Thermal Propagators in dS Spacetime

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Abstract

- We consider a scalar field propagating into dS spacetime and try to systematize its various thermal corrections into the field's correlator via the use of Thermo-Field Dynamics.
- We fine tune the free parameters of the system via the experimental value of the spectral index n_S[1], and use the result in order to calculate the running of n_S, the running of the running of n_S and the non-Gaussianity parameter f_{NL}[2].
- We observe an LCP for the free parameters that keep the value of n_S fixed.

[1] Planck Collaboration, Y. Akrami et al., Planck 2018 results. X. Constraints on inflation, Astron. Astrophys. 641 (2020), 1807.06211 [astro-ph.CO]

[2] Planck Collaboration, Y. Akrami et al., Planck 2018 results. IX. Constraints on primordial non-Gaussianity, Astron. Astrophys. 641 (2020), 1905.05697 [astro-ph.CO]

Outline

Real Scalar Field in the PP of dS spacetime

We study the **Poincare Patch** of the d+1 dS spacetime described by the metric:

$$ds^2 = \alpha^2(d\tau^2 - dx^2), \quad \alpha = -\frac{1}{H\tau}, \quad \tau \in (-\infty, 0) \tag{1.1}$$

along with a real scalar field ϕ whose propagation satisfies the action:

$$\mathcal{S} = \int d^4x \sqrt{-g}\Big[\frac{1}{2} g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi - (m^2 + \xi R)\phi^2\Big]. \tag{1.2}$$

Defining

$$\Phi_{\mathrm{k}} = \frac{\chi_{\mathrm{k}}}{\alpha}, \quad \Phi_{|\mathrm{k}|} = \int d^4x \; \phi(x) \; e^{ikx} \tag{1.3}$$

the following E.O.M holds:

$$\ddot{\chi}_{\mathrm{k}} + \omega^2_{|\mathrm{k}|}\chi_{\mathrm{k}} = 0, \quad \omega^2_{|\mathrm{k}|} = |\mathrm{k}|^2 + m^2_{dS} \tag{1.4}$$

with

$$m^2_{dS} = \frac{1}{\tau^2}(M^2 - \frac{d^2-1}{4}), \quad M^2 = \frac{m^2}{H^2} + 12\xi. \tag{1.5}$$

Real Scalar Field in the PP of dS spacetime

In terms of the mode functions $u_{|\mathrm{k}|}(\tau)$:

$$\chi_{\mathrm{k}}(\tau) = a_{\mathrm{k}}^{-} u_{|\mathrm{k}|}^{*}(\tau) + a_{\mathrm{k}}^{+} u_{|\mathrm{k}|}(\tau), \tag{1.6}$$

the general solution of the E.O.M. can be wrtten as a linear combination of the Bessel functions $J_\nu(\tau \mathrm{k})$, $Y_\nu(\tau \mathrm{k})$

$$u_{|\mathrm{k}|}(\tau) = \sqrt{|\tau|}\Big[A_{|\mathrm{k}|} J_\nu(|\mathrm{k}||\tau|) + B_{|\mathrm{k}|} Y_\nu(|\mathrm{k}||\tau|)\Big] \tag{1.7}$$

with:

$$\nu = \frac{d}{2}\sqrt{1 - \frac{4M^2}{d^2}}, \quad A_{|\mathrm{k}|} B_{|\mathrm{k}|}^{*} - A_{|\mathrm{k}|}^{*} B_{|\mathrm{k}|} = \frac{i\pi}{2}. \tag{1.8}$$

For the choice $A_{|\mathrm{k}|} = \sqrt{\pi/4}$,

$$u_{|\mathrm{k}|,BD} = \frac{\sqrt{\pi|\tau|}}{2}\mathcal{H}_\nu^{(2)}(|\mathrm{k}||\tau|). \tag{1.9}$$

Quantisation

Due to curvature, quantazing the above system results to a time-depedent vacuum defined in each instance by:

$$a_{\mathrm{k}}^{-}\,|0(\tau_1)\rangle = 0, \quad [a_{\mathrm{k}}^{-} a_{\mathrm{q}}^{+}] = \delta^{(3)}(\mathrm{k}-\mathrm{q}) \tag{2.1}$$

while at another instance

$$b_{\mathrm{k}}^{-}\,|0(\tau_2)\rangle = 0, \quad [b_{\mathrm{k}}^{-} b_{\mathrm{q}}^{+}] = \delta^{(3)}(\mathrm{k}-\mathrm{q}). \tag{2.2}$$

