Valence parton distribution of pion from lattice QCD at physical point

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LaMET 2021 Online, Dec 7 – 9, 2021

Pion valence quark PDF

Pion play a central role in the study of the strong interactions.

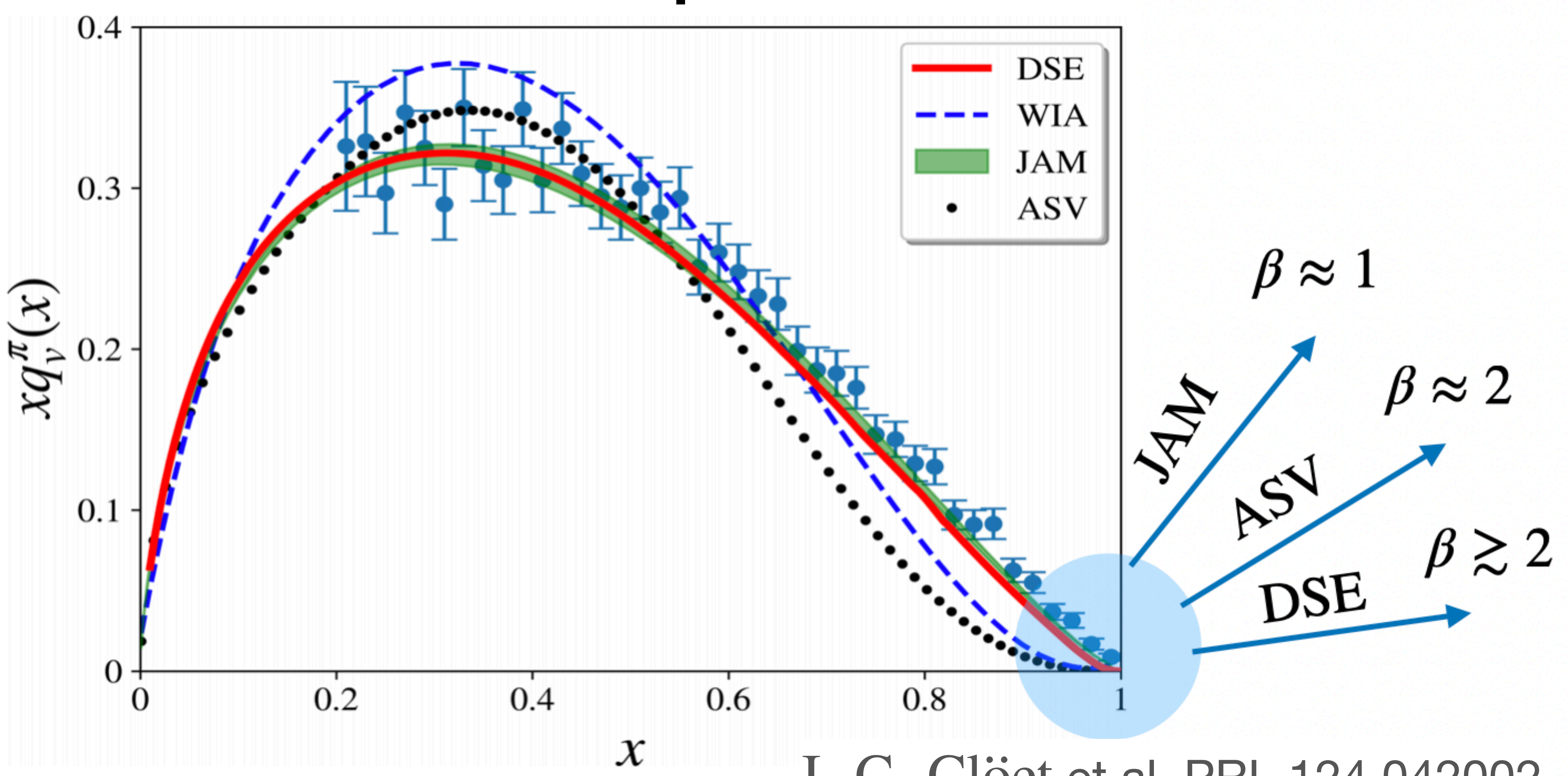
$$m_\pi \approx 140 \text{ MeV} \xrightarrow[\mathrm{m_q=0}]{\text{chiral limit}} 0$$

- Critical ingredient for understanding the dynamical chiral symmetry breaking in QCD.
- Quarks and gluons in massless NG bosons.
- …

However, the absence of fixed pion targets has made it difficult to determine the pion's structure experimentally. One of the key physics issue is x=1 behavior:

$$\lim_{x\to 1} f_v^\pi(x) \sim (1-x)^\beta$$

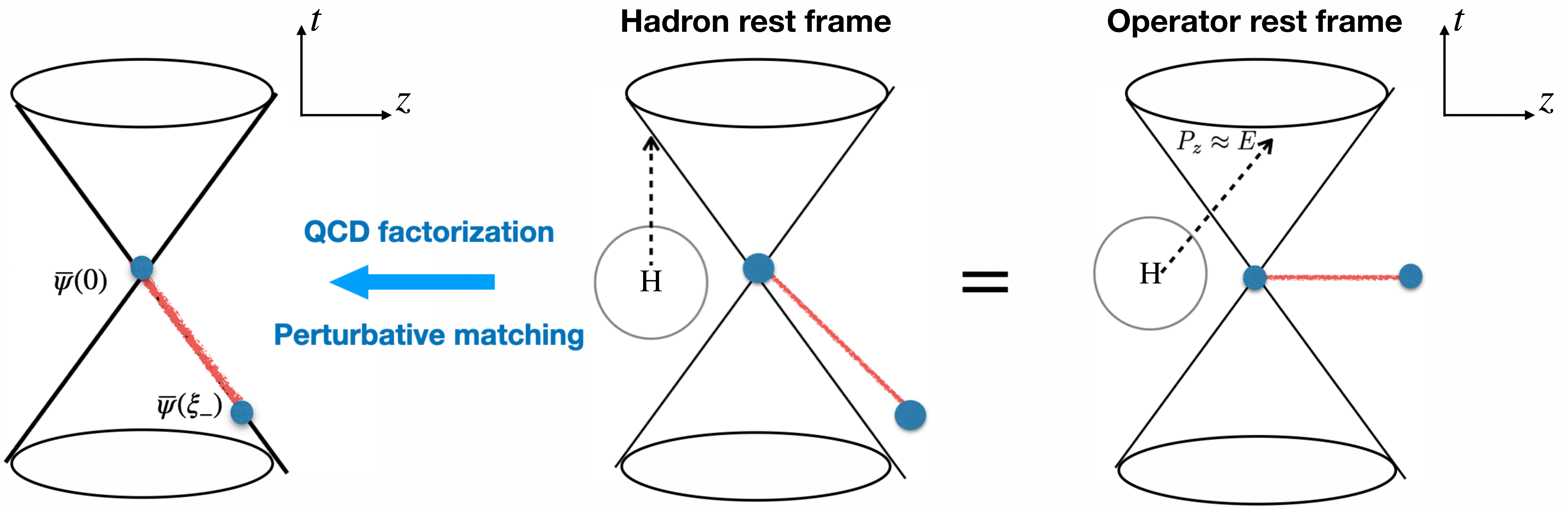
Pion valence quark PDF



I. C. Clöet et al. PRL.124.042002

Equal-time correlators and qPDF factorization

$z + ct = 0, \qquad z - ct \neq 0$ $\qquad\qquad$ $t = 0, \qquad z \neq 0$



Conventional PDF definition:

$$q(x) \equiv \int \frac{d\xi_{-}}{4\pi} e^{-ixP^{+}\xi_{-}} \langle P | O_{\gamma^\mu}(\xi_{-}, \epsilon) | P \rangle,$$

$$O_\Gamma(\xi_{-}, \epsilon) = \overline{\psi}(0)\Gamma W_{-}(0,\xi_{-})\psi(\xi_{-})$$

$$\xi_{-} = z - ct$$

Quasi-PDF:

$$\tilde{q}(x) \equiv \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \tilde{O}_{\gamma^\mu}(z, \epsilon) | P \rangle,$$

$$\tilde{O}_\Gamma(z, \epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Ji, Y. Zhao, et al, arXiv:2004.03543

Equal-time correlators and qPDF factorization

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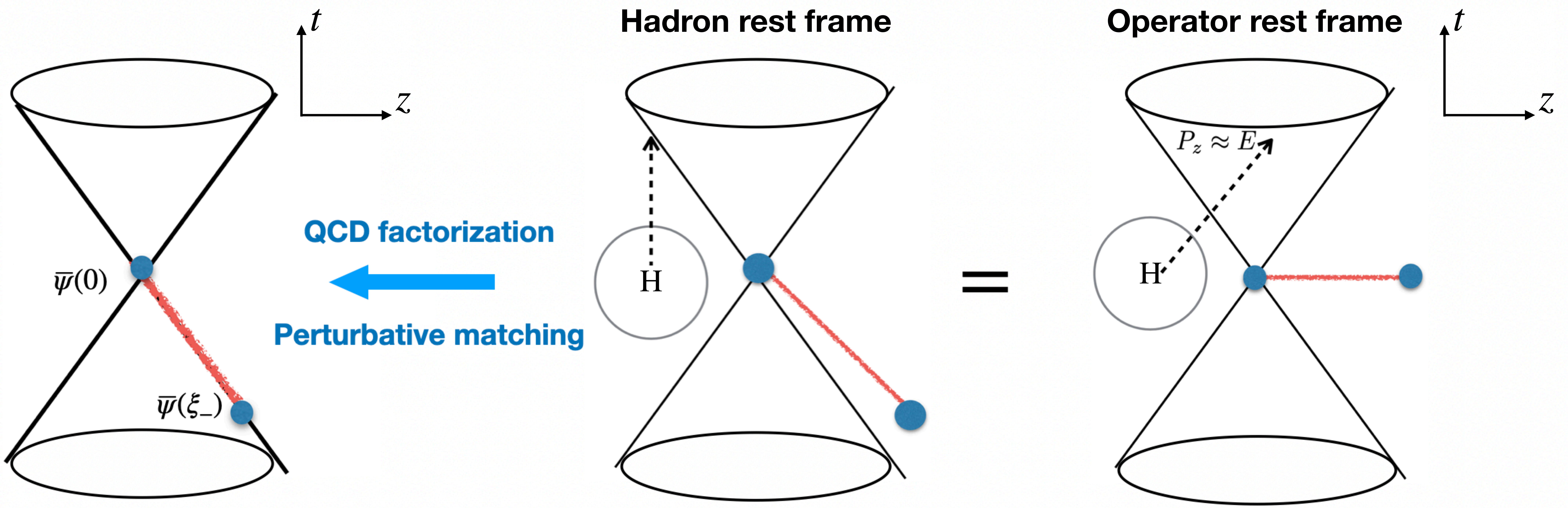


See Yong Zhao's talk for more details.

