Topics in LaMET

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Topics

- Renormalon (w/ WY Liu 2010.06623)
- Finite volume effect w/ ChPT (w/ WY Liu 2011.13536)
- Matching in hybrid renormalization (w/ CY Chou)

Renormalon in LaMET

$$\tilde{Q}(x, P_z, \mu') = \int_{-1}^{1} \frac{dy}{|y|} Z(\frac{x}{y}, yP_z, \mu', \mu) Q(y, \mu) + \mathcal{O}(\frac{1}{P_z^2})$$

In $\overline{\rm MS}$ and RI/MOM scheme, renormalon ambiguity arises. Braun, Vladimirov and Zhang (1810.00048): power correction $\mathcal{O}(\Lambda_{\rm QCD}^2/x^2P_z^2)$

Renormalon Ambiguity

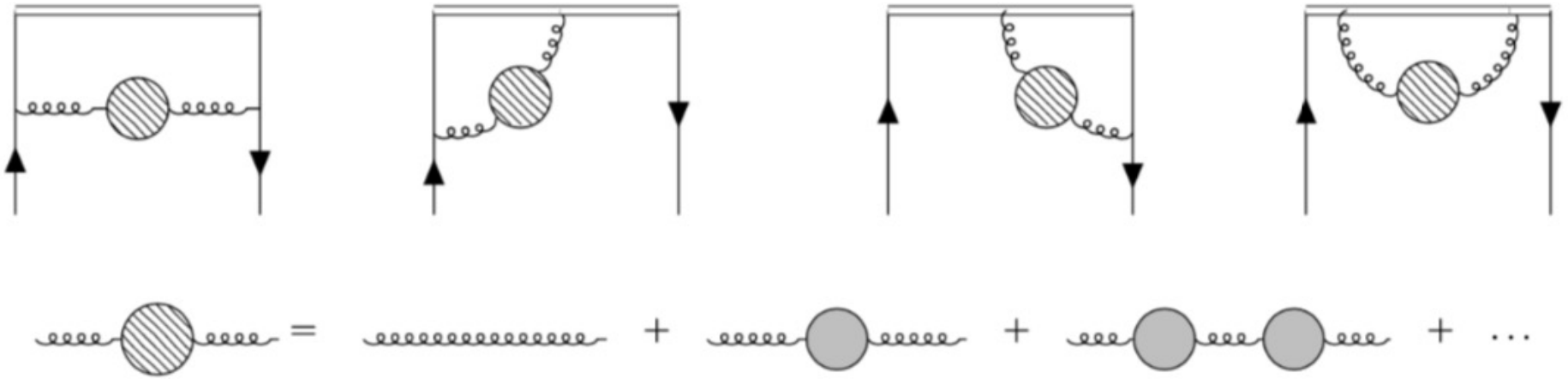
In an OPE,

(1) use Borel transform to improve the convergence of the Wilson coefficients

(2) sum the series

(3) then perform inverse Borel transform. Poles in the integrant (renormalons) lead to ambiguity in the contour integrals which can be absorbed by the power corrections.

