Polarized pseudodistributions at short distances

Wayne Morris

Ian Balitsky, Anatoly Radyushkin

as part of the

HadStruc Collaboration

based on

arXiv:2112.02011

December 8, 2021

Quasidistributions

- X. Ji [2013] proposed to calculate equal time correlation functions at purely spacelike separations
- qPDFs are defined through matrix elements of bilocal operators with purely spacelike ($z = (0, 0, 0, z)$) separation

$$q\left(x, \mu^2, P^z\right) = \int \frac{\mathrm{d}z}{4\pi} e^{ixzP^z} \langle P| \bar{\psi}(z)\gamma^z \exp\left(-ig\int_0^z \mathrm{d}z' A^z(z')\right) \psi(0) |P\rangle + \mathcal{O}\left(\Lambda^2/x^2P_z^2, M^2/x^2P_z^2\right)$$

- Approach PDF in large P^z limit
- Matching relation:

$$q\left(x, \mu^2, P^z\right) = \int_x^1 \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda^2/x^2P_z^2, M^2/x^2P_z^2\right)$$

[X. Ji, 2013]

Pseudodistributions

- A. Radyushkin introduced a coordinate-space oriented approach [Radyushkin, 2017]

$$\langle p| \phi(z) \phi(0) |p\rangle = \mathcal{M}\left(\nu, z_3^2\right) = \int_{-1}^{1} \mathrm{d}x e^{ix\nu} \mathcal{P}\left(x, z_3^2\right)$$

$$\nu = -(pz) = p_3 z_3 \quad [\text{Ioffe}, 1969]$$

- Ioffe-time pseudodistribution (ITD) $\mathcal{M}\left(\nu, z_3^2\right)$
- PDFs are obtained from $z_3 \to 0$ limit of psuedo-PDFs: $\mathcal{P}(x, 0) = f(x)$
- UV divergences handled by reduced ITD:

$$\mathfrak{M}\left(\nu, z_3^2\right) = \frac{\mathcal{M}\left(\nu, z_3^2\right)}{\mathcal{M}\left(0, z_3^2\right)}$$

- z_3^2 analogous to renormalization paramater μ^2 in LC PDFs
- Matching relation:

$$\mathfrak{M}\left(\nu, z_3^2\right) = \int_0^1 \mathrm{d}u K\left(u, z_3^2 \mu^2, \alpha_s\right) \mathcal{I}\left(u\nu, \mu^2\right)$$

- Light cone (LC) ITD $\mathcal{I}\left(\nu, \mu^2\right)$ [Braun, *et al.*, 1995]

Lorentz decomposition of matrix element

- Dual field defined by: $\widetilde{G}_{\lambda\beta} = \frac{1}{2}\epsilon_{\lambda\beta\rho\gamma}G^{\rho\gamma}$

$$m_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p,s|\, G_{\mu\alpha}(z)\, \tilde{E}(z,0;A)\widetilde{G}_{\lambda\beta}(0)|p,s\rangle$$

- Spin dependence in z-odd combination:

$$\widetilde{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv \widetilde{m}_{\mu\alpha;\lambda\beta}(z,p) - \widetilde{m}_{\mu\alpha;\lambda\beta}(-z,p)$$

- Lorentz decomposition:

$$\begin{aligned}
\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = &\left(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\right)\widetilde{\mathcal{M}}_{sp} \\
&+\left(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\right)\widetilde{\mathcal{M}}_{ps} \\
&+\left(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\right)\widetilde{\mathcal{M}}_{sz} \\
&+\left(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\right)\widetilde{\mathcal{M}}_{zs} \\
&+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz} + (p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps} \\
&+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz} + (p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}
\end{aligned}$$

Lorentz decomposition of matrix element

- Dual field defined by: $\widetilde{G}_{\lambda\beta} = \frac{1}{2}\epsilon_{\lambda\beta\rho\gamma}G^{\rho\gamma}$

$$m_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p,s|\, G_{\mu\alpha}(z)\, \tilde{E}(z,0;A)\widetilde{G}_{\lambda\beta}(0)|p,s\rangle$$

- Spin dependence in z-odd combination:

$$\widetilde{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv \widetilde{m}_{\mu\alpha;\lambda\beta}(z,p) - \widetilde{m}_{\mu\alpha;\lambda\beta}(-z,p)$$

