

Sifting through the SM for the hints of an ALP

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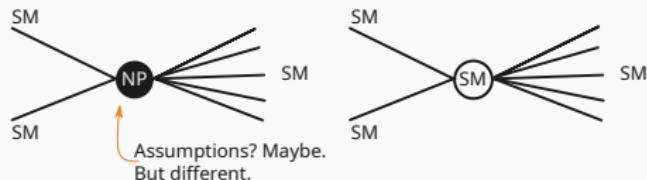
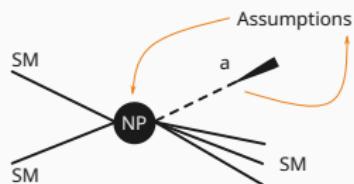


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Direct vis à vis Indirect



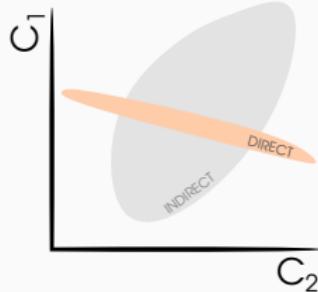
Exotic Signatures

- Missing
- Displaced etc

Precision SM

- Branching Frac
- Shape etc

EFT approach:
Term by Term



Overview

Shift Symmetric $a \rightarrow a + c$

- $aG\tilde{G}$ $aW\tilde{W}$ $aB\tilde{B}$
- $\frac{1}{f_a} \partial_\mu a [C_L^i \bar{q}_L^i t_{ij}^a q_L^j + L \leftrightarrow R]$
- $\bar{q}_L \left[e^{ix_L^a T_a a / f_a} M_q \right] e^{ix_R^b T_b a / f_a} q_R$

- $q_{L/R} \rightarrow SU(3)_{L/R}$ 3
- Mass from QCD anomaly
- Rotate $aG\tilde{G}$ etc away
 - $q_R \rightarrow e^{i \frac{a}{f_a}} q_R$
- Redef Invariance

The ‘bare’ mass

- $\frac{1}{2} m_a^2 a^2$

Periodic Symmetry? $a \rightarrow a + \frac{2\pi}{n}$

- aGG aWW aBB , $\frac{1}{f_a} a \bar{q} \gamma^\mu q j_\mu$ Maybe Later!
! e.g., Leading terms of $\sin(a), \cos(a)$

The Lagrangian we want

Theory of light mesons and an ALP: $A\chi\text{PT}$

- $\mathcal{L}_{A\chi PT} = \mathcal{L}_{A\chi PT}^{SM} + \mathcal{L}_{A\chi PT}^a$
 - In flavour basis
 - $a - \pi_0$ two-point functions \rightarrow redefined π^0
 - $a \mathcal{O}(\pi_0, \pi_+, j_\pm^\mu, \dots)$
 - $\implies \pi^0 \mathcal{O}(\pi_0, \pi_+, j_\pm^\mu, \dots)$ In mass basis
 - Modifies the $\langle K^+/\pi^+ | \mathcal{O} | \pi^0 \rangle$ form factors
 - Goes on to mess up light-Meson mass spectrum
 - Modifies Meson decay Sum rules

Ongoing: The same exercise for HQET

To the chiral Lagrangian

- The symmetry breaking:

$$SU(3)_L \times SU(3)_R \xrightarrow{\langle 0 | \bar{q}_L^i q_R^i | 0 \rangle \sim \Lambda^3 \delta^{ij}} SU(3)_V$$

- Non-linear representation of the Goldstones:

$$U_\pi \equiv e^{2t_i \frac{\pi^i}{f_\pi}} \xrightarrow{L \times R} L U_\pi R^\dagger$$

