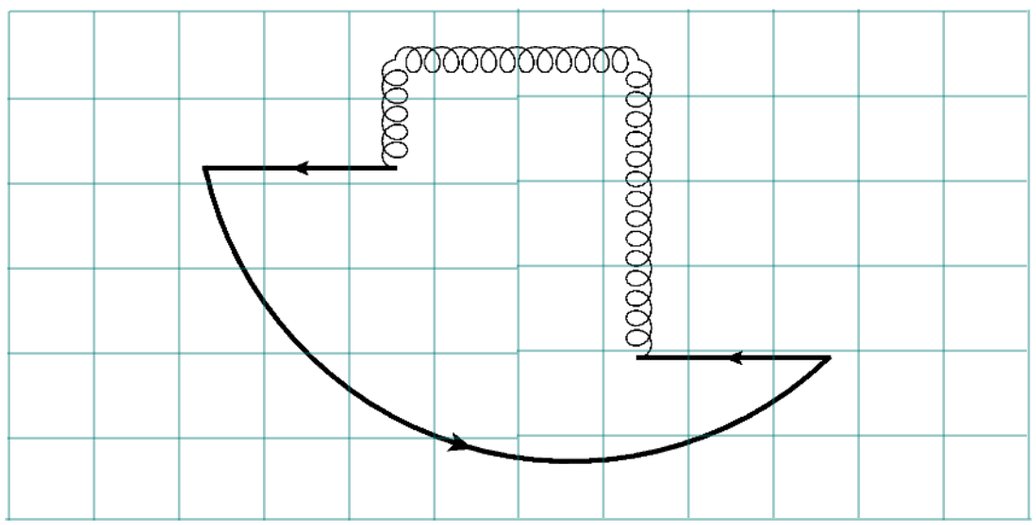
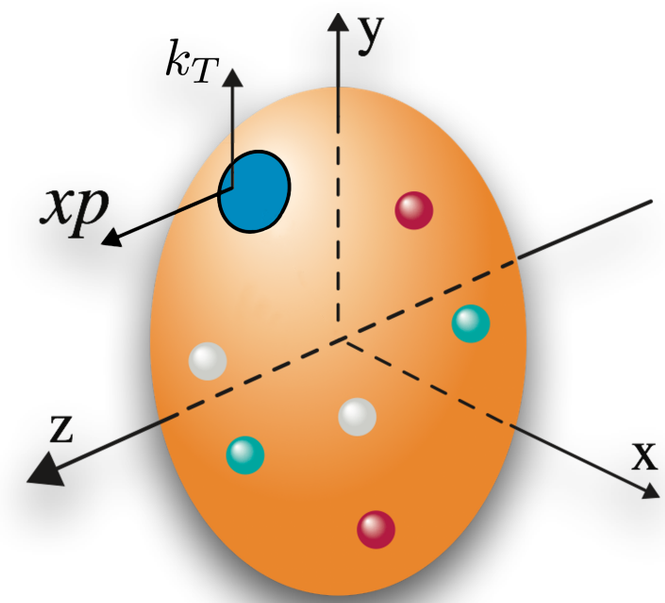
Lattice QCD calculation of the Collins-Soper kernel from quasi TMDPDFs





Michael Wagman

in collaboration with

Phiala Shanahan, Yong Zhao

Fermilab

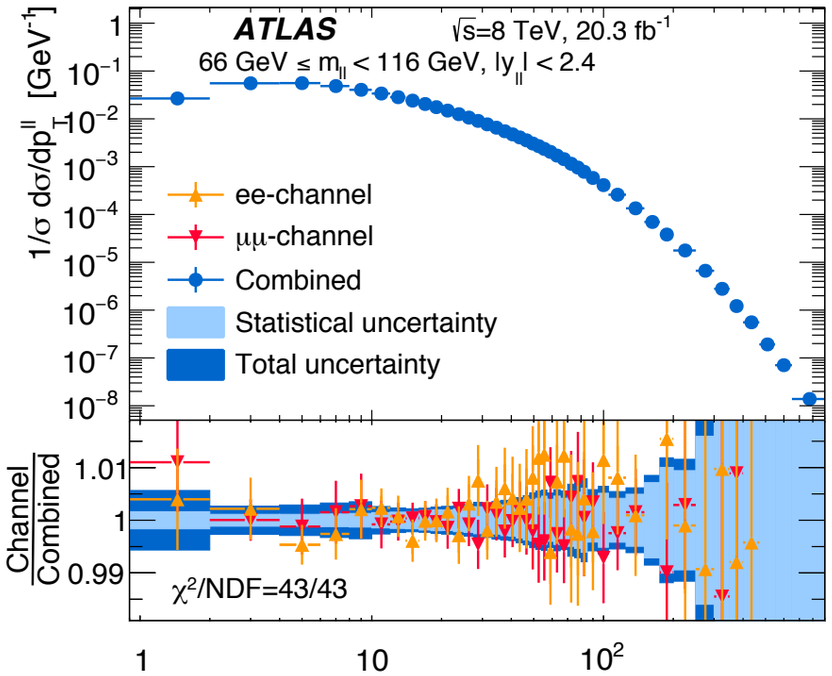
Shanahan, MW, Zhao, PRD 101 (2020) PRD 102 (2020) **PRD 104 (2021)**

LaMET 2021 Online

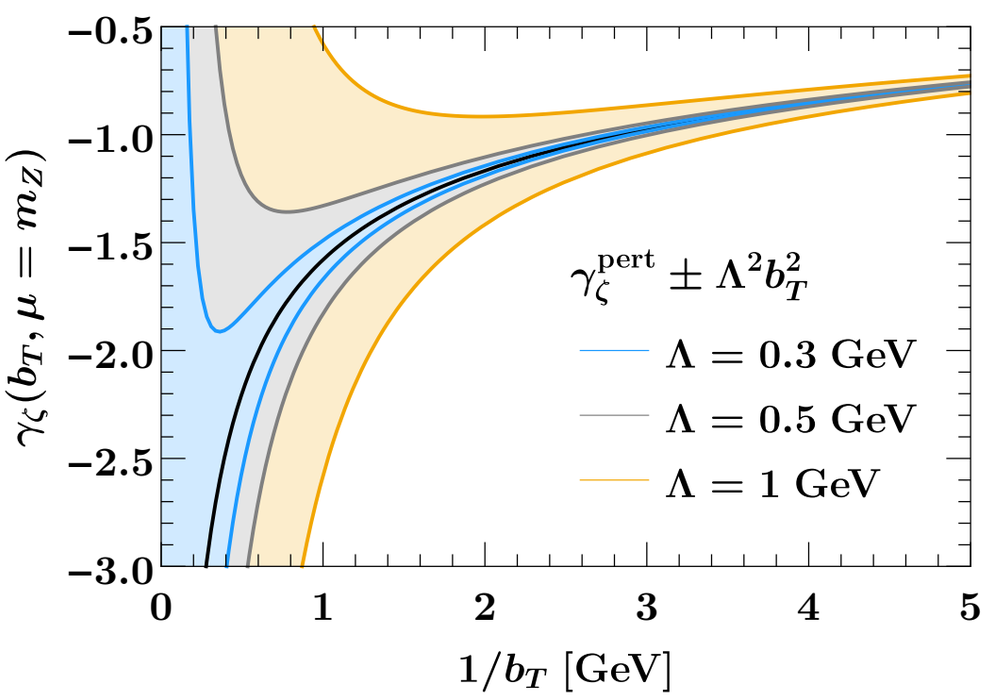
TMD evolution

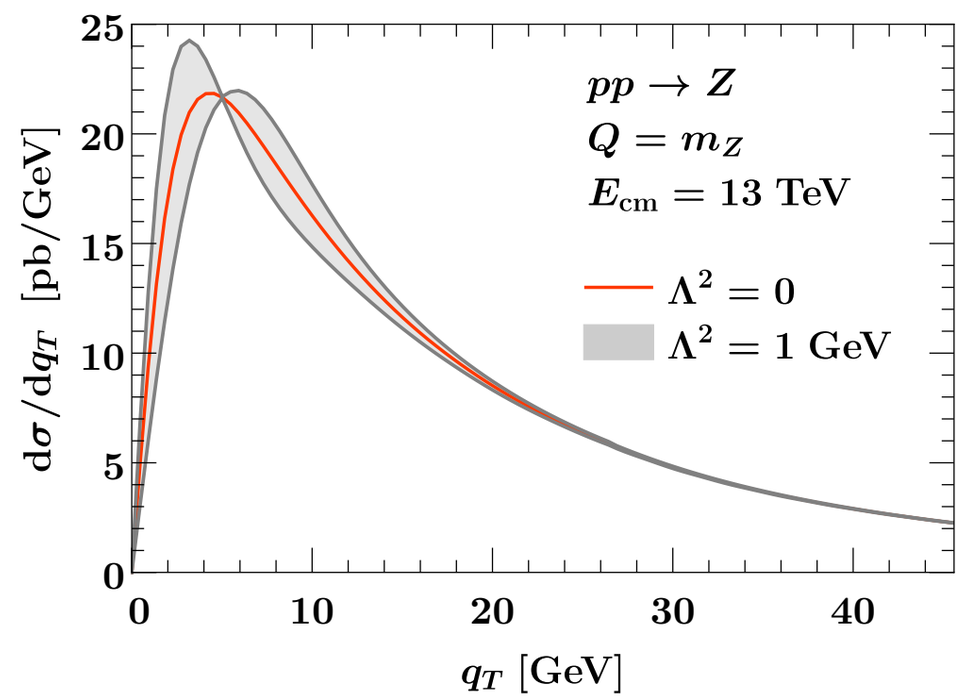
TMDPDFs encode 3D structure of hadrons, q_T dependence of hadronic cross-sections

Drell-Yan q_T dependence measured at LHC with <1% precision



Uncertainty in nonperturbative contributions to TMDPDF evolution limits QCD theory predictions to much lower precision





The Collins-Soper kernel

TMDPDFs depend on UV renormalization scale μ as well as a scale ζ associated with the renormalization of rapidity divergences

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu_0, \zeta_0)$$

$$\times \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^i(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\right]$$

Fourier conjugate to $\vec{q}_T$

UV anomalous dimension

Collins-Soper kernel

(rapidity anomalous dimension)

Changing hard momentum scales requires evolving TMDPDFs in μ and $\zeta \sim (2xP^z)^2$

Unlike UV anomalous dimension, CS kernel is nonperturbative for large b_T

CS kernel phenomenology

Fits to SIDIS and Drell-Yan datasets with multiple energy scales are sensitive to evolution effects and therefore the CS kernel

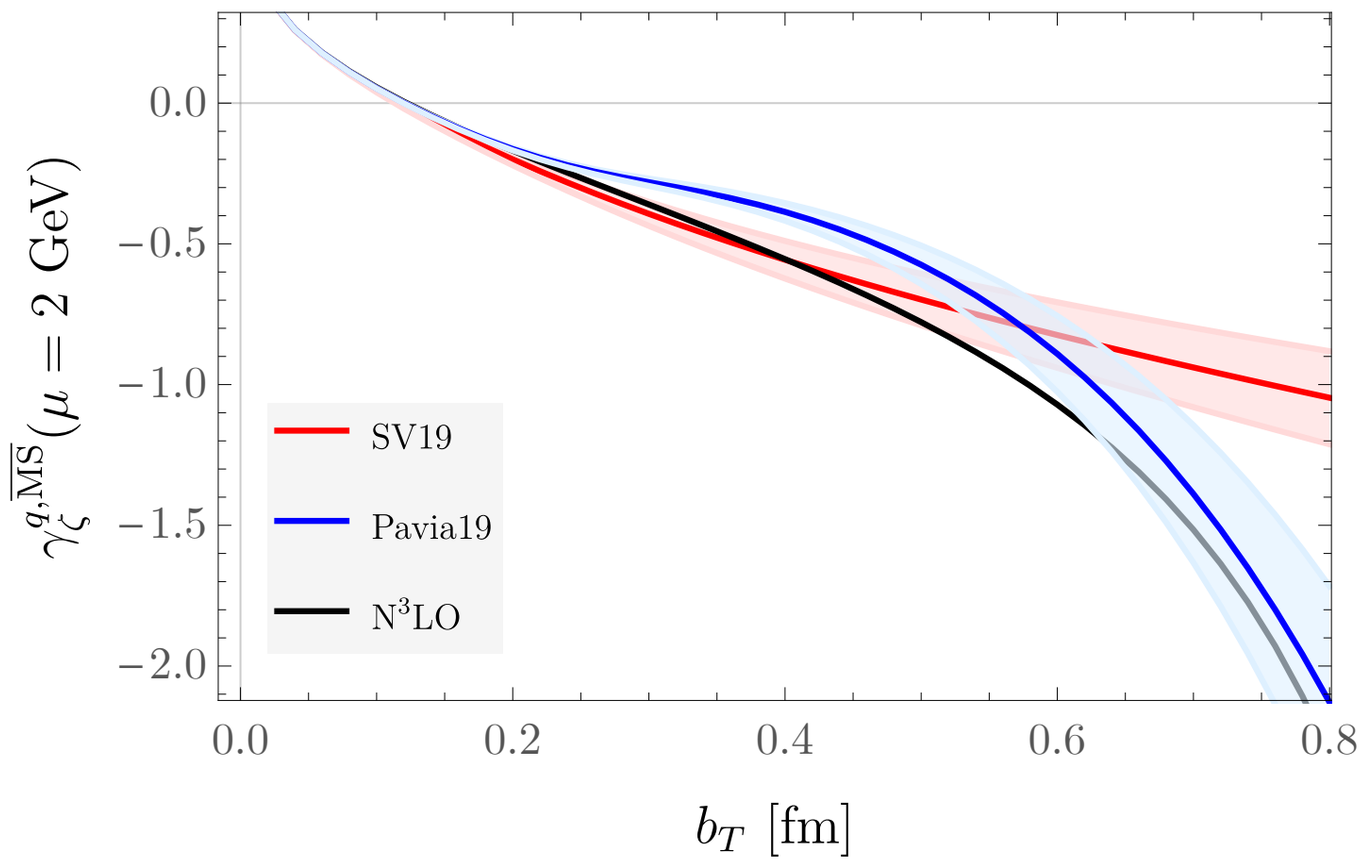
CS kernel can be extracted along with TMDPDF in global fit