- Lorentz decomposition:

$$\begin{aligned}
\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) =& (sz)\left(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\right)\widetilde{\mathcal{M}}_{pp} \\
&+(sz)\left(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\right)\widetilde{\mathcal{M}}_{zz} \\
&+(sz)\left(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\right)\widetilde{\mathcal{M}}_{zp} \\
&+(sz)\left(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\right)\widetilde{\mathcal{M}}_{pz} \\
&+(sz)\left(p_\mu z_\alpha - p_\alpha z_\mu\right)\left(p_\lambda z_\beta - p_\beta z_\lambda\right)\widetilde{\mathcal{M}}_{ppzz} \\
&+(sz)\left(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\right)\widetilde{\mathcal{M}}_{gg}\ .
\end{aligned}$$

Polarized gluon LC ITD

- The light-cone distribution is obtained from $(z = (0, z^-, 0_\perp))$

$$g^{\alpha\beta}\widetilde{M}_{+\alpha;\beta+}(z_-, p) = -2p_+ s_+ \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu, 0) \; + \; p_+ z_- \widetilde{\mathcal{M}}_{pp}(\nu, 0)\right]$$

$$\widetilde{\mathcal{M}}^{(+)}_{sp} = \widetilde{\mathcal{M}}_{sp} + \widetilde{\mathcal{M}}_{ps}$$

- Related to LC ITD and polarized distribution

$$i\left(\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\widetilde{\mathcal{M}}_{pp}\right) \equiv \mathcal{I}_p(\nu) = \frac{i}{2}\int_{-1}^{1} dx\, e^{-ix\nu} x\Delta g(x) = \int_0^1 dx\, \sin(x\nu) x\Delta g(x)$$

Polarized gluon pseudo-ITD

- The following multiplicatively renormalizable [Zhang et. al, 2019] quantities

$$
\begin{aligned}
\widetilde{M}_{0i;0i} &= -2s_0p_0\widetilde{\mathcal{M}}^{(+)}_{sp} + 2p_0^2 s_3 z_3 \widetilde{\mathcal{M}}_{pp} + 2s_3 z_3 \widetilde{\mathcal{M}}_{gg} \\
\widetilde{M}_{3i;3i} &= -2p_3 s_3 \widetilde{\mathcal{M}}^{(+)}_{sp} - 2z_3 s_3 \widetilde{\mathcal{M}}^{(+)}_{sz} \\
&\quad + 2s_3 z_3 [p_3^2 \widetilde{\mathcal{M}}_{pp} - \widetilde{\mathcal{M}}_{gg} + z_3^2 \widetilde{\mathcal{M}}_{zz} + z_3 p_3 \widetilde{\mathcal{M}}^{(+)}_{zp}] \ , \\
\widetilde{M}_{0i;3i} &= -2\left(s_0 p_3 \mathcal{M}_{sp} + s_3 p_0 \mathcal{M}_{ps}\right) - 2s_0 z_3 \mathcal{M}_{sz} - 2(sz)\left(p_0 p_3 \mathcal{M}_{pp} + p_0 z_3 \mathcal{M}_{pz}\right) \\
\widetilde{M}_{3i;0i} &= -2\left(s_3 p_0 \mathcal{M}_{sp} + s_0 p_3 \mathcal{M}_{ps}\right) - 2s_0 z_3 \mathcal{M}_{zs} - 2(sz)\left(p_3 p_0 \mathcal{M}_{pp} + z_3 p_0 \mathcal{M}_{zp}\right)
\end{aligned}
$$

- Polarization pseudo-vector defined by: $s^2 = -m^2$, $s = (p_3, 0, 0, p_0)$, $(ps) = 0$

- Leading twist terms can be isolated by taking $\widetilde{M}_{00} = \widetilde{M}_{0i;0i} + \widetilde{M}_{ij;ij}$

$$
-\widetilde{M}_{00}/(2p_3p_0) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu \widetilde{\mathcal{M}}_{pp}\right] - \frac{m^2 z_3^2}{\nu}\widetilde{\mathcal{M}}_{pp} \qquad \widetilde{M}_{ij;ij} = -2s_3 z_3 \widetilde{\mathcal{M}}_{gg}
$$