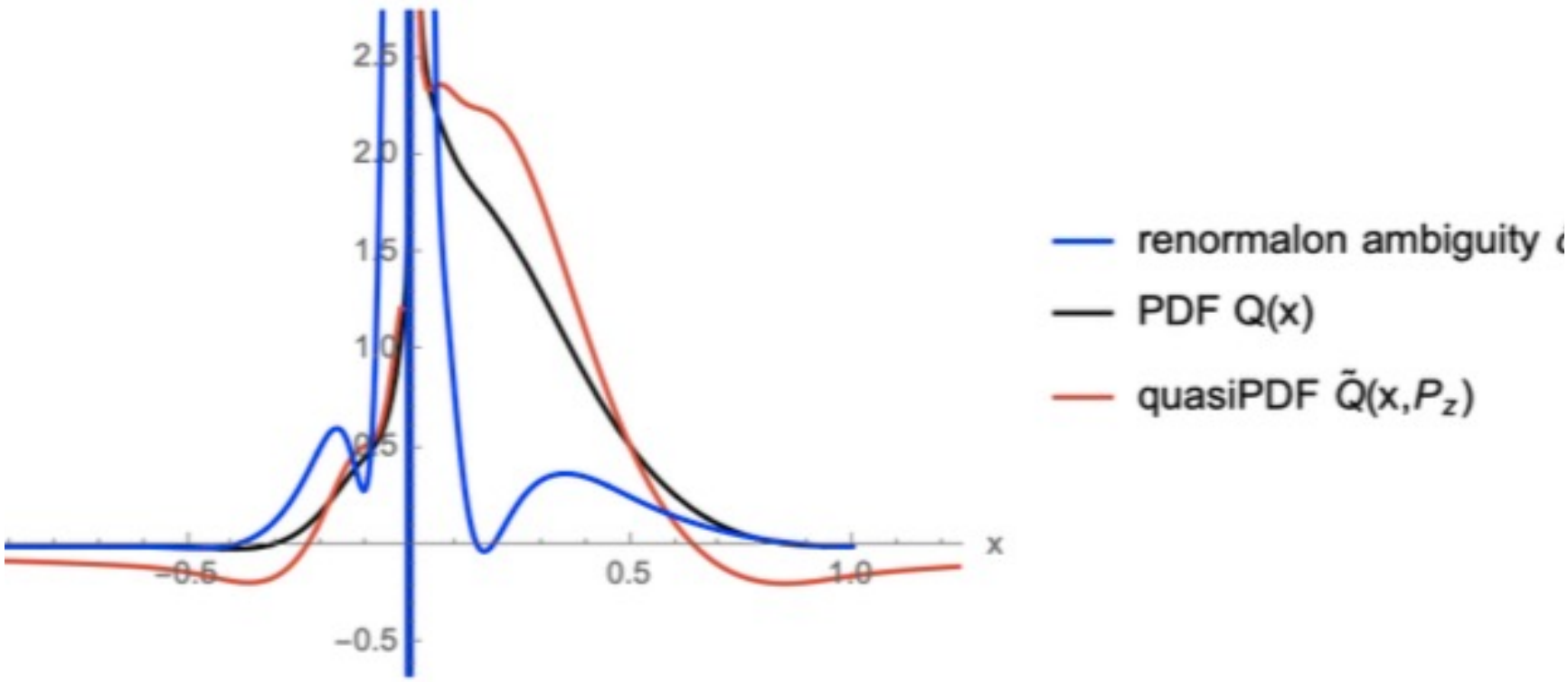
Studied by bubble chain diagrams



$$\alpha_s n_f \quad \text{as} \quad \mathcal{O}(1)$$

$$\frac{1}{-k^2 - i\epsilon} \to \frac{(\Lambda^2_{\rm QCD})^w}{(-k^2 - i\epsilon)^{1+w}}$$

