Pion and Kaon Distribution Amplitudes with LaMET

LaMET 2021 Online

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Contents

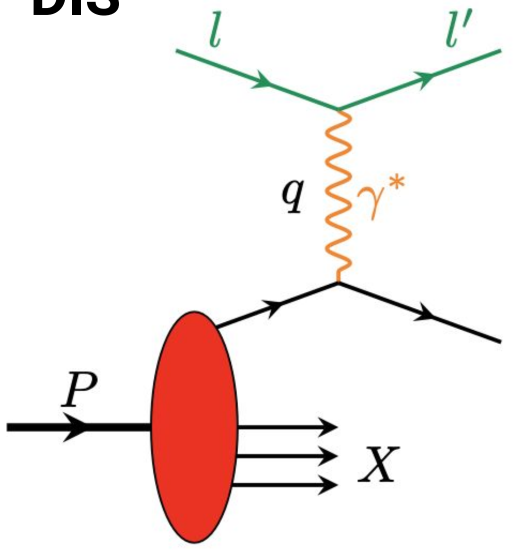
Motivation

LCDAs: Inner structure of hadrons

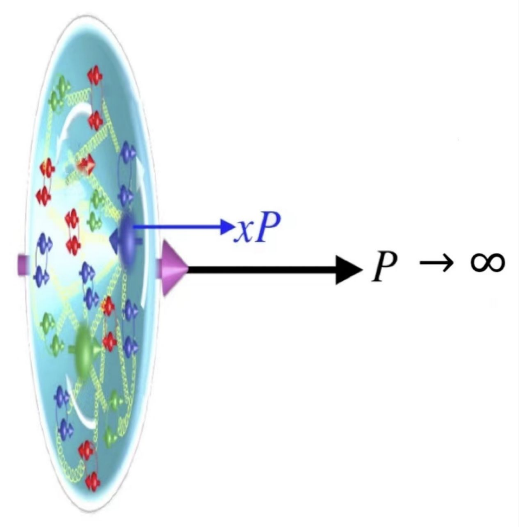
- **PDFs: describe the distribution of a single parton**
- **LCDAs: describe the distribution of all partons**

$$\phi_\pi(x) = \frac{1}{if_\pi}\int \frac{d\lambda}{2\pi P^+} e^{-i\left(x-\frac{1}{2}\right)\lambda} \left\langle 0 \left| O_{\gamma^+\gamma_5}(\lambda n) \right| \pi(P) \right\rangle$$

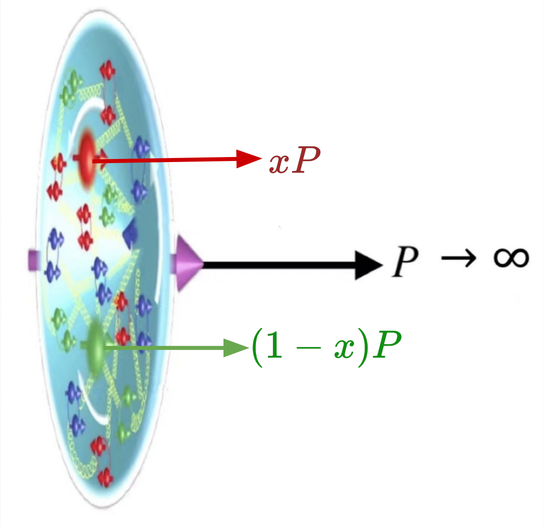
DIS



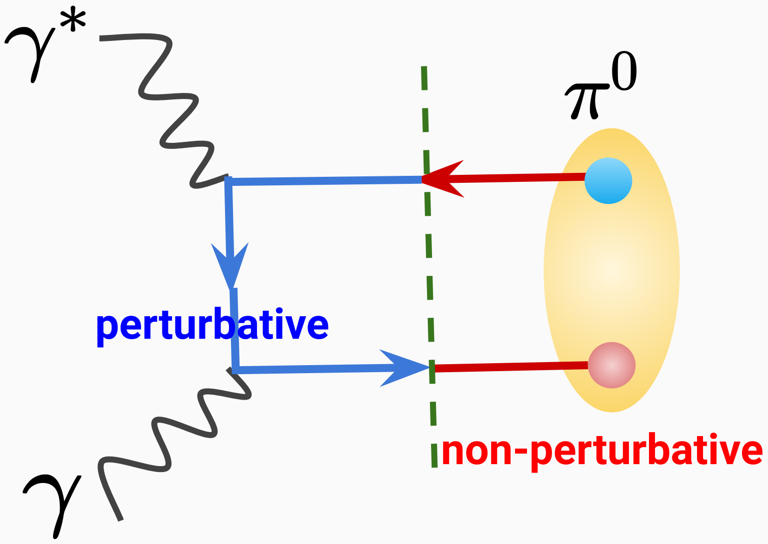
PDFs



LCDAs



LCDAs: essential input for exclusive processes



Simplest exclusive process to understand QCD factorization

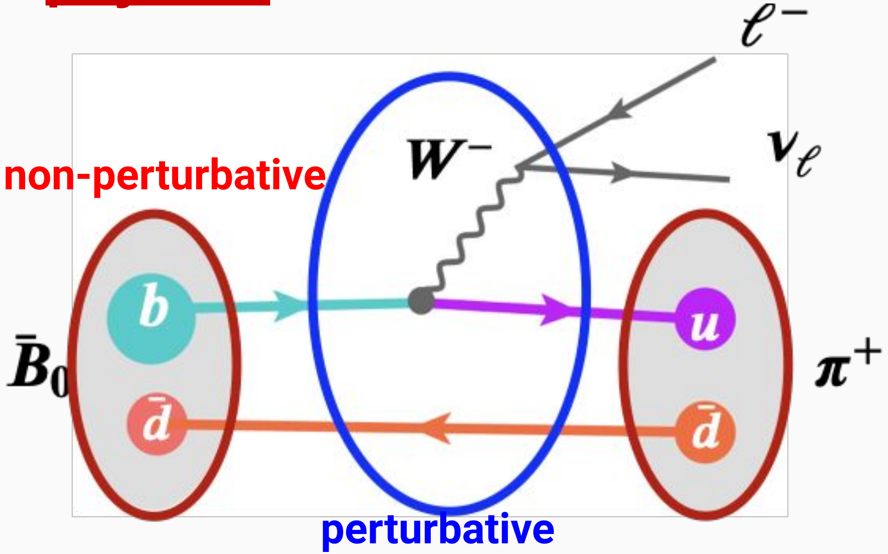
QCD factorization: separate the processes at different energy scales

Decay amplitudes of exclusive process

= hard scattering kernels $\otimes$ light-cone distribution amplitudes

LCDAs: essential input for exclusive processes

Exclusive processes provide an ideal platform to search for new physics.



- **CKM matrix elements**

The breaking of the unitarity of CKM matrix will gives the indication of new physics.

- **Rare decays**

Sensitive to the beyond-SM contribution.

Decay amplitudes of exclusive process

= hard scattering kernels $\otimes$ light-cone distribution amplitudes

LCDAs: theoretical efforts

Four decades

1980 → 2021

- **Asymptotic LCDAs**

  A.V. Efremov et. al., Theor.Math.Phys.42 (1980)

- **Sum rules**

  V.L. Chernyak et. al., Nucl.Phys.B 201 (1982)
  Vladimir M. Braun et. al., Z.Phys.C 44 (1989)
  Patricia Ball et. al., JHEP 08 (2007)

- **Lattice calculation with OPE**

  G. Martinelli et. al., Phys.Lett.B 190 (1987)
  RQCD Collaboration, JHEP 11 (2020)

- **Quark model**

  Choi, Phys.Rev.D 75 (2007)

- **Dyson-Schwinger Equation**

  Fei Gao, Phys.Rev.D 90 (2014)
  Craig D.et.al., Prog.Part.Nucl.Phys. (2021)

- **Lattice calculation with LaMET**

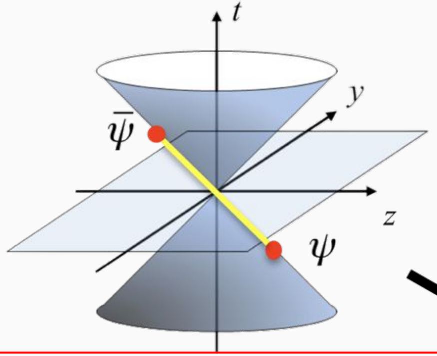
  Zhang, et. al., Phys.Rev.D 95 (2017)
  R. Zhang et.al., Phys.Rev.D 102 (2020)

- ...

