

# Pion and Nucleon Gluon Distribution Function on Lattice

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in collaboration with Huey-Wen Lin and Alejandro Salas  
based on work: 2007.16113, 2104.06372, 2112.03124  
and on-going work

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``CAREER: Constraining Parton Distribution Functions for New-Physics Searches''



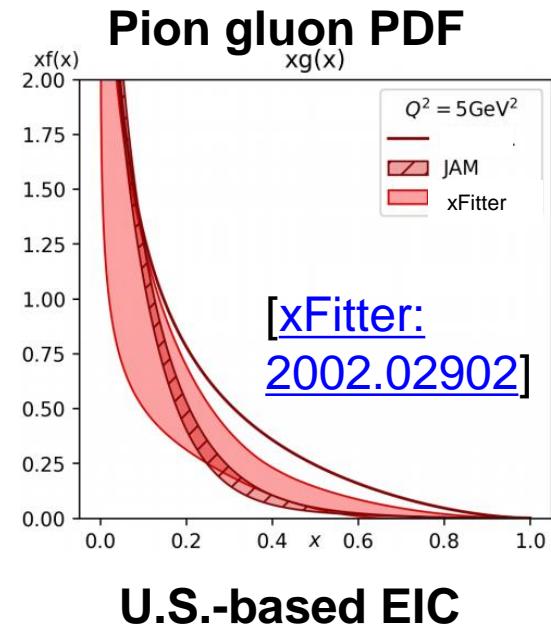
Studying the parton distribution functions (PDFs) is important to characterize the structure of the hadron and nonperturbative QCD.

Global fits use different experimental data and fit strategies

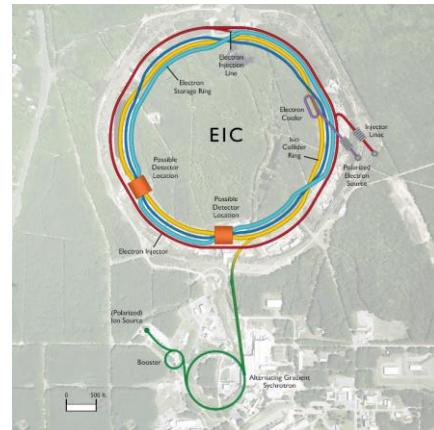
- The error at large x-region is large comparing to the valence quark PDFs
- Current gluon PDFs from different global analyses vary by the input experiment data

To improve the gluon PDFs,

- Experimentally, for example, there are Electron-Ion Collider (EIC) will aim at gluon PDF
- Theoretically, lattice QCD is an independent approach to calculate gluon PDF



**U.S.-based EIC**



[BNL News]

<https://www.bnl.gov/newsroom/news.php?a=116998>

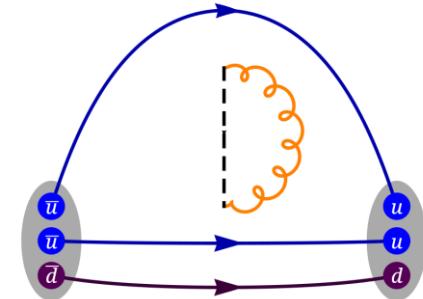
The reduced Ioffe-time pseudo-distribution (pITDs) definition [[Orginos et al, 1706.05373](#)],

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0, 0)}$$

where  $M(\nu, z^2) = \langle 0 | O_g(z) | 0 \rangle$ , Ioffe-time  $\nu = zP_z$ .

$$O_g(z) = \sum_{i \neq z, t} O(F^{ti}, F^{ti}; z) - \sum_{i \neq z, t} O(F^{ij}, F^{ij}; z),$$

$$O(F^{\mu\nu}, F^{\rho\sigma}; z) = F^{\mu\nu}(z)U(z, 0)F^{\rho\sigma}(0)$$



The gluon pseudo-PDF matching condition,

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x \rangle_g} (R_1(x\nu, z^2 \mu^2) + R_2(x\nu))$$

[Balitsky et al, 1910.13963]

evolution      scheme conversion

The evolved pITD (EpITD) definition and matching from EpITD to  $xg(x, \mu^2)$ ,

$$G(\nu, z^2, \mu) = \mathcal{M}(\nu, z^2) + \int_0^1 du R_1(u, z^2 \mu^2) \mathcal{M}(u\nu, z^2)$$

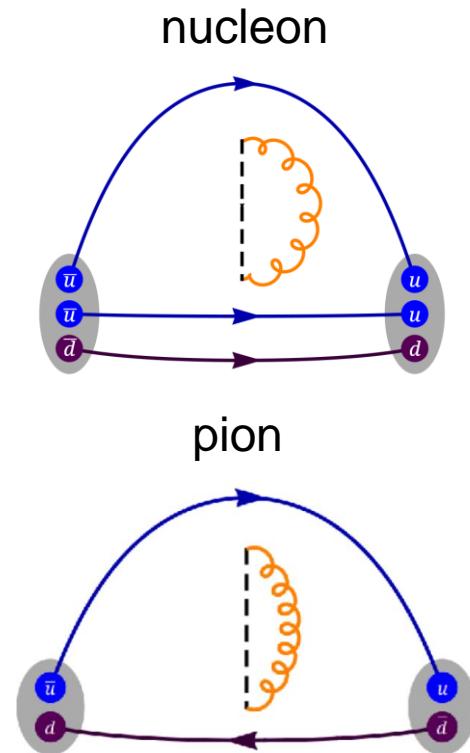
$$= \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x \rangle_g} R_2(x\nu)$$



# Lattice setups

We use clover valence fermions on  $N_f = 2 + 1 + 1$  highly improved staggered quarks (HISQ) lattices generated by the MILC Collaboration [[MILC 1212.4768](#)].

ensemble	a09m310	a12m220	a12m310	a15m310
a (fm)	0.0888(8)	0.1184(10)	0.1207(11)	0.1510(20)
$M_\pi^{sea}$ (MeV)	312.7(6)	216.9(2)	305.3(4)	306.9(5)
$M_\pi^{val}$ (MeV)	313.1(13)	226.6(3)	309.0(11)	319.1(31)
$M_{\eta_s}^{val}$ (MeV)	698.0(7)	696.9(2)	684.1(6)	684.1(6)
$L^3 \times T$	$32^3 \times 96$	$32^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
$N_{meas}^{pion}$	N/A	731,200	143,680	21,600
$N_{meas}^{nucleon}$	145,296	N/A	324,160	21,600



We thank MILC Collaboration for sharing the lattices used to perform this study.



