

Scale-invariance, dynamically induced Planck scale and inflation in the Palatini formulation

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Based on: [2006.09124](#), I.D.G., A. Karam and A. Racioppi

[2104.04550](#), I.D.G., A. Karam, T.D. Pappas and V.C. Spanos

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Inflation

FRW metric:

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right) \quad (1)$$

- Inflation is a theory of exponential expansion of space in the early universe, i.e. $a \sim e^{Ht}$.
- $\sim 10^{-33} - 10^{-32}$ seconds after the Big Bang.
- Solves the **horizon** and **flatness** problems.
- It can also provide a mechanism for the generation of the perturbations that have resulted in the anisotropies observed in the CMB.

Minimal Inflation

Action: EH + a scalar field

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (M_{\text{Pl}}^2 \equiv 1) \quad (2)$$

Friedmann equations:

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3}, \quad H^2 + \dot{H}^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \quad (3)$$

Density & Pressure:

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (4)$$

Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (5)$$

Inflationary Observables

The scalar (\mathcal{P}_ζ) and tensor (\mathcal{P}_T) power spectrum is

$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1}, \quad A_s = \frac{1}{24\pi^2} \frac{V(\phi_*)}{\epsilon_V(\phi_*)}, \quad \mathcal{P}_T = 8 \left(\frac{H}{2\pi} \right)^2 \simeq \frac{2V}{3\pi^2} \quad (6)$$

Spectral tilt (n_s) and tensor-to-scalar ratio (r)

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_\zeta(k)}{d \ln k} \simeq -6\epsilon_V + 2\eta_V, \quad r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \simeq 16\epsilon_V \quad (7)$$

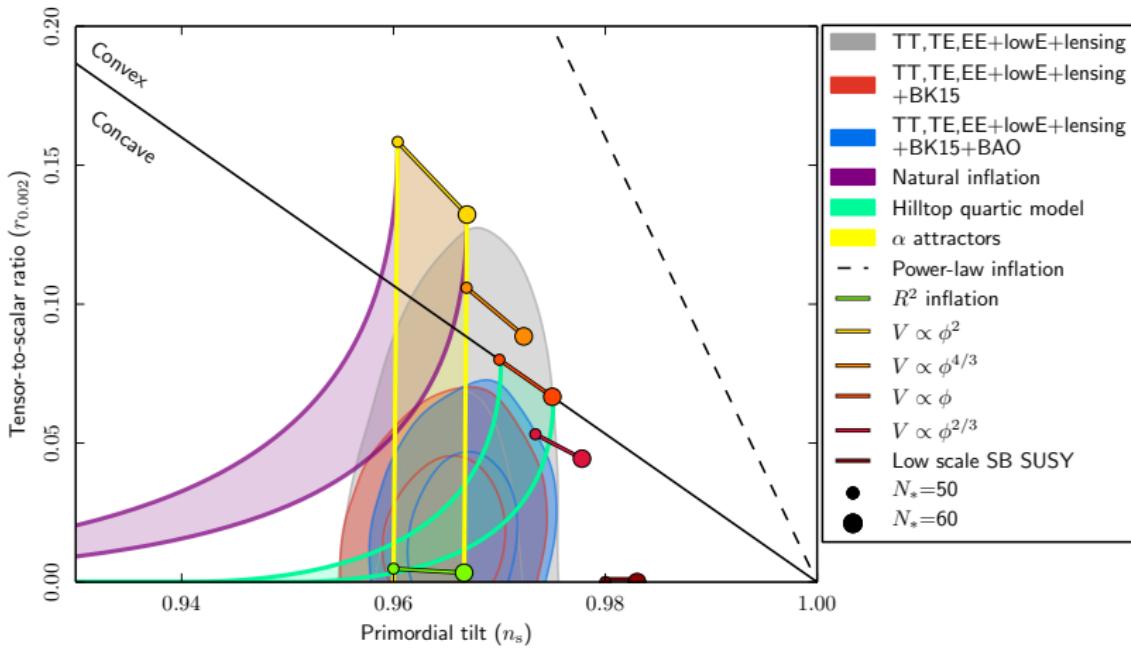
We have used the potential slow-roll parameters:

$$\epsilon_V = \frac{1}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \quad \eta_V = \frac{V''(\phi)}{V(\phi)}. \quad (8)$$

Number of e -folds

$$N(\phi) = \int_t^{t_{\text{end}}} H dt = \int_{\phi_{\text{end}}}^\phi \frac{d\phi}{\sqrt{2\epsilon_H}} \approx \int_{\phi_{\text{end}}}^\phi \frac{d\phi}{\sqrt{2\epsilon_V}} \simeq 50 - 60 \quad (9)$$

Inflationary Observables (Planck 2018 1807.06211)



$$n_s = 0.9649 \pm 0.0042, \quad r < 0.056 \text{ and } A_s = (2.10 \pm 0.03) \times 10^{-9}$$

Starobinsky Inflation (Starobinsky, 1980)

The action is

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\text{Pl}}^2 R}{2} + \frac{R^2}{12M^2} \right), \quad M \sim 10^{-5}. \quad (10)$$

After a Weyl rescaling of the metric $g_{\mu\nu}$ and a field redefinition

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^2 \tilde{R}}{2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (11)$$

where

$$V(\phi) = \frac{3}{4} M_{\text{Pl}}^4 M^2 \left[1 - \exp \left(-\sqrt{\frac{2}{3}} \phi / M_{\text{Pl}} \right) \right]^2, \quad (12)$$

We find for $N_* = 60$

$$n_s \simeq 1 - \frac{2}{N_*} \simeq 0.966$$

$$r \simeq \frac{12}{N_*^2} \simeq 0.0033$$

Metric vs. Palatini

- In **metric formulation**, the metric is the only dynamical degree of freedom and the connection is the Levi-Civita: $R_{\mu\nu} = R_{\mu\nu}(g, \partial g, \partial^2 g)$.
- In **Palatini formulation**, both the metric and the connection are independent dynamical degrees of freedom $R_{\mu\nu} = R_{\mu\nu}(\Gamma, \partial\Gamma)$.

$$S_J = \int d^4x \sqrt{-g} \left(\frac{1}{2} A(\phi) g^{\mu\nu} R_{\mu\nu}(\Gamma) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \quad (13)$$

Variation with respect to Γ gives

$$\Gamma_{\alpha\beta}^\lambda = \left\{ \begin{array}{c} \lambda \\ \alpha\beta \end{array} \right\} + (1-\kappa) \left[\delta_\alpha^\lambda \partial_\beta \omega(\phi) + \delta_\beta^\lambda \partial_\alpha \omega(\phi) - g_{\alpha\beta} \partial^\lambda \omega(\phi) \right], \quad \omega(\phi) = \ln \sqrt{A(\phi)}$$

where $\kappa = 1$ in metric and $\kappa = 0$ in Palatini. Performing a Weyl rescaling

$$\tilde{g}_{\mu\nu} \equiv A(\phi) g_{\mu\nu} \quad \rightarrow \quad \sqrt{-g} = A^{-2} \sqrt{-\tilde{g}}, \quad R = A \left(1 - \kappa \times 6 A^{1/2} \tilde{\nabla}^\mu \tilde{\nabla}_\mu A^{-1/2} \right) \tilde{R},$$

the action becomes

$$S_E^{Pal \ or \ metric} = \int d^4x \sqrt{-\tilde{g}} \left(\frac{1}{2} \tilde{R} - \frac{1}{2} \left(\frac{1}{A} + \kappa \times \frac{3}{2} \frac{A_{,\phi}}{A^2} \right) \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{V(\phi)}{A^2} \right). \quad (14)$$

Higgs Inflation: (0710.3755 Bezrukov & Shaposhnikov) (0803.2664 Bauer & Demir)

We consider the Higgs-like inflationary potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2, \quad A(\phi) = 1 + \xi\phi^2. \quad (15)$$

Canonical field redefinition gives

$$\phi(\chi) \simeq \frac{1}{\sqrt{\xi}} \exp\left(\sqrt{\frac{1}{6}}\chi\right) \quad (\text{Metric}), \quad \phi(\chi) = \frac{1}{\sqrt{\xi}} \sinh(\sqrt{\xi}\chi) \quad (\text{Palatini}) \quad (16)$$

