Semillero de
Investigación
SEM18-02

GUT based on the conformal superalgebra

(aka “Unconventional Supersymmetry”)

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Punchline 1/2

- We study a general recipe to implement models for gravity, gauge theories and matter using the **adjoint reresentation** of the superconformal algebra

$$SU(2,2|N)$$

- Fermion/boson matching of d.o.f. is not mandatory.
- Standard gauge kinetic terms are included.
- Models are highly predictive, no free parameters in the action besides one global constant.
- Also included: fermion quartic terms, torsion coupling.

Punchline 2/2

- We constructed a GUT based on

$$SU(2,2|5)_{\text{diag}} = [SU(2,2|5) \times SU(2,2|10)]_{\text{diag}}$$

- That embeds the SU(5) Georgi-Glashow model into the conformal superalgebra.

  The model contains:

- all the quarks and leptons of the SM in the 5* + 10
- Gluons, W and B bosons plus X,Y bosons of the GG model in a 24 of SU(5)
- Also extra Z = (5,5*,ynew) + (5*,5,-ynew), extra SU(5), extra U(1)_ynew

Outline

Why?

- Unification and predictability

How?

- Standard SUSY vs SUSY in the adjoint rep. (aka USUSY)
- Specifics of the GUT model

Conclusions, What is next?

My collaborators in USUSY

- J. Zanelli (CECs)
- P. Pais (U. Austral)
- M. Valenzuela (CECs)
- E. Rodriguez (U. Nac. Colombia)
- P. Salgado (PUCV)
- L. Delage (U. Talca)
- A. Chavez (phd student)

Unification?

MSSM ~ 100 free parameters

SUGRA MSSM ~ 20 free parameters

Grand Unified Theories [Georgi, Glasgow '74]

- Standard model: 15 left-handed fermions

$$(\nu_e, e^-)_L : (1,2)$$
$$e^+_L : (1,1)$$
$$(u_\alpha, d_\alpha)_L : (3,2)$$
$$u^{c\alpha}_L : (3^*,1)$$
$$d^{c\alpha}_L : (3^*,1)$$

- Can be accommodated in the SU(5) reps

The fundamental conjugate rep ψ^i $\quad \mathbf{5^*} = (3^*, 1) + (1, 2^*)$ $\qquad \mathbf{5^*}: (\psi^i)_L = (d^{c1} d^{c2} d^{c3}\ e^- \ -\nu_e)_L$

The antisymmetric 5×5 $\psi_{ij} = -\psi_{ji}$ $\quad \mathbf{10} = (3^*, 1) + (3, 2) + (1, 1).$ $\qquad \mathbf{5}: (\psi_i)_R = (d_1\ d_2\ d_3\ e^+ \ -\nu_e^c)_R$

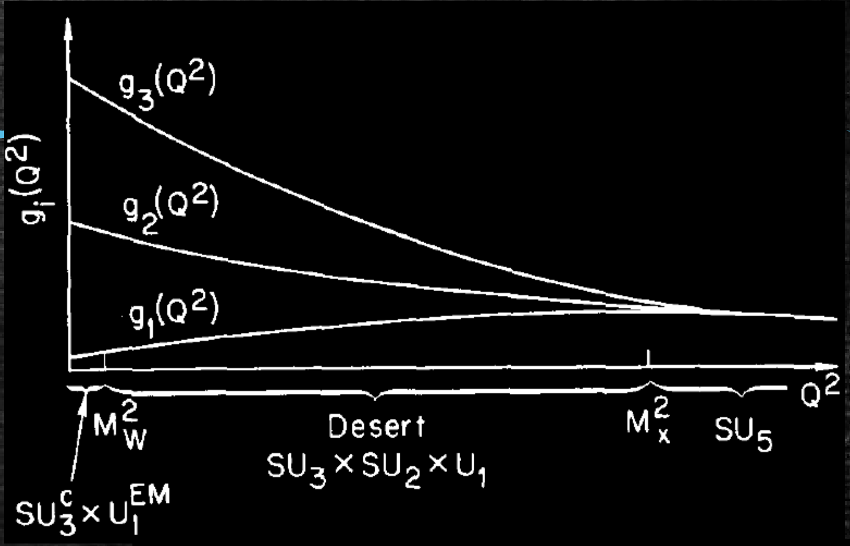
$$\varepsilon^{\alpha\beta\gamma}\psi_{\alpha\beta} \sim (3^*, 1) \qquad \varepsilon_{rs}\psi^{rs} \sim (1, 1)$$

$l^a = (\nu, e)_L$ as a 2 under SU(2) $l^b = \varepsilon^{ab} l_a$

$$\mathbf{10}: (\chi_{ij})_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & u^{c3} & -u^{c2} & u_1 & d_1 \\ -u^{c3} & 0 & u^{c1} & u_2 & d_2 \\ u^{c2} & -u^{c1} & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{bmatrix}_L$$