Background field method

- Introduced by DeWitt in 1965
- Calculation based on techniques introduced by [I. Balitsky, V. Braun, 1989]
- Technique maintains explicit gauge invariance $\rightarrow$ useful for gluon correlators
- Fields are divided into a fluctuating quantum field ($[\mathcal{A}, \phi]$, virtualities μ_2^2 to μ_1^2) and a "classical" background field ($[A, \psi]$, virtualities $< \mu_1^2$).

$$A_\mu^a \rightarrow \mathcal{A}_\mu^a + A_\mu^a, \quad \psi \rightarrow \phi + \psi$$

- Idea is to integrate over the quantum fields
- Modified Lagrangian

$$\mathcal{L} = -\frac{1}{4g^2}\left(G_{\mu\nu}^a + D_\mu \mathcal{A}_\nu^a - D_\nu \mathcal{A}_\mu^a + f^{abc}\mathcal{A}_\mu^b \mathcal{A}_\nu^c\right)^2 - \frac{1}{2g^2}\left(D^\mu \mathcal{A}_\mu^a\right)^2 + \left(\bar{\phi} + \bar{\psi}\right)\left(i\not{D} + \mathcal{A}_\mu^a \gamma^\mu t^a\right)\left(\phi + \psi\right) + \mathcal{L}_{gh}$$

$$gA_\mu \rightarrow A_\mu, \quad D_\mu = \partial_\mu - iA_\mu$$

Gluon propagator in external gluon fields

- Background fields use Fock-Schwinger gauge: $z^\mu A_\mu(z) = 0$

$$A_\mu(z) = \int_0^1 \mathrm{d}u\, u z^\nu G_{\nu\mu}(uz)$$

- Schwinger representation for the propagator:

$$\frac{i}{P^2 + i\epsilon} = \int_0^\infty \mathrm{d}s \exp\left[is\left(P^2 + i\epsilon\right)\right]$$

- Gluon propagator in terms of external gluon fields (omitting ϵ):

$$g^{-2} i\mathcal{A}^a_\mu(z)\mathcal{A}^b_\nu(0) = \langle z| \left(\frac{1}{P^2 g_{\mu\nu} + 2iG_{\mu\nu}}\right)^{ab} |0\rangle = -i\int_0^\infty \mathrm{d}s\, \langle z| e^{is\left(P^2 g_{\mu\nu} + 2iG_{\mu\nu}\right)} |0\rangle$$

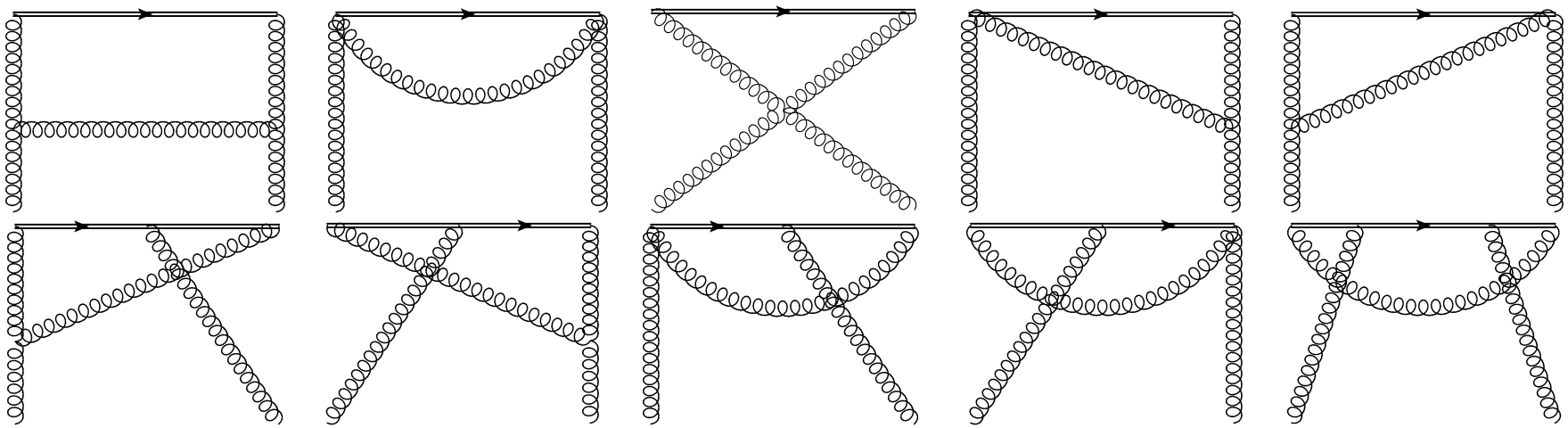
$$= -ig_{\alpha\beta}\frac{\Gamma(d/2-1)}{4\pi^2\left(-z^2\right)^{d/2-1}} + \frac{\Gamma(d/2-2)}{16\pi^2\left(-z^2\right)^{d/2-2}}\int_0^1 \mathrm{d}u \left\{2G_{\alpha\beta}(uz) - \bar{u}uD_\sigma G^{\sigma\rho}(uz) z_\rho g_{\alpha\beta}\right.$$