The Einstein-frame potential in terms of χ can be expressed as

$$U(\chi) \simeq \frac{\lambda}{4\xi^2} \left(1 + \exp\left(-\sqrt{\frac{2}{3}}\chi\right)\right)^{-2}, \quad (\text{Metric}), \quad (17)$$

$$U(\chi) = \frac{\lambda}{4\xi^2} \tanh^4\left(\sqrt{\xi}\chi\right), \quad (\text{Palatini}) \quad (18)$$

$$n_s \simeq 1 - \frac{2}{N_*} + \frac{3}{2N_*^2}, \quad r \simeq \frac{12}{N_*^2}, \quad A_s \simeq \frac{\lambda N_*^2}{72\pi^2\xi^2} \quad (\text{Metric}),$$

$$n_s \simeq 1 - \frac{2}{N_*} - \frac{3}{8\xi N_*^2}, \quad r \simeq \frac{2}{\xi N_*^2}, \quad A_s \simeq \frac{\lambda N_*^2}{12\pi^2\xi} \quad (\text{Palatini}).$$

Palatini inflation with an R^2 term (1810.05536 Enckell et al)

1810.10418 Antoniadis et al, 1901.01794 Tenkanen, 1911.11513 I.D.G & A.B. Lahanas
2012.06831 Dimopoulos et al

Action

$$S_J = \int d^4x \sqrt{-g} \left[\frac{A(\phi)}{2} R + \frac{\alpha}{2} R^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \quad (19)$$

Introducing an auxiliary field we eliminate the R^2 term and after a Weyl rescaling we obtain

$$S_E = \int d^4x \sqrt{-g} \left(\frac{\mathcal{R}}{2} + K(\phi) X + L(\phi) X^2 - U(\phi) \right), \quad (20)$$

with $X = -1/2 \partial_\mu \phi \partial^\mu \phi$ and $K(\phi) = \frac{A(\phi)}{A^2(\phi) + 8\alpha V(\phi)}$, $L(\phi) = \frac{2\alpha}{A^2(\phi) + 8\alpha V(\phi)}$,

$$U(\phi) = \frac{V(\phi)}{A^2(\phi) + 8\alpha V(\phi)}.$$

- Equation of motion

$$(K + 3L\dot{\phi}^2)\ddot{\phi} + 3H(K + L\dot{\phi}^2)\dot{\phi} + U'(\phi) + \frac{1}{4}(2K' + 3L'\dot{\phi}^2)\dot{\phi}^2 = 0, \quad (21)$$

- Speed of sound

$$c_s^2 = \frac{\partial p / \partial X}{\partial \rho / \partial X} = \frac{1 + L\dot{\phi}^2/K}{1 + 3L\dot{\phi}^2/K}, \quad (22)$$

Dynamical generation of Planck scale and inflation

1502.01334, 1509.05423, 1710.04853, 2006.09124 I.D.G., A. Karam & A. Racioppi

$$S = \int d^4x \sqrt{-g} \left[\frac{\xi \phi^2 R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad V(\phi) = \frac{1}{4} \lambda(\phi) \phi^4 + \Lambda^4 \quad (23)$$

At the minimum

$$V(v) = \frac{1}{4} \lambda(v) v^4 + \Lambda^4 = 0, \quad v = \frac{M_P}{\sqrt{\xi}} \quad (24)$$

Minimization: $\beta(v) + 4\lambda(v) = 0$. This implies

- a) $\beta(v) > 0, \lambda(v) < 0$
- b) $\beta(v) = \lambda(v) = 0$

Taylor expansion around the VEV

$$\lambda(\phi) = \lambda(v) + \beta(v) \ln \frac{\phi}{v} + \frac{1}{2!} \beta'(v) \ln^2 \frac{\phi}{v} + \frac{1}{3!} \beta''(v) \ln^3 \frac{\phi}{v} + \dots, \quad (25)$$

For the two cases we get

$$\lambda^a(\phi) \simeq \lambda(v) + \beta(v) \ln \frac{\phi}{v}, \quad V(\phi) = \Lambda^4 \left\{ 1 + \left[4 \ln \left(\frac{\phi}{v} \right) - 1 \right] \frac{\phi^4}{v^4} \right\} \quad (26)$$

$$\lambda^b(\phi) \simeq \frac{\beta'(v)}{2} \ln^2 \frac{\phi}{v}, \quad V(\phi) = \frac{1}{8} \beta' \phi^4 \ln^2 \left(\frac{\phi}{v} \right) \quad (27)$$

1st and 2nd order Coleman-Weinberg potentials

1st order:

$$\text{JF } V(\phi) = \Lambda^4 \left\{ 1 + \left[4 \ln \left(\frac{\phi}{v} \right) - 1 \right] \frac{\phi^4}{v^4} \right\}, \quad \text{EF } \bar{U}(\bar{\zeta}) = \Lambda^4 \left(4 \frac{\bar{\zeta}}{v} + e^{-4 \frac{\bar{\zeta}}{v}} - 1 \right) \quad (28)$$

For $v \ll 1$ (i.e. $\xi \gg 1$) and $\bar{\zeta} > 0$, the potential becomes

$$\bar{U}(\bar{\zeta}) \approx a_\zeta \bar{\zeta}, \quad \text{with} \quad a_\zeta = 4 \frac{\Lambda^4}{v}. \quad (29)$$

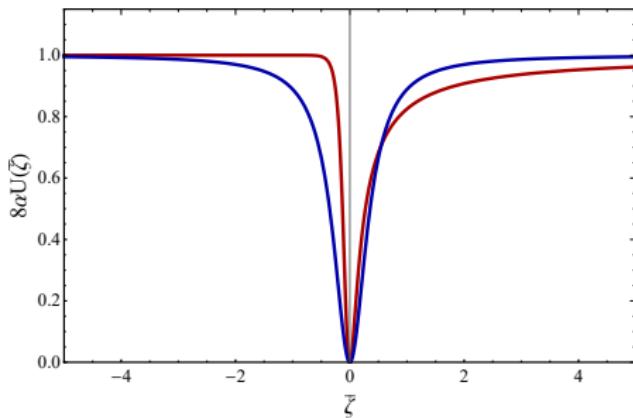
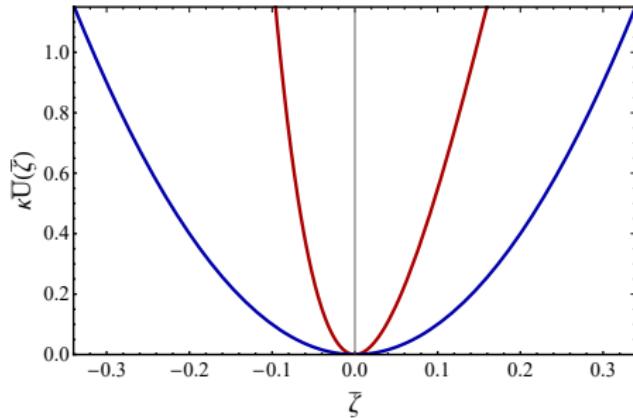
For $v \gg 1$ (i.e. $\xi \ll 1$), the potential reduces to

$$\bar{U}(\bar{\zeta}) \approx \frac{m^2}{2} \bar{\zeta}^2, \quad \text{with} \quad m = m_1 = 4 \frac{\Lambda^2}{v}. \quad (30)$$

2nd order:

$$\text{JF } V(\phi) = \frac{1}{8} \beta' \phi^4 \ln^2 \left(\frac{\phi}{v} \right), \quad \text{EF } \bar{U}(\bar{\zeta}) = \frac{m^2}{2} \bar{\zeta}^2, \quad \text{with} \quad m^2 = m_2^2 = \frac{\beta' v^2}{4}. \quad (31)$$

Adding an $\frac{\alpha}{2}R^2$

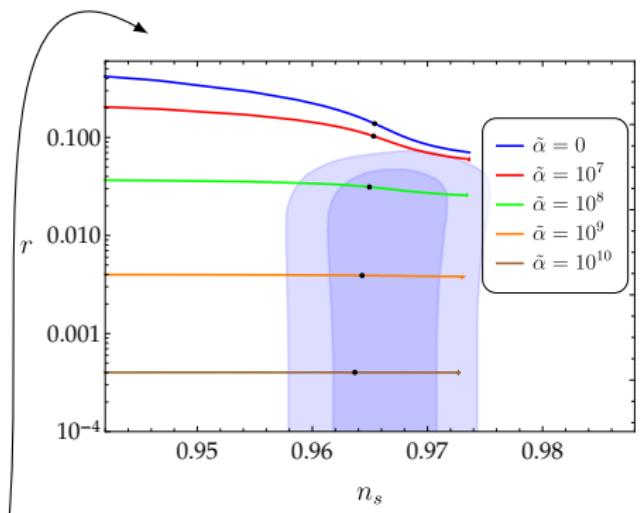


Red \rightarrow 1st order

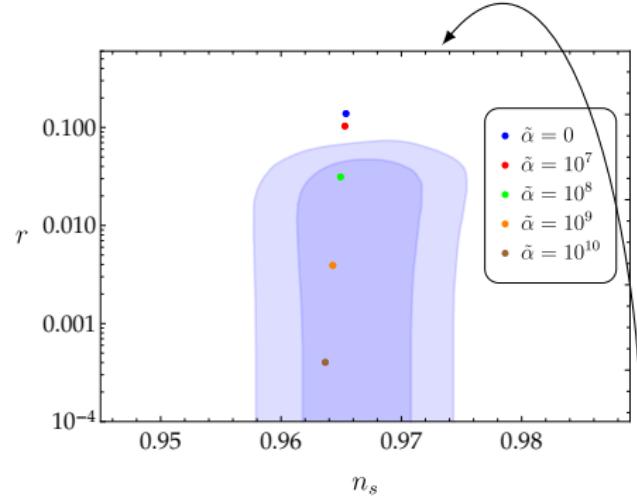
Blue \rightarrow 2nd order

$$U = \frac{\bar{U}}{1 + 8\alpha\bar{U}} \quad \text{with} \quad \bar{U} = V/A^2.$$