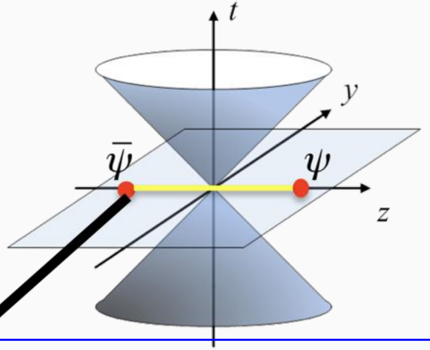
LaMET and high precision analysis

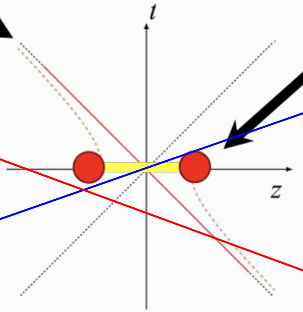
Large-Momentum Effective Theory(LaMET)

First principle calculations of entire LCDAs became feasible



X. Ji. Parton Physics on a Euclidean Lattice, Phys.Rev.Lett. 110, 262002 (2013).





Light-cone distribution:

Separated on the time axis;

Cannot be calculated on the lattice

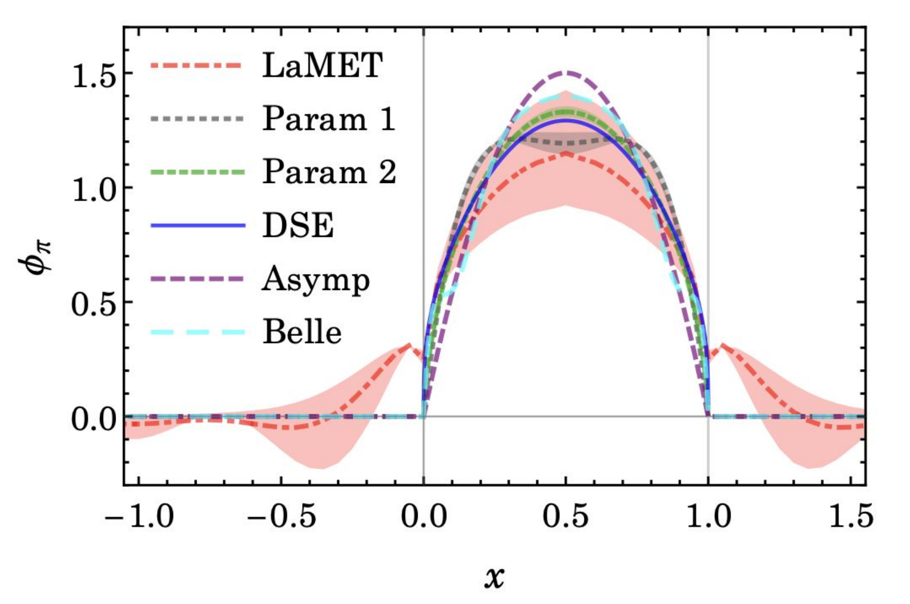
Quasi distribution:

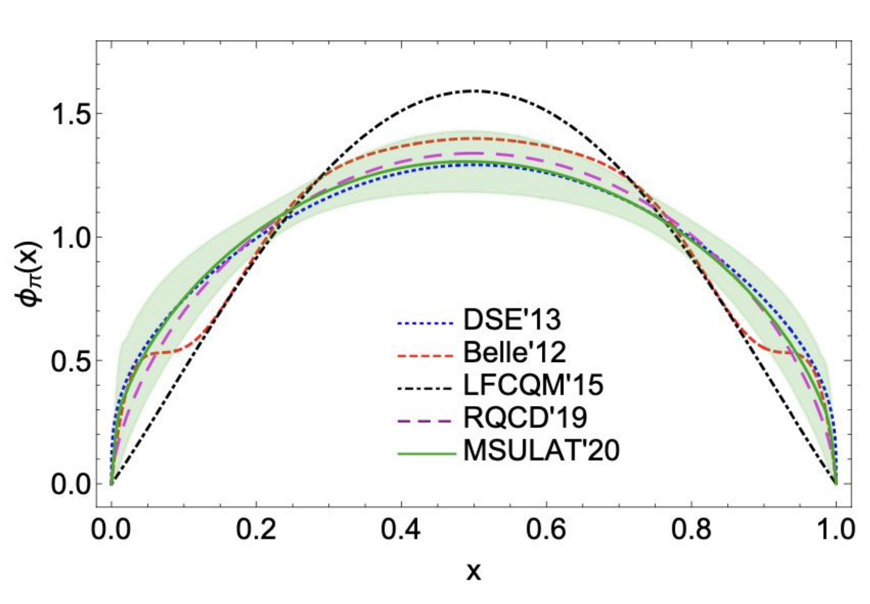
Equal time;

Directly calculable on the lattice

$$\boxed{\tilde{\phi}_\pi}\left(y, P^z, \mu_R, p^z_R\right) = \int_0^1 dx C_\pi\left(y, x, r, \frac{P^z}{\mu}, \frac{P^z}{p^z_R}\right) \boxed{\phi_\pi}(x, \mu) + \dots$$

Lattice calculation of LCDAs with LaMET





<u>J. H. Zhang, et. al., Phys.Rev.D 95 (2017)</u>

Pion mass = 310 MeV; a = 0.12 fm

Wilson-line renormalization

<u>R. Zhang et.al., Phys.Rev.D 102 (2020)</u>

Pion mass = 310 MeV

RI/MOM renormalization

High precision study with LaMET

1. Physical pion/kaon mass
2. Self-renormalization
3. Extrapolation in the coordinate space
4. Two loop matching... (for further study)

LCDAs on lattice in LaMET

Lattice Setup

- **2+1+1 flavors of HISQ action (MILC collaboration)**
- **3 lattice spacings: 0.06fm, 0.09fm, 0.12fm**
- **Physical pion/kaon mass**
- **Momentum: 1.29GeV, 1.72GeV, 2.15GeV**
- **Gamma factor: 9.21, 12.29, 15.36**

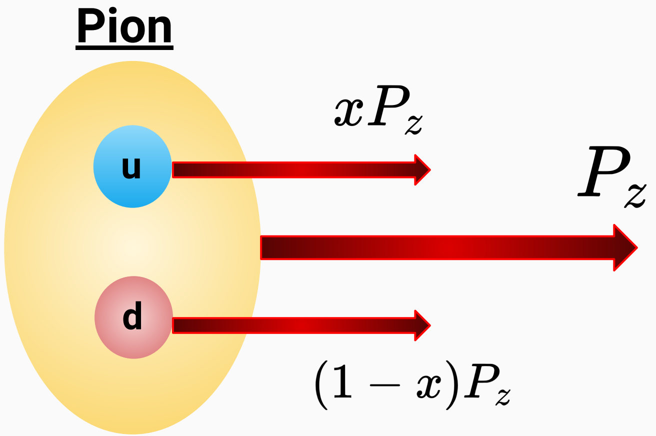
Ensemble	a(fm)	$L^3 \times T$	c_{SW}	$m_{u/d}$	m_s
a06m130	0.057	$\boxed{96^3 \times 192}$	1.03493	-0.0439	-0.0191
a09m130	0.088	$64^3 \times 96$	1.04239	-0.0580	-0.0174
a12m130	0.121	$48^3 \times 64$	1.05088	-0.0785	-0.0191

Leading-twist LCDAs

The leading-twist LCDAs of a pseudoscalar meson:

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0 | \bar{\psi}_1(0) \not{n} \gamma_5 U\left(0, \xi^-\right) \psi_2\left(\xi^-\right) | M(P) \rangle$$
$$= i f_M (p \cdot n) \Phi_M(x)$$

$$U\left(0, \xi^-\right) = P \exp\left[i g_s \int_{\xi_-}^{0} ds n_+ \cdot A\left(s n_+\right)\right]$$



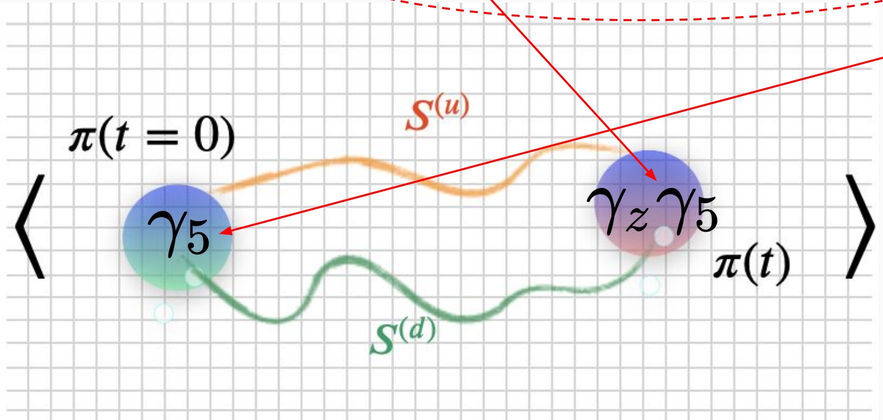
LCDAs: $\Phi_\pi(x, \mu^2)$

x is the longitudinal momentum fraction

Lattice calculation

We simulate on the lattice the quasi-LF correlation:

$$C_2^m(z,\vec{P},t) = \int d^3y e^{-i\vec{P}\cdot\vec{y}} \langle 0 | \bar{\psi}_1(\vec{y},t)\boxed{\Gamma_1}U(\vec{y},\vec{y}-z\hat{z})\psi_2(\vec{y}-z\hat{z},t)\bar{\psi}_2(0,0)\boxed{\Gamma_2}\psi_1(0,0)|0\rangle$$

