# Calculation of the Second Moment of the Pion Light-Cone Distribution Amplitude

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December 7, 2021

# Light-Cone Distribution Amplitude

- Factor experimentally measurable processes into non-perturbative structure function times perturbative parton physics
- LCDA $\varphi_\pi(\xi)$ represents amplitude for $\pi$ transitioning into $q\bar{q}$ pair with momenta $(1+\xi)p/2$, $(1-\xi)p/2$
- Formally defined via

$$\langle 0|\bar{d}(-z)\gamma_\mu\gamma_5\mathcal{W}[-z,z]u(z)|\pi^+(p)\rangle = ip_\mu f_\pi \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot z}\varphi_\pi(\xi)$$

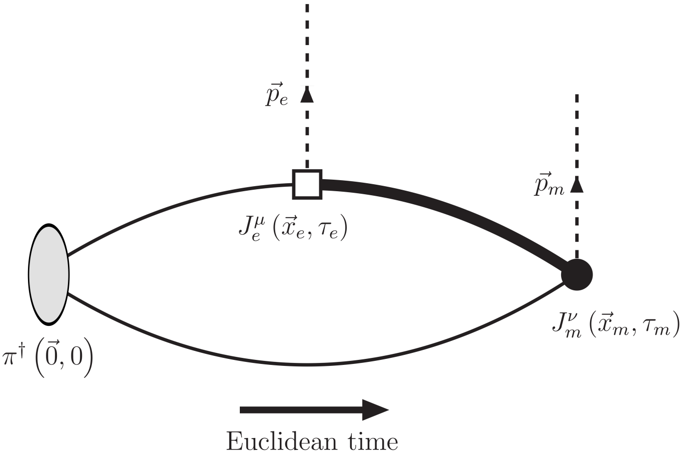
# Lattice Determination of LCDA

- Our approach: expand LCDA into Mellin moments

$$\langle \xi^n \rangle = \int_{-1}^{1} d\xi \, \xi^n \varphi_\pi(\xi)$$

    - This talk: Computation of $\langle \xi^2 \rangle$
    - Next talk (Robert Perry): Exploratory computation of $\langle \xi^4 \rangle$
- Previous lattice calculations
    - Local matrix elements
    - Light-quark operator product expansion
    - Quasi-PDF and pseudo-PDF (determine $\varphi_\pi(\xi)$ without recourse to moments)

# Heavy-Quark Operator Product Expansion (HOPE)



# Heavy-Quark Operator Product Expansion (HOPE)

- Hadronic tensor can be expanded in terms of moments

$$V^{\mu\nu}(p,q) = \frac{2if_\pi \varepsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} \sum_{\substack{n=0\\ \text{even}}}^{\infty} \frac{\tilde{\omega}^n}{2^n(n+1)} C_W^{(n)}(\tilde{Q}, m_\Psi, \mu)\langle \xi^n \rangle(\mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{\tilde{Q}}\right)$$

with $\tilde{\omega} = 2p\cdot q/\tilde{Q}^2$ and $\tilde{Q}^2 = -q^2 - m_\Psi^2$

- Heavy quark mass $m_\Psi$ suppresses higher-twist effects

# Hadronic Tensor

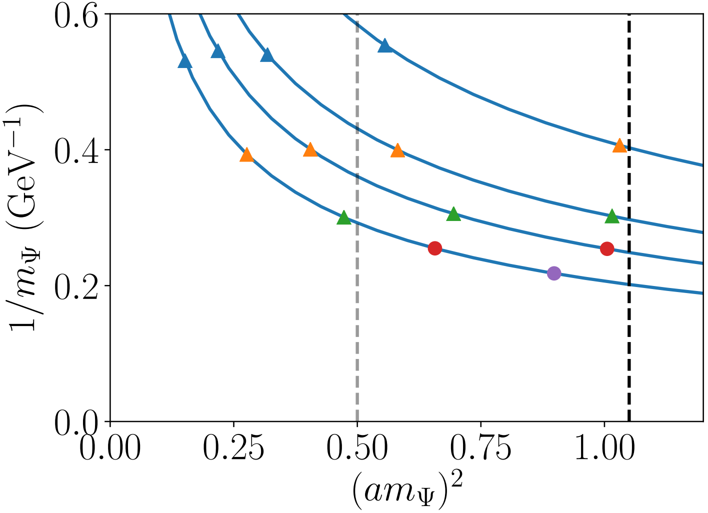
$$V^{\mu\nu}(q,p) = \int d^4x\, e^{iq\cdot x} \left\langle 0 \left| \mathcal{T}\left[A^\mu\left(\frac{x}{2}\right) A^\nu\left(-\frac{x}{2}\right)\right]\right| \pi^+(p)\right\rangle$$

$$\int dq_4 e^{-iq_4\tau} V^{\mu\nu}(q,p) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \left\langle 0 \left| \mathcal{T}\left[A^\mu\left(\frac{\mathbf{x}}{2},\frac{\tau}{2}\right) A^\nu\left(-\frac{\mathbf{x}}{2},-\frac{\tau}{2}\right)\right]\right| \pi^+(\mathbf{p})\right\rangle$$

