



Collins-Soper kernel from transverse momentum-dependent wave functions in LaMET

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(Lattice Parton Collaboration)

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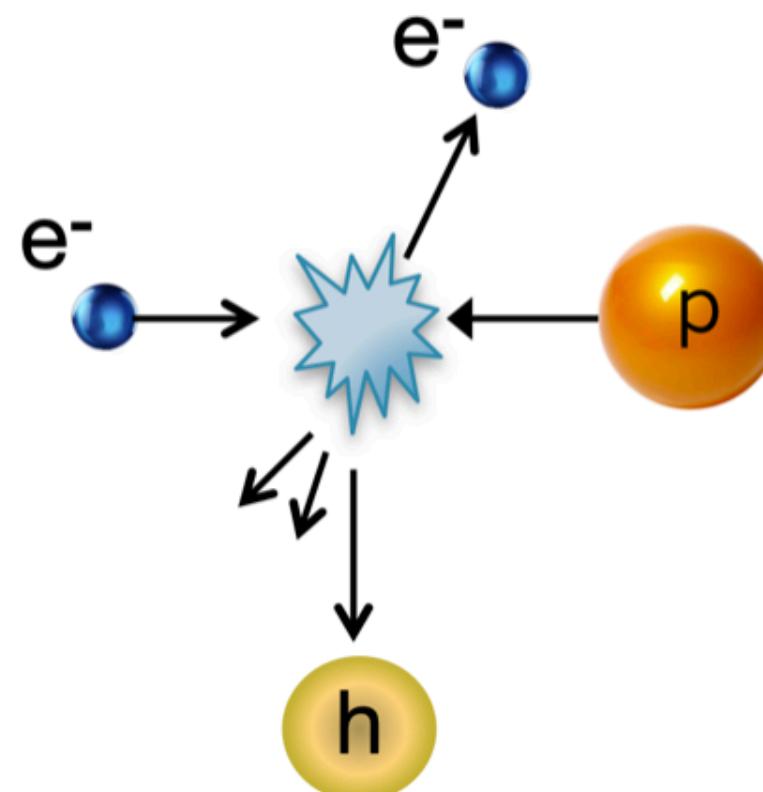


Outline

- Motivation: Collins-Soper kernel
- Collins-Soper kernel from lattice QCD in LaMET
- Numerical results
- Summary and outlook

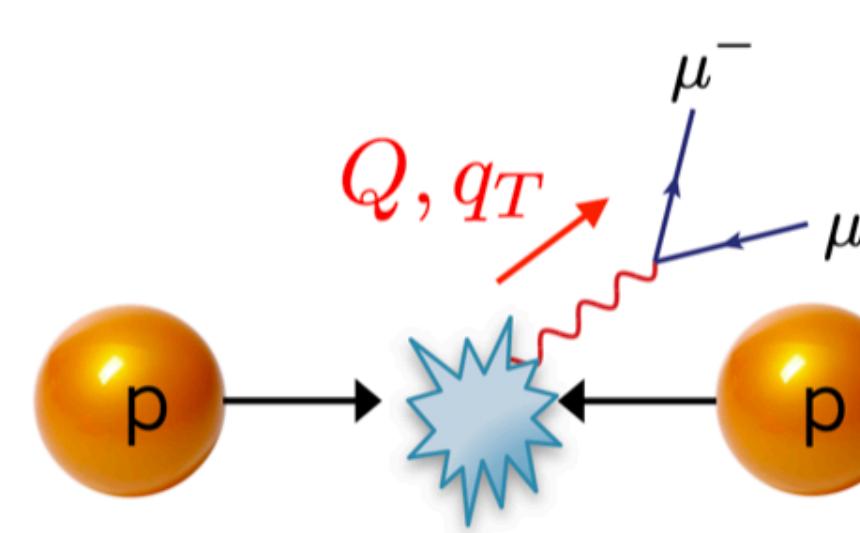
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



R. Angeles-Martinez et. al, arxiv 2507.05267 (2015)

- TMDPDFs/TMDWFs as important **inputs**

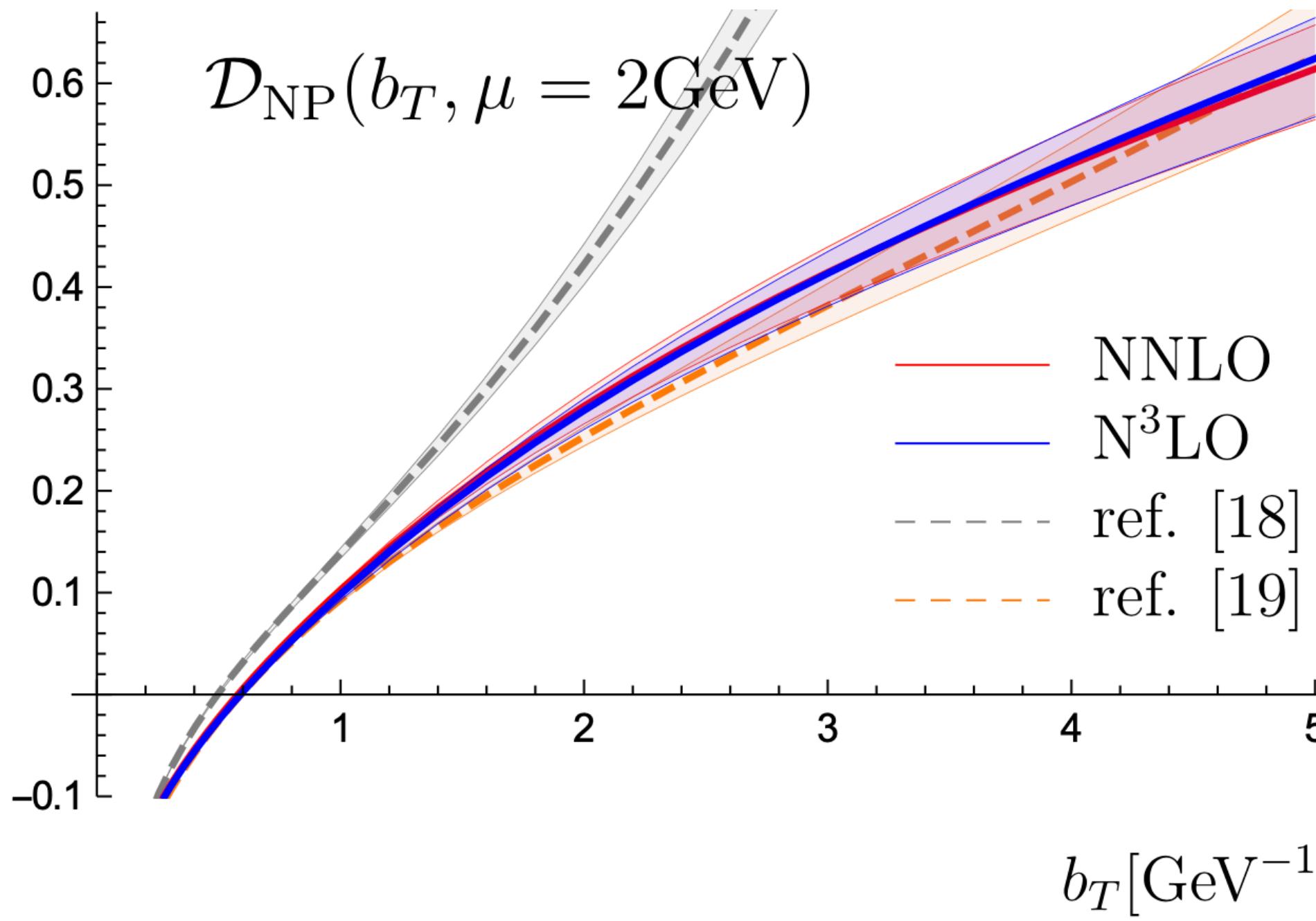
- RG evolution (μ)

$$\mu \frac{d}{d\mu} f_{q/P}^{TMD}(x, b_\perp, \mu, \zeta) = \gamma(\mu, \zeta)$$

- Rapidity evolution (ζ)

$$2\zeta \frac{d}{d\zeta} \ln f_{q/P}^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu)$$

