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Generalization of light-cone PDFs onto space-like intervals w/ Lorentz covariant parton momentum fraction

- *non-trivial light-cone limit*
- *short-distance factorization matches ITD to pseudo-ITD*

$$\mathcal{M}(\nu,z^2) = C(z^2\mu^2,\alpha_s(\mu^2))\otimes Q(\nu,\mu^2) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

Nucleon Pseudo-ITDs - Numerical Study

Spatial smearing increases operator-state overlaps onto low-lying modes

$$\hat{q}\left(\vec{x}, T\right) = \sum_{\vec{y}} S\left[U\right]\left(\vec{x}, \vec{y}\right) q\left(\vec{y}, T\right)$$

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Distillation - advantageous spatial smearing kernel

M. Peardon et al., Phys. Rev. D80, 054506 (2009)

- low-rank approximation of a gauge-covariant smearing kernel

$$J_{\sigma,n_\sigma}=e^{\sigma\nabla^2}=\sum_\lambda e^{-\sigma\lambda}\,|\lambda\rangle\langle\lambda|$$

$$\square\left(\vec{x},\vec{y};t\right)_{ab}=\sum_{k=1}^{R_{\mathcal{D}}}\xi_a^{(k)}\left(\vec{x},t\right)\xi_b^{(k)\dagger}\left(\vec{y},t\right)$$

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Why Distillation ?

- *explicit momentum projections - all times*
- *excited-state control*
- *reusability*
- *efficient realization of variational method*

R. Briceno et al., Phys.Rev.D 97 (2018) 5, 054513
J. Dudek et. al., Phys.Rev.D 88 (2013) 9, 094505
L. Liu, et. al., JHEP 07, (2012) 126
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CE, D. Richards, F. Winter, Phys. Rev. D 99 (2019) 3, 034506
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Goal: Can distillation isolate pseudo-ITDs with better precision than conventional smearing kernels & shed light on any systematics?

JLab/WM/LANL 2+1 Flavor Isotropic Lattices

ID	a (fm)	m_π (MeV)	$L^3\times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$	$N_{\rm vec}$
$a094m358$	$0.094(1)$	$358(3)$	$32^3\times 64$	349	4	64

Parameters/Statistics

t_{sep}/a	$p_z\ \left(\times\frac{2\pi}{L}\right)$	z/a
$4,6,\cdots,14$	$0,\pm1,\cdots,\pm6$	$0,\pm1,\cdots,\pm12,\cdots$
$0.38,\cdots 1.32$ fm	$0,0.411,\cdots,2.47$ GeV	$0,0.094,\cdots,1.13$ fm

Obtaining the Ioffe-time Pseudo-Distribution

Needed correlation functions:

$$C_2\left(p_z,T\right)=\left\langle\mathcal{N}\left(-p_z,T_f\right)\overline{\mathcal{N}}\left(p_z,T_0\right)\right\rangle=\sum_n\left|\mathcal{A}_n\right|^2e^{-E_nT}$$

$$\begin{aligned}C_3\left(p_z,T,\tau;z_3\right)&=V_3\left\langle\mathcal{N}\left(-p_z,T_f\right)\mathring{\mathcal{O}}_{\mathrm{WL}}^{[\gamma_4]}\left(z_3,\tau\right)\overline{\mathcal{N}}\left(p_z,T_0\right)\right\rangle\\&=V_3\sum_{n,n'}\left\langle\mathcal{N}|n'\right\rangle\left\langle n|\overline{\mathcal{N}}\right\rangle\left\langle n'\right|\mathring{\mathcal{O}}_{\mathrm{WL}}^{[\gamma_4]}\left(z_3,\tau\right)\left|n\right\rangle e^{-E_{n'}(T-\tau)}e^{-E_nT}\end{aligned}$$

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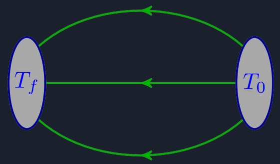

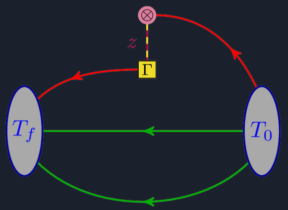

Irrep. projection

"Perambulators" $\tau^{kl}_{\alpha\beta}\left(t_f,t_0\right)=\xi^{(k)\dagger}\left(t_f\right)M^{-1}_{\alpha\beta}\left(t_f,t_0\right)\xi^{(l)}\left(t_0\right)$

"Elementals" $\Phi^{(i,j,k)}_{\mu\nu\sigma}\left(t\right)=\epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c\left(t\right)S_{\mu\nu\sigma}$

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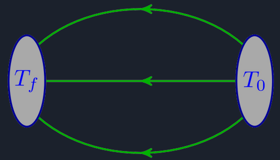
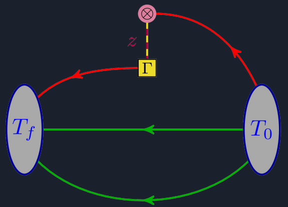
$\pm 2\cdot\frac{2\pi}{L}\hat{z}$





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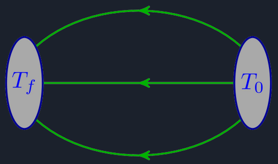

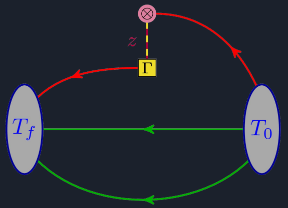

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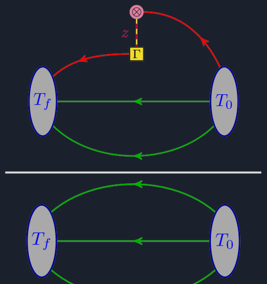
G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
C. Egerer et al., Phys. Rev. D 103 (2021) 3, 034502

Summation method

L. Maiani et al., Nucl. Phys. B293 (1987)
C. Bouchard et al., Phys. Rev. D 96, no. 1, 014504 (2017)

$$R\left(p_z, z_3; T\right) = \sum_{\tau/a=1}^{T-1} \frac{C_3\left(p_z, T, \tau; z_3\right)}{C_2\left(p_z, T\right)}$$



$$R_{\mathrm{fit}}\left(p_z, z_3; T\right) = \mathcal{A} + M_4\left(p_z, z_3\right) T + \mathcal{O}\left(e^{-\Delta E T}\right)$$

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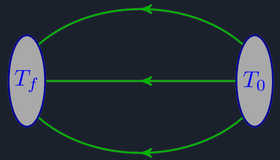
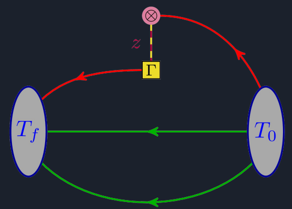
Needed correlation functions:

$$C_2\left(p_z, T\right) = \langle \mathcal{N}\left(-p_z, T_f\right) \overline{\mathcal{N}}\left(p_z, T_0\right)\rangle = \sum_n \left|\mathcal{A}_n\right|^2 e^{-E_n T}$$

$$C_3\left(p_z, T, \tau; z_3\right) = V_3 \langle \mathcal{N}\left(-p_z, T_f\right) \mathring{\mathcal{O}}_{\mathrm{WL}}^{[\gamma_4]}\left(z_3, \tau\right) \overline{\mathcal{N}}\left(p_z, T_0\right)\rangle$$
$$= V_3 \sum_{n,n'} \langle \mathcal{N}|n'\rangle \langle n|\overline{\mathcal{N}}\rangle \langle n'| \mathring{\mathcal{O}}_{\mathrm{WL}}^{[\gamma_4]}\left(z_3, \tau\right)|n\rangle e^{-E_{n'}(T-\tau)} e^{-E_n T}$$





"Perambulators" $\tau_{\alpha\beta}^{kl}\left(t_f, t_0\right) = \xi^{(k)\dagger}\left(t_f\right) M_{\alpha\beta}^{-1}\left(t_f, t_0\right) \xi^{(l)}\left(t_0\right)$

Irrep. projection

"Elementals" $\Phi_{\mu\nu\sigma}^{(i,j,k)}\left(t\right) = \epsilon^{abc}\left(\mathcal{D}_1 \xi^{(i)}\right)^a \left(\mathcal{D}_2 \xi^{(j)}\right)^b \left(\mathcal{D}_3 \xi^{(k)}\right)^c \left(t\right) S_{\mu\nu\sigma}$

$$\Xi_{\alpha\beta}^{(l,k)}\left(T_f, T_0; \tau, z_3\right) = \sum_{\vec{y}} \xi^{(l)\dagger}\left(T_f\right) D_{\alpha\sigma}^{-1}\left(T_f, \tau; \vec{y} + z_3\hat{z}\right) \boxed{\left[\gamma^4\right]}_{\sigma\rho} \boxed{\Phi_{\hat{z}}^{(f)}\left(\{\vec{y} + z_3\hat{z}, \vec{y}\}\right)} D_{\rho\beta}^{-1}\left(\tau, T_0; \vec{y}\right) \xi^{(k)}\left(T_0\right)$$

Unpolarized PDFs

Space-like Wilson line

High momenta & short-distance factorization

$$\xi_{\pm}^{(k)}\left(\vec{z}, t\right) \equiv e^{i\vec{\zeta}_{\pm}\cdot\vec{z}} \xi^{(k)}\left(\vec{z}, t\right)$$

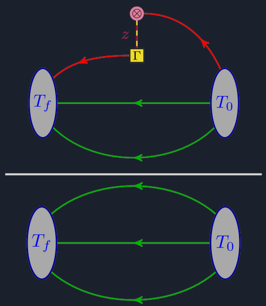
$\pm 2 \cdot \frac{2\pi}{L}\hat{z}$

G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
C. Egerer et al., Phys. Rev. D 103 (2021) 3, 034502

Summation method

L. Maiani et al., Nucl. Phys. B293 (1987)
C. Bouchard et al., Phys. Rev. D 96, no. 1, 014504 (2017)

$$R\left(p_z, z_3; T\right) = \sum_{\tau/a=1}^{T-1} \frac{C_3\left(p_z, T, \tau; z_3\right)}{C_2\left(p_z, T\right)}$$



$$R_{\mathrm{fit}}\left(p_z, z_3; T\right) = \mathcal{A} + M_4\left(p_z, z_3\right) T + \mathcal{O}\left(e^{-\Delta E T}\right)$$

Reduced Distribution (reduced pseudo-ITD)

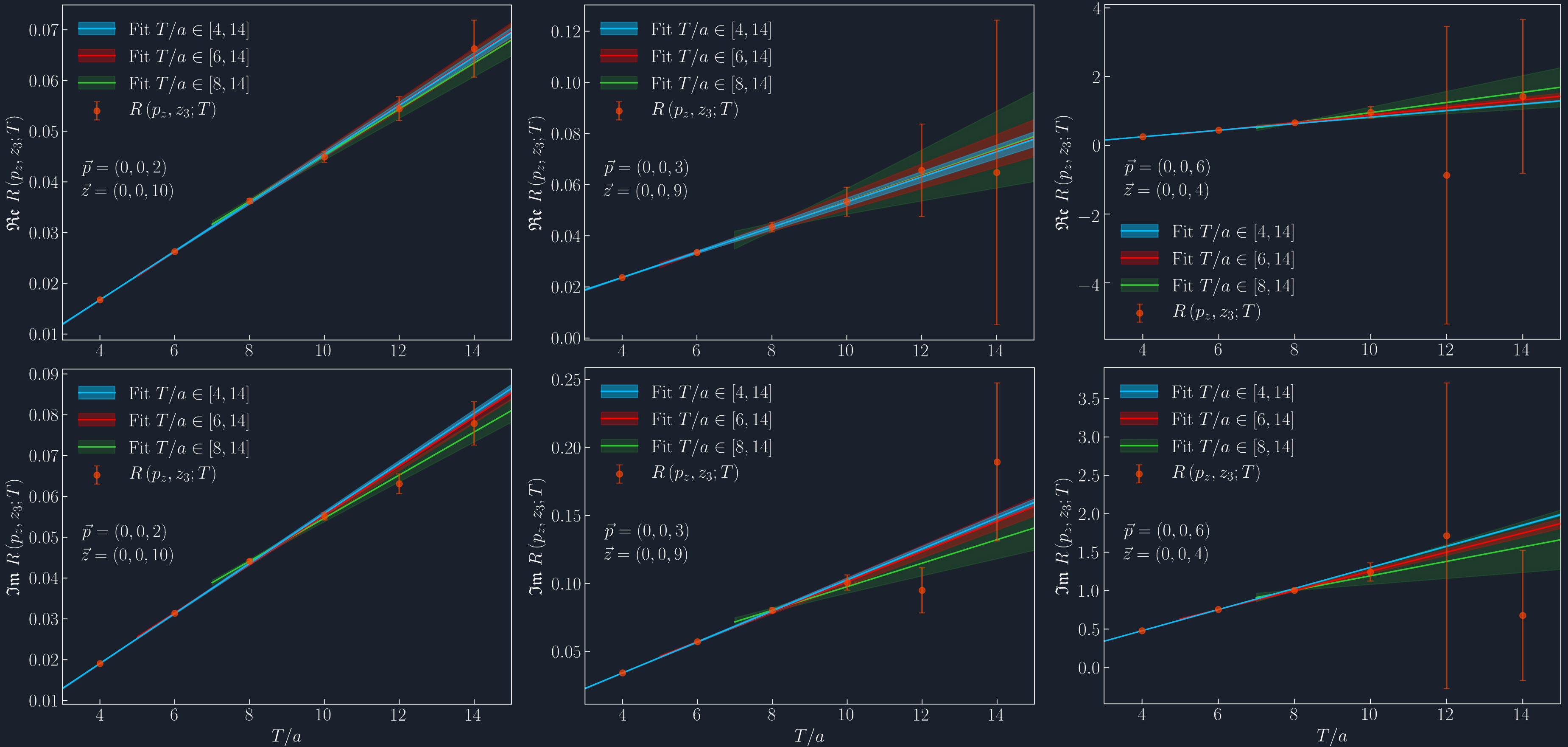
➢ cancel Wilson line UV divergences

K. Orginos, et al., Phys. Rev. D96, 094503 (2017)

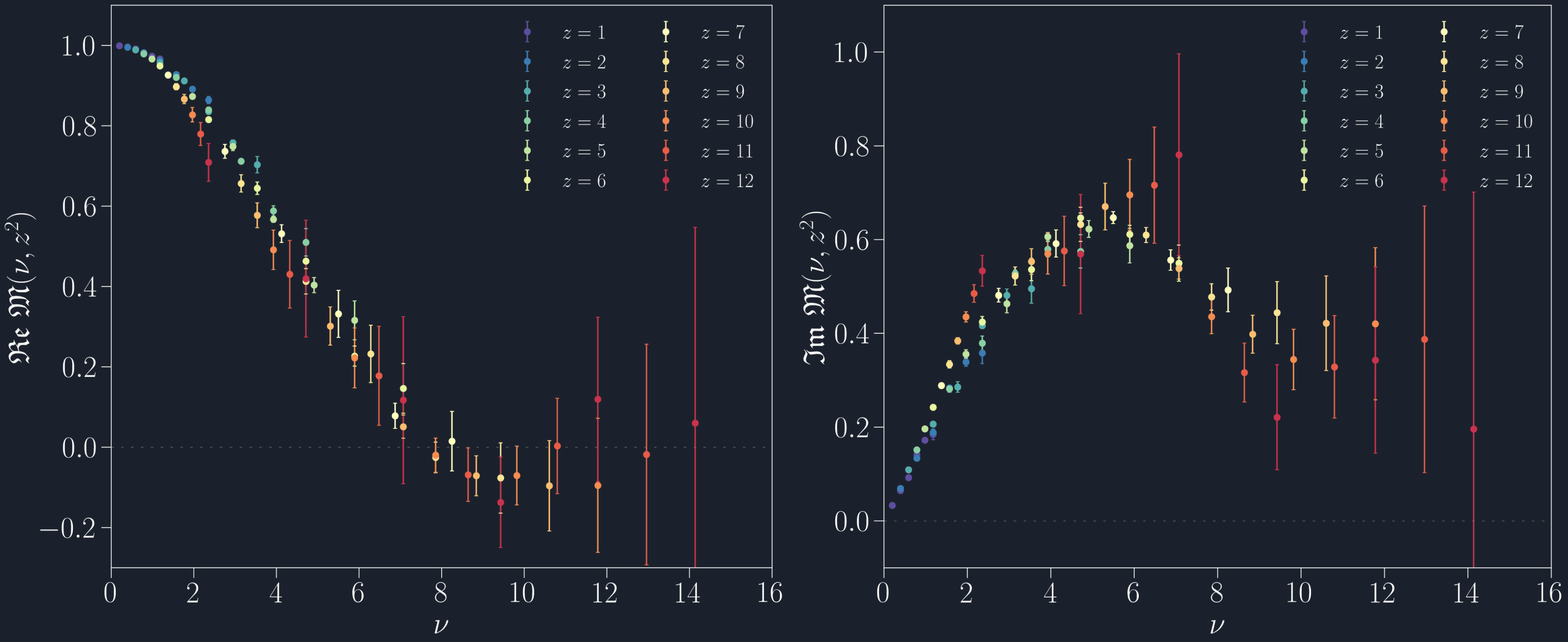
Local vector current in zero sep. limit (not conserved)

$$\mathfrak{M}\left(\nu, z_3^2\right) \equiv \frac{\mathcal{M}_p\left(\nu, z_3^2\right)}{\mathcal{M}_p\left(0, z_3^2\right)}$$
$$= \left(\frac{\mathcal{M}_p\left(\nu, z_3^2\right)}{\mathcal{M}_p\left(\nu, 0\right)|_{z_3=0}}\right) \times \underbrace{\left(\frac{\mathcal{M}_p\left(0,0\right)|_{p=0,z_3=0}}{\mathcal{M}_p\left(0, z_3^2\right)|_{p=0}}\right)}_{\texttt{RGI!}}$$