SV19 - Scimemi and Vladimirov, JEHP 06 (2020)

(582 SIDIS + 457 DY data points)

Pavia19 - Bacchetta et al, JEHP 07 (2020)

(353 DY data points)



Modeling significant for

$$b_T \gtrsim 0.2 \text{ fm}$$

(nonperturbative region)

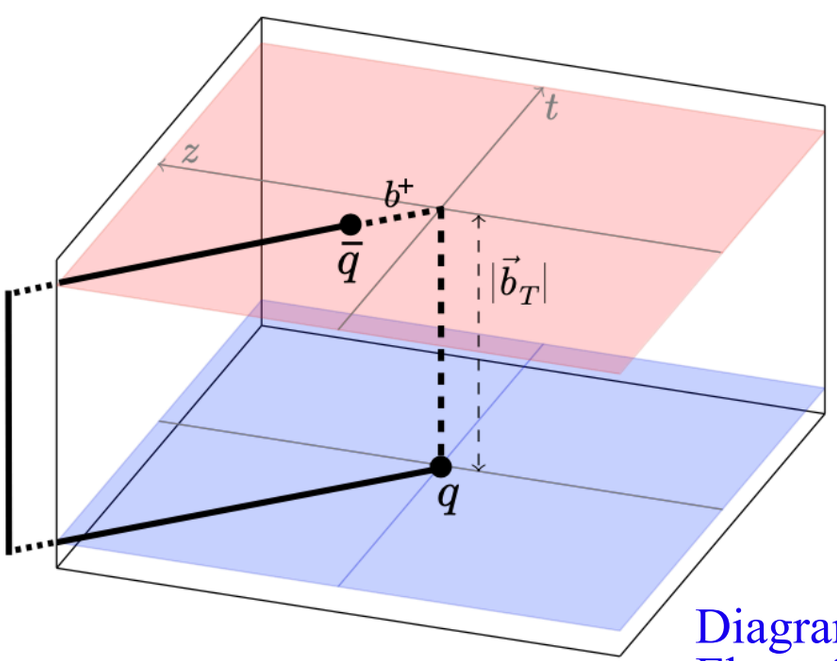
Light-cone structure and LQCD

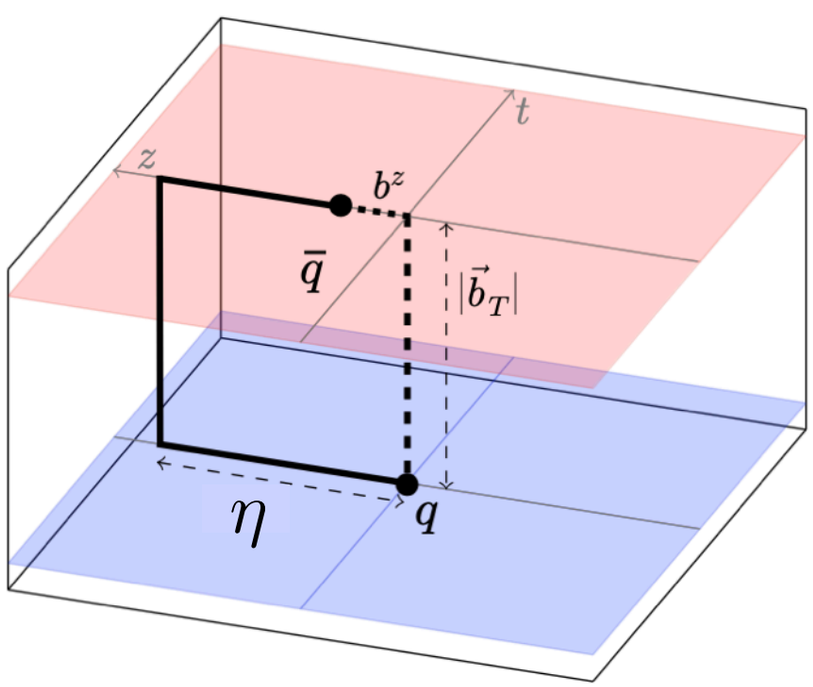
Lattice QCD provides a UV regularized definition and means of numerically calculating Euclidean QCD path integrals

Euclidean matrix elements containing hard scales can be factorized into perturbative coefficients and structure functions

Large momentum effective theory review: Ji et al, arXiv:2004.03543

For colinear PDFs and some aspects of TMDPDFs, Euclidean and light-cone structure functions are related by Lorentz invariance





Diagrams:
Ebert, Stewart, Zhao, JHEP 1909 (2019)

The CS kernel from LQCD

TMDPDF soft factor not related in Euclidean and Minkowski spacetime by boosting, more complicated to determine from LQCD

Recent progress: Ji, Liu, and Liu, Nucl Phys B 955 (2020)

Zhang et al [LPC], PRL 125 (2020)

See talk by Yizhuang Liu

Ratios of TMDPDFs free from soft factors and can be calculated with LQCD

Musch et al, PRD 85 (2012) Engelhardt et al, PRD 93 (2016) Yoon et al, PRD 96 (2017)

CS kernel determination using quasi-TMDPDFs suggested

Ji, Sun, Xiong, Yuan PRD 91 (2015)

Method concretely relating CS kernel to quasi TMDPDF ratios proposed and derived

Ebert, Stewart, Zhao, PRD 99 (2019)

Euclidean quasi-beam function

$$\gamma_\zeta^{q,\overline{\rm MS}}(b_T,\mu) = 2\zeta \frac{d}{d\zeta} \ln f_q^{\overline{\rm MS}}(x,b_T,\mu,\zeta)$$

$$= \frac{1}{\ln(p_1^z/p_2^z)} \ln \frac{C_{\rm TMD}^{\overline{\rm MS}}(\mu, xP_2^z) \int db^z e^{ib^z x p_1^z} \widetilde{B}_q^{\overline{\rm MS}}(b^z, b_T, \eta, \mu, p_1^z)}{C_{\rm TMD}^{\overline{\rm MS}}(\mu, xp_1^z) \int db^z e^{ib^z x p_2^z} \widetilde{B}_q^{\overline{\rm MS}}(b^z, b_T, \eta, \mu, p_2^z)}$$

Quenched results

CS kernel's independence from external state used to enable efficient calculations in quenched QCD

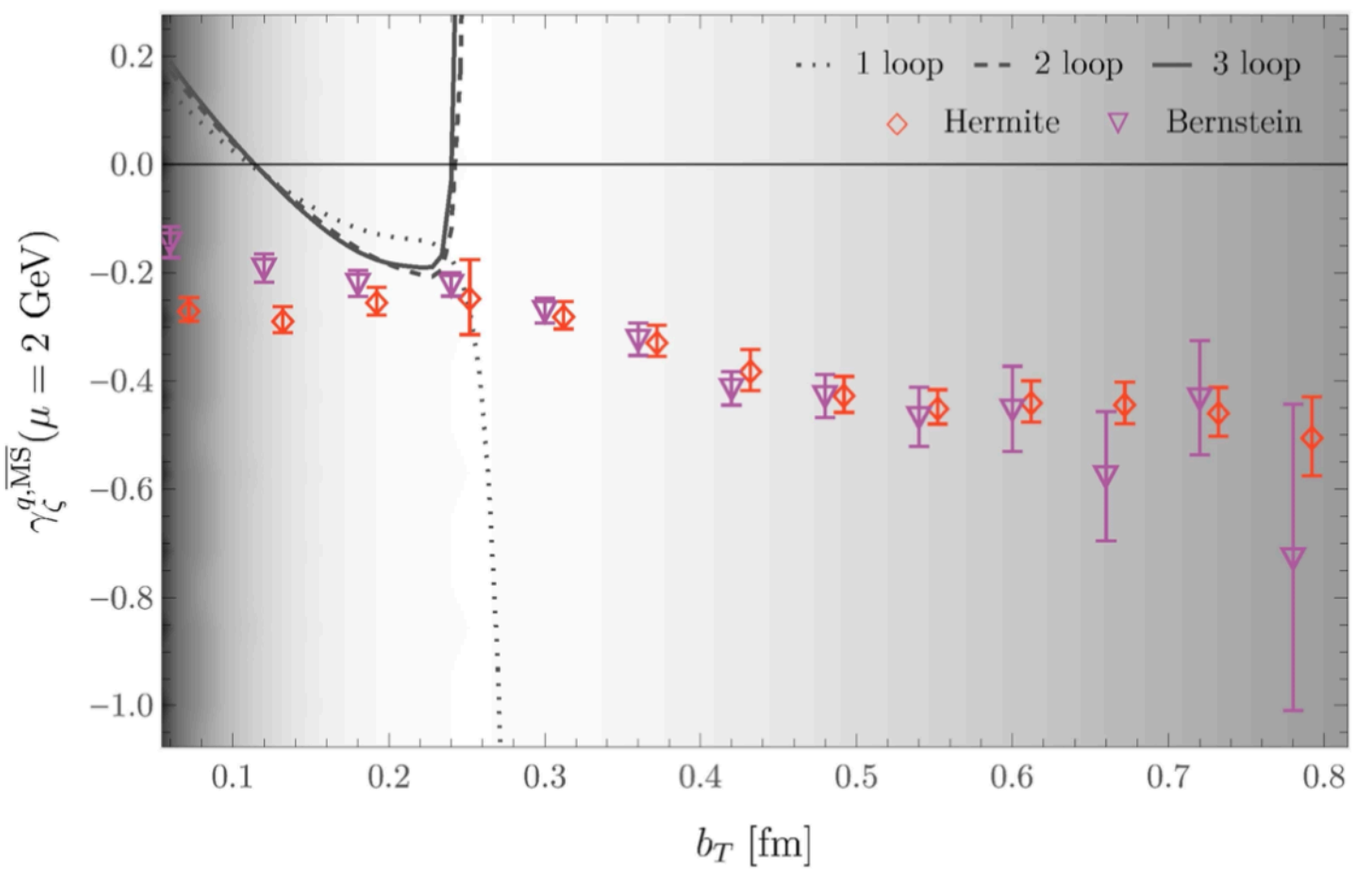
Valence pion mass

Label	β	a [fm]	$L^3 \times T$	κ	$n_{\rm src}$	$n_{\rm cfg}$
E_{32}	6.3017	0.06	$32^3 \times 64$	0.1222	2	200

$$m_\pi = 1.2 \text{ GeV}$$

$$P^z \in \{1.3,\ 1.9,\ 2.6\} \text{ GeV}$$

$$\eta \leq 0.8 \text{ fm}$$

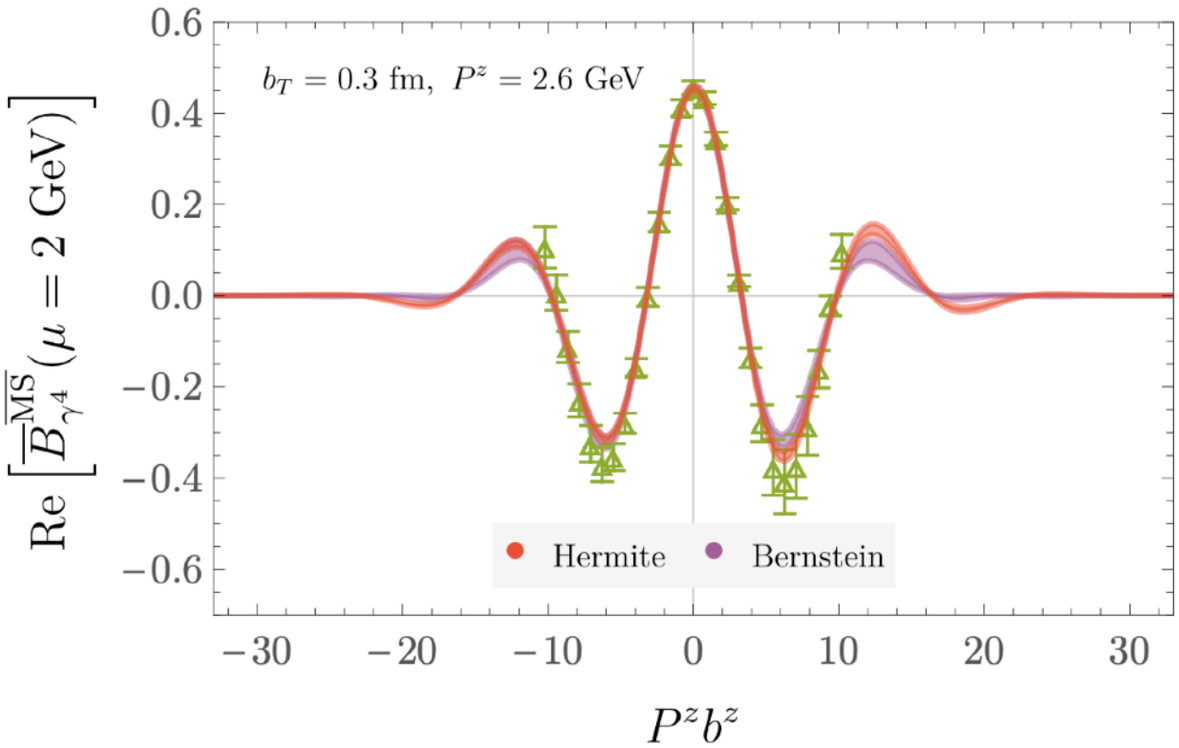


CS kernel determined precisely for b_T extending into nonperturbative regime

Shanahan, MW, Zhao, PRD 102 (2020)