- Inverse FT of $V^{\mu\nu}$ calculable on lattice in terms of 2-point and 3-point functions

$$C_2(\tau) = \langle \mathcal{O}_\pi(\tau)\mathcal{O}_\pi^\dagger(0)\rangle$$
$$C_3(\tau_e,\tau_m) = \langle A^\mu(\tau_e)A^\nu(\tau_m)\mathcal{O}_\pi^\dagger(0)\rangle$$

# Ensembles Used



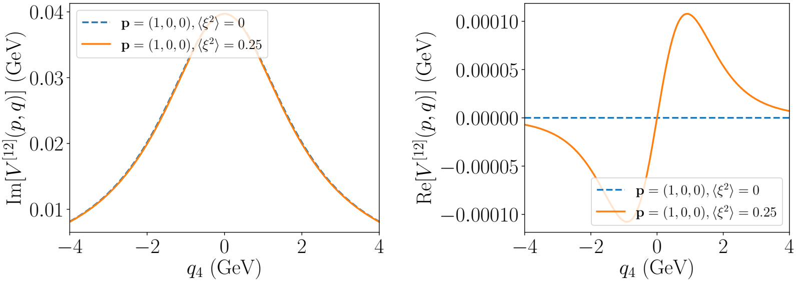
# Choice of Kinematics

$$V^{\mu\nu}(p,q) = \frac{2if_\pi \varepsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} \sum_{\substack{n=0 \\ \text{even}}}^{\infty} \frac{\tilde{\omega}^n}{2^n(n+1)} C_W^{(n)}(\tilde{Q}, m_\Psi, \mu) \langle \xi^n \rangle (\mu) + O\left(\frac{\Lambda_{\text{QCD}}}{\tilde{Q}}\right)$$

- Wilson coefficients $C_W^{(n)}(\mu = 2 \text{ GeV})$ calculated to 1-loop
- Fit parameters: $f_\pi$, $m_\Psi$, $\langle \xi^2 \rangle$
- Contribution of second moment $\langle \xi^2 \rangle$ suppressed by

$$\frac{\tilde{\omega}^2}{2^2 \times 3} = \frac{1}{3}\left(\frac{p \cdot q}{\tilde{Q}^2}\right)^2 \lesssim 10^{-2}$$

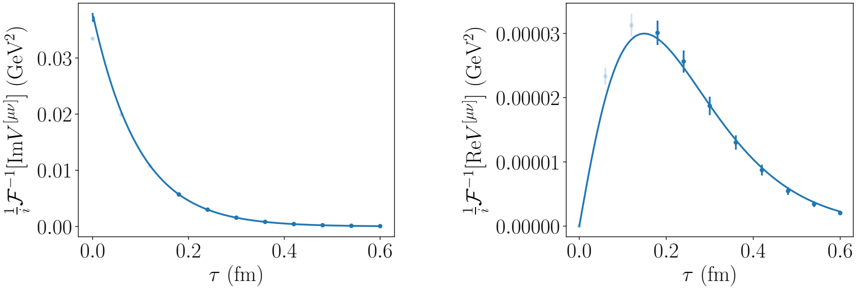
# Choice of Kinematics




$$\mathbf{p} = (1,0,0) = (0.64\ \text{GeV}, 0, 0)$$
$$2\mathbf{q} = (1,0,2) = (0.64\ \text{GeV}, 0, 1.28\ \text{GeV})$$

# Fitting Hadronic Tensor

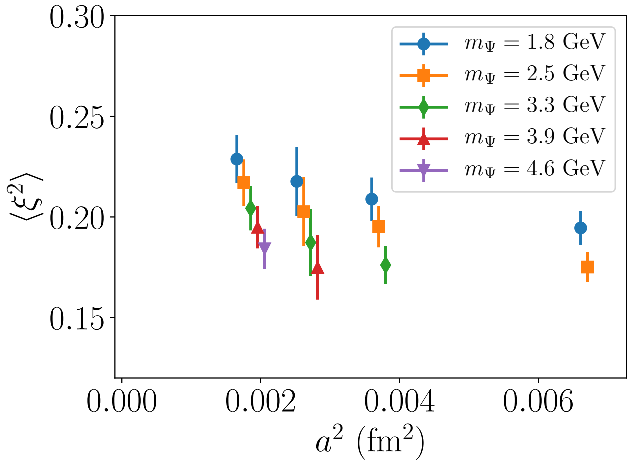
- Fit ratio of 2- and 3-point correlators to inverse FT of OPE