Inflationary predictions



1st order



2nd order

Scale invariant quadratic gravity

1403.4226 A. Salvio & A. Strumia, 1512.05890 A. Farzinnia & S. Kouwn,
2104.04550 I.D.G., A. Karam, T.D. Pappas & V.C. Spanos

The model: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$

Extra particles: 3 RH neutrinos, 1 gauge boson, 1 scalar field

- $\mathcal{L}_{\text{Yukawa}}^{\text{BSM}} = -y_D^{ij} \bar{\ell}_L^i H N_R^j - \frac{1}{2} y_M^i \Phi \bar{N}_R^{iC} N_R^i + h.c$
- $\mathcal{L}_{\text{scalar}}^{\text{BSM}} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{4} \lambda_\phi \phi^4 + \frac{1}{4} \lambda_{h\phi} h^2 \phi^2$
- \mathcal{L}^{SM} with no Higgs mass term
- $\mathcal{L}_{\text{gravity}} = \frac{1}{2} (\xi_\phi \phi^2 + \xi_h h^2) g^{\mu\nu} R_{\mu\nu}(\Gamma) + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu} R^{\mu\nu}$

→ Dynamical generation

$$M_P^2 = \xi_\phi v_\phi^2 + \xi_h v_h^2$$

Inflationary action

$$S_J = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} [(\xi_\phi \phi^2 + \xi_h h^2) g^{\mu\nu} R_{\mu\nu} + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}] - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - V^{(0)}(\phi, h) \right\}, \quad (32)$$

with $V^{(0)}(\phi, h) = \frac{1}{4} (\lambda_\phi \phi^4 - \lambda_{h\phi} h^2 \phi^2 + \lambda_h h^4)$. After a Weyl rescaling
 $g^{\mu\nu} \rightarrow \Omega^2 g^{\mu\nu}$, $\Omega^2 = \xi_\phi \phi^2 + \xi_h h^2$, $R \rightarrow \Omega^2 R$ we obtain

$$S_{IF} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} [g^{\mu\nu} R_{\mu\nu} + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}] - \frac{1}{2\Omega^2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2\Omega^2} g^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{V^{(0)}(\phi, h)}{\Omega^4} \right\}. \quad (33)$$

Gildener-Weinberg approach Gildener & Weinberg, 1976

The intermediate frame potential is

$$U^{(0)}(\phi, h) \equiv \frac{V^{(0)}(\phi, h)}{\Omega^4} = \frac{(\lambda_\phi \phi^4 - \lambda_{h\phi} h^2 \phi^2 + \lambda_h h^4)}{4 (\xi_\phi \phi^2 + \xi_h h^2)^2}. \quad (34)$$

Flat direction (FD): $\partial_\phi U^{(0)}(\phi, h) = \partial_h U^{(0)}(\phi, h) = 0$ gives the extremization

condition, $v_h = \sqrt{\frac{\lambda_{h\phi} \xi_\phi + 2\lambda_\phi \xi_h}{\lambda_{h\phi} \xi_h + 2\lambda_h \xi_\phi}} v_\phi$

Along the FD $U_{\min}^{(0)} \equiv U^{(0)}(v_\phi, v_h) = \frac{(4\lambda_h \lambda_\phi - \lambda_{h\phi}^2) M_P^4}{16 [\lambda_\phi \xi_h^2 + \xi_\phi (\lambda_{h\phi} \xi_h + \lambda_h \xi_\phi)]}. \quad (35)$

Orthogonal rotation: $\begin{pmatrix} \phi \\ h \end{pmatrix} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} s \\ \sigma \end{pmatrix}.$

Mixing angle: $\omega \equiv \arctan(v_h/v_\phi).$

One-loop effective potential

The one-loop corrections along the flat direction for the canonical field s_c at the scale Λ may be written as

$$U^{(1)}(s_c) = \mathbb{A} s_c^4 + \mathbb{B} s_c^4 \ln \frac{s_c^2}{\Lambda^2}, \quad (36)$$

where in our model

$$\begin{aligned} \mathbb{A} &= \frac{1}{64\pi^2 v_s^4} \left\{ M_h^4 \left(\ln \frac{M_h^2}{v_s^2} - \frac{3}{2} \right) + 6M_W^4 \left(\ln \frac{M_W^2}{v_s^2} - \frac{5}{6} \right) + 3M_Z^4 \left(\ln \frac{M_Z^2}{v_s^2} - \frac{5}{6} \right) \right. \\ &\quad \left. + 3M_X^4 \left(\ln \frac{M_X^2}{v_s^2} - \frac{5}{6} \right) - 6M_{N_R}^4 \left(\ln \frac{M_{N_R}^2}{v_s^2} - 1 \right) - 12M_t^4 \left(\ln \frac{M_t^2}{v_s^2} - 1 \right) \right\}, \\ \mathbb{B} &= \frac{\mathcal{M}^4}{64\pi^2 v_s^4}, \quad \mathcal{M}^4 \equiv M_h^4 + 3M_X^4 + 6M_W^4 + 3M_Z^4 - 6M_{N_R}^4 - 12M_t^4, \end{aligned}$$

Minimizing (36), we can determine the scale Λ as $\Lambda = v_s \exp \left[\frac{\mathbb{A}}{2\mathbb{B}} + \frac{1}{4} \right]$. Then, we can express the one-loop correction as

$$U^{(1)}(s_c) = \frac{\mathcal{M}^4}{64\pi^2 v_s^4} s_c^4 \left[\ln \frac{s_c^2}{v_s^2} - \frac{1}{2} \right]. \quad (37)$$

One-loop effective potential

We now require that the full one-loop effective potential is zero at v_s . Then

$$U_{\text{eff}}(v_s) = U_{\text{min}}^{(0)} + U^{(1)}(v_s) = 0, \quad (38)$$

which finally yields

$$U_{\text{eff}}(s_c) = \frac{\mathcal{M}^4}{128\pi^2} \left[\frac{s_c^4}{v_s^4} \left(2 \ln \frac{s_c^2}{v_s^2} - 1 \right) + 1 \right].$$

Don't forget that

$$v_s^2 = \frac{M_P^2}{\xi_s}, \quad \text{with } \xi_s \equiv \xi_\phi \cos^2 \omega + \xi_h \sin^2 \omega.$$

Finally, the effective action along the FD written explicitly in terms of the inflaton field reads

$$S_{IF} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} [g^{\mu\nu} R_{\mu\nu} + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}] - \frac{1}{2} g^{\mu\nu} \partial_\mu s_c \partial_\nu s_c - U_{\text{eff}}(s_c) \right\}. \quad (39)$$

Einstein frame representation

The IF action can be cast in the form

$$S_{IF} = \int d^4x \sqrt{-g} \left[\frac{1}{2} C(g_{\mu\nu}, R_{\mu\nu}) + \mathcal{L}_m(g_{\mu\nu}, s_c, \partial_\mu s_c) \right], \quad (40)$$

where we have defined

- $C(g_{\mu\nu}, R_{\mu\nu}) = g^{\mu\nu} R_{\mu\nu} + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}$
- $\mathcal{L}_m(g_{\mu\nu}, s_c, \partial_\mu s_c) = -\frac{1}{2} g^{\mu\nu} \partial_\mu s_c \partial_\nu s_c - U_{\text{eff}}(s_c)$

Einstein frame representation

See Jaakko Annala's 2020 Master thesis for more details on this calculation.