Unification, quantization of electric charge, anomaly cancellation

- Gauge coupling unification



- Q_{elec} and Y quantization rationale:

$$3Q_d + Q_{e^+} = 0$$

$$Q(\psi_i) = \begin{bmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1 & \\ & & & & 0 \end{bmatrix}$$

- Anomaly cancellation rationale:

$$\frac{A(5^*)}{A(10)} = \frac{\mathrm{tr}\, Q^3(\psi^i)}{\mathrm{tr}\, Q^3(\psi_{ij})} = \frac{3(1/3)^3 + (-1)^3 + 0^3}{3(-2/3)^3 + 3(2/3)^3 + 3(-1/3)^3 + 1^3} = -1$$

$$A(5^*) + A(10) = 0.$$

Interactions

- SU(5) adjoint rep.: SU(3) × SU(2) × U(1) decomposition

$$\mathbf{24} = (\mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{2}) + (\mathbf{3}^*, \mathbf{2}).$$

$$A = \frac{1}{\sqrt{2}} \begin{bmatrix} & & & X_1 & Y_1 \\ & [G - 2B/\sqrt{30}]^\alpha_\beta & & X_2 & Y_2 \\ & & & X_3 & Y_3 \\ X^1 & X^2 & X^3 & W^3/\sqrt{2} + 3B/\sqrt{30} & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -W^3/\sqrt{2} + 3B/\sqrt{30} \end{bmatrix}$$

- Charge assignation

$$Q_X = -4/3 \quad \text{and} \quad Q_Y = -1/3.$$

Interactions

- Lagrangian

$$\mathcal{L}_f = i(\bar{\psi}^c_R)_a(\not{D}\psi^c_R)^a + i(\bar{\psi}_L)_{ac}(\not{D}\psi_L)^{ac}$$

$$= (\bar{\psi}^c_R)_a\left[i\not{\partial}\delta^a_b + \frac{g_5}{\sqrt{2}}\not{A}^a_b\right](\psi^c_R)^b + (\bar{\psi}_L)_{ac}\left[i\not{\partial}\delta^a_b + \frac{2g_5}{\sqrt{2}}\not{A}^a_b\right]\psi^{bc}_L,$$

Gauge kinetic term

$$\mathcal{L}_G = g_5\sum_{i=1}^{8}\left[\bar{u}\not{G}^i\frac{\lambda^i}{2}u + \bar{d}\not{G}^i\frac{\lambda^i}{2}d\right] + g_5\sum_{i=1}^{3}\left[(\bar{u}\bar{d})_L\not{W}^i\frac{\tau^i}{2}\begin{pmatrix}u\\d\end{pmatrix}_L + (\bar{\nu}_e\bar{e}^-)_L\not{W}^i\frac{\tau^i}{2}\begin{pmatrix}\nu_e\\e^-\end{pmatrix}_L\right]$$

$$+\sqrt{\tfrac{3}{5}}g_5[-\tfrac{1}{2}(\bar{\nu}_L\not{B}\nu_L + \bar{e}_L\not{B}e_L) + \tfrac{1}{6}(\bar{u}_L\not{B}u_L + \bar{d}_L\not{B}d_L)$$

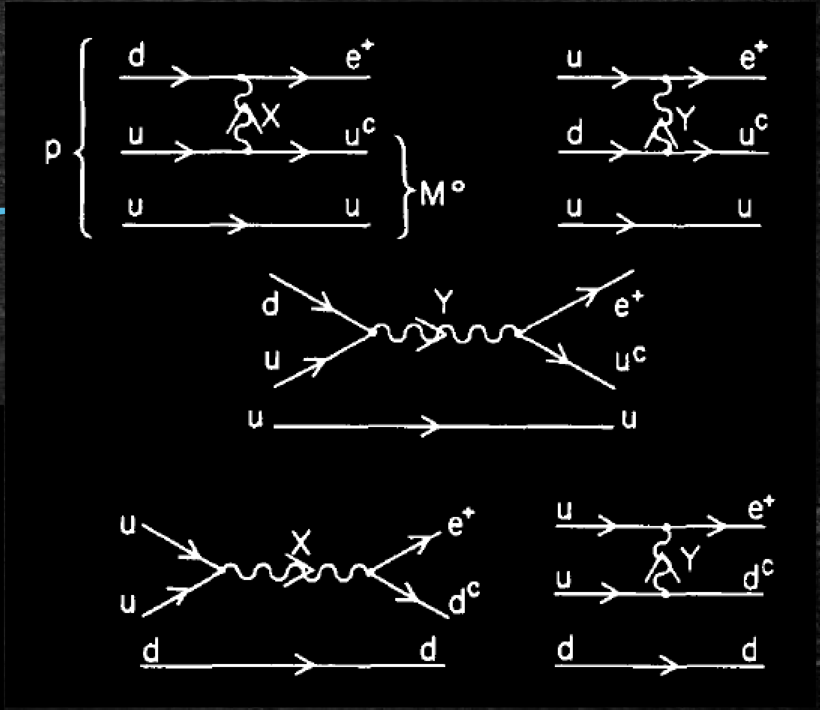
$$+\tfrac{2}{3}\bar{u}_R\not{B}u_R - \tfrac{1}{3}\bar{d}_R\not{B}d_R - \bar{e}_R\not{B}e_R]$$