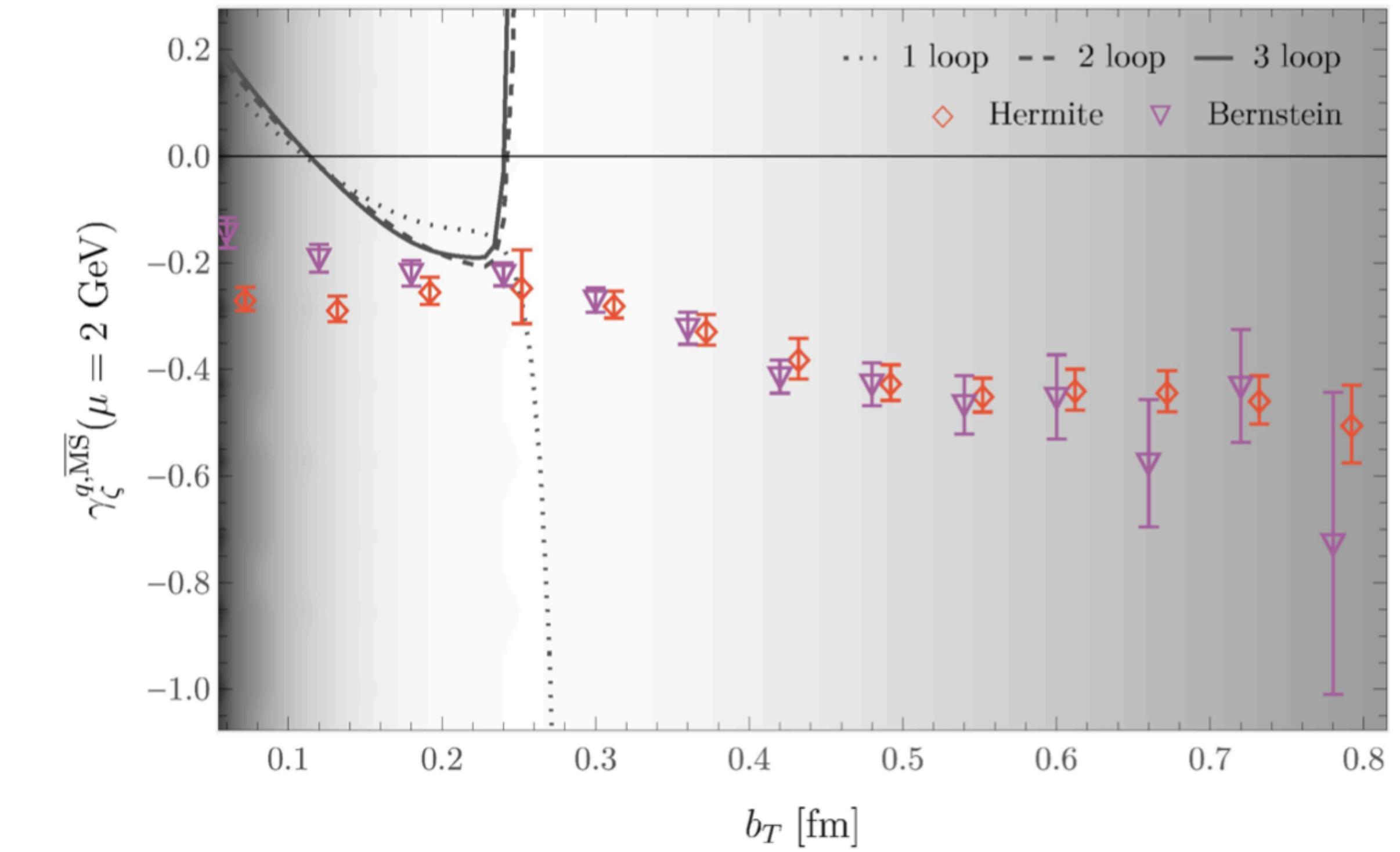
phenomenological (traditional)



$$K(b_\perp, \mu) = -2(D_{NP} + \sum_{n=0}^{\infty} \alpha_s(\mu)^n d_n)$$

I. Scimemi, A. Vadimirov, arxiv 1912.06532 (2020)

lattice



P. Shanahan, M. Wangman, Y. Zhao, arxiv 2003.06063 (2020)



Motivation: CS kernel through TMDs

From TMDPDFs

$$2\zeta \frac{d}{d\zeta} \ln f^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu)$$

$$f^{TMD}(x, b_\perp, \mu, \zeta) \xrightarrow{\text{lattice}} \langle 0 | \hat{O}(0) \hat{O}(b_\perp, z) \hat{O}(0) | 0 \rangle$$

$C_3(b_\perp, z)$

From TMDWFs

$$2\zeta \frac{d}{d\zeta} \ln \psi^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu)$$

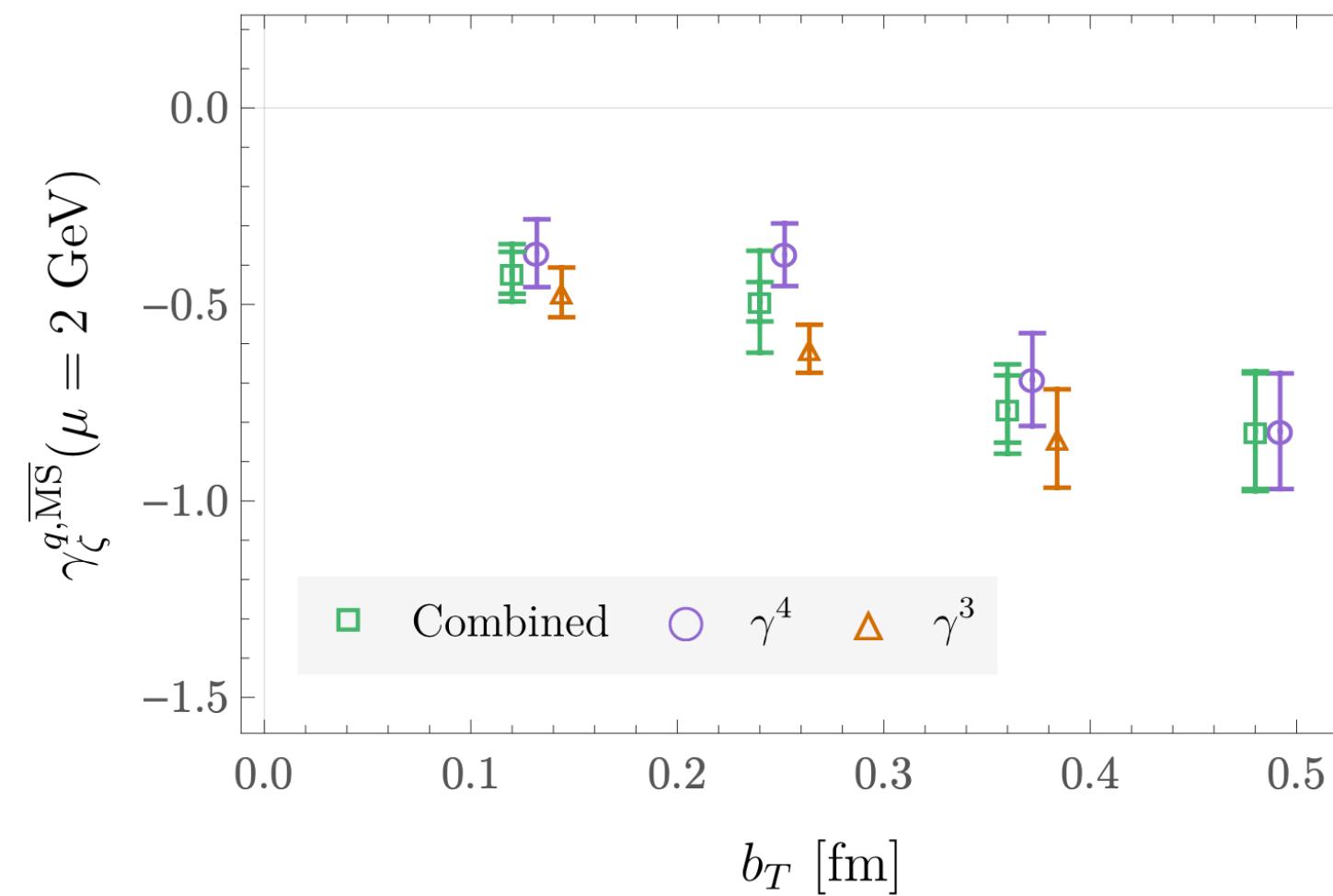
$$\psi^{TMD}(x, b_\perp, \mu, \zeta) \xrightarrow{\text{lattice}} \langle 0 | \hat{O}(b_\perp, z) \hat{O}(0) | 0 \rangle$$

$C_2(b_\perp, z)$

Simpler and has better signal for
lattice calculation!