Now, upon introducing the auxiliary field $\Sigma_{\mu\nu}$ the action becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} C(g_{\mu\nu}, \Sigma_{\mu\nu}, s_c) + \frac{1}{2} \frac{\partial C}{\partial \Sigma_{\mu\nu}} (R_{\mu\nu} - \Sigma_{\mu\nu}) + \mathcal{L}_m(g_{\mu\nu}, s_c) \right]. \quad (41)$$

We introduce the new variable $\sqrt{-q} q^{\mu\nu} = \sqrt{-g} \frac{\partial C}{\partial \Sigma_{\mu\nu}}$ thus the action can be written as

$$\begin{aligned} S &= \int d^4x \left\{ \frac{\sqrt{-q}}{2} q^{\mu\nu} R_{\mu\nu} \right. \\ &\quad \left. - \frac{\sqrt{-g}}{2} \left[\frac{\partial C}{\partial \Sigma_{\mu\nu}} \Sigma_{\mu\nu}(q_{\mu\nu}, g_{\mu\nu}, s_c) - C(q_{\mu\nu}, g_{\mu\nu}, s_c) - 2\mathcal{L}_m(g_{\mu\nu}, s_c) \right] \right\} \end{aligned} \quad (42)$$

EH sector matter sector

Einstein frame representation

Varying the previous action with respect to $g_{\mu\nu}$ will give us $g_{\mu\nu}$ as a function of $q_{\mu\nu}$, s_c and $\partial_\mu s_c$. This way we obtain that

$$\begin{aligned}\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = & - \frac{1}{4(\beta + 4\alpha)} \frac{\sqrt{-q}}{\sqrt{-g}} q^{\sigma\lambda} g_{\sigma\mu} g_{\lambda\nu} \\ & + \frac{1}{4\beta} \frac{q}{g} \left(q^{\sigma\lambda} q^{\rho\delta} g_{\lambda\delta} g_{\rho\nu} g_{\sigma\mu} - \frac{\alpha}{\beta + 4\alpha} q^{\delta\rho} g_{\delta\rho} q^{\sigma\lambda} g_{\sigma\mu} g_{\lambda\nu} \right) \\ & + \frac{1}{2} g_{\mu\nu} \left[\frac{1}{\beta + 4\alpha} \left(\frac{1}{2} + \frac{\alpha}{8\beta} \frac{q}{g} q^{\lambda\sigma} g_{\lambda\sigma} q^{\rho\delta} g_{\rho\delta} \right) - \frac{q}{g} \frac{1}{8\beta} q^{\lambda\sigma} q^{\delta\rho} g_{\lambda\delta} g_{\sigma\rho} \right] \\ & + \frac{1}{2} g_{\mu\nu} \left(\frac{1}{2} g^{\lambda\sigma} \partial_\lambda s_c \partial_\sigma s_c + U_{\text{eff}}(s_c) \right) - \frac{1}{2} \partial_\mu s_c \partial_\nu s_c = 0.\end{aligned}$$

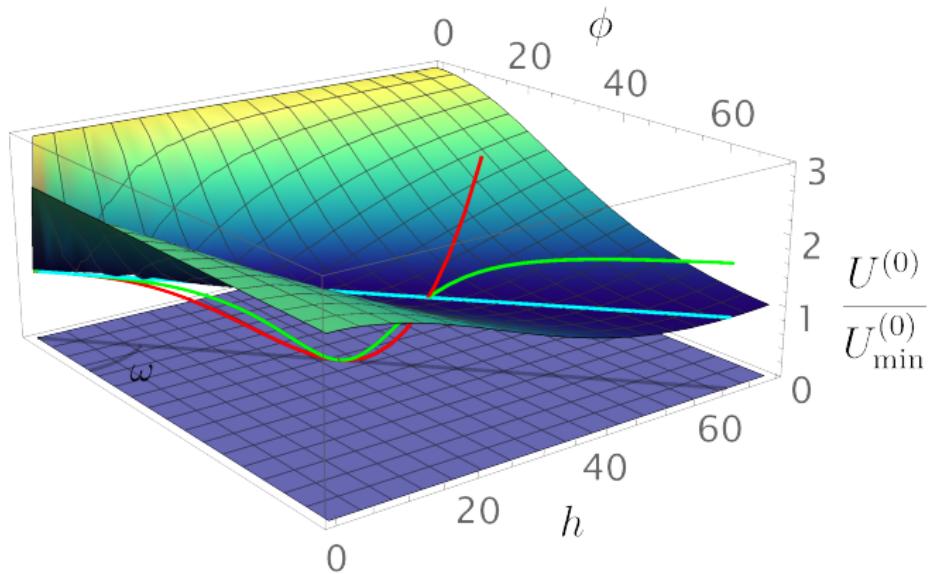
which will help us to solve the metric $g_{\mu\nu}$ in terms of the metric $q_{\mu\nu}$ and the inflaton field by applying a disformal transformation [gr-qc/9211017 J.D. Bekenstein](#)

$$g_{\mu\nu} = A q_{\mu\nu} + B \partial_\mu s_c \partial_\nu s_c \tag{43}$$

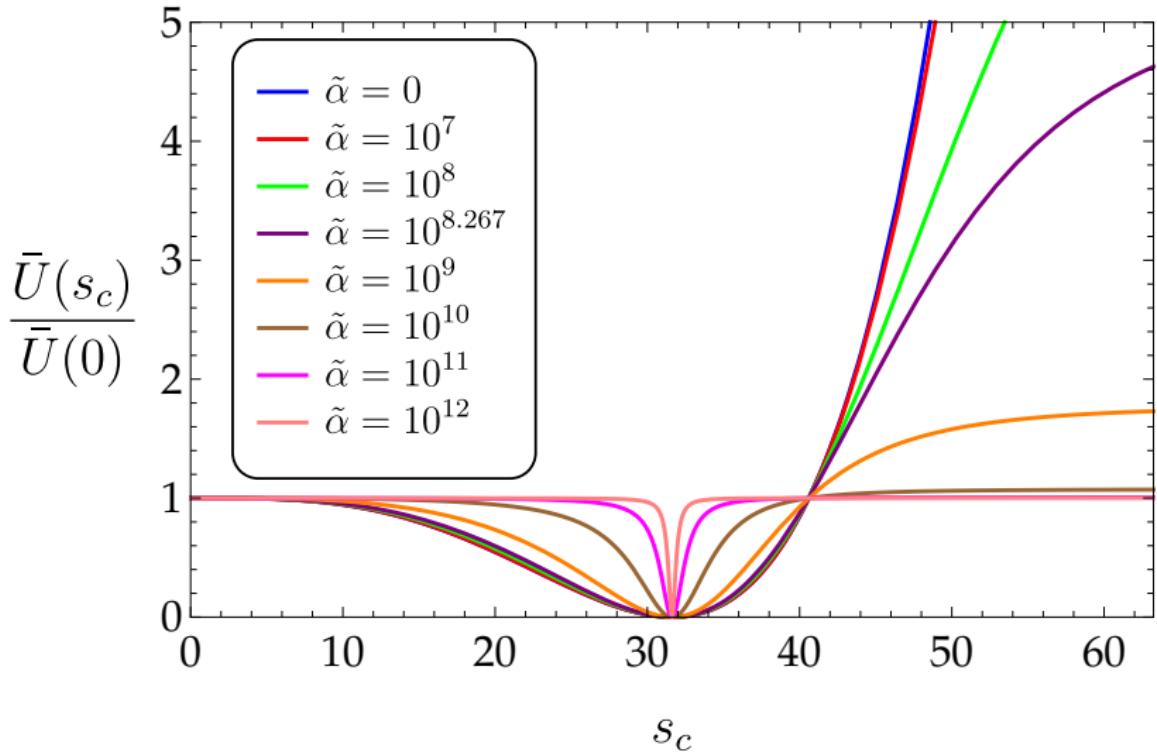
Final Einstein frame action

$$S_E = \int d^4x \sqrt{-q} \left[\frac{1}{2} q^{\mu\nu} R_{\mu\nu} + K(s_c) X_q - \bar{U}(s_c) + \mathcal{O}(X_q^2) \right], \quad (44)$$

with $K(s_c) = \frac{1}{1 + \tilde{\alpha} U_{\text{eff}}(s_c)}$, $\bar{U}(s_c) = \frac{U_{\text{eff}}(s_c)}{1 + \tilde{\alpha} U_{\text{eff}}(s_c)}$ and $\tilde{\alpha} = 2\beta + 8\alpha$.