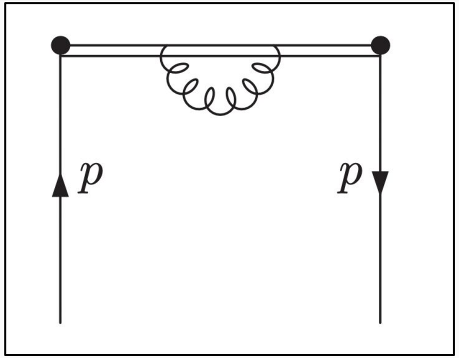


The g.s. quasi-DA can be extracted:

Normalized g.s.
Matrix element

$$\frac{C_2^m(z,\vec{P},t)}{C_2^m(z{=}0,\vec{P},t)} = \frac{\boxed{H_m^b(z)}(1+c_m(z)e^{-\Delta E t})}{(1+c_m(0)e^{-\Delta E t})}$$

Linear divergence



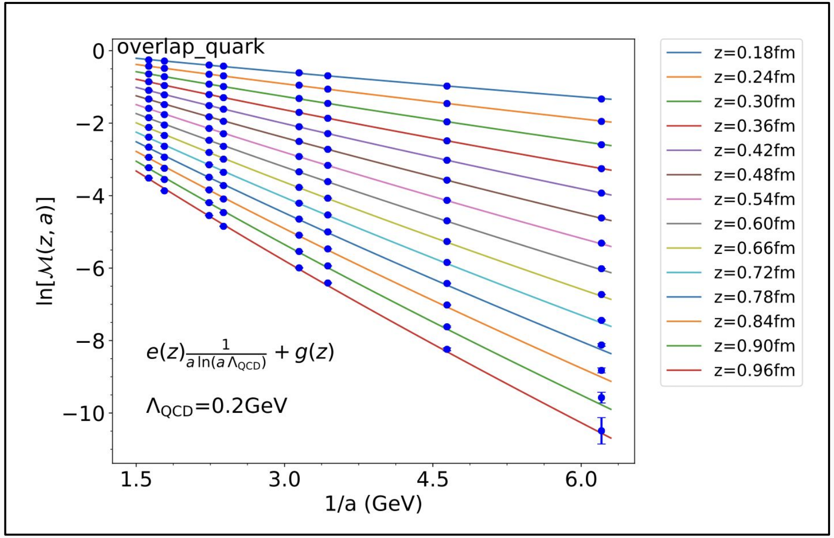
Irrelevant with the current

Linear divergence: comes from self-energy of Wilson link

Mass counterterm: $\delta m = -\frac{\alpha_s C_F}{2\pi}(\pi\Lambda), \Lambda = \frac{1}{a}$

Lattice spacing

J. Chen, et.al, Nucl. Phys. B915, 1 (2017)



Y. Huo, et al. Nucl. Phys. B (2021)

Self-renormalization: hybrid scheme

Self-renormalization: One fits the divergence structure to zero momentum hadron matrix element and uses it for renormalization.

Linear divergence

Renormalon ambiguity

Residual

Discretization error

$$\ln \mathcal{M}(z,a) = \frac{kz}{a\ln\left[a\Lambda_{QCD}\right]} + m_0 z + r(z) + f_z a + \frac{3C_F}{b_0}\ln\left[\frac{\ln\left[1/\left(a\Lambda_{QCD}\right)\right]}{\ln\left[\mu/\Lambda_{QCD}\right]}\right] + \ln\left[1+\frac{d}{\ln\left(a\Lambda_{QCD}\right)}\right]$$

Resummation of log divergence

Renormalization factor:

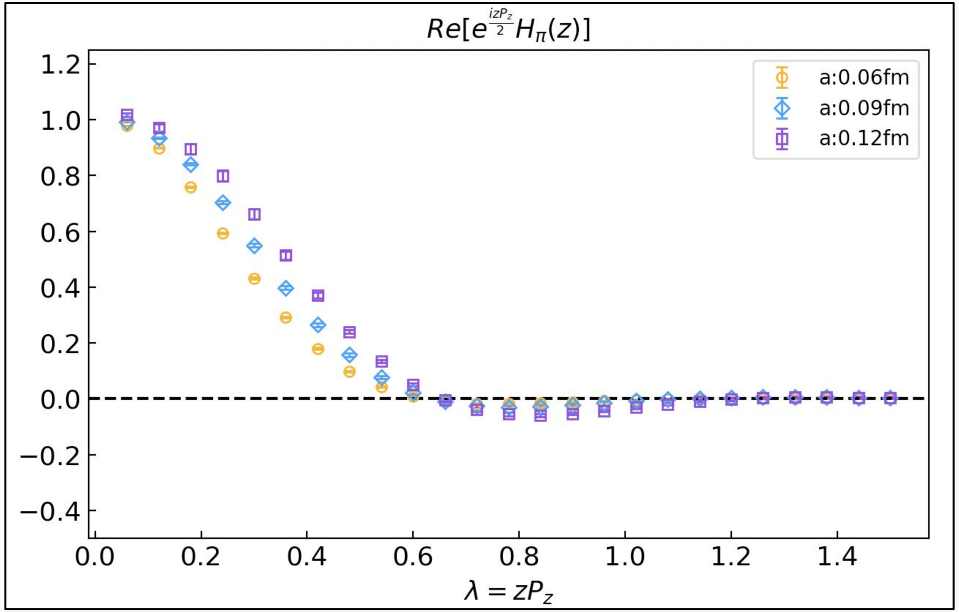
$$\ln(Z(z,a)_R) = \ln\mathcal{M}(z,a) - r(z)$$

Renormalized matrix element:

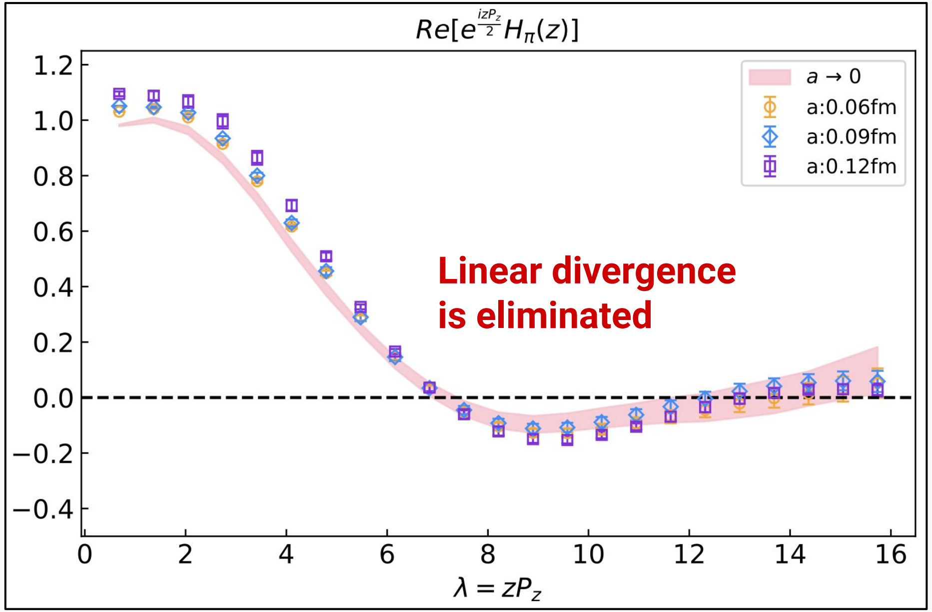
$$\boxed{H_m^R(z) = \frac{H_m^b(z)}{Z(z,a)_R \cdot Z_{\overline{\mathrm{MS}}}^{\gamma_z\gamma_5}\left(z,\mu,\Lambda_{\overline{\mathrm{MS}}}\right)}}$$