$$+\left\{\frac{g_5}{\sqrt{2}}\bar{X}^\alpha_\mu[\bar{d}_{R\alpha}\gamma^\mu e^+_R + \bar{d}_{L\alpha}\gamma^\mu e^+_L + \varepsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_L\gamma^\mu u^\beta_L]\right.$$

$$\left.+\frac{g_5}{\sqrt{2}}\bar{Y}^\alpha_\mu[-\bar{d}_{R\alpha}\gamma^\mu\nu^c_R - \bar{u}_{L\alpha}\gamma^\mu e^+_L + \varepsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_L\gamma^\mu d^\beta_L] + \text{H.C.}\right\}.$$

Interactions

- Proton and bound neutron decay



$$+\left\{\frac{g_5}{\sqrt{2}}\bar{X}^{\alpha}_{\mu}[\bar{d}_{R\alpha}\gamma^{\mu}e^{+}_{R}+\bar{d}_{L\alpha}\gamma^{\mu}e^{+}_{L}+\varepsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_{L}\gamma^{\mu}u^{\beta}_{L}]\right.$$

$$\left.+\frac{g_5}{\sqrt{2}}\bar{Y}^{\alpha}_{\mu}[-\bar{d}_{R\alpha}\gamma^{\mu}\nu^{c}_{R}-\bar{u}_{L\alpha}\gamma^{\mu}e^{+}_{L}+\varepsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_{L}\gamma^{\mu}d^{\beta}_{L}]+\text{H.C.}\right\}.$$

- Demand: two stage SSB

$$\mathrm{SU}(5)\xrightarrow{v_1}\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\xrightarrow{v_2}\mathrm{SU}(3)\times\mathrm{U}(1).$$

$$v_1 \gtrsim 10^{12}\, v_2$$

Doublet-triplet problem

SUSY in the adjoint representation

Unconventional matter coupling

- Matter in the adjoint $$\psi^\alpha \in A_\mu$$

- Red. Reps. $$\Psi_\mu^\alpha = 1 \otimes 1/2 = 3/2 \oplus 1/2$$

(a) Gravitino $$\xi_\mu^\alpha : \gamma^\mu \xi_\mu^\alpha = 0 \quad P_{(1/2)} \xi_\mu^\alpha = 0$$

(b) USUSY

$$\psi_\mu^\alpha = \gamma_\mu \psi^\alpha = 0 \quad P_{(3/2)} \psi_\mu^\alpha = 0$$

Unconventional SUSY: **fields in the adjoint rep**

$$\mathbb{A}_\mu \supset \overline{Q} e^a{}_\mu \gamma_a \psi$$

- We choose a basis of the conformal superalgebra where the Q's carry an R-symmetry rep

→ Correct gauge transformations

(see Trigiante's lectures on supergravity)

- From:

$$\delta \mathbb{A} = DG$$

→

$$\begin{cases} \delta A_{SU(N)} = D\lambda_{SU(N)} \\ \delta\psi = [\lambda_{SU(N)}, \psi] \end{cases}$$

SUGRA a la MacDowell-Mansouri

- The action is written as

$$S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

MacDowell, Mansouri '77
Castellani 1802.03407
Trigiante 1609.09745

- Obvious resemblance to Yang-Mills

$$S = \int \langle \mathbb{F} * \mathbb{F} \rangle$$

Similarity can be exploited to study field equations and symmetries [PA, Chavez Zanelli 2111.09845 hep-th]

YM action

- Wanted: $$\langle \mathbb{F} \circledast \mathbb{F} \rangle \propto -\frac{1}{2} F * F + d^4x|e|\overline{\psi} \not{D} \psi$$

Implying

1) Matter in the adjoint rep. $$\psi^\alpha \in A_\mu$$

2) Generalized dual operator $$\circledast = ?$$

Unconventional SUSY: Dirac kinetic term?