$$\left. -2ig_{\alpha\beta}\int_0^u \mathrm{d}v \bar{u}v z^\lambda G_{\lambda\xi}(uz) z^\rho G_\rho^{\ \xi}(vz)\right\} - \frac{i\Gamma(d/2-3)}{16\pi^2\left(-z^2\right)^{d/2-3}}$$

$$\times \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v \left[G_{\alpha\xi}(uz) G_\beta^{\ \xi}(vz) - \tfrac{1}{2} i\bar{u} D^2 G_{\alpha\beta}(uz)\right]$$

$$+ \mathcal{O}(\text{twist } 3)$$

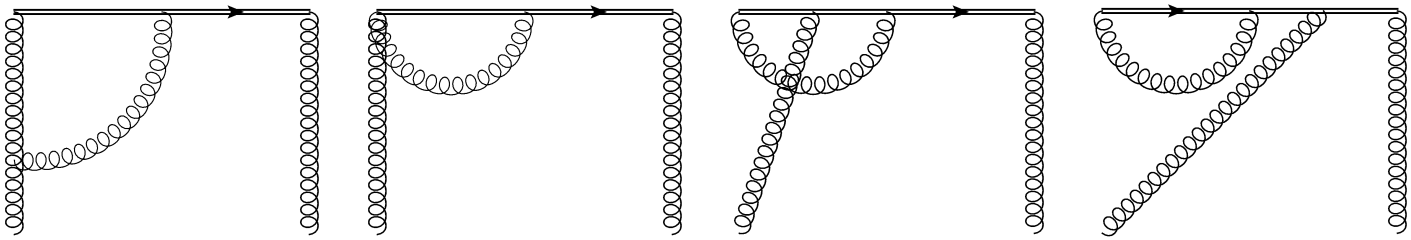
Gluon bilocal operator, handbag



$$\mathcal{O}^H_{\mu\alpha;\rho\sigma}(z) = \frac{1}{2}\epsilon_{\rho\sigma}{}^{\nu\beta} i(D_\mu \mathcal{A}_\alpha - D_\alpha \mathcal{A}_\mu)^a(z)(D_\nu \mathcal{A}_\beta - D_\beta \mathcal{A}_\nu)^b(0)$$
$$= \frac{1}{2}\epsilon_{\rho\sigma}{}^{\nu\beta} \langle z| \left(P_\mu \delta^\xi_\alpha - P_\alpha \delta^\xi_\mu\right) \frac{1}{P^2 g_{\xi\eta} + 2iG_{\xi\eta}} \left(P_\nu \delta^\eta_\beta - P_\beta \delta^\eta_\nu\right) |y\rangle^{ab} \Big|_{y=0}$$

- Expansion of $\frac{1}{P^2 g_{\xi\eta} + 2iG_{\xi\eta}}$ using Schwinger representation