Inflationary potential



Inflationary observables

- For $\xi_s \ll 1$ and $\tilde{\alpha} = 0$ for both small field inflation (SFI) and large field inflation (LFI)

$$n_s \simeq 1 - \frac{2}{N_*}, \quad r_0 \simeq \frac{8}{N_*}, \quad \text{quadratic} \quad (45)$$

where r_0 denotes the tensor-to-scalar ratio for $\tilde{\alpha} = 0$.

- For $\xi_s \gg 1$ and $\tilde{\alpha} = 0$ we find for both SFI and LFI,

$$n_s \simeq 1 - \frac{3}{N_*}, \quad \text{quartic} \quad (46)$$

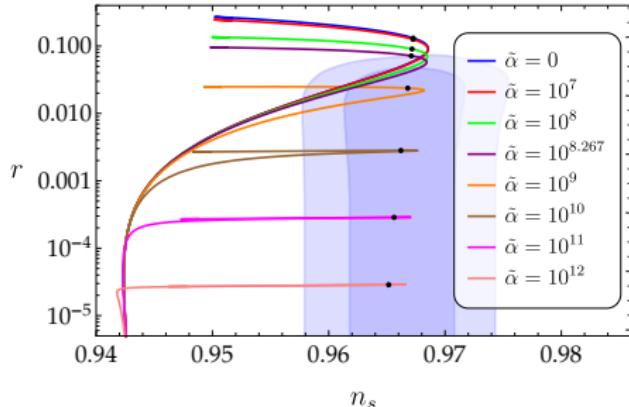
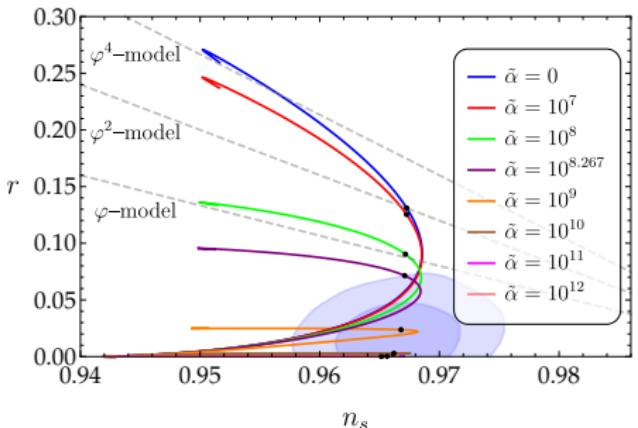
while

$$r_0 \simeq \frac{16}{N_*} \quad (\text{for LFI}), \quad \text{quartic} \quad r_0 \simeq 0 \quad (\text{for SFI}), \quad (47)$$

When $\tilde{\alpha} \neq 0$, the predictions for n_s remain the same but r gets modified as (1810.05536 Enckell et al)

$$r = \frac{r_0}{1 + \tilde{\alpha} U_{\text{eff}}^*} = \frac{r_0}{1 + \frac{3}{2} \pi^2 \tilde{\alpha} A_s r_0}. \quad (48)$$

Inflationary observables



In the Table $\xi_s \rightarrow 0$

$\tilde{\alpha}$	0	10^7	10^8	1.85×10^8	10^9	10^{10}	10^{11}	10^{12}
r	0.13090	0.12526	0.09022	0.07134	0.02368	0.00282	0.00029	0.00003
n_s	0.96727	0.96726	0.96717	0.96711	0.96681	0.96621	0.96563	0.96517
N_*	60.6	60.6	60.4	60.3	59.8	58.8	58.0	57.3

For viable inflation $\xi_s \lesssim 4 \times 10^{-3} \Rightarrow v_s \gtrsim 15 M_P \Rightarrow v_s \simeq v_\phi$ as $v_h \sim \mathcal{O}(10^{-16}) M_P$.

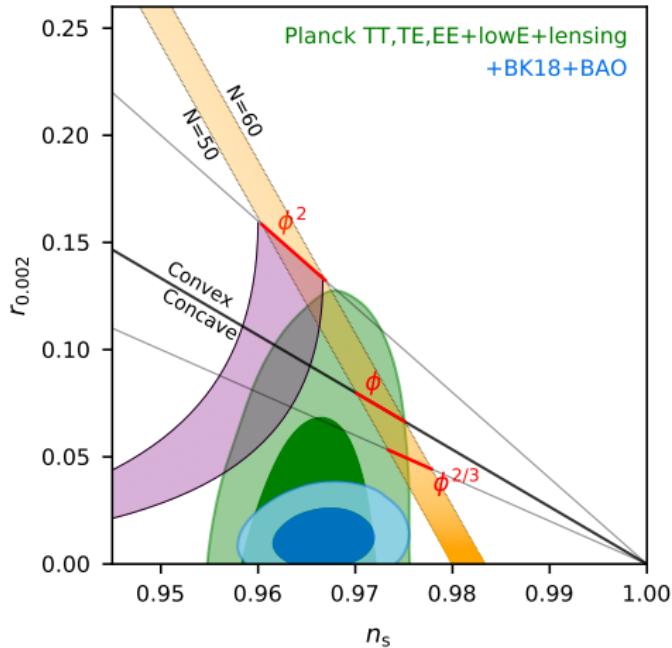
Summary

- 1 scalar field $+ R^2$ in **metric** leads to two-field inflation
- 1 scalar field $+ R^2$ (or/and $R_{\mu\nu}R^{\mu\nu}$) in **Palatini** leads to one-field inflation
- The effective potential is asymptotically flat
- The value of r becomes smaller
- The values of A_s and n_s are "unaffected"
- The Planck scale can be dynamically generated through the VEVs of the scalar fields

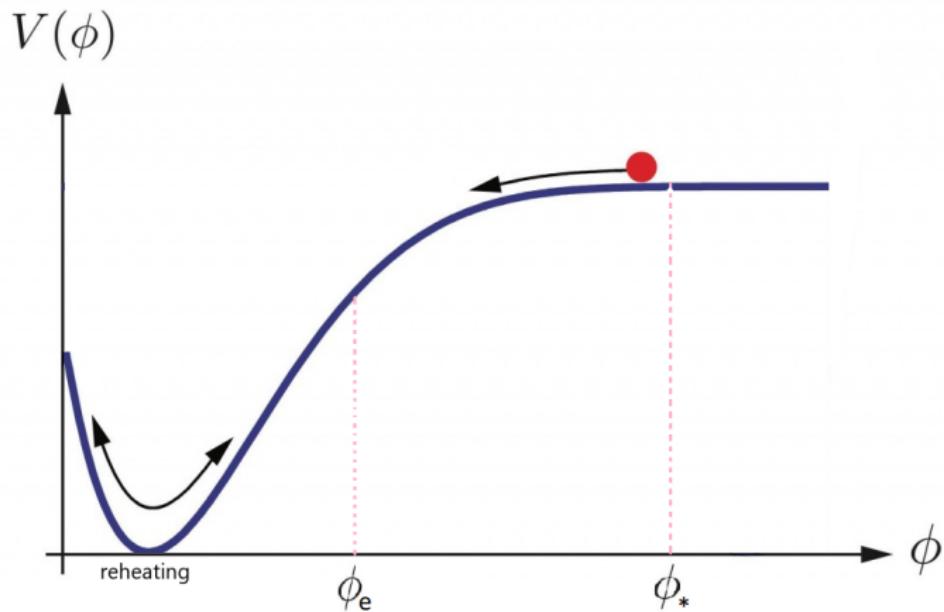
Thank you!

BACKUP SLIDES

BICEP / Keck XIII: Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season, (arXiv:2110.00483)



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Slow-roll approximation:

$$V(\phi) \gg \dot{\phi}^2, \quad |\ddot{\phi}| \ll |3H\dot{\phi}|, |V'|. \quad (49)$$

The potential slow-roll parameters:

$$\epsilon_V = \frac{1}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \quad \eta_V = \frac{V''(\phi)}{V(\phi)}. \quad (50)$$

→ During inflation $\epsilon_V \ll 1$ and $|\eta_V| \ll 1$.

Hubble slow-roll parameter (HSRP):

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V}, \quad \frac{\ddot{a}}{a} = H^2(1 - \epsilon_1) \quad (51)$$

→ Inflation ends when $\epsilon_1 = 1$.