Through **time-independent** Bogolyubov tranformations, one can connect between the two states via a linear transformation:

$$b_{\mathrm{k}}^{+} = c_{|\mathrm{k}|} a_{\mathrm{k}}^{+} - d_{|\mathrm{k}|} a_{-|\mathrm{k}|}^{-}, \quad b_{\mathrm{k}}^{-} = c_{|\mathrm{k}|}^{*} a_{\mathrm{k}}^{-} - d_{|\mathrm{k}|}^{*} a_{-|\mathrm{k}|}^{+}. \tag{2.3}$$

of their respective ladder operators. We study the case:

$$|\mathrm{in},(\beta)\rangle \equiv \tau \to -\infty \quad \text{(BD vacuum)}^{34}, \tag{2.4a}$$

$$|\mathrm{out},(\beta)\rangle \equiv \tau = \delta \to 0^{-}. \tag{2.4b}$$

[3] N. A. Chernikov and E. A. Tagirov, Quantum theory of scalar field in de Sitter space-time, Annales de l'I. H. P., section A, tome 9, no 2 (1968), p. 109-141

[4] B. Allen, Vacuum States in de Sitter Space, Phys. Rev. D32 (1985) 3136

Quantisation

The general scalar Feynman propagator between two spacetime points can be characterized by

$$\mathcal{D}^I_J(\tau_1, \mathrm{x}_1; \tau_2, \mathrm{x}_2) = \langle J; (\beta)|\mathcal{T}\Big\{\Phi^I(\tau_1, \mathrm{x}_1)\Phi^I(\tau_2, \mathrm{x}_2)\Big\}|J; (\beta)\rangle \qquad (2.5)$$

with

$$\Phi^I = \begin{pmatrix} \Phi^{+,I} \\ \Phi^{-,I} \end{pmatrix}, \quad I, J = \text{in,out}. \qquad (2.6)$$

We consider a doubled Hilbert spacetime which via the Schwinger-Keldysh path integral[5][6] describes a forward and a backward branch in conformal time τ evolution.

[5] J. Schwinger. Brownian motion of a quantum oscillator. Journal of Mathematical Physics, 2(3):407–432, 1961.

[6] L. V. Keldysh. Diagram technique for nonequilibrium processes. Soviet Physics, JETP, 20(4):1018–1026, 1965.

Quantisation

Then, the field propagator $\mathcal{D}$ in this formalism has a 2×2 matrix structure:

$$\mathcal{D} = \begin{pmatrix} \mathcal{D}_{++} & \mathcal{D}_{<} \\ \mathcal{D}_{>} & \mathcal{D}_{--} \end{pmatrix} \tag{2.7}$$

with

$$\mathcal{D}_{<}(\tau_1;\tau_2) = \langle 0|\Phi^{+}(\tau_2)\Phi^{-}(\tau_1)|0\rangle \quad \mathcal{D}_{>}(\tau_1;\tau_2) = \langle 0|\Phi^{-}(\tau_1)\Phi^{+}(\tau_2)|0\rangle \tag{2.8}$$

$$\mathcal{D}_{++}(\tau_1;\tau_2) = \langle 0|\mathcal{T}\Big\{\Phi^{+}(\tau_2)\Phi^{+}(\tau_1)\Big\}|0\rangle \tag{2.9}$$

$$\mathcal{D}_{--}(\tau_1;\tau_2) = \langle 0|\mathcal{T}^{*}\Big\{\Phi^{-}(\tau_2)\Phi^{-}(\tau_1)\Big\}|0\rangle \tag{2.10}$$

where $\mathcal{T}$ and $\mathcal{T}^{*}$ describes time and anti-time ordering.

Thermo-Field Dynamics in dS spacetime

We can describe temperature $T = 1/\beta$ in our system with either the insertion of an explicit density matrix

$$|I;\alpha\rangle = \mathrm{U}_{\alpha}|I\rangle, \quad \mathrm{U}_{\alpha}^{\intercal}\mathrm{U}_{\alpha} = 1, \quad T_{dm} = \frac{1}{\alpha} \tag{3.1}$$

or via the or the Gibbons-Hawking effect[7]

$$|I\rangle = |J;\delta\rangle, \quad I \neq J, \quad T_{GH} = \frac{1}{\delta}. \tag{3.2}$$

The thermal generalization of the propagator, in TFD is seen as a matrix transformation:

$$\mathcal{D}_{\beta} = \mathrm{U}_{\beta}\mathcal{D}_{SK}\mathrm{U}_{\beta}^{\intercal}. \tag{3.3}$$

while its zero-temperature flat spacetime limit of the Feynman propagator gives

$$\mathcal{D}_{SK} = \begin{pmatrix} \frac{i}{k^2-m^2-i\epsilon} & 0 \\ 0 & \frac{-i}{k^2-m^2-i\epsilon} \end{pmatrix} \tag{3.4}$$

[7] G. W. Gibbons and S. W. Hawking, Cosmological Event Horizons, Thermodynamics, and Particle Creation, Phys. Rev. D15 (1977) 2738–2751