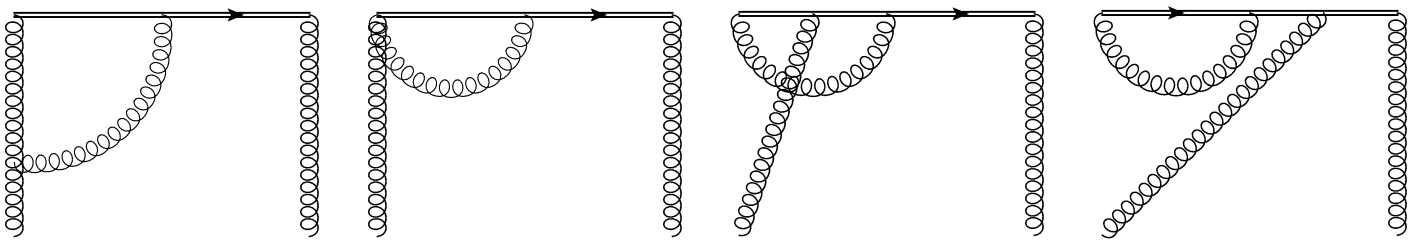
Gluon bilocal operator, vertex



- Linear divergences are 'hidden' inside the vertex diagram:

$$
\begin{aligned}
&\mathcal{O}^V_{\mu\alpha;\rho\sigma}(z) \\
&\to \frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2(-z^2)^{d/2-1}} \int_0^1 \mathrm{d}u \int_0^{\bar{u}} \mathrm{d}v \\
&\quad \times \left\{ \delta(u) \left(\frac{v^{3-d}-v}{d-2} \right) \epsilon_{\rho\sigma}{}^{\nu\beta} z_\beta G_{\mu\alpha}(\bar{u}z) G_{z\nu}(vz) \right. \\
&\qquad \left. + \delta(v) \left(\frac{u^{3-d}-u}{d-2} \right) \left(z_\alpha G_{z\mu}(\bar{u}z) - z_\mu G_{z\alpha}(\bar{u}z) \right) \tilde{G}_{\rho\sigma}(vz) \right\} \\
&\quad + \frac{N_c \Gamma(d/2-2)}{8\pi^2(-z^2)^{d/2-2}} \int_0^1 \mathrm{d}u \int_0^{\bar{u}} \mathrm{d}v \left\{ \delta(u) \left[\frac{v^{3-d}-1}{d-3} \right]_+ + \delta(v) \left[\frac{u^{3-d}-1}{d-3} \right]_+ \right\} G_{\mu\alpha}(\bar{u}z) \tilde{G}_{\rho\sigma}(vz)
\end{aligned}
$$

Gluon bilocal operator, vertex



- UV divergent part

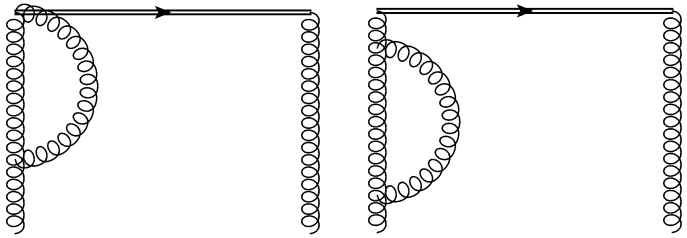
$$\mathcal{O}^V_{0i;0i}(z) + \mathcal{O}^V_{ji;ji}(z)$$
$$\overset{\text{UV}}{\to} \frac{\alpha_s N_c}{4\pi}\left(\frac{1}{\epsilon_{UV}} + \ln\left(z_3^2\mu_{UV}^2\right)\right)\int_0^1 \mathrm{d}u\left(G_{0i}(uz)\tilde{G}_{0i}(0) + G_{ji}(uz)\tilde{G}_{ji}(0)\right)$$

- For lattice, replace $(\alpha_s N_c/4\pi)\ln z_3^2\mu_{\rm UV}^2$ with $(\alpha_s N_c/4\pi)\ln\left(1+\pi z_3^2/a_L^2\right)$

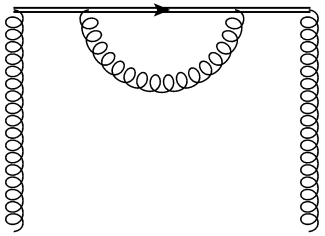
- At higher orders, exponentiates into:

$$Z_{\rm L}(z_3/a_L) = \left(1+\pi^2 z_3^2/a_L^2\right)^{\alpha_s N_c/4\pi}$$

Gluon bilocal operator, link and self energy



$$O^{\text{self}}_{\mu\alpha;\nu\beta}(z) = \frac{g^2 N_c}{8\pi^2}\frac{1}{2-d/2}\left[2-\frac{\beta_0}{2N_c}\right]G_{\mu\alpha}(z)\tilde{G}_{\nu\beta}(0)\ , \qquad \beta_0 = \frac{11N_c}{3}$$



$$O^{\text{link}}_{\mu\alpha;\nu\beta}(z) = \frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2(-z^2)^{d/2-2}}\frac{-1}{(d-3)(d-4)}G_{\mu\alpha}(z)\tilde{G}_{\nu\beta}(0)$$