- The Lagrangian:

$$\begin{aligned}\mathcal{L} &\supset \frac{f_\pi^2}{4} (D_\mu U_\pi)^\dagger (D^\mu U_\pi) + \Lambda \frac{f_\pi^2}{2} \text{Tr}[M U_\pi^\dagger] + h.c. + \dots \\ D_\mu U_\pi &= \partial_\mu U_\pi - i L_\mu U_\pi + i U_\pi R_\mu\end{aligned}$$

$$J_\mu^{L^a} = \frac{\delta \mathcal{L}}{\delta (\partial^\mu q_L)} \frac{\delta q_L}{\delta \alpha_L^a} + q_L \leftrightarrow \bar{q}_L \text{ (upstairs)}$$

$$J_\mu^{L^a} = \frac{\delta \mathcal{L}}{\delta (\partial^\mu U_\pi)} \frac{\delta U_\pi}{\delta \alpha_L^a} + U_\pi \leftrightarrow \bar{U}_\pi = -i \frac{f_\pi^2}{2} \text{Tr} \left[U_\pi^\dagger t^a \partial^\mu U_\pi \right] \text{ (downstairs)}$$

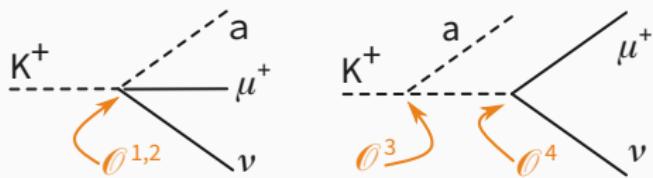
Let's focus: $K^+ \rightarrow \pi^0 \ell \nu$

$$\mathcal{L} = iG_F V_{\bar{s}u} \left[f_{SM}^+(q^2) \{ K^+ \partial_\mu \pi_0 - \partial_\mu K^+ \pi_0 \} + f_{SM}^-(q^2) \partial_\mu (K^+ \pi_0) \right] j_{-\ell}^\mu$$

- $\langle \pi^0(p_\pi) | \bar{s} \gamma_\mu u | K^+(p_K) \rangle \equiv \frac{1}{\sqrt{2}} \left[f_{+,SM}^{K^+\pi^0}(q^2) Q_\mu + f_{-,SM}^{K^+\pi^0}(q^2) q_\mu \right]$
- $\langle \pi^+(p_\pi) | \bar{s} \gamma_\mu u | K^0(p_K) \rangle \equiv f_{+,SM}^{K^0\pi^-}(q^2) Q_\mu + f_{-,SM}^{K^0\pi^-}(q^2) q_\mu$
 - $Q^\mu = p_K^\mu + p_\pi^\mu; \quad q_\mu = p_K^\mu - p_\pi^\mu$
- $\frac{f_{+}^{K^+\pi^0}(0)}{f_{+}^{K^0\pi^-}(0)} = 1 - \sqrt{3} \epsilon - \xi^2 \frac{C_3}{8} [C_A^3 + C_{LR}^3 + 2\sqrt{3} C_{LR}^8]$
- $f_-^{K^+\pi^0}(0) = 0$ at LO in the SM

$K^+ \rightarrow \pi^0 \ell \nu$: A χ PT

- $\mathcal{O}_{K_{\ell_3}^+}^1 : (K^+ \partial_\mu a - \partial_\mu K^+ a) j_{-,\ell}^\mu$ $i G_F V_{\bar{s}u} \frac{\xi}{2} (\mathcal{C}_R - 2i\mathcal{C}_W)$
- $\mathcal{O}_{K_{\ell_3}^+}^2 : (K^+ \partial_\mu a + \partial_\mu K^+ a) j_{-,\ell}^\mu$ $i G_F V_{\bar{s}u} \frac{\xi}{2} (\mathcal{C}_R + 2i\mathcal{C}_W)$
- $\mathcal{O}_{K_{\ell_3}^+}^3 : \partial^\mu a (\partial_\mu K^+ K^- - K^+ \partial_\mu K^-)$ $\frac{i}{4} \frac{1}{f_\pi} \xi (\mathcal{C}_R + \sqrt{3} \mathcal{C}_L^8)$
- $\mathcal{O}_{K_{\ell_3}^+}^4 : \partial_\mu K^+ j_{-}^\mu$ $- 2f_\pi G_F V_{\bar{s}u}$



$$! \quad \mathcal{C}_R = \mathcal{C}_R^3 + \sqrt{3} \mathcal{C}_R^8$$

Distortion of Shape of diff. distribution

SM+ALP χ PT

$$\mathcal{L} = iG_F V_{\bar{s}u} \left[\tilde{f}^+(q^2) \{K^+ \partial_\mu \pi_0 - \partial_\mu K^+ \pi_0\} + \tilde{f}^-(q^2) \partial_\mu (K^+ \pi_0) \right] j_{-,\ell}^\mu$$