Selected Matrix Element Extractions

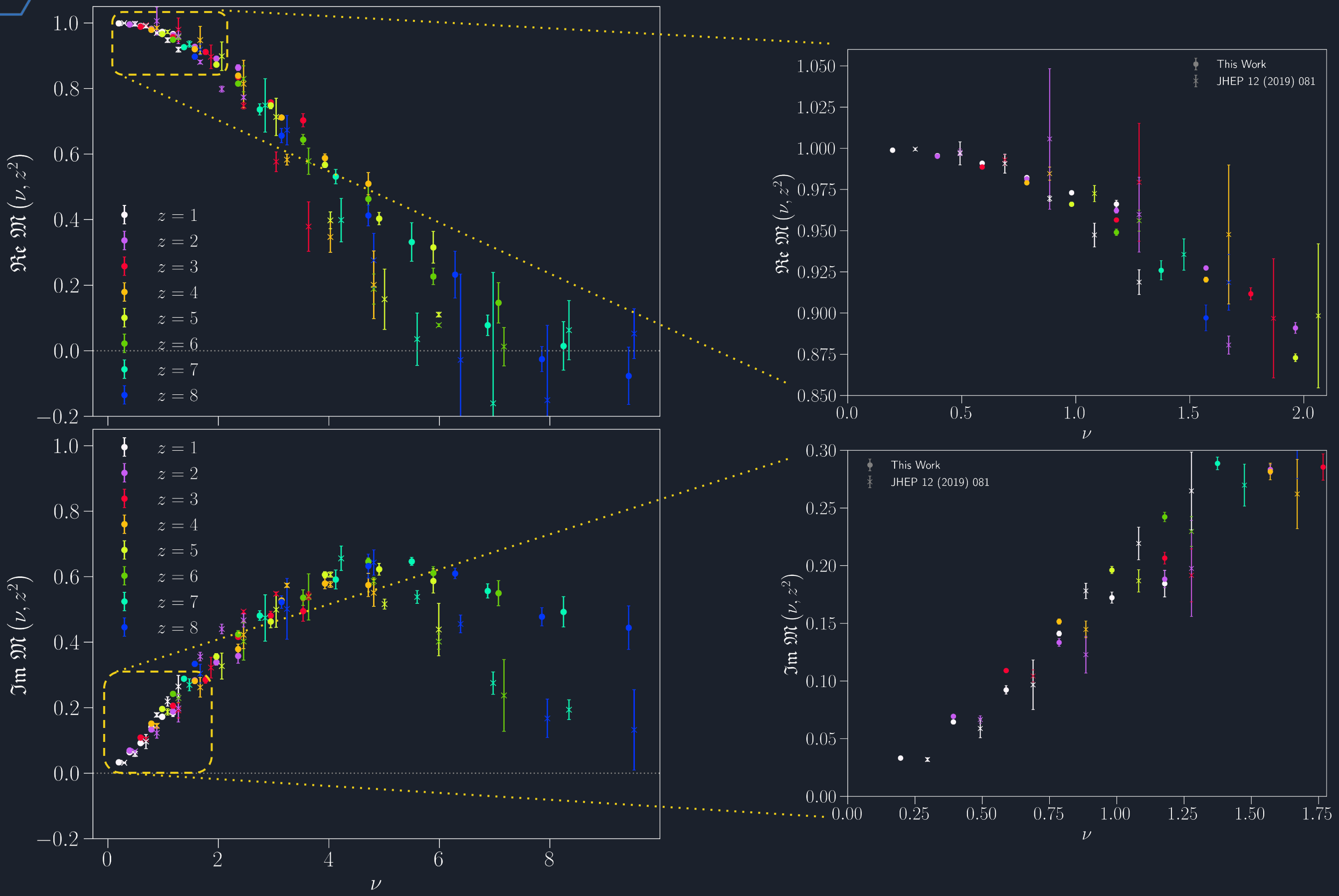


Unpolarized Ioffe-time Pseudo-Distribution



Efficacy of Distillation

Efficacy of Distillation



B. Joó et al., JHEP 12 (2019) 081
[Gaussian smearing]

$N_{\rm cfg} = 417 \qquad N_{\rm src} = 8 \qquad N_{\zeta} = 5$

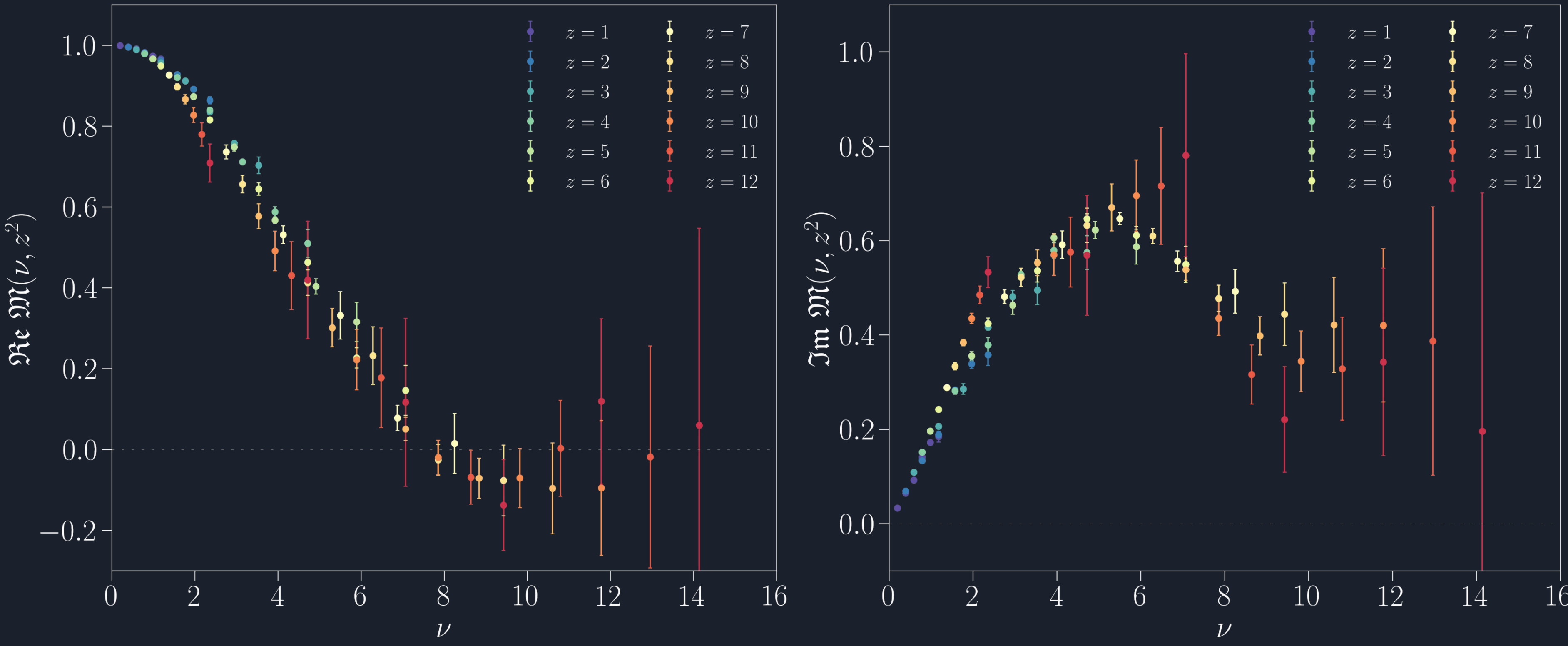
$N_{\rm inv}/{\rm cfg} \simeq 8.6{\rm k}$

This Work
[Distillation]

$N_{\rm cfg} = 349 \qquad N_{\rm src} = 4 \qquad N_{\zeta} = 3$

$N_{\rm inv}/{\rm cfg} \simeq 16{\rm k}$

Unpolarized Ioffe-time Pseudo-Distribution



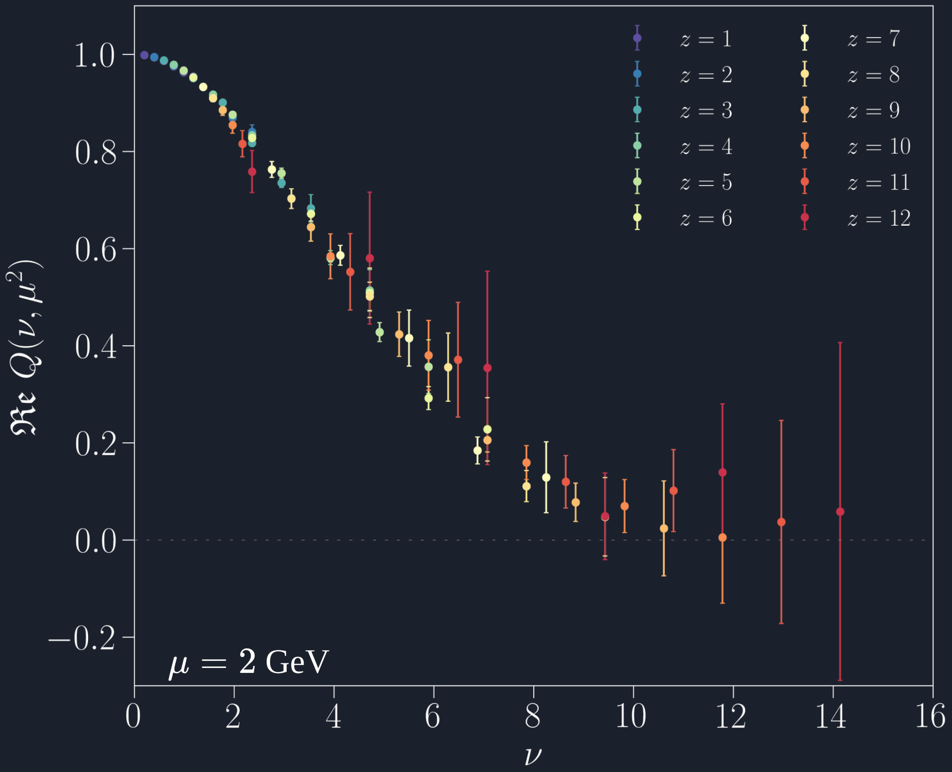
Unpolarized Ioffe-time Distribution

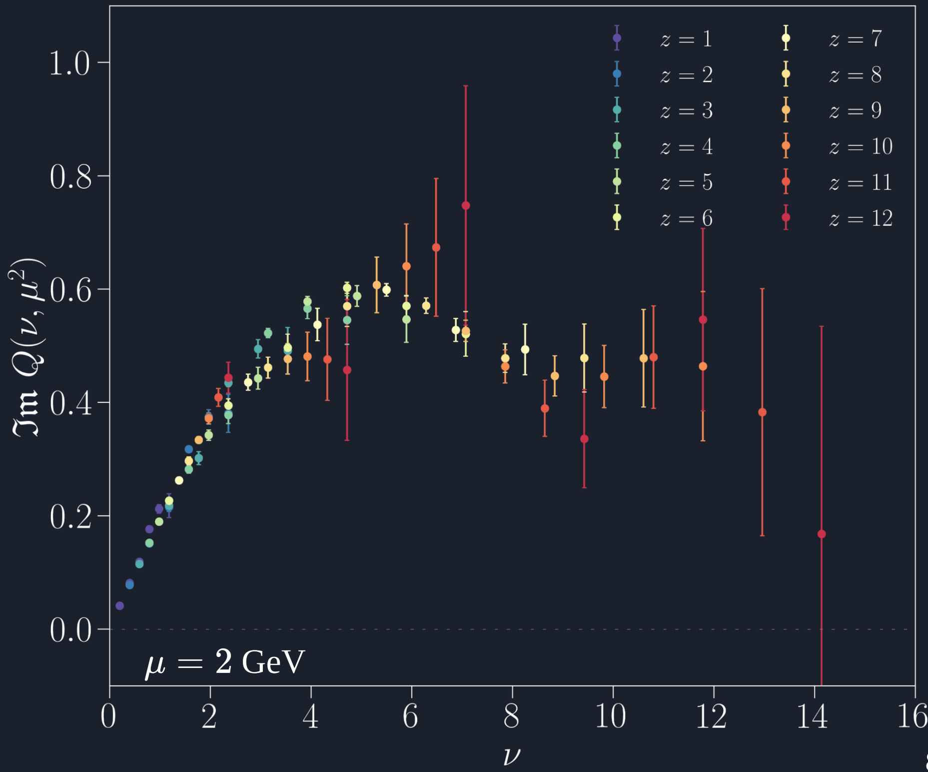
T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004
A. Radyushkin, Phys. Rev. D 98 (2018) no.1, 014019
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508

$$B(u) = \left(\frac{1+u^2}{1-u}\right)_+$$

$$L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

$$\mathcal{Q}(\nu,\mu^2) = \mathfrak{M}(\nu,z^2) + \frac{\alpha_s C_F}{2\pi}\int_0^1 du \left[\ln\left(\frac{e^{2\gamma_E+1}z^2\mu^2}{4}\right)\boxed{B(u)} + \boxed{L(u)}\right]\mathfrak{M}(u\nu,z^2)$$





Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $$\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\ \mathcal{K}\left(x\nu, z^2\mu^2\right) f_{q/h}\left(x, \mu^2\right) + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\ \mathcal{K}\left(x\nu, z^2\mu^2\right) \boxed{f_{q/h}\left(x, \mu^2\right)} + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\ \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta\ \Omega_n^{(\alpha,\beta)}\left(x\right)}$$

$$\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\ \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta\ \Omega_n^{(\alpha,\beta)}\left(x\right)}$$

Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\, \mathcal{K}\left(x\nu, z^2\mu^2\right) \boxed{f_{q/h}\left(x, \mu^2\right)} + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$

$$\begin{aligned}
\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) &= \int_0^1 dx\, \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)} \\
&= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} c_{2k}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha + 2k + j + 1, \beta + 1\right) \nu^{2k} \\
\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) &= \int_0^1 dx\, \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)} \\
&= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} c_{2k+1}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha + 2k + j + 2, \beta + 1\right) \nu^{2k+1}
\end{aligned}$$

Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\ \mathcal{K}\left(x\nu, z^2\mu^2\right) \boxed{f_{q/h}\left(x, \mu^2\right)} + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\ \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha \left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} c_{2k}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+1, \beta+1\right) \nu^{2k}$$
$$\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\ \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha \left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} c_{2k+1}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+2, \beta+1\right) \nu^{2k+1}$$

$$\mathfrak{Re}\ \mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{\mathrm{v},n}^{lt\,(\alpha,\beta)}$$

$$\mathfrak{Im}\ \mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{+,n}^{lt\,(\alpha,\beta)}$$

Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\, \mathcal{K}\left(x\nu, z^2\mu^2\right) \boxed{f_{q/h}\left(x, \mu^2\right)} + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$

$$\begin{aligned}
\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) &= \int_0^1 dx\, \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)} \\
&= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} c_{2k}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+1, \beta+1\right)\nu^{2k} \\
\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) &= \int_0^1 dx\, \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)} \\
&= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} c_{2k+1}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+2, \beta+1\right)\nu^{2k+1}
\end{aligned}$$

$$\mathfrak{Re}\,\mathfrak{M}_{\mathrm{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{\mathrm{v},n}^{lt\,(\alpha,\beta)} + \boxed{\frac{a}{|z|}} \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{\mathrm{v},n}^{az\,(\alpha,\beta)}$$

$$\mathfrak{Im}\,\mathfrak{M}_{\mathrm{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{+,n}^{lt\,(\alpha,\beta)} + \boxed{\frac{a}{|z|}} \sum_{n=0}^{\infty} \eta_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{+,n}^{az\,(\alpha,\beta)}$$

Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $\mathfrak{M}\left(\nu, z^2\right)=\int_{-1}^{1} dx\ \mathcal{K}\left(x\nu, z^2\mu^2\right)\boxed{f_{q/h}\left(x,\mu^2\right)}+\sum_{k=1}^{\infty}\mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$

$$
\begin{aligned}
\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) &= \int_0^1 dx\ \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right)\boxed{x^\alpha\left(1-x\right)^\beta\,\Omega_n^{(\alpha,\beta)}\left(x\right)} \\
&= \sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{\left(-1\right)^k}{(2k)!}c_{2k}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)}B\left(\alpha+2k+j+1,\beta+1\right)\nu^{2k} \\
\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) &= \int_0^1 dx\ \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right)\boxed{x^\alpha\left(1-x\right)^\beta\,\Omega_n^{(\alpha,\beta)}\left(x\right)} \\
&= \sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{\left(-1\right)^k}{(2k+1)!}c_{2k+1}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)}B\left(\alpha+2k+j+2,\beta+1\right)\nu^{2k+1}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{Re}\ \mathfrak{M}_{\mathrm{fit}}\left(\nu, z^2\right) = {} & \sum_{n=0}^{\infty}\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right)C_{\mathrm{v},n}^{lt\,(\alpha,\beta)}+\boxed{\frac{a}{|z|}}\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{\mathrm{v},n}^{az\,(\alpha,\beta)} \\
& +\boxed{z^2\Lambda_{\mathrm{QCD}}^2}\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{\mathrm{v},n}^{t4\,(\alpha,\beta)}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{Im}\ \mathfrak{M}_{\mathrm{fit}}\left(\nu, z^2\right) = {} & \sum_{n=0}^{\infty}\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right)C_{+,n}^{lt\,(\alpha,\beta)}+\boxed{\frac{a}{|z|}}\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{az\,(\alpha,\beta)} \\
& +\boxed{z^2\Lambda_{\mathrm{QCD}}^2}\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{t4\,(\alpha,\beta)}
\end{aligned}
$$

Expand Pseudo-ITD in Orthogonal Polynomials

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$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\ \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta\,\Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} c_{2k}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+1, \beta+1\right)\nu^{2k}$$
$$\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\ \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta\,\Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} c_{2k+1}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+2, \beta+1\right)\nu^{2k+1}$$

$$\mathfrak{Re}\ \mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{\mathrm{v},n}^{lt\,(\alpha,\beta)} + \frac{a}{|z|}\sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{\mathrm{v},n}^{az\,(\alpha,\beta)}$$
$$+ z^2\Lambda_{\text{QCD}}^2 \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{\mathrm{v},n}^{t4\,(\alpha,\beta)} + z^4\Lambda_{\text{QCD}}^4 \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{\mathrm{v},n}^{t6\,(\alpha,\beta)}$$

$$\mathfrak{Im}\ \mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{+,n}^{lt\,(\alpha,\beta)} + \frac{a}{|z|}\sum_{n=0}^{\infty} \eta_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{+,n}^{az\,(\alpha,\beta)}$$
$$+ z^2\Lambda_{\text{QCD}}^2 \sum_{n=0}^{\infty} \eta_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{+,n}^{t4\,(\alpha,\beta)} + z^4\Lambda_{\text{QCD}}^4 \sum_{n=0}^{\infty} \eta_{0,n}^{(\alpha,\beta)}\left(\nu\right) C_{+,n}^{t6\,(\alpha,\beta)}$$