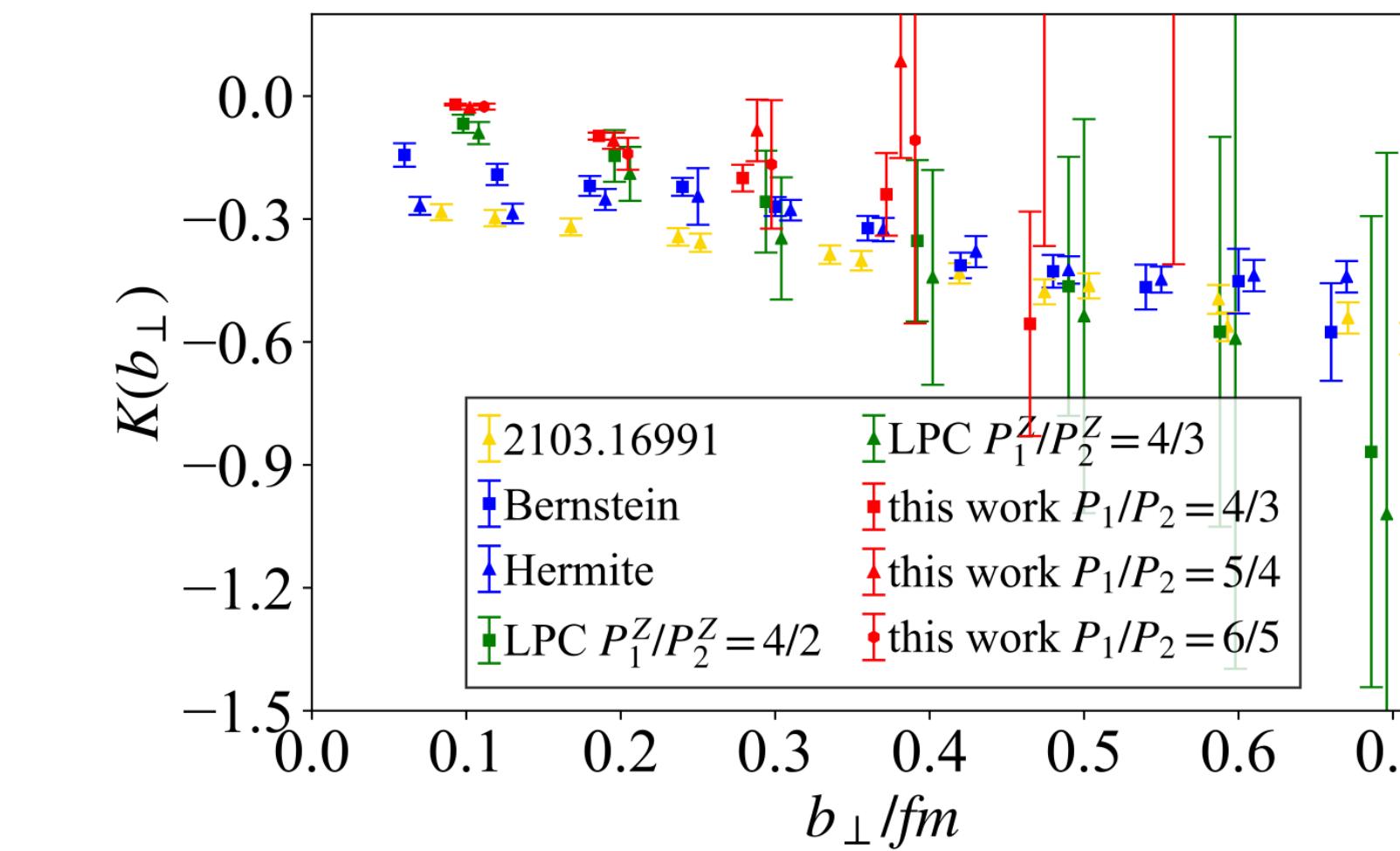
Motivation: Previous LQCD results

$K(b_\perp, \mu)$ from TMDPDFs at **1-loop**

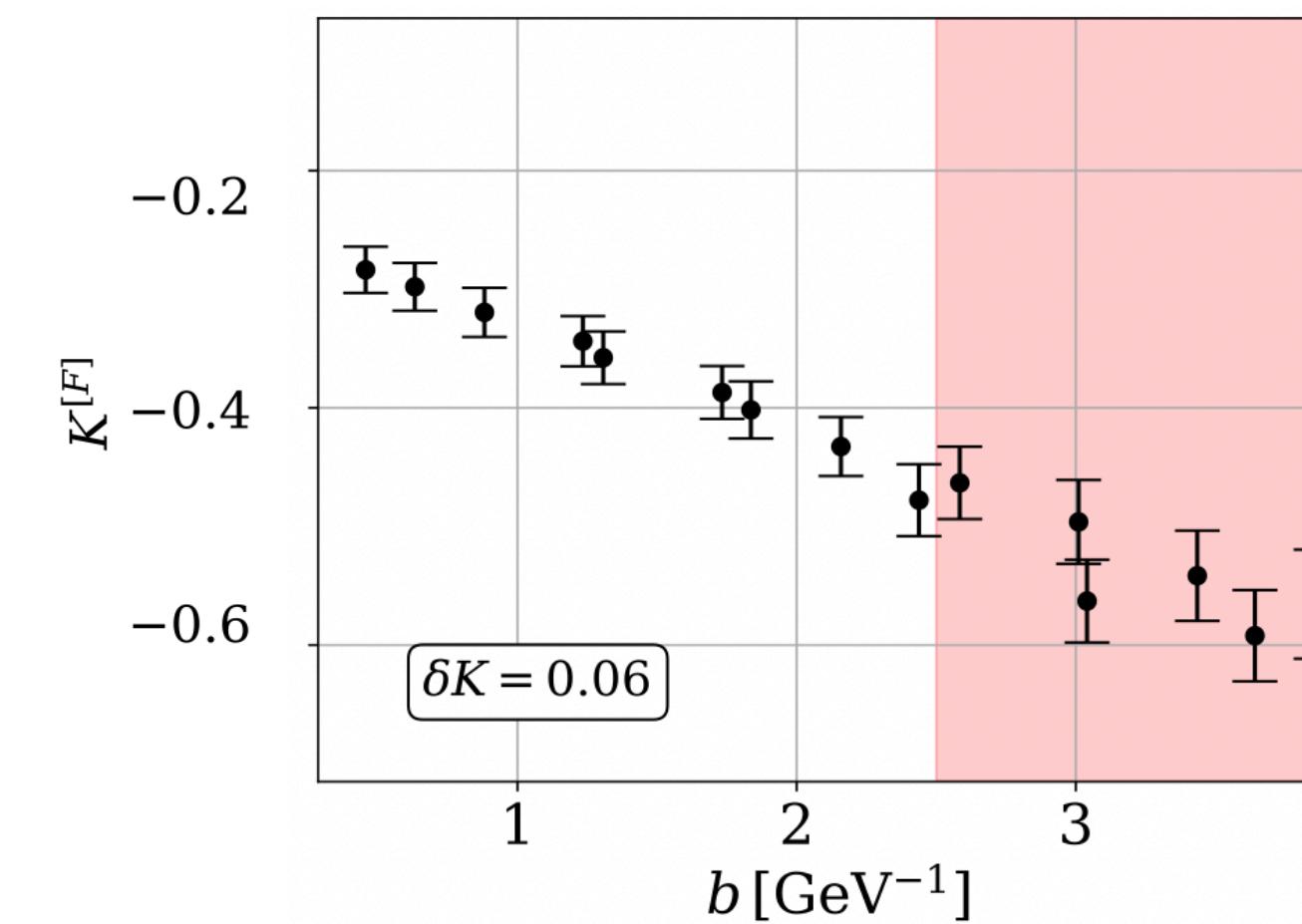


P. Shanahan, M. Wangman, Y. Zhao, arxiv 2017.11903 (2021)

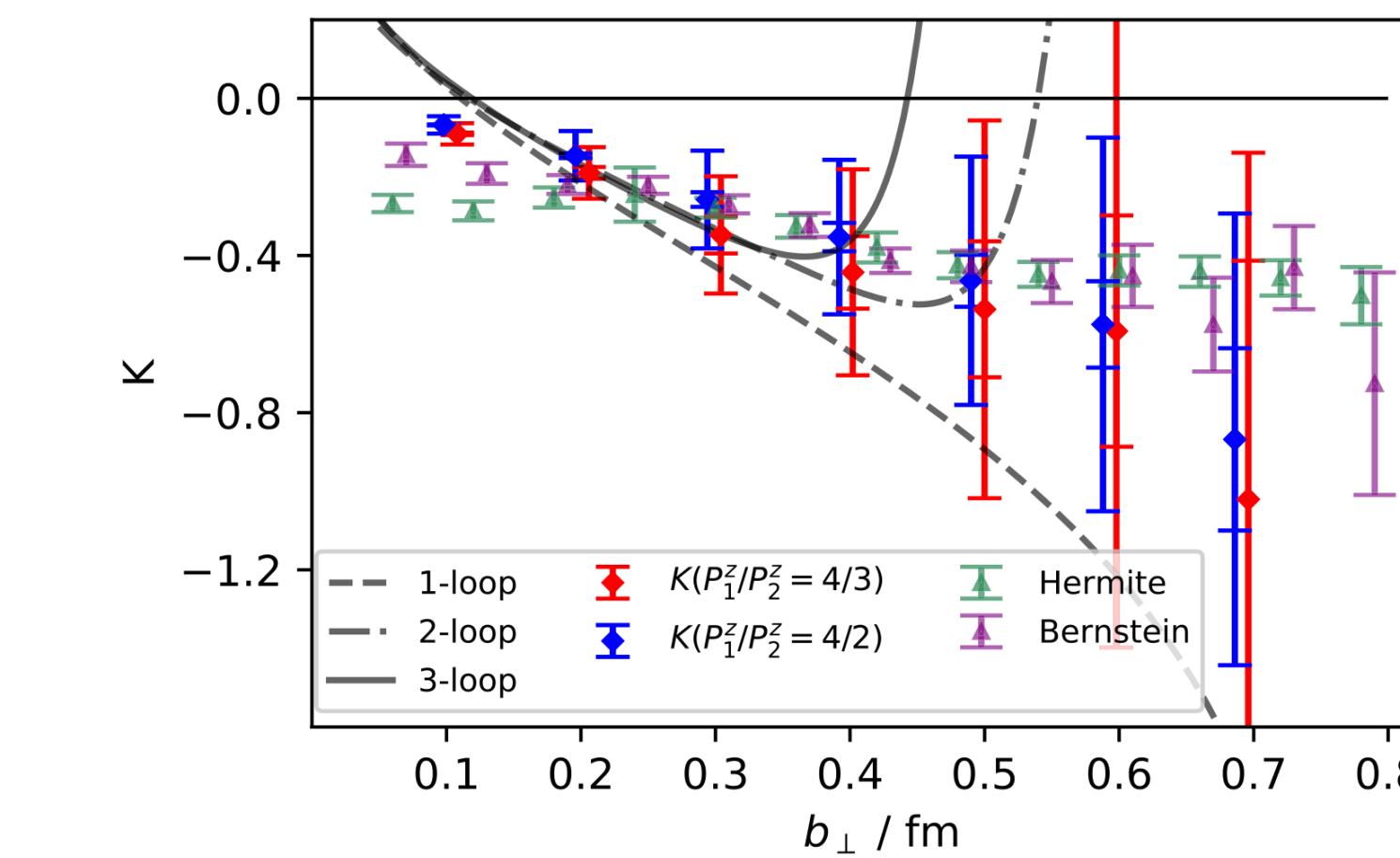
$K(b_\perp, \mu)$ from TMDWFs at **tree level**