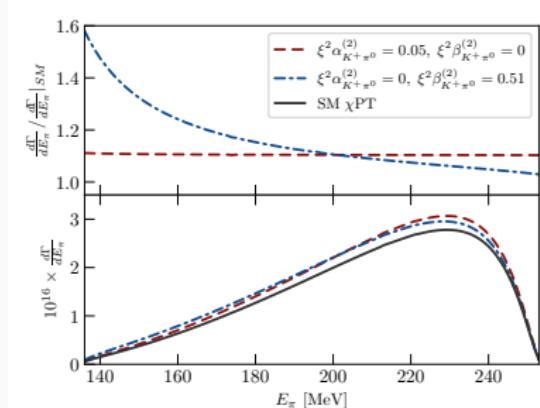
$\partial_\mu (K^+ \pi^0) \rightarrow$ Brings out lepton mass

$$\tilde{f}_+(q^2) \rightarrow \left[1 + \frac{\xi^2 \alpha_{K^+\pi^0}^{(2)}}{f^+(0)} \right] f_{\text{SM}}^+(q^2)$$

$$\tilde{f}_-(q^2) \rightarrow \left[1 + \frac{\xi^2 \beta_{K^+\pi^0}^{(2)}}{f^-(0)} \right] f_{\text{SM}}^-(q^2)$$

$$\alpha_{K^+\pi^0}^{(2)} \equiv \alpha_{K^+\pi^0}^{(2)}(C_i)$$

$$\beta_{K^+\pi^0}^{(2)} \equiv \beta_{K^+\pi^0}^{(2)}(C_i)$$



Distortion of differential spectrum

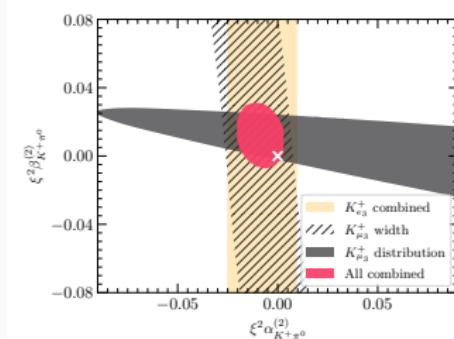
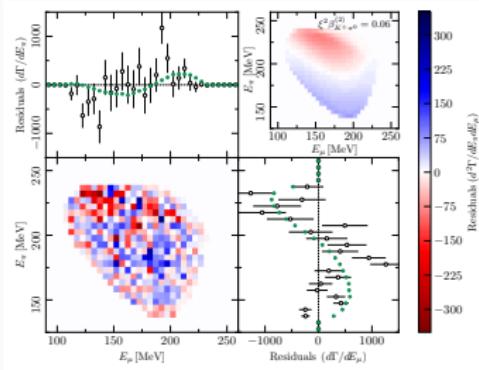
$$\overline{|\mathcal{A}|}_{K_{l3}}^2 = 2G_F^2 |V_{\bar{s}u}|^2 C_{\text{cor}} \left[1 + 2\xi^2 \frac{\alpha_{K^+\pi^0}^{(2)}}{f_{SM}^+(0)} \right] (2H \cdot p_\ell H \cdot p_{\nu_\ell} - H^2 p_\ell \cdot p_{\nu_\ell}),$$
$$H_\mu \equiv f_{SM}^+(q^2) Q_\mu + \left[1 + \xi^2 \left(\frac{\beta_{K^+\pi^0}^{(2)}}{f_{SM}^-(0)} - \frac{\alpha_{K^+\pi^0}^{(2)}}{f_{SM}^+(0)} \right) \right] f_{SM}^-(q^2) q_\mu.$$