PDF

$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{m_h^2}{x^2P_z^2}, \frac{\Lambda_{QCD}^2}{x^2P_z^2}\right)$$

qPDF

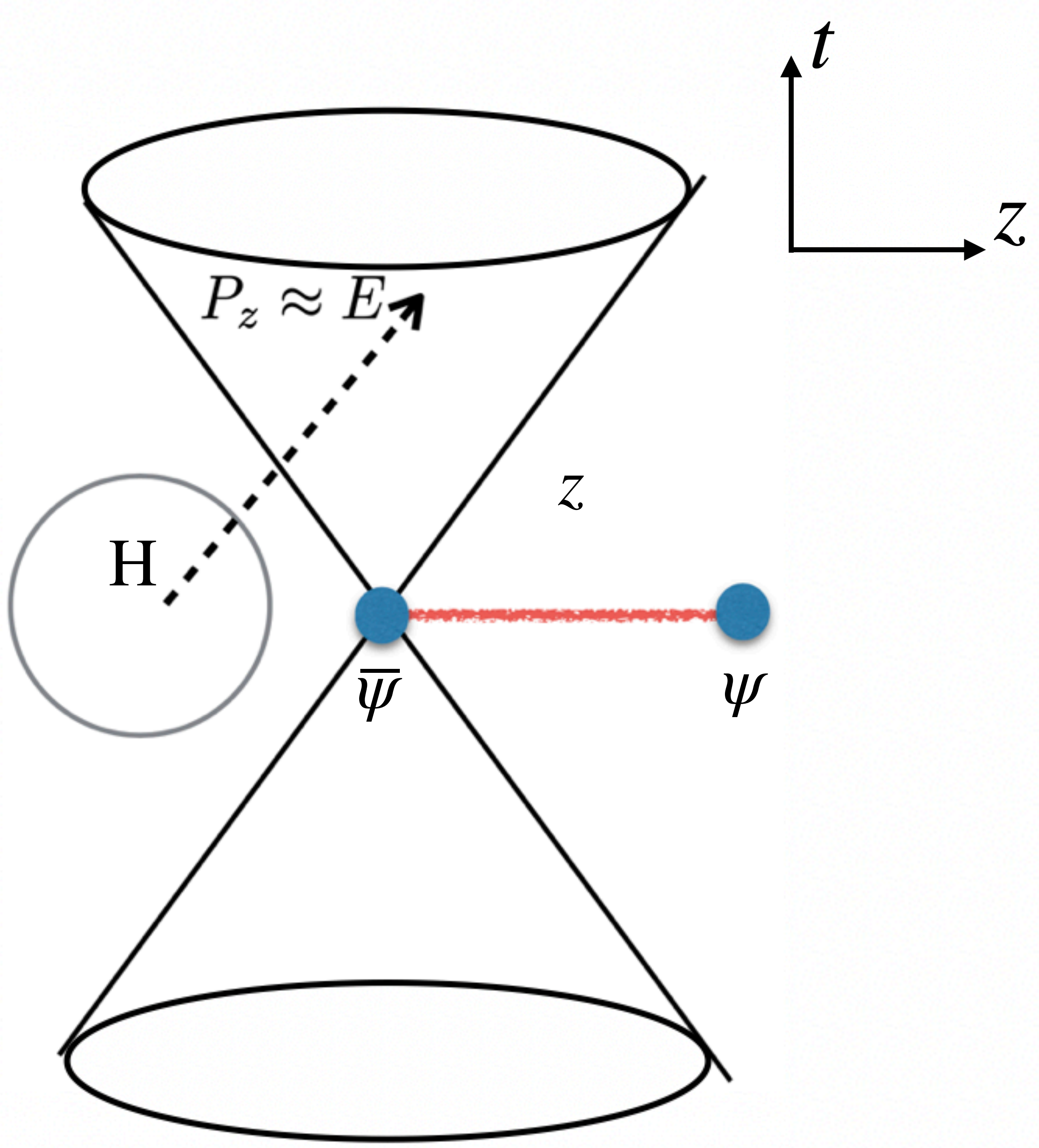
Perturbative kernel

Large momentum xP_z is essential

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Ji, Y. Zhao, et al, arXiv:2004.03543

Equal-time correlators and SDF

$t = 0, \qquad z \neq 0$



$\langle P | \tilde{O}_\Gamma(z,\epsilon) | P \rangle$

$\tilde{O}_\Gamma(z,\epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$

Short distance Factorization in coordinate space:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

Ioffe time $\zeta = zP_z$

$$\tilde{Q}(\zeta, z^2\mu^2) = \langle P | \tilde{O}_{\gamma^\mu}(z,\mu) | P \rangle / (2P^\mu)$$

$$= \int_{-1}^{1} d\alpha\, \mathscr{C}(\alpha, \mu^2 z^2) Q(\alpha\zeta, \mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

Perturbative kernel

Ioffe-time distribution (ITD)

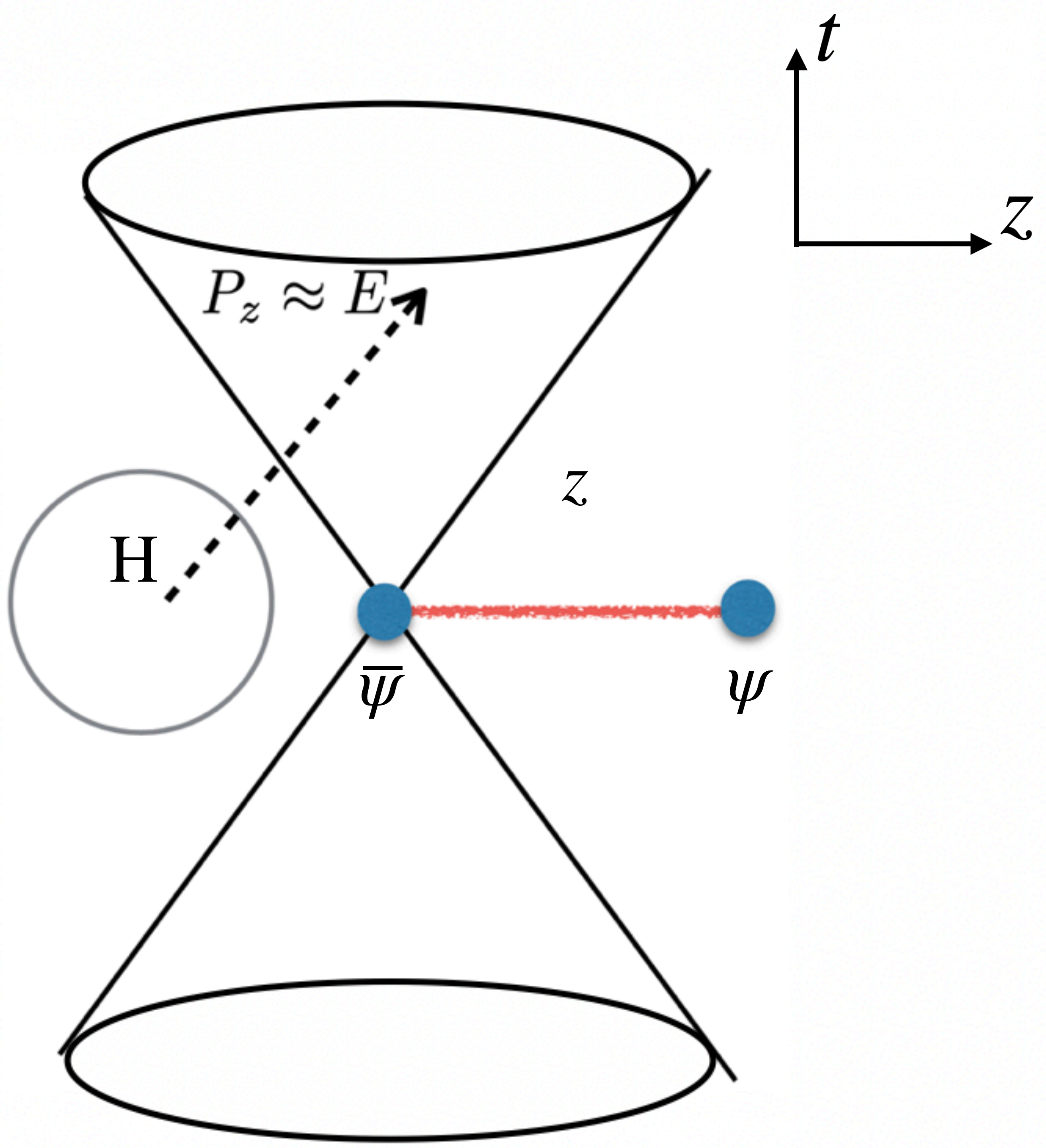
$$Q(\zeta, \mu) = \int_{-1}^{1} dy e^{-iy\zeta} q(y,\mu)$$

- The lattice data in coordinate space can be directly related to the F.T. of PDFs.
- The perturbative matching is valid in short range of z.
- The information that lattice data contains is limited by the range of finite $\zeta = zP_z$.