# Extracted bare matrix element

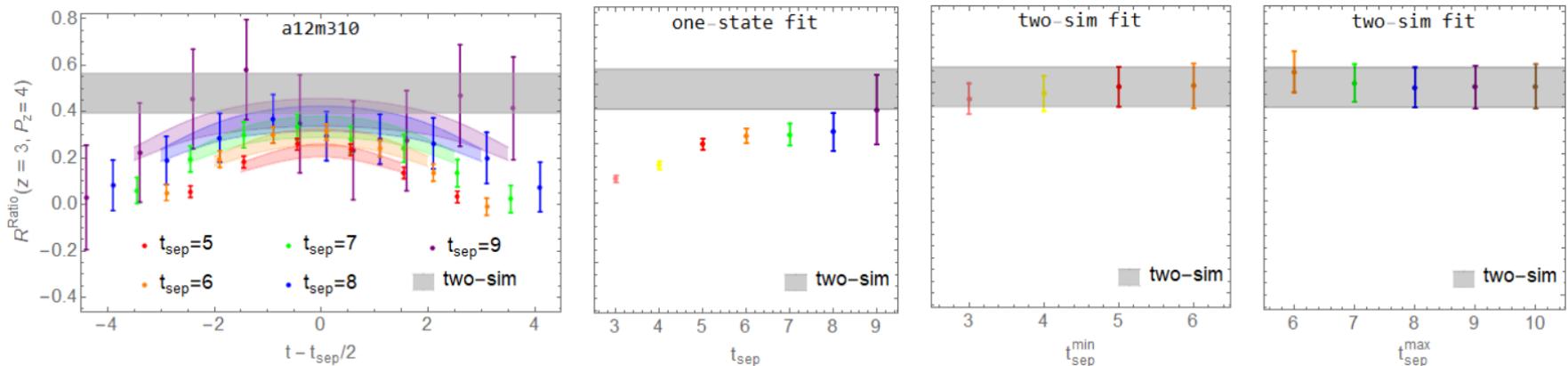
Following the work [[Bhattacharya et al, 1306.5435](#)], the correlators  $C_{3pt}$  and  $C_{2pt}$  can be decomposed as,

$$\begin{aligned} C_{3pt}(z, P_z; t_{sep}, t) = & |A_0|^2 \langle 0|O|0 \rangle e^{-E_0 t_{sep}} \\ & + |A_1||A_0| \langle 1|O|0 \rangle e^{-E_1(t_{sep}-t)} e^{-E_0 t} \\ & + |A_0||A_1| \langle 0|O|1 \rangle e^{-E_0(t_{sep}-t)} e^{-E_1 t} + \dots \end{aligned}$$

$$C_{2pt}(z, P_z; t_{sep}) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t} + \dots$$

where the ground state matrix element is  $\langle 0|O|0 \rangle$ , assuming  $\langle 1|O|0 \rangle = \langle 0|O|1 \rangle$ .

a12m310, **nucleon**



- The bare matrix element (ME) extracted from the two-sim fits are stable with the  $t_{sep}$  fit input choices around our final choice  $t_{sep} = [5,9]$



# Extracted bare matrix element

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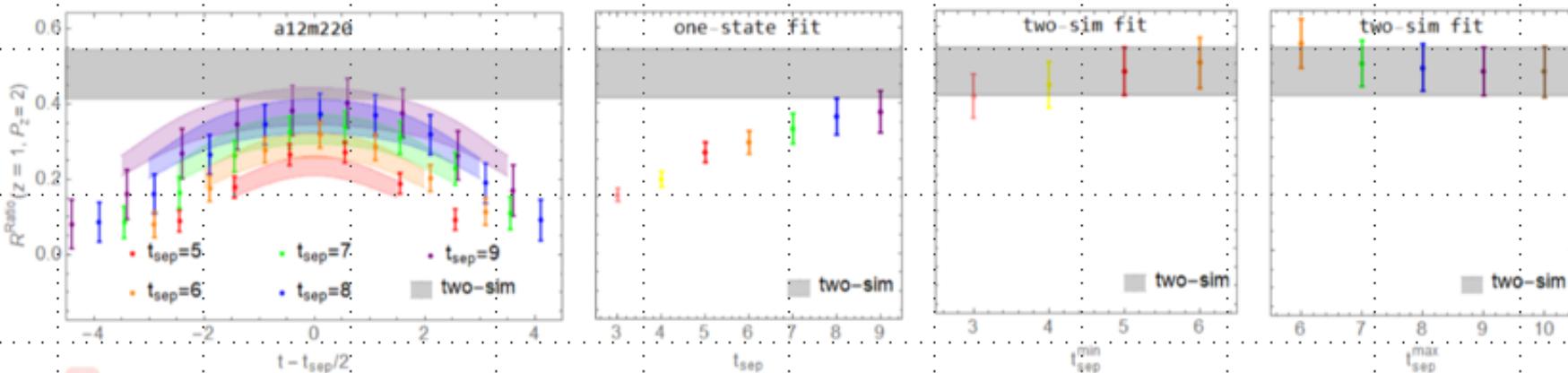
$$\begin{aligned} C_{3pt}(z, P_z; t_{sep}, t) &= |A_0|^2 \langle 0|O|0 \rangle e^{-E_0 t_{sep}} \\ &\quad + |A_1||A_0| \langle 1|O|0 \rangle e^{-E_1(t_{sep}-t)} e^{-E_0 t} \\ &\quad + |A_0||A_1| \langle 0|O|1 \rangle e^{-E_0(t_{sep}-t)} e^{-E_1 t} + \dots \end{aligned}$$

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a12m220, Pion

[Fan et al, 2104.06372](#)