Thermo-Field Dynamics in dS Spacetime

We choose[8]:

$$\mathrm{U}_\beta = \begin{pmatrix} \cosh\theta_{|\mathsf{k}|} & \sinh\theta_{|\mathsf{k}|} \\ \sinh\theta_{|\mathsf{k}|} & \cosh\theta_{|\mathsf{k}|} \end{pmatrix}, \quad \cosh\theta_{|\mathsf{k}|} = \frac{1}{\sqrt{1 - e^{-\beta\omega_{|\mathsf{k}|}}}} \tag{3.5}$$

so that:

$$\mathcal{D}^I_{J,\beta} = \langle J;\beta|\mathcal{T}\left\{\Phi^I\left(\Phi^I\right)^{\mathsf{T}}\right\}|J;\beta\rangle\,. \tag{3.6}$$

Near the Horizon $m^2_{dS} \to \infty$,

$$\alpha < \delta: \quad \beta = \alpha + \delta e^{-\frac{|\alpha-\delta|}{2} m^2_{dS}} + \ldots \tag{3.7a}$$

$$\alpha > \delta: \quad \beta = \delta + \alpha e^{-\frac{|\alpha-\delta|}{2} m^2_{dS}} + \ldots \tag{3.7b}$$

which results to the maximum temperature of dS spacetime:

$$T_{dS,max} = \frac{H}{2\pi}. \tag{3.7c}$$

[8] A. Das, Finite Temperature Field Theory, Singapore: World Scientific (1997).

Thermo-Field Dynamics in dS Spacetime

In the case $\beta = \delta$:

$$\mathcal{D}_\beta = \mathcal{D}_{SK} + \left(\mathcal{D}_{++} + \mathcal{D}^*_{++}\right)\left(\sinh^2\theta_{|\mathsf{k}|} + \sinh\theta_{|\mathsf{k}|}\cosh\theta_{|\mathsf{k}|}\right)\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \quad (3.8)$$

with the flat limit of the above propagator being diagonal along with a therma $i\epsilon$ shift:

$$i\epsilon \to i\epsilon\cosh\left(\frac{\beta\omega_{|\mathsf{k}|}}{2}\right). \quad (3.9)$$

At time of horizon exit $|\tau|H \to 1$ and equal-spacetime points we define the **scalar** power spectrum:

$$\mathcal{P}_{S,\beta} = \mathcal{D}_\beta\Big|_{\tau_1=\tau_2}, \quad |\mathsf{k}\tau| = 1 \quad (3.10)$$

for each component of the 2×2 matrix.

Parametrizing the Thermal Corrections

The scalar power spectrum $\mathcal{P}_\beta$ can determine various cosmological indices where we systemize their definitions with the introduction of a single mathematical parameter:

$$\kappa = \omega_{|\mathrm{k}|}|\tau|\Big|_{|\mathrm{k}\tau|=1} = \sqrt{\frac{5-d^2}{4} + M^2}. \tag{4.1}$$

We are mainly interested in the $d = 3$ boundary, with

$$\mathcal{P}_S = \frac{|\mathrm{k}|^3}{2\pi^2}\left|\frac{\Phi_{|\mathrm{k}|}}{z}\right|^2 = \frac{|\mathrm{k}|^3}{2\pi^2}\left|\mathcal{R}_\mathrm{k}\right|^2, \quad z \equiv \alpha\frac{\psi'}{H} \tag{4.2}$$

where $\mathcal{R}_\mathrm{k}$ are the curvature perturbations.

Note that when $M = 0$ (or $\kappa = i$) in zero temperature, $\mathcal{P}_S$ is scale invariant

$$\kappa = i \Leftrightarrow \nu = \frac{3}{2}. \tag{4.3}$$

Parametrizing the Thermal Corrections

In general, Bogolyubov transformations can be **time-dependent**, resulting into the change of the observed frequency[9]

$$\Omega_{|k|} = \omega_{|k|}\Big(|\cosh\theta_{|k|}|^2 + |\sinh\theta_k|^2\Big) \tag{4.4}$$

with the thermal corrections being encoded into the new one. Correspondingly, the order of the Bessel solution ν changes

$$\nu \to \nu'$$

and so does the κ parameter into:

$$\Lambda = \kappa\Big(1 + 2\frac{e^{-2x\kappa}}{1-e^{-2x\kappa}}\Big) = \kappa\coth(x\kappa), \quad x = \frac{\pi H}{2\pi T}. \tag{4.5}$$

[9] B. Garbrecht, T. Prokopec and M. G. Schmidt, Particle number in kinetic theory, Eur. Phys. J. C38 (2004) 135-143, hep-th/0211219 [hep-th].