Self-renormalization: hybrid scheme

Bare Pion DA



Renormalized Pion DA

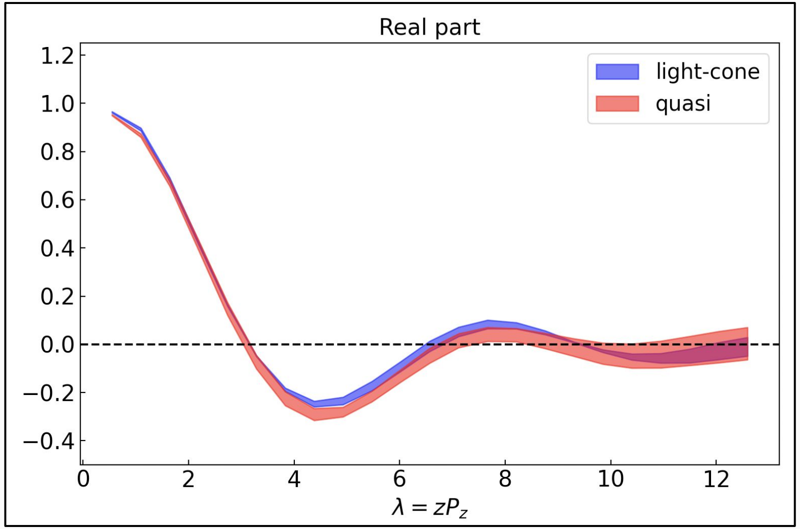


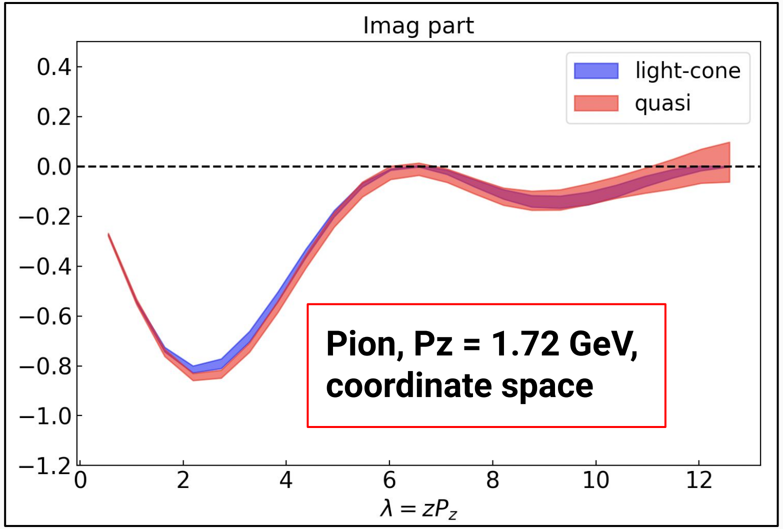
Inverse matching

Factorization $\boxed{\tilde{h}}(\lambda, z, \mu_R) = \int_0^\lambda d\lambda' \boxed{h}(\lambda', \mu)\, C(\lambda, \lambda', \mu) + \mathcal{O}\left(\frac{M^2}{z^2}, \frac{\Lambda_{\rm QCD}^2}{z^2}\right)$

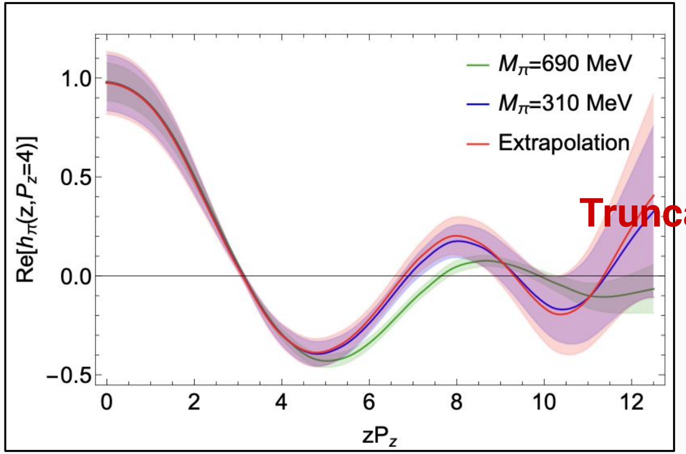
Kernel $C(\lambda, \lambda', \mu) = \delta(\lambda - \lambda') + C^{(1)}(\lambda, \lambda', \mu) + \mathcal{O}(\alpha_s^2)$

Inverse Matching $\boxed{h}(\lambda) \approx \boxed{\tilde{h}}(\lambda) - \int_0^\lambda d\lambda' \boxed{\tilde{h}}(\lambda')\, C^{(1)}$

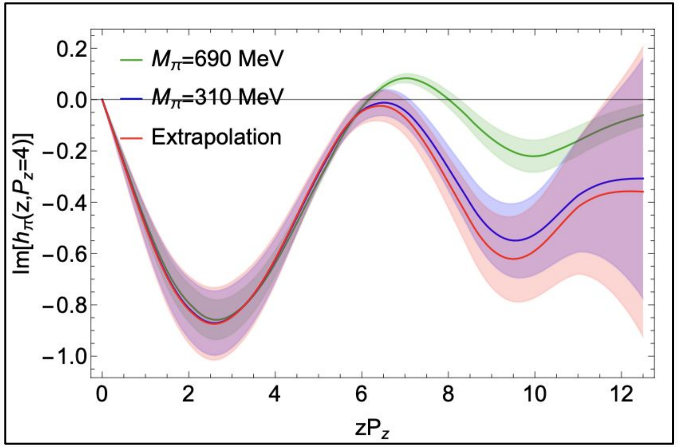




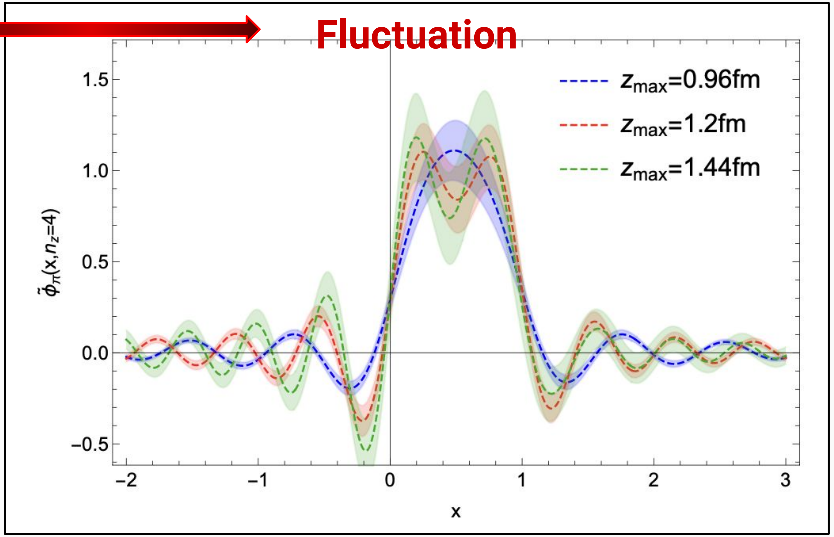
Why we need to extrapolate in the coordinate space?



Truncation



Fluctuation



[**R. Zhang et.al., Phys.Rev.D 102 (2020)**]

Extrapolation in the coordinate space

- LCDAs in momentum space

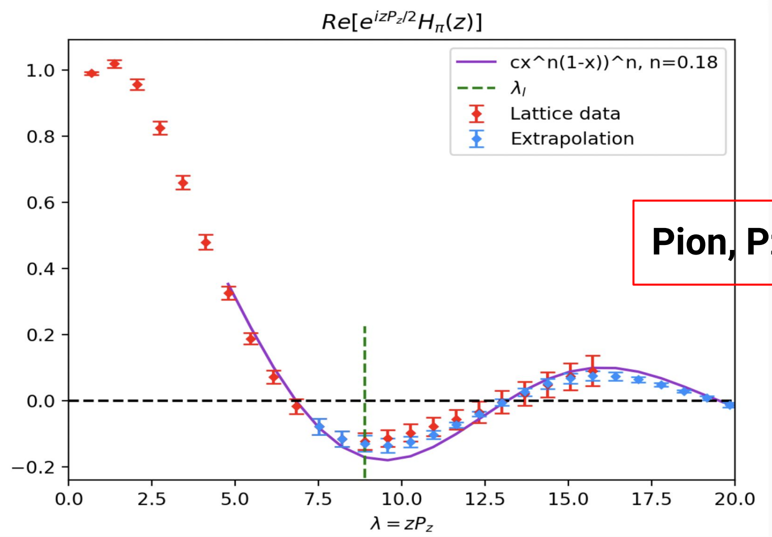
$$\Psi(x) \sim x^a(1-x)^b$$

- FT to the coordinate space

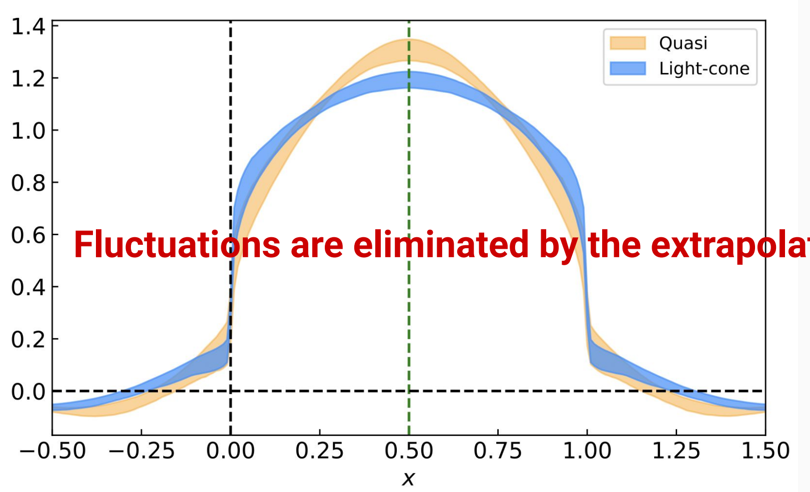
$$h(\lambda) = \int_0^1 dx\, e^{-ix\lambda} x^a (1-x)^b$$