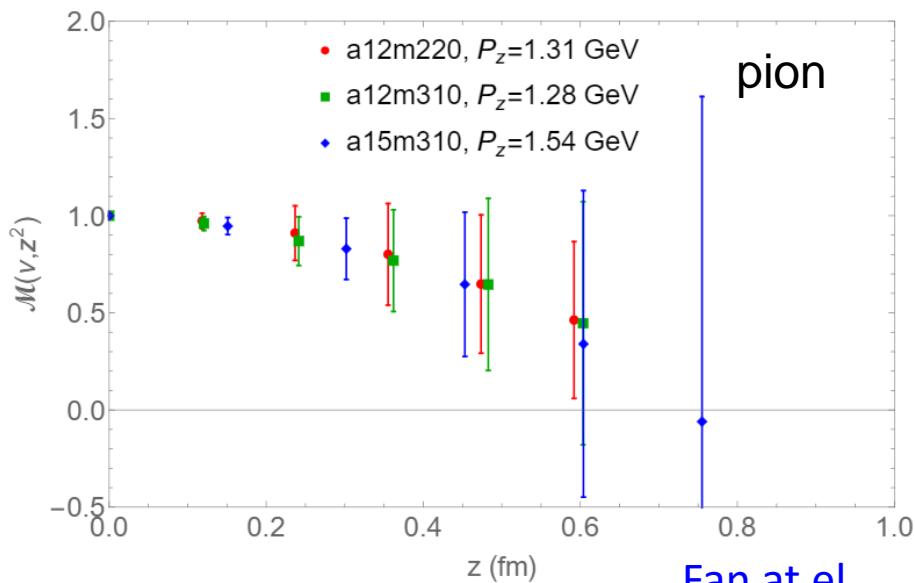
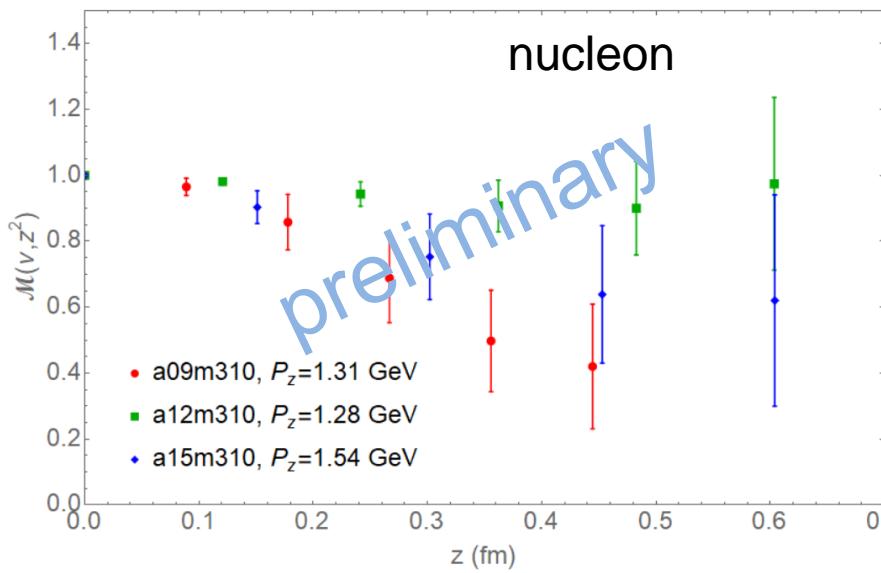


- The bare matrix element (ME) extracted from the two-sim fits are stable with the  $t_{sep}$  fit input choices around our final choice  $t_{sep} = [5,9]$

# RpITD from different ensembles

The RpITDs are obtained by taking the double-ratio of ground-state matrix element,

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0, 0)}$$



Pion&nucleon RpITDs at boost momentum  $P_z \sim 1.3$  GeV

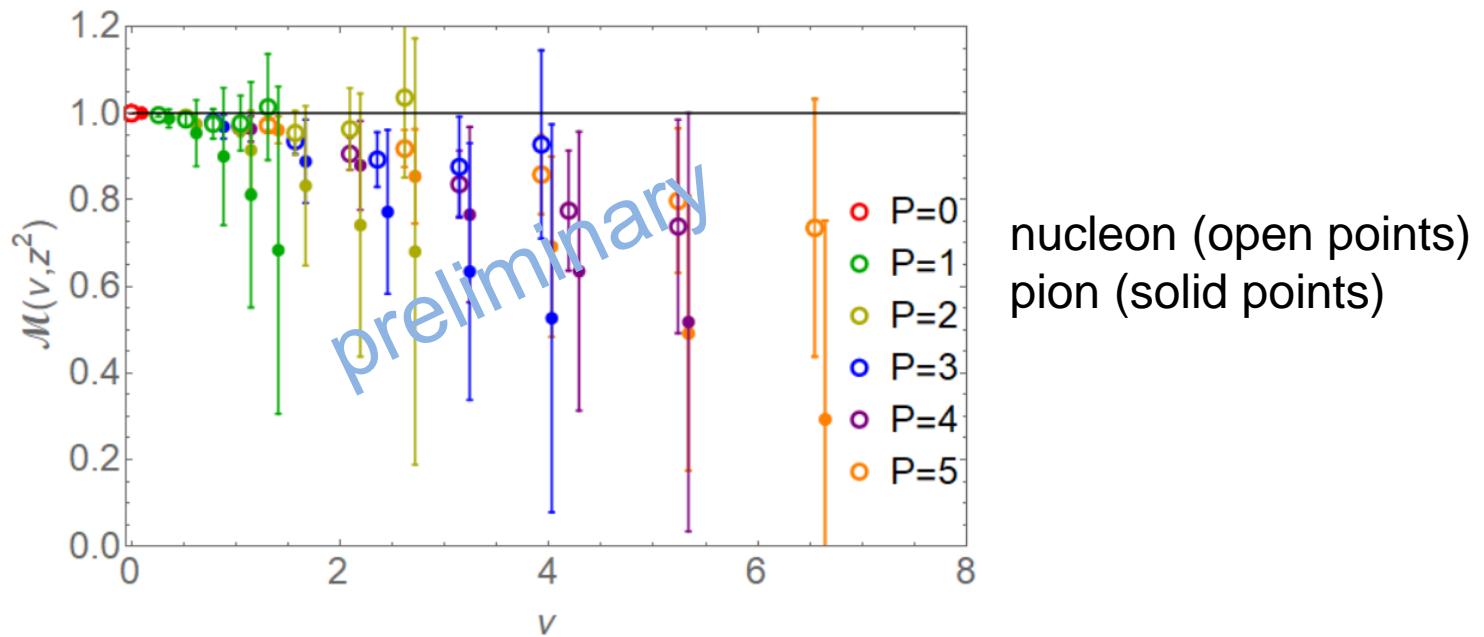
[Fan et al,  
2104.06372](#)

- The RpITDs obtained from bare matrix elements have weak Lattice-spacing and pion-mass dependence

# Nucleon and pion results comparison

We compare the nucleon and pion RpITD under the same measurements and 2-sim fit method.