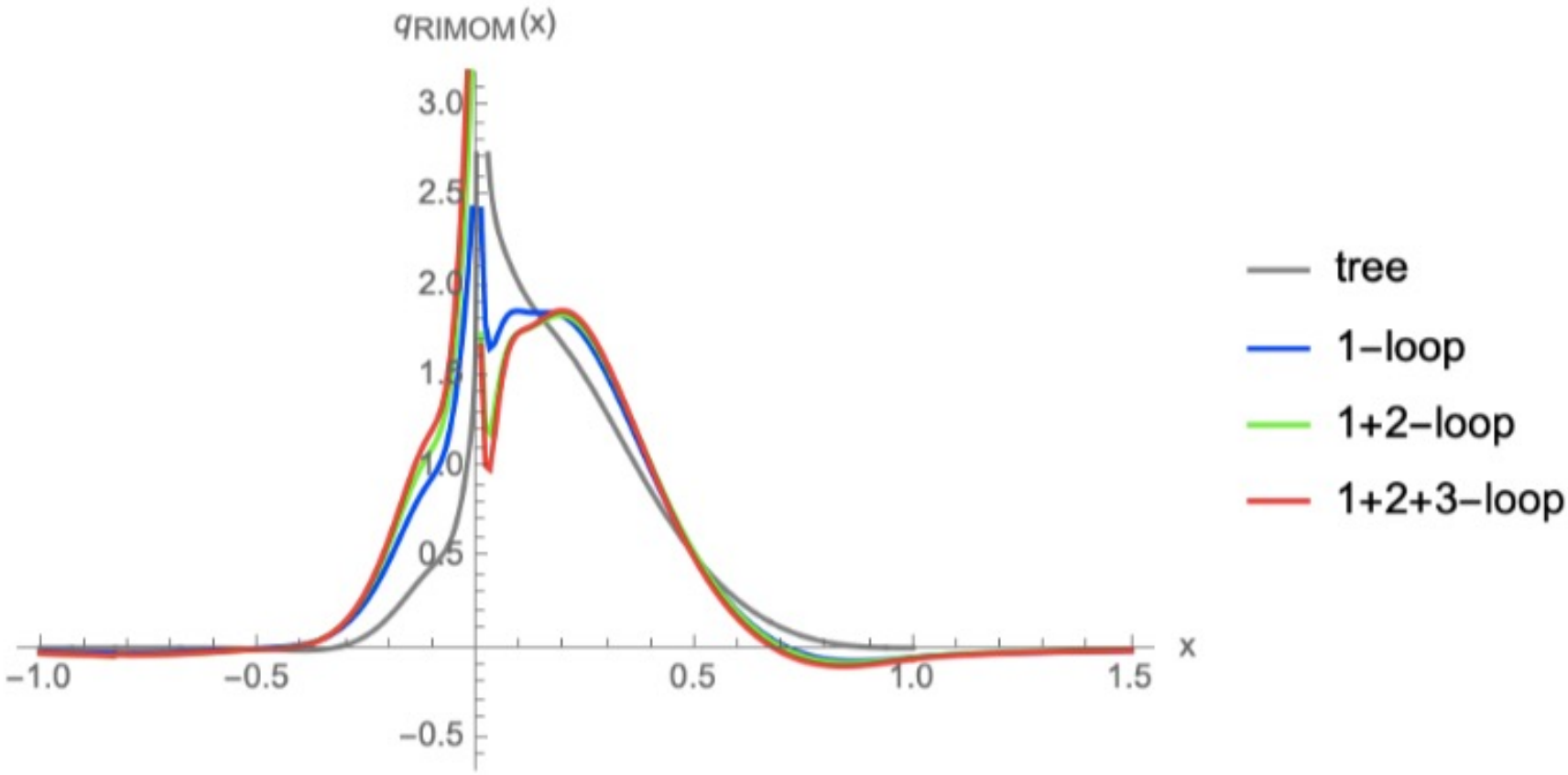
Power Corrections suggested by Renormalon Ambiguity



$$\delta\tilde{Q}_{ren}(x,P_z) = \frac{\pi}{\beta_0} e^{5/3} C_F \frac{\Lambda^2_{\rm QCD}}{P_z^2} \int_{-1}^{1} \frac{dy}{|y|y^2} \left[\frac{\theta(1-\frac{x}{y})\theta(\frac{x}{y}) - \delta(1-\frac{x}{y})}{1-\frac{x}{y}} \right]_+ Q(y)$$

$$= \frac{\pi}{\beta_0} e^{5/3} C_F \frac{\Lambda^2_{\rm QCD}}{x^2 P_z^2} \left\{ \int_0^1 d\xi \frac{1}{1-\xi} \left[\xi Q(x/\xi) - Q(x) \right] + Q(x) - xQ'(x) \right\}$$

Bubble diagram contribution up to 3-loops (RI/MOM to MS-bar)