Y. Li et al, arxiv 2106.13027 (2021)



M. Shlemmer et al, arxiv 2103.16991 (2021)



Q. Zhang et al, arxiv 2005.14572 (2020)



Outline

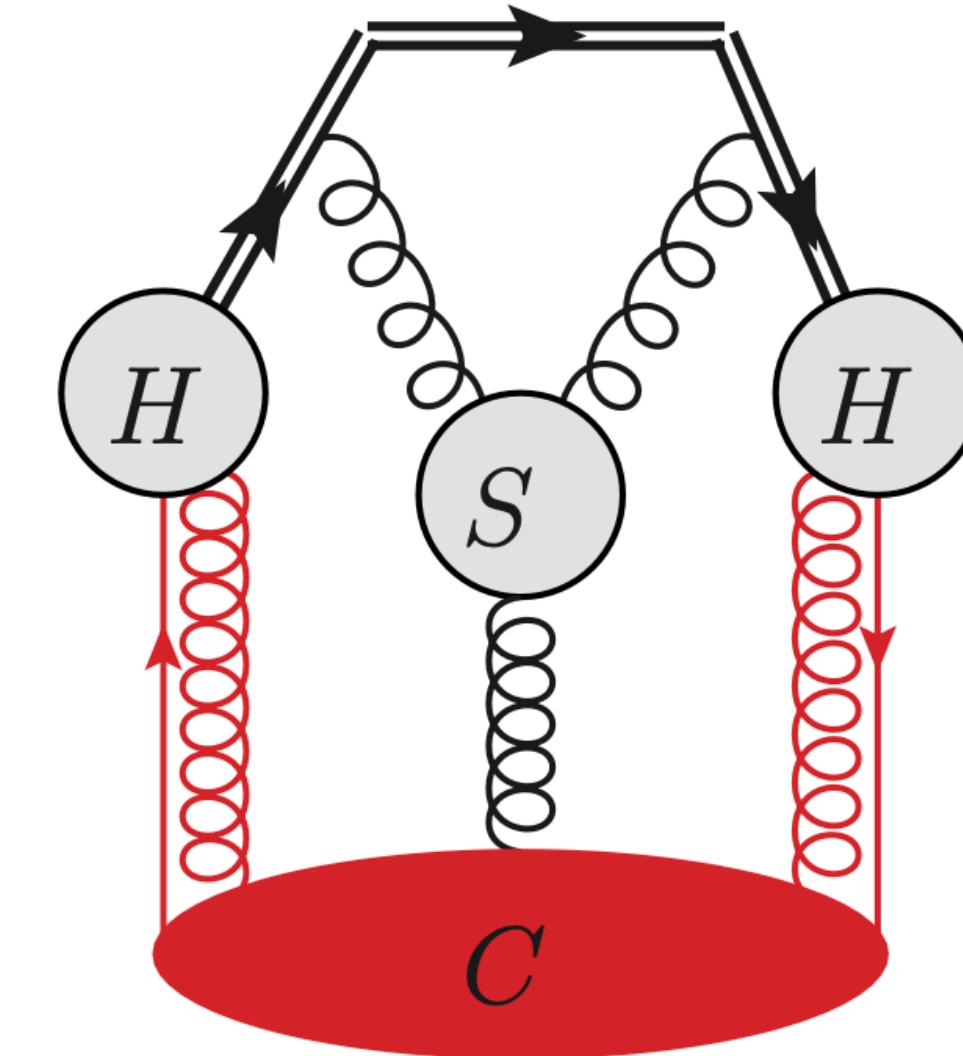
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Factorization

$$\psi^\pm(b, x, \zeta) = H_1^{-1}(\zeta^z, \bar{\zeta}_z) e^{-\frac{1}{2} \ln\left(\frac{\zeta^z}{\zeta}\right) K(b)} S_r^{\frac{1}{2}}(b) \tilde{\psi}^\pm(b, x, \zeta^z)$$

Collins-Soper kernel

$$K(b_\perp, \mu) = \frac{1}{\ln(P_2^z/P_1^z)} \ln \left[\frac{H_N^\pm(\zeta_1^z, \bar{\zeta}_1^z) \tilde{\psi}_N^\pm(b, x, \zeta_2^z)}{H_N^\pm(\zeta_2^z, \bar{\zeta}_2^z) \tilde{\psi}_N^\pm(b, x, \zeta_1^z)} \right]$$

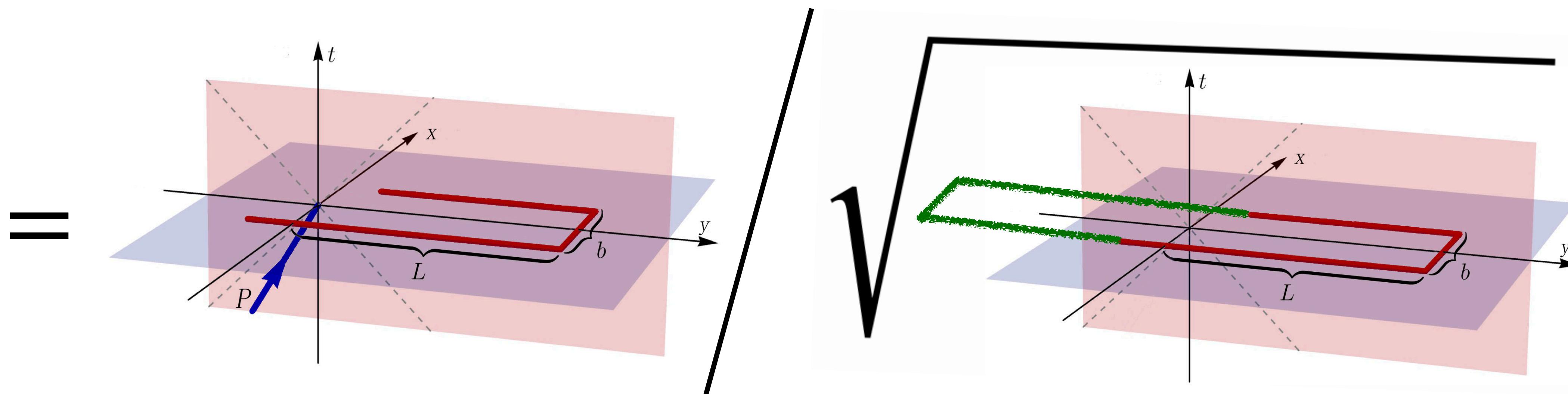


Leading-order reduced graph for
quasi-TMDWFs of a pseudo-scalar meson.



Collins-Soper kernel from lattice QCD in LaMET

$$\text{Quasi-TMDWF: } \tilde{\psi}^\pm(z, b_\perp, \zeta^z) = \frac{\langle 0 | \bar{\Psi}_{\mp n^z}(zn^z/2 + b_\perp n^x) \Gamma \Psi_{\mp n^z}(-zn^z/2) | \pi \rangle}{\sqrt{Z_E(2L, b_\perp)}} = \frac{C_2(L, b_\perp, z, t, P^z)}{\sqrt{Z_E(2L, b_\perp)}}$$