Quenched challenges

Fourier transform truncation effects: challenging systematic uncertainties to quantify

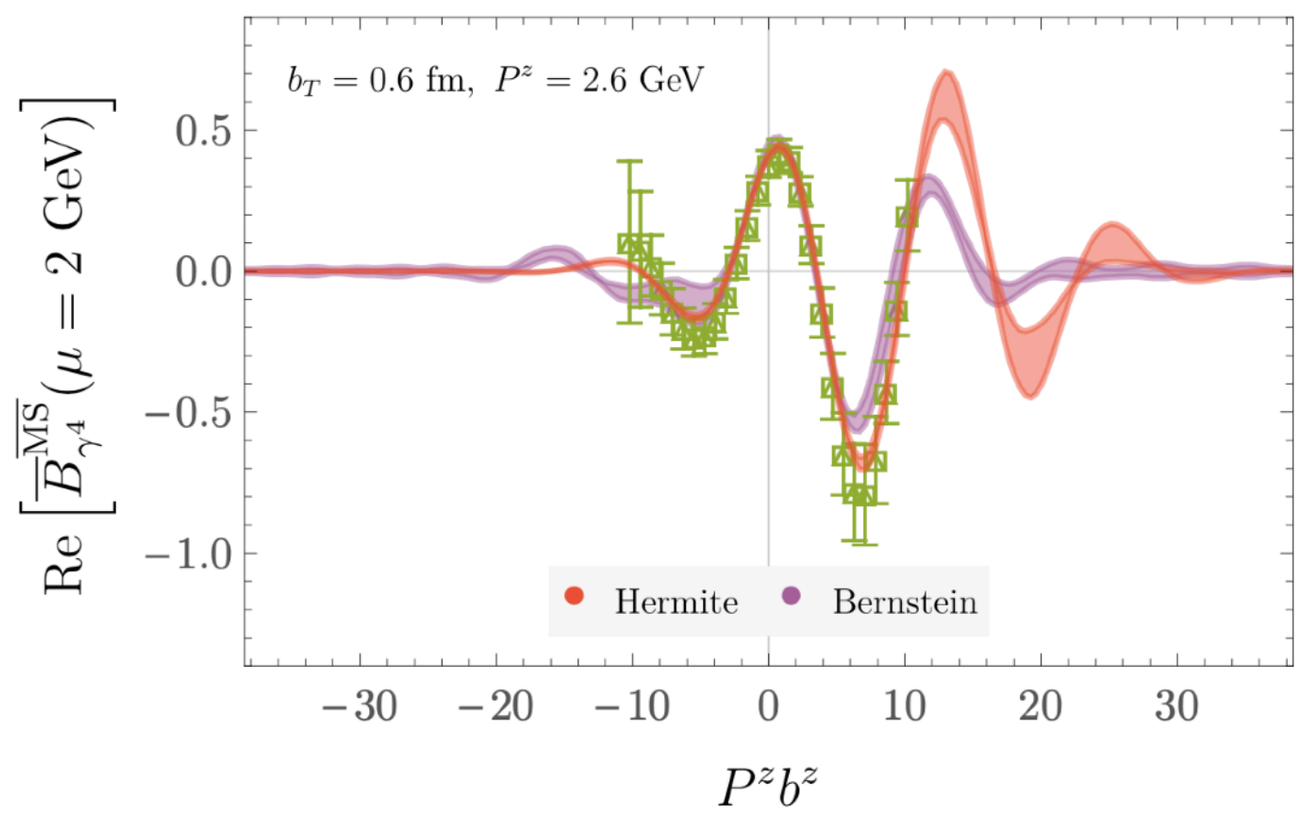


Modeling of beam functions outside region with LQCD results necessary to cover region where Fourier transform integrand matters

Different analytic models agree for small b_T

Broadening of the (physical) beam function with increasing b_T leads to larger truncation effects

Extrapolation effects (and unexpected asymmetry) more pronounced for larger b_T



Dynamical LQCD setup

Mixed action: $N_f = 2+1+1$ MILC ensembles with ~physical quark masses

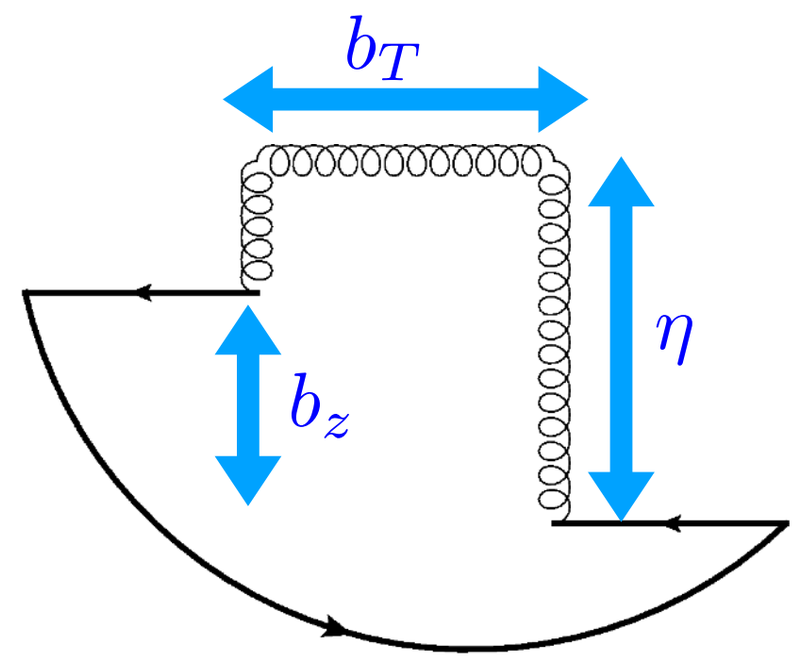
$a = 0.12$ fm $\quad L = 48a = 5.6$ fm

Bazavov et al [MILC] PRD 87 (2013)

Wilson valence quarks with tree-level clover improvement,
Wilson flow $t = 1.0$ used as smearing in valence action

$m_\pi = 538(1)$ MeV

n_z	P^z [GeV]	η/a	$n_{\rm src}$	$n_{\rm cfg}$
3	0.65	{12,14}	4	96
3	0.65	23	16	100
5	1.1	{12,14}	4	449
7	1.5	{12,14}	16	596



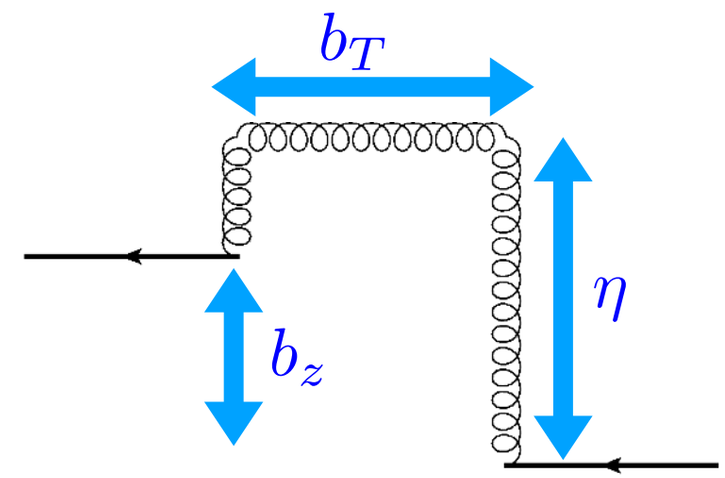
Larger volumes enable larger staple extents than in quenched calculation $\quad \eta \leq 1.7$ fm

$\eta P^z_{\rm max} = 14.5$ vs quenched $\eta P^z_{\rm max} = 11.0$

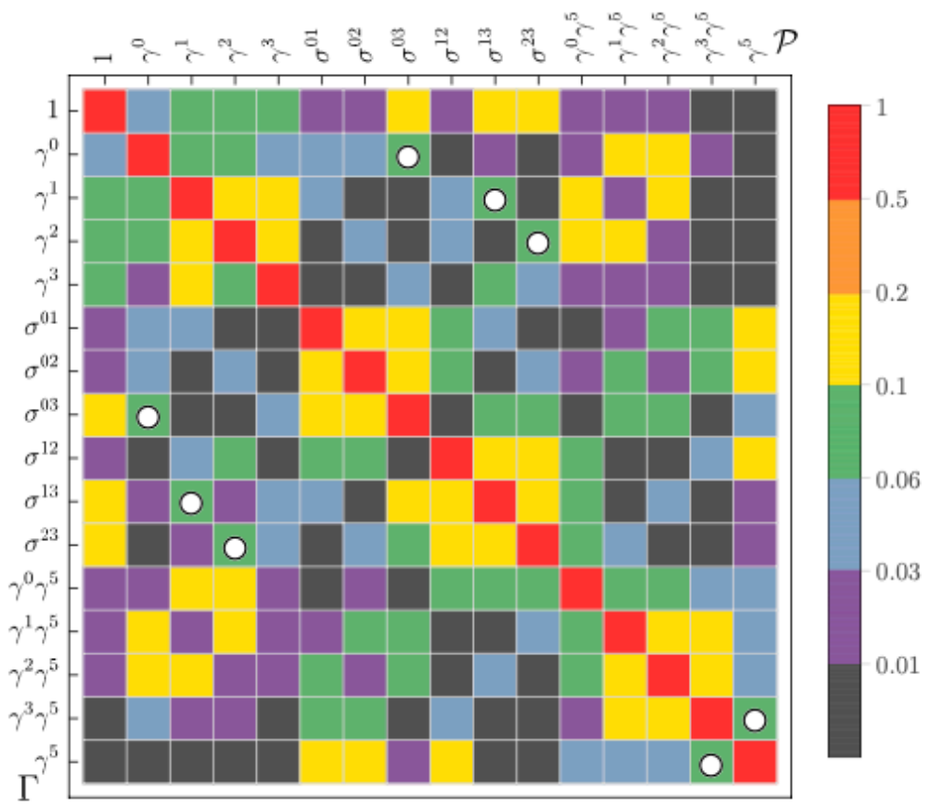
Renormalization and mixing

Nonlocal quark bilinear operators with stapled shaped Wilson lines renormalized using high-momentum quark vertex function (RI/MOM scheme)

$$\mathcal{O}^q_\Gamma(b^\mu,\eta) = \overline{q}(b^\mu)\frac{\Gamma}{2}W_{\hat{z}}(b^\mu;\eta - b^z)$$
$$\times W^\dagger_T(\eta\hat{z};b_T)W^\dagger_{\hat{z}}(0;\eta)q(0)$$



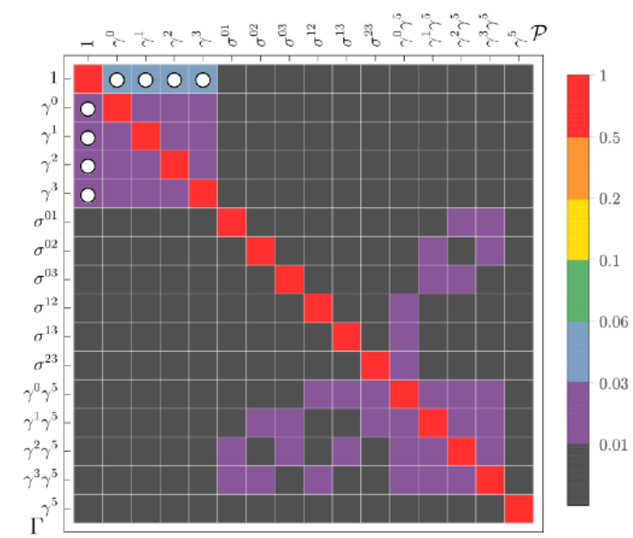
NLO (one-loop) matching used to convert to $\overline{\text{MS}}$ scheme: Ebert, Stewart, Zhao, JHEP 099 (2020)