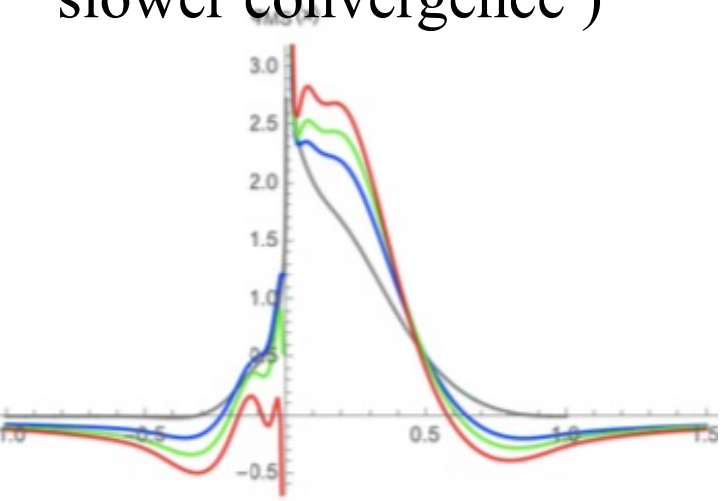


R-Scheme

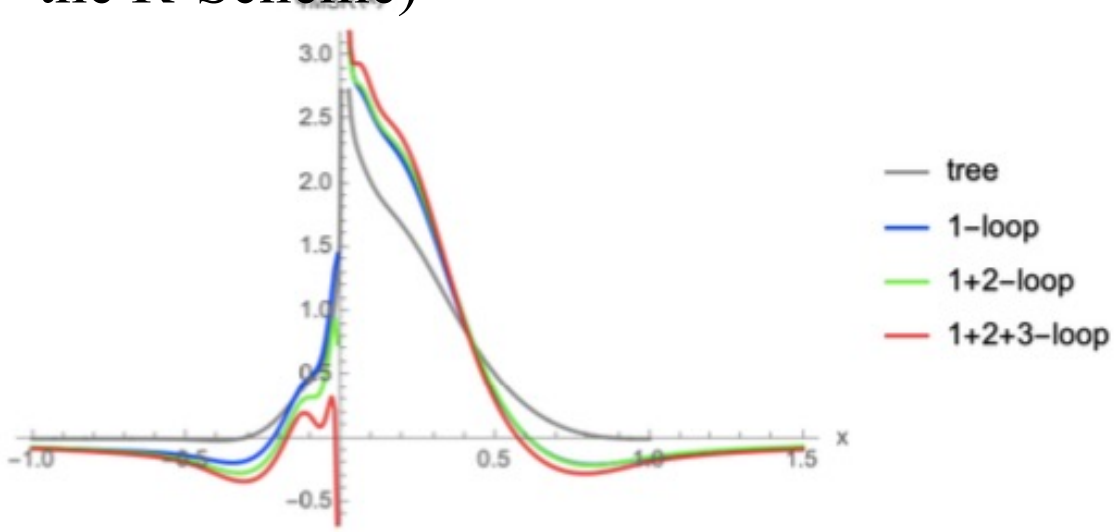
(Hoang, Jain, Scimemi, Stewart, 0908.3189)

$$\tilde{Q}_R(x, P_z, P_z', \Lambda') = \frac{P_z^2 \tilde{Q}(x, P_z, \Lambda') - P_z'^2 \tilde{Q}(x, P_z', \Lambda')}{P_z^2 - P_z'^2}$$

(MS-bar to MS-bar slower convergence)



(faster convergence by adding the R-Scheme)



$P_z' = 3$ GeV, $\alpha_s = 0.283$, $P_z = 1.5$ GeV, and $\mu = 3$ GeV

ChPT for LaMET

- Idea: Heavy Baryon ChPT can be used for a baryon with a large momentum, as long as its off-shellness in the loop is much smaller than the baryon mass.
- Equal time correlator is dominated by the symmetric traceless (twist-2) terms under OPE. Trace terms are suppressed by the baryon momentum.
- Matching of the twist-2 operators standard by now: JWC, Ji, **PLB**523 (2001) 107; **PRL** 87 (2001) 152002; **PRL** 88 (2002) 052003; JWC, Stewart, **PRL** 92 (2004) 202001; Arndt, Savage, **NPA**697 (2002) 429

ChPT for LaMET

$$\lambda_\mu \bar{\psi}(z)\Gamma^\mu W(z,0)\psi(0) \simeq \sum_{n=0}^{\infty} \frac{(iz)^n}{n!}\lambda_\mu\lambda_{\mu_1}\lambda_{\mu_2}\ldots\lambda_{\mu_n}\bar{\psi}\Gamma^\mu iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n}\psi,$$

