Collins-Soper kernel from transverse momentum-dependent wave functions in LaMET

Min-Huan Chu, Shanghai Jiao Tong University
(Lattice Parton Collaboration)

9/12/2021

LaMET 2021
Motivation: Collins-Soper kernel

Collins-Soper kernel from lattice QCD in LaMET

Numerical results

Summary and outlook
Motivation: TMD process and evolution

- TMDPDFs/TMDWFs as important inputs

- RG evolution ($\mu$)

\[ \frac{d}{d\mu} f_{q/P}^{TMD}(x, b_\perp, \mu, \zeta) = \gamma(\mu, \zeta) \]

- Rapidity evolution ($\zeta$)

\[ 2\zeta \frac{d}{d\zeta} \ln f_{q/P}^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu) \]
**Motivation: CS kernel through TMDs**

**phenomenological (traditional)**

\[ D_{NP}(b_T, \mu = 2 \text{GeV}) \]

\[ K(b_{\perp}, \mu) = -2(D_{NP} + \sum_{n=0}^{\infty} \alpha_s(\mu)^n d_n) \]

I. Scimemi, A. Vadimirov, arxiv 1912.06532 (2020)

**lattice**

From TMDPDFs

\[ 2\zeta \frac{d}{d\zeta} \ln f^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu) \]

\[ f^{TMD}(x, b_\perp, \mu, \zeta) \xrightarrow{\text{lattice}} \langle 0 | \hat{O}(0)\hat{O}(b_\perp, z)\hat{O}(0) | 0 \rangle \]

\[ C_3(b_\perp, z) \]

From TMDWFs

\[ 2\zeta \frac{d}{d\zeta} \ln \psi^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu) \]

\[ \psi^{TMD}(x, b_\perp, \mu, \zeta) \xrightarrow{\text{lattice}} \langle 0 | \hat{O}(b_\perp, z)\hat{O}(0) | 0 \rangle \]

\[ C_2(b_\perp, z) \]

**Simpler and has better signal for lattice calculation!**
**Motivation: Previous LQCD results**

**$K(b_\perp, \mu)$ from TMDPDFs at 1-loop**

![Graph showing $K(b_\perp, \mu)$ from TMDPDFs at 1-loop]

*P. Shanahan, M. Wangman, Y. Zhao, arxiv 2017.11903 (2021)*

**$K(b_\perp, \mu)$ from TMDWFs at tree level**

![Graph showing $K(b_\perp, \mu)$ from TMDWFs at tree level]

*Y. Li et al, arxiv 2106.13027 (2021)*

**$K(P)$**

![Graph showing $K(P)$]

*M. Shlemmer et al, arxiv 2103.16991 (2021)*

**$\kappa$**

![Graph showing $\kappa$]

*Q. Zhang et al, arxiv 2005.14572 (2020)*
• Motivation: Collins-Soper kernel

• Collins-Soper kernel from lattice QCD in LaMET

• Numerical results

• Summary and outlook
Factorization

$$\psi^\pm(b, x, \zeta) = H_1^{-1}(\zeta^z, \bar{\zeta}^z) e^{-\frac{1}{2} \ln\left(\frac{\zeta^z}{\bar{\zeta}^z}\right) K(b)} \tilde{S}^\pm(b) \psi^\pm(b, x, \zeta^z)$$

Collins-Soper kernel

$$K(b_\perp, \mu) = \frac{1}{\ln(P_2^z/P_1^z)} \ln \left[ \frac{H_N^\pm(\zeta_1^z, \bar{\zeta}_1^z) \tilde{\psi}_N^\pm(b, x, \zeta_2^z)}{H_N^\pm(\zeta_2^z, \bar{\zeta}_2^z) \tilde{\psi}_N^\pm(b, x, \zeta_1^z)} \right]$$

Leading-order reduced graph for quasi-TMDWFs of a pseudo-scalar meson.

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)
Quasi-TMDWF: $\tilde{\psi}^\pm(z, b_\perp, \xi^z) = \frac{\langle 0 \mid \bar{\Psi}_{\mp n^z}(zn^z/2 + b_\perp n^x)\Gamma\Psi_{\mp n^z}(-zn^z/2) \mid \pi \rangle}{\sqrt{Z_E(2L, b_\perp)}} = \frac{C_2(L, b_\perp, z, t, P \xi)}{\sqrt{Z_E(2L, b_\perp)}}$
• Motivation: Collins-Soper kernel

• Collins-Soper kernel from lattice QCD in LaMET

• Numerical results

• Summary and outlook
### CLS configurations (previous calculation)