a12m310,  $M_\pi^{val} = 305.3(4)$  MeV, 324160 measurements



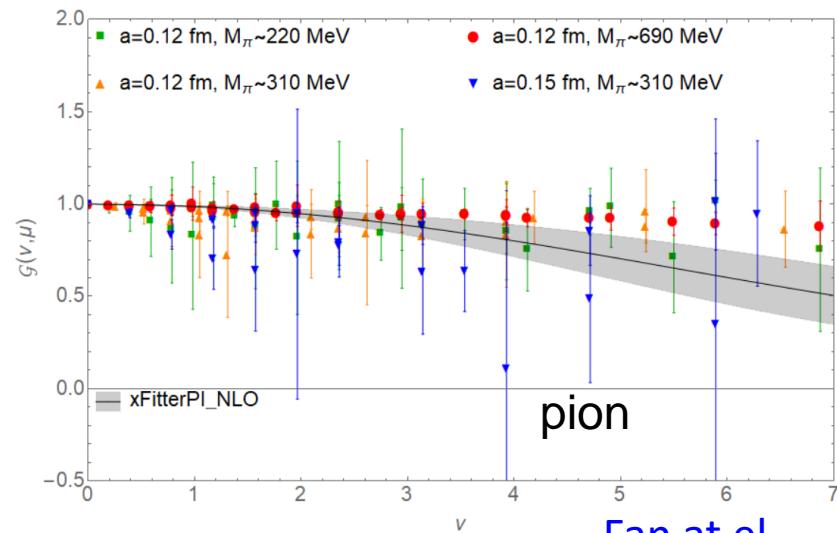
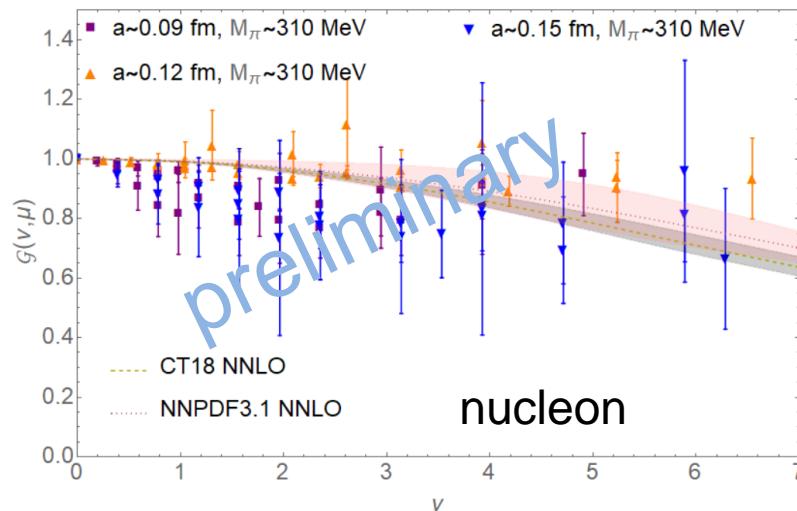
- The nucleon RpITDs have smaller relative errors than pion RpITD for the same ensembles



# Evolved Ioffe-time pseudo-distribution (EpITD)

Through the EpITD  $G$ , the lattice calculated RpITD can be connected with gluon PDF with these two steps,

$$G(v, z^2, \mu) = \mathcal{M}(v, z^2) + \int_0^1 du R_1(u, z^2 \mu^2) \mathcal{M}(uv, z^2) = \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x \rangle_g} R_2(xv)$$



[Fan et al,  
2104.06372](#)

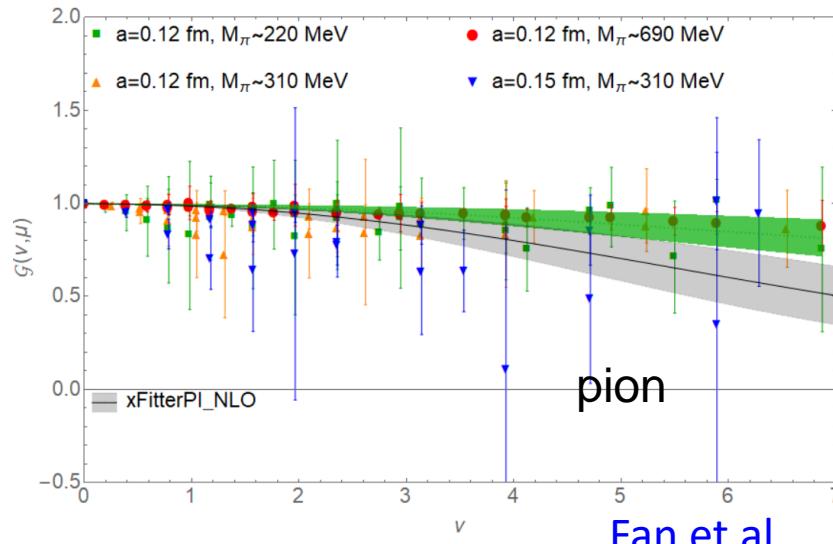
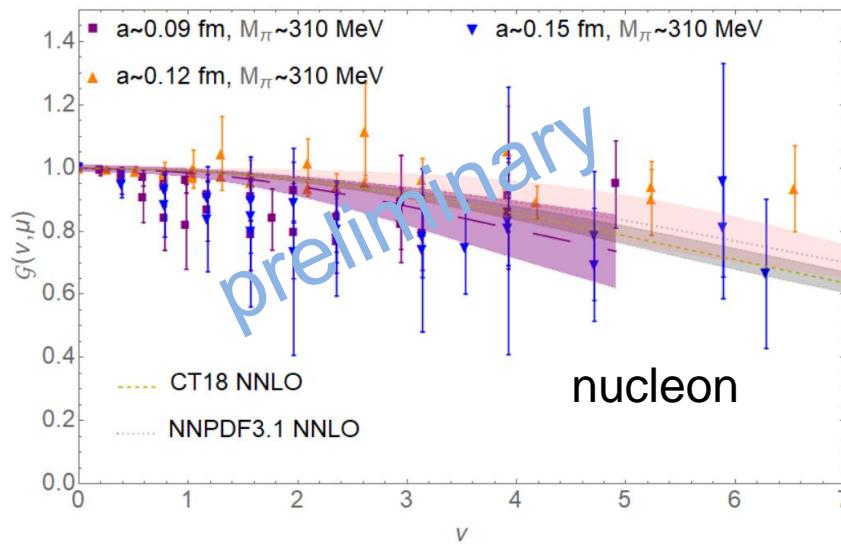
- The EpITDs from global fit PDFs go through the EpITD points from lattice calculations



# Evolved Ioffe-time pseudo-distribution (EpITD)

Functional form light-cone PDF,

$$f_g(x, \mu) = \frac{xg(x, \mu)}{\langle x \rangle_g(\mu)} = \frac{x^A(1-x)^C}{B(A+1, C+1)},$$