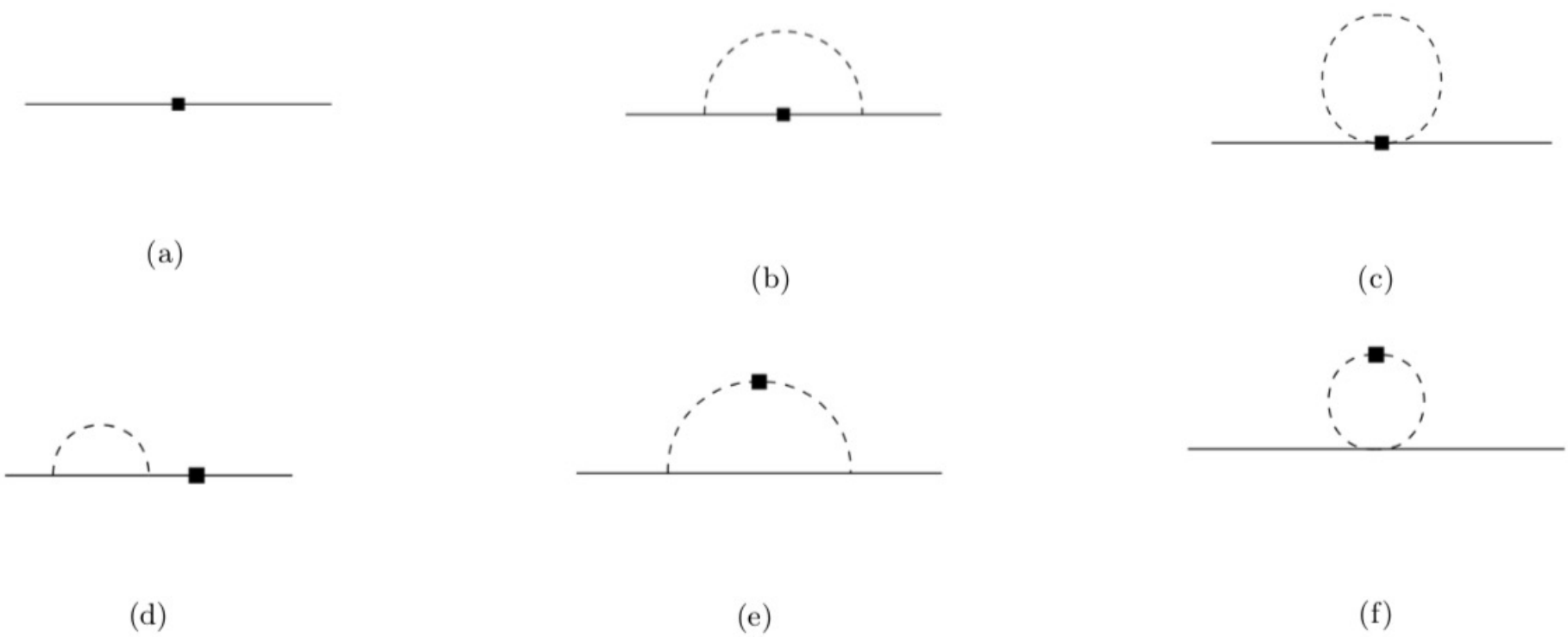
$$\mathcal{O}_q^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{(\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n)}\psi,$$

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger) + \eta\mathrm{tr}(\mathcal{M}\Sigma^\dagger + \mathcal{M}^\dagger\Sigma) + \overline{N}iv\cdot DN + 2g_A\overline{N}S\cdot AN + \ldots,$$

$$\Sigma = e^{\frac{i}{F_\pi}\Pi}, \quad \Pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\mathcal{M} = \mathrm{diag}(m_u, m_d) \qquad u^2 = \Sigma$$