Small z and large P_z are essential

Equal-time correlators and SDF

$t = 0, \qquad z \neq 0$



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$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) Q(\alpha\zeta, \mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

What we can get from finite $\zeta = zP_z$:

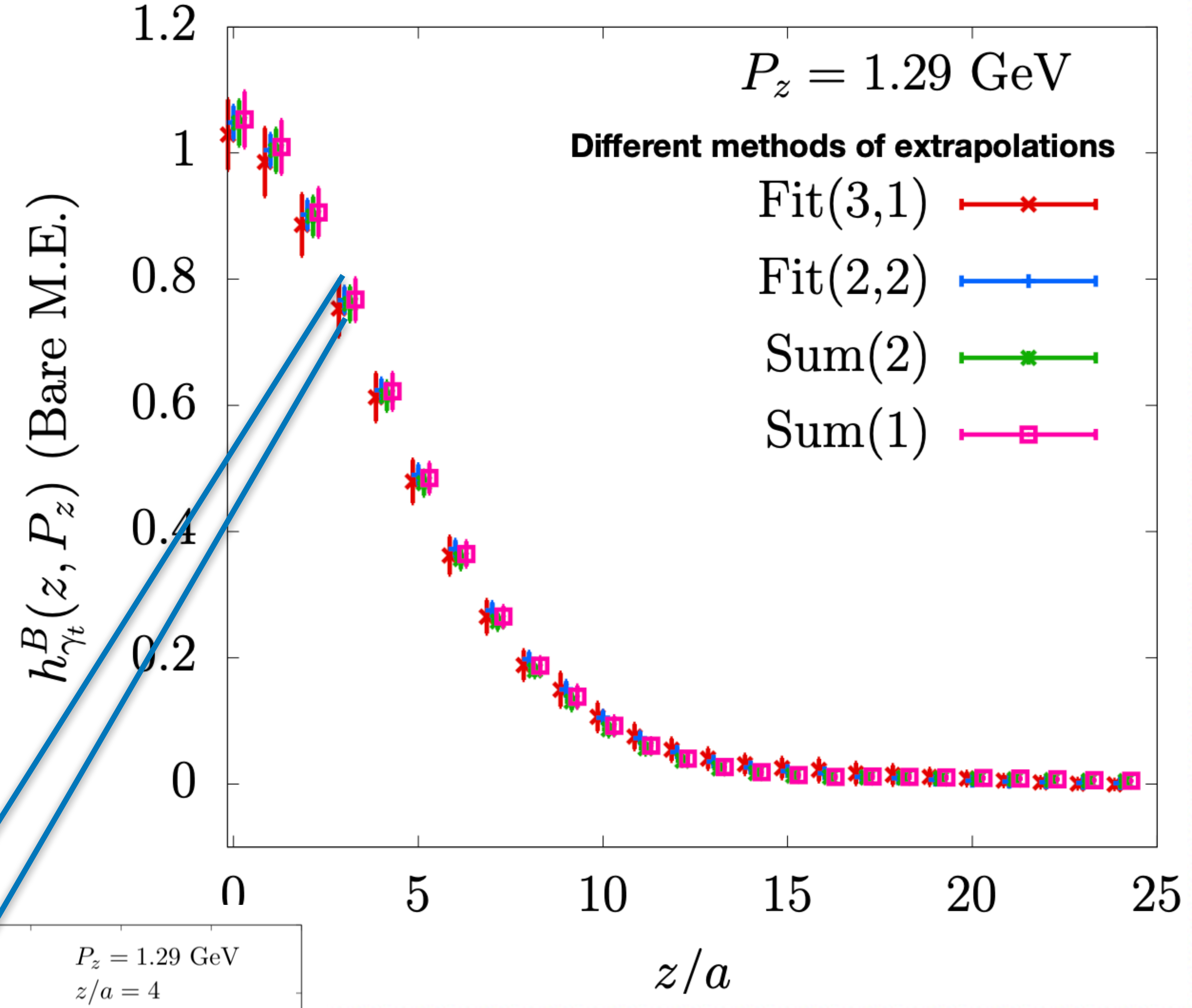
- Extracted $Q(\zeta, \mu)$ up to the largest $\zeta = zP_z$ achieved on lattice by solving the inverse problem above.
- Extract the moments of PDFs model independently.

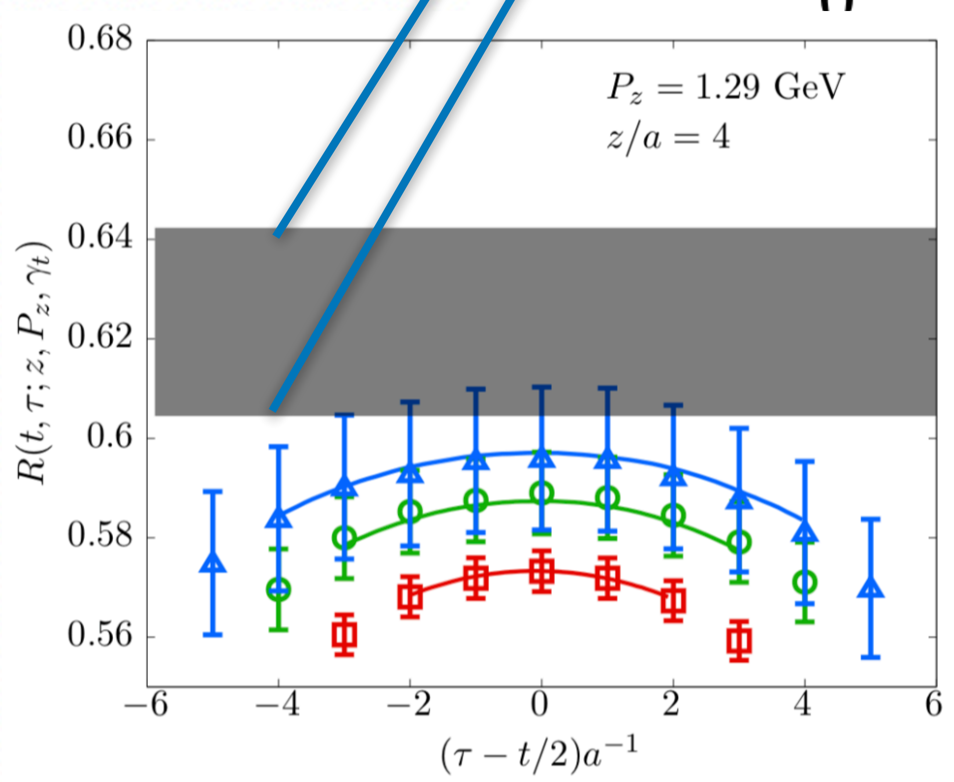
$$\tilde{Q}(\zeta, z^2\mu^2) = \sum_n \underbrace{C_n(\mu^2 z^2)}_{\text{Wilson coefficients}} \frac{(-i\zeta)^n}{n!} \underbrace{\int_{-1}^{1} dy y^n q(y, \mu)}_{\text{Moments of PDF}} + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

- Reconstruct the PDFs using certain models.

Bare matrix elements and Renormalization

Bare matrix elements of boosted pion state





The operator can be multiplicatively renormalized

- X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
- J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m(a)|z|} Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

- **Ratio scheme renormalization**
  - A. V. Radyushkin et al., PRD 96 (2017)
  - BNL, PRD 102 (2020)