Expand Pseudo-ITD in Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation: $$\mathfrak{M}\left(\nu,z^2\right)=\int_{-1}^{1}dx\,\mathcal{K}\left(x\nu,z^2\mu^2\right)\boxed{f_{q/h}\left(x,\mu^2\right)}+\sum_{k=1}^{\infty}\mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

$$\begin{aligned}\sigma_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)&=\int_0^1 dx\,\mathcal{K}_{\mathrm{v}}\left(x\nu,z^2\mu^2\right)\boxed{x^\alpha\left(1-x\right)^\beta\Omega_n^{(\alpha,\beta)}\left(x\right)}\\&=\sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}c_{2k}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)}B\left(\alpha+2k+j+1,\beta+1\right)\nu^{2k}\\\eta_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)&=\int_0^1 dx\,\mathcal{K}_{+}\left(x\nu,z^2\mu^2\right)\boxed{x^\alpha\left(1-x\right)^\beta\Omega_n^{(\alpha,\beta)}\left(x\right)}\\&=\sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}c_{2k+1}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)}B\left(\alpha+2k+j+2,\beta+1\right)\nu^{2k+1}\end{aligned}$$

$$\begin{aligned}\mathfrak{Re}\,\mathfrak{M}_{\mathrm{fit}}\left(\nu,z^2\right)=&\sum_{n=0}^{\infty}\sigma_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)C_{\mathrm{v},n}^{lt\,(\alpha,\beta)}+\boxed{\frac{a}{|z|}}\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{\mathrm{v},n}^{az\,(\alpha,\beta)}\\&+\boxed{z^2\Lambda_{\mathrm{QCD}}^2}\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{\mathrm{v},n}^{t4\,(\alpha,\beta)}+\boxed{z^4\Lambda_{\mathrm{QCD}}^4}\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{\mathrm{v},n}^{t6\,(\alpha,\beta)}\end{aligned}$$

$$\begin{aligned}\mathfrak{Im}\,\mathfrak{M}_{\mathrm{fit}}\left(\nu,z^2\right)=&\sum_{n=0}^{\infty}\eta_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)C_{+,n}^{lt\,(\alpha,\beta)}+\boxed{\frac{a}{|z|}}\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{az\,(\alpha,\beta)}\\&+\boxed{z^2\Lambda_{\mathrm{QCD}}^2}\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{t4\,(\alpha,\beta)}+\boxed{z^4\Lambda_{\mathrm{QCD}}^4}\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{t6\,(\alpha,\beta)}\end{aligned}$$

$$\Downarrow$$

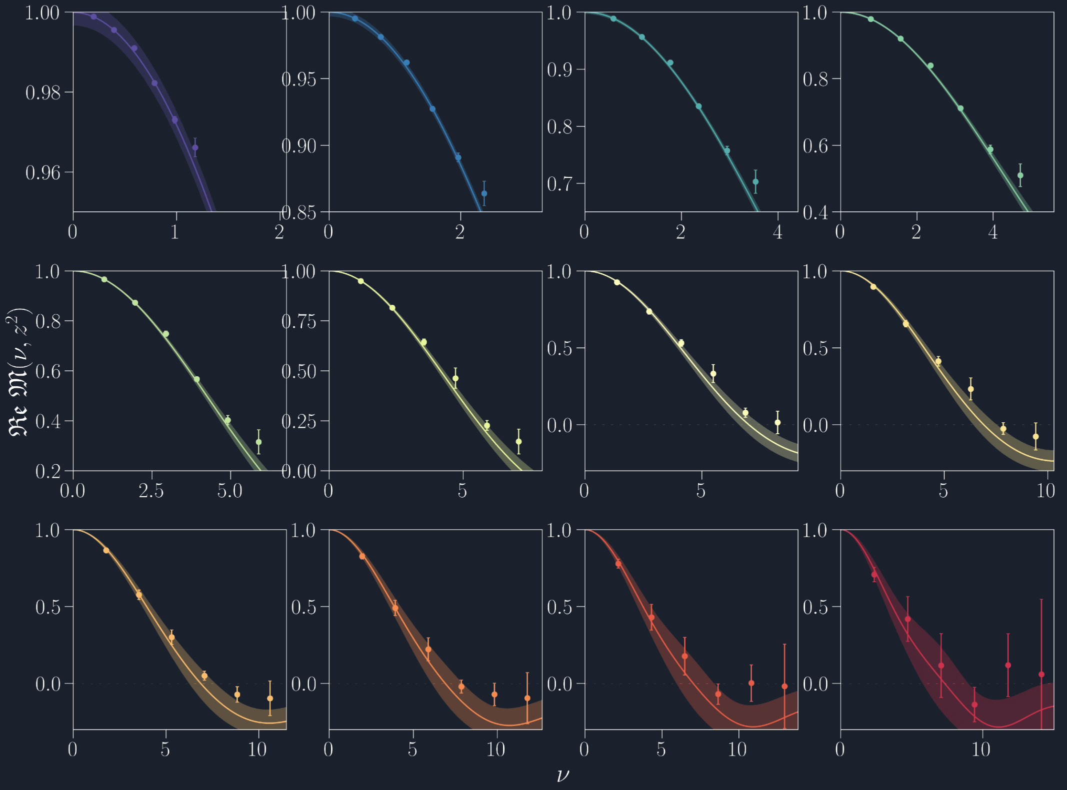
Jacobi polynomial basis are only non-linear terms
Separable non-linear optimization → variable projection

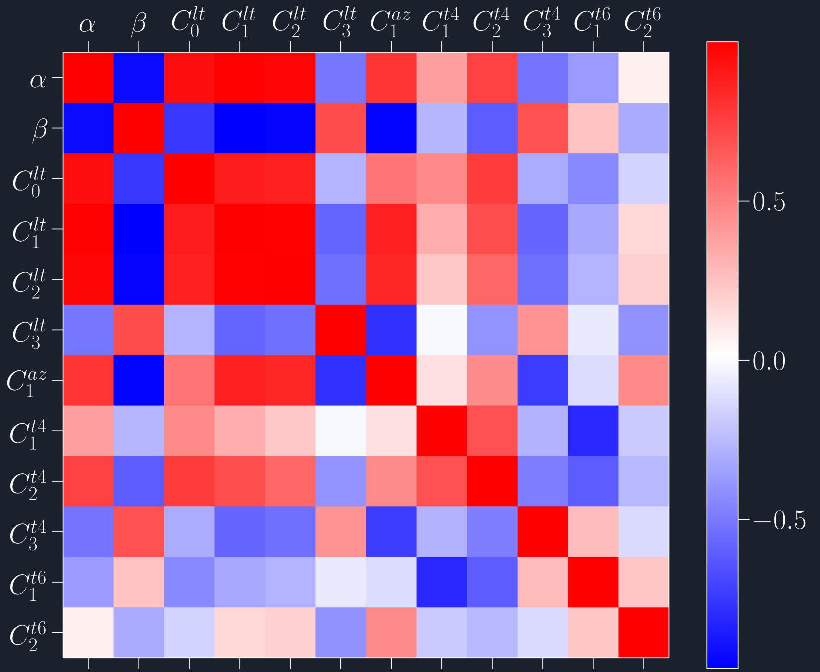
G. Golub and V. Pereyra, SIAM Journal on Numerical Analysis 10, 413 (1973)

Strategy of parametric fits with Jacobi polynomials

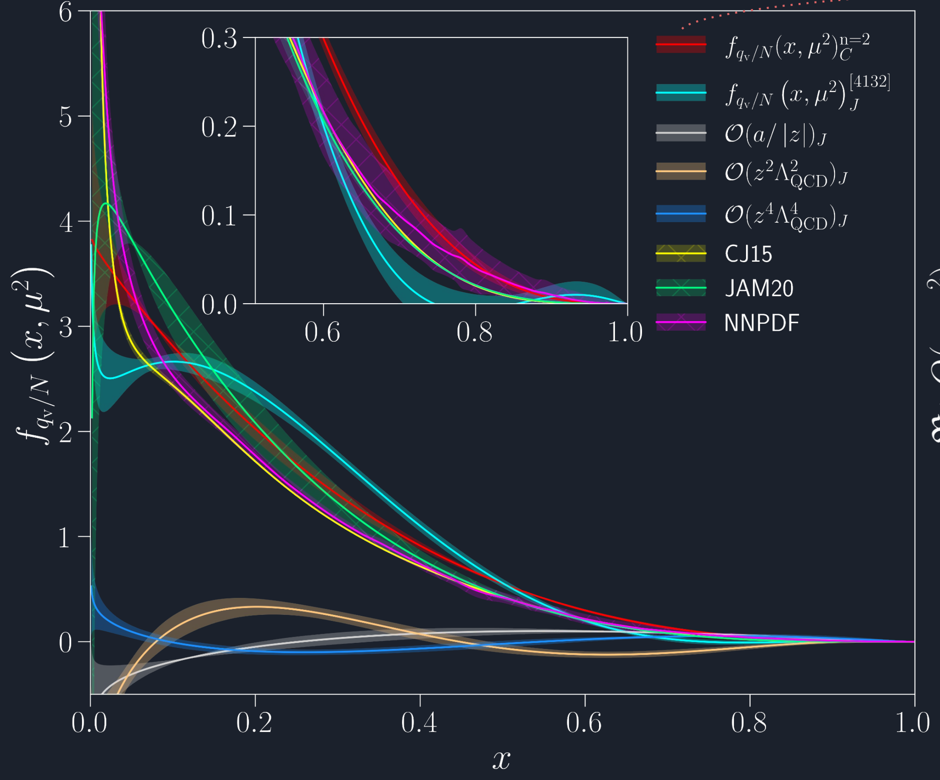
1. scan over truncation orders
   a. search for optimal expansion coefficients for each
2. establish polynomial hierarchy
   a. preference given to low-order polynomials
   b. restrict x-space containating distributions to be sub-leading to leading-twist PDF
   c. Bayesian priors (gaussian)
3. separability of non-linear optimization

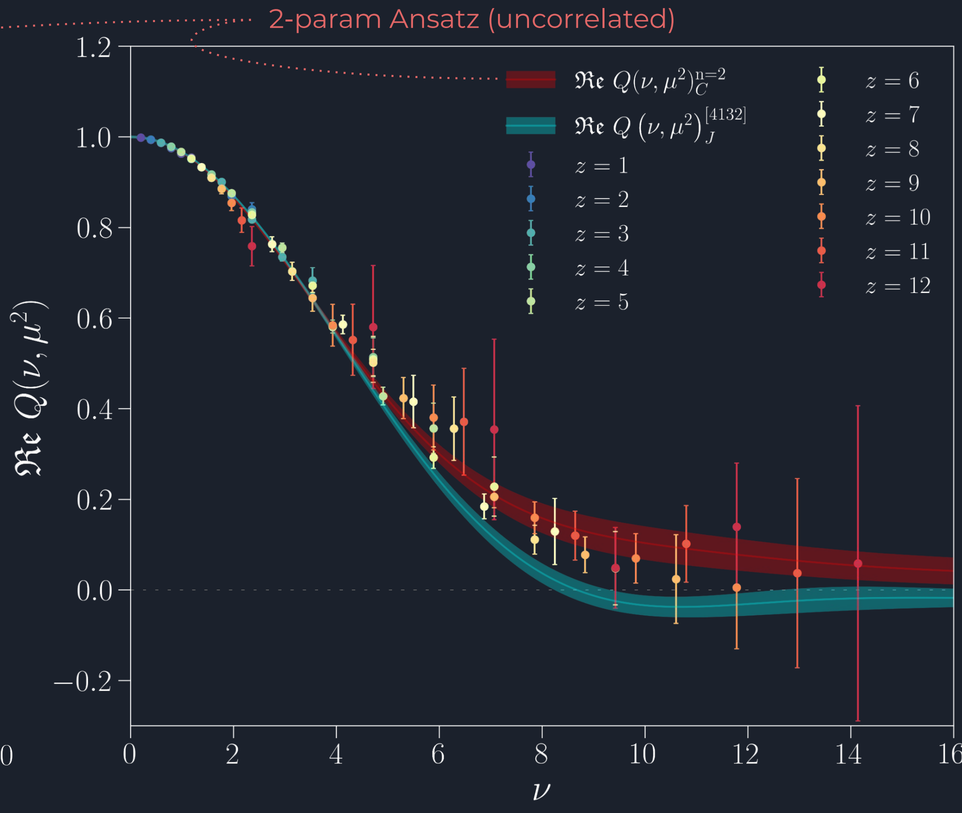
Optimal Fit for Valence PDF





Valence Quark PDF and Leading-Twist ITD





$$f_{q_v/N}\left(x,\mu^2\right)_J = x^\alpha \left(1-x\right)^\beta \sum_{n=0}^{n_{lt}} C_{v,n}^{(\alpha,\beta)} \Omega_n^{(\alpha,\beta)}\left(x\right)$$

$$\mathcal{O}\left(corr\right)_J = x^\alpha \left(1-x\right)^\beta \sum_{n=1}^{n_{corr}} C_{v,n}^{corr(\alpha,\beta)} \Omega_n^{(\alpha,\beta)}\left(x\right)$$

Short-Distance Tension

Dramatic effect of a discretization correction

$\{n_{lt}, n_{az}, n_{t4}, n_{t6}\}_{\mathrm{v}/+}$	α	β	$C^{lt}_{\mathrm{v},0}$	$C^{lt}_{\mathrm{v},1}$	$C^{lt}_{\mathrm{v},2}$	$C^{lt}_{\mathrm{v},3}$
$\{4,1,3,2\}_{\mathrm{v}}$	$-0.209(147)$	$1.330(415)$	$1.606(257)$	$0.427(752)$	$-0.880(409)$	$-0.675(122)$
$\{4,0,3,2\}_{\mathrm{v}}$	$-0.376(37)$	$2.032(496)$	$1.340(165)$	$0.335(261)$	$-0.125(100)$	$-0.651(140)$

$C^{az}_{\mathrm{v},1}$	$C^{t4}_{\mathrm{v},1}$	$C^{t4}_{\mathrm{v},2}$	$C^{t4}_{\mathrm{v},3}$	$C^{t6}_{\mathrm{v},1}$	$C^{t6}_{\mathrm{v},2}$	χ^2_r
$-0.279(48)$	$0.052(53)$	$-0.371(106)$	$-0.407(122)$	$-0.045(37)$	$0.228(52)$	$2.620(345)$
–	$-0.090(52)$	$-0.112(77)$	$0.274(99)$	$0.011(39)$	$0.397(84)$	$45.68(1.72)$

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Visualize scale dependence in reduced pseudo-ITD via mock pseudo-PDF fit

$$\frac{\Gamma(5+\alpha)}{\Gamma(1+\alpha)\Gamma(4)} x^{\alpha}(1-x)^3$$

$$\mathfrak{Re}\ \mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \int_0^1 dx \cos(x\nu)\, \mathfrak{Re}\ \mathcal{P}\left(x, z^2; \alpha, 3\right)$$

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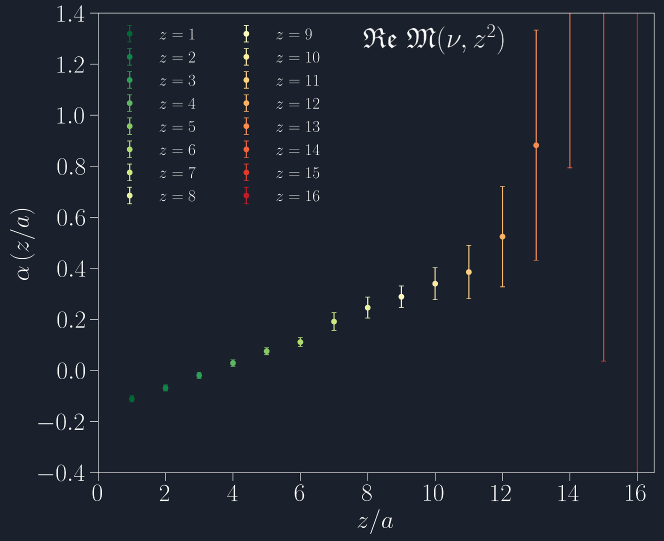

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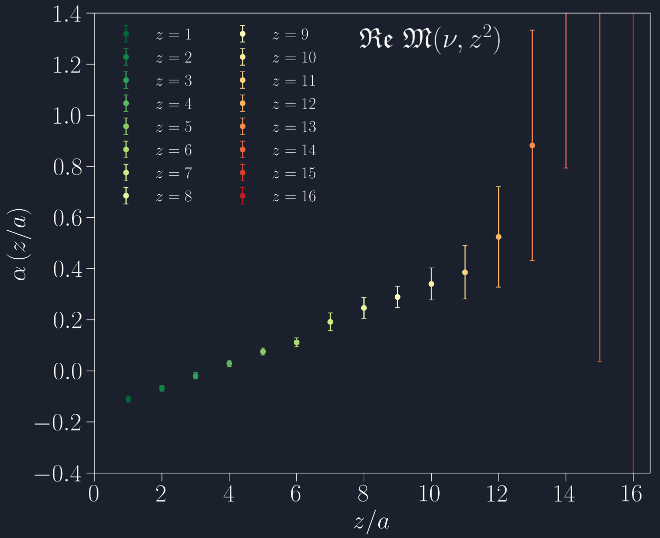

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Evolution/matching with pseudo-PDF fit

$$\mathfrak{Re}\ \mathcal{Q}(\nu, \mu^2) = \mathfrak{Re}\ \mathfrak{M}(\nu, z^2) + \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, \mathfrak{P}(u\nu, z^2; \alpha, \beta = 3) \left[\ln\left(\frac{z^2 \mu^2 e^{2\gamma_E + 1}}{4} \right) B(u) + L(u) \right]$$

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➢ redo two-parameter fits to matched ITD

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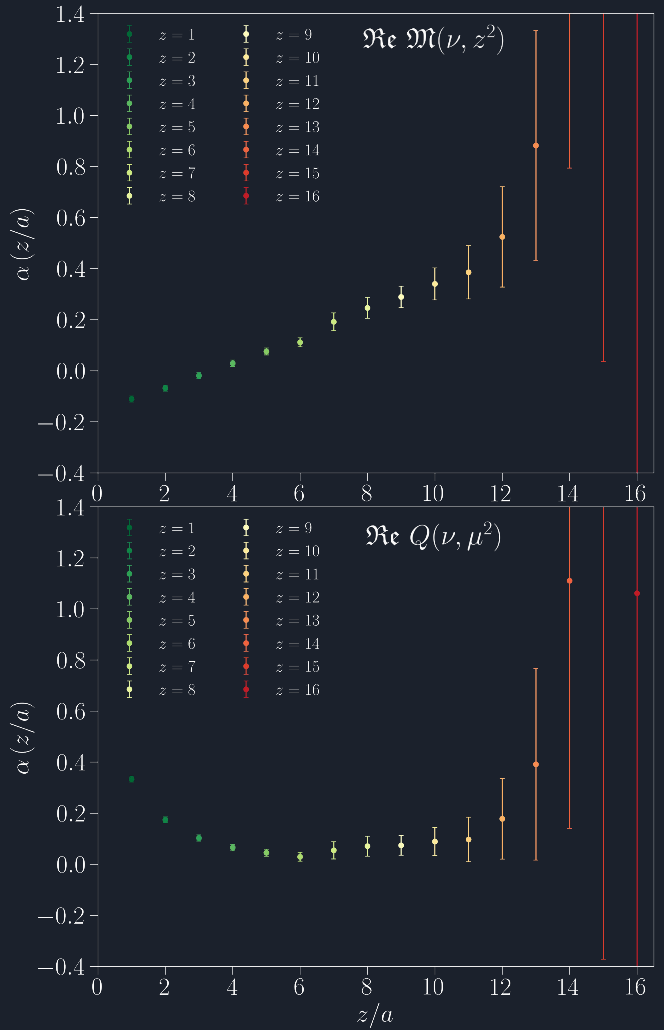