[Fan et al.,  
2104.06372](#)

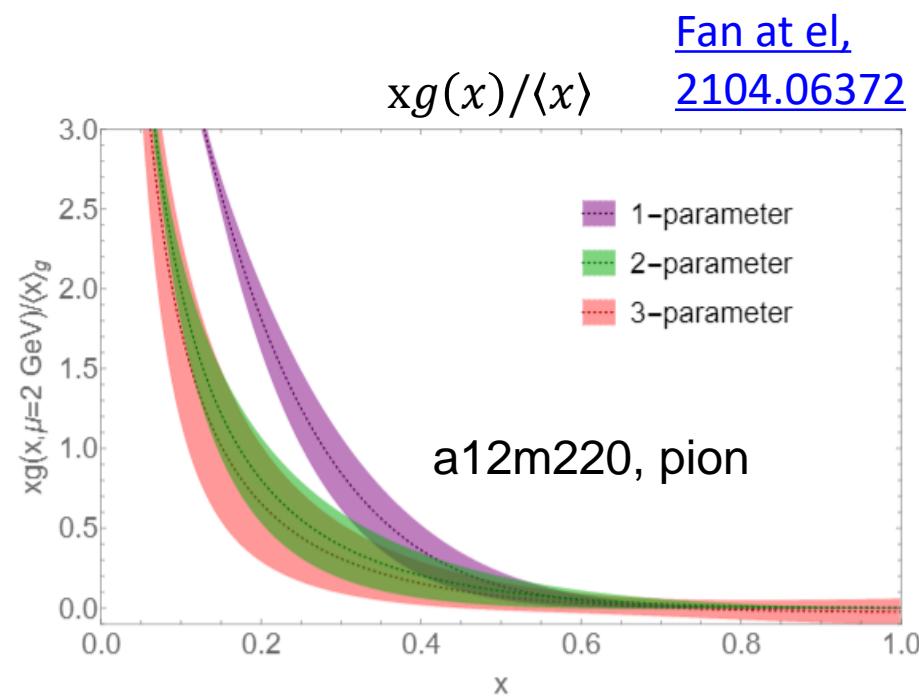
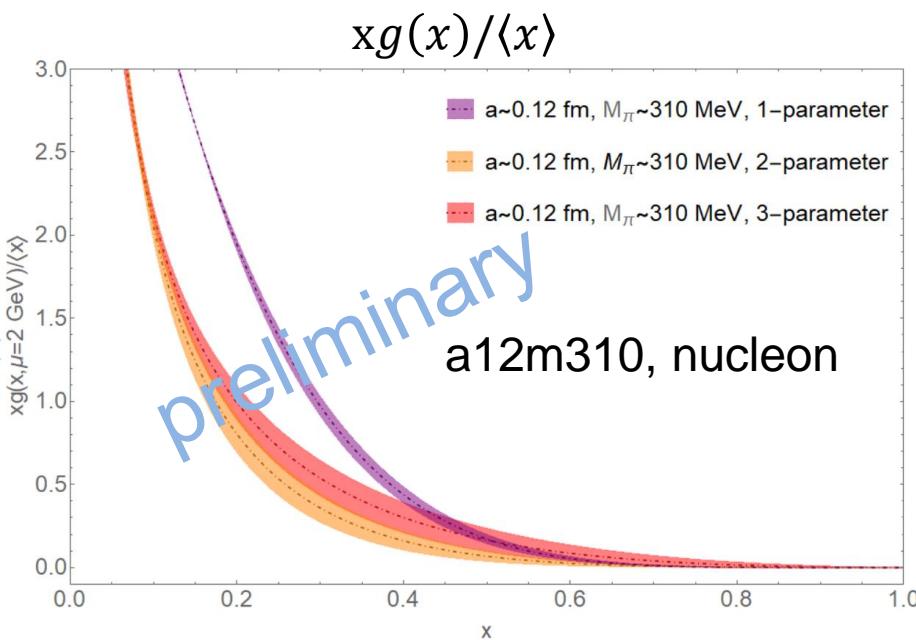
- RpITD  $\rightarrow$  EpITD  $\rightarrow$  Gluon PDF
- Assuming a function form to fit the EpITD instead of a direct Fourier transformation



# Fit form comparison

The 1-, 2-, 3-parameter fit forms,

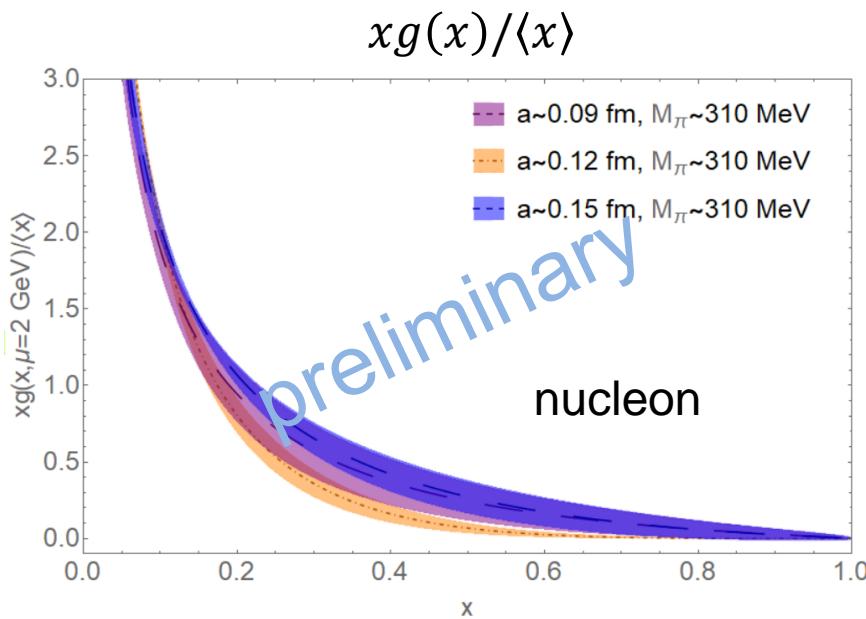
$$\begin{aligned}xg(x) &\sim (1-x)^c \\&\sim x^A(1-x)^c \\&\sim x^A(1-x)^c(1+D\sqrt{x})\end{aligned}$$