$$Q \equiv p_K + p_\pi; \quad q \equiv p_K - p_\pi$$

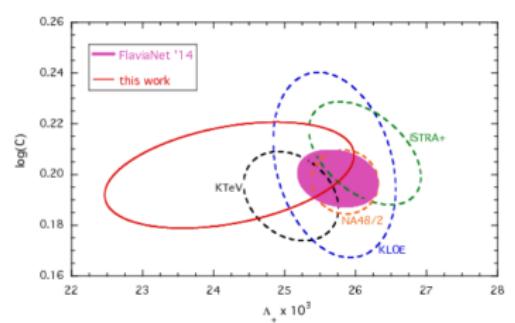
C_{cor} → EW and EM corrections

Bounds

NA48/2: $K_{e_3}^+$, $K_{\mu_3}^+$



ETM Coll. (Lattice)



- Indirect: 3 GeV
- Direct ~ 30 GeV
- Natural Next step:
 - $K \rightarrow a\mu\nu$ as bg for $K \rightarrow \mu\nu$ (.3%)
 - $\tau \rightarrow aK\nu$ as bg for $\tau \rightarrow K\nu$ (1%)

! SCALE ! → HE Colliders

Flat Directions: “Phobias”

$$\mathcal{A}(K^+ \rightarrow a\ell\nu) \propto \alpha_{K^+ a}^{(1)}$$

$$\mathcal{A}(K^+ \rightarrow \pi^0 \ell\nu) \propto \alpha_{K^+ a}^{(2)}$$

-
- $$\alpha_{K^+\pi^0}^{(2)} = \frac{C_3}{2} \left(\alpha_{K^+ a}^{(1)} + \frac{C_3}{4} \right)$$
- Strength of $K^+ \rightarrow \pi^0 e^+ \nu$
 - Strength of $K^+ \rightarrow a e^+ \nu$

Sum Rules

- Add up amplitudes to get an idea of underlying theory
- In the SM:
$$\frac{1}{4} \left| f_{+, \text{SM}}^{K^+\pi^0}(0) \right|^2 + \frac{3}{4} \left| f_{+, \text{SM}}^{K^+\eta}(0) \right|^2 = 1$$
 - Completeness of basis (Also think of Cabibbo angle etc)
- The same sum in the $A\chi\text{PT}$:
 - $$\frac{1}{4} \left| \tilde{f}_{+}^{K^+\pi^0}(0) \right|^2 + \frac{3}{4} \left| \tilde{f}_{+}^{K^+\eta}(0) \right|^2 = 1 - \frac{\xi^2}{16} (C_3 + \sqrt{3}C_8)^2 + \xi^2 \frac{3}{16} (C_L^8)^2$$
- The first term is expected
- These sums can tell us about the *structure* of the theory