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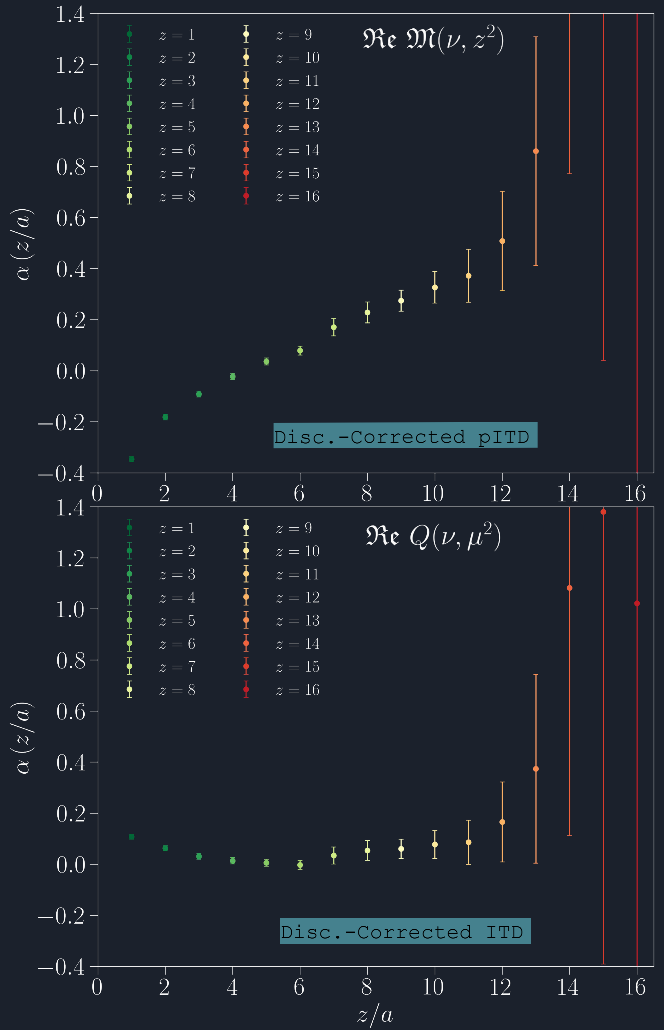
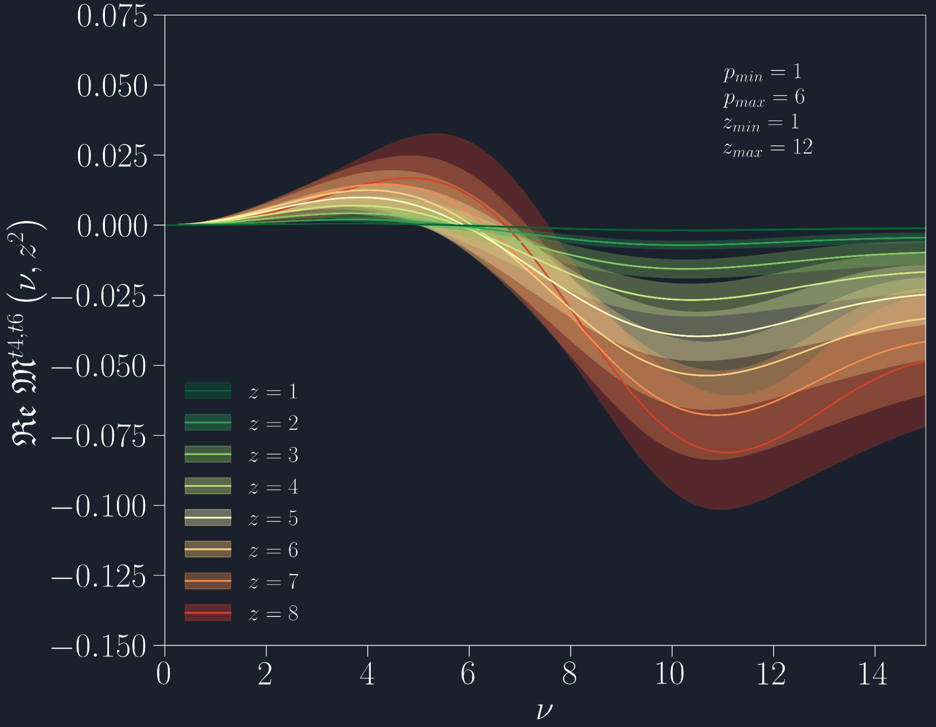


Parameterized Higher-Twist Distribution



$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda_{\rm QCD}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\rm v,n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\rm QCD}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\rm v,n}^{t6\ (\alpha,\beta)}$$

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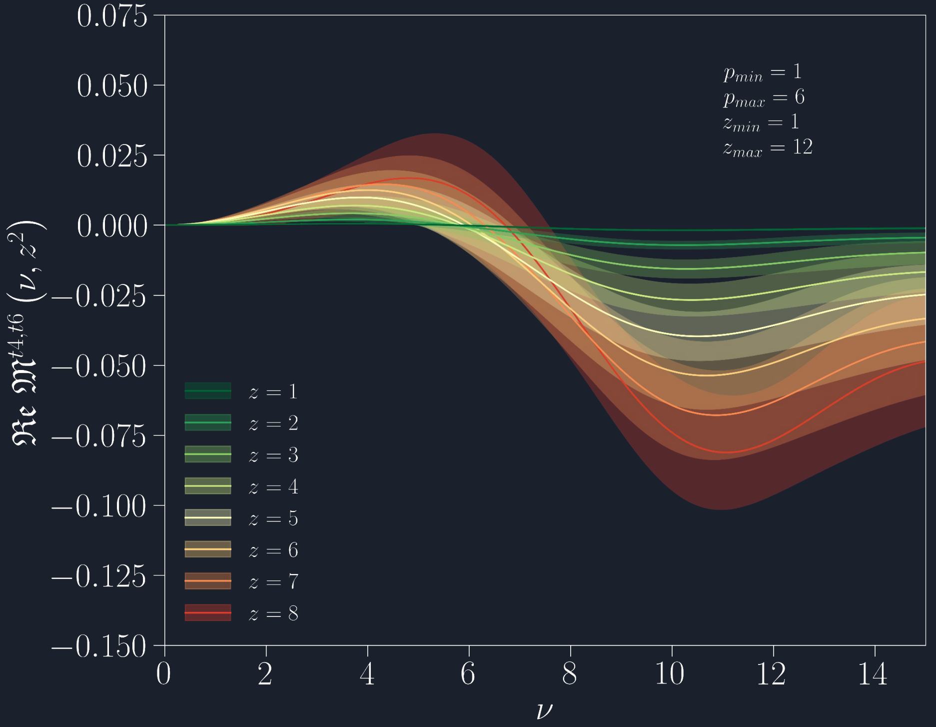
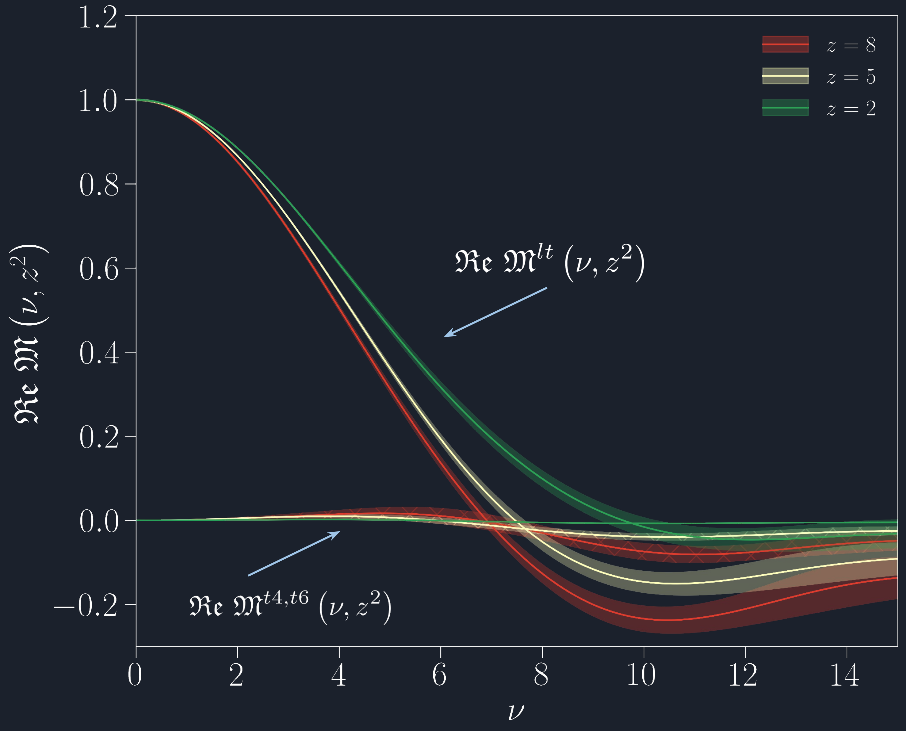
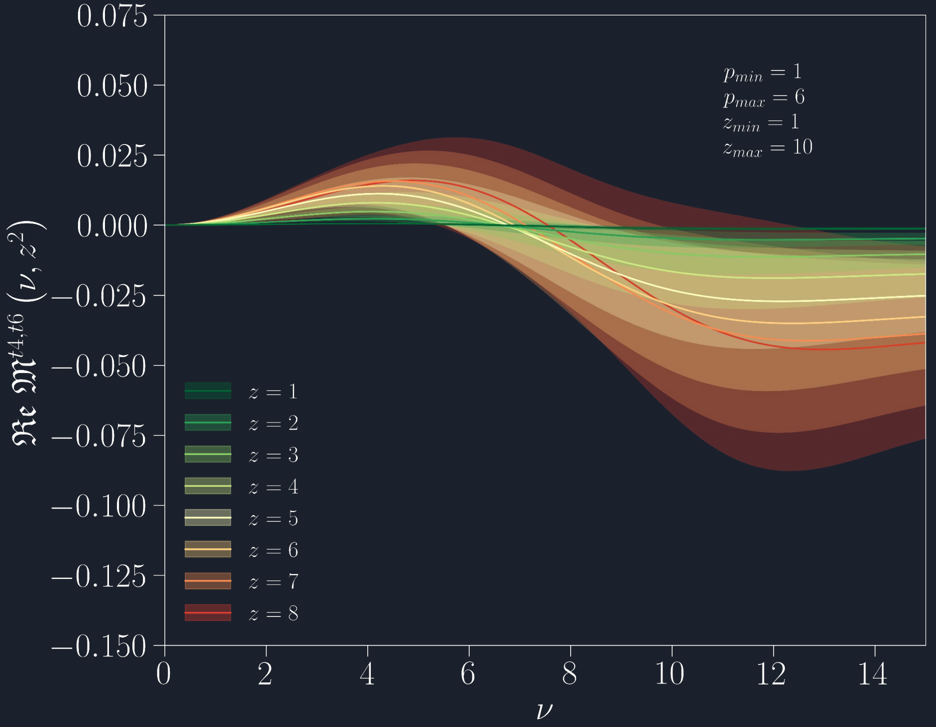
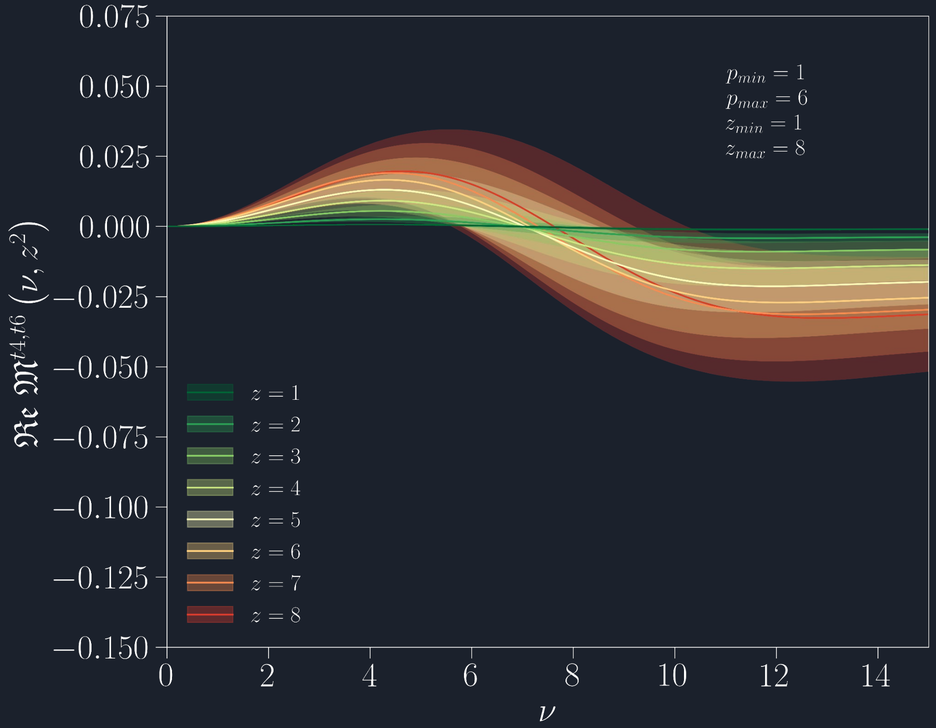




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda^2_{\mathrm{QCD}} \sum_{n=1}^{n_{t4}} \sigma^{(\alpha,\beta)}_{0,n}(\nu)\, C^{t4\ (\alpha,\beta)}_{\mathrm{v},n} + z^4 \Lambda^4_{\mathrm{QCD}} \sum_{n=1}^{n_{t6}} \sigma^{(\alpha,\beta)}_{0,n}(\nu)\, C^{t6\ (\alpha,\beta)}_{\mathrm{v},n}$$

Higher-Twist Variability - Cuts on Wilson Line





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Next Steps - Expanded Interpolator Basis

Excited-state contamination

- ➢ optimize operator/state overlaps - saturate correlation functions at early temporal separations

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Generic light-quark nucleon interpolator smeared with distillation

$$\mathcal{O}_i\left(t\right) = \epsilon^{abc}\left(\boxed{\mathcal{D}_1}\Box u\right)_a^\alpha \left(\boxed{\mathcal{D}_2}\Box d\right)_b^\beta \left(\boxed{\mathcal{D}_3}\Box u\right)_c^\gamma \left(t\right) S_i^{\alpha\beta\gamma}$$

Dirac structure/covariant derivatives

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Discretized continuum-like interpolators of definite permutational symmetries

$$\mathcal{O}_B=\left(\mathcal{F}_{\mathcal{P}(\mathrm{F})}\otimes\mathcal{S}_{\mathcal{P}(\mathrm{S})}\otimes\mathcal{D}_{\mathcal{P}(\mathrm{D})}\right)\{q_1q_2q_3\} \qquad (N_M\otimes(\tfrac{1}{2}^+)^1_M\otimes D^{[2]}_{L=1,A})^{J^P=\frac{1}{2}^+}\equiv N^2P_A\tfrac{1}{2}^+$$

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(Generally) Continuum spins reducible under octahedral group

Canonical subductions

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$$\mathcal{O}_{^n\Lambda,r}^{\{J\}}=\sum_m S_{^n\Lambda,r}^{J,m}\;\mathcal{O}^{\{J,m\}}$$

R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)
J. Dudek and R. Edwards, Phys. Rev. D85, 054016 (2012)

Helicity subductions

C. Thomas, et al., Phys. Rev. D85, 014507 (2012)
C. Thomas, private communication

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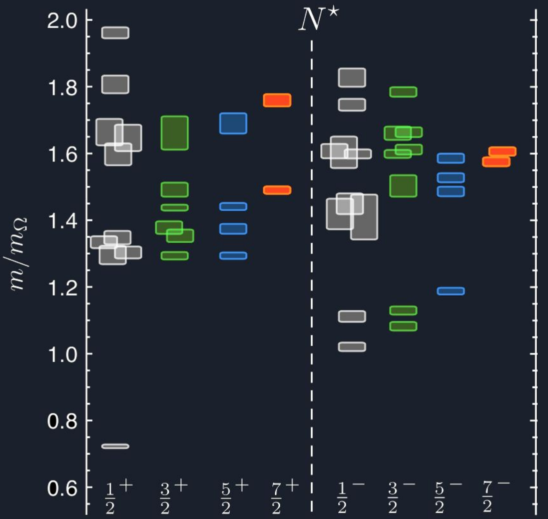

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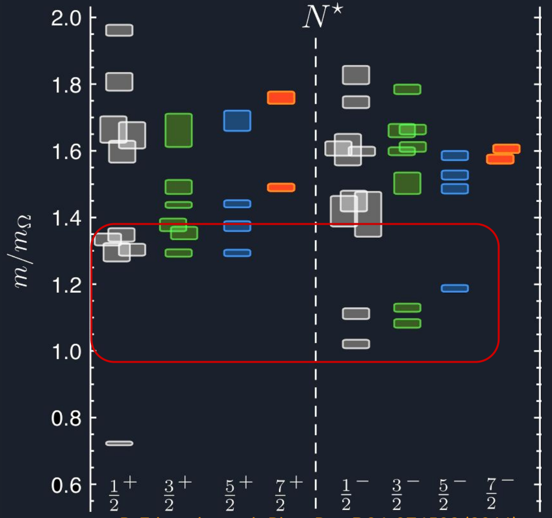

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$$\left[\mathbb{O}^{J^P,\lambda}\left(\vec{p}\right)\right]^\dagger = \sum_m \mathcal{D}^{(J)}_{m,\lambda}\left(R\right)\left[O^{J^P,m}\left(\vec{p}\right)\right]^\dagger$$

- ➢ subduce into little groups

$$\left[\mathbb{O}^{J^P,|\lambda|}_{\Lambda,\mu}\left(\vec{p}\right)\right]^\dagger = \sum_{\hat{\lambda}=\pm|\lambda|} S^{\tilde{\eta},\hat{\lambda}}_{\Lambda,\mu}\left[\mathbb{O}^{J^P,\hat{\lambda}}\left(\vec{p}\right)\right]^\dagger$$