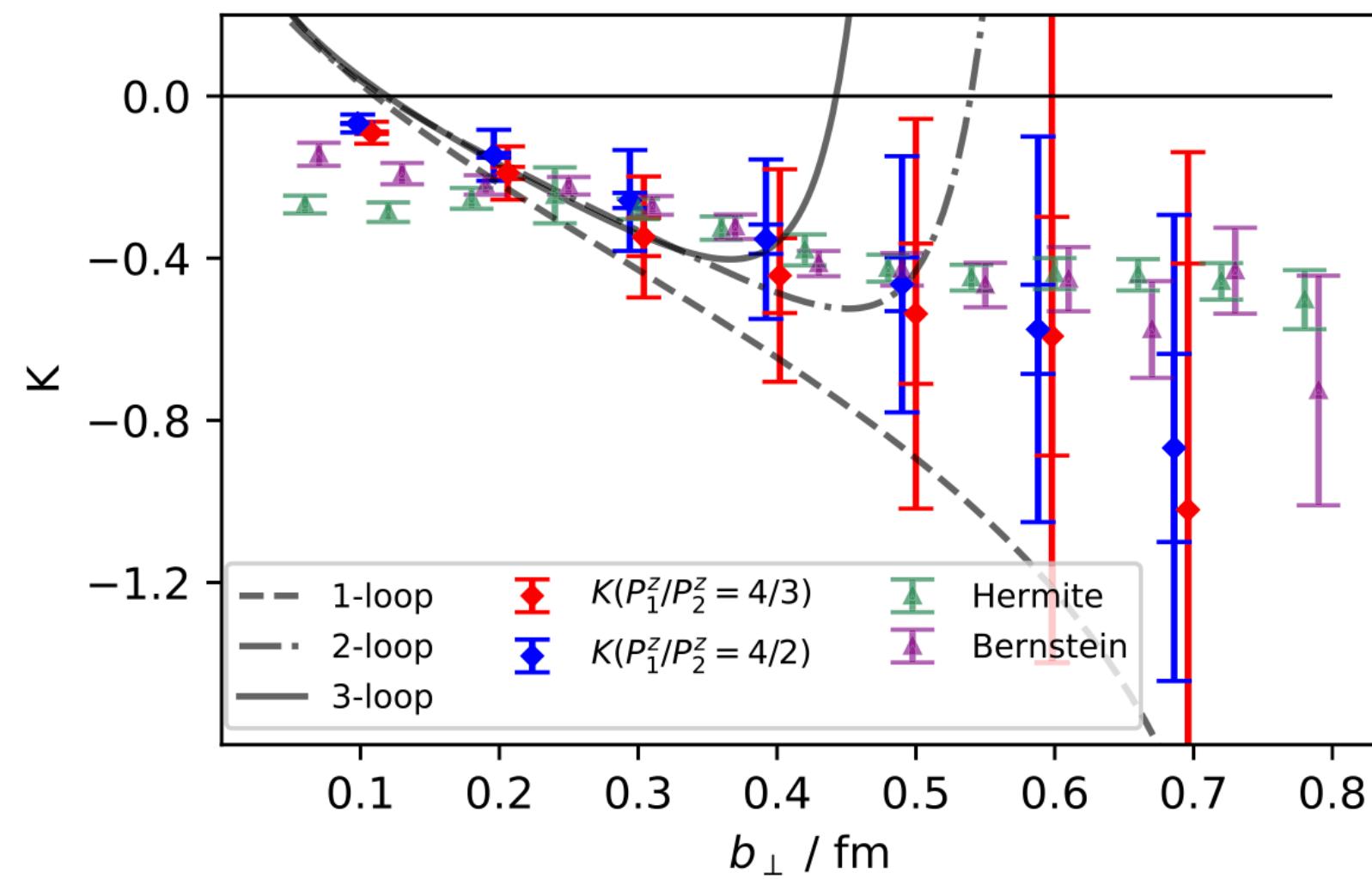


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CLS configurations (previous calculation)

$L^3 \times T$	a (fm)	c_{sw}	κ_l^{sea}	m_π^{sea} (MeV)
$24^3 \times 48$	0.098	2.066 86	0.136 75	333
2.35 fm		N_{cfg}	κ_l^ν	m_π^ν (MeV)
		864	0.136 22	547



Q. Zhang et al, arxiv 2005.14572 (2020)

MILC configurations (this work)

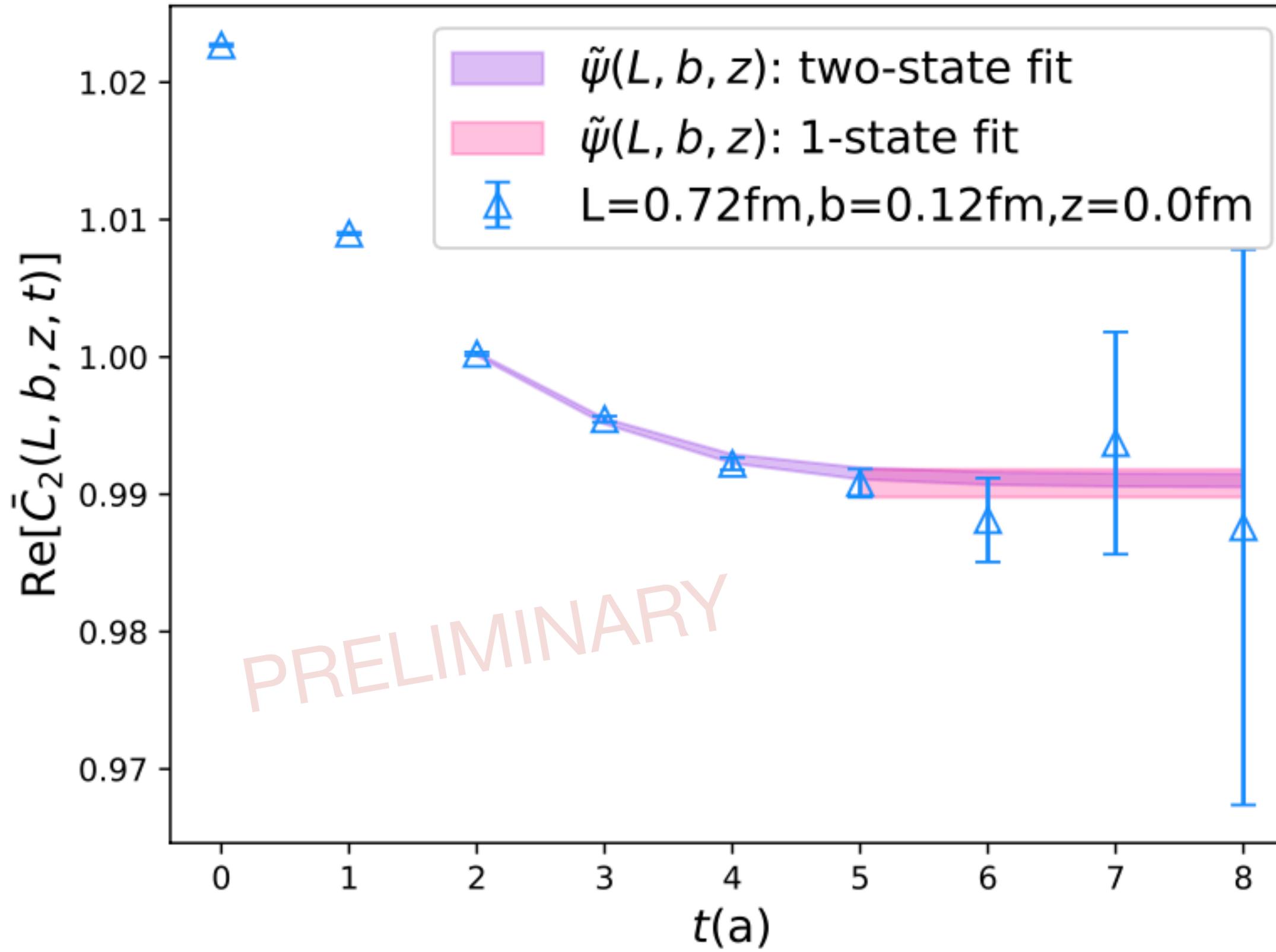
$L^3 \times T$	a (fm)	c_{sw}	κ_l^{sea}	m_π^{sea} (MeV)
$48^3 \times 64$	0.12	1.05088	0.12750	130
5.76 fm		N_{cfg}	κ_l^ν	m_π^ν (MeV)
		382	0.12560	670

- 2+1+1 flavors of HISQ action (MILC)
- Momenta: 1.72GeV, 2.15GeV, 2.58GeV
- Gamma factor: 2.57, 3.21, 3.85



Numerical Result: fit t

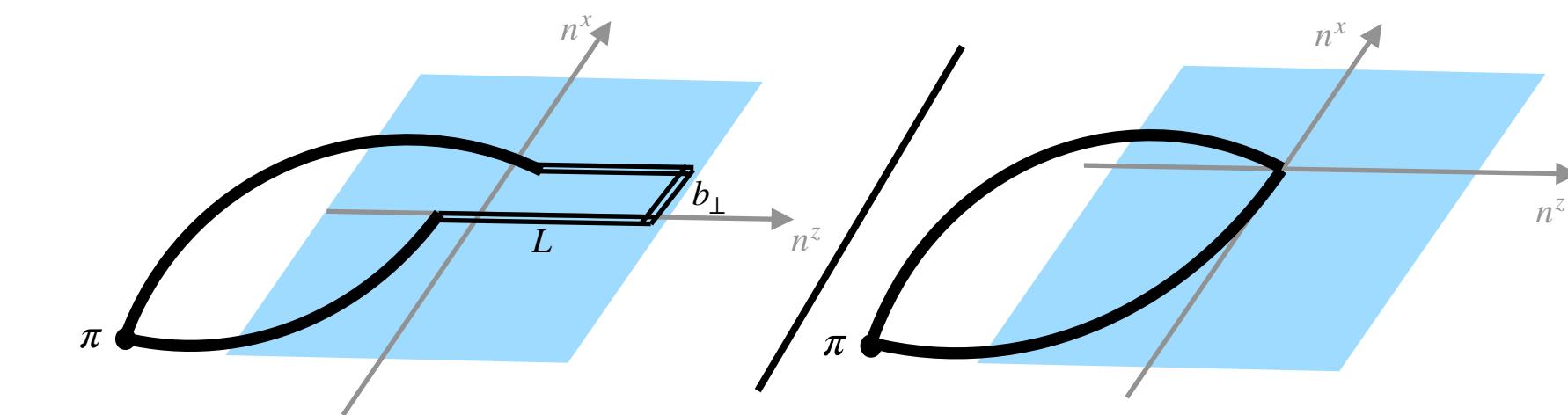
12



1-state fit is **consistent** with two-state fit.