- Extrapolation form

$$\tilde{H}(z, P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right]$$



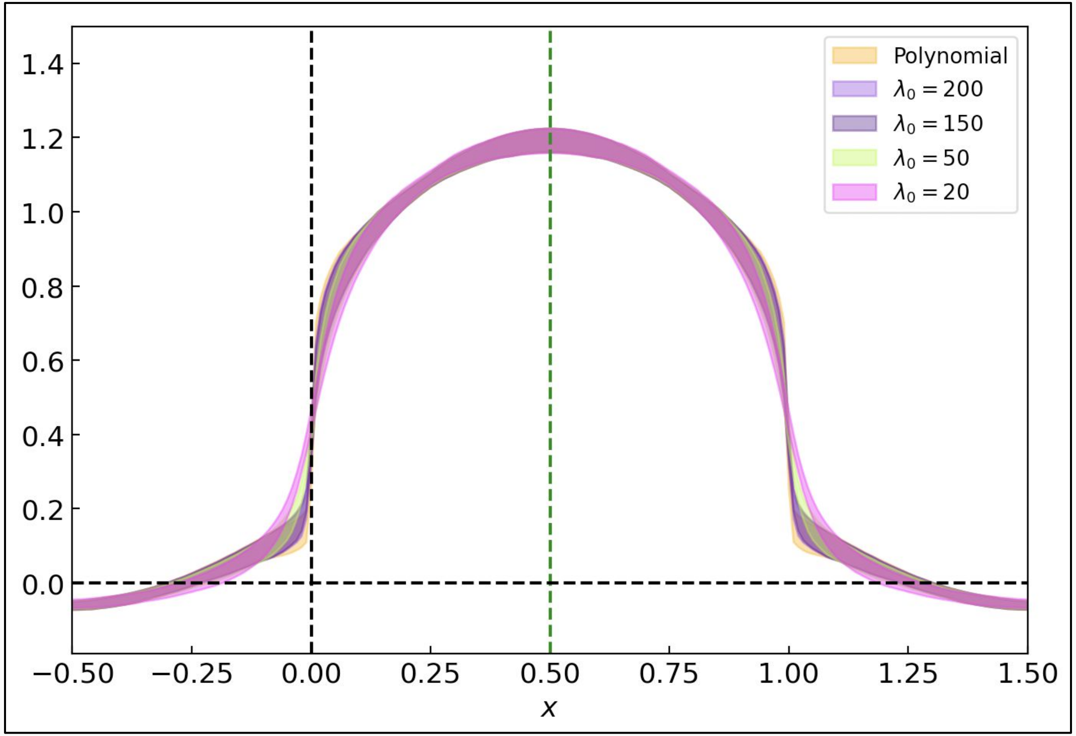
Pion, Pz = 2.15 GeV



Fluctuations are eliminated by the extrapolation

Extrapolate in the coordinate space

Extrapolate with different forms: the results of LCDAs are consistent.



Pion, Pz = 2.15 GeV, LCDAs in mom space

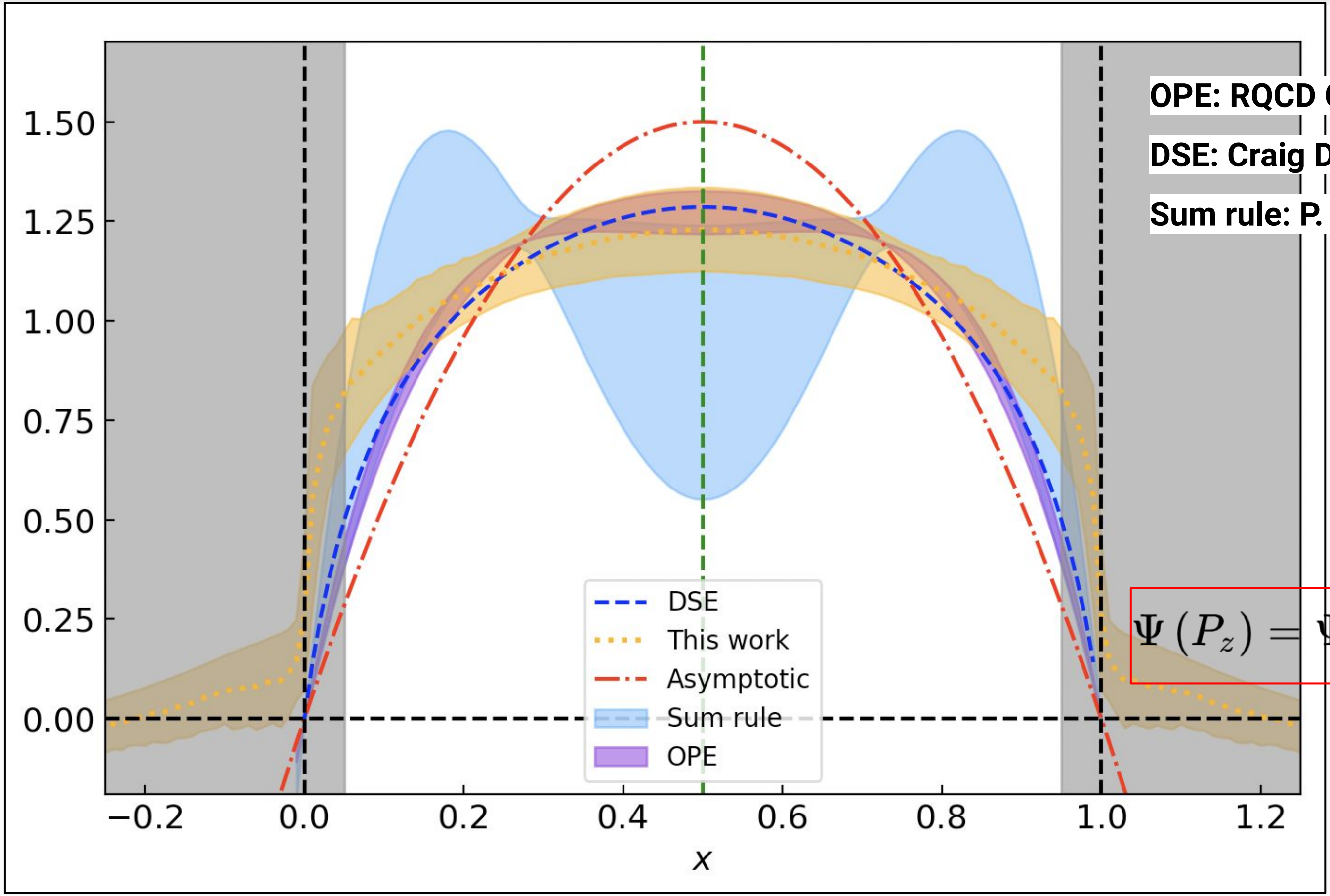
Polynomial form

$$\tilde{H}(z, P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right]$$

Exponential form

$$\tilde{H}(z, P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right] e^{-\frac{\lambda}{\lambda_0}}$$

Final results of Pion LCDAs



OPE: RQCD Collaboration, JHEP 11 (2020)

DSE: Craig D.et.al., Prog.Part.Nucl.Phys. (2021)

Sum rule: P. Ball et. al., JHEP08 (2007)