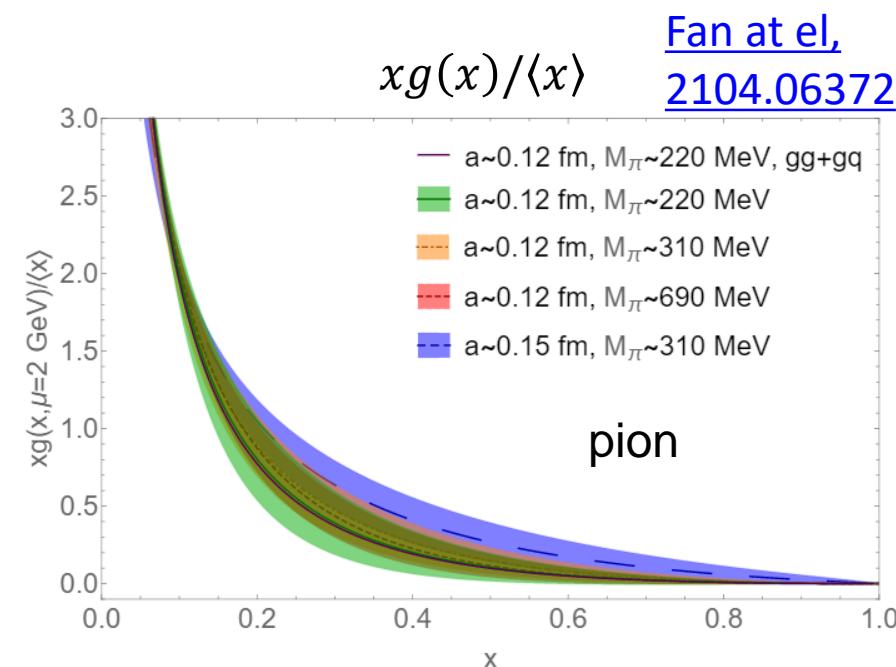
- We conclude that 1-parameter fit on lattice data here is not quite reliable, and the fit results converge at the 2- and 3-parameter fits

# Comparing the different ensembles results

Nucleon  $xg(x)/\langle x \rangle$  from different ensembles with 3 different lattice spacings



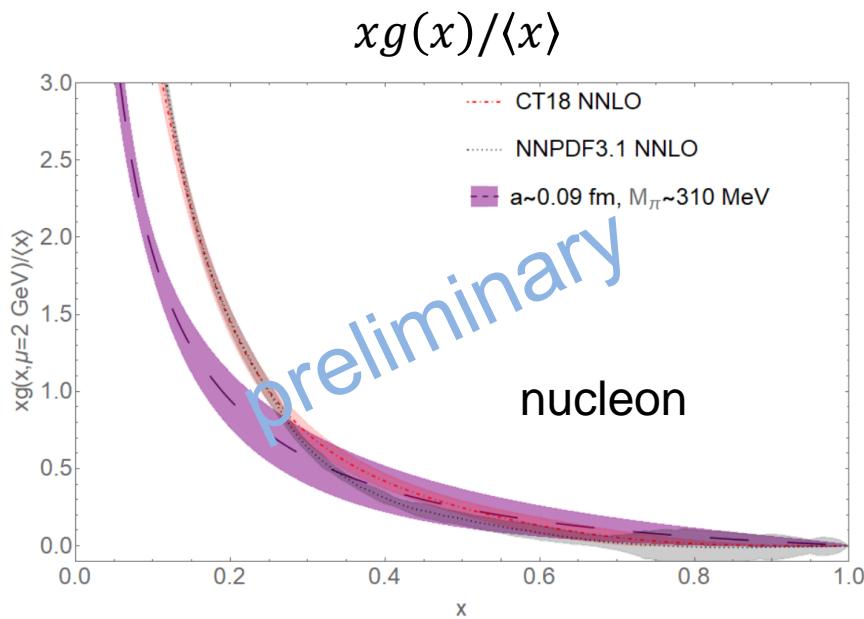
Pion  $xg(x)/\langle x \rangle$  from different ensembles with 2 different lattice spacings and 3 pion masses



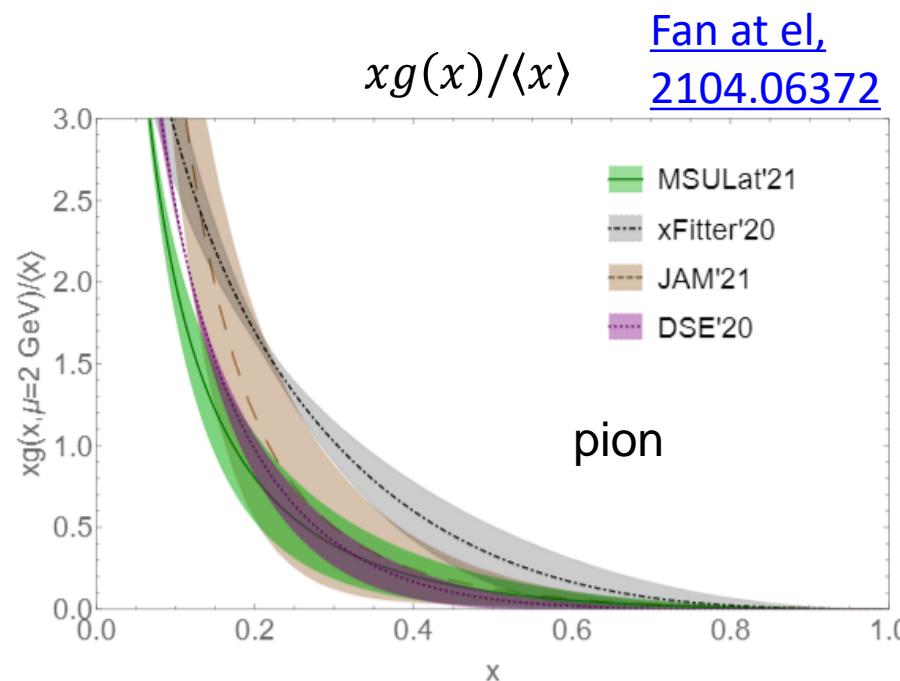
- The different lattice spacings and pion masses  $xg(x)/\langle x \rangle$  results are consistent with each other within one sigma error

# Comparing with the global fit results

Nucleon  $xg(x)/\langle x \rangle$  comparing with the CT18 and NNPDF3.1 NNLO unpolarized gluon PDFs.



Pion  $xg(x)/\langle x \rangle$  comparing with the DSE, JAM and xFitter pion gluon PDFs.