- Curvature

$$\mathbb{F} = d\mathbb{A} + \mathbb{A}^2 \supset \overline{Q} D(\not{e}\psi)$$

$$\langle \mathbb{F} \circledast \mathbb{F} \rangle$$

$$\cup$$

$$(\text{here } \overline{\psi}^1 = \overline{\psi}\not{e} \text{ and } \chi^1 = \not{e}\psi)$$

$$\begin{aligned}
\hat{\mathcal{F}}^i \gamma_5 \mathcal{F}_i &= -\overline{\psi}^1 \overleftarrow{D}_\Omega \gamma_5 D_\Omega \chi^1 \\
&= \overline{\psi}^1 (-\overleftarrow{D}^+ + \Omega^-) \gamma_5 (D^+ + \Omega^-) \chi^1 \\
&= -\overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 D^+ \chi^1 - \overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 D^+ \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1 \\
&= d[\overline{\psi}^1 \gamma_5 D^+ \chi^1] + \overline{\psi}^1 \gamma_5 (D^+)^2 \chi^1 + \overline{\psi}(\overleftarrow{D}\not{e} + \not{T}) \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 (-\not{e} D + \not{T}) \psi + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1
\end{aligned}$$

- Grading

$$\Omega = \Omega_+ + \Omega_-$$

Ω_+:

Even generators	Odd generators
$[\mathbb{J}_{ab}, S] = 0$	$\{\mathbb{J}_a, S\} = 0$
$[\mathbb{Z}, S] = 0$	$\{\mathbb{K}_a, S\} = 0$
$[\mathbb{T}_I, S] = 0$	
$[\mathbb{D}, S] = 0$	$\Omega_- = \frac{1}{2} f^a \mathbb{J}_a + \frac{1}{2} g^a \mathbb{K}_a$

- Identification with frames

$$f^a = \rho e^a , \qquad g^a = \sigma e^a ,$$

→ **Dirac kinetic term + GR + CC**

Identification with frames $f^a = \rho e^a\,, \qquad g^a = \sigma e^a\,,$

f^a and g^a are nondynamical thanks to the S grading!

$$\left\{\begin{aligned} &\langle \mathbb{E} \circledast \mathbb{O} \rangle = 0 = \langle \mathbb{O} \circledast \mathbb{E} \rangle\,, \\ &\langle \mathbb{E}_1 \circledast \mathbb{E}_2 \rangle = \langle \mathbb{E}_2 \circledast \mathbb{E}_1 \rangle\,, \\ &\langle \mathbb{O}_1 \circledast \mathbb{O}_2 \rangle = -\langle \mathbb{O}_2 \circledast \mathbb{O}_1 \rangle\,. \end{aligned}\right.$$

Very transparent:
- Field equations and integrability conditions
- splitting of genuine gauge transformations and on-shell symmetries
- Natural definition of self-dual condition

- Cancellation of Pauli-like couplings

Conformal superalgebra for SU(2,2|N) GUT

Superconformal algebra for Unified theories
SU(2,2)×SU(N)×U(1) ⊂ SU(2,2|N)

$$[\mathbb{J}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{J}_c - \eta_{ac}\mathbb{J}_b \,,$$

$$[\mathbb{J}_{ab}, \mathbb{J}_{cd}] = -(\eta_{ac}\mathbb{J}_{bd} - \eta_{ad}\mathbb{J}_{bc} - \eta_{bc}\mathbb{J}_{ad} + \eta_{bd}\mathbb{J}_{ac})$$

$$[\mathbb{T}_I, \mathbb{T}_J] = f^{IJK}\mathbb{T}_K$$

$$[\mathbb{K}_a, \mathbb{K}_b] = -\mathbb{J}_{ab} \,.$$

$$[\mathbb{J}_a, \mathbb{K}_b] = s\eta_{ab}\mathbb{D} \,.$$

$$[\mathbb{K}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{K}_c - \eta_{ac}\mathbb{K}_b \,.$$

$$[\mathbb{D}, \mathbb{K}_a] = -s^{-1}\mathbb{J}_a \,.$$

$$[\mathbb{D}, \mathbb{J}_a] = -s\mathbb{K}_a \,.$$

Superconformal algebra for Unified theories
SU(2,2)×SU(N)×U(1) ⊂ SU(2,2|N)

$$[\mathbb{T}_I, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{i}{2}\overline{\mathbb{Q}}^j_\alpha (\lambda_I)_j{}^i\,, \quad [\mathbb{T}_I, \mathbb{Q}^\alpha_i] = \tfrac{i}{2}(\lambda_I)_i{}^j \mathbb{Q}^\alpha_j\,,$$

$$[\mathbb{Z}, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{iz}{3}\overline{\mathbb{Q}}^i_\alpha\,, \quad [\mathbb{Z}, \mathbb{Q}^\alpha_i] = \tfrac{iz}{3}\mathbb{Q}^\alpha_i\,,$$

$$\{\mathbb{Q}^\alpha_i, \overline{\mathbb{Q}}^j_\beta\} = \left(\tfrac{1}{2s}(\gamma^a)^\alpha{}_\beta \mathbb{J}_a - \tfrac{1}{2}(\Sigma^{ab})^\alpha{}_\beta \mathbb{J}_{ab} - \tfrac{1}{2}(\tilde{\gamma}^a)^\alpha{}_\beta \mathbb{K}_a + \tfrac{1}{2}(\gamma^5)^\alpha{}_\beta \mathbb{D}\right)\delta^j_i + \delta^\alpha_\beta\left(-i(\lambda_I)_i{}^j \mathbb{T}_I - \tfrac{i}{4z}\delta^j_i \mathbb{Z}\right)$$