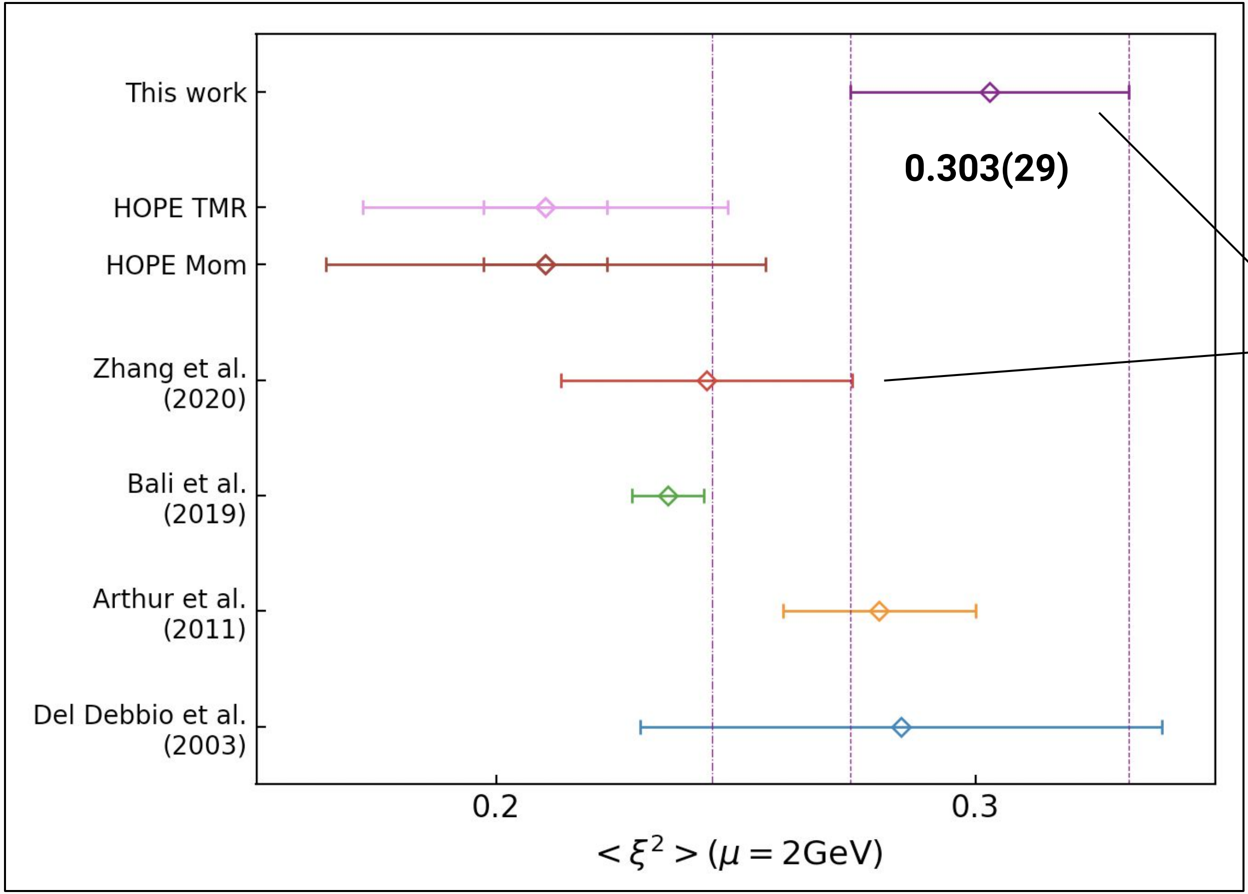
Pz = 1.29 GeV

Pz = 1.72 GeV

Pz = 2.15 GeV

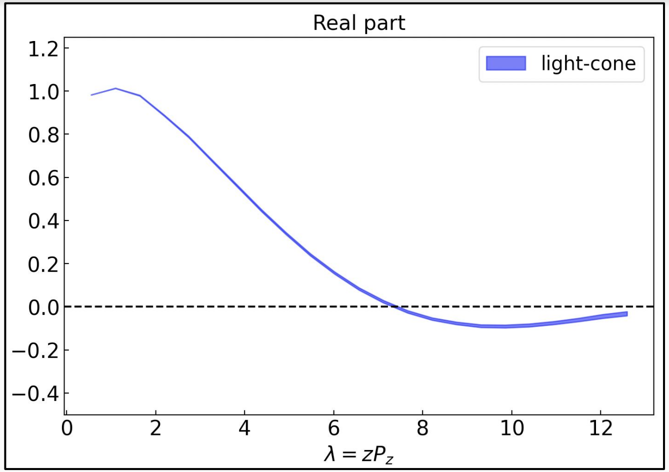
$$\Psi\left(P_z\right) = \Psi\left(P_z \to \infty\right) + \frac{c_2}{P_z^2} + O\left(\frac{1}{P_z^4}\right)$$

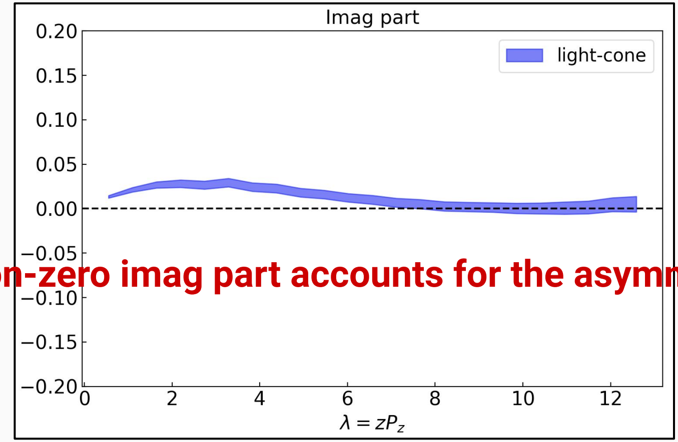
Mellin moment $< \xi^2 >$



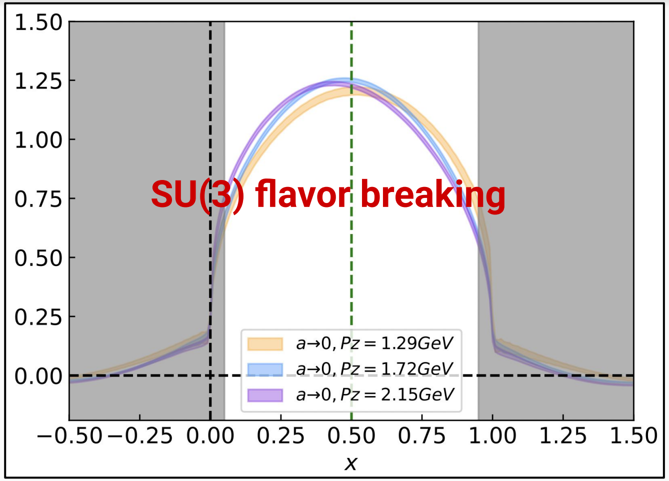
- **Renormalization method**
- **Extrapolation in the coordinate space**

Final results of Kaon LCDAs

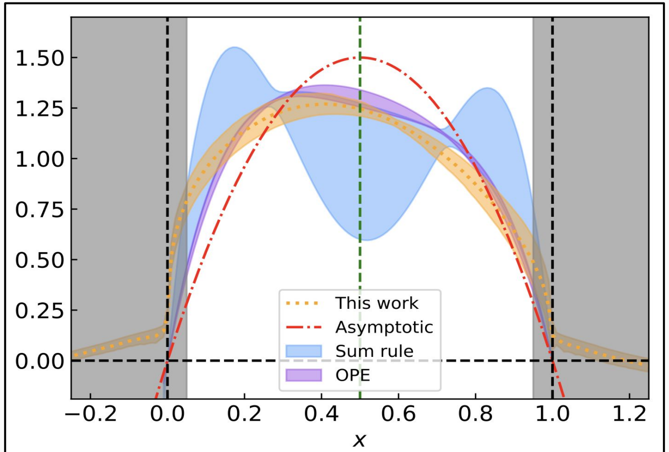




Non-zero imag part accounts for the asymmetry



SU(3) flavor breaking



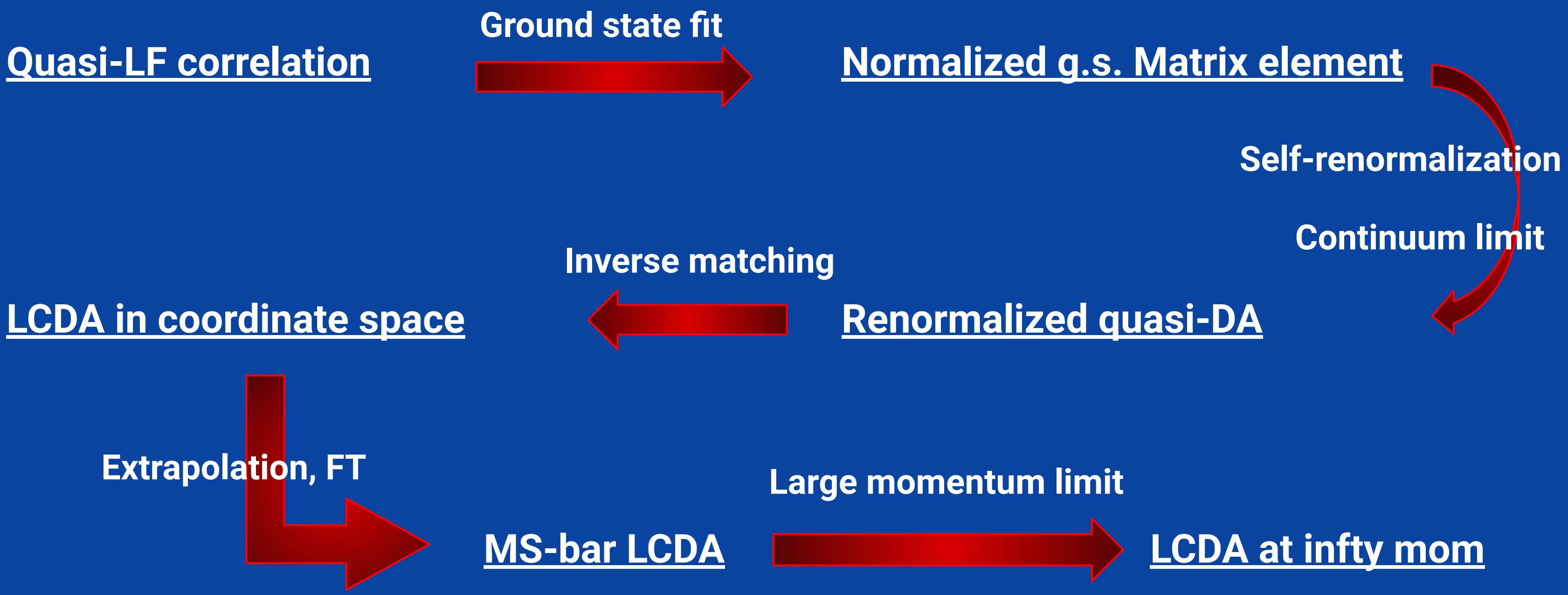
Summary

- **LCDAs are essential inputs for exclusive processes**
- **We calculated LCDAs on lattice at physical mass with LaMET:**
  - **Hybrid scheme based on self renormalization is firstly adopted**
  - **Extrapolation in the coordinate space to avoid fluctuation in the momentum space**
- **For further study:**
  - **High order, Higher twist**
  - **...**