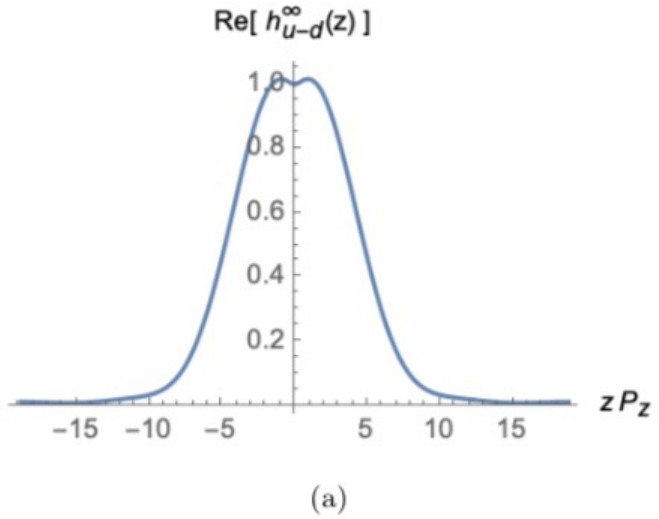
ChPT for LaMET



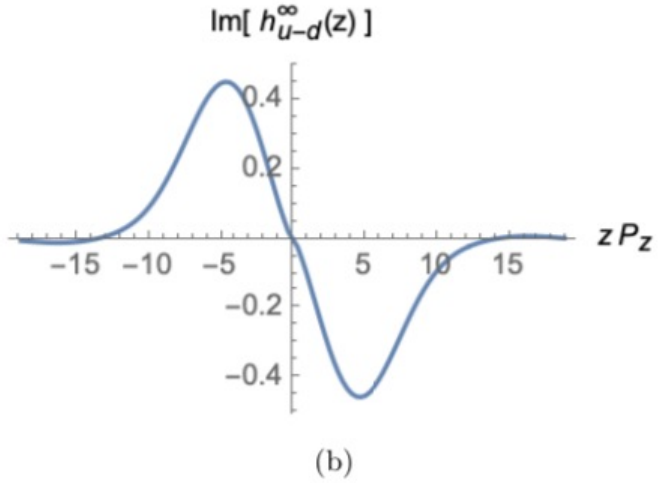
$$\mathcal{O}^{\mu\mu_1\mu_2\cdots\mu_n}_{u-d} = c_1^{(n)}\bar{N}v^{(\mu}v^{\mu_1}\ldots v^{\mu_n)}(u\tau^3u^\dagger + u^\dagger\tau^3u)N$$
$$+ \tilde{c}_1^{(n)}\bar{N}S^{(\mu}v^{\mu_1}\ldots v^{\mu_n)}(u\tau^3u^\dagger - u^\dagger\tau^3u)N + \cdots$$

$$\mathcal{O}^{\mu}_{u-d,\pi} \simeq a^{(0)}F_\pi^2\mathrm{tr}\left(\Sigma^\dagger\tau^3 i\partial^\mu\Sigma + \Sigma\tau^3 i\partial^\mu\Sigma^\dagger\right)$$

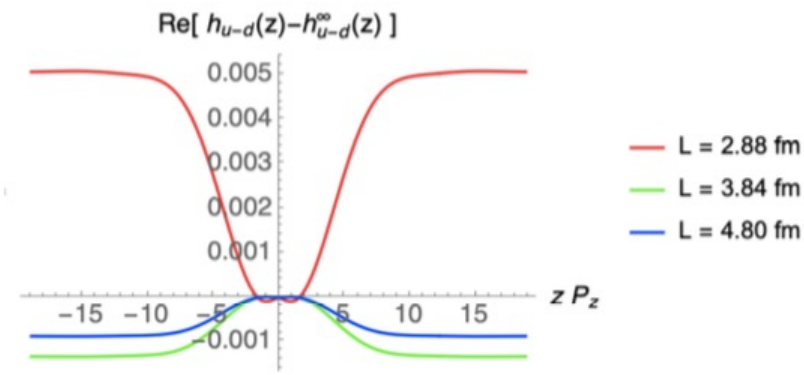
ChPT for LaMET



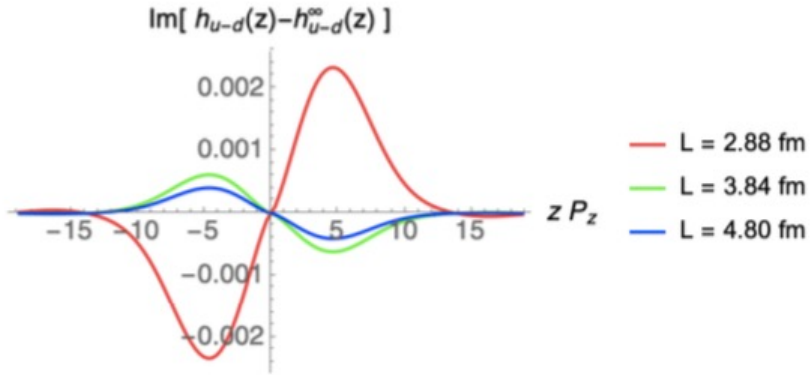
(a)



(b)



(c)



(d)