$$[\mathbb{J}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{J}_c - \eta_{ac}\mathbb{J}_b\,,$$

$$[\mathbb{J}_{ab}, \mathbb{J}_{cd}] = -(\eta_{ac}\mathbb{J}_{bd} - \eta_{ad}\mathbb{J}_{bc} - \eta_{bc}\mathbb{J}_{ad} + \eta_{bd}\mathbb{J}_{ac})$$

$$[\mathbb{T}_I, \mathbb{T}_J] = f^{IJK}\mathbb{T}_K$$

$$[\mathbb{K}_a, \mathbb{K}_b] = -\mathbb{J}_{ab}\,.$$

$$[\mathbb{J}_a, \mathbb{K}_b] = s\eta_{ab}\mathbb{D}\,.$$

$$[\mathbb{K}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{K}_c - \eta_{ac}\mathbb{K}_b\,.$$

$$[\mathbb{D}, \mathbb{K}_a] = -s^{-1}\mathbb{J}_a\,.$$

$$[\mathbb{D}, \mathbb{J}_a] = -s\mathbb{K}_a\,.$$

$$[\mathbb{J}_a, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{s}{2}\overline{\mathbb{Q}}^i_\beta(\gamma_a)^\beta{}_\alpha\,, \quad [\mathbb{J}_a, \mathbb{Q}^\alpha_i] = -\tfrac{s}{2}(\gamma_a)^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

$$[\mathbb{J}_{ab}, \overline{\mathbb{Q}}^i_\alpha] = \overline{\mathbb{Q}}^i_\beta(\Sigma_{ab})^\beta{}_\alpha\,, \quad [\mathbb{J}_{ab}, \mathbb{Q}^\alpha_i] = -(\Sigma_{ab})^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

$$[\mathbb{K}_a, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}^i_\beta(\tilde{\gamma}_a)^\beta{}_\alpha\,, \quad [\mathbb{K}_a, \mathbb{Q}^\alpha_i] = -\tfrac{1}{2}(\tilde{\gamma}_a)^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

$$[\mathbb{D}, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}^i_\beta(\gamma_5)^\beta{}_\alpha\,, \quad [\mathbb{D}, \mathbb{Q}^\alpha_i] = -\tfrac{1}{2}(\gamma_5)^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

Superconformal algebra for Unified theories
SU(2,2)×SU(N)×U(1) ⊂ SU(2,2|N)

$$[\mathbb{T}_I, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{i}{2}\overline{\mathbb{Q}}^j_\alpha(\lambda_I)_j{}^i\,, \quad [\mathbb{T}_I, \mathbb{Q}^\alpha_i] = \tfrac{i}{2}(\lambda_I)_i{}^j\mathbb{Q}^\alpha_j\,,$$

GG model →

$$\mathbf{5}^*: (\psi^i)_L = (d^{c1} d^{c2} d^{c3}\; e^- \; - \nu_e)_L$$

Wanted: Q_{ij}

$$\Psi = \overline{Q}^{ij} \chi_{ij} \subset \mathbb{A}$$

GG model →

$$\mathbf{10}: (\chi_{ij})_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & u^{c3} & -u^{c2} & u_1 & d_1 \\ -u^{c3} & 0 & u^{c1} & u_2 & d_2 \\ u^{c2} & -u^{c1} & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{bmatrix}_L$$

Embedding Q_{ij} supercharges in the conformal algebra

$$[\mathbb{T}_I, \mathbb{Q}^\alpha_{ij}] = i(t_I)_{ij}{}^{kl}\mathbb{Q}^\alpha_{kl}\,, \qquad [\mathbb{T}_I, \overline{\mathbb{Q}}^{ij}_\alpha] = -i\overline{\mathbb{Q}}^{kl}_\alpha(t_I)_{kl}{}^{ij}\,,$$

$$(t_I)_{ij}{}^{kl} = \Delta^{mn}_{ij}(\lambda_I)_m{}^p\delta^q_n\Delta^{kl}_{pq}\,,$$

$$[t_I, t_J]_{ij}{}^{kl} \equiv (t_I)_{ij}{}^{pq}(t_J)_{pq}{}^{kl} - (t_J)_{ij}{}^{pq}(t_I)_{pq}{}^{kl}\,.$$

$$(\mathbb{Q}^\alpha_{ij})^A{}_B = 2\Delta^A_{ij}\delta^\alpha_B\,, \qquad (\overline{\mathbb{Q}}^{ij}_\alpha)^A{}_B = 2\delta^A_\alpha\Delta^{ij}_B\,.$$

Embedding Q_{ij} supercharges in the conformal algebra

There is catch!

$$(\mathbb{Q}^{\alpha}_{ij})^{A}{}_{B} = 2\Delta^{A}_{ij}\delta^{\alpha}_{B}\,, \qquad (\overline{\mathbb{Q}}^{ij}_{\alpha})^{A}{}_{B} = 2\delta^{A}_{\alpha}\Delta^{ij}_{B}\,.$$

