

# Emergence of Kinematic Space from Quantum Modular Geometric Tensor

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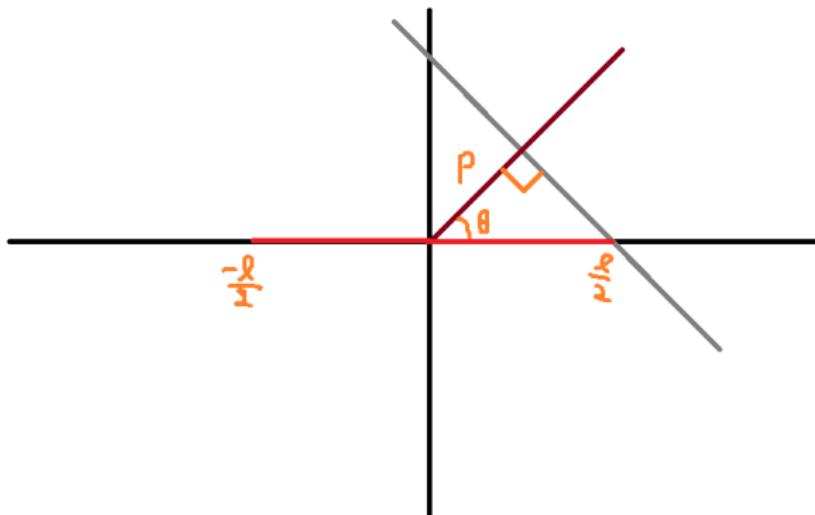
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## Crofton Formula

- Length of a curve=summation of all intersection points
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$$\frac{1}{2} \int_0^{2\pi} \int_0^{\frac{l}{2}|\cos(\theta)|} dp d\theta = l. \quad (1)$$



## Ball Region

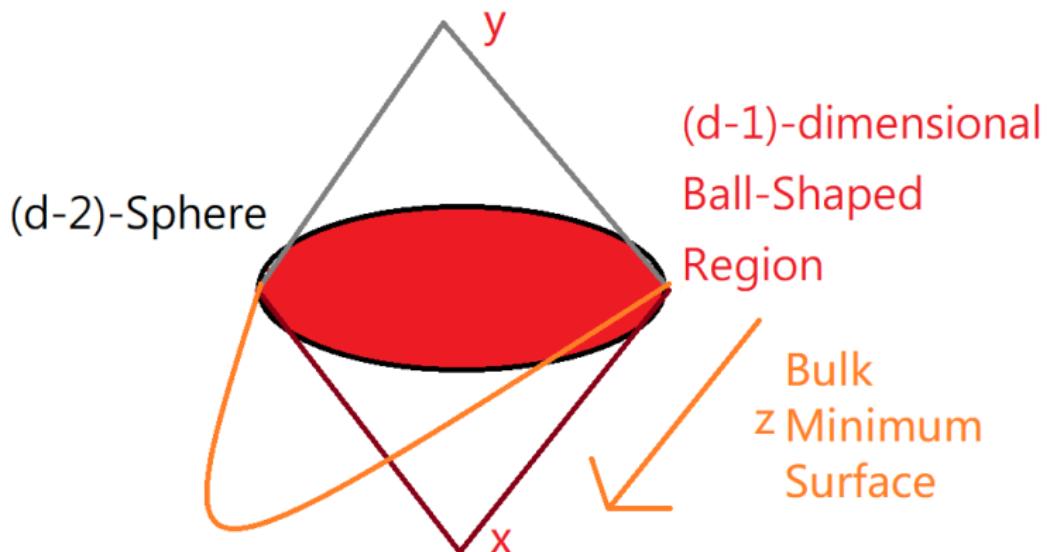


Figure: A causal diamond in  $d = 3$ . The spherical region is specified by a pair of time-like separated points  $x^\mu$  and  $y^\mu$ .