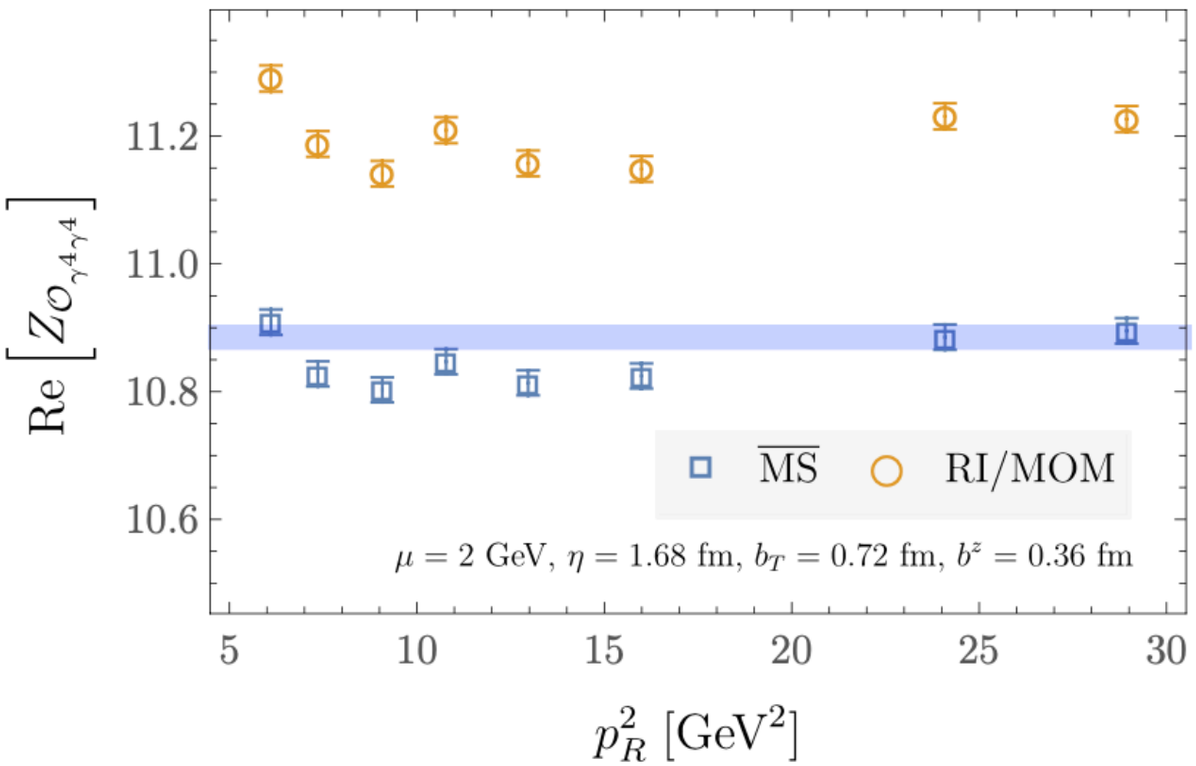
Shanahan, MW, Zhao, PRD 101 (2020)

Nonperturbative operator mixing significant for operators with large staples

Much smaller mixing observed e.g. for local quark bilinears:

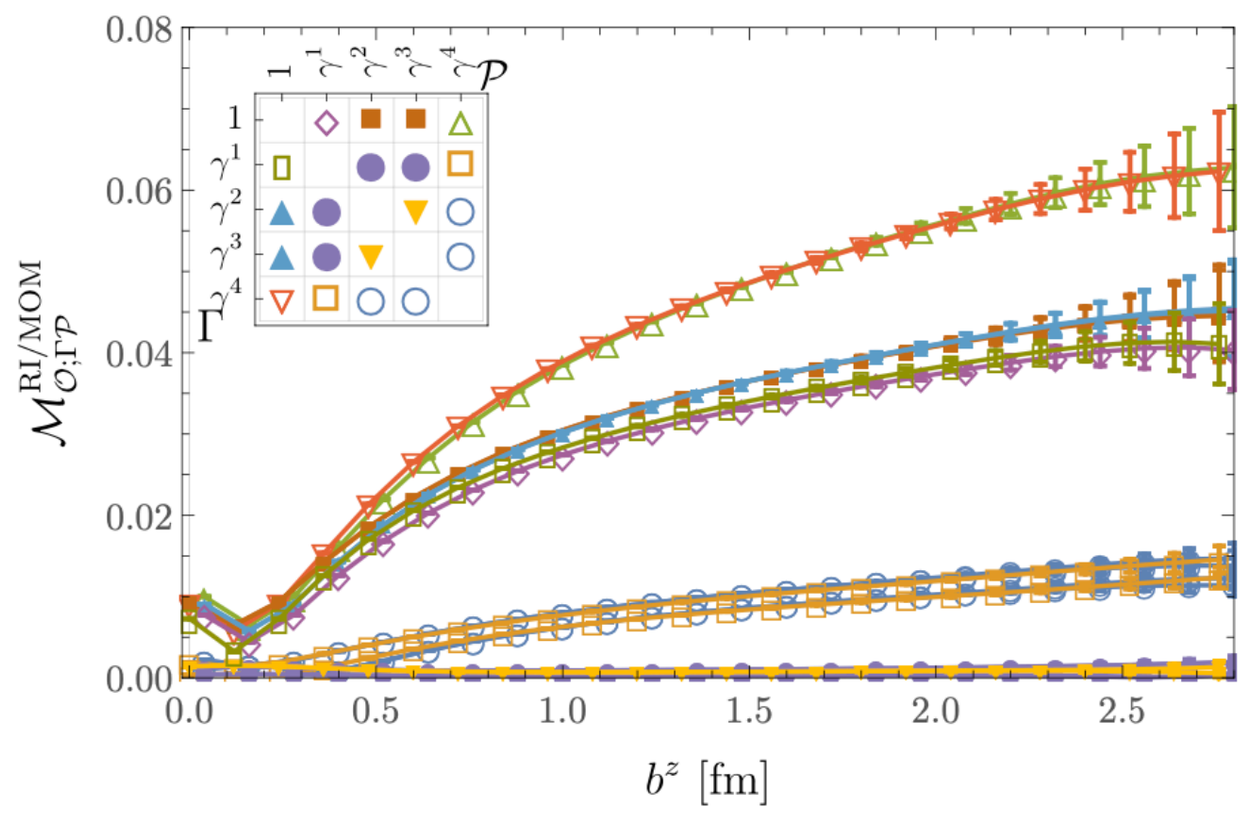


RI/MOM renormalization



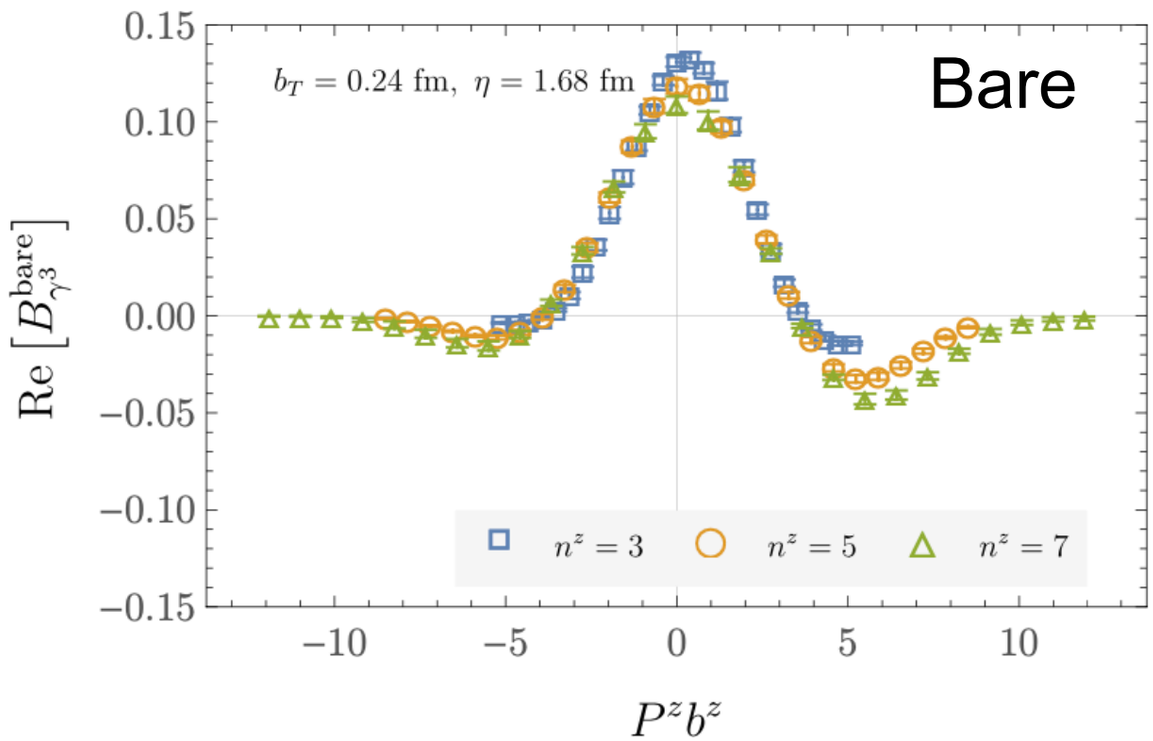
Renormalization factor results show mild dependence on p_R quark momentum after NLO matching to $\overline{\mathrm{MS}}$

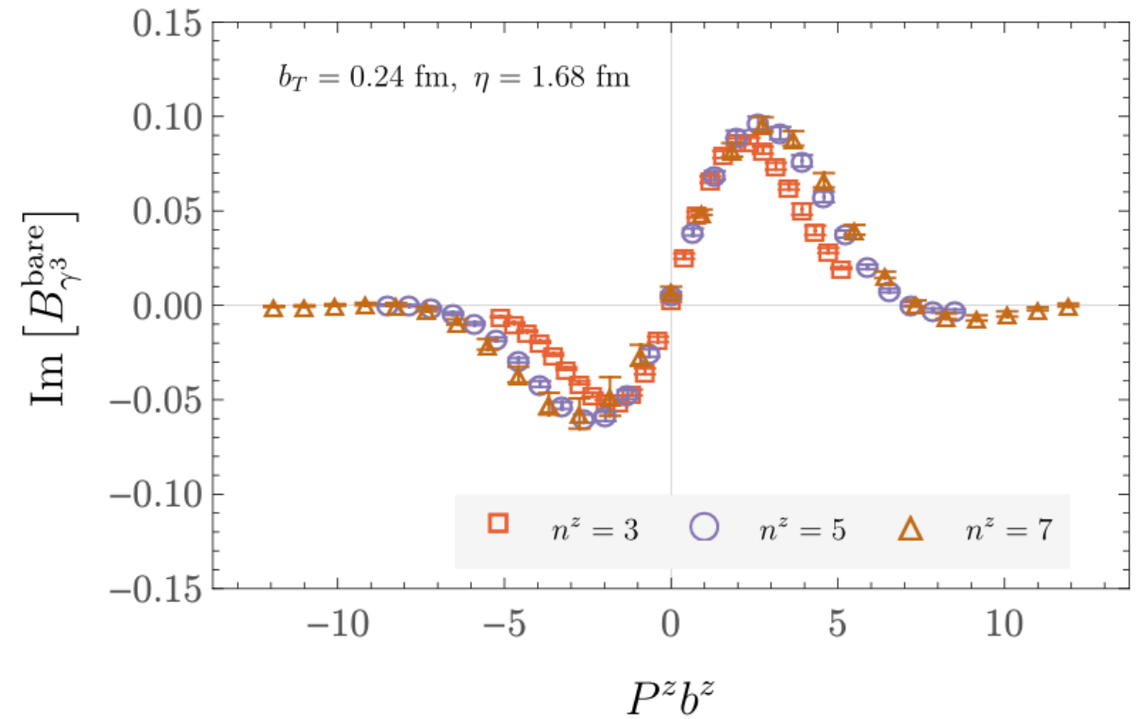
Including mixing between operators with different Dirac structures (16 x 16 matrix) leads to $\lesssim 5\%$ corrections to renormalized matrix elements that depend on staple geometry



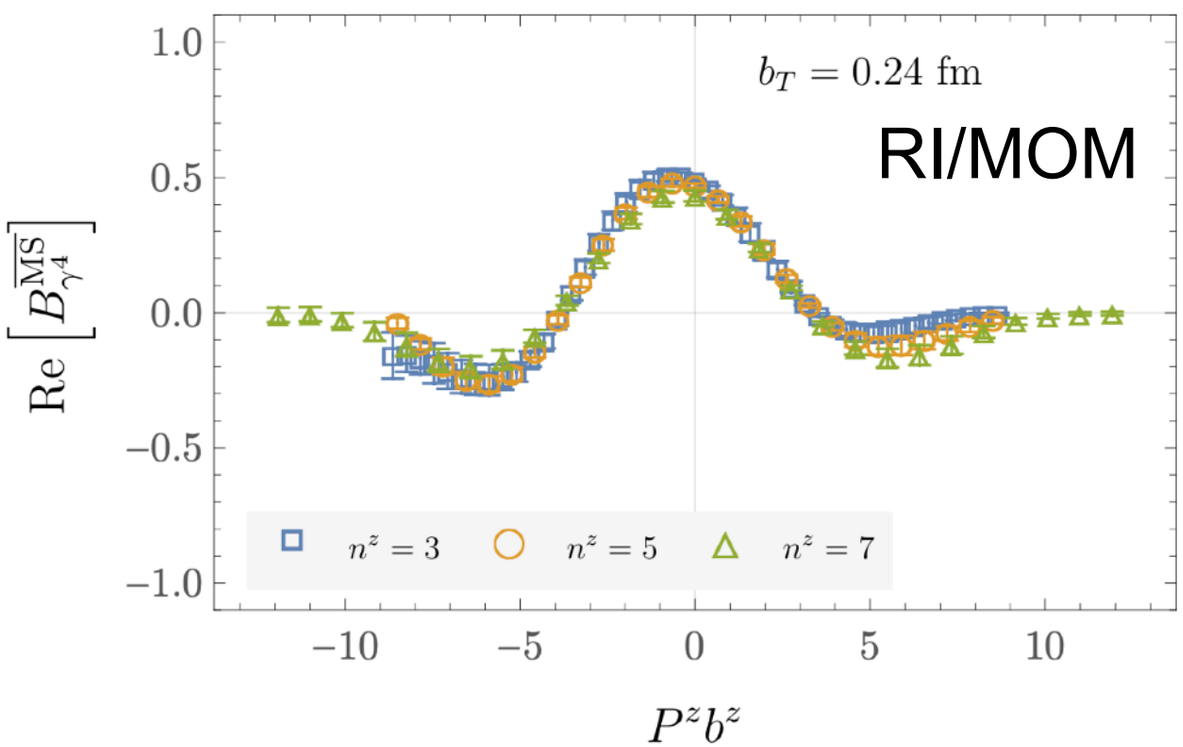
Shanahan, MW, Zhao, PRD 104 (2021)