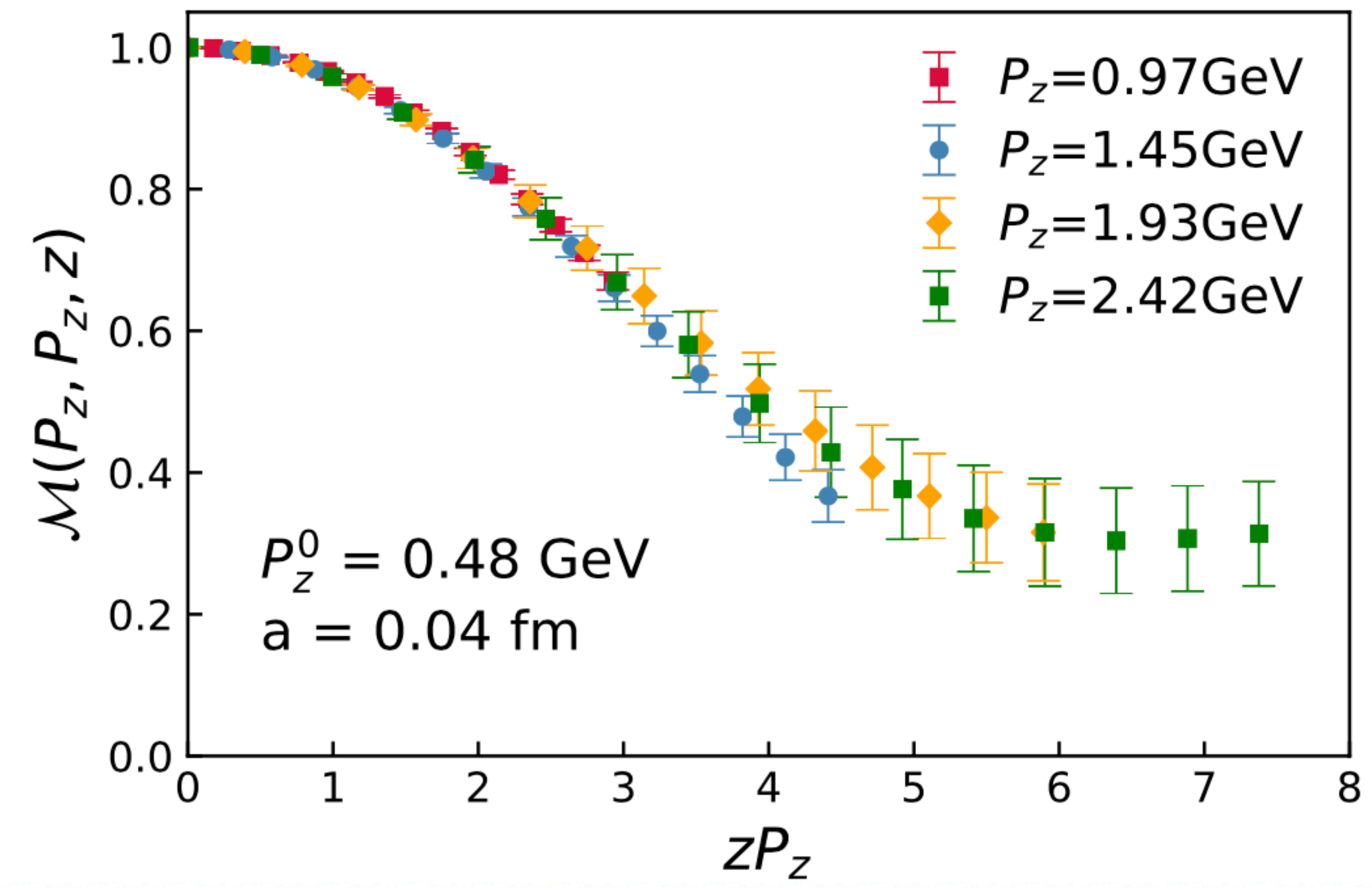
$$h_B(z, P_z, a) = \langle P_z | [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B | P_z \rangle$$

$$\frac{h_B(z, P_z, a)}{h_B(z, P_z^0, a)} = \frac{\tilde{Q}(z, P_z, \mu)}{\tilde{Q}(z, P_z^0, \mu)}$$

$$= \frac{\sum_n C_n(\mu^2 z^2)\frac{(-izP_z)^n}{n!}\int_{-1}^{1} dy y^n q(y,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)}{\sum_n C_n(\mu^2 z^2)\frac{(-izP_z^0)^n}{n!}\int_{-1}^{1} dy y^n q(y,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)}$$

Pion valence quark PDF: $Q(\zeta, \mu)$

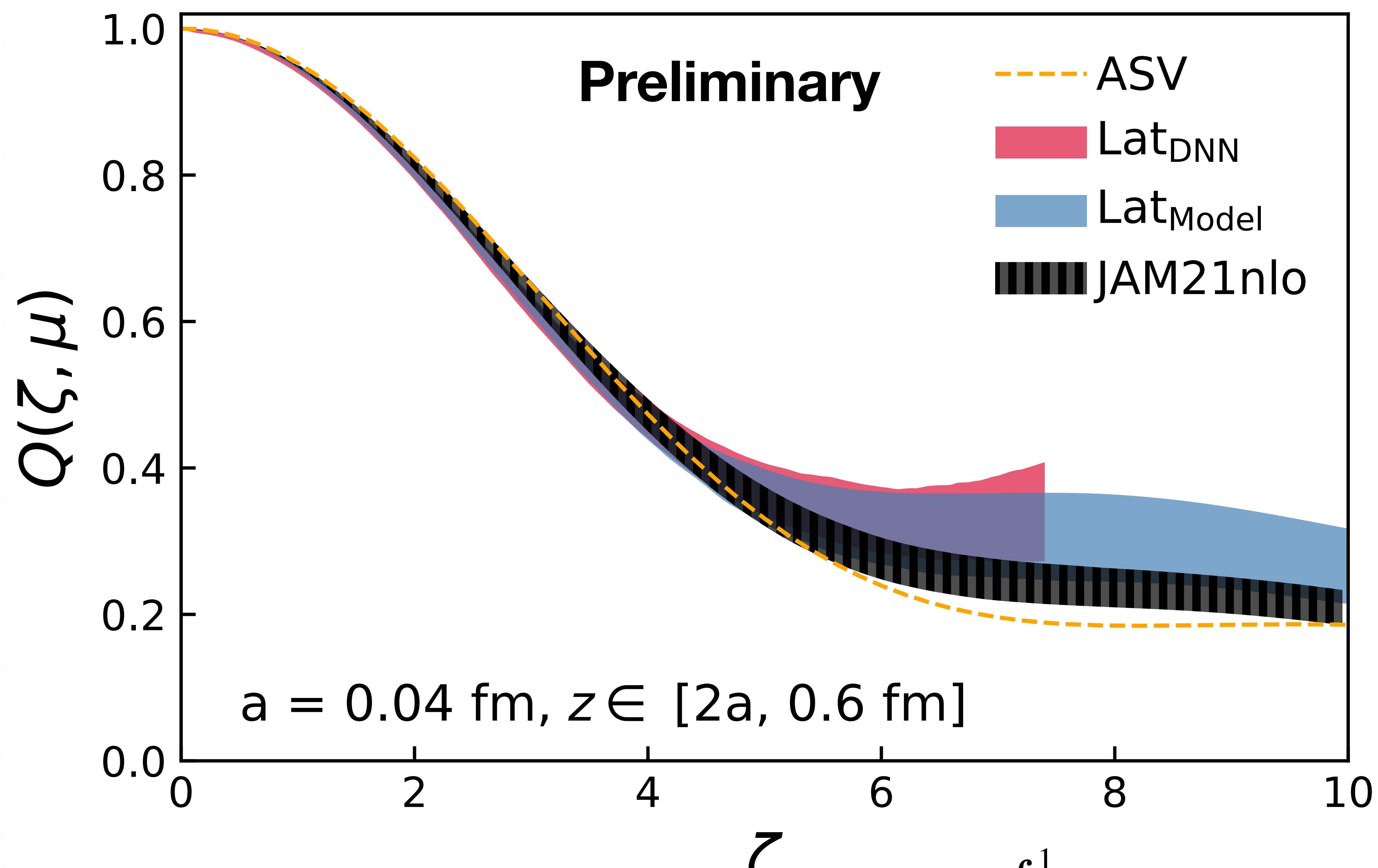
Ratio scheme renormalized matrix elements



$$\tilde{Q}(\zeta, z^2\mu^2) = \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) Q(\alpha\zeta, \mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

- $Q(\zeta, \mu)$ can be reconstructed by solving the inverse problem above.
- DNN (Deep Neural Network) is probably the best tool achieve this in an unbiased fashion.