## Parallel Transport

- the tangent vector  $V_{\delta\lambda}$  of a listed curve, which generates a **parallel transport**:

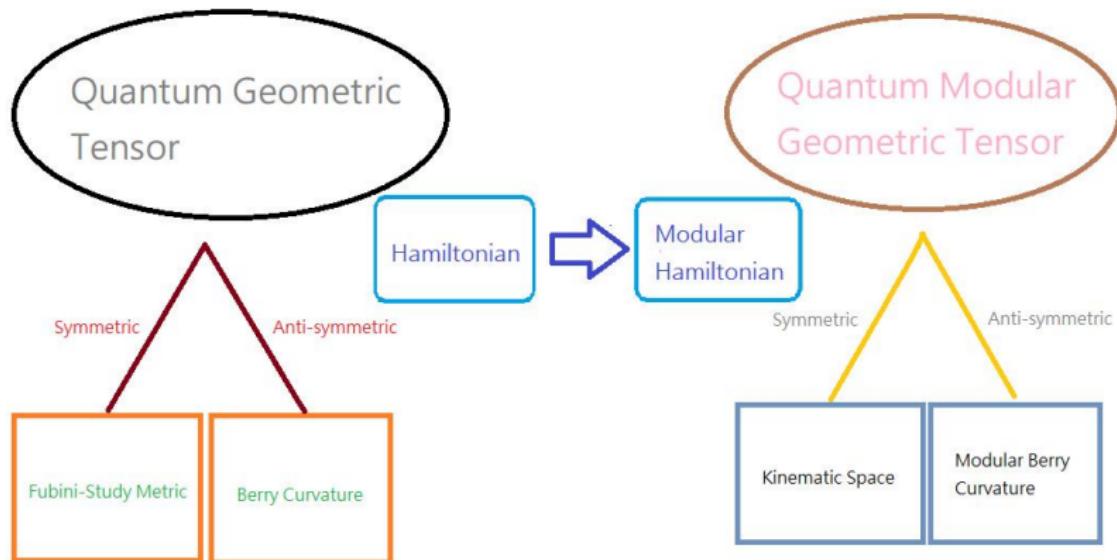
$$\partial_\lambda H_{\text{mod}} = [V_{\delta\lambda}, H_{\text{mod}}]; \quad P_0[V_{\delta\lambda}] = 0, \quad (2)$$

where

$$\begin{aligned} \partial_\lambda H_{\text{mod}} &= (\partial_\lambda x^\mu)(\partial_{\mu,x} H_{\text{mod}}) + (\partial_\lambda y^\mu)(\partial_{\mu,y} H_{\text{mod}}); \\ V_{\delta\lambda} &\equiv \frac{1}{2\pi i} ((\partial_\lambda x^\mu)(\partial_{\mu,x} H_{\text{mod}}) - (\partial_\lambda y^\mu)(\partial_{\mu,y} H_{\text{mod}})), \end{aligned} \quad (3)$$

and the  $P_0$  is a projection operator to the space of **zero modes**

# QGT



## QMGT in CFT

- **QMGT**

$$g_{jk}^{(n)} = \sum_{m \neq n} \frac{\langle n | \partial_j H_{\text{mod}} | m \rangle \langle m | \partial_k H_{\text{mod}} | n \rangle}{(E_{\text{mod}}^{(n)} - E_{\text{mod}}^{(m)})^2}, \quad (4)$$

where  $H_{\text{mod}}$  is diagonal for the eigenstate  $|n\rangle$ , and  $E_{\text{mod}}^{(n)}$  is an eigenvalue of  $H_{\text{mod}}$  for  $|n\rangle$

- $E_{\text{mod}}^{(n)} = E_{\text{mod}}^{(m)} \pm 2\pi i$  when  $\langle m | \partial_{\nu,x;y} H_{\text{mod}} | n \rangle \neq 0$  as in the following:

$$\begin{aligned} & \langle m | [H_{\text{mod}}, \partial_{\nu,x;y} H_{\text{mod}}] | n \rangle \\ &= (E_{\text{mod}}^{(m)} - E_{\text{mod}}^{(n)}) \langle m | \partial_{\nu,x;y} H_{\text{mod}} | n \rangle \\ &= \mp 2\pi i \langle m | \partial_{\nu,x;y} H_{\text{mod}} | n \rangle, \end{aligned} \quad (5)$$

where we use  $[H_{\text{mod}}, \partial_{\nu,x;y} H_{\text{mod}}] = \mp 2\pi i \partial_{\nu,x;y} H_{\text{mod}}$