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$a$ (fm)</th>
<th>$c_{sw}$</th>
<th>$\kappa_{l}^{\text{sea}}$</th>
<th>$m_{\pi}^{\text{sea}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24^3 \times 48$</td>
<td>0.098</td>
<td>2.06686</td>
<td>0.13675</td>
<td>333</td>
</tr>
</tbody>
</table>

2.35 fm

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$a$ (fm)</th>
<th>$c_{sw}$</th>
<th>$\kappa_{l}^{\text{sea}}$</th>
<th>$m_{\pi}^{\text{sea}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24^3 \times 48$</td>
<td>0.098</td>
<td>2.06686</td>
<td>0.13675</td>
<td>333</td>
</tr>
</tbody>
</table>

### MILC configurations (this work)

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$a$ (fm)</th>
<th>$c_{sw}$</th>
<th>$\kappa_{l}^{\text{sea}}$</th>
<th>$m_{\pi}^{\text{sea}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$48^3 \times 64$</td>
<td>0.12</td>
<td>1.05088</td>
<td>0.12750</td>
<td>130</td>
</tr>
</tbody>
</table>

5.76 fm

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$a$ (fm)</th>
<th>$c_{sw}$</th>
<th>$\kappa_{l}^{\text{sea}}$</th>
<th>$m_{\pi}^{\text{sea}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$48^3 \times 64$</td>
<td>0.12</td>
<td>1.05088</td>
<td>0.12750</td>
<td>130</td>
</tr>
</tbody>
</table>

- 2+1+1 flavors of HISQ action (MILC)
- Momenta: 1.72GeV, 2.15GeV, 2.58GeV
- Gamma factor: 2.57, 3.21, 3.85
1-state fit is consistent with two-state fit.

Two point correlation function

\[ \tilde{C}_2(L, b, z, t) = \frac{\tilde{\psi}(L, b, z)(1 + c_1(L, b, z)e^{-\Delta E t})}{1 + c_0(0)e^{-\Delta E t}} \]

1-state fit \[ \tilde{C}_2(L, b, z, t) = \tilde{\psi}(L, b, z) \]
Numerical Results: Large L limit

\[ n^z = 12, \ b_\perp = 0.24\text{fm}, z = 0.24\text{fm} \]

\[ \tilde{\psi}(b, z) = \lim_{l \to \infty} \tilde{\psi}(l, b, z) \]

- \( \tilde{\psi}_l \) and \( \sqrt{Z_E} \) contain linear divergence.
- Linear divergence is cancelled at large \( L \).
- \( L = L_0 = 6a = 0.72\text{fm} \) as asymptotic.
Numerical Results: Operator mixing

quasi-TMDWFs in coordinate space

- \( \langle 0 | \bar{\Psi}(x)\gamma^\mu\gamma_5\Psi(y) | \pi(P) \rangle \propto i\gamma_\mu P^\mu \)
- \( \gamma^t\gamma_5 \) and \( \gamma^z\gamma_5 \) give similar results.
- \( \bar{\psi} = \frac{1}{2} [\bar{\psi}(\Gamma = \gamma^t\gamma_5) + \bar{\psi}(\Gamma = \gamma^z\gamma_5)] \)
Fourier transformation

\[ \tilde{\psi}(x, b_{\perp}, \mu, \zeta_{\perp}) = \lim_{L \to \infty} \int e^{i(x-1/2)P_{\perp}} \tilde{\psi}(z, b_{\perp}, \zeta_{\perp}) \]

quasi-WF \( \tilde{\psi}(x, b_{\perp}, \mu, \zeta_{\perp}) \) is complex!
• $K(b_{\perp}, \mu)$ from average of $\tilde{\psi}(L \to \infty)$ and $\tilde{\psi}(L \to -\infty)$

• Joint fit of different momentum combinations

\[
K(b_{\perp}, \mu, x, P_{1z}, P_{2z}) = K(b_{\perp}, \mu) + A \left[ \frac{1}{x^2(1-x)^2(P_{1z})^2} - \frac{1}{x^2(1-x)^2(P_{2z})^2} \right]
\]
• **Perturbative calculation:**
  1loop, 2loop, 3loop: Y. Li et al, arxiv 1604.01404 (2017)

• From TMDWFs at **tree level:**
  ETMC/PKU 21: Y. Li et al, arxiv 2106.13027 (2021)

• From TMDPDFs at **1-loop:**
  SVZES 21: M. Schlemmer et al, arxiv 2103.16991 (2021)
• Parameterization of $K(b_⊥, \mu)$:

• Parameterization of TMDPDFs:
• Collins-Soper kernel describes the \textit{rapidity evolution} of TMDPDFs/TMDWFs.

• This is the \textit{first attempt} for extracting Collins-Soper kernel from TMDWFs with \textit{1-loop level} matching.

• This has added evidences for the extraction of \textit{partonic structure} from LQCD in LaMET.

Thanks for your attention!