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We know that $N_k = \ln \frac{a_{end}}{a(t)}$. This can be written as

$$\begin{aligned} N_* &= 66.89 - \ln c_s^* - \ln \left(\frac{k^*}{a_0 H_0} \right) + \frac{1}{4} \left(\ln \frac{9H_*^4}{\rho_{end}} \right) - \frac{1}{12} \ln g_s^{*(reh)} \\ &+ \frac{1 - 3w}{3(1 + w)} \left(\ln \frac{T_{reh}}{M_{Pl}} - \frac{1}{4} \ln \frac{\rho_{end}}{M_{Pl}^4} - \frac{1}{4} \ln \frac{30}{\pi^2} + \frac{\ln g^{*(reh)}}{4} \right). \quad (52) \end{aligned}$$

- T_{reh} reheating temperature
- H_0 Hubble constant today
- a_0 scale factor constant today
- g, g_s energy and entropy dofs
- w equation of state parameter

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The potential is given by

$$V(\phi) = \lambda_n \phi^n. \quad (53)$$

The first two PSRPs are easily computed to be

$$\epsilon_V = \frac{n^2}{2} \frac{1}{\phi^2}, \quad \eta_V = n(n-1) \frac{1}{\phi^2}. \quad (54)$$

In the slow-roll approximation, inflation ends when $\epsilon_1 \simeq \epsilon_V = 1$, so $\phi_{\text{end}} = n/\sqrt{2}$. Then

$$n_s = 1 - \frac{n+2}{2N_*}, \quad r = \frac{4n}{N_*} \quad (55)$$

Let us now consider $N_* = 60$

• $V = \frac{1}{2}m^2\phi^2 \Rightarrow$ $n_s \simeq 0.966$ and $r \simeq 0.13$.

1σ

• $V = \frac{1}{4}\lambda\phi^4 \Rightarrow$ $n_s \simeq 0.950$ and $r \simeq 0.26$.

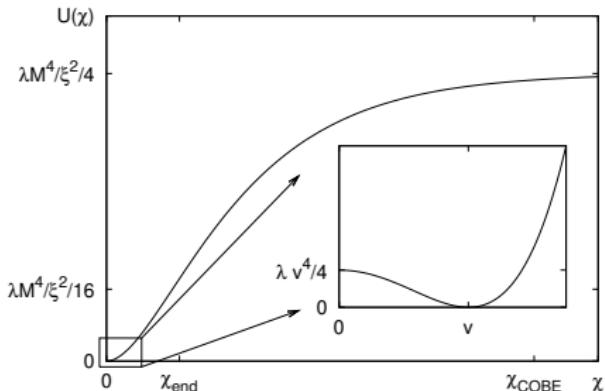
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The action is

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2 R}{2} + \frac{\xi h^2 R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} (h^2 - v^2)^2 \right], \quad (56)$$

After a Weyl rescaling $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega^2 = 1 + \xi h^2 / M_{\text{Pl}}^2$ and a field redefinition we obtain

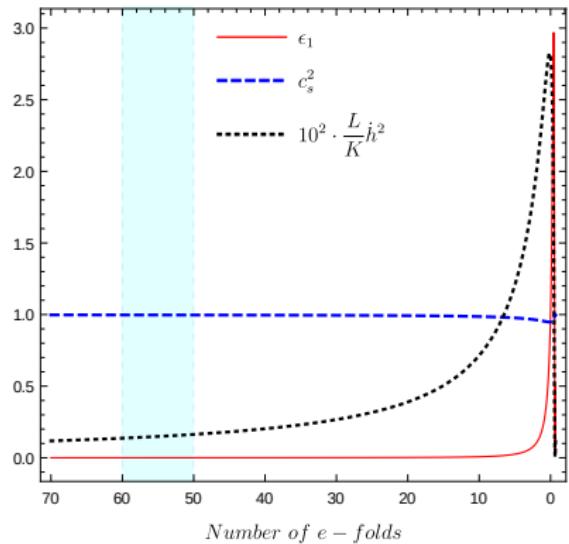
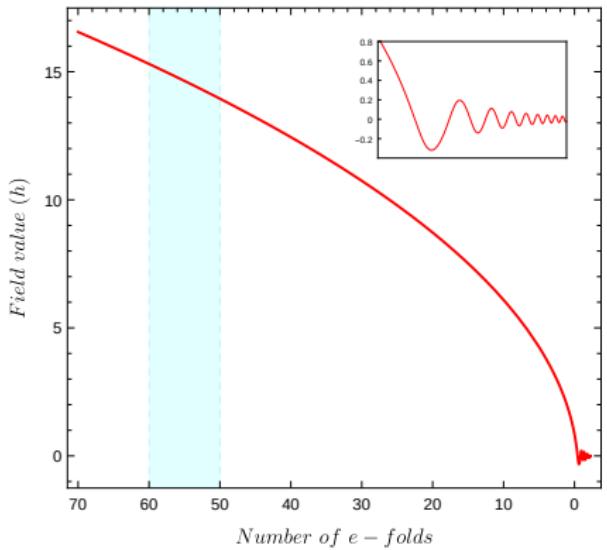
$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^2 \tilde{R}}{2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right], \quad (57)$$



$$n_s \simeq 1 - \frac{2}{N_*} \simeq 0.966$$

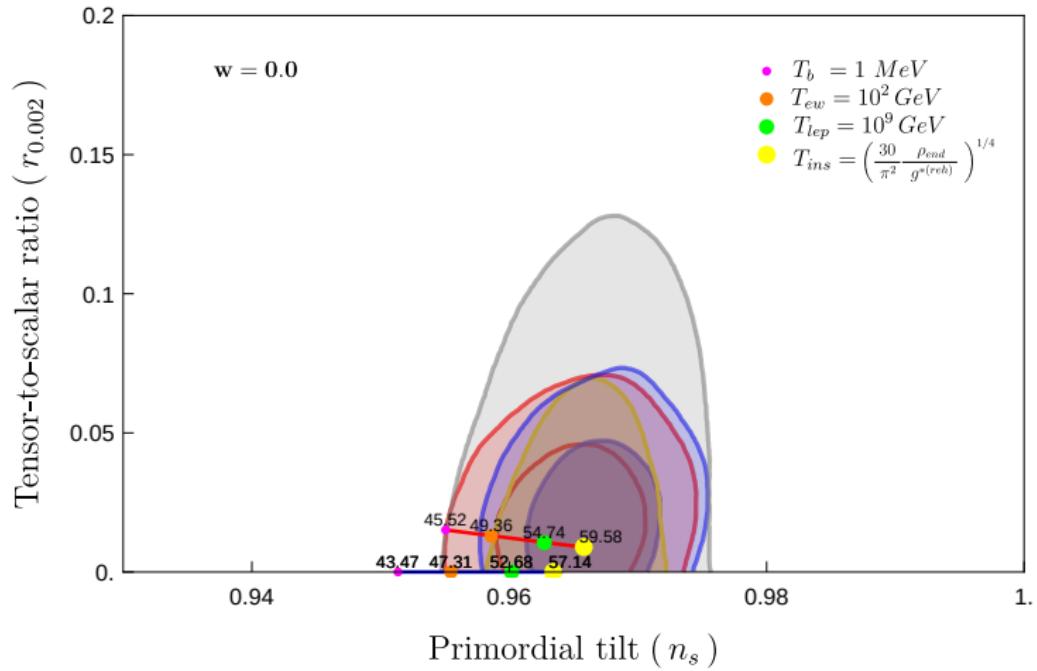
$$r \simeq \frac{12}{N_*^2} \simeq 0.0033$$

Negligible kinetic terms

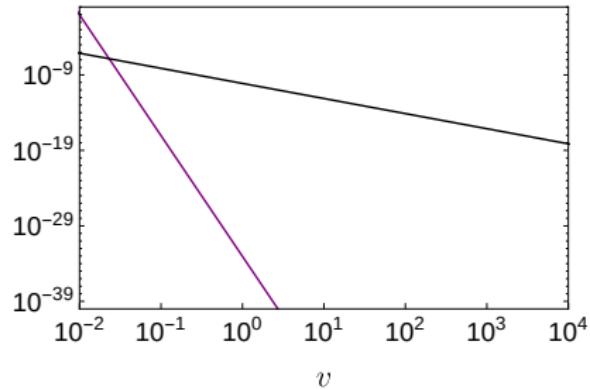
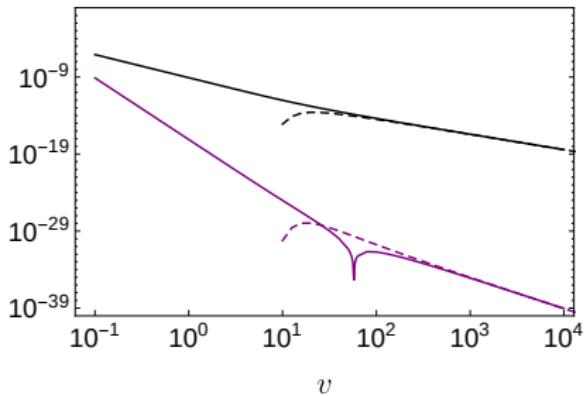