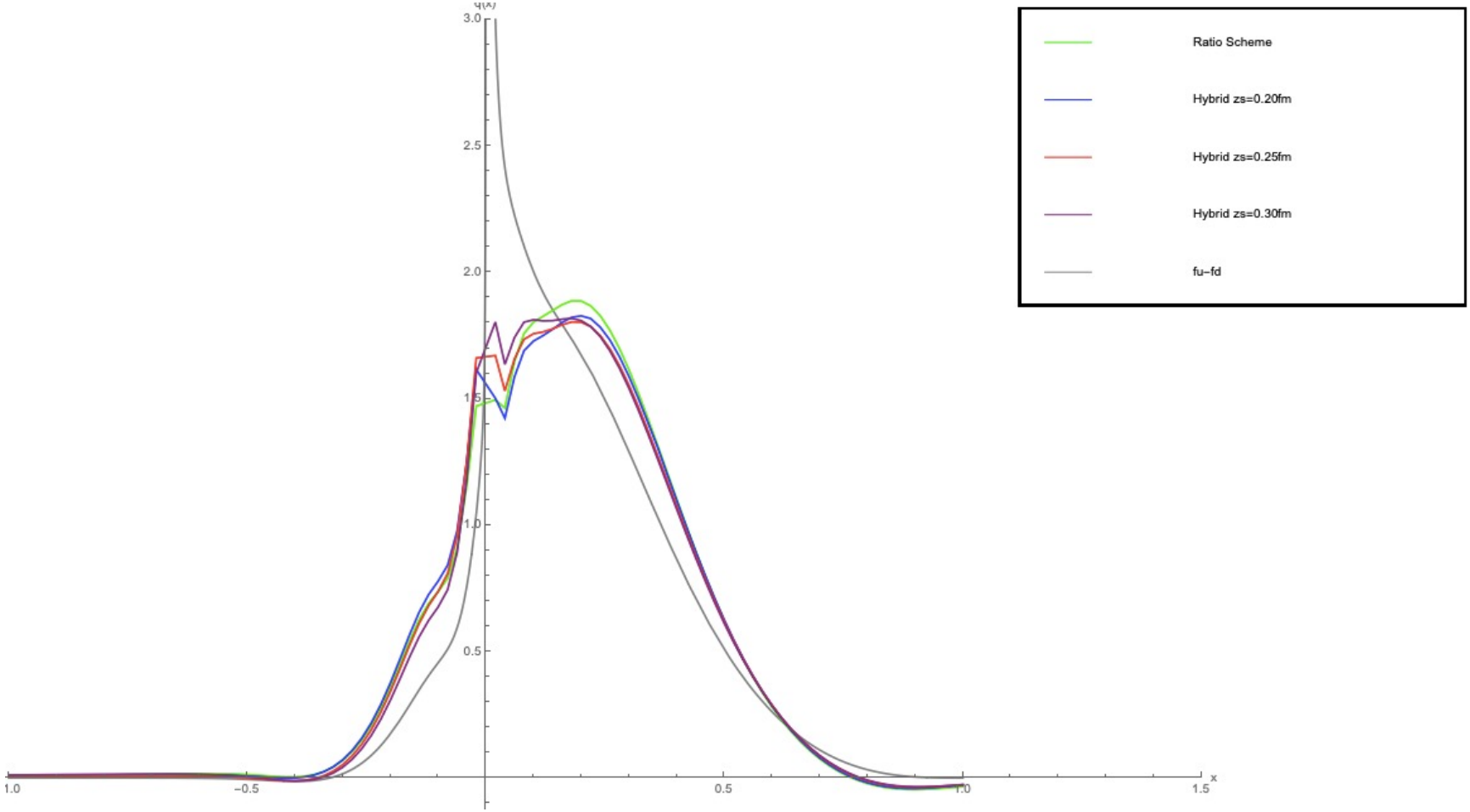
Finite volume effect less than 1% when $P_z/M \geq 1$ and $m_\pi L \geq 3$ consistent w/ Lin & Zhang (2019).

Hybrid Renormalization
(Ji et al. 2008.03886)

- RI/MOM might introduce IR contamination in the matching kernel in longer distance
- Hence replaced by Wilson line mass subtraction beyond Zs (and model the long tail beyond Zl)

Some progress (w/ Chien-Yu Chou)

- MS-bar to MS-bar matching: Fourier transform and epsilon expansion was now commute. The delta function in infinite x is cancelled in RI/MOM or ratio schemes.
- Matching kernel in the hybrid renormalization now satisfies particle number conservation.


q(x)
3.0
2.5
2.0
1.5
1.0
0.5
1.0
−0.5
0.5
1.0
1.5
x
Ratio Scheme
Hybrid zs=0.20fm
Hybrid zs=0.25fm
Hybrid zs=0.30fm
fu−fd

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