$\rightarrow$ $SU(2,2|10)$

$\frac{N(N-1)}{2}$

$[i,j]$	i'	A
$1,2$	1	$\alpha+1$
$1,3$	2	$\alpha+2$
$\vdots$	$\vdots$	$\vdots$
$1,n$	n	$\alpha+n$
$2,1$	$n+1$	$\alpha+n+1$
$\vdots$	$\vdots$	$\vdots$
$n-1,n$	d_n	$\alpha+d_n$

$$(\mathbb{T}_I)^{A}{}_{B} = 2i(t^t_I)^{A}{}_{B} = 2i(t_I)_{B}{}^{A} \equiv 2i\Delta^{A}_{kl}(t_I)_{ij}{}^{kl}\Delta^{ij}_{B}\,, \quad \in \quad \mathbb{T}_X$$

$(\mathbb{T}_I)$: $su(5)$

$\mathbb{T}_X$: $su(10)$

New bosons

$$A_X = \{A_I, A_{\tilde{X}}\}$$

$$\not{\nabla}' \chi_L^{\text{phys}} = \not{\nabla}_{su(5)} \chi_L^{\text{phys}} - ig \not{A}^{\tilde{X}} t_{\tilde{X}} \chi_L^{\text{phys}} - ig_{(U(1))}^{(\text{rank } 2)} \not{A} \chi_L^{\text{phys}} ,$$

- Group theory decomposition

$$\mathbf{99} = (\mathbf{24}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{24}, 0) + (\mathbf{1}, \mathbf{1}, 0) + (\mathbf{5}, \mathbf{5}^*, -y_{\text{new}}) + (\mathbf{5}^*, \mathbf{5}, y_{\text{new}})$$

Charge assignation 5^*

$$(\psi_i)_L = \begin{pmatrix} d_1^c \\ d_1^c \\ d_1^c \\ e^- \\ -\nu_e \end{pmatrix}_L = \begin{pmatrix} d_1^c \\ d_1^c \\ d_1^c \\ 0 \\ 0 \end{pmatrix}_L + \begin{pmatrix} 0 \\ 0 \\ 0 \\ e^- \\ 0 \end{pmatrix}_L + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\nu_e \end{pmatrix}_L$$

$$= (\psi_i(d^c))_L + (\psi_i(e^-))_L + (\psi_i(\nu_e))_L$$

Commutators in the superalgebra!

$$\Psi(x) = \overline{Q}\psi(x)$$

$$[Q_{elec}, \Psi(x)] = q_{elec}\Psi(x) \qquad [Y, \Psi(x)] = y\Psi(x)$$

Charge assignation 10

$$(\chi_{ij})_L = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}_L = \begin{pmatrix} 0 & 0 & 0 & 0 & -d^1 \\ 0 & 0 & 0 & 0 & -d^2 \\ 0 & 0 & 0 & 0 & -d^3 \\ 0 & 0 & 0 & 0 & 0 \\ d^1 & d^2 & d^3 & 0 & 0 \end{pmatrix}_L + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^+ \\ 0 & 0 & 0 & e^+ & 0 \end{pmatrix}_L$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -u^1 & 0 \\ 0 & 0 & 0 & -u^2 & 0 \\ 0 & 0 & 0 & -u^3 & 0 \\ u^1 & u^2 & u^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_L + \begin{pmatrix} 0 & u_3^c & -u_2^c & 0 & 0 \\ -u_3^c & 0 & u_1^c & 0 & 0 \\ u_2^c & -u_1^c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_L$$

$$= (\chi_{ij}(d))_L + (\chi_{ij}(e^+))_L + (\chi_{ij}(u))_L + (\chi_{ij}(u^c))_L \qquad (2.11)$$

Commutators in the superalgebra!

$$\Psi(x) = \overline{Q}\chi(x)$$

$$[Q_{elec}, \Psi(x)] = q_{elec}\Psi(x) \qquad [Y, \Psi(x)] = y\Psi(x)$$

GUT model action

$$\mathcal{S} = -\int \left(\langle \xi \mathbb{F} \circledast \mathbb{F} \rangle + \langle \xi' \mathbb{F}' \circledast \mathbb{F}' \rangle \right)$$

$$\begin{aligned}\circledast \mathbb{F} =& (\varepsilon_s S) \left(\frac{1}{2} \mathcal{F}^{ab} \mathbb{J}_{ab} + \mathcal{F}^a \mathbb{J}_a + \mathcal{G}^a \mathbb{K}_a \right) \\ &+ (\varepsilon_1 *) \mathcal{H} \mathbb{D} + (\varepsilon_2 *) \mathcal{F}^I \mathbb{T}_I + (\varepsilon_3 *) \mathcal{F} \mathbb{Z} \\ &+ \overline{\mathbb{Q}} (-i \varepsilon_\psi \gamma_5) \mathcal{X} + \overline{\mathcal{X}} (-i \varepsilon_\psi \gamma_5) \mathbb{Q} \,.\end{aligned}$$