- **Extract the Ioffe-time distribution (ITD) $Q(\zeta, \mu)$.**

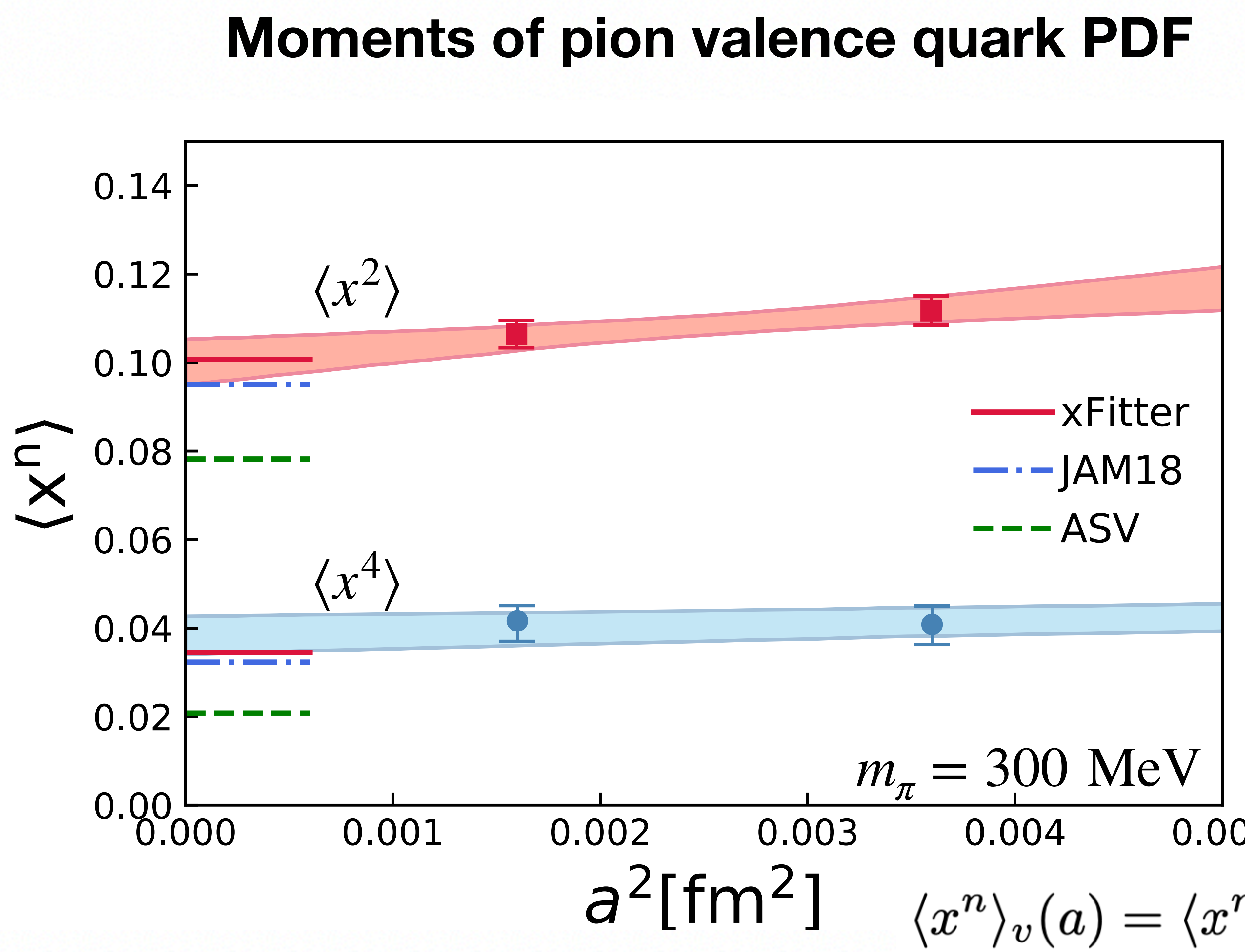


$$Q(\zeta, \mu) = \int_{-1}^{1} dy e^{-iy\zeta} q(y, \mu)$$

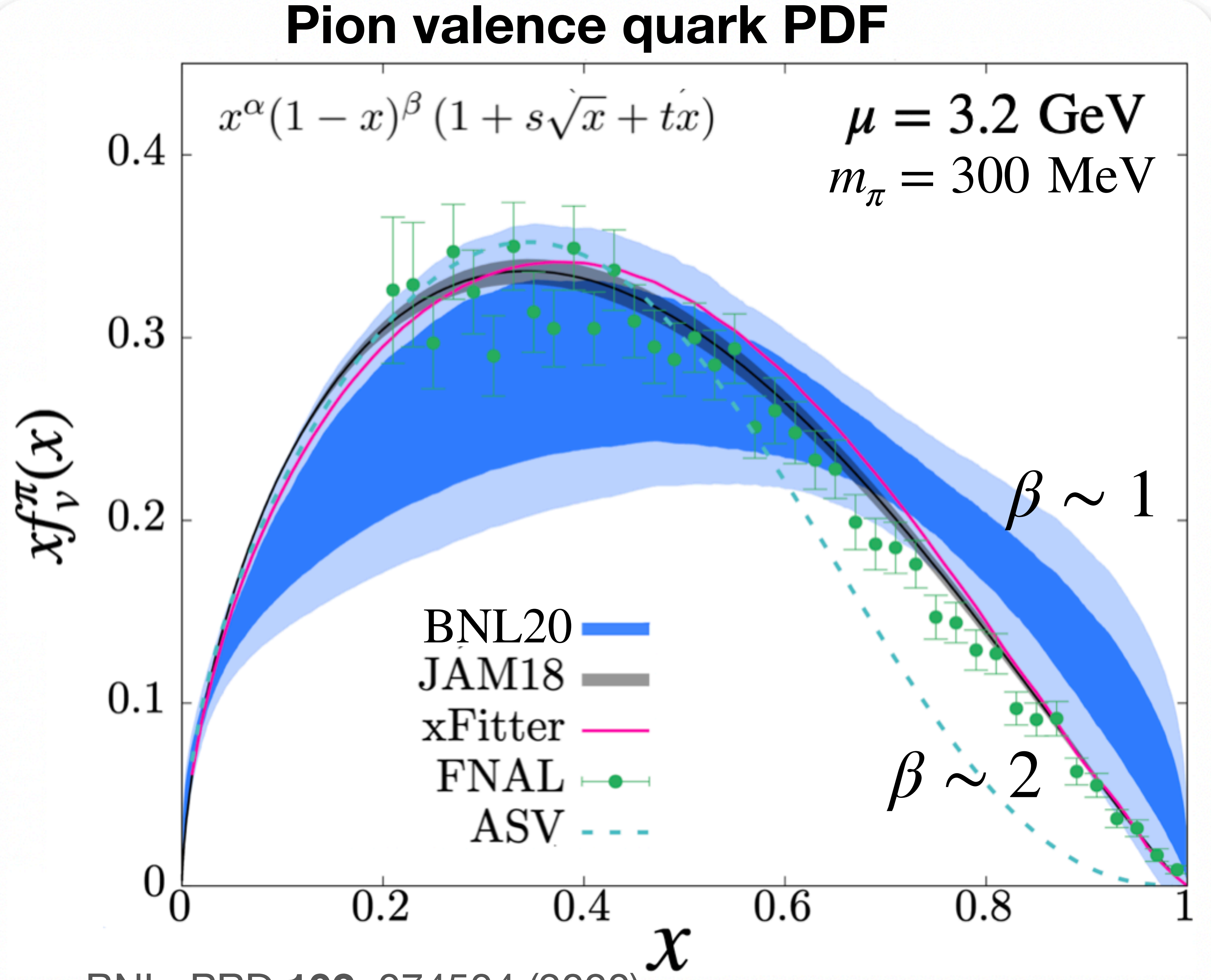
$$\zeta = zP_z$$

- $\mathrm{Lat}_{\mathrm{DNN}}$: DNN represented $Q(\zeta, \mu)$.
- $\mathrm{Lat}_{\mathrm{Model}}$: PDF model $q(x) = Ax^{\alpha}(1-x)^{\beta}$ reconstructed $Q(\zeta, \mu)$.

Pion valence quark PDF: NLO results



$$\langle x^n \rangle_v(a) = \langle x^n \rangle_v + d_n a^2$$



BNL, PRD **102**, 074504 (2020)

$$\tilde{Q}(\zeta, z^2\mu^2) = \sum_n C_n(\mu^2 z^2)\frac{(-i\zeta)^n}{n!}\int_{-1}^{1} dy y^n q(y,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

Improvement:

- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

Pion valence quark PDF: NNLO matching

Improvement:

- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

- **NNLO matching**

  - Li, Ma and Qiu, PRL 126 (2021)

$$C_n(z^2\mu^2) = 1 + \alpha_s(\mu)C_n^{(1)}(z^2\mu^2) + \alpha_s^2(\mu)C_n^{(2)}(z^2\mu^2) + \mathcal{O}(\alpha_s^3)$$

$$= 1 + \frac{\alpha_s(\mu)C_F}{2\pi}[(\frac{3+2n}{2+3n+n^2} + 2H_n)\ln(z_0^2\mu^2) + \ldots] + \ldots$$

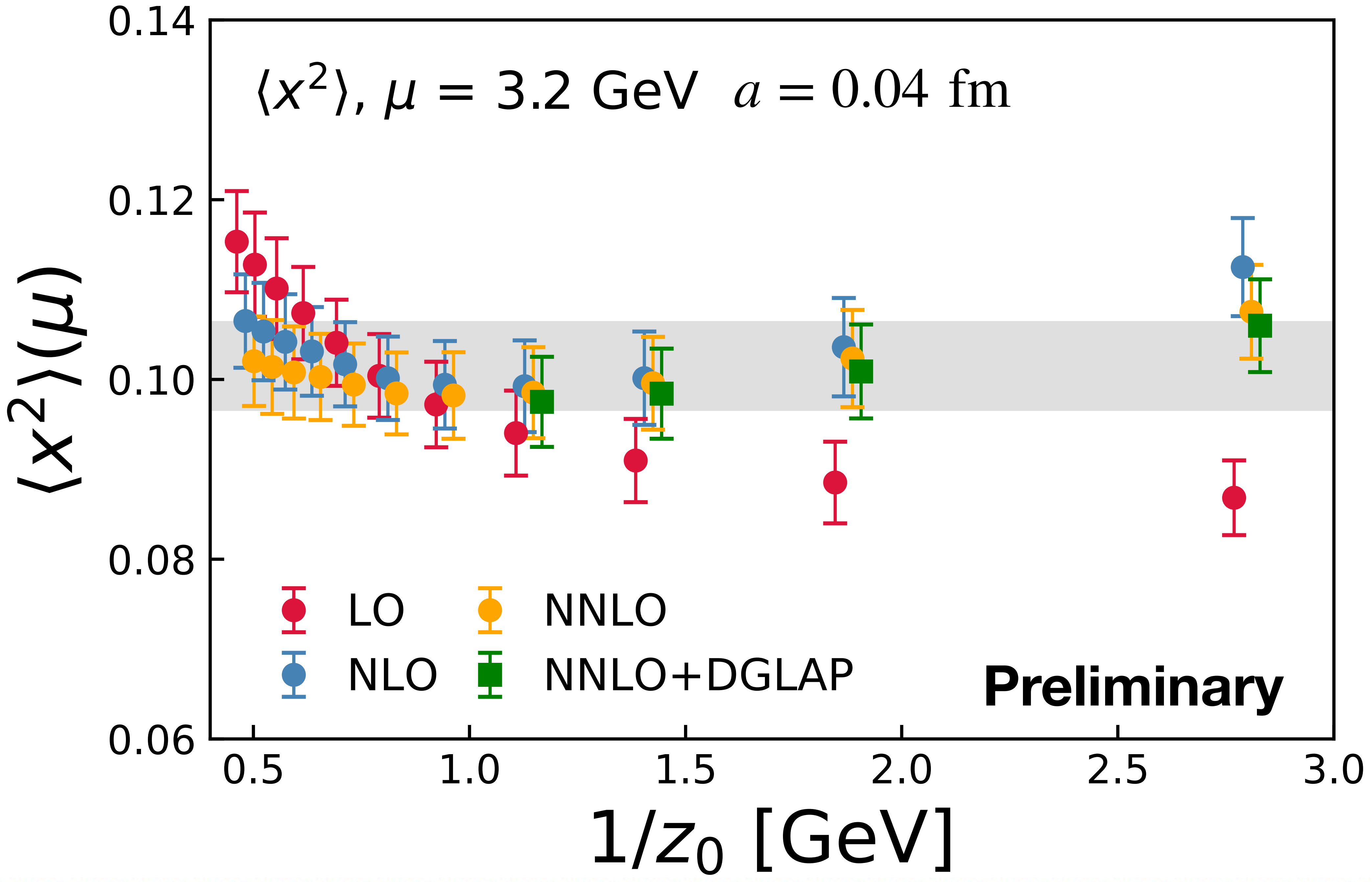
$$z_0^2 = z^2 e^{2\gamma_E}/4$$

When $\ln(z_0^2\mu^2)$ become large, one may need to include the DGLAP evolution:

$$\left[\frac{\partial}{\partial \ln \mu^2} + \beta(\alpha_s(\mu))\frac{\partial}{\partial \alpha_s} - \gamma_n\right] C_n^{evo} = 0$$