$$f_\pi = 149 \pm 1 \text{ MeV} \qquad \langle \xi^2 \rangle = 0.209 \pm 0.011$$

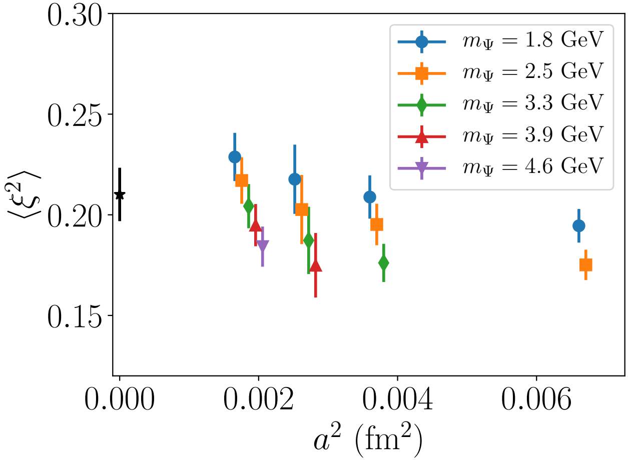
$$m_\Psi = 1.85 \pm 0.01 \text{ GeV}$$

# Fits to Various Ensembles




Masses are (left to right) $\{1.8, 2.5, 3.3, 3.9, 4.6\}$ GeV

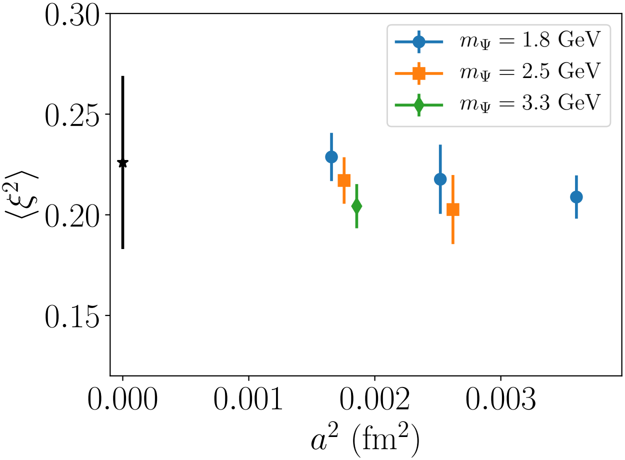
# Continuum Extrapolation




$$\text{data} = \langle \xi^2 \rangle + \frac{A}{m_\Psi} + Ba^2 + Ca^2 m_\Psi + Da^2 m_\Psi^2$$

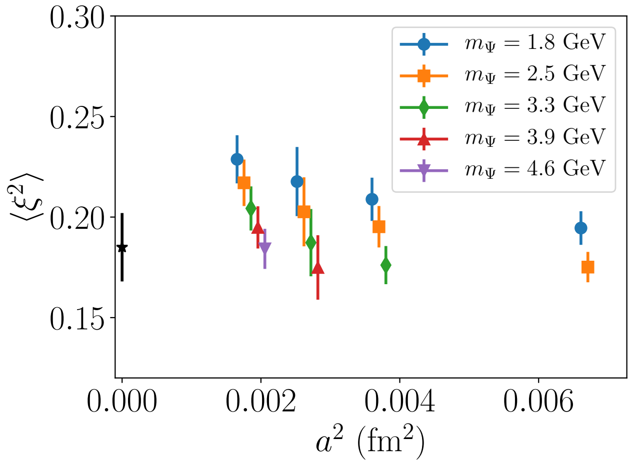
Extrapolate away both discretization errors and twist-3 effects

# Uncertainty in Continuum Extrapolation




- Original fit restricted $am_\Psi$ to $< 1.05$
- Could take a more conservative threshold, e.g. $am_\Psi < 0.7$

# Uncertainty in Higher-Twist Effects




- Could add twist-4 term to fit as well

$$\text{data} = \langle \xi^2 \rangle + Am_\Psi^{-1} + Bm_\Psi^{-2} + Ca^2 + Da^2 m_\Psi + Ea^2 m_\Psi^2$$

# Remaining Uncertainties

- Excited state contamination: estimated at **1%**
- Finite volume effects: $m_\pi L = 5.4 \Rightarrow \frac{1}{m_\pi L} e^{-m_\pi L} = \mathbf{0.08\%}$
- Unphysical pion mass ($m_\pi = 550$ MeV): Likely a $\sim$ **5%** error (V. M. Braun et al., hep-lat/1503.03656)
- Fit range: Excluding $\tau = 3a$ as well gives discrepancy of **1%**
- Wilson coefficients: Performing fit at $\mu = 4$ GeV and running back to 2 GeV gives discrepancy of **4%**
- Quenching: Formally uncontrolled, typically around **10–20%**

# Combined Uncertainty

$$
\begin{aligned}
\langle \xi^2 \rangle = 0.210 &\pm 0.013 \text{ (statistical)} \\
&\pm \mathbf{0.016} \text{ (continuum)} \\
&\pm \mathbf{0.025} \text{ (higher twist)} \\
&\pm 0.002 \text{ (excited states)} \\
&\pm 0.0002 \text{ (finite volume)} \\
&\pm 0.014 \text{ (unphysically heavy pion)} \\
&\pm 0.002 \text{ (fit range)} \\
&\pm 0.008 \text{ (running coupling)} \\
\hline
\langle \xi^2 \rangle = 0.210 &\pm 0.036 \text{ (total, exc. quenching)}
\end{aligned}
$$

# Comparison to Literature