Summary and Outlook

Hadronic structure accessible from certain lattice calculable matrix elements

- ➢ short-distance factorization

Nucleon valence (plus) quark PDF

- ➢ distillation (+phasing) - first use in structure studies
  - ○ precise pseudo-ITDs & PDFs
- ➢ systematic effects can be reliably addressed

Fidelity of PDFs extracted from Lattice QCD hinges on control of systematic effects

- ➢ observed deviation from expected DGLAP evolution of pseudo-PDF at small distances
- ➢ neglect of data correlations/Ioffe-time cuts
  - ○ useful, but can yield erroneous results

Repeat calculations at lighter pion masses & finer lattice spacings underway

HadStruc Collaboration

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Jianwei Qiu, David Richards, Eloy Romero, Frank Winter [1]

Balint Joó[2]

Carl Carlson, Tanjib Khan, Christopher Monahan,
Kostas Orginos, Raza Sufian[3]

Wayne Morris, Anatoly Radyushkin [4]

Joe Karpie [5]

Savvas Zafeiropoulos [6]

Yan-Qing Ma [7]

Jefferson Lab [1], Oak Ridge [2], William and Mary [3], Old Dominion University [4], Columbia University [5], Aix Marseille University [6], Peking University [7]

Supplements

An Ill-Posed Inverse

(pseudo-)ITD/PDF matching relations are ill-posed

$$\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\, \mathcal{K}\left(x\nu, z^2\mu^2\right) f_{q/h}\left(x, \mu^2\right) + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

$$\mathcal{Q}\left(\nu, \mu^2\right) = \int_{-1}^{1} dx\, e^{i\nu x} f_{q/h}\left(x, \mu^2\right)$$

- commonly: matched ITD fit by F.T. of parameterized PDF $f_{q/h}\left(x, \mu^2\right) = N x^{\alpha}\left(1-x\right)^{\beta} P\left(x\right)$

Jacobi (hypergeometric) polynomials

$$P_n^{(\alpha,\beta)}\left(z\right) = \frac{\Gamma\left(\alpha+n+1\right)}{n!\Gamma\left(\alpha+\beta+n+1\right)} \sum_{j=0}^{n} \binom{n}{j} \frac{\Gamma\left(\alpha+\beta+n+j+1\right)}{\Gamma\left(\alpha+j+1\right)} \left(\frac{z-1}{2}\right)^j$$

$z \in [-1, 1]$	$\left(1-z\right)^{\alpha}\left(1+z\right)^{\beta}$	$\alpha, \beta > -1$
Interval	Metric	Validity

$$\int_{-1}^{1} dz\, \left(1-z\right)^{\alpha}\left(1+z\right)^{\beta} P_n^{(\alpha,\beta)}\left(z\right) P_m^{(\alpha,\beta)}\left(z\right) = \delta_{n,m} h_n\left(\alpha, \beta\right)$$

A convenient change of variables: $z \mapsto 1 - 2x$

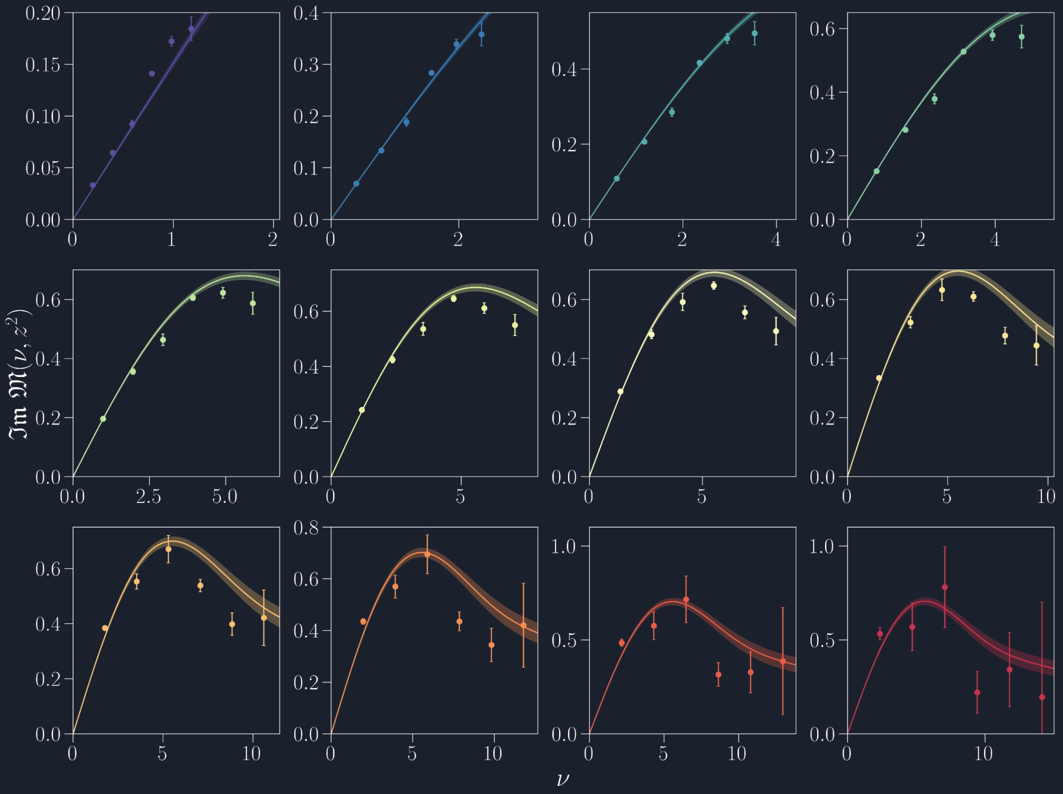
$$\Omega_n^{(\alpha,\beta)}\left(x\right) = \sum_{j=0}^{n} \underbrace{\frac{\Gamma\left(\alpha+n+1\right)}{n!\Gamma\left(\alpha+\beta+n+1\right)} \binom{n}{j} \frac{\left(-1\right)^j \Gamma\left(\alpha+\beta+n+j+1\right)}{\Gamma\left(\alpha+j+1\right)}}_{\omega_{n,j}^{(\alpha,\beta)}} x^j$$

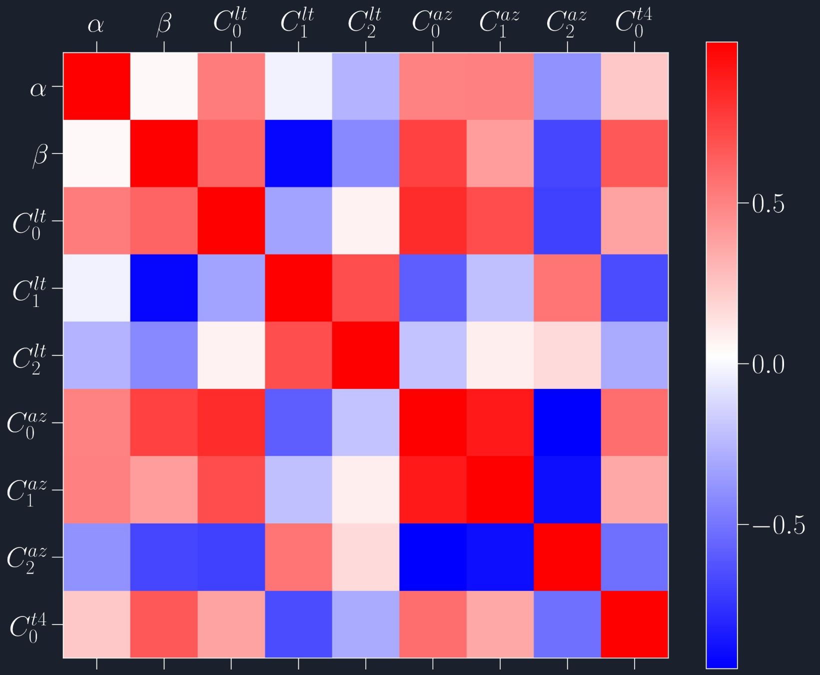
$x \in [0, 1]$	$x^{\alpha}\left(1-x\right)^{\beta}$	$\alpha, \beta > -1$
Interval	Metric	Validity

Flexibility of PDF functional form captured without bias via $\left\{\Omega_n^{(\alpha,\beta)}\right\}$

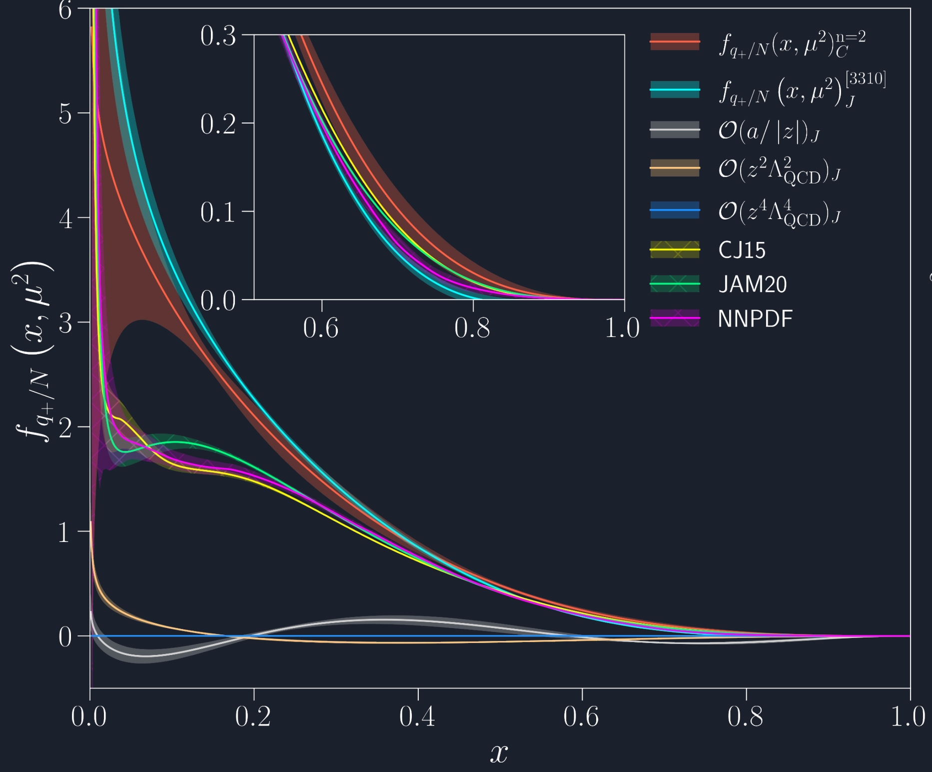
$$f_{q/h}\left(x\right) = x^{\alpha}\left(1-x\right)^{\beta} \sum_{n=0}^{\infty} C_{q,n}^{(\alpha,\beta)} \Omega_n^{(\alpha,\beta)}\left(x\right)$$

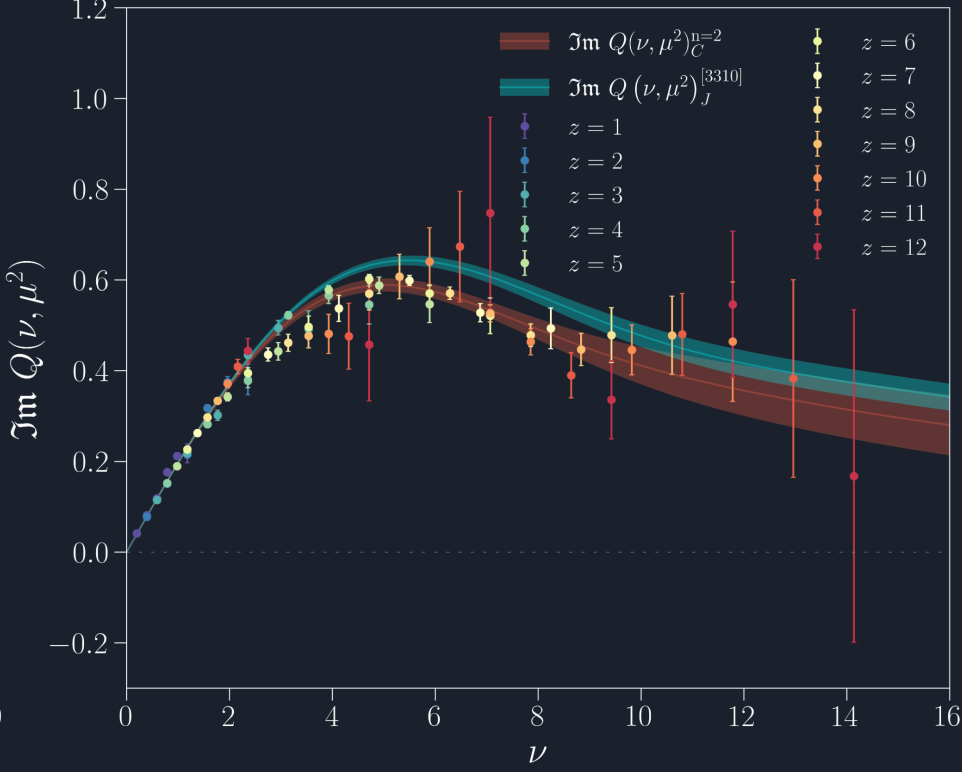
Optimal Fit for Plus PDF





Plus Quark PDF and Leading-Twist ITD





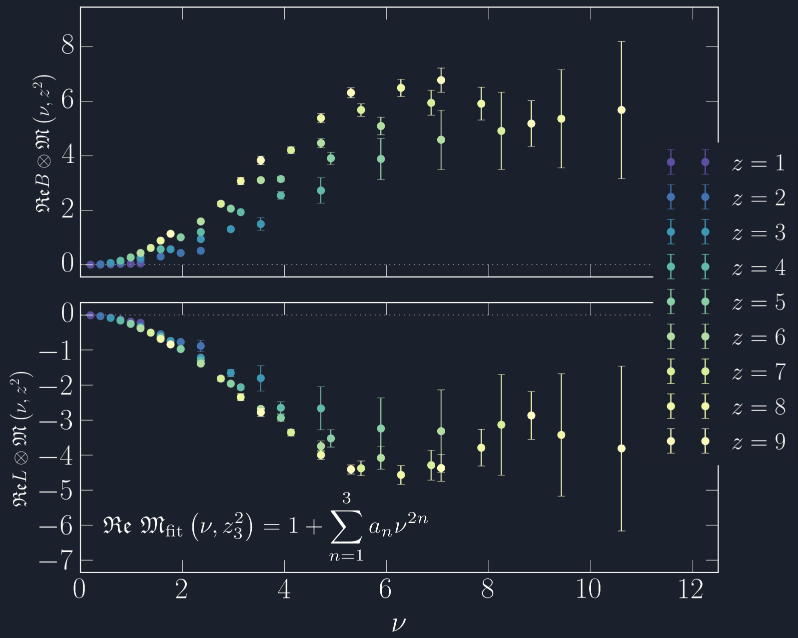
Evolution and Scheme Conversion

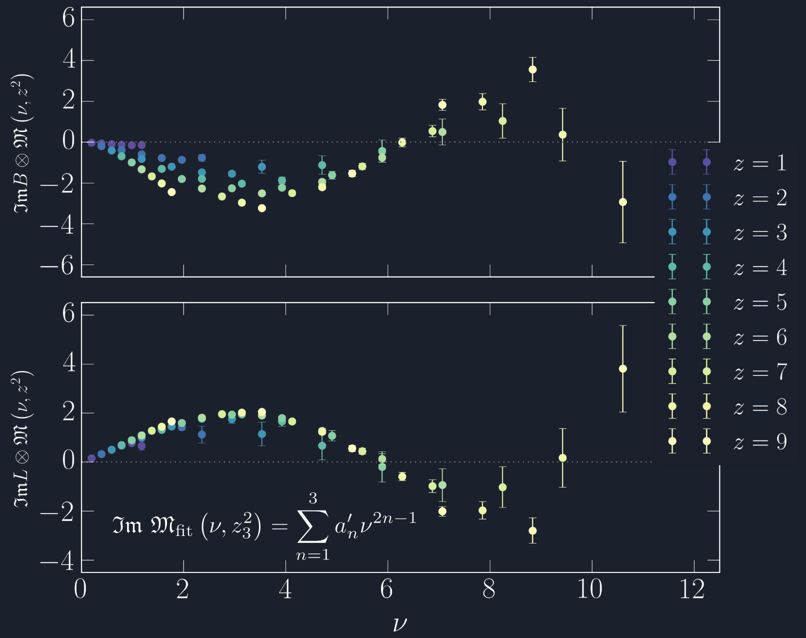
Matching reduced pseudo-ITD to ITD requires a continuous description

$$B(u) = \left(\frac{1+u^2}{1-u}\right)_+ \qquad L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

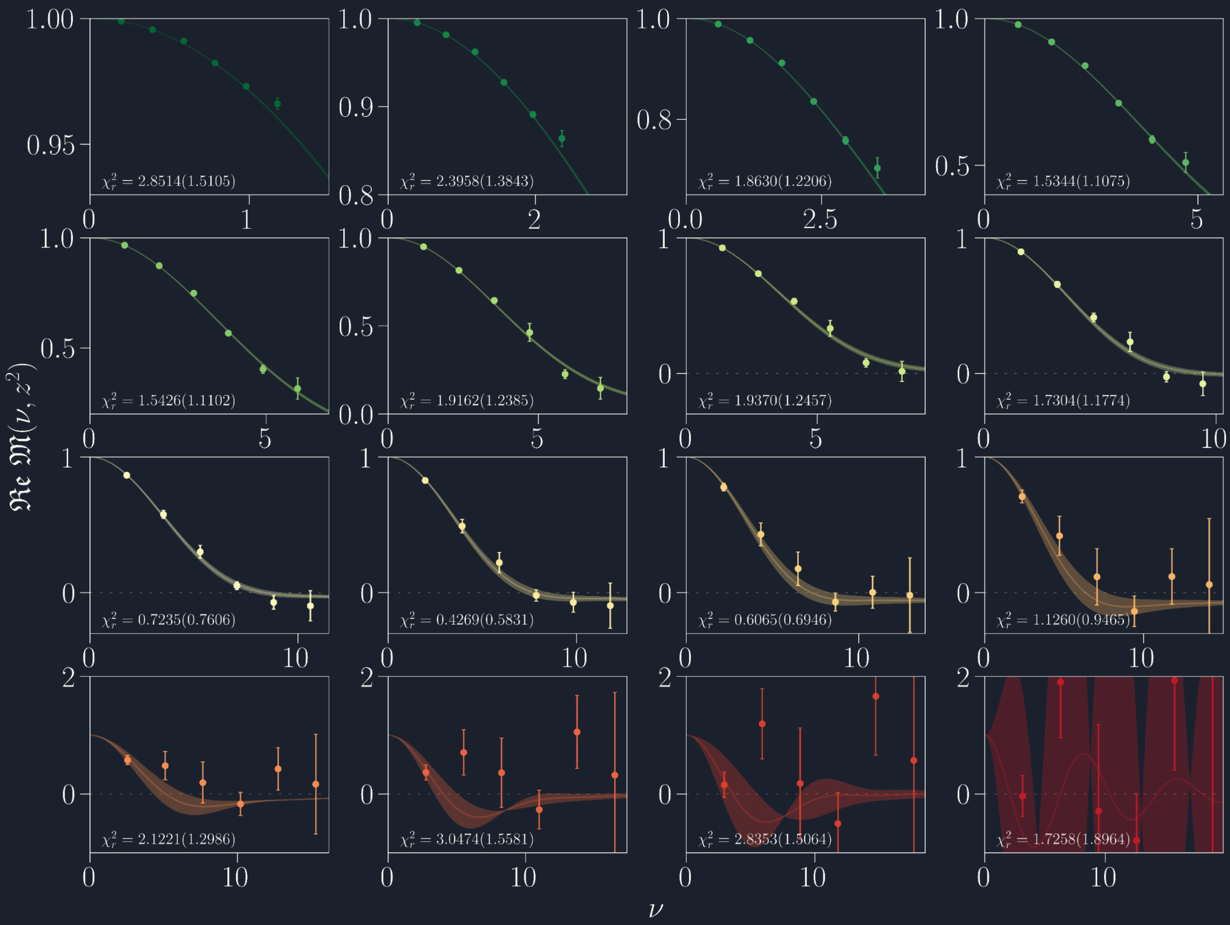
$$\mathcal{Q}(\nu,\mu^2) = \mathfrak{M}(\nu,z^2) + \frac{\alpha_s C_F}{2\pi}\int_0^1 du \left[\ln\left(\frac{e^{2\gamma_E+1}z^2\mu^2}{4}\right)\boxed{B(u)} + \boxed{L(u)}\right]\mathfrak{M}(u\nu,z^2)$$