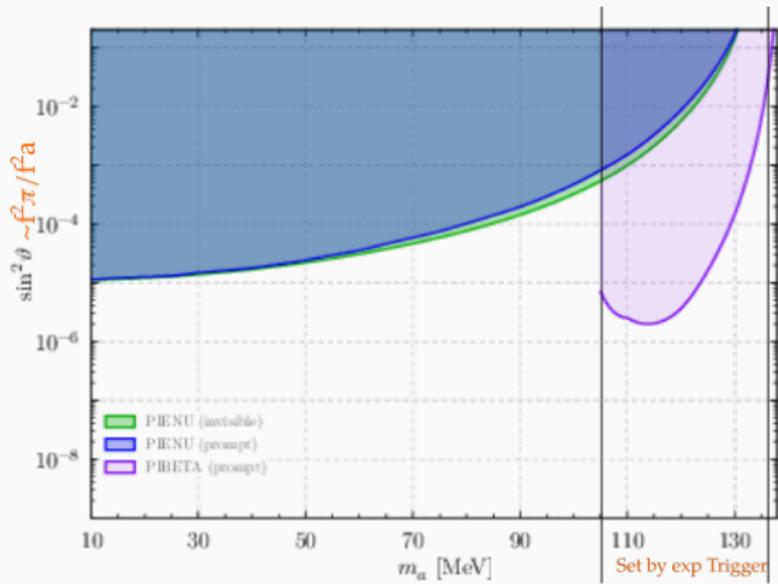
Summary

- Low lying ALPs modify χ PT in non-trivial ways
- These modifications can be observed and categorized
 - Meson mass spectrum
 - Differential widths (also sum rules)
- These modifications will complement direct searches
 - More precise computations of SM parameters needed
- Interplay between high-energy and flavour experiments

Thank You
Questions/Input/Critique?

For Context

$$\pi^+ \rightarrow ae^+\nu$$



Form Factors etc

$$f_{+/0,\text{SM}}^{K^+\pi^0}(t) = f_{+/0,\text{SM}}^{K^+\pi^0}(0) \left[1 + \lambda_{K^+\pi^0}^{+/0,(0)} \frac{t}{M_\pi^2} + \frac{1}{2} \lambda_{K^+\pi^0}^{t+/0,(0)} \frac{t^2}{M_\pi^4} \right] + \dots, \quad (3.9)$$

$$f_{-,\text{SM}}^{K^+\pi^0}(t) = \left[f_{0,\text{SM}}^{K^+\pi^0}(t) - f_{+,\text{SM}}^{K^+\pi^0}(t) \right] \frac{M_K^2 - M_\pi^2}{t}.$$

We use the dispersive parametrization of the FFs, where the slope parameters are determined in terms of the slope of the vector, $f_+^{K^+\pi^0}(t)$, FF at $t = 0$ and the slope of the scalar, $f_0^{K^+\pi^0}(t)$, FF at the Callan-Treiman point ($t_{CT} = M_K^2 - M_\pi^2$), Λ_+ and C respectively [58]:

$$\begin{aligned} \lambda_{K^+\pi^0}^{+(0)} &= \Lambda_+ ; \\ \lambda_{K^+\pi^0}^{+(0)} &= \left(\lambda_{K^+\pi^0}^{+(0)} \right)^2 + 5.79(97) \times 10^{-4} ; \\ \lambda_{K^+\pi^0}^{(0)(0)} &= \frac{M_Z^2}{t_{CT}} [\log(C) - 0.0398(44)] ; \\ \lambda_{K^+\pi^0}^{(0)(0)} &= \left(\lambda_{K^+\pi^0}^{(0)(0)} \right)^2 + 4.16(56) \times 10^{-4}. \end{aligned} \quad (3.10)$$

We tabulate the lattice determination [52] of Λ_+ , C , and $f_{+,\text{SM}}^{K^+\pi^0}(0) (= f_{0,\text{SM}}^{K^+\pi^0}(0))$ in Table 3. We also note the correlations among these parameters, which we include in our computations.

Parameter	Correlation
$\Lambda_+ = 24.22(1.16) \times 10^{-3}$	$\rho[\Lambda_+, \log(C)] = 0.376$
$\log(C) = 0.1998(138)$	$\rho[f_{+,\text{SM}}^{K^+\pi^0}(0), \log(C)] = -0.719$
$f_{+/0,\text{SM}}^{K^+\pi^0}(0) = 0.9709(46)$	$\rho[f_{+/0,\text{SM}}^{K^+\pi^0}(0), \Lambda_+] = -0.228$

Table 3. Lattice determined values (left) and correlations (right) of the FF parameters in the dispersive formalism [52]. The quantities $\rho[i, j]$ are the correlation coefficients between the i -th and the j -th parameters.