$$\varepsilon_s = +1 = \varepsilon_1 = \varepsilon_2 = -\varepsilon_3$$

Explicit computation gives:

$$\begin{aligned}\mathcal{L} =& \frac{1}{4} \varepsilon_s (\xi + \xi') \epsilon_{abcd} \mathcal{F}^{ab} \mathcal{F}^{cd} - \varepsilon_1 (\xi + \xi') \mathcal{H} * \mathcal{H} \\ &- \frac{1}{2} \varepsilon_2 \left(\xi \mathcal{F}^I * \mathcal{F}^I + \xi' (n-2) \mathcal{F}'^X * \mathcal{F}'^X \right) \\ &- 4 \varepsilon_3 \left[\xi (4/n - 1) + \xi' (4/d_n - 1) \right] \mathcal{F} * \mathcal{F} \,, \\ &- 2 i \varepsilon_\psi \overline{\mathcal{X}} \gamma_5 \mathcal{X} - \frac{i}{2} \varepsilon_\chi \overline{\mathcal{Y}} \gamma_5 \mathcal{Y} \,.\end{aligned}$$

Dirac terms

$$-2i\varepsilon_{\psi}\overline{\mathcal{X}}\gamma_5\mathcal{X}-\frac{i}{2}\varepsilon_{\chi}\overline{\mathcal{Y}}\gamma_5\mathcal{Y}\,.$$

- Action

$$\mathscr{L}_{\mathrm{f}}=\mathrm{i}(\bar{\psi}^{\mathrm{c}}_{\mathrm{R}})_a(\not{D}\psi^{\mathrm{c}}_{\mathrm{R}})^a+\mathrm{i}(\bar{\psi}_{\mathrm{L}})_{ac}(\not{D}\psi_{\mathrm{L}})^{ac}$$

$$\begin{aligned}\not{\nabla}\psi_L^{\mathrm{phys}} &=\not{\nabla}_{su(5)}\psi_L^{\mathrm{phys}}-ig^{(\mathrm{rank}\ 1)}_{(U(1))}\not{A}\psi_L^{\mathrm{phys}}\,,\\ \not{\nabla}'\chi_L^{\mathrm{phys}} &=\not{\nabla}_{su(5)}\chi_L^{\mathrm{phys}}-ig\not{A}^{\tilde{X}}t_{\tilde{X}}\chi_L^{\mathrm{phys}}-ig^{(\mathrm{rank}\ 2)}_{(U(1))}\not{A}\chi_L^{\mathrm{phys}}\,,\end{aligned}$$