- A. V. Radyushkin, PLB 781 (2018)
- BNL, ANL, arXiv: 2102.01101

Moments extracted at fixed z by varying P_z



- Clear z_0 dependence can be observed at **LO**.
- Moments evolved from $1/z_0$ to μ from **NNLO** are consistent with **NLO** with current statistics but more flat, and agree with the **DGLAP** improved case.

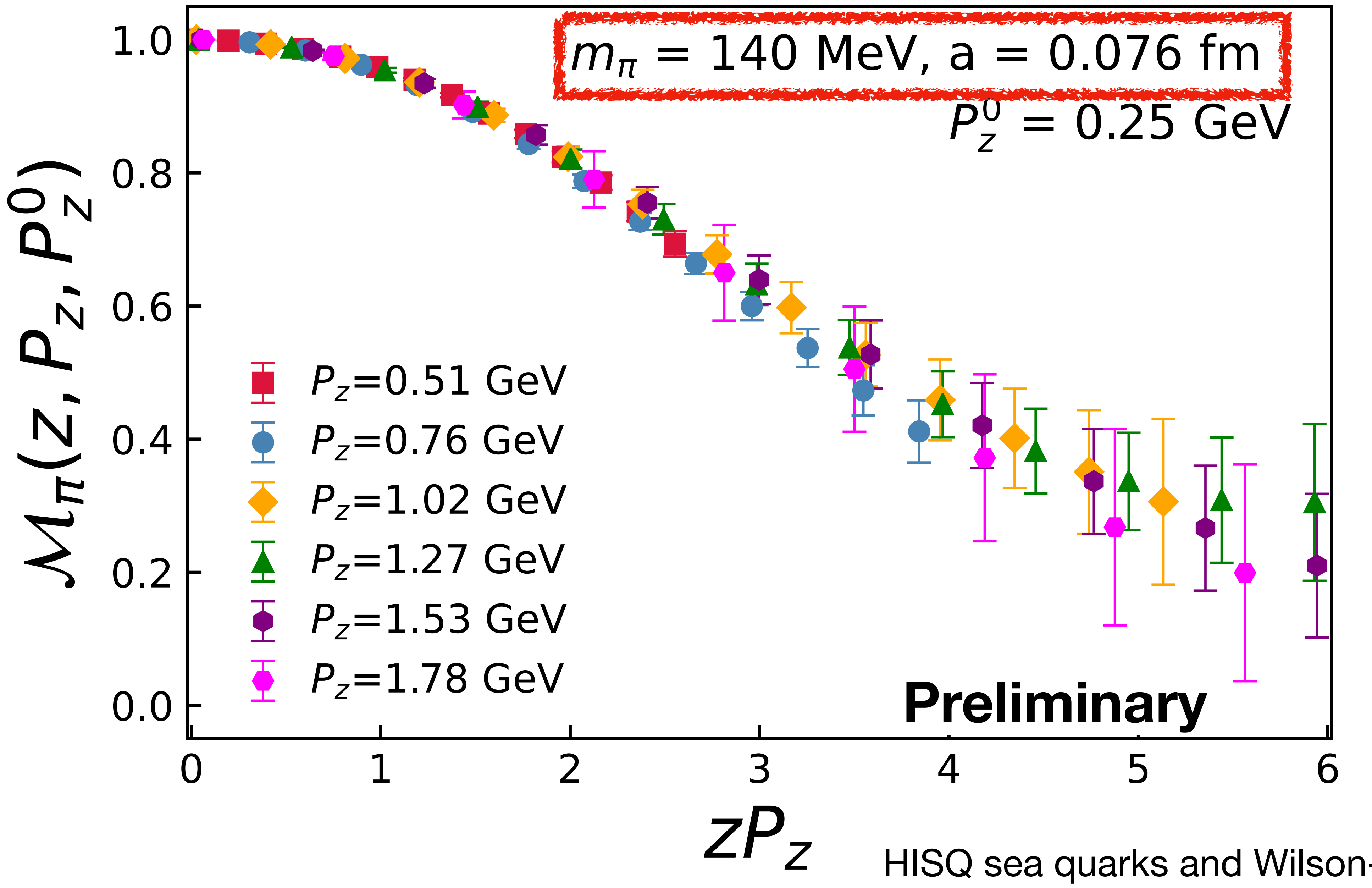
Pion valence quark PDF: Physical point

Improvement:

- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

- **Physical pion mass**

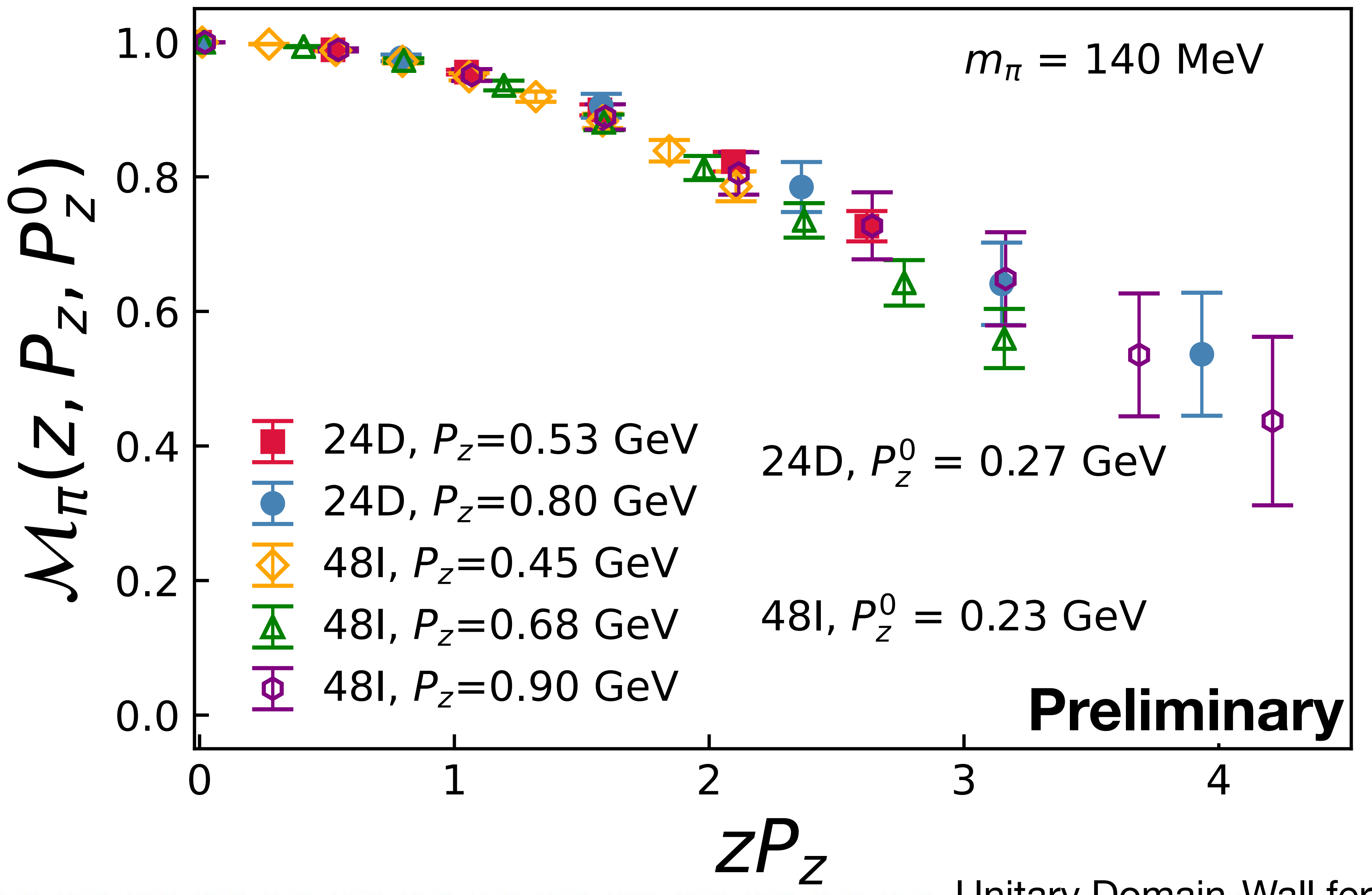
Ratio scheme renormalized matrix elements



HISQ sea quarks and Wilson-Clover valence quarks.