## Connected Correlator of Wilson Line

- define a new quantity from the **connected correlator of Wilson lines**:

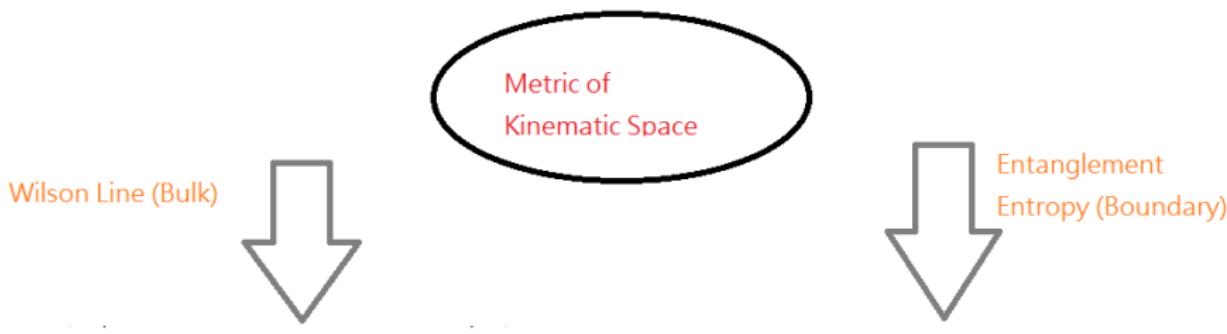
$$\begin{aligned} I[A : B] &= S_{EE}[A] + S_{EE}[B] + \frac{c}{6} \ln \langle 0 | W(x_1, x_2) W(x_3, x_4) | 0 \rangle \\ &= \frac{c}{6} \frac{\langle 0 | W(x_1, x_2) W(x_3, x_4) | 0 \rangle_{\text{connected}}}{\langle 0 | W(x_1, x_2) | 0 \rangle \langle 0 | W(x_3, x_4) | 0 \rangle} + \mathcal{O}\left(\frac{1}{c}\right), \end{aligned}$$

where the region  $A$  is an interval between  $x_1$  and  $x_2$ , and the region  $B$  is an interval between  $x_3$  and  $x_4$

- connected correlator

$$\begin{aligned} &\frac{c}{6} \frac{\langle W(x_1, x_2) W(x_3, x_4) \rangle_{\text{connected}}}{\langle W(x_1, x_2) \rangle \langle W(x_3, x_4) \rangle} \\ &= \frac{1}{12} z^2 {}_2F_1(2, 2; 4, z) + \frac{1}{12} \bar{z}^2 {}_2F_1(2, 2; 4, \bar{z}) + \mathcal{O}\left(\frac{1}{c}\right) \end{aligned}$$

# Mutual Information



$$\begin{aligned} & \langle 0 | H_{\text{mod}}(x - \delta x, y) H_{\text{mod}}(x, y + \delta y) | 0 \rangle \\ & - \langle 0 | H_{\text{mod}}(x - \delta x, y) H_{\text{mod}}(x, y) | 0 \rangle \\ & - \langle 0 | H_{\text{mod}}(x, y) H_{\text{mod}}(x, y + \delta y) | 0 \rangle \\ & + \langle 0 | H_{\text{mod}}(x, y) H_{\text{mod}}(x, y) | 0 \rangle. \end{aligned}$$

$$\begin{aligned} & S_{EE}(x - \delta x, y) + S_{EE}(x, y + \delta y) \\ & - S_{EE}(x, y) - S_{EE}(x - \delta x, y + \delta y) \end{aligned}$$

$$\frac{c}{6} I[(x_1, x_2) : (x_3, x_4)] = \langle H_{\text{mod}}(x_1, x_2) H_{\text{mod}}(x_3, x_4) \rangle + \mathcal{O}(c^0).$$

$(c/6)I(A : B) \rightarrow \text{Mutual Information}$

Introduction

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Modular Berry Geometry

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Quantum Modular Geometric Tensor

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Holographic Entanglement Formula

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# Thank you!