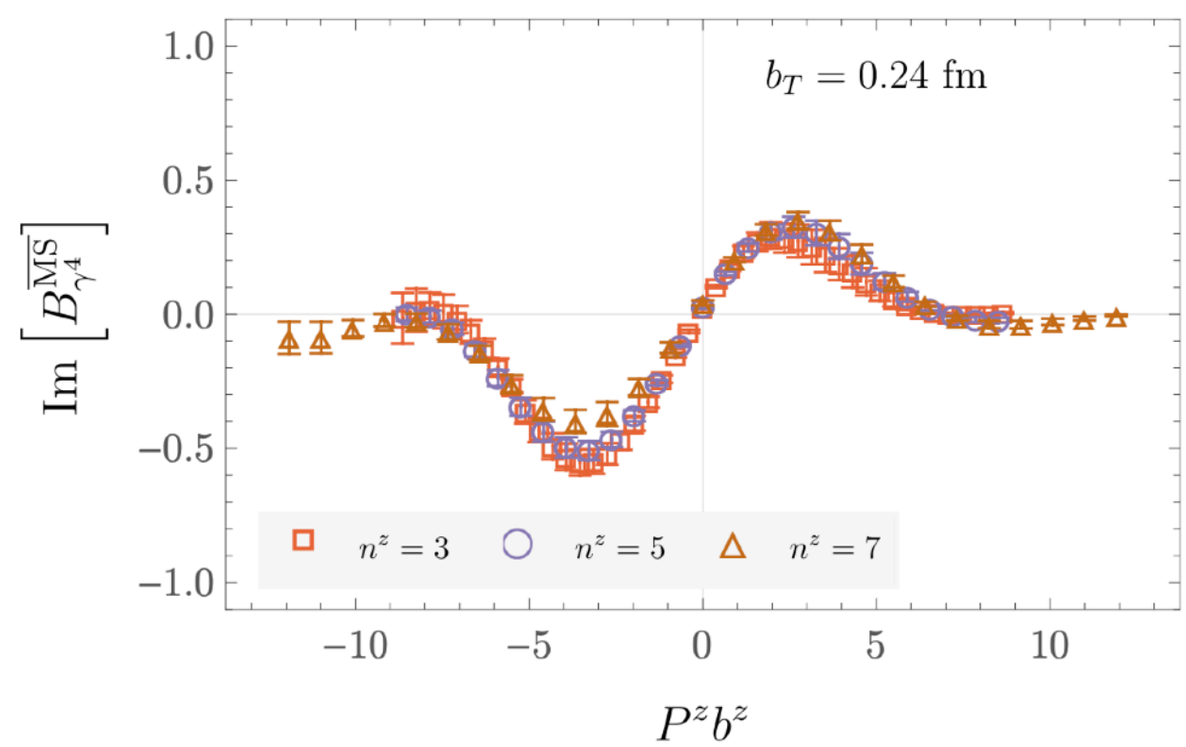
Trouble with RI/MOM





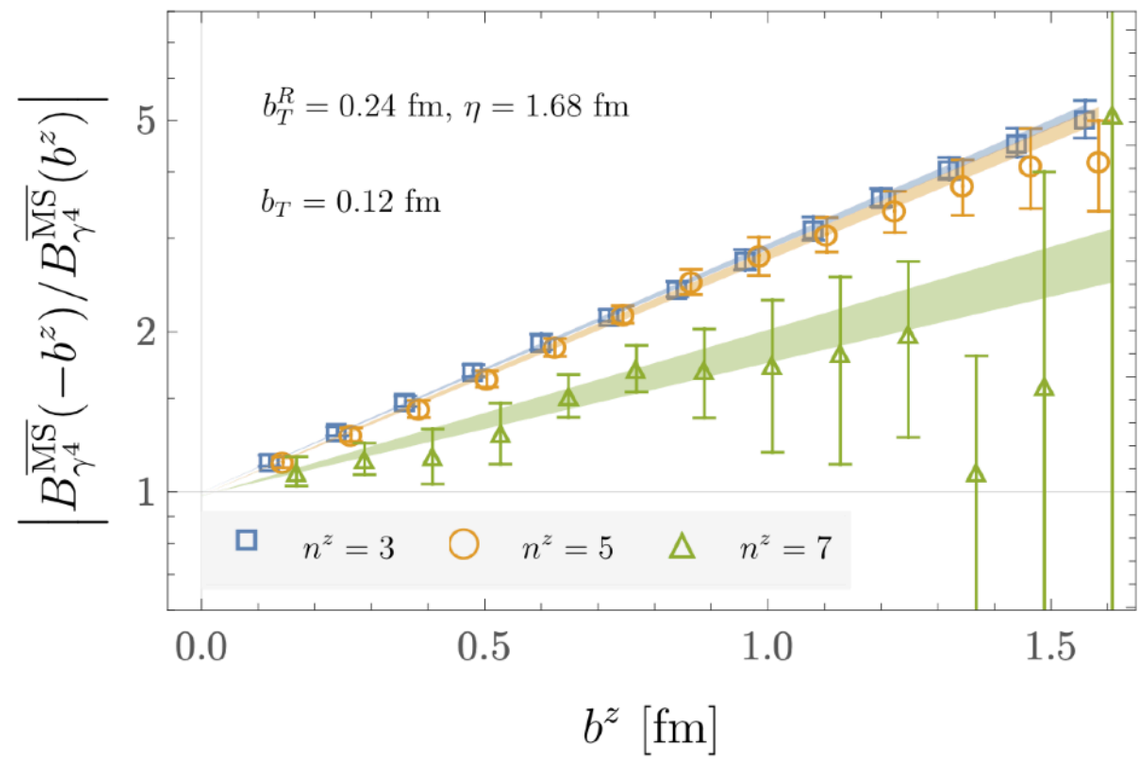
Asymmetry visible in beam functions after RI/MOM renormalization



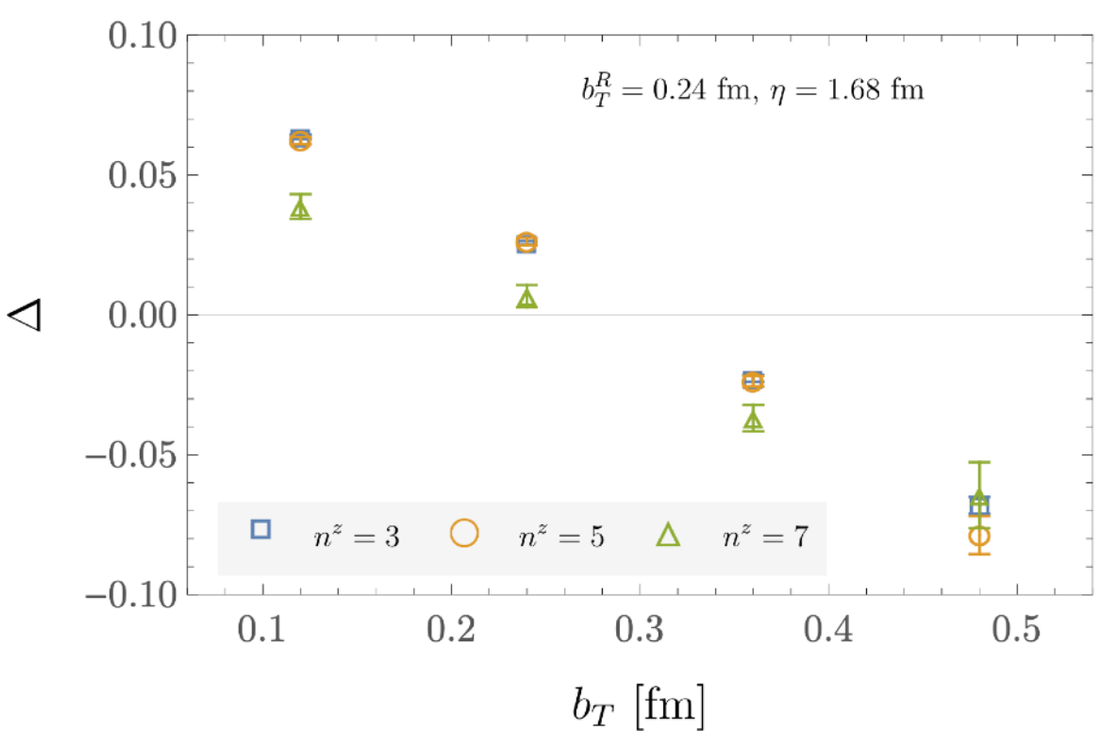


Beam function asymmetry

Asymmetry visible after RI/MOM renormalization could arise from state-dependence of static quark potential



State dependence of static quark potential visible in previous calculations



Correction for difference in static quark potentials applied

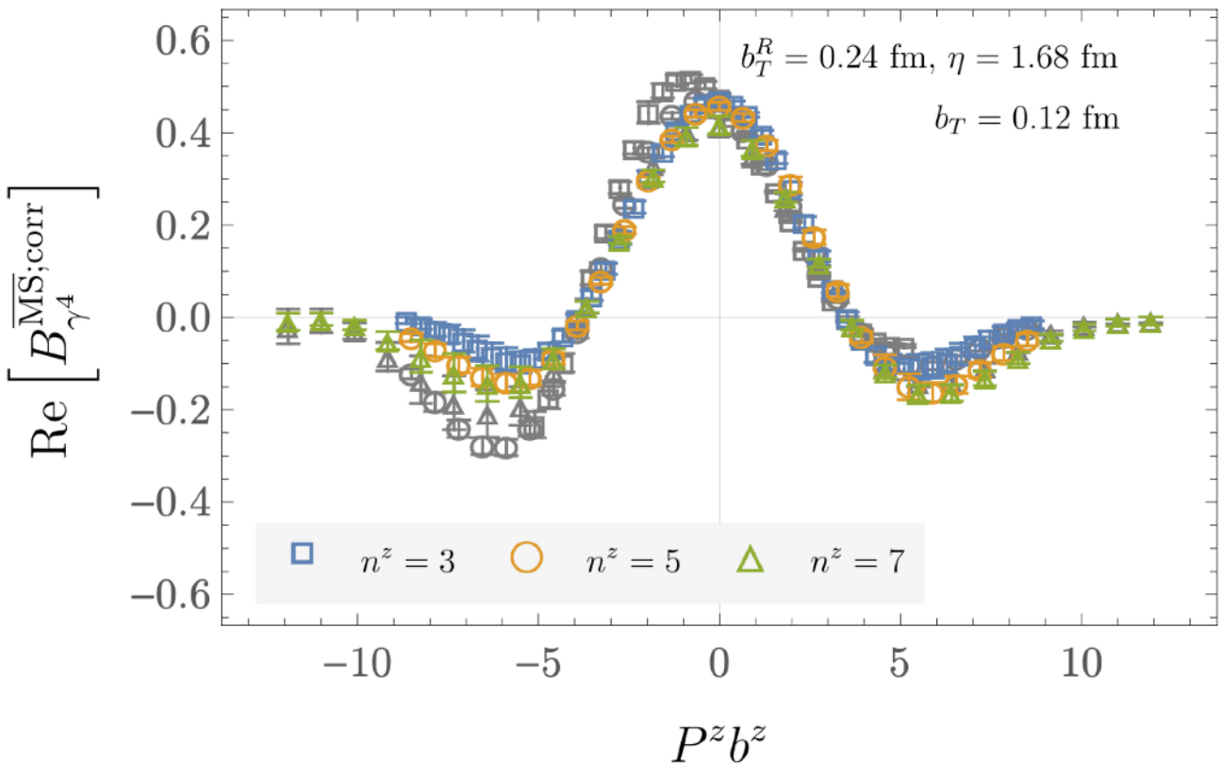
$$B^{\overline{\rm MS};{\rm corr}}_{\gamma_4}(b^z, b_T) = e^{\Delta(b_T)|b^z|} B^{\overline{\rm MS}}_{\gamma_4}(b^z, b_T)$$

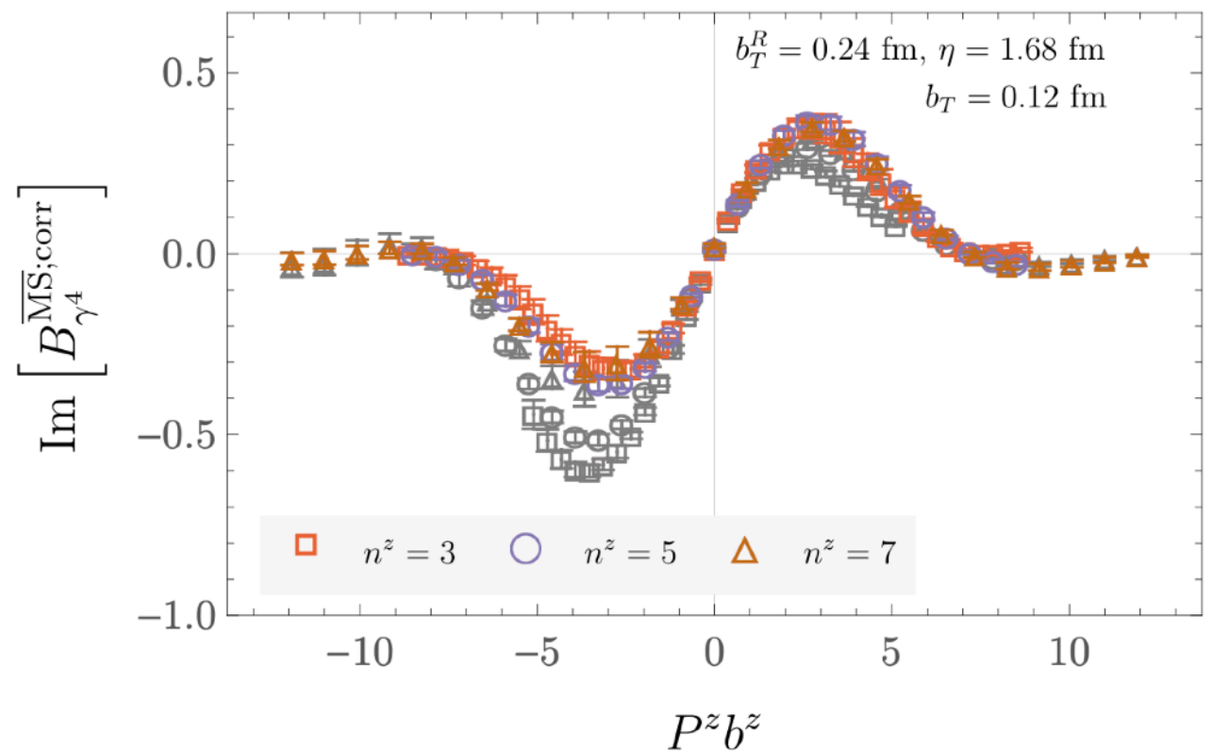
Roughly linear trend in b_T observed

$$\Delta(b_T) = V(b_T)_{\rm quark} - V(b_T)_{\rm pion} \sim \sigma\, b_T$$

Asymmetry correction

After correcting for state dependence of static quark potential, expected (anti)symmetrization of beam function emerges