- **Chiral fermion**

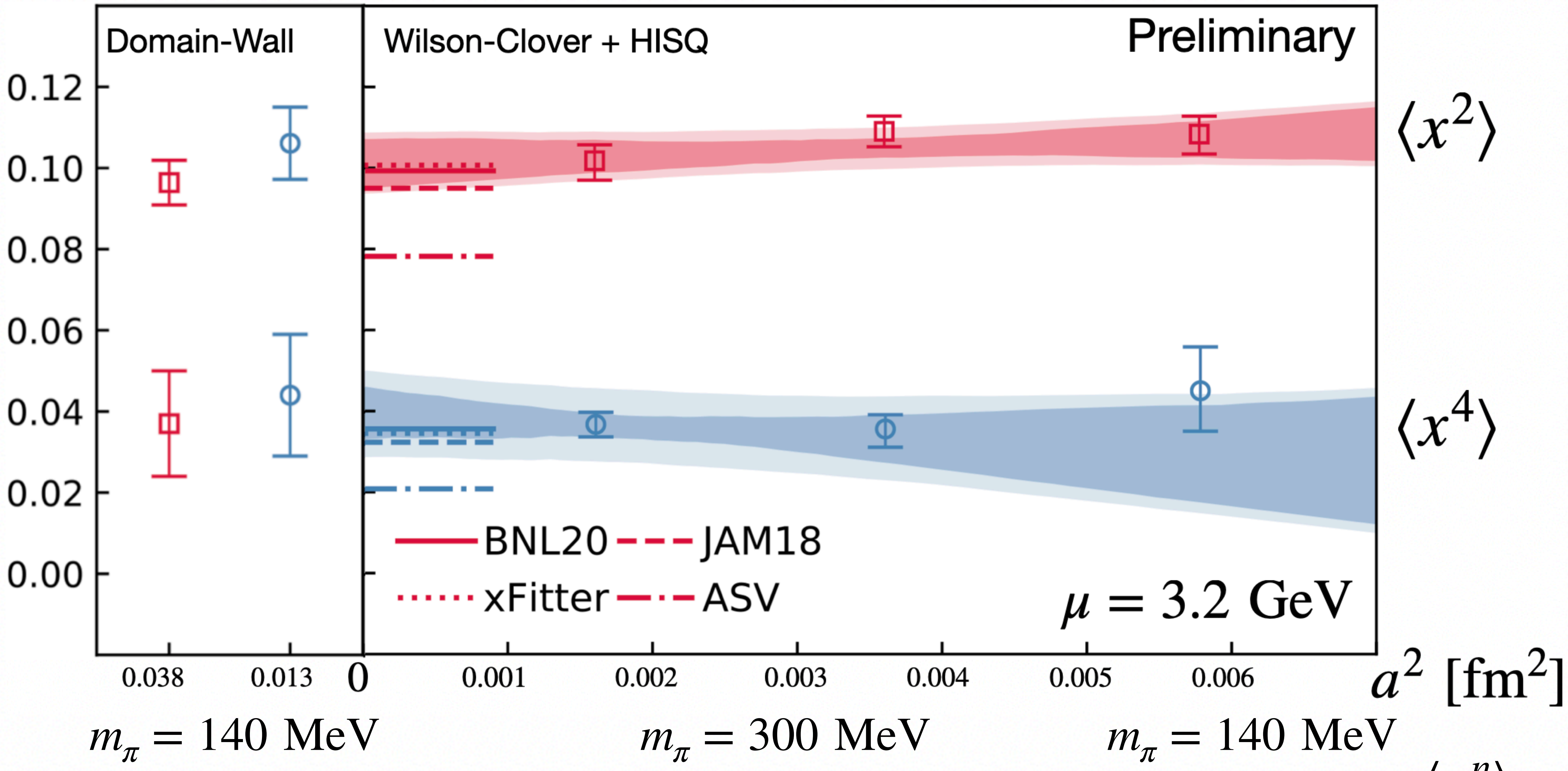
Ratio scheme renormalized matrix elements



Unitary Domain-Wall fermion calculation.

Pion valence quark PDF: Moments

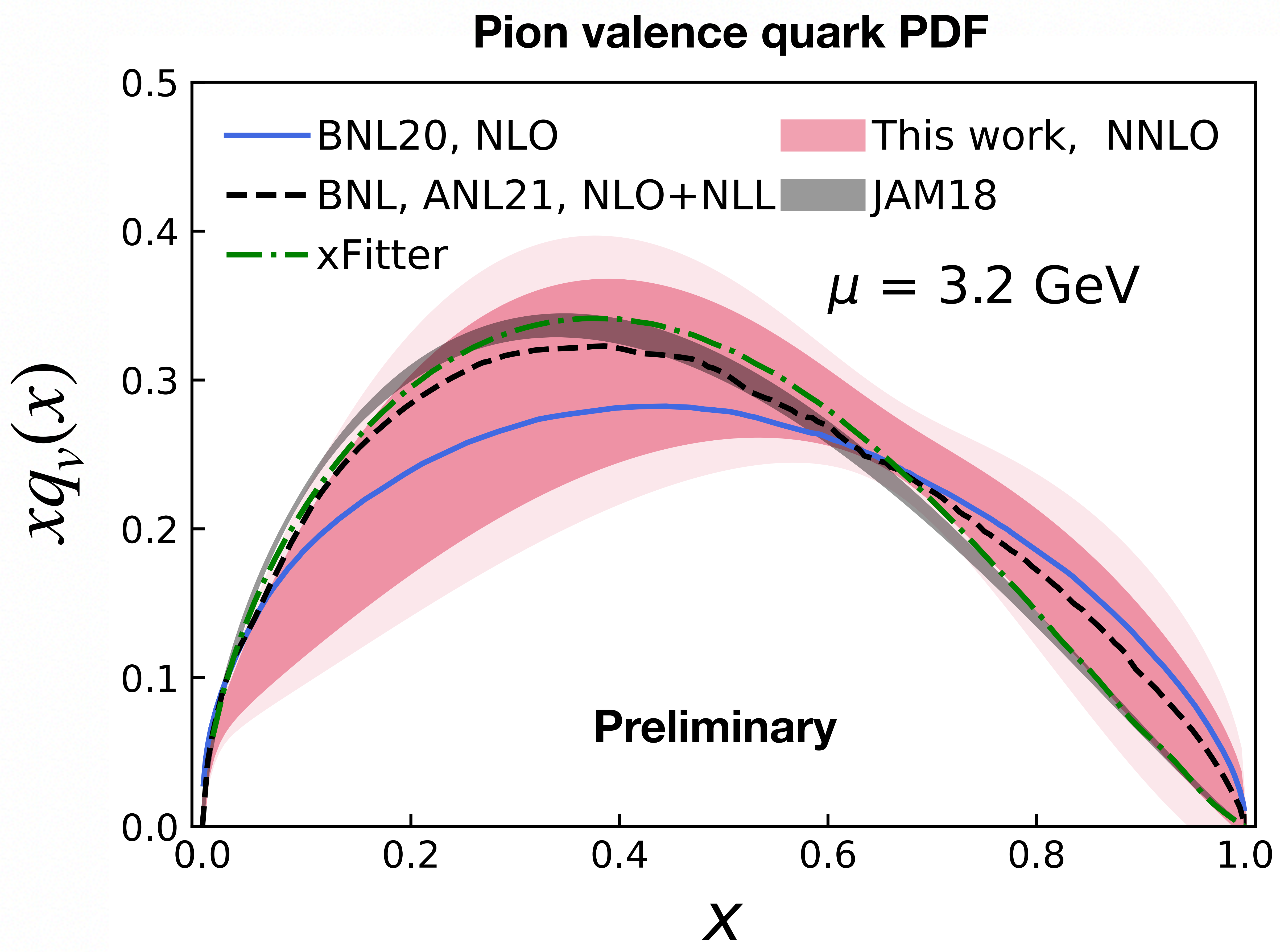
Moments: NNLO matching, physical point, chiral fermion



$$\langle x^n \rangle = \langle x^n \rangle_{\mathrm{Ext}} + d_n a^2$$

- The mass dependence is mild for pion valence PDF.
- Chiral fermion shows good agreement with Wilson-Clover + HISQ fermion with fine lattice spacings.

Pion valence quark PDF: model reconstruction



We reconstruct the x-dependent pion valence PDF using model

$$q_v(x) = x^\alpha(1-x)^\beta(1+t\sqrt{x}+sx),$$

which shows good agreement with JAM18, xFitter as well as our previous determination using NLO matching. This reconstruction favors large-x behavior $\beta \sim 1$.

More improvement:

- Resummation in perturbative matching. For example, NLO+NLL **threshold resummation** (BNL, ANL PRD 103 (2021)).
- More statistics and large momentum P_z to extract **higher moments**.