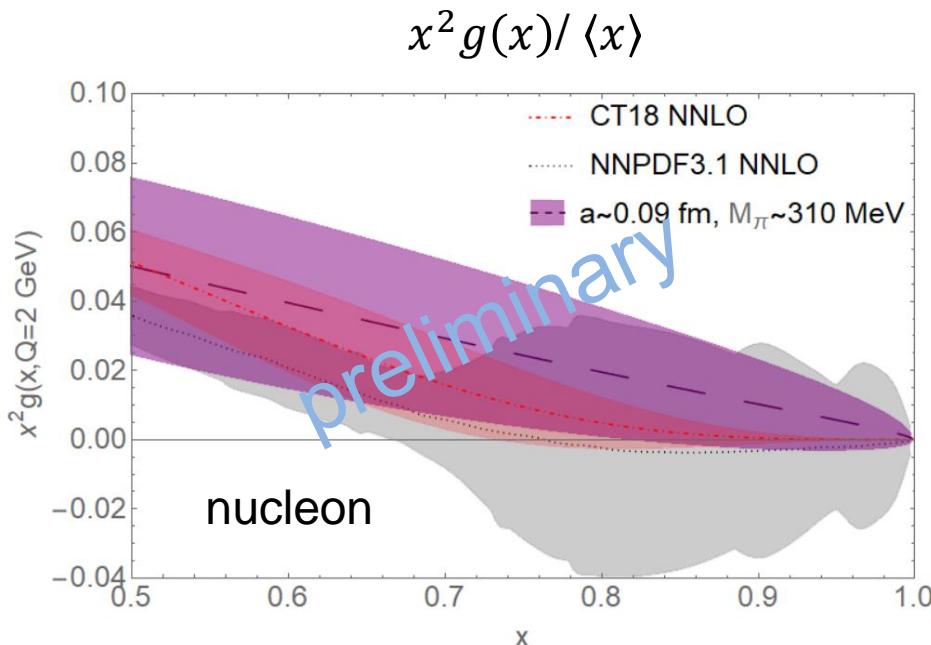


- Our smallest lattice spacing  $a09m310$  PDF result is consistent with global fit PDFs at  $x>0.3$  region

- Our smallest pion mass  $a12m220$  PDF result is consistent with global fit PDFs at  $x>0.2$  region

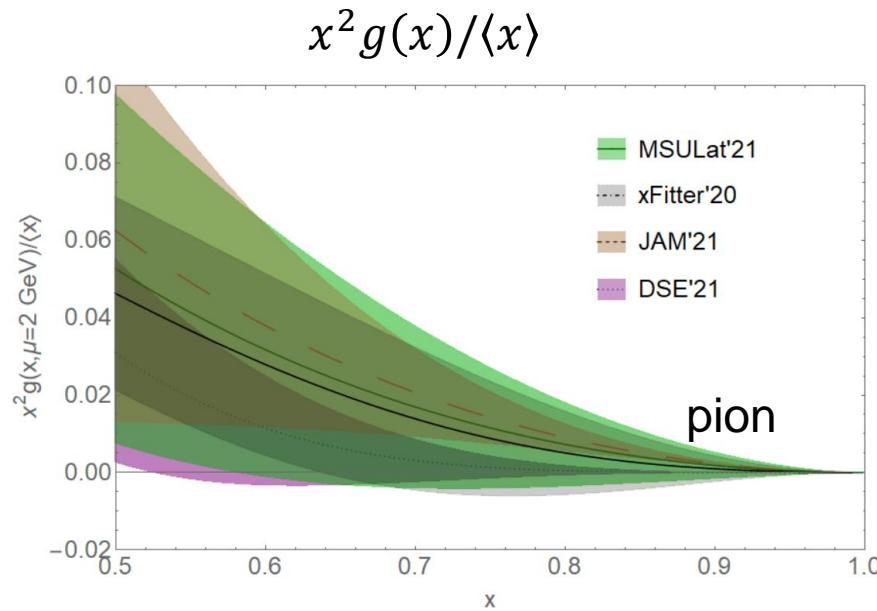
# Comparing with the global fit results

Nucleon  $xg(x)/\langle x \rangle$  comparing with the CT18 and NNPDF3.1 NNLO unpolarized gluon PDFs.



- Consistent but slightly larger than the global fit PDFs

Pion  $xg(x)/\langle x \rangle$  comparing with the DSE, JAM and xFitter pion gluon PDFs.



- Despite the differences in the fit form, the large- $x$  behaviors are quite consistent



# 1-step fit on RpITD

- 2-step fit

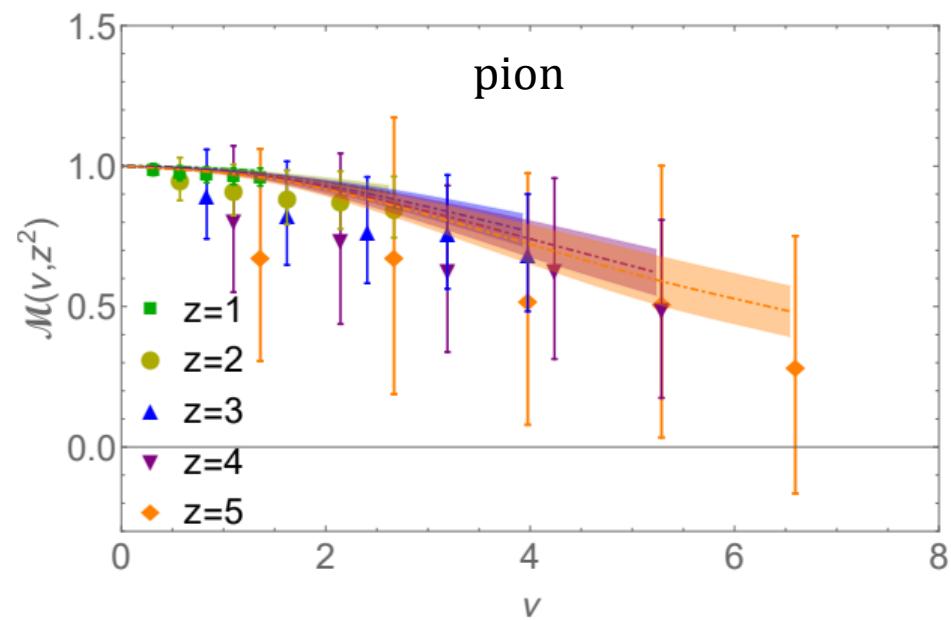
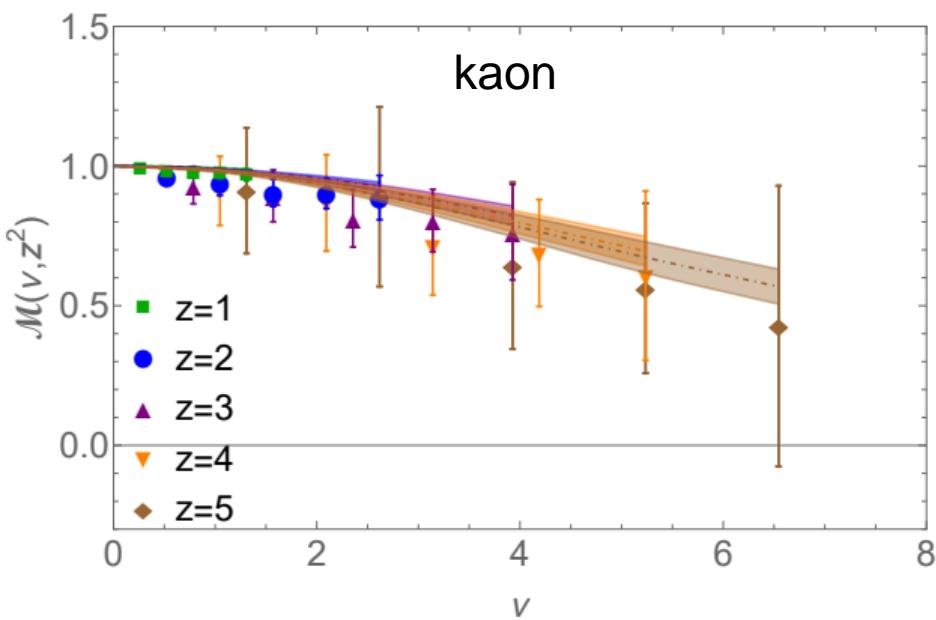