T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004
A. Radyushkin, Phys.Lett. B781 (2018) 433-442
A. Radyushkin, Phys. Rev. D 98 (2018) no.1, 014019
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508

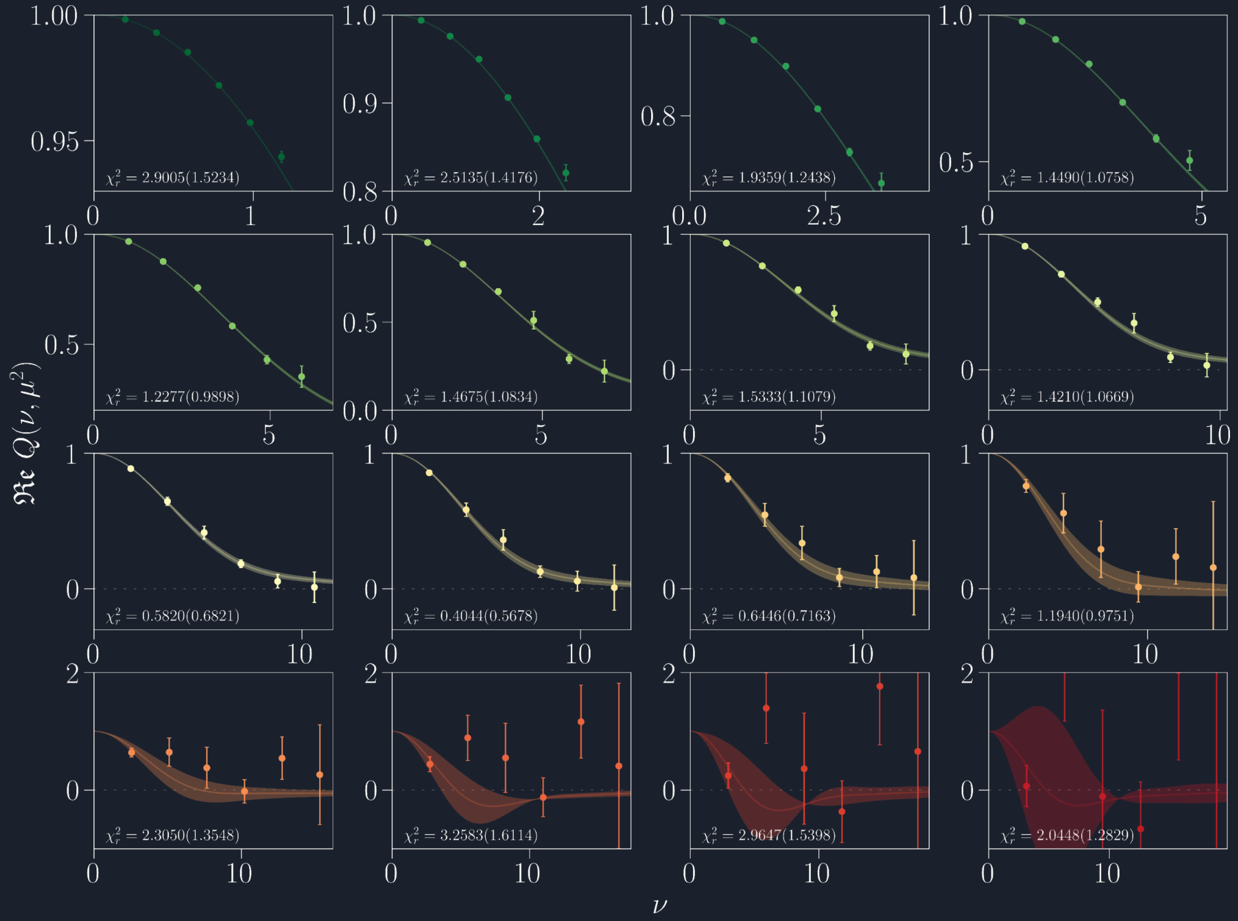




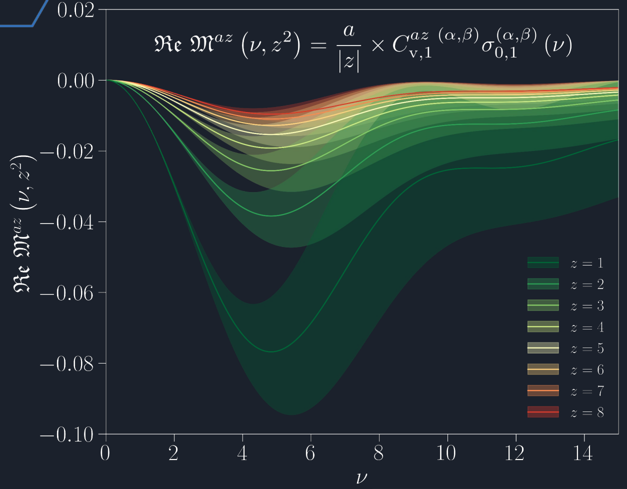
Pseudo-PDF Fit of Reduced Pseudo-ITD



ITD Fit - Pseudo-PDF Form for Convolution

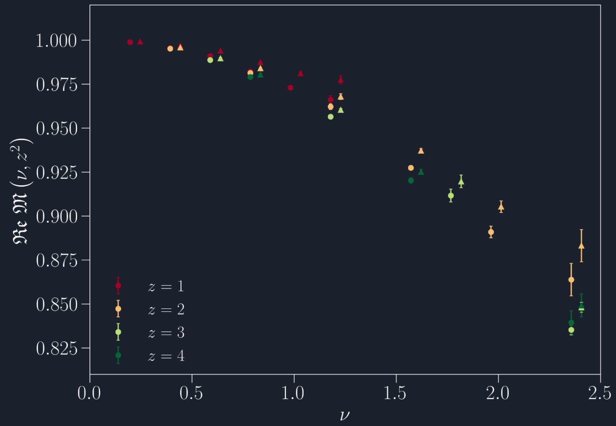


Discretization Effect



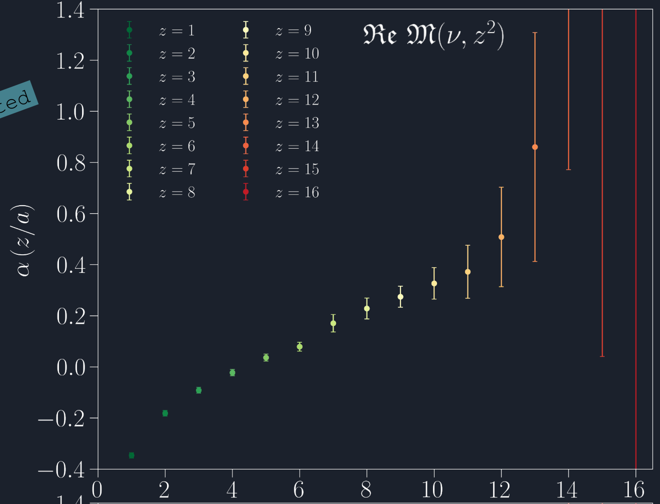
Removal of parameterized discretization effect

- (numerically) slight vertical shift of small-z data
- effect most pronounced for $\nu \lesssim 4.5$

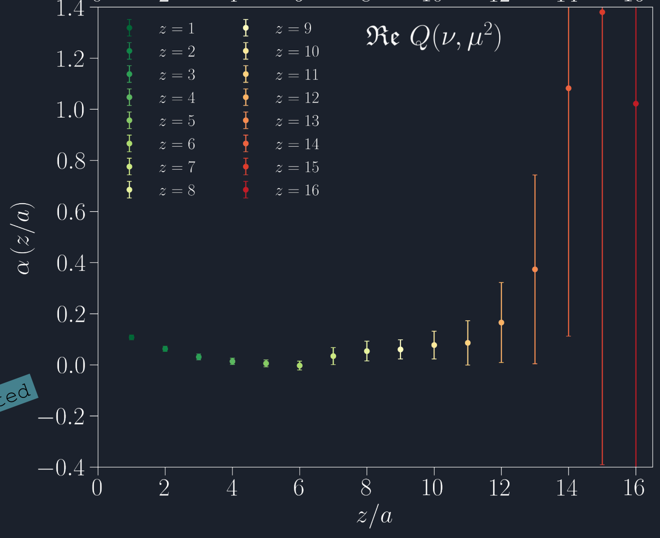


Repeat DGLAP check in (pseudo-)ITD

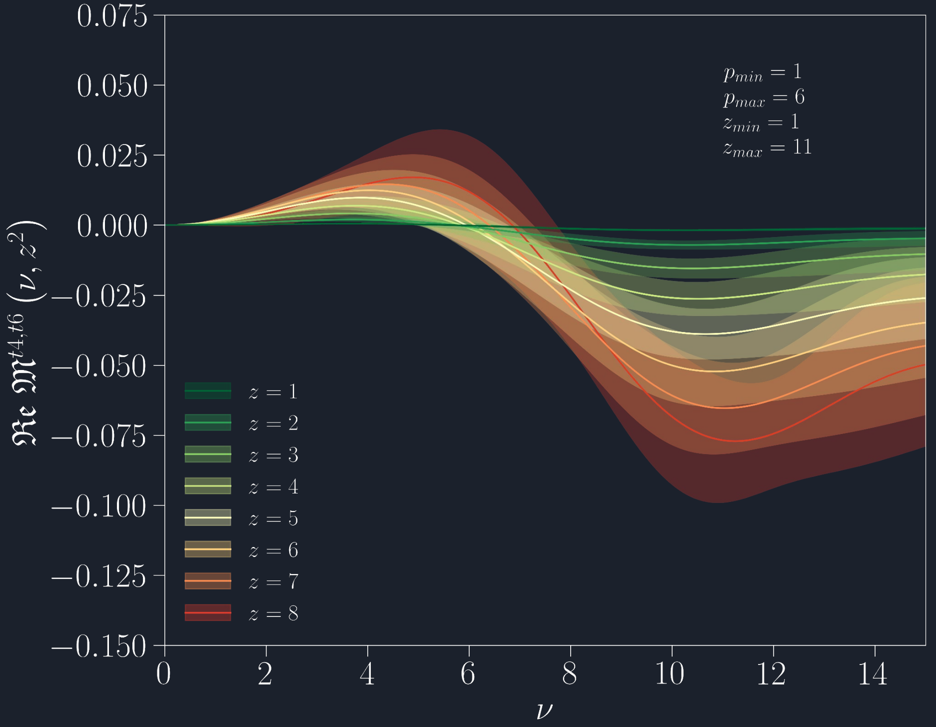
Disc.-Corrected pITD

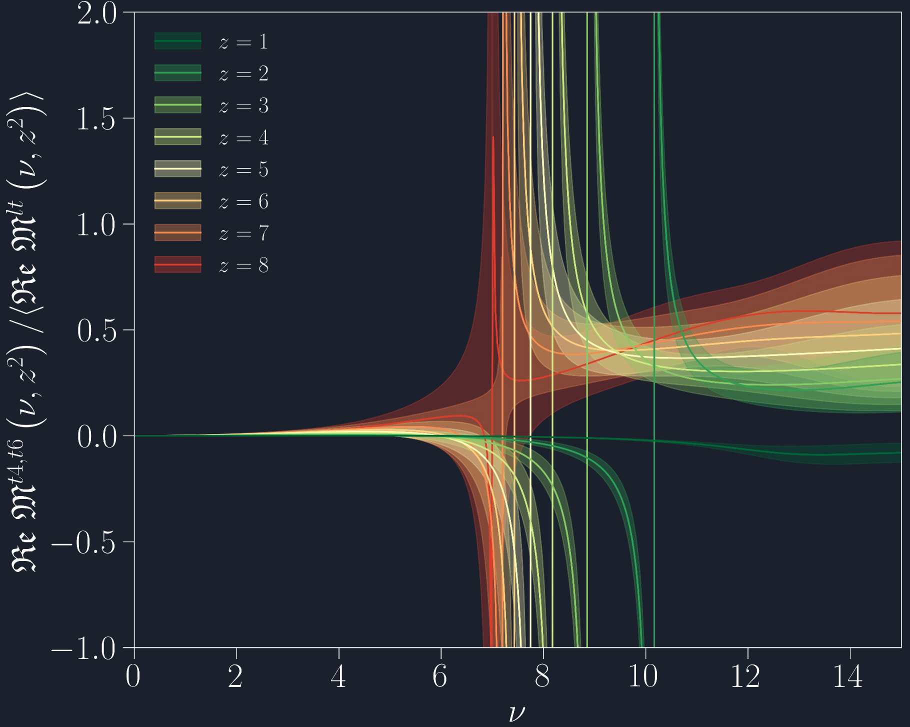


Disc.-Corrected ITD



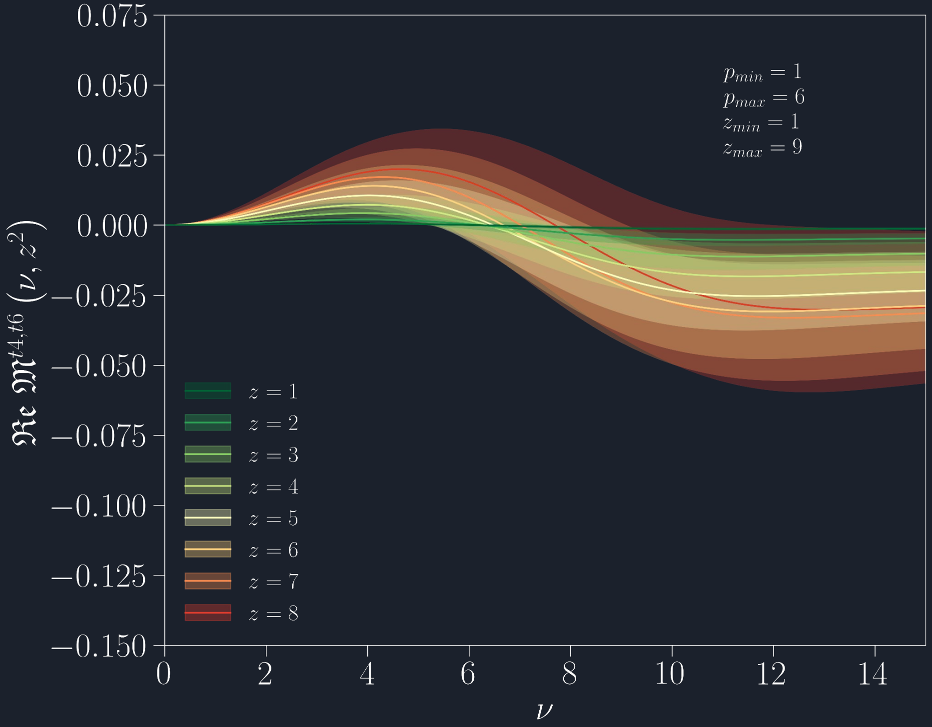
Higher-Twist Variability - Cuts on Wilson Line

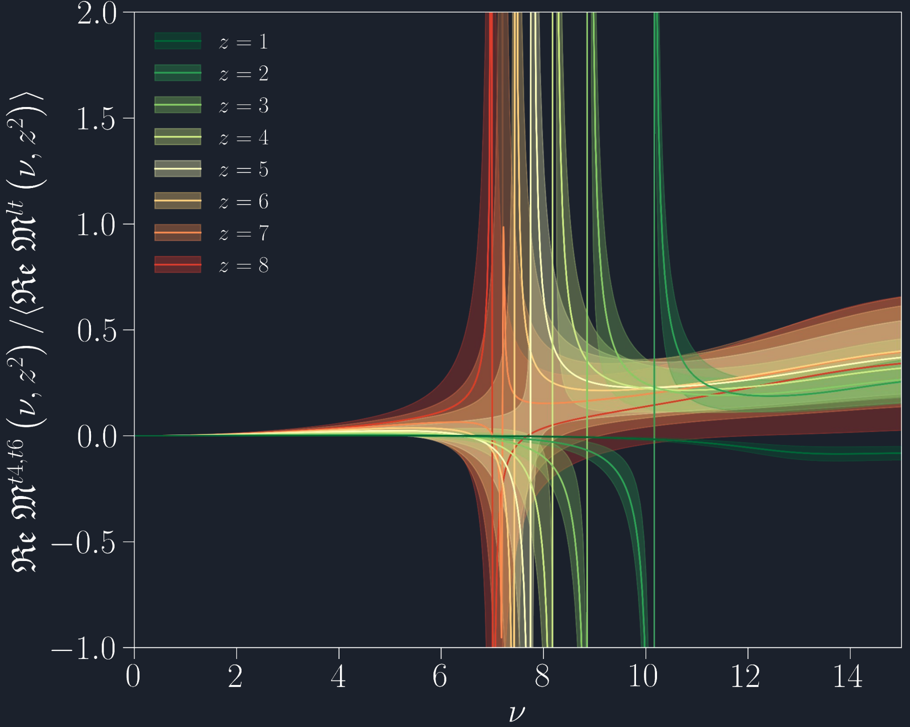




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda^2_{\mathrm{QCD}} \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda^4_{\mathrm{QCD}} \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