Pion valence quark PDF: large-x behavior

Model independent estimate of large-x behavior β

The moments $\langle x^n \rangle$ approach zero in the large-n limit in a manner dependent on β as:

- BNL, PRD **102**, 074504 (2020)

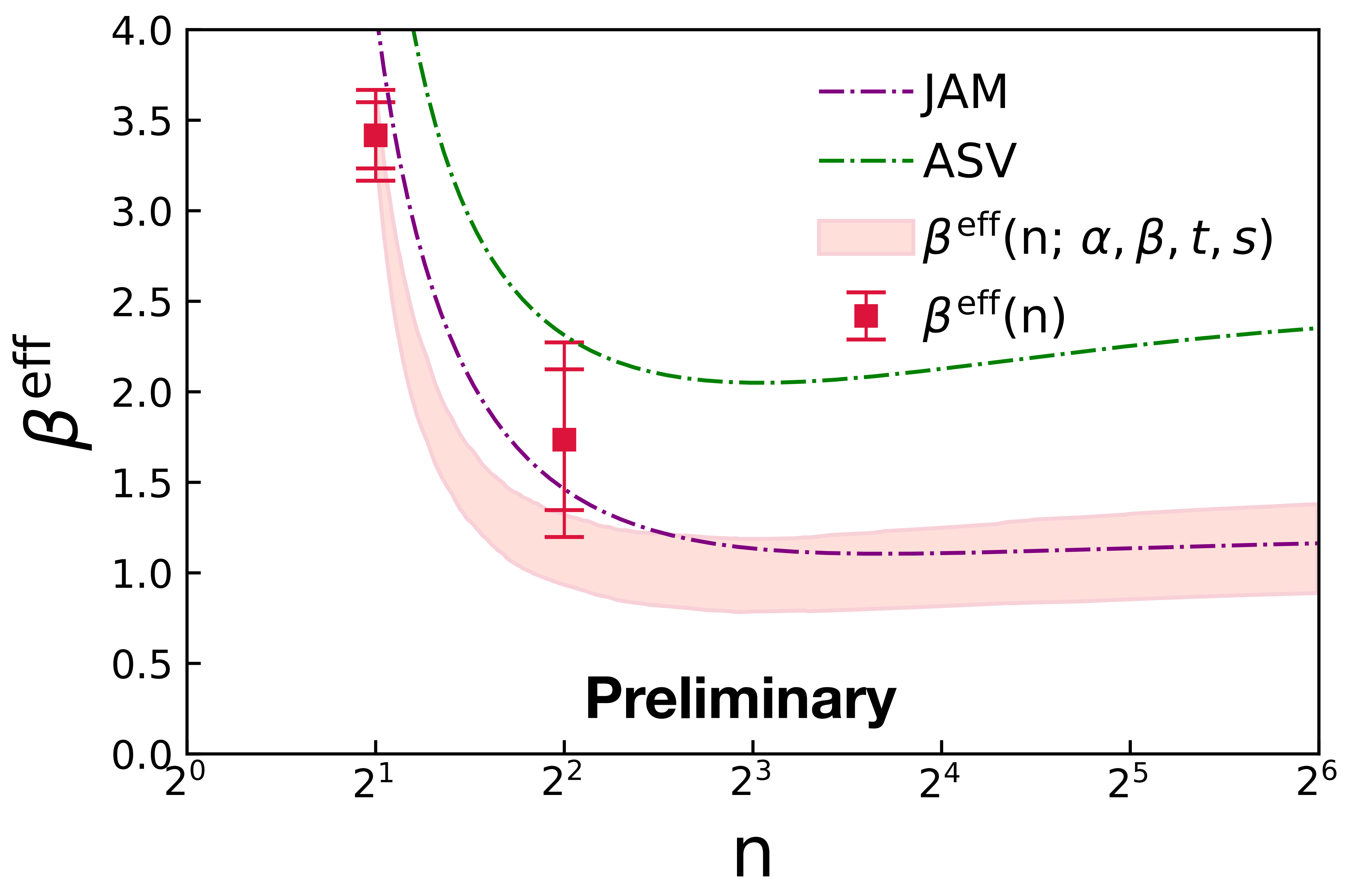
$$\langle x^n \rangle \propto n^{-\beta-1}(1 + \mathcal{O}(1/n))$$

then one can determine β in a model independent way:

$$\beta + 1 = -\frac{d\log(\langle \mathrm{x^n} \rangle)}{d\log(\mathrm{n})} + \mathcal{O}(1/n)$$

A discretised form of the above expression that is suitable for a practical implementation:

$$\beta_{\rm eff}(n) \equiv -1 + \frac{\langle x^{n-2} \rangle - \langle x^{n+2} \rangle}{\langle x^n \rangle}\frac{n}{4}$$



The $\beta_{\rm eff}(n)$ computed from the model independent moments as well as the model dependent $\beta_{\rm eff}(n;\alpha,\beta,t,s)$ are shown.

Current lattice data **can't rule out** $\beta \sim 1$ or 2.

Summary

- We studied pion valence quark PDF by computing the equal-time correlators from lattice QCD.
- We applied the ratio scheme renormalization and coordinate-space factorization scheme with matching formula up to NNLO level.
- Our calculations used several pion mass including the physical one, we observed the mass dependence of the pion valence PDF is mild.
- Our current lattice data can’t rule out any of the large-x behavior as $\beta \sim 1$ or $\beta \sim 2$, for which higher momentum are needed.

Thanks for your attention

Pion valence quark PDF: Higher-twist effect

To estimate the **higher-twist/non-perturbative** effect as a function of z, one can define an effective δm^{eff}

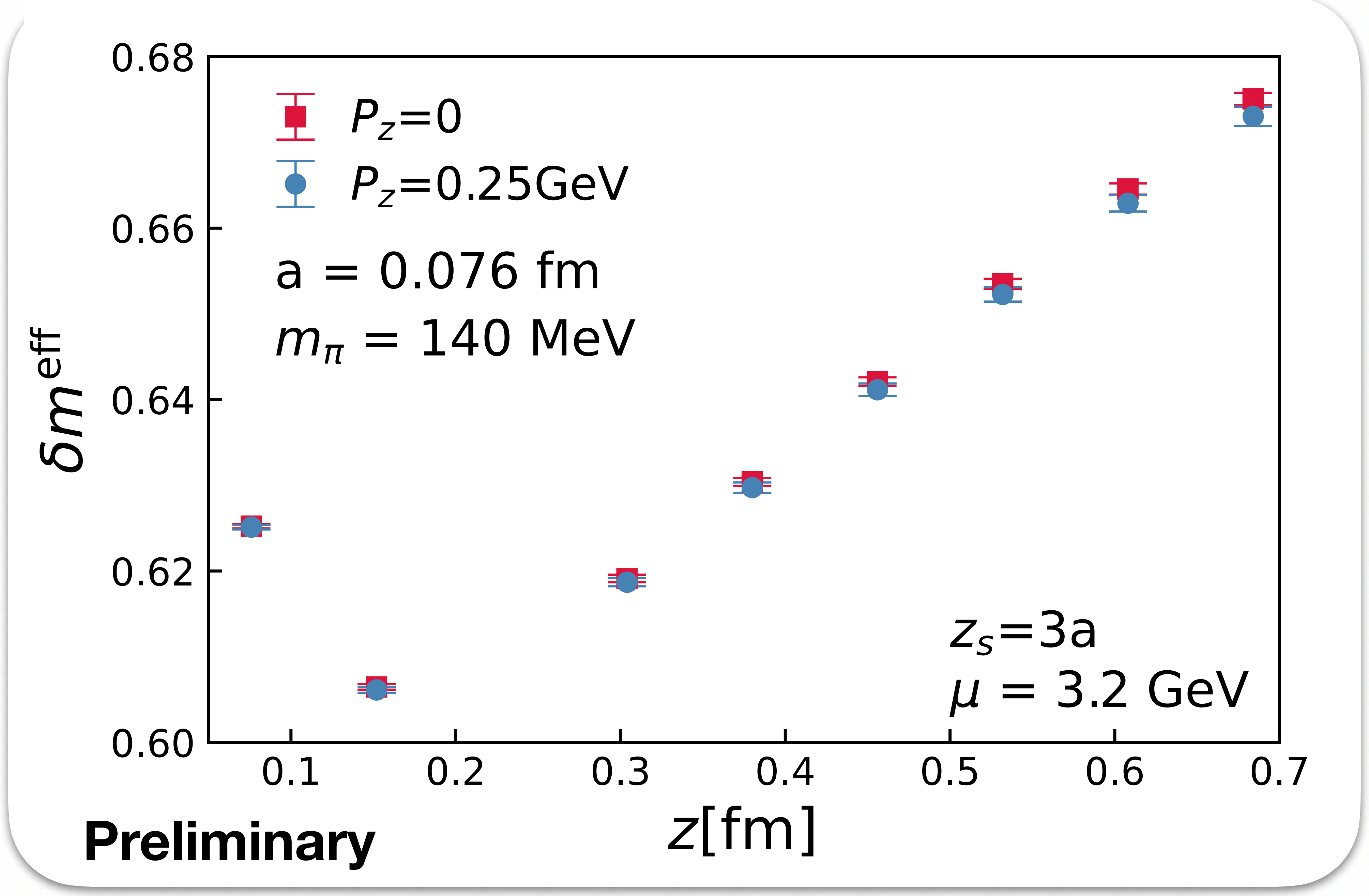
$$-\delta m^{eff}|z-z_s|$$

$$= \ln \frac{h_B(z, P_z, a)}{h_B(z_s, P_z^0=0, a)} - \ln \frac{C_0(\mu^2 z^2) - C_2(\mu^2 z^2)\frac{(zP_z)^2}{2}\langle x^2\rangle_{z_s}}{C_0(\mu^2 z_s^2)}$$

$$= -\delta m|z-z_s| + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

z_s is a fixed small distance to cancel Z(a).

- Subtract the **twist-2** contribution from the matrix elements.
- Matrix elements of **non-zero** P_z contains information of the moments of the PDFs.
- Limit $zP_z < 1$ where the data is only sensitive to the 2nd moment $\langle x^2\rangle$, which can be extracted at **small** z.



- δm^{eff} doesn't show a plateau, suggesting the **higher-twist/non-perturbative** effects as a function of z.
- Two different momentum produce consistent results at least up to 0.6 fm, where we can still apply the short distance factorization based on **ratio scheme renormalization**.