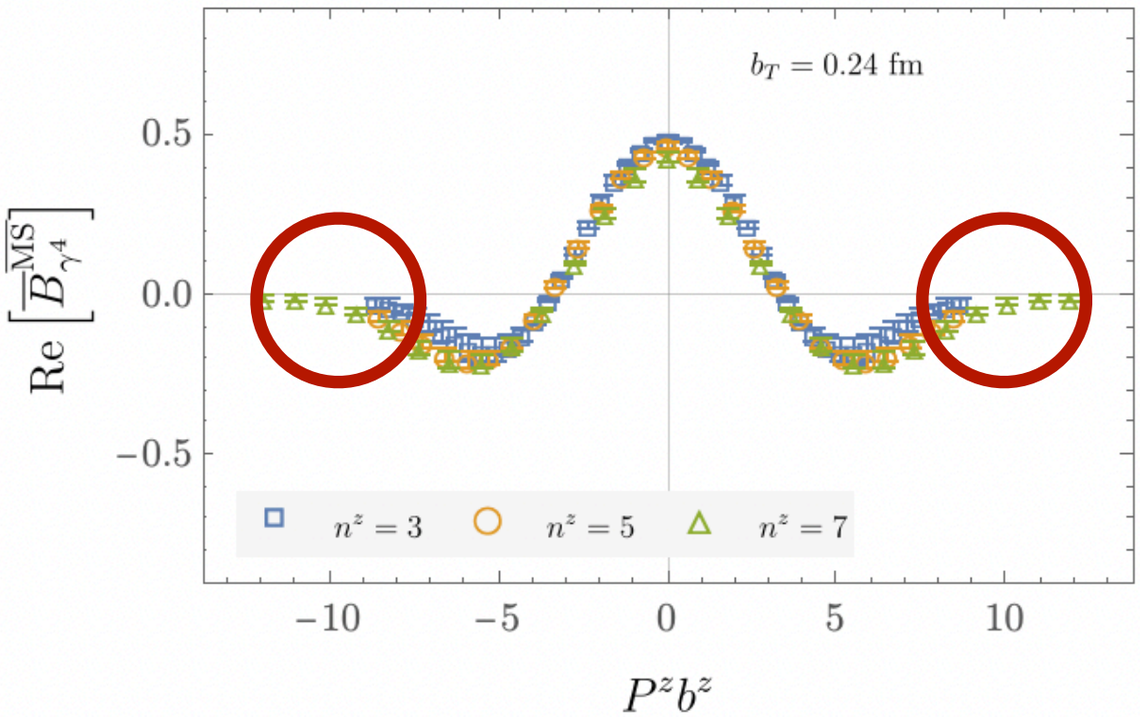




Extrapolation to large η (by a constant) and averaging over choice of b_T^R used in renormalization performed after including corrections independently

Systematics included to reflect variation in η and b_T^R reduced after corrections included

Inverse problems



x is Fourier conjugate to $b \cdot P$

Inverse problem:

Fourier transforming data from a finite interval is a formally ill-posed problem

LQCD results span a finite range of

$$b^z, \qquad P^z, \qquad b \cdot P = b^z P^z$$

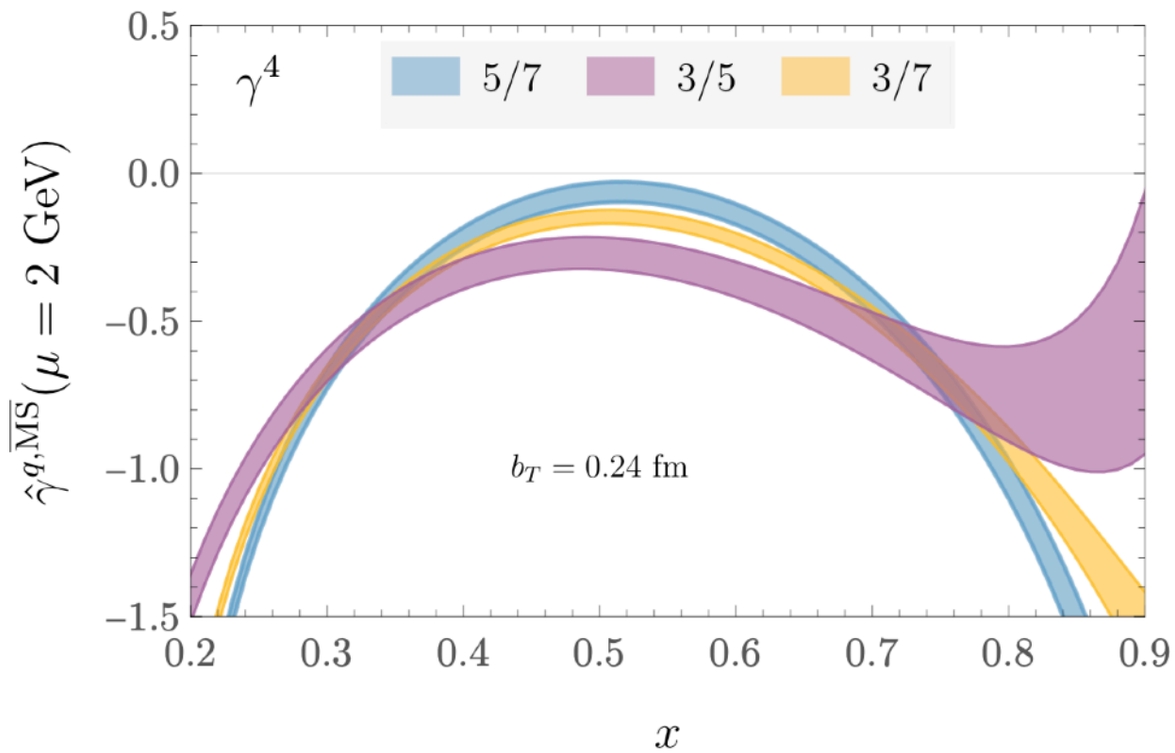
Curves approximately collapse to functions of $b \cdot P$, approach zero for $b \cdot P \to \infty$

Rapid FT convergence as $b \cdot P \to \infty$

Inverse problem is (not) **severe** in practice if LQCD results are (not) **truncated** to values of $b \cdot P$ where the Fourier transform **integrand** is **non-negligible**

Any method to fit function of x given data with limited $b \cdot P$ is an attempt to work around the inverse problem and should be judged by practicailities

CS kernel systematics

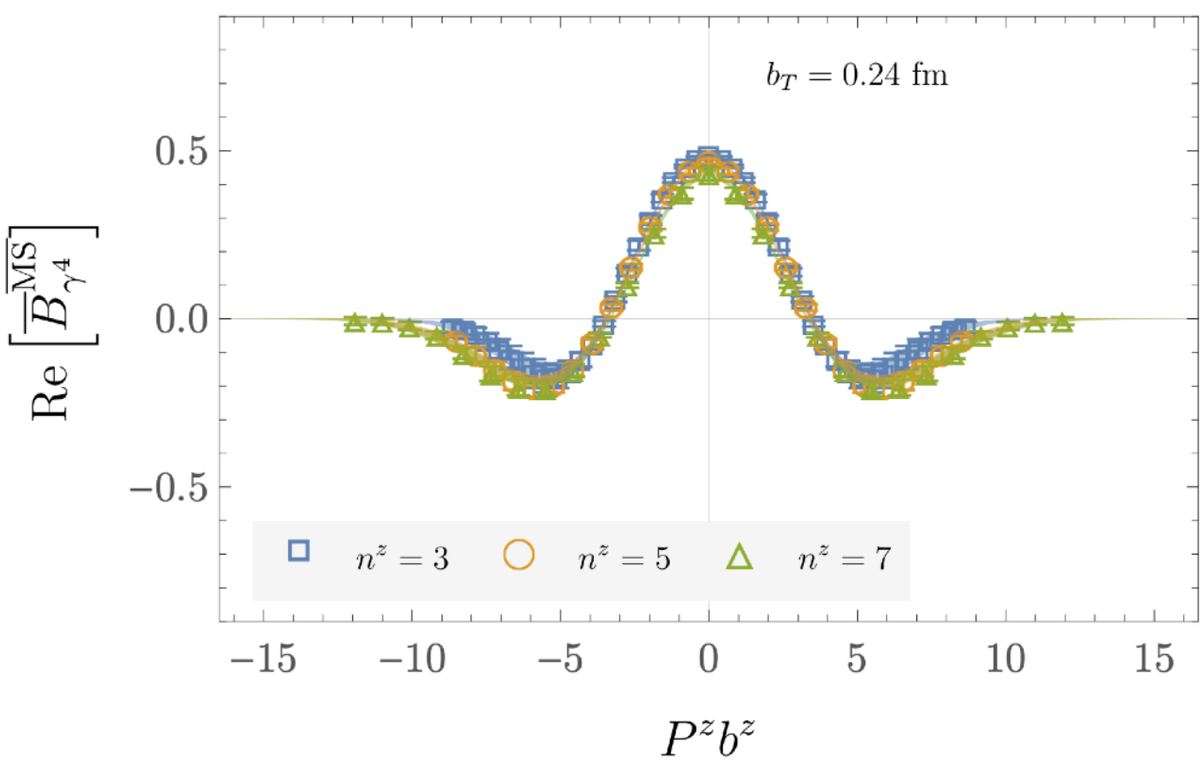


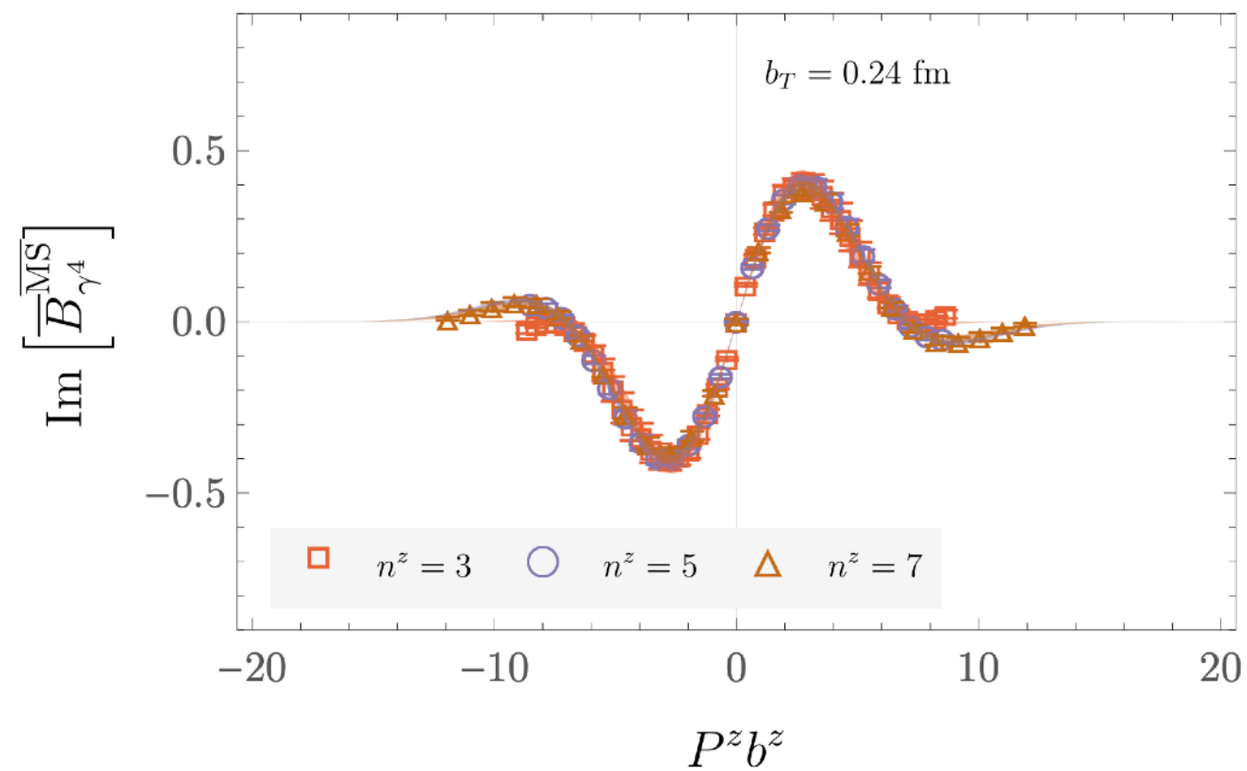
Discrete Fourier transform leads to significant x dependence of (asymptotically flat) CS kernel estimate