Quadratic model

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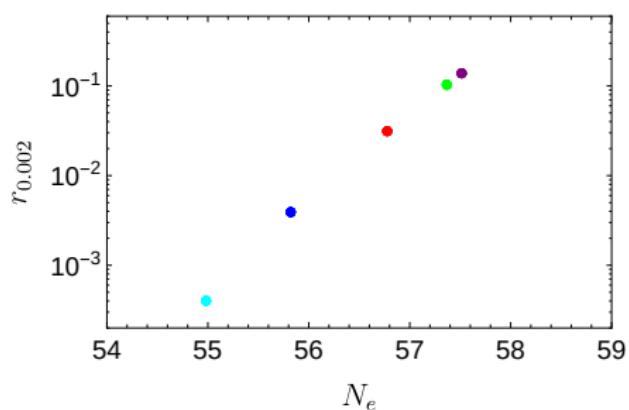
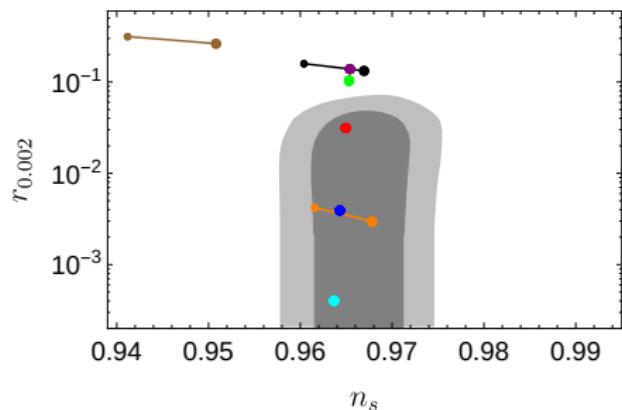
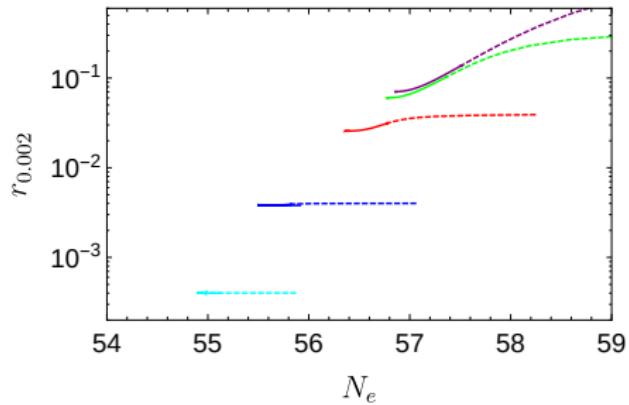
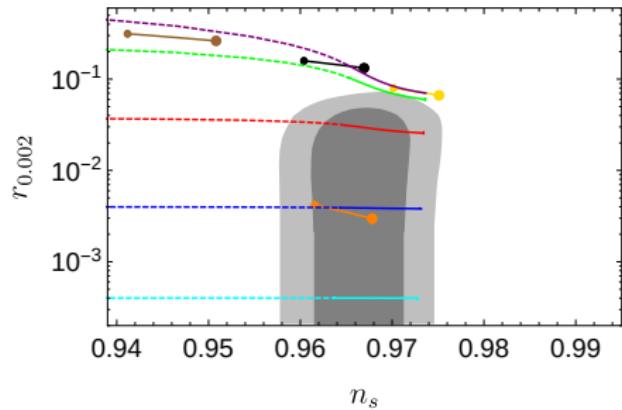
The 1st order CW potential is linear in the logarithmic term, therefore it corresponds to a 1-loop effective potential. As such the validity of the approximation is ensured by the requirement

$$\beta_\lambda(\phi) \approx \frac{\lambda(\phi)^2}{\pi^2} \ll \beta .$$

On the other hand, the 2nd order CW potential is quadratic in the logarithmic term, therefore it corresponds to a 2-loop effective potential. As such the validity of the approximation is ensured by the requirement

$$\beta_\lambda^2(\phi) \approx \frac{\lambda(\phi)^4}{\pi^4} \ll \beta' .$$

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- Mass matrix: $M_{ij}^2 \equiv \frac{\partial^2 U^{(0)}}{\partial \Phi^i \partial \Phi^j} \Big|_{\Phi^i = v_{\Phi^i}, \Phi^j = v_{\Phi^j}}$ where $(\Phi^1, \Phi^2) = (\phi, h)$
- Mixing angle: $\omega \equiv \arctan \left(\frac{v_h}{v_\phi} \right) = \arctan \left(\sqrt{\frac{\lambda_{\phi h} \xi_\phi + 2\lambda_\phi \xi_h}{\lambda_{\phi h} \xi_h + 2\lambda_h \xi_\phi}} \right)$
- Orthogonal rotation: $\begin{pmatrix} \phi \\ h \end{pmatrix} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} s \\ \sigma \end{pmatrix}$
- Mass eigenvalues: $m_s^2 = 0$,

$$m_\sigma^2 = \frac{M_P^4 (\lambda_{h\phi} \xi_h + 2\lambda_h \xi_\phi) (2\lambda_\phi \xi_h + \lambda_{h\phi} \xi_\phi)^2 [(\lambda_{h\phi} + 2\lambda_\phi) \xi_h + (2\lambda_h + \lambda_{h\phi}) \xi_\phi]}{8 v_h^2 [\lambda_\phi \xi_h^2 + \xi_\phi (\lambda_{h\phi} \xi_h + \lambda_h \xi_\phi)]^3}$$

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Along the FD $\sigma = 0$, so

$$s^2 = \phi^2 + h^2, \quad s = \frac{\phi}{\cos \omega} = \frac{h}{\sin \omega}. \quad (58)$$

and

$$\frac{1}{\Omega^2} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h \right] = \frac{1}{\Omega^2} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s \right],$$

where, the nonminimal coupling functional expressed in terms of s has the following form:

$$\frac{1}{\Omega^2} = \frac{1}{\xi_\phi \phi^2 + \xi_h h^2} = \frac{1}{\xi_s s^2}. \quad (59)$$

In the last equation, we have defined an effective nonminimal coupling constant for the scalaron as

$$\xi_s \equiv \xi_\phi \cos^2 \omega + \xi_h \sin^2 \omega. \quad (60)$$

Finally, we perform the following field redefinition in order to render the kinetic term of s canonical:

$$s_c - v_c = \int_{v_s}^s \frac{1}{\sqrt{\xi_s}} \frac{ds'}{s'} = \frac{1}{\sqrt{\xi_s}} \ln \frac{s}{v_s}. \quad (61)$$

BACKUP SLIDES

Minimizing (36), we can determine the scale Λ as

$$\Lambda = v_s \exp \left[\frac{\mathbb{A}}{2\mathbb{B}} + \frac{1}{4} \right]. \quad (62)$$

Then, we can express the one-loop correction as

$$U^{(1)}(s_c) = \frac{\mathcal{M}^4}{64\pi^2 v_s^4} s_c^4 \left[\ln \frac{s_c^2}{v_s^2} - \frac{1}{2} \right]. \quad (63)$$

From the one-loop corrections we can obtain the radiatively-generated mass for the s scalar

$$m_s^2 = \frac{\mathcal{M}^4}{8\pi^2 v_s^2}. \quad (64)$$

BACKUP SLIDES

In contrast to the metric case there is now a plethora of invariants that can be constructed out of the Ricci and Riemann tensors:

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}, \quad \hat{R}^\mu{}_\nu = g^{\lambda\sigma} R^\mu{}_{\sigma\nu\lambda} \quad \text{and} \quad R'_{\mu\nu} = R^\lambda{}_{\lambda\mu\nu}.$$

The most general Lagrangian second order in the Riemann tensor contains 16 possible contractions and can be written as

$$\begin{aligned} S = & \int d^4x \sqrt{-g} \left[\alpha R^2 + \beta_1 R_{\mu\nu} R^{\mu\nu} + \beta_2 R_{\mu\nu} R^{\nu\mu} + \beta_3 R_{\mu\nu} \hat{R}^{\mu\nu} + \beta_4 R_{\mu\nu} \hat{R}^{\nu\mu} + \right. \\ & \beta_5 \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + \beta_6 \hat{R}_{\mu\nu} \hat{R}^{\nu\mu} + \beta_7 \hat{R}_{\mu\nu} R'^{\mu\nu} + \beta_8 R'_{\mu\nu} R'^{\mu\nu} + \beta_9 R_{\mu\nu} R'^{\mu\nu} + \\ & \gamma_1 R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda} + \gamma_2 R_{\mu\nu\sigma\lambda} R^{\mu\sigma\nu\lambda} + \gamma_3 R_{\mu\nu\sigma\lambda} R^{\nu\mu\sigma\lambda} + \gamma_4 R_{\mu\nu\sigma\lambda} R^{\nu\sigma\mu\lambda} + \\ & \left. \gamma_5 R_{\mu\nu\sigma\lambda} R^{\sigma\nu\mu\lambda} + \gamma_6 R_{\mu\nu\sigma\lambda} R^{\sigma\lambda\mu\nu} \right]. \end{aligned}$$