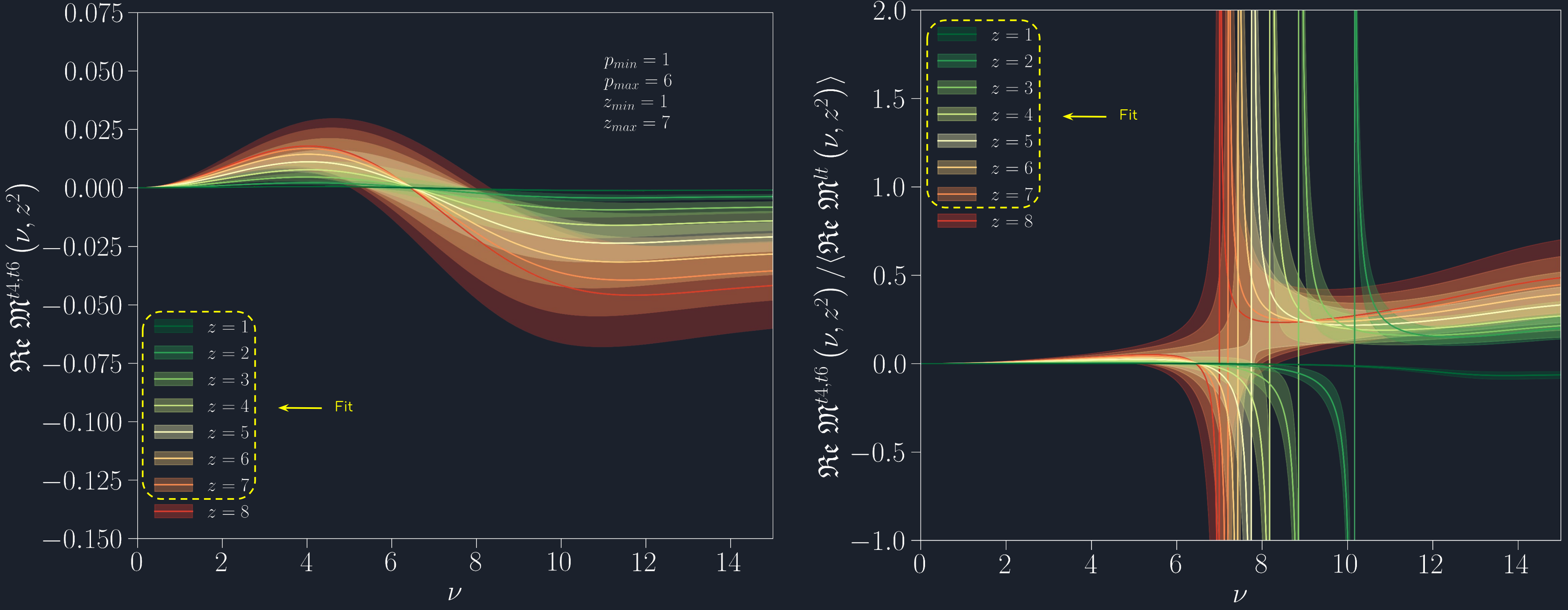
Higher-Twist Variability - Cuts on Wilson Line





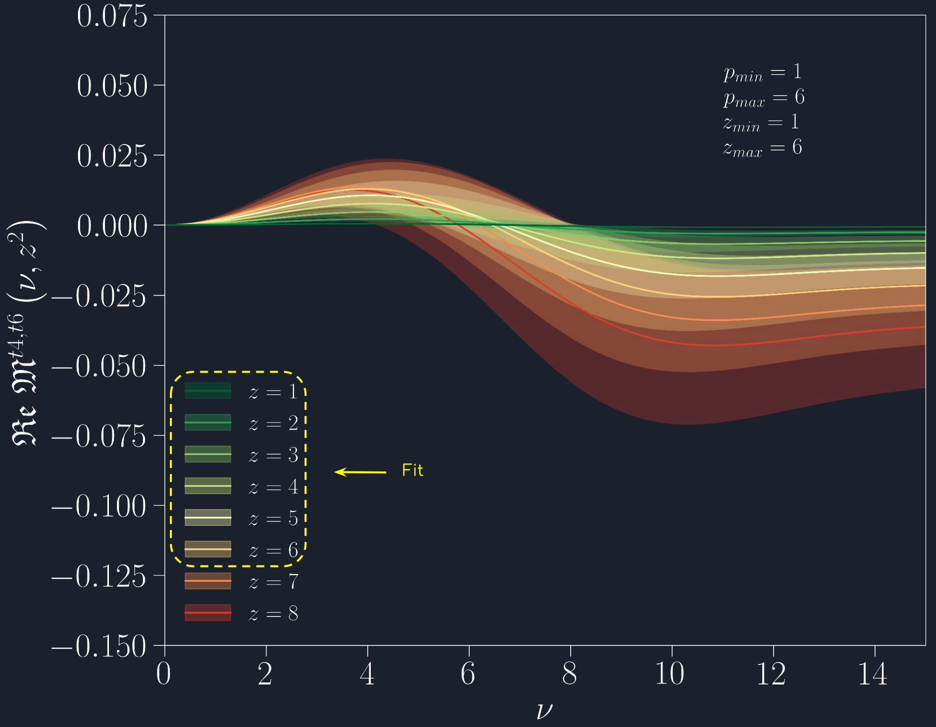
$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda^2_{\mathrm{QCD}} \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda^4_{\mathrm{QCD}} \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

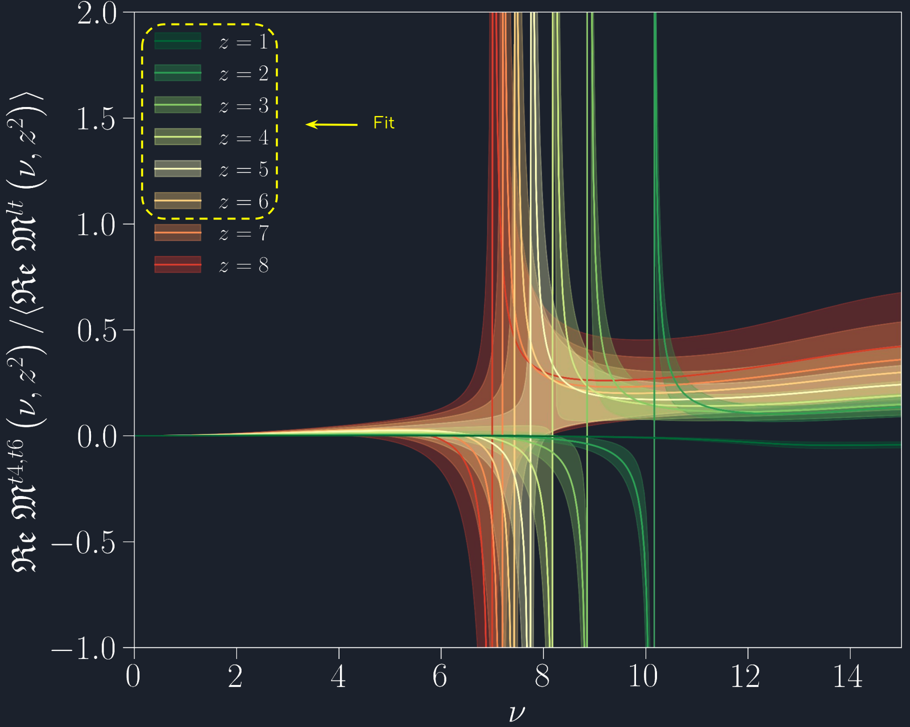
Higher-Twist Variability - Cuts on Wilson Line



$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}\left(\nu, z^2\right) \equiv z^2 \Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\mathrm{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

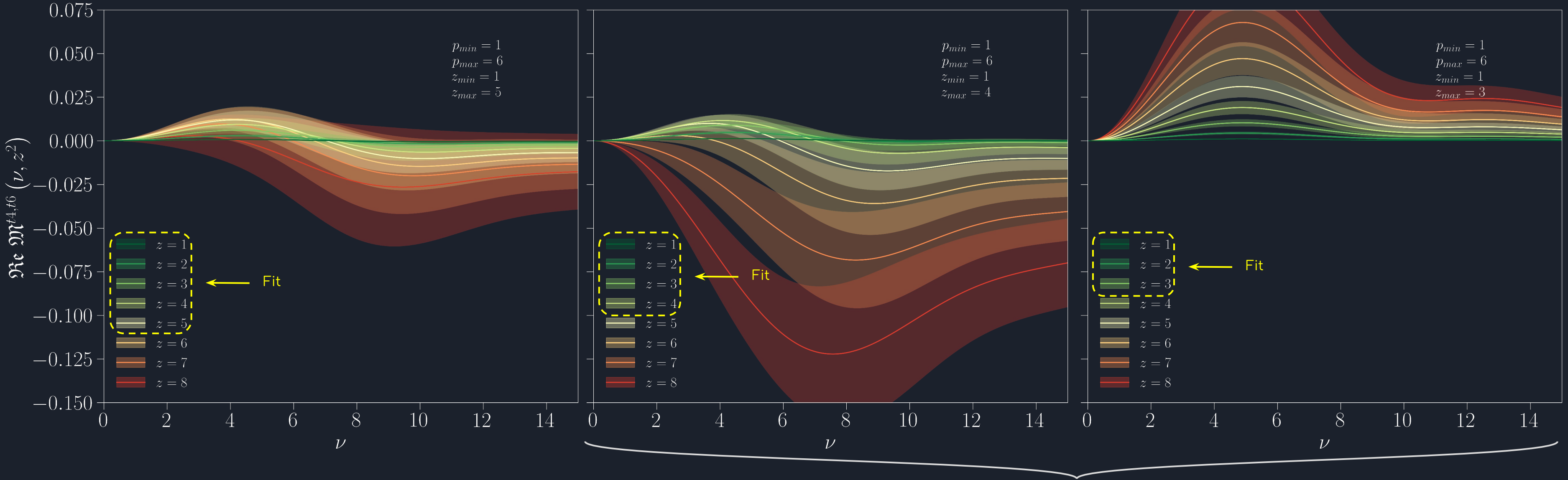
Higher-Twist Variability - Cuts on Wilson Line





$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}\left(\nu, z^2\right) \equiv z^2 \Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\mathrm{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

Higher-Twist Variability - Cuts on Wilson Line

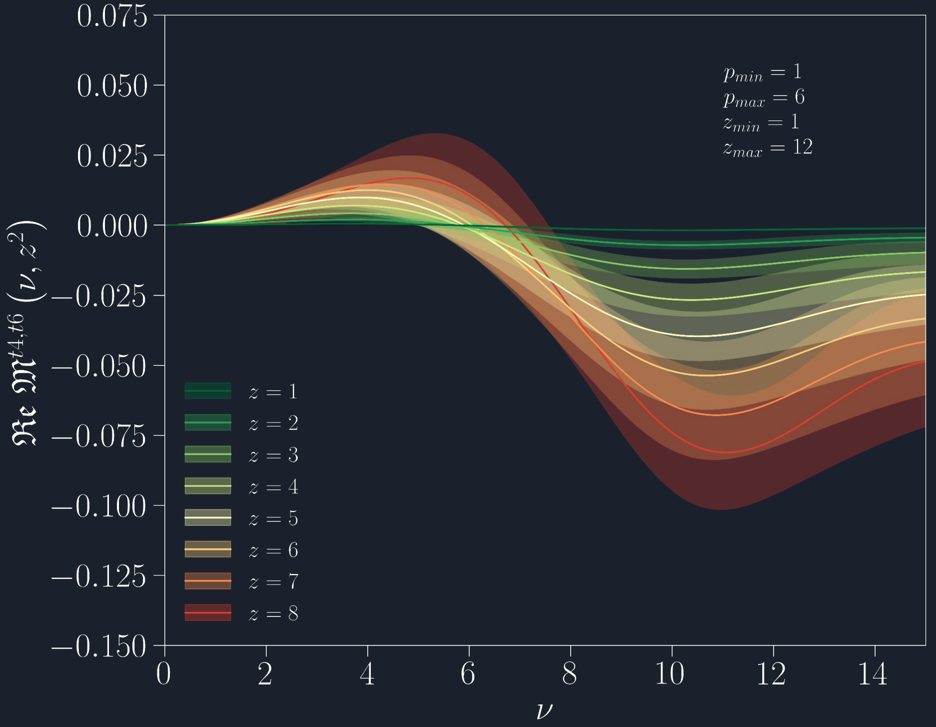


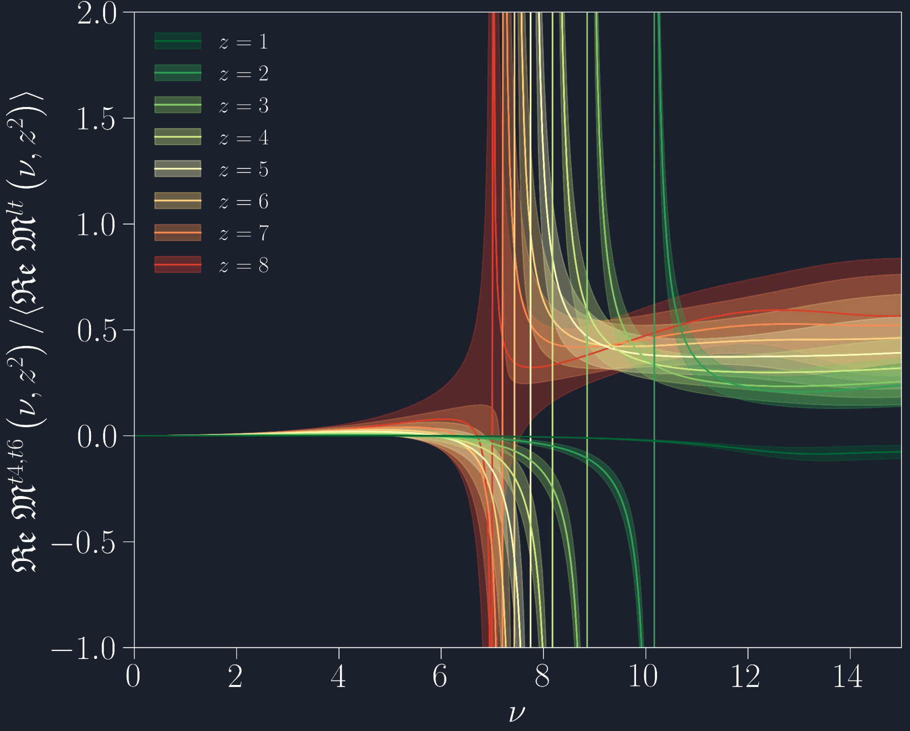
Large variability of parameterized higher-twist effects beyond Wilson line cut

➢ effect no longer constrained

$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}\left(\nu, z^2\right) \equiv z^2 \Lambda^2_{\mathrm{QCD}} \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda^4_{\mathrm{QCD}} \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

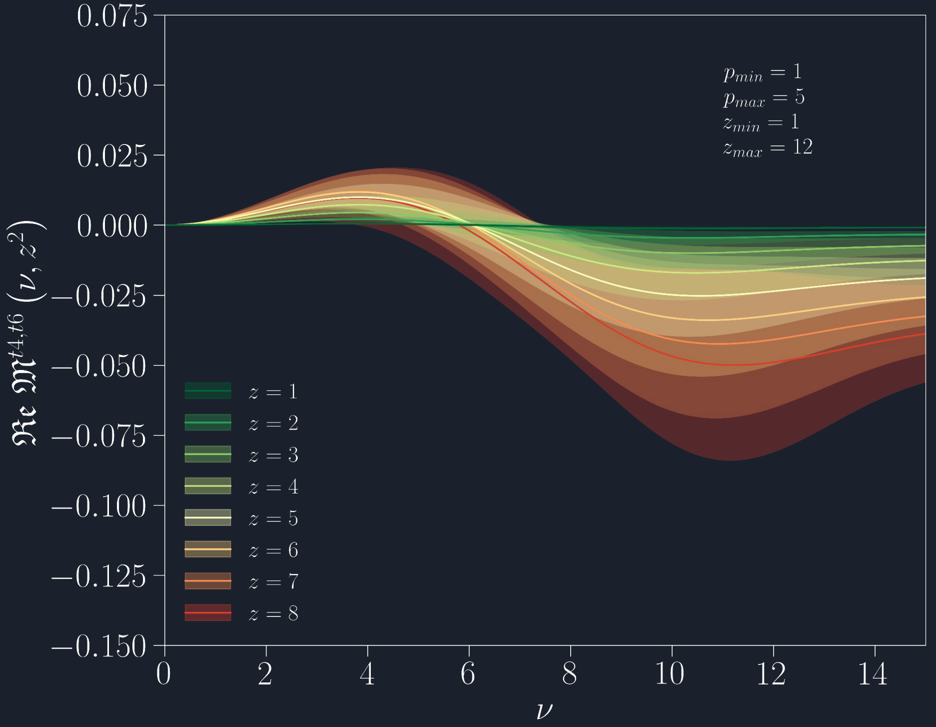
Higher-Twist Variability - Cuts on Momenta

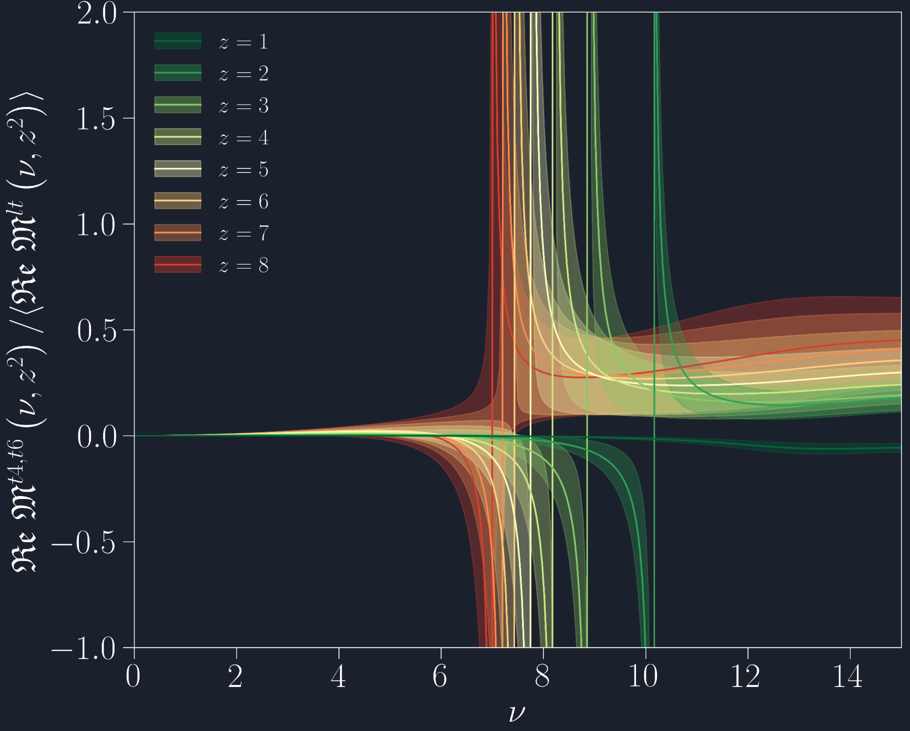




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\mathrm{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