Differences between estimates with different momentum pairs visible

Large-distance extrapolation

Fits performed independently for each b_T, P^z to analytic model in order to extrapolate to larger b^z

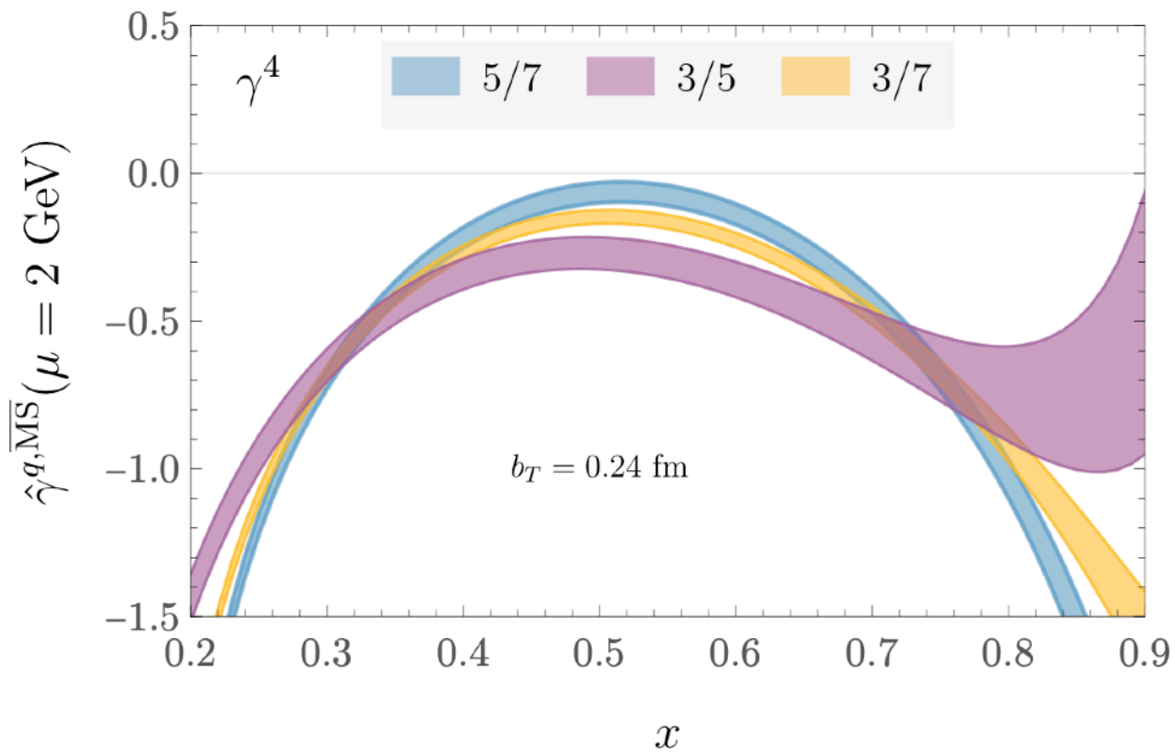




Untruncated Fourier transformations performed analytically after fitting

Improvement from quenched calculation: modeling in coordinate space instead of momentum space permits x dependent extraction of CS kernel and NLO quasi/light-cone matching

CS kernel systematics



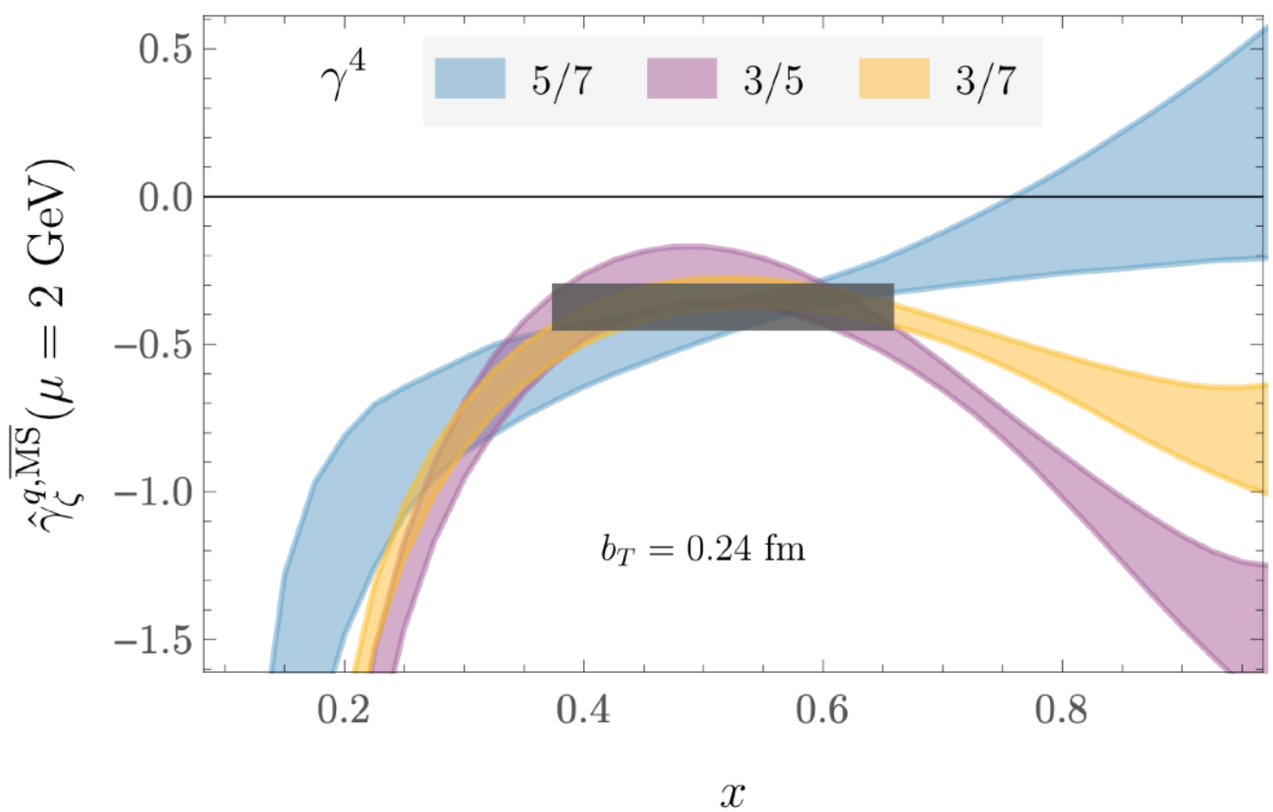
Discrete Fourier transform leads to significant x dependence of (asymptotically flat) CS kernel estimate

Differences between estimates with different momentum pairs visible

Fourier transforming the analytically extrapolated model leads to smaller (though still visible) x and P^z dependence

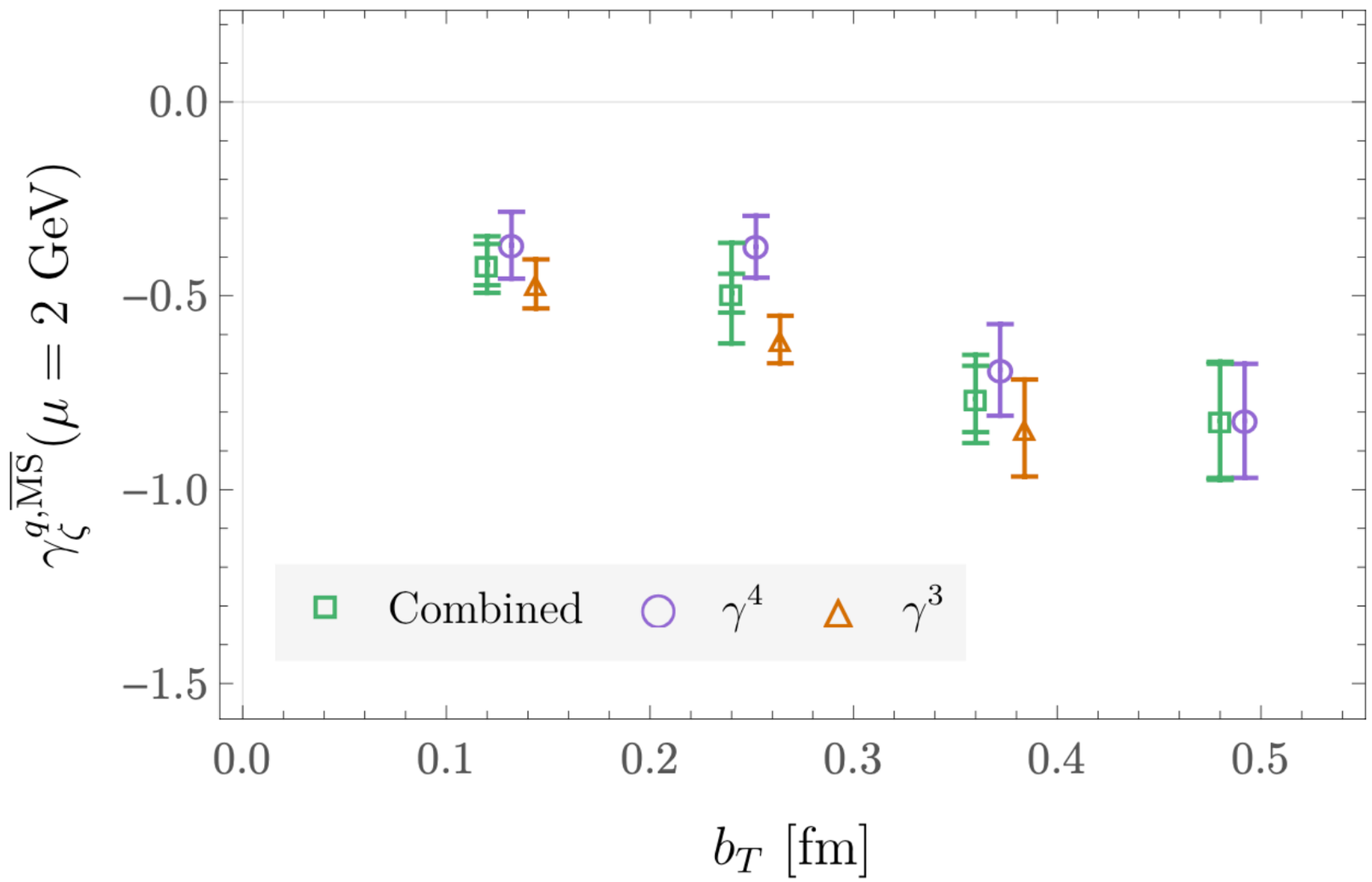
"Plateau region" identified by automated search for overlap between different P^z pairs

Fits of $1/P^z$ artifacts also attempted



CS kernel results

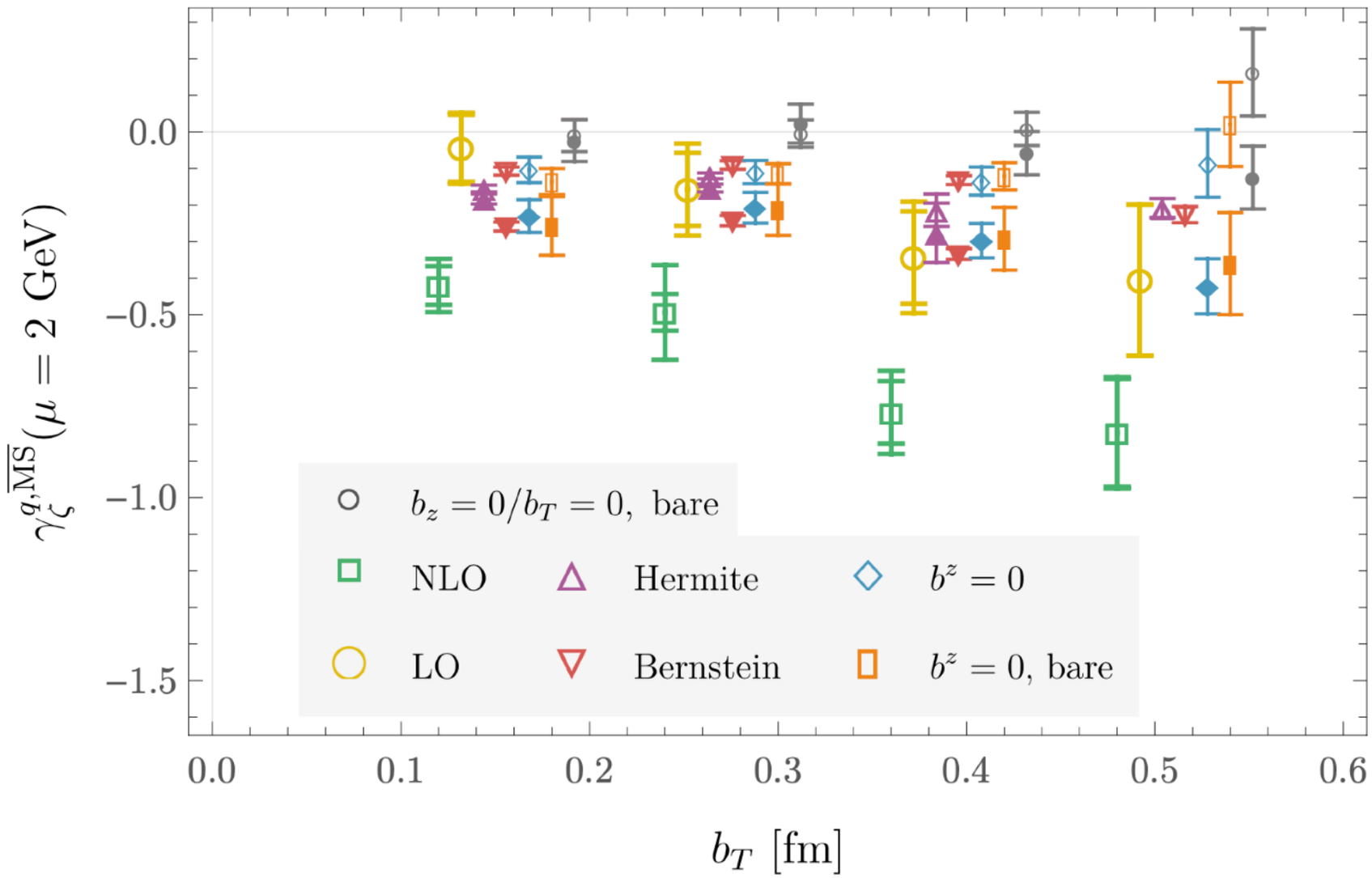
Plateau regions of x-dependent CS kernel used to give final results and (bootstrap confidence interval) uncertainties