Two point correlation function

$$\bar{C}_2(L, b, z, t) =$$



two-state fit

$$\bar{C}_2(L, b, z, t) = \frac{\tilde{\psi}(L, b, z)(1 + c_1(L, b, z)e^{-\Delta Et})}{1 + c_0(0)e^{-\Delta Et}}$$

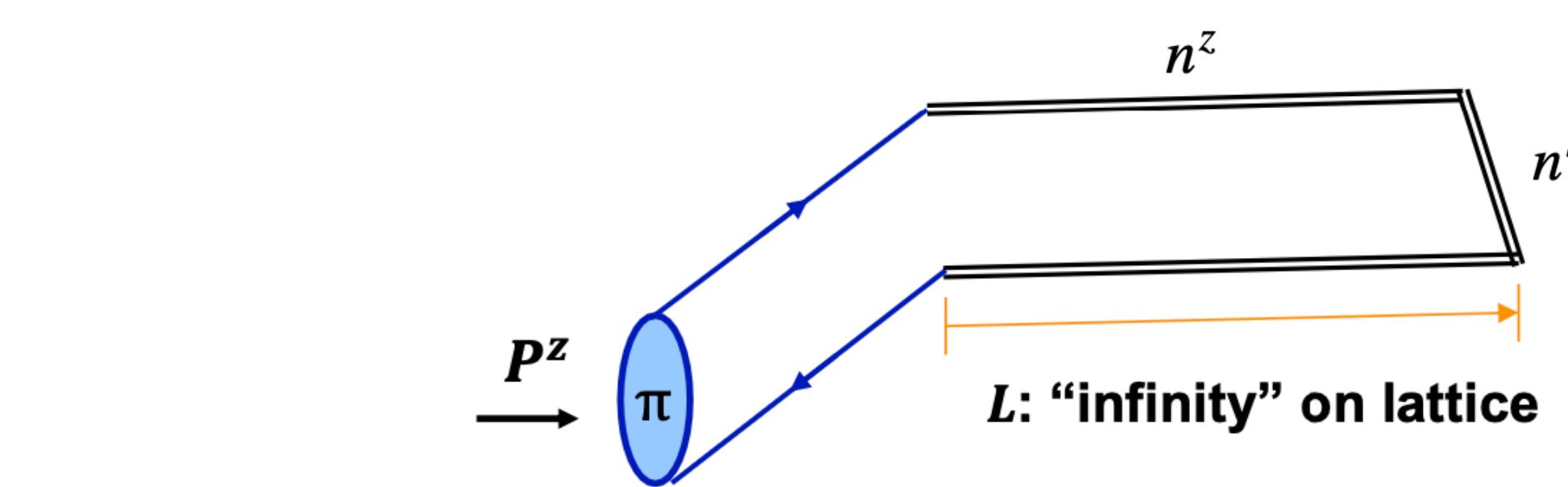
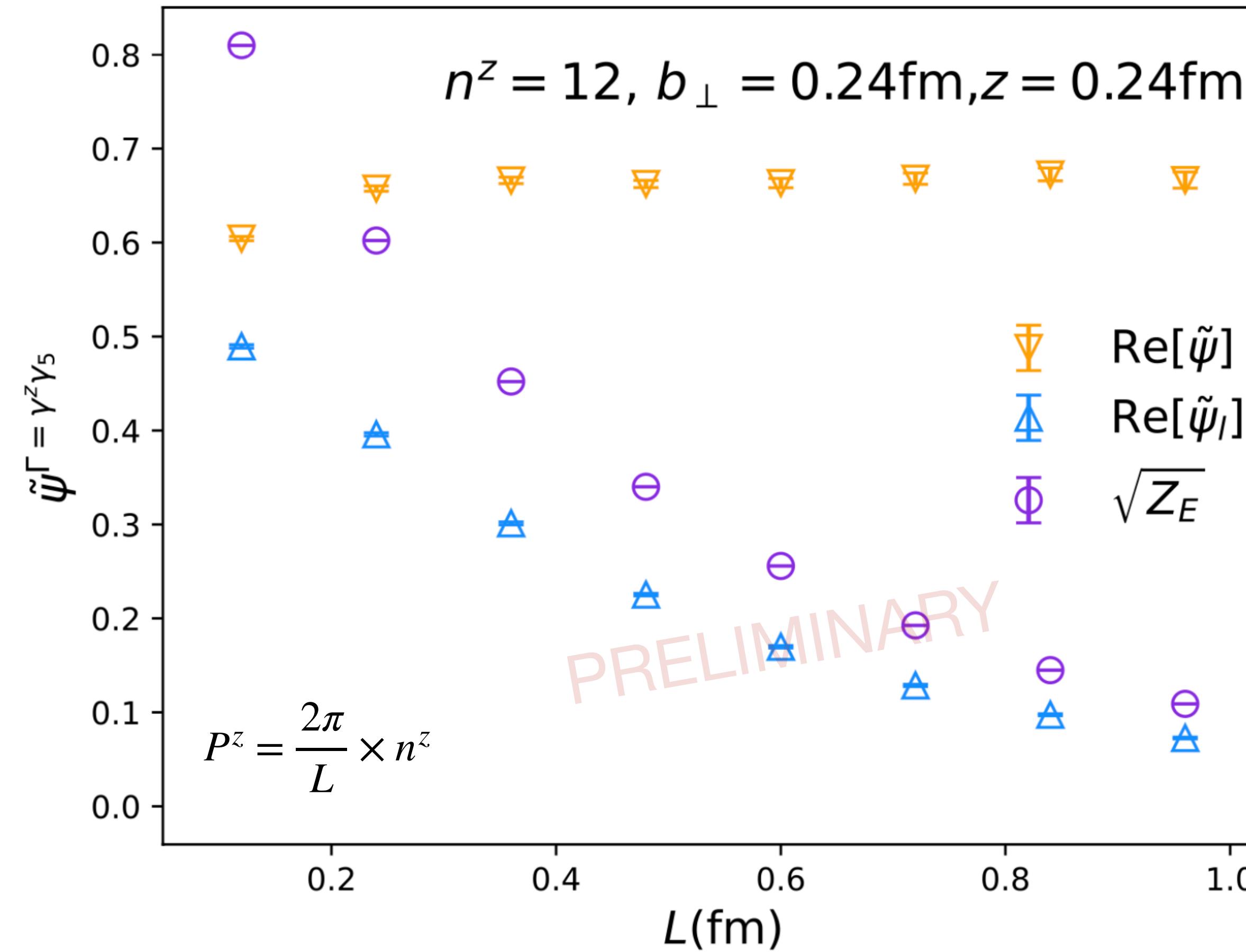
1-state fit