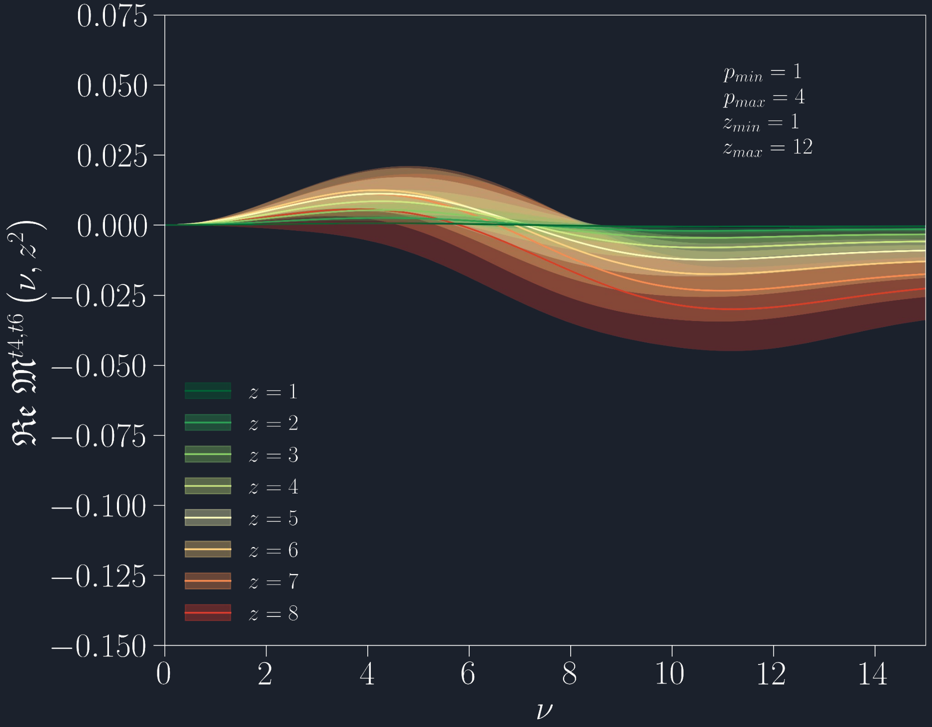
Higher-Twist Variability - Cuts on Momenta

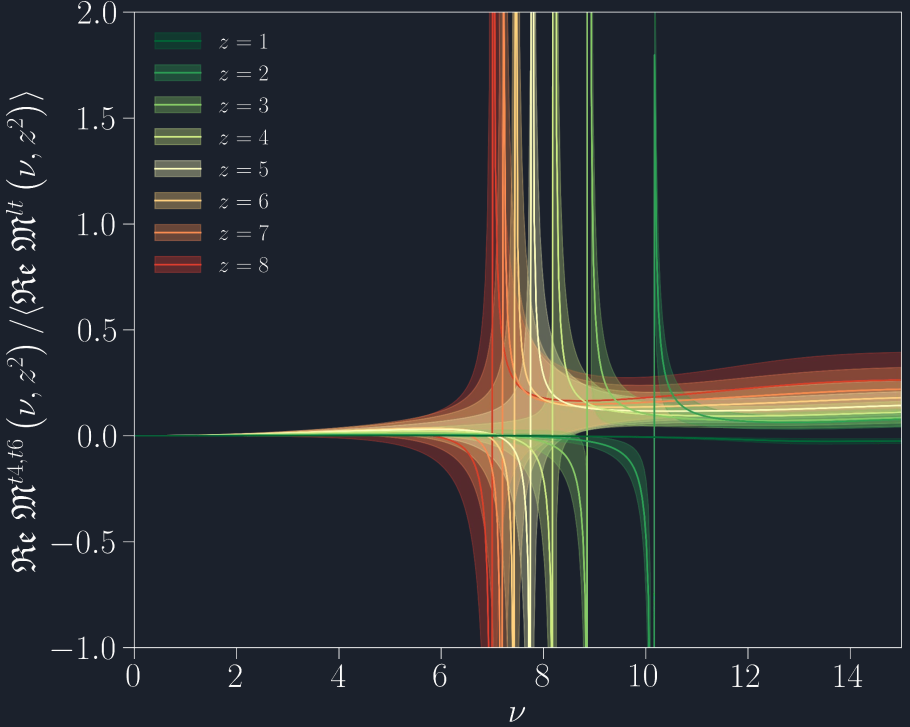




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\mathrm{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

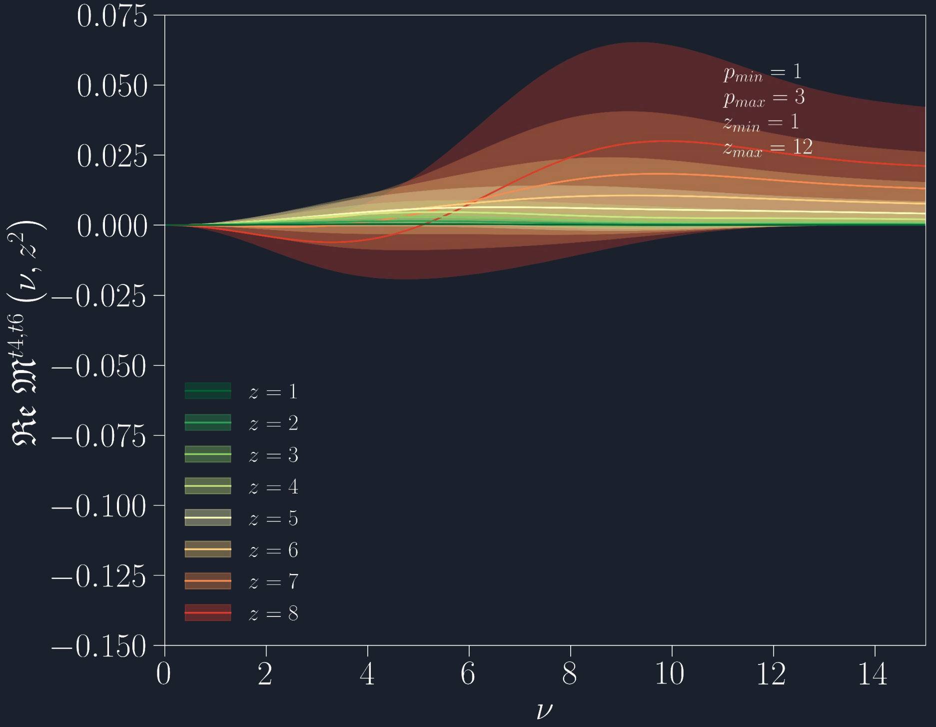
Higher-Twist Variability - Cuts on Momenta

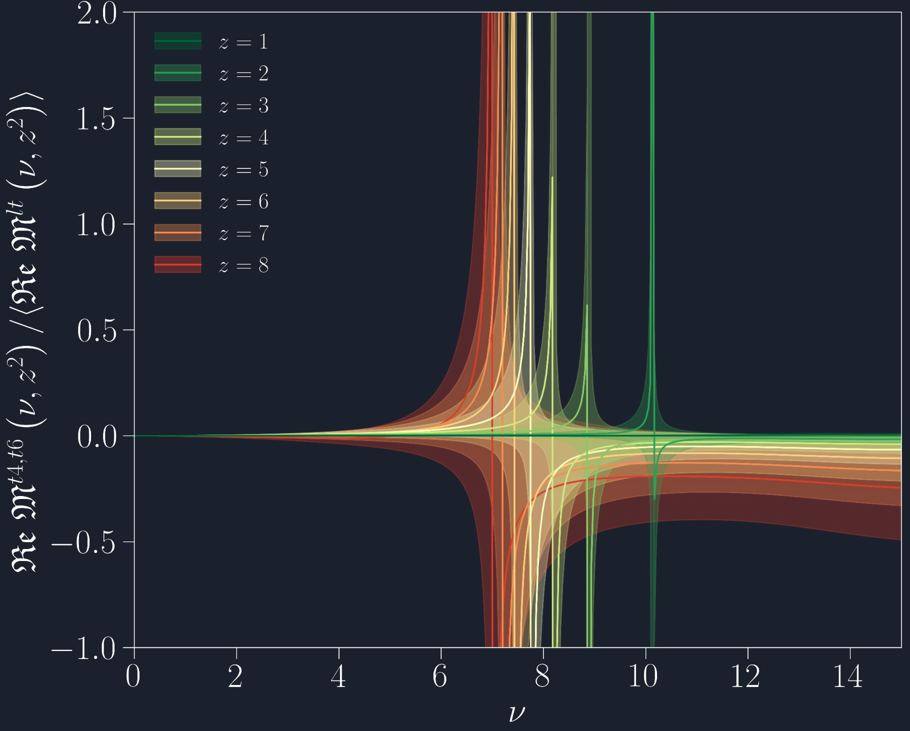




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda_{\rm QCD}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\rm QCD}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t6\ (\alpha,\beta)}$$

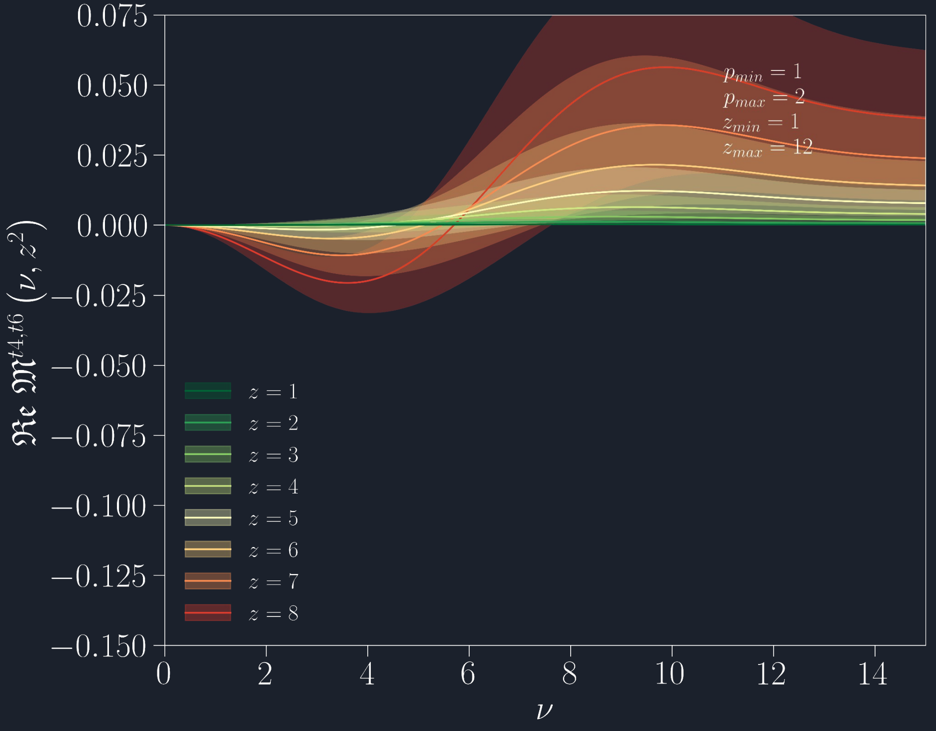
Higher-Twist Variability - Cuts on Momenta

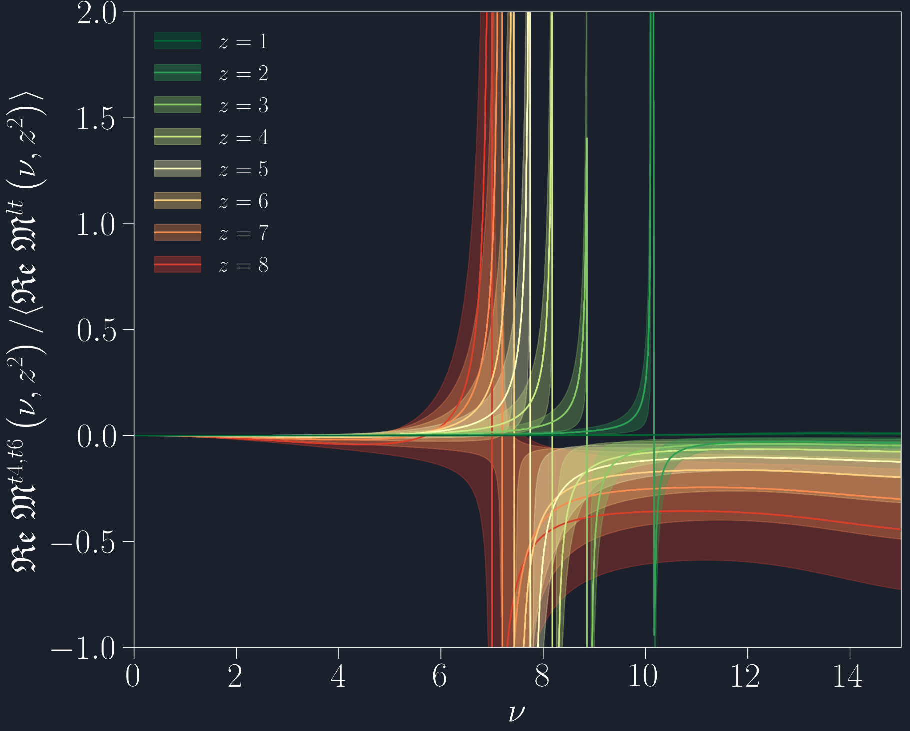




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda^2_{\mathrm{QCD}} \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda^4_{\mathrm{QCD}} \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

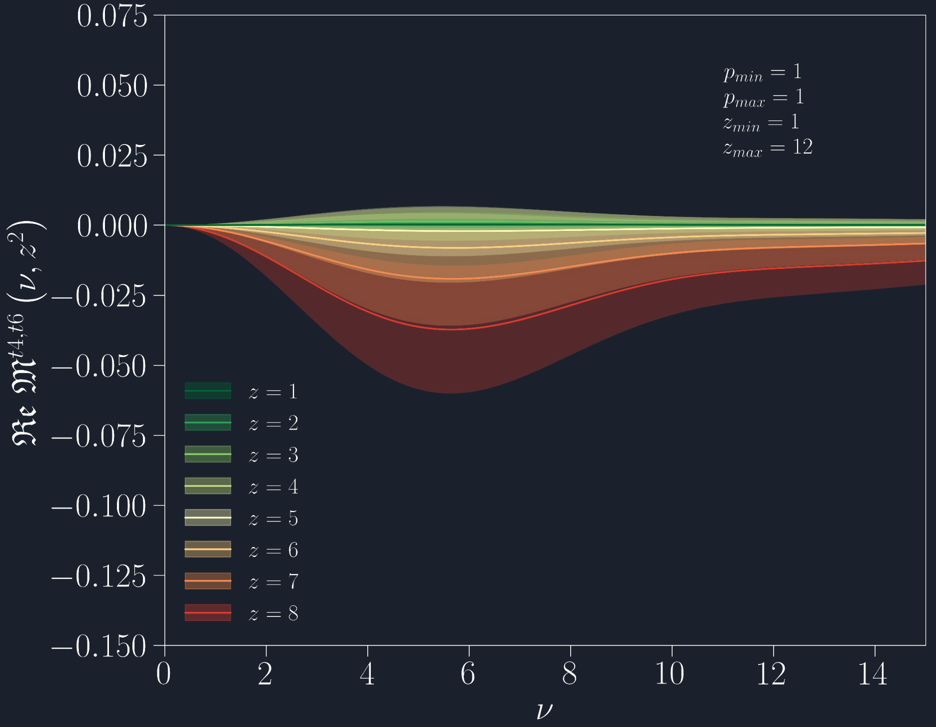
Higher-Twist Variability - Cuts on Momenta

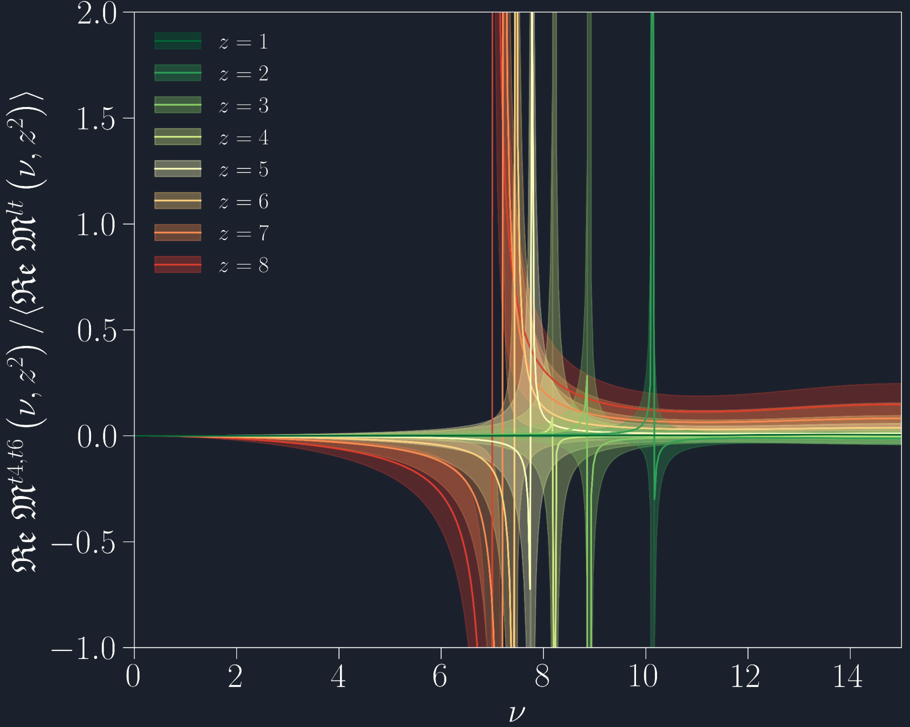




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}\left(\nu, z^2\right) \equiv z^2 \Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\ (\alpha,\beta)} + z^4 \Lambda_{\mathrm{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\ (\alpha,\beta)}$$

Higher-Twist Variability - Cuts on Momenta





$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2 \Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\,(\alpha,\beta)} + z^4 \Lambda_{\mathrm{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t6\,(\alpha,\beta)}$$