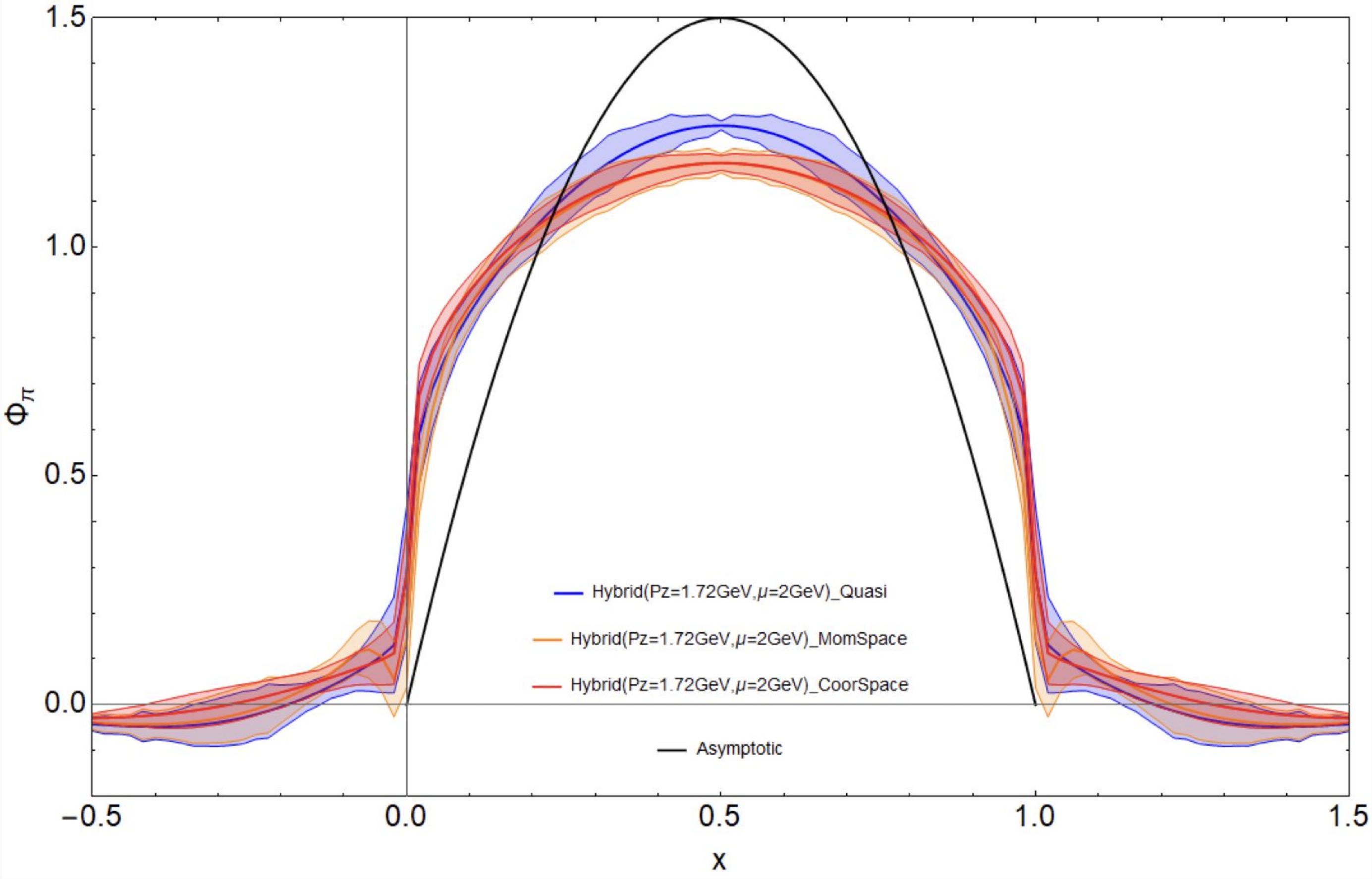
Thanks for your attention!