$$\bar{C}_2(L, b, z, t) = \tilde{\psi}(L, b, z)$$



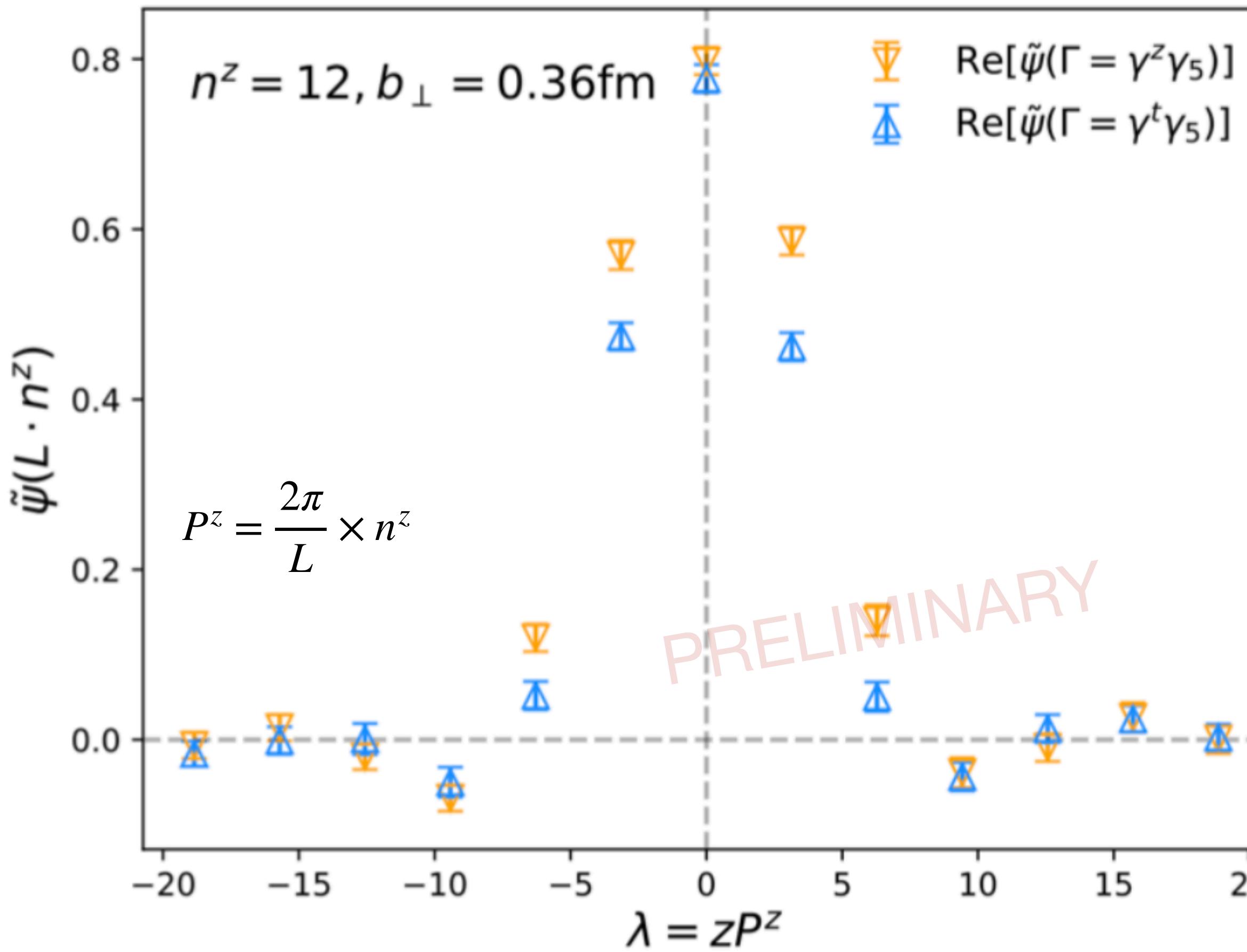
Numerical Results: Large L limit

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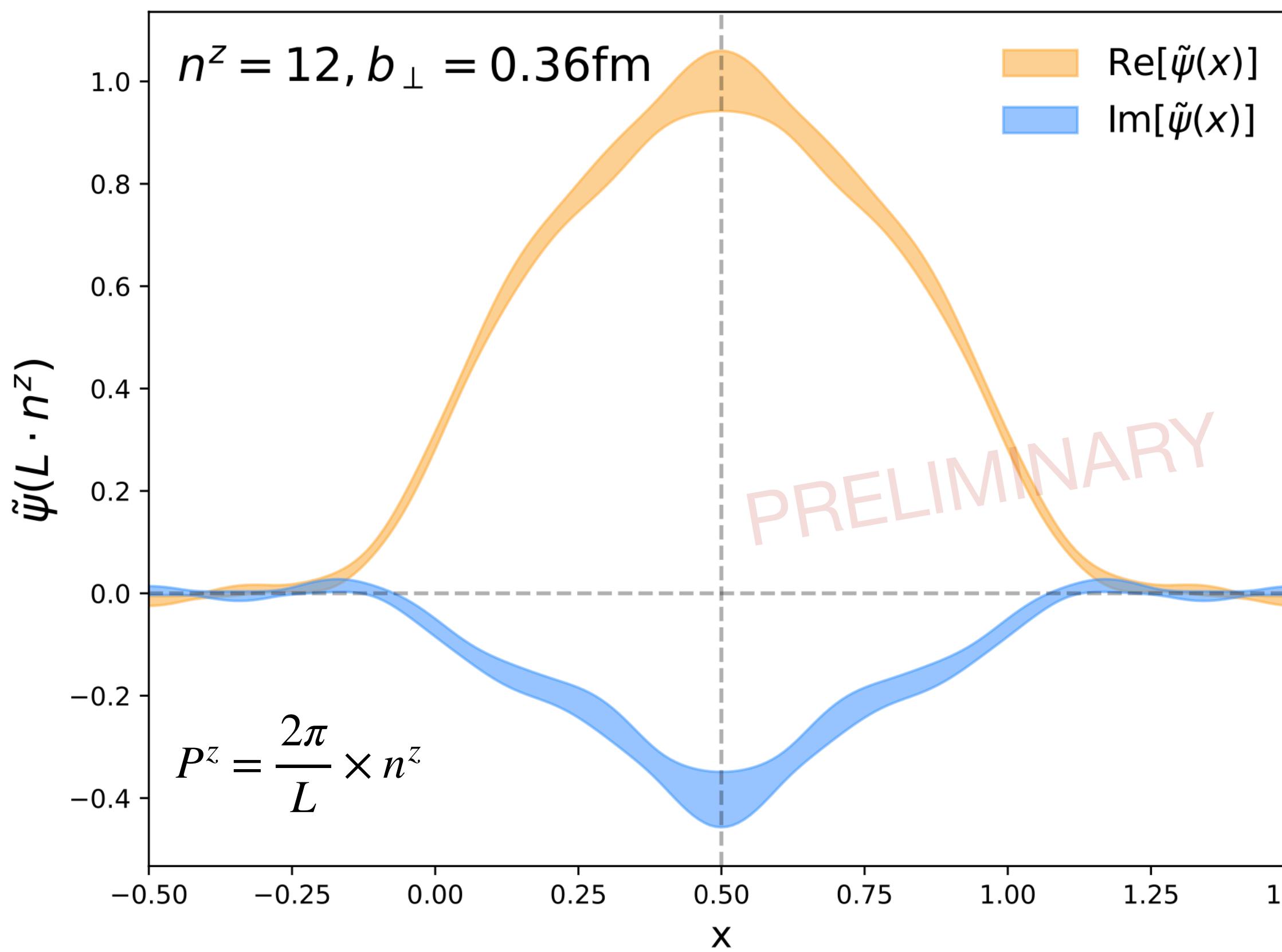
$$\tilde{\psi}(b, z) = \lim_{l \rightarrow \infty} \tilde{\psi}(l, b, z)$$

- $\tilde{\psi}_l$ and $\sqrt{Z_E}$ contain linear divergence.
- linear divergence is **cancelled** at large L .
- $L = L_0 = 6a = 0.72 \text{ fm}$ as asymptotic



quasi-TMDWFs in coordinate space

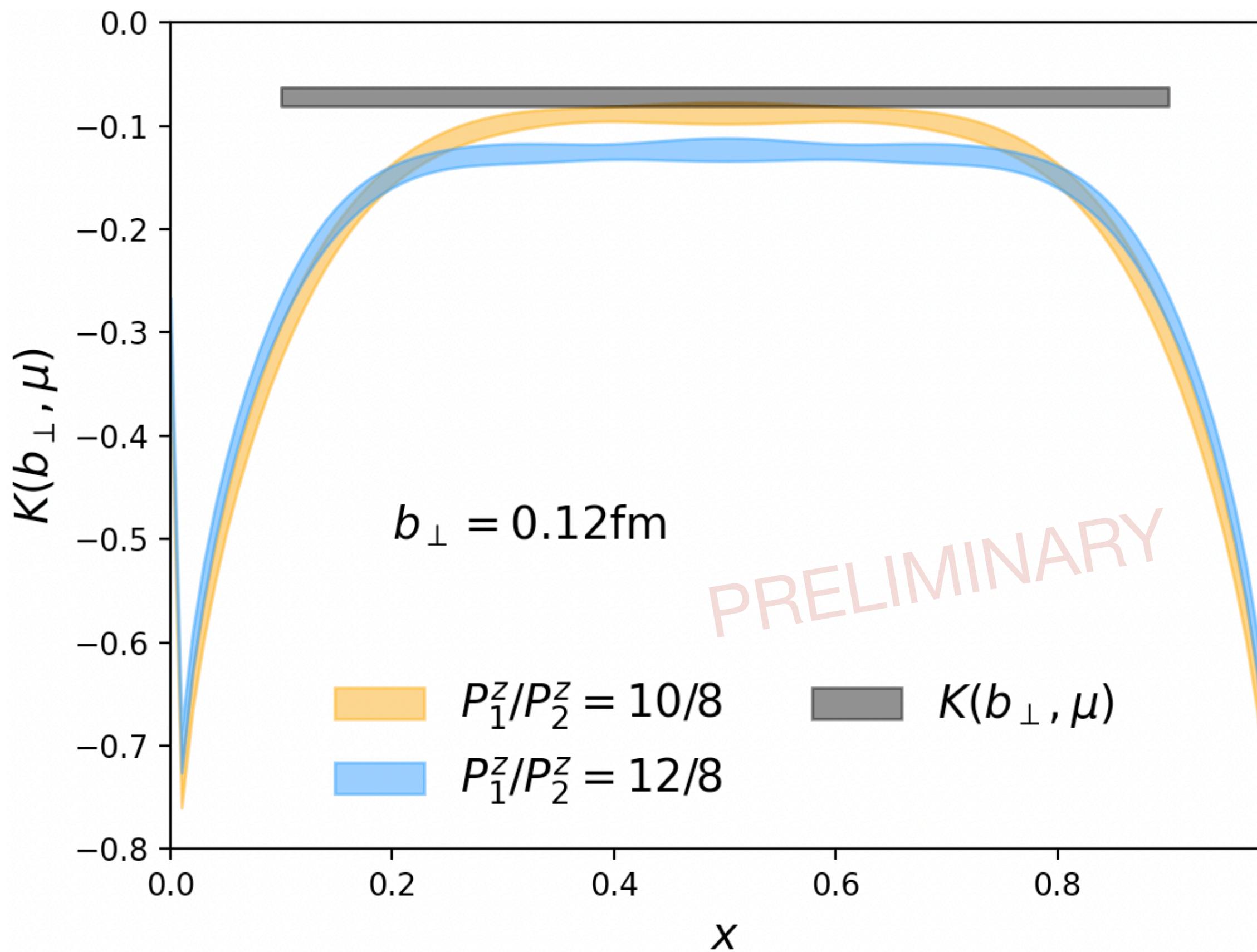
- $\langle 0 | \bar{\Psi}(x) \gamma^\mu \gamma_5 \Psi(y) | \pi(P) \rangle \propto i f_\pi P^\mu$
- $\gamma^t \gamma_5$ and $\gamma^z \gamma_5$ give **similar** results.
- $\tilde{\psi} = \frac{1}{2} [\tilde{\psi}(\Gamma = \gamma^t \gamma_5) + \tilde{\psi}(\Gamma = \gamma^z \gamma_5)]$



Fourier transformation

$$\tilde{\psi}(x, b_\perp, \mu, \zeta^z) = \lim_{L \rightarrow \infty} \int \frac{P^z dz}{4\pi} e^{i(x-1/2)P^z z} \tilde{\psi}(z, b_\perp, \zeta^z)$$