- Link has linear and logarithmic divergence

One-loop result

$$
\begin{aligned}
&\widetilde{M}_{0i;0i}(z,p) + \widetilde{M}_{ij;ij}(z,p) \\
&\to \frac{g^2 N_c}{8\pi^2}\left[\frac{4}{3}\left(\frac{1}{\epsilon_{\mathsf{UV}}} + \log\left(z_3^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\right) + 2\right]\left(\widetilde{M}_{0i;0i}(z,p) + \widetilde{M}_{ij;ij}(z,p)\right) \\
&\quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left\{-2\bar{u}u + \left(\frac{1}{\bar{u}} - \bar{u}\right)_+ - 4\left[\frac{u+\log(1-u)}{\bar{u}}\right]_+\right. \\
&\quad \left. + \left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\right)\left[\left\{4u\bar{u} + 2\left[u^2/\bar{u}\right]_+\right\} + \frac{1}{2}\left(\frac{\beta_0}{N_c} - 6\right)\delta(\bar{u})\right]\right\} \\
&\quad \times \left(\widetilde{M}_{0i;0i}(uz,p) + \widetilde{M}_{ij;ij}(uz,p)\right) \ .
\end{aligned}
$$

- Altarelli-Parisi (AP) kernel

$$
\widetilde{\mathcal{B}}_{gg}(u) = \left\{4u\bar{u} + 2\left[u^2/\bar{u}\right]_+\right\} + \frac{1}{2}\left(\frac{\beta_0}{N_c} - 6\right)\delta(\bar{u})
$$

Gluon-quark mixing

- Result that corresponds to $\widetilde{M}_{0i;0i} + \widetilde{M}_{ij;ij}$ in the $\overline{\mathrm{MS}}$ scheme at the operator level is:

$$-\frac{g^2 C_F}{8\pi^2}\int_0^1 \mathrm{d}u\; 2\bar{u}u\,\partial_0 \mathcal{O}_q^0\left(uz\right) - \frac{g^2 C_F}{8\pi^2}\log\left(z_3^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\int_0^1 \mathrm{d}u\left(1-\bar{u}^2\right)\partial_0\mathcal{O}_q^0\left(uz\right)$$

- Singlet combination of quark fields is defined as

$$\mathcal{O}_q^0\left(z\right) = \frac{1}{2}\sum_f\left(\bar{\psi}_f(z)\gamma^0\gamma_5\psi_f(0) + \bar{\psi}_f(0)\gamma^0\gamma_5\psi_f(z)\right)$$

- Paremetrization in terms of ITD:

$$\partial_0\left\langle p,s\right|\mathcal{O}_q^0\left(z\right)\left|p,s\right\rangle = -\,2p_0p_3\,i\Delta\mathcal{I}_S\left(\nu\right) \qquad \Delta\mathcal{I}_S\left(\nu\right) = \int_0^1 x\sin\left(x\nu\right)\Delta f_S\left(x\right)$$

$$\begin{aligned}&\widetilde{M}_{0i;0i}(z,p) + \widetilde{M}_{ij;ij}(z,p)\\ \to &2p_0p_3\frac{g^2C_F}{8\pi^2}\int_0^1 \mathrm{d}u\left[\log\left(z_3^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\widetilde{\mathcal{B}}_{gq}(u) + 2\bar{u}u\right]i\Delta\mathcal{I}_S\left(u\nu\right)\end{aligned}$$