New w.r.t. X, Y of the GG model

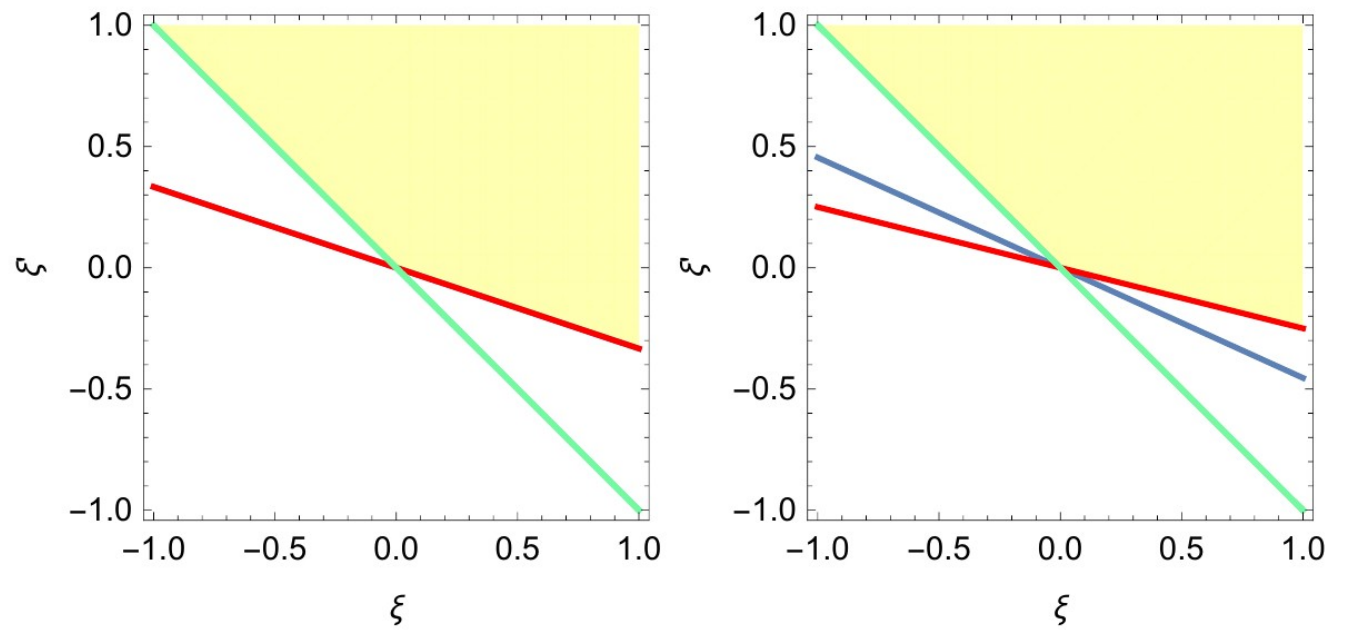
Coupling constants

- Bosonic part

$$\mathcal{L}_{\rm b} = \frac{1}{4}\varepsilon_s(\xi+\xi')\epsilon_{abcd}\mathcal{R}^{ab}\mathcal{R}^{cd} - \varepsilon_1(\xi+\xi')H*H - \frac{1}{2}\varepsilon_2\left(\xi+\xi'(n-2)\right)F^I*F^I - 4\varepsilon_3\left[\xi(4/n-1)+\xi'(4/d_n-1)\right]F*F - \frac{(n-2)}{2}\varepsilon_2\xi'\left[2F^I*F^I_1 + F^I_1*F^I_1 + F^{\widetilde{X}}*F^{\widetilde{X}}\right],$$

- No ghost conditions

$$\xi+\xi' > 0\,,$$
$$\xi(4/n-1)+\xi'(4/d_n-1) < 0\,,$$
$$\xi+\xi'(n-2) > 0\,.$$



We overcome technical difficulties encountered by Ferrara, Kaku, Townsend, van Nieuwenhuizen ‘77

Coupling constants

- Canonical normalization of the fields

$$-aF * F = -\frac{1}{2}F' * F',$$

$$D = d - ig_0\ \rho(T_r)A^r,$$

$$A' = \sqrt{2a}A$$

$$g_{(SU(n))} = g_{(SU(d_n))} = \frac{1}{\sqrt{\xi + \xi'(n-2)}},$$

$$g^{(\text{rank }1)}_{(U(1))} = \frac{4/n - 1}{\sqrt{-8(\xi(4/n-1) + \xi'(4/d_n - 1))}},$$

$$g^{(\text{rank }2)}_{(U(1))} = \frac{4/d_n - 1}{\sqrt{-8(\xi(4/n-1) + \xi'(4/d_n - 1))}}.$$

Summary of the model

- Symmetry group

$$SU(2,2|5)_{\mathrm{diag}} = [SU(2,2|5) \times SU(2,2|10)]_{\mathrm{diag}}$$

- All fields in the adjoint rep.

$$\mathbb{A} = \Omega + \overline{\mathbb{Q}}^i \not{e} \psi_i + \overline{\psi}^i \not{e} \mathbb{Q}_i \,,$$

$$\mathbb{A}' = \Omega' + \frac{1}{2} \overline{\mathbb{Q}}^{ij} \not{e} \chi_{ij} + \frac{1}{2} \overline{\chi}^{ij} \not{e} \mathbb{Q}_{ij} \,,$$

$$\Omega = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + f^a\mathbb{J}_a + g^a\mathbb{K}_a + h\mathbb{D} + A^I\mathbb{T}_I + A\mathbb{Z} \,,$$

$$\Omega' = \frac{1}{2}\omega'^{ab}\mathbb{J}_{ab} + f'^a\mathbb{J}_a + g'^a\mathbb{K}_a + h'\mathbb{D} + A'^X\mathbb{T}_X + A'\mathbb{Z} \,.$$

- Diagonal symmetry group

$$\omega'^{ab} = \omega^{ab} \,, \quad f'^a = f^a \,, \quad g'^a = g^a \,, \quad h' = h \,, \quad A' = A \,,$$

$$A'^X\big|_{X=I} = A^I \,.$$

- Highly predictive
- Embedding of SU(5) GG model + new gauge fields
- Chiral theory from a L-R handed symmetric theory

Outlook

- Pheno. SSB: SU(10) $\rightarrow$ SU(5) $\rightarrow$ SU(3) $\times$ SU(2) $\times$ U(1) $\rightarrow$ SU(2) $\times$ U(1)

$$\mathbf{99} = (\mathbf{24}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{24}, 0) + (\mathbf{1}, \mathbf{1}, 0) + (\mathbf{5}, \mathbf{5}^*, -y_{\mathrm{new}}) + (\mathbf{5}^*, \mathbf{5}, y_{\mathrm{new}}) \,.$$

- Embedding of other GUT schemes that are phenomenologically more successful (Pati-Salam SO(10)?)
- Model with gravitini: full theory and study of the on-shell symmetries (horizontal symmetries)
- USUSY non-renormalization theorems?
- Nieh-Yang-Weyl symmetry anomaly?

ευχαριστώ

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