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$$g_{\mu\nu} = A q_{\mu\nu} + B \partial_\mu s_c \partial_\nu s_c \quad (65)$$

- Inverse: $g^{\mu\nu} = \bar{A} q^{\mu\nu} + \bar{B} q^{\mu\lambda} q^{\nu\sigma} \partial_\lambda s_c \partial_\sigma s_c$ with
 $\bar{A} = \frac{1}{A}$, $\bar{B} = -\frac{B}{A^2 - 2ABX_q}$.
- Determinant: $g = qA^3 (A - 2BX_q)$
- Kinetic q: $X_q \equiv -\frac{1}{2} q^{\mu\nu} \partial_\mu s_c \partial_\nu s_c$
- Kinetic g: $X_g = \bar{A} X_q - 2\bar{B} X_q^2$

Substituting all of these in $\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 0$ and requiring that the coefficients of

$q_{\mu\nu}$,

and

$\partial_\mu s_c \partial_\nu s_c$

must vanish identically we obtain

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$$\frac{1}{16\beta(4\alpha + \beta)R_5} \left(\begin{array}{l} 4(4\alpha + \beta)A^2 - 4\beta A\sqrt{R_5} - 4\alpha AR_2 - (4\alpha + \beta)R_3 \\ + 4\beta R_5 + \alpha R_2^2 \end{array} \right) + \frac{U_{\text{eff}}(s_c)}{2} - \frac{X_g}{2} = 0, \quad (66)$$

$$\frac{1}{16\beta(4\alpha + \beta)R_5} \left(\begin{array}{l} 4(4\alpha + \beta)R_4 - 4\beta R_1\sqrt{R_5} - 4\alpha R_2 R_1 - (4\alpha + \beta)BR_3 \\ + 4\beta BR_5 + \alpha BR_2^2 \end{array} \right) + \frac{BU_{\text{eff}}(s_c)}{2} - \frac{BX_g}{2} - \frac{1}{2} = 0, \quad (67)$$

with $R_1 = B(2A - 2BX_q), \dots$

In slow-roll $A = a_0 + a_1 X_q + \mathcal{O}(X_q^2), \quad B = b_0 + b_1 X_q + \mathcal{O}(X_q^2).$

$$a_0 = \frac{1}{1 + \tilde{\alpha}U_{\text{eff}}}, \quad b_0 = \frac{(\tilde{\beta} - \tilde{\alpha})}{(1 + \tilde{\alpha}U_{\text{eff}})(1 + \tilde{\beta}U_{\text{eff}})},$$

$$a_1 = \frac{\tilde{\beta}}{2(1 + \tilde{\beta}U_{\text{eff}})}, \quad b_1 = \frac{(\tilde{\beta} - \tilde{\alpha}) \left(3\tilde{\beta} - 2\tilde{\alpha} + (2\tilde{\beta} - \tilde{\alpha})(\tilde{\alpha} + \tilde{\beta})U_{\text{eff}} + \tilde{\alpha}\tilde{\beta}^2 U_{\text{eff}}^2 \right)}{(1 + \tilde{\alpha}U_{\text{eff}})(1 + \tilde{\beta}U_{\text{eff}})^3},$$

where we have defined $\tilde{\alpha} = 2\beta + 8\alpha, \tilde{\beta} = 4\beta + 8\alpha.$

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The functions R_i which has been displayed in Eqs. (66)-(67) are listed below

$$R_1 = B(2A - 2BX_q),$$

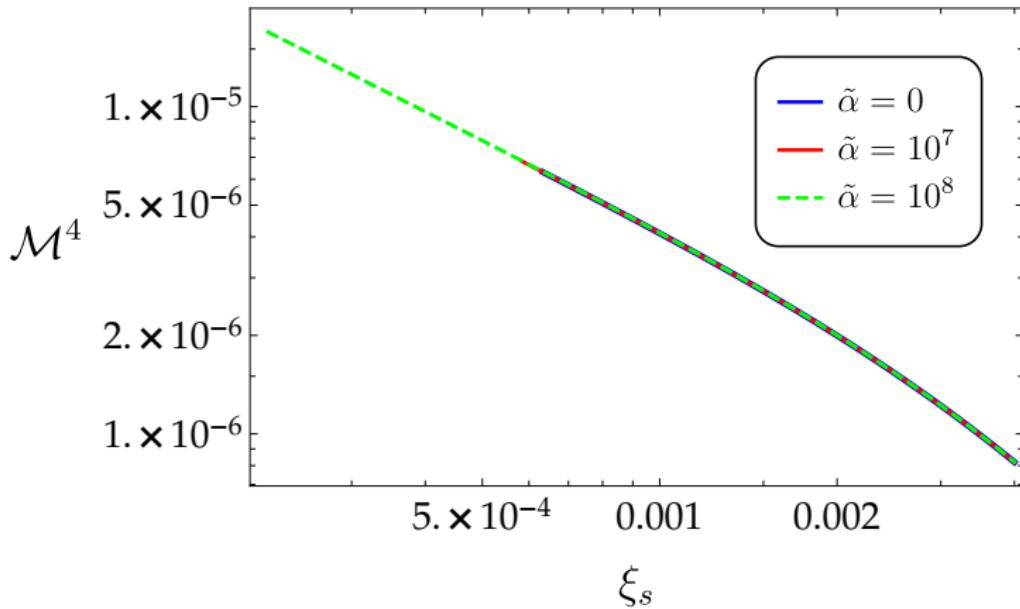
$$R_2 = 4A - 2BX_q,$$

$$R_3 = 4A^2 - 4ABX_q + 4B^2X_q^2,$$

$$R_4 = A(R_1 + AB) - 2BR_1X_q,$$

$$R_5 = A^3(A - 2BX_q).$$

BACKUP SLIDES



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Small field inflation					
$\tilde{\alpha}$	$\xi_s^{(\min)}$	\mathcal{M}	r	n_s	N_*
0	0.0006267	0.0502432	0.0729636	0.968159	60.3
10^7	0.0005830	0.0510926	0.0730490	0.968233	60.3
10^8	0.0002017	0.0651665	0.0732724	0.968439	60.3

$\tilde{\alpha}$	$\xi_s^{(\max)}$	\mathcal{M}	r	n_s	N_*
0	0.0041417	0.0297085	0.0161109	0.957741	59.6
10^7	0.0041389	0.0297168	0.0160355	0.957747	59.6
10^8	0.0041367	0.0297308	0.0152745	0.957739	59.6

Table: For $\tilde{\alpha} \lesssim 10^{8.267} \simeq 1.85 \times 10^8$, only small field inflation yields viable values for r and n_s . Here, we give the minimum and maximum values of ξ_s for which we obtain viable predictions for various $\tilde{\alpha}$. We also give the values of \mathcal{M} , r , n_s and N_* for these marginal values of ξ_s .

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Small field inflation					
$\tilde{\alpha}$	$\xi_s^{(\max)}$	\mathcal{M}	r	n_s	N_\star
10^9	0.0040967	0.0299033	0.0103843	0.957734	59.4
10^{10}	0.0039033	0.0306853	0.0024263	0.957835	58.8
10^{11}	0.0036767	0.0316817	0.0002763	0.957919	58.0
10^{12}	0.0035200	0.0324432	0.0000280	0.957921	57.3

Large field inflation					
$\tilde{\alpha}$	$\xi_s^{(\max)}$	\mathcal{M}	r	n_s	N_\star
10^9	0.0028733	0.0245332	0.0248280	0.958142	60.0
10^{10}	0.0025667	0.0259176	0.0027631	0.957819	59.0
10^{11}	0.0020250	0.0286194	0.0002796	0.957922	58.1
10^{12}	0.0017108	0.0306805	0.0000280	0.957918	57.4

Table: For various $\tilde{\alpha} \gtrsim 10^{8.267} \simeq 1.85 \times 10^8$, and for both small and large field inflation, we give the corresponding maximum values of ξ_s that yield predictions that comply with the observational bounds.