Implications to Cosmology

The thermal spectral index $n_{S,\beta}$ is defined by $\mathcal{P}_S$ as:

$$n_{S,\beta} \equiv 1 + \frac{d \ln\left(|\mathrm{k}|^3 \mathcal{P}_{S,\beta}\right)}{d \ln |\mathrm{k}|} \tag{5.1}$$

so that its thermal corrections δn_S are equal to:

$$\delta n_S \equiv n_{S,\beta} - 1 = -\frac{2x}{\Lambda}\left[\frac{e^{-x\Lambda}}{1 - e^{-2x\Lambda}}\right]. \tag{5.2}$$

Since de Sitter temperature T_{dS} corresponds to $x = \pi$, the only free parameter is Λ which we fix by the constraint:

$$n_{S,\beta} = 0.964. \tag{5.3}$$

Implications to Cosmology

Imagine now the following Scenario:

1. An observer measures the $\nu = 3/2$ BD vacuum energy at $\tau \to -\infty$ (UV- CFT[10]) and starts a journey with his spaceship in order to reach the Horizon.

2. As he approaches close the Horizon, he lands on the zero temperature conformal boundary (IR-CFT) which corresponds to $Im\kappa = 0$.

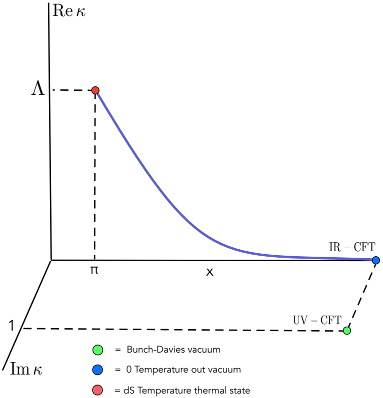
3. However, he continues his journey approaching the Horizon via a time-dependent Bogolyubov transformation as he observes thermal effects breaking the CFT as result.

[10] I. Antoniadis, P. O. Mazur and E. Mottola, Conformal Invariance, Dark Energy, and CMB Non- Gaussianity, JCAP 09 (2012) 024, arXiv:1103.4164 [gr-qc].

Implications to Cosmology

Suppose that the observer's spaceship was equipped with a device that could measure the spectral index.

Λ	x
$\to 0$	$\to \infty$
10^{-6}	$3.5 \cdot 10^{7}$
0.01	1600
0.5	14.8
1.5117	π



After the conformal point, we find an LCP that suggests that the device will keep on giving the same $n_{S,\beta}$.

Implications to Cosmology

We compute the runnings of $n_{S,\beta}$ defined by the relations:

$$n^{(1)}_{S,\beta} = \frac{dn_{S,\beta}}{d\ln|\mathrm{k}|}, \quad n^{(2)}_{S,\beta} = \frac{dn(1)_{S,\beta}}{d\ln|\mathrm{k}|} \tag{5.4}$$

which at the dS temperature ($x = \pi \Leftrightarrow \Lambda = 1.5117$) give:

$$n^{(1)}_{S,\beta} = 0.0186, \quad n^{(2)}_{S,\beta} = 0.125. \tag{5.5}$$

In addition, we compute the non-Gaussianity parameter f_{NL}[11][12]

$$f_{NL} = -\frac{5\left[x(-1+\Lambda^2)^2\left(1 + x\Lambda\cot(\frac{x\Lambda}{2})\right) + 2\Lambda^3\sinh(x\Lambda)\right]}{6\Lambda^2\left[x(-1+\Lambda^2) + \Lambda\sinh(x\Lambda)\right]} \tag{5.6}$$

which results at T_{dS} results to

$$f_{NL} = -1.7138. \tag{5.7}$$

[11] P. Creminelli and M. Zaldarriaga, Single field consistency relation for the 3-point function, JCAP 10 (2004) 006, astro-ph/0407059 [astro-ph].

[12] A. Kehagias and A. Riotto, The Four-point Correlator in Multifield Inflation, the Operator Product Expansion and the Symmetries of de Sitter, Nucl. Phys. B868 (2013)

Summary

1. We used the TFD formalism in order to describe thermal effects at the PP of dS spacetime.
2. Even if we mix explicit thermal effects along with a GH temperature, the maximum dS temperature is maintained.
3. We systematized the thermal corrections of the scalar power spectrum $\mathcal{P}_{S,\beta}$ and the spectral index $n_{S,\beta}$.
4. By fixing the value of $n_{S,\beta}$, a LCP emerged keeping $n_{S,\beta}$ fixed as our system heats up.
5. We calculated additional cosmological parameters where $n_{S,\beta}^{(1)}$ and f_{NL} are well within experimental data, while $n_{S,\beta}^{(2)}$ is not.

Thank You!