- gq component of the evolution kernel:

$$\widetilde{\mathcal{B}}_{gq}(u) = 1-(1-u)^2$$

Reduced pseudo-ITD

- $\widetilde{M}_{00}$ is proportional to p_3, cannot use $\widetilde{M}_{00}(z_3, p_3 = 0)$in denominator for ratio method
- Use $M_{00} \equiv M_{0i;i0} + M_{ij;ji} = 2p_0^2 \mathcal{M}(\nu, z_3^2)$ from unpolarized case instead
- M_{00} cancels link renormalization and self energy related UV divergences, but not vertex
- Also use:

$$Z_{\mathrm{L}}(z_3/a_L) = \left(1 + \pi^2 z_3^2/a_L^2\right)^{\alpha_s N_c/4\pi}$$

$$\mathfrak{M}\left(\nu, z_3^2\right) \equiv i \frac{\{\widetilde{M}_{00}(z_3, p_3)/(p_3 p_0)\}/Z_L\left(z_3/a_L\right)}{M_{00}(z_3, p_3 = 0)/m^2}$$

Matching relation

$$\widetilde{\mathfrak{M}}\left(\nu, z_3^2\right)\langle x_g\rangle_{\mu^2} = \mathcal{I}_p(\nu,\mu^2) - \frac{\alpha_s N_c}{2\pi}\int_0^1 \mathrm{d}u\, \mathcal{I}_p(u\nu,\mu^2)\left\{\log\left(z_3^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\right.$$
$$\left(\left[\frac{2u^2}{\bar{u}} + 4u\bar{u}\right]_+ - \left(\frac{1}{2} + \frac{4}{3}\frac{\langle x_S\rangle_{\mu^2}}{\langle x_g\rangle_{\mu^2}}\right)\delta(\bar{u})\right)$$
$$\left. +4\left[\frac{u+\log(1-u)}{\bar{u}}\right]_+ - \left(\frac{1}{\bar{u}} - \bar{u}\right)_+ - \frac{1}{2}\delta(\bar{u}) + 2\bar{u}u\right\}$$
$$- \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}u\, \Delta\mathcal{I}_S\left(u\nu,\mu^2\right)\left\{\log\left(z_3^2\mu^2\frac{e^{2\gamma_E}}{4}\right)\widetilde{\mathcal{B}}_{gq}(u) + 2\bar{u}u\right\}$$

$$\widetilde{B}_{gg}(u) = \left[\frac{2u^2}{\bar{u}} + 4u\bar{u}\right]_+\Bigg|_{u\neq 1} \qquad \widetilde{\mathcal{B}}_{gq}(w) = 1 - (1-u)^2$$

$$\underbrace{\langle x_g\rangle_{\mu^2} \equiv \int_0^1 \mathrm{d}x\, x f_g(x,\mu^2)}_{\text{gluon momentum fraction}} \qquad \underbrace{\langle x_S\rangle_{\mu^2} \equiv \sum_f \int_0^1 \mathrm{d}x\, x\left(f_f(x,\mu^2) + f_{\bar{f}}(x,\mu^2)\right)}_{\text{singlet quark momentum fraction}}$$

- LC ITD directly relatable to LC PDF through:

$$\mathcal{I}_p\left(\nu,\mu^2\right) = \frac{i}{2}\int_{-1}^1 \mathrm{d}x e^{ix\nu} x\Delta g\left(x,\mu^2\right) = \int_0^1 \mathrm{d}x \sin(x\nu) x\Delta g\left(x,\mu^2\right)$$

Conclusion

- Formulated basic points of pseudo-PDF approach to lattice calculation of polarized gluon PDFs
- Presented the results of our calculations of the one-loop corrections for the bilocal $G_{\mu\alpha}(z)\widetilde{G}_{\lambda\beta}(0)$ correlator of gluonic fields
- Specified combinations of indices giving three matrix elements that contain the structures corresponding to the twist-2 invariant amplitude related to the polarized PDF
- Derived matching relations between Euclidean and light-cone Ioffe-time distributions that are necessary for extraction of the polarized gluon distributions from lattice data

Thank you

HadStruc Collaboration

- **Jefferson Lab:** Robert Edwards, Christos Kallidonis, Nikhil Karthik, Jianwei Qiu, David Richards, Eloy Romero, Frank Winter
- **Oak Ridge:** Balint Joó
- **William and Mary:** Carl Carlson, Colin Egerer, Tanjib Khan, Christopher Monahan, Kostas Orginos, Raza Sufian
- **Old Dominion University:** WM, Anatoly Radyushkin
- **Columbia University:** Joe Karpie
- **Aix Marseille University:** Savvas Zafeiropoulos
- **Peking University:** Yan-Qing Ma