Results computed using both γ^4 and γ^3, difference used to estimate systematics due to finite P^z

Comparing approximations

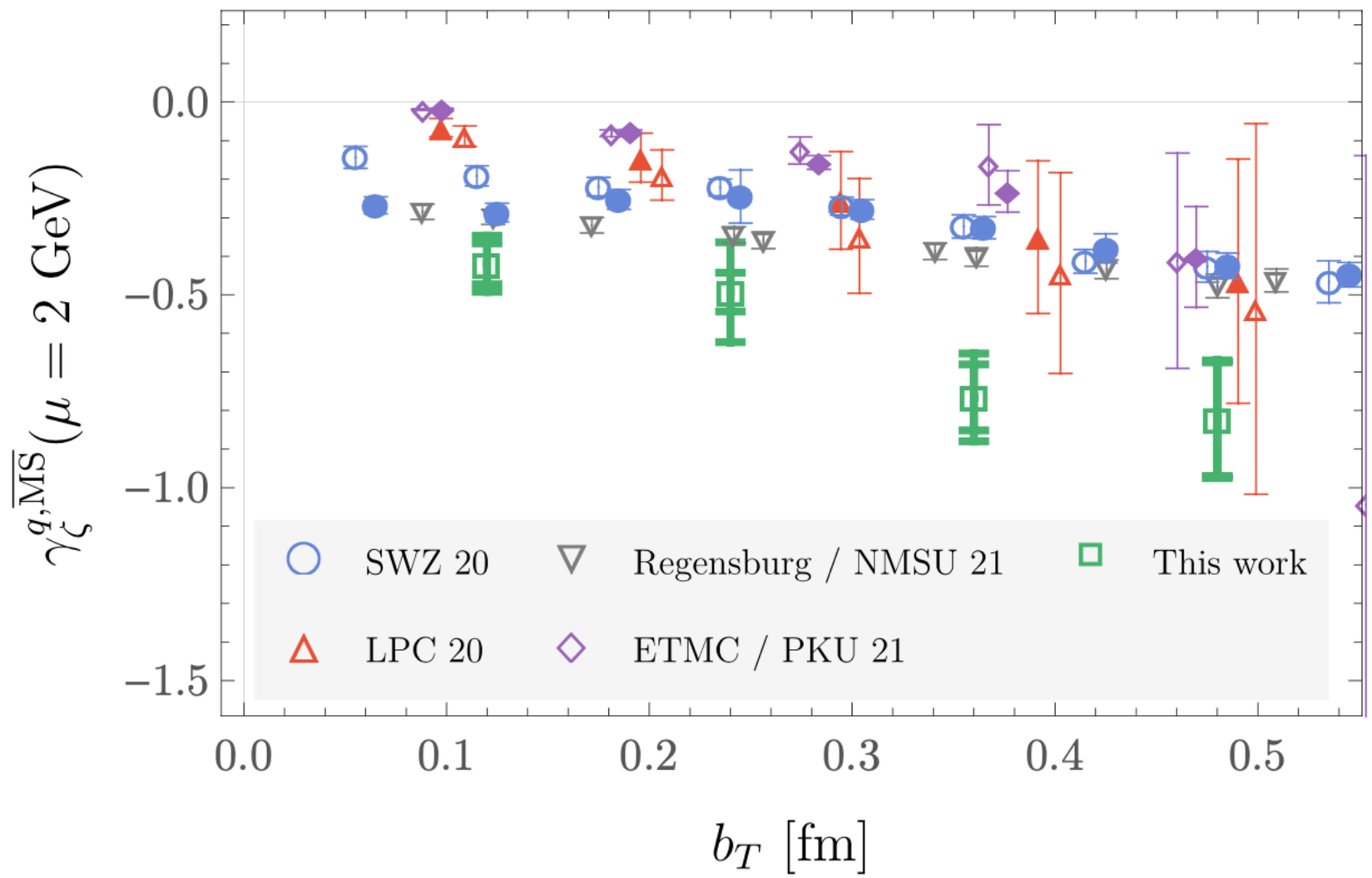
NLO matching leads to significant effects on CS kernel determination



LO results using ratios of $b^z = 0$ beam functions or the momentum-space models used in quenched calculation are consistent with LO results using average over x dependence but give smaller uncertainty estimates

Lattice comparison

Results are broadly consistent with other LQCD calculations (different actions and systematics)

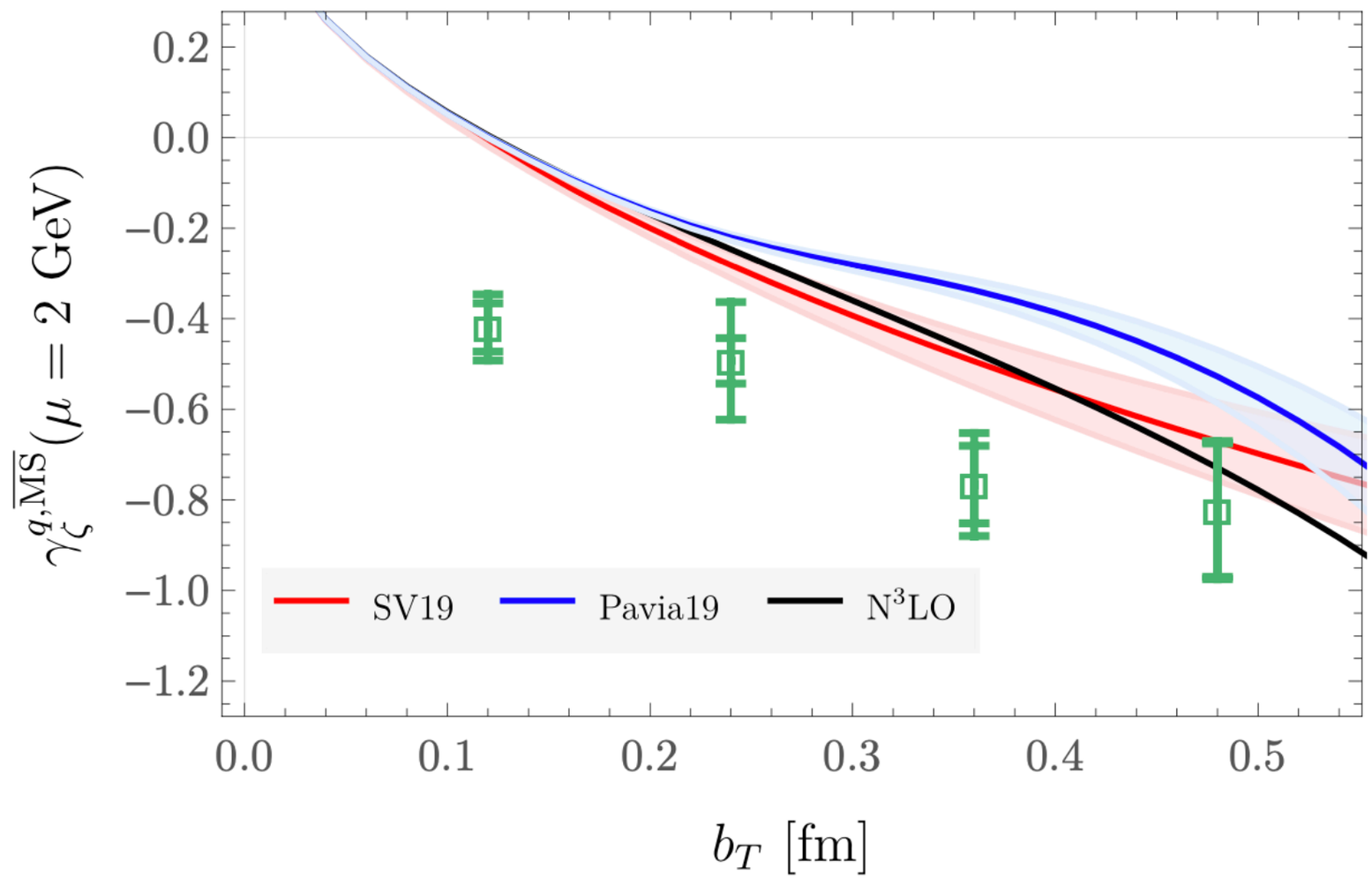


Differences with previous LO calculations (SWZ 20, LPC 20, ETMC / PKU 21) consistent with differences between Fourier transform schemes

Phenomenological comparison

Results can be compared with phenomenology

Warning - no continuum extrapolation, unquantified systematics remain

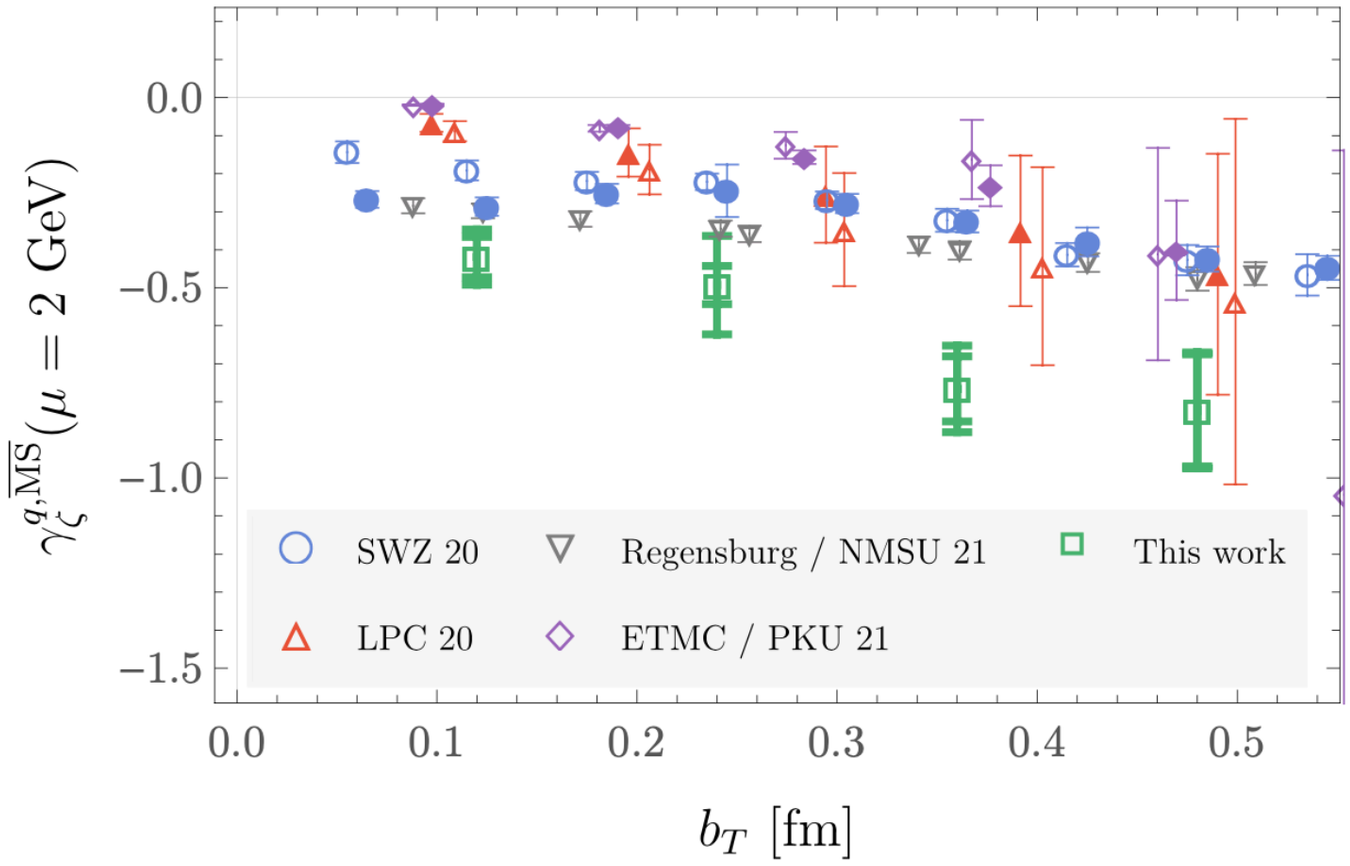


Lattice artifacts at small b_T ? Underestimated Fourier transform systematics?
Further studies needed!

Outlook

Nonperturbative QCD input is required to determine the Collins-Soper kernel governing TMDPDF evolution and improve precision of SIDIS and Drell-Yan predictions / TMDPDF extractions

Recent LQCD results by several groups demonstrate increasing control over challenging systematics in CS kernel calculations



Further calculations using larger $b^z P^z$ are needed to control Fourier transform systematics

Studies of lattice spacing dependence and other schemes such as hybrid renormalization also needed