Lattice Form Factors: 1602.04113 (ETM collaboration)

Short Distance EW, EM, Isospin: 0801.1817 (FlaviaNet), NPB 196 (1982), 83 (Sirlin), 1704.04104 (Moulson)

$$\begin{aligned} \mathcal{L} &\supset \frac{\mathfrak{f}_\pi^2}{4} \left\langle D_\mu U^\dagger D^\mu U \right\rangle \\ &= \frac{1}{2} \left(\partial_\mu \Pi_a \right)^2 + \frac{\mathfrak{f}_\pi^2}{8} \left(V_5^\mu \right)^2 + \frac{\mathfrak{f}_\pi^2}{2} \partial_\mu \Pi_a V_5^\mu + \frac{1}{4} \int_{abc} \Pi_{[a} \partial_\mu \Pi_{b]} V_c^\mu + \frac{1}{8 \mathfrak{f}_\pi} \int_{abc} \Pi_{[a} V_{b]} V_{c]}^\mu \quad \boxed{6} \end{aligned}$$

$$\begin{aligned} \text{Re} \left(\bar{f}_{+}^{K^+\pi^0}(t) \right) &= \left(\alpha_{K^+\pi^0}^{(0)} + \xi^2 \alpha_{K^+\pi^0}^{(2)} + \delta \alpha_{K^+\pi^0}^{(0)} + \xi^2 \delta \alpha_{K^+\pi^0}^{(2)} \right) \\ &\times \left[1 + \left(\lambda_{K^+\pi^0}^{+(0)} + \xi^2 \lambda_{K^+\pi^0}^{+(2)} \right) \frac{t}{M_\pi^2} + \left(\lambda_{K^+\pi^0}^{t+(0)} + \xi^2 \lambda_{K^+\pi^0}^{t+(2)} \right) \frac{t^2}{2M_\pi^4} \right] + \dots \\ &\simeq \left[1 + \xi^2 \frac{\alpha_{K^+\pi^0}^{(2)}}{\alpha_{K^+\pi^0}^{(0)}} \right] f_{+,\text{SM}}^{K^+\pi^0}(t) , \\ \text{Re} \left(\bar{f}_{-}^{K^+\pi^0}(t) \right) &= \left(\delta \beta_{K^+\pi^0}^{(0)} + \xi^2 \beta_{K^+\pi^0}^{(2)} + \xi^2 \delta \beta_{K^+\pi^0}^{(2)} \right) \\ &\times \left[1 + \left(\lambda_{K^-\pi^0}^{-(0)} + \xi^2 \lambda_{K^-\pi^0}^{-(2)} \right) \frac{t}{M_\pi^2} + \left(\lambda_{K^-\pi^0}^{t-(0)} + \xi^2 \lambda_{K^-\pi^0}^{t-(2)} \right) \frac{t^2}{2M_\pi^4} \right] + \dots \\ &\simeq \left[1 + \xi^2 \frac{\beta_{K^+\pi^0}^{(2)}}{\delta \beta_{K^+\pi^0}^{(0)}} \right] f_{-,\text{SM}}^{K^+\pi^0}(t) , \end{aligned}$$

The factor C_{cor} encapsulates effects that are not captured by the lattice computations of the FFs that we use in our numerical analyses. It is defined as:

$$C_{\text{cor}} = S_{\text{EW}} (1 + \delta_{SU(2)}^{K^+} + \delta_{em}^{K^+})^2. \quad (3.7a)$$

$$\text{Here, } S_{\text{EW}} = 1 + \frac{2\alpha_{\text{EM}}}{\pi} \left(1 - \frac{\alpha_s}{4\pi} \right) \log \frac{M_Z}{M_\rho} + \mathcal{O}\left(\frac{\alpha \alpha_s}{\pi^2}\right) = 0.0232 \pm 0.0003, \quad (3.7b)$$

encodes the short-distance contribution to the EM corrections [55, 56]. The other corrections, viz. $\delta_{SU(2)}^{K^+} = (2.45 \pm 0.19)\%$ [57] and $\delta_{\text{EM}}^{K^+} = (0.016 \pm 0.25) \times 10^{-2}$ [56] are the isospin breaking and long distance electromagnetic corrections respectively. In our calculations, We drop the $\delta_{\text{EM}}^{K^+}$ corrections as it is much smaller than the order we are working up to.

(7.9) (7.9)