- 1-step fit



a12m310, 324160 measurements

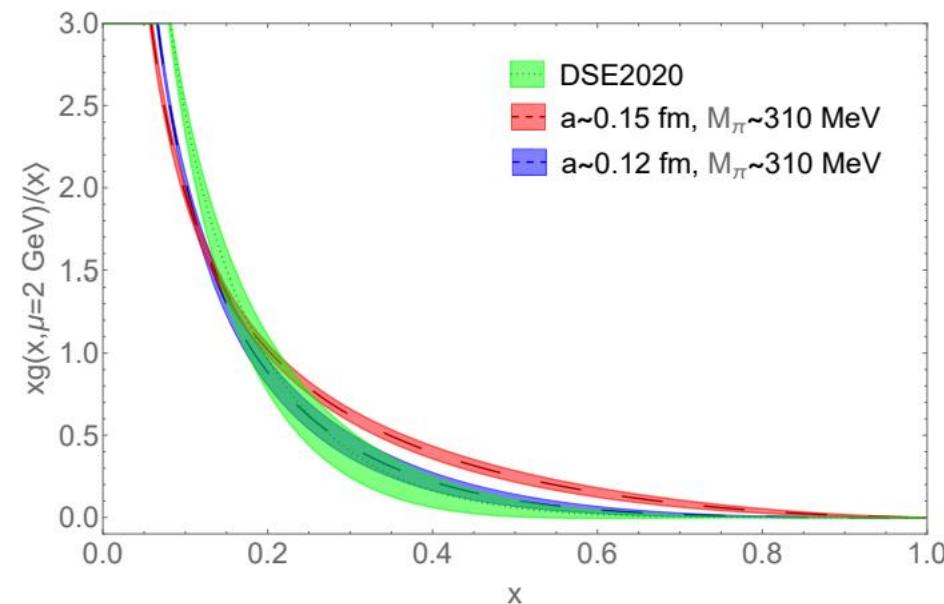
[Alejandro Salas](#)  
[et al, 2112.03124](#)



# Kaon gluon PDF

Kaon  $xg(x)/\langle x \rangle$  comparing with the DSE2020 [Z.-F. Cui et al, 2020](#) kaon gluon PDF

$xg(x)/\langle x \rangle$

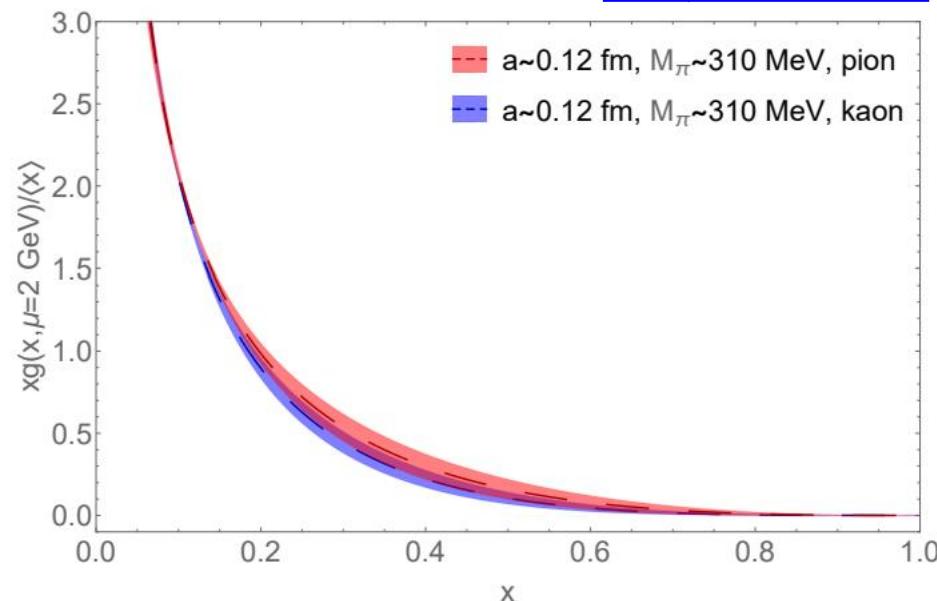


- The smaller lattice spacing  $a_{12m310}$  band is consistent with DSE2020 band at  $x > 0.15$  region

Kaon  $xg(x)/\langle x \rangle$  comparing with the pion  $xg(x)/\langle x \rangle$

$xg(x)/\langle x \rangle$

[Alejandro Salas et al, 2112.03124](#)



- Pion  $xg(x)/\langle x \rangle$  is slightly larger than kaon's except the small- $x$  region



- We extract the pion, kaon and nucleon  $x$ -dependent gluon PDF.
- The pion mass and lattice spacing dependents are weak under the current statistics for pion and nucleon
- Nucleon gluon PDF: 1-step fit study,  $\langle x \rangle$  calculation, more measurements
- There are systematics yet to be studied for nucleon gluon PDF (quark contribution, finite  $\nu$  in EpITDs or RpITDs)