Supplement

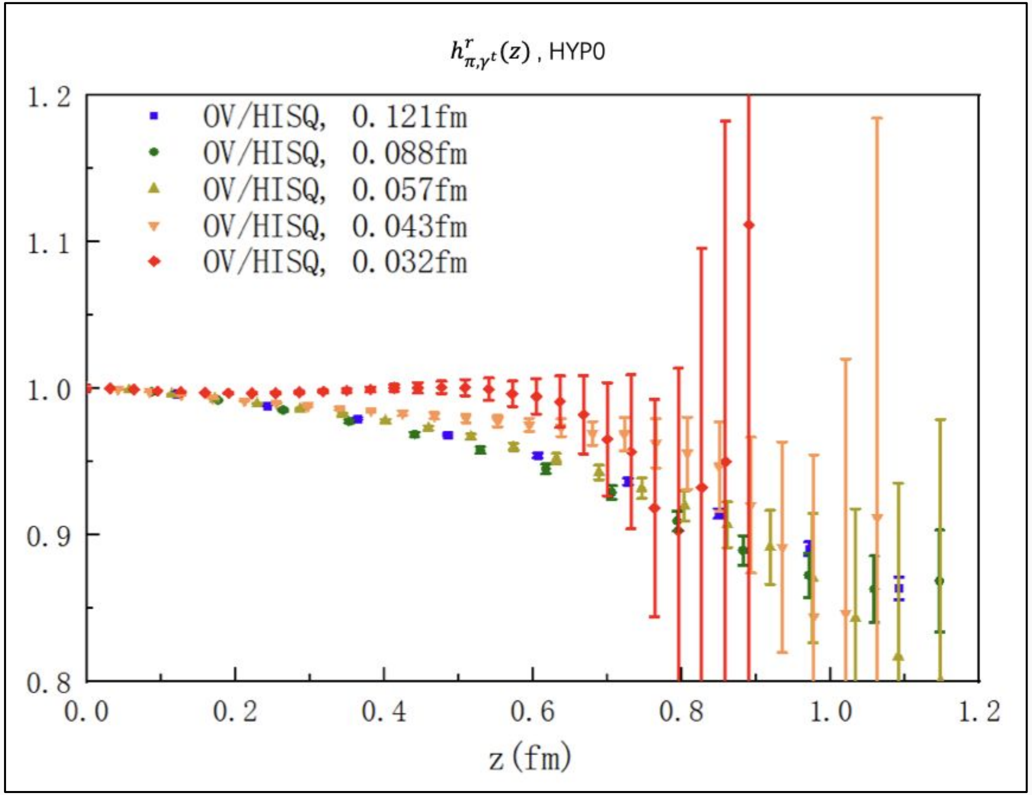
Pion LCDA in LaMET



Matching in the coordinate space v.s. matching in the momentum space



RI/MOM: residual linear divergence

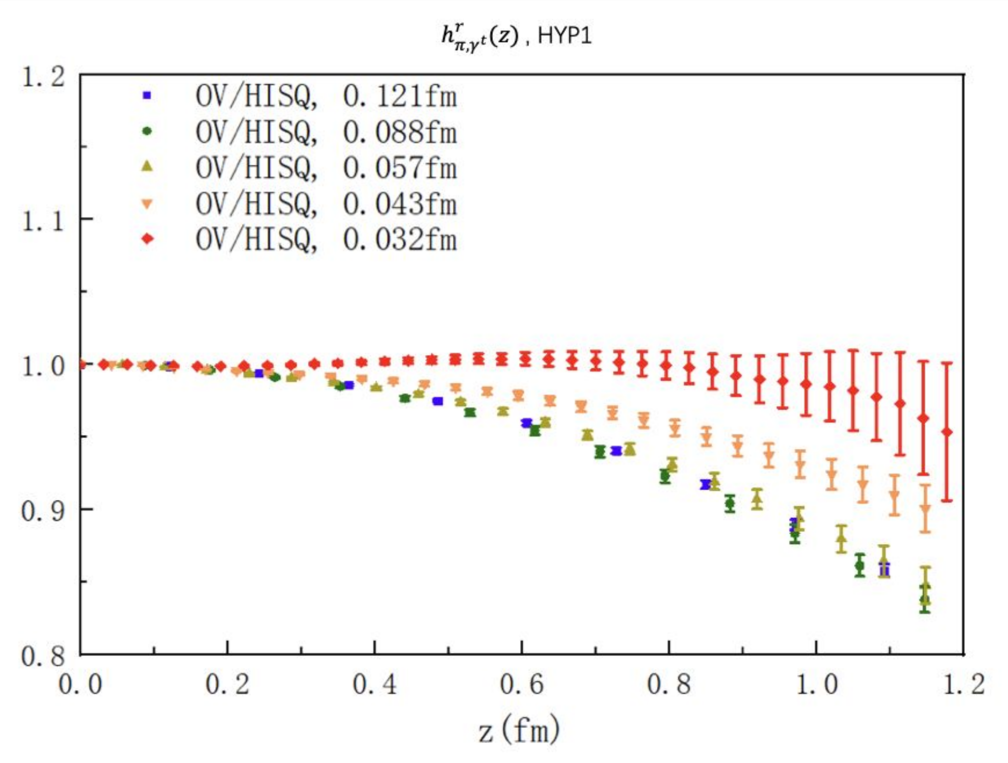


RI/MOM cannot eliminate all linear divergence

χQCD Collaboration • Kuan Zhang et al.
DOI: 10.1103/PhysRevD.104.074501

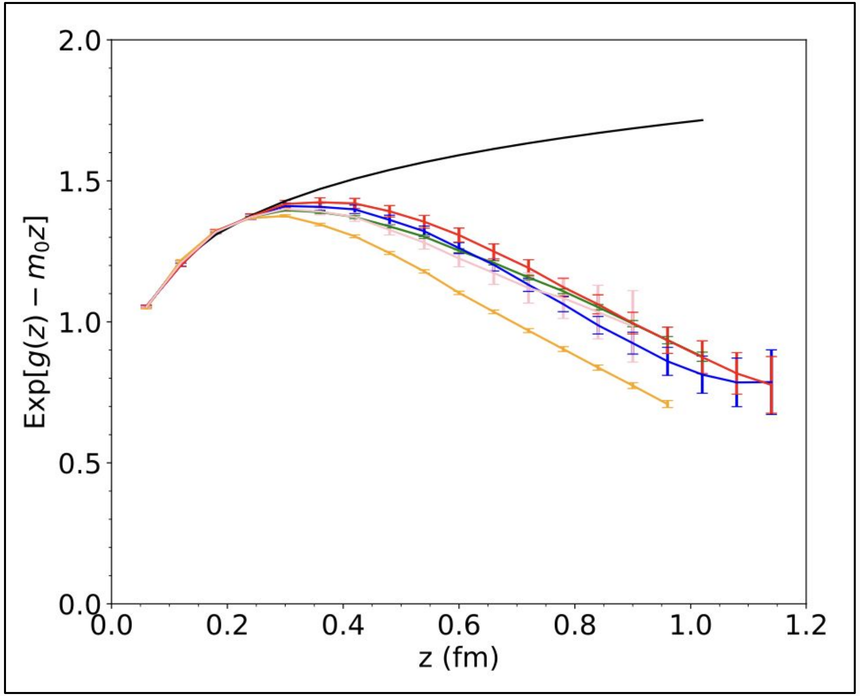
RI/MOM: residual linear divergences

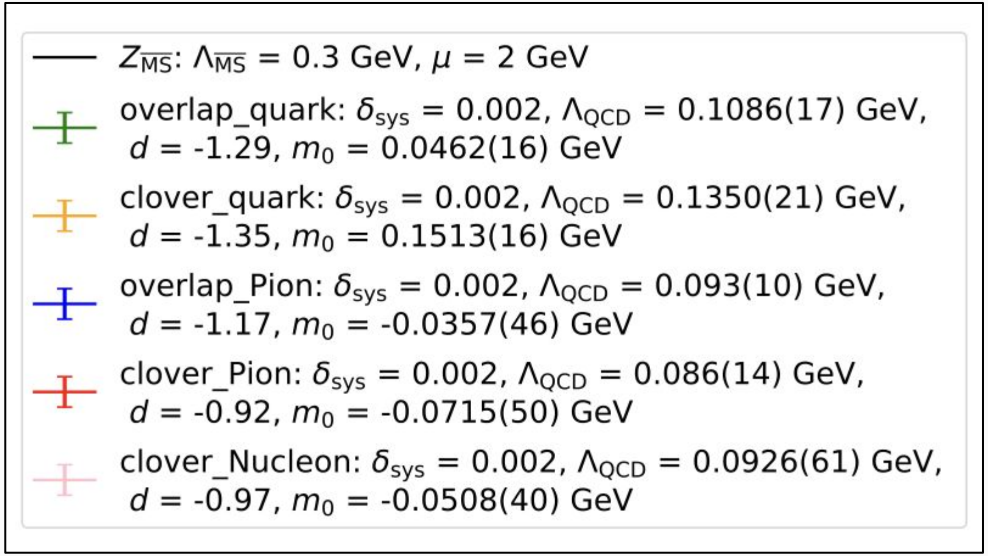
After RI/MOM renormalization, there are residual linear divergences, which make the continuum extrapolation misleading or even problematic.



χQCD Collaboration • Kuan Zhang et al.
DOI: 10.1103/PhysRevD.104.074501

RI/MOM renormalization: non-perturbative effects





Quark and hadron have different non-perturbative structure at large z

Yi-Kai Huo, Yushan Su et al. Nuclear Physics B (2021): 115443

$$\Psi\left(P_z\right) = \Psi\left(P_z \to \infty\right) + \frac{c_2}{P_z^2} + O\left(\frac{1}{P_z^4}\right)$$

Large momentum limit: Why start from $\frac{1}{P_z^2}$

$$\tilde{\Psi}' = C * \Psi = \tilde{\Psi} + O(\frac{1}{P_z^2})$$

$$C^{-1} * \tilde{\Psi} = C^{-1} * (\tilde{\Psi}' - O(\frac{1}{P_z^2})) = \Psi - C^{-1} * O(\frac{1}{P_z^2})$$

Strategy of g.s. Fit: two-state fit cannot extract e.s. successfully

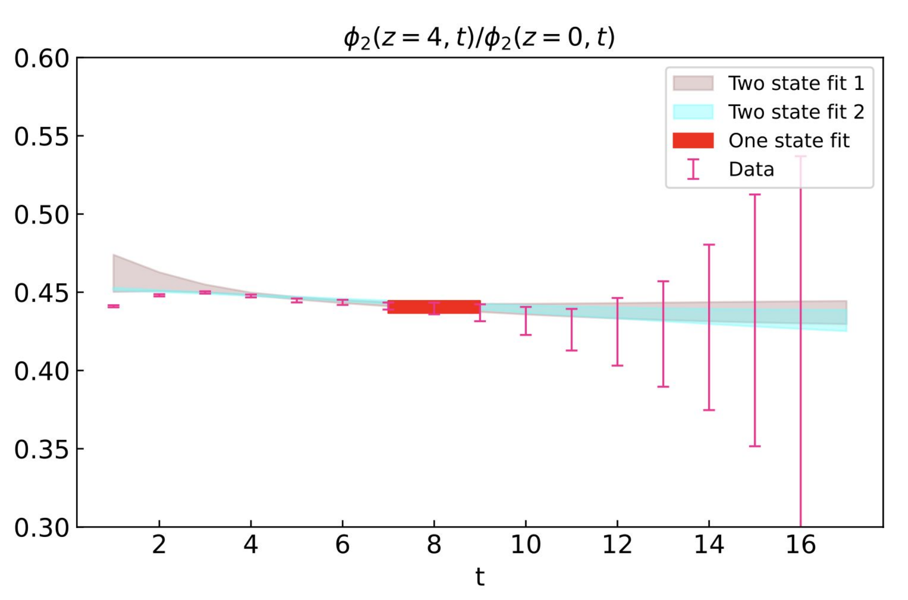
Two-state fit

1.72GeV >> 0.14 GeV

$$\frac{C_2^m(z,\vec{P},t)}{C_2^m(z=0,\vec{P},t)} = \frac{\boxed{H_m^b(z)}(1+c_m(z)e^{-\Delta E t})}{(1+c_m(0)e^{-\Delta E t})}$$

One-state fit

$$\frac{C_2^m(z,\vec{P},t)}{C_2^m(z=0,\vec{P},t)} = \boxed{H_m^b(z)}$$




```
z=4
Least Square Fit:
  chi2/dof [dof] = 0.083 [26]    Q = 1    logGBF = 226.45

Parameters:
             c0    3.5(1.0)e-06      [  0.0 (1.0) ]
             c1        0.20 (34)     [      0 (10) ]
             c2     1e-19 +- 10      [      0 (10) ]
             E0       0.471 (30)     [      0 (10) ]
           cr_1        0.28 (34)     [  0.0 (5.0) ]
           ci_1        0.30 (33)     [  0.0 (5.0) ]
        deltaE1        0.27 (25)     [  0.0 (5.0) ]
        phi2_re      0.4366 (89)     [      0 (10) ]
        phi2_im      -0.843 (17)     [      0 (10) ]
```

The fit results of coefficients of e.s. cover zero within error