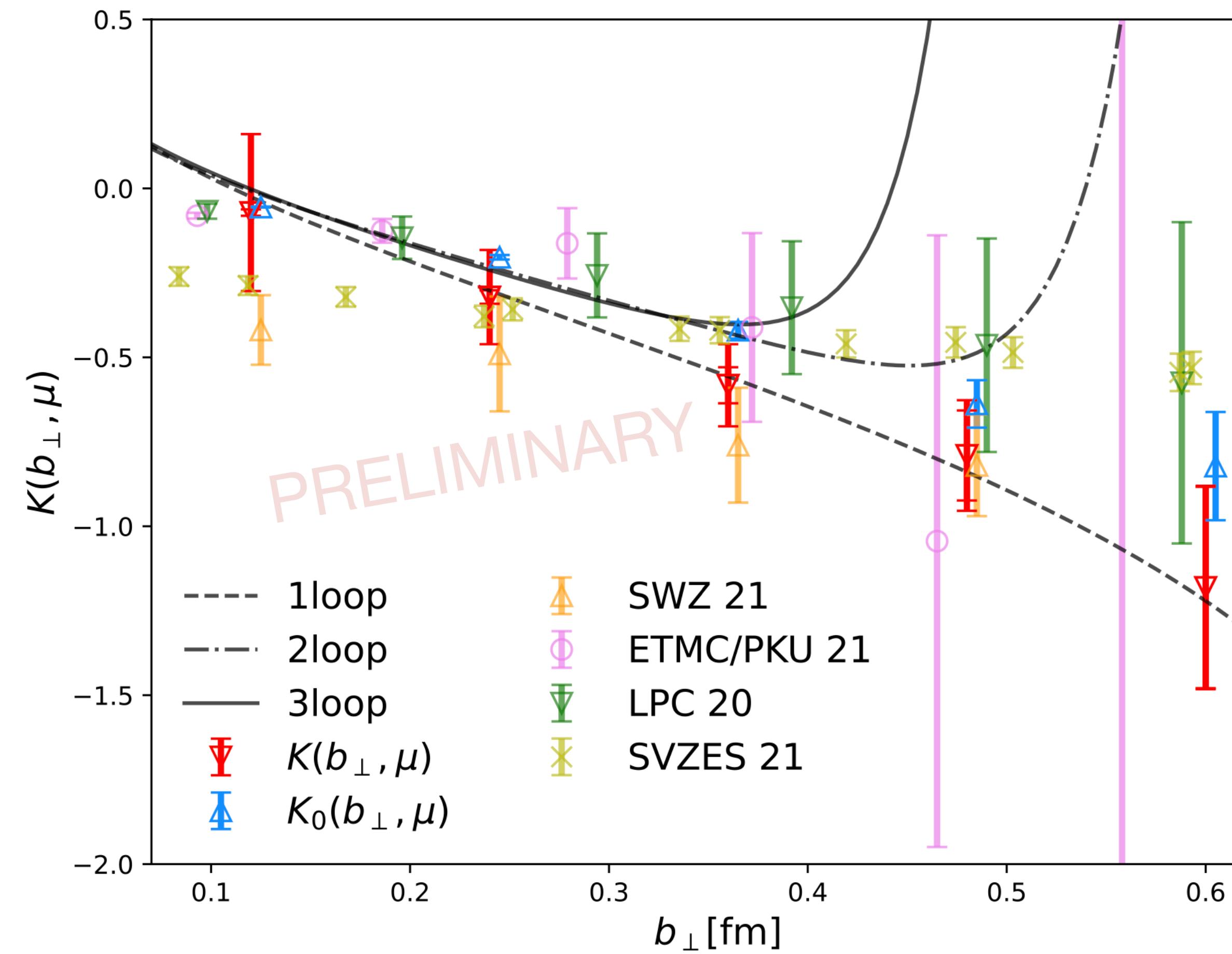
quasi-WF $\tilde{\psi}(x, b_\perp, \mu, \zeta^z)$ is **complex** !



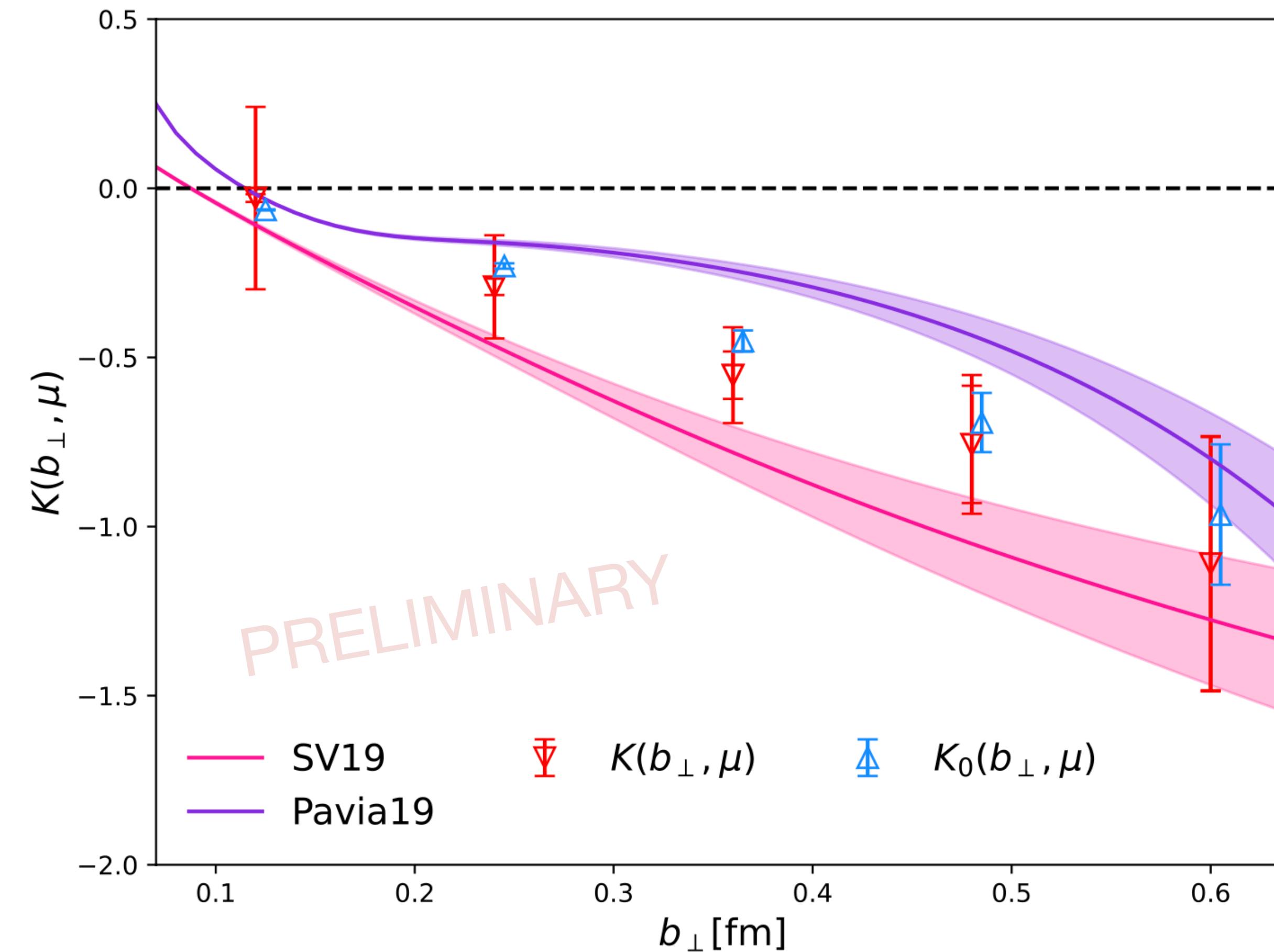
- $K(b_{\perp}, \mu)$ from average of $\tilde{\psi}(L \rightarrow \infty)$ and $\tilde{\psi}(L \rightarrow -\infty)$
- Joint fit of different momentum combinations

$$K(b_{\perp}, \mu, x, P_1^z, P_2^z)$$

$$= K(b_{\perp}, \mu) + A \left[\frac{1}{x^2(1-x)^2(P_1^z)^2} - \frac{1}{x^2(1-x)^2(P_2^z)^2} \right]$$



- Perturbative calculation:
1loop, 2loop, 3loop: Y. Li et al, arxiv 1604.01404 (2017)
- From TMDWFs at tree level:
ETMC/PKU 21: Y. Li et al, arxiv 2106.13027 (2021)
LPC 20: Q. Zhang et al, arxiv 2005.14572 (2020)
- From TMDPDFs at 1-loop:
SWZ 21: P. Shanahan et al, arxiv 2017.11903 (2021)
SVZES 21: M. Schlemmer et al, arxiv 2103.16991 (2021)



- Parameterization of $K(b_\perp, \mu)$:
SV19: I. Scimemi et al, arxiv: 1912.06532 (2020)
- Parameterization of TMDPDFs:
Pavia19: A. Bacchetta et al, arxiv: 1912.07550 (2020)



Summary and outlook

- Collins-Soper kernel describes the **rapidity evolution** of TMDPDFs/TMDWFs.
- This is the **first attempt** for extracting Collins-Soper kernel from TMDWFs with **1-loop level** matching.
- This has added evidences for the extraction of **partonic structure** from LQCD in LaMET.

Thanks for your attention!