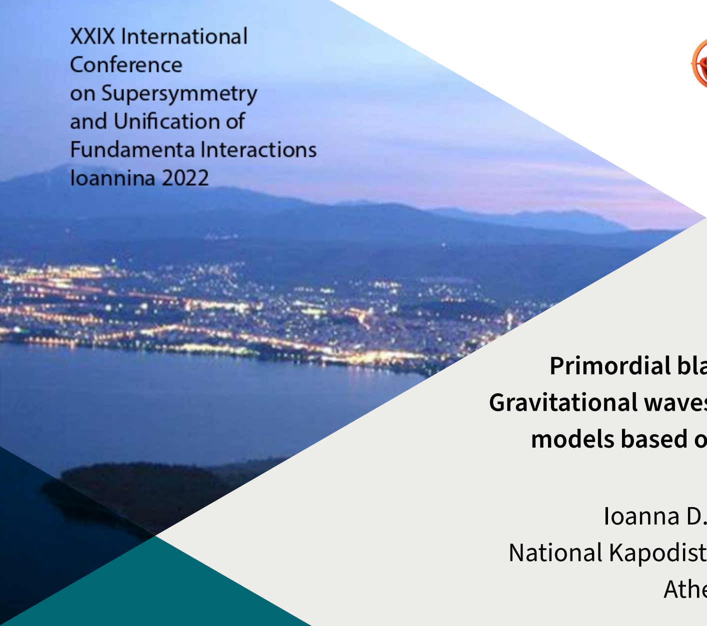

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Primordial black holes and Gravitational waves from inflationary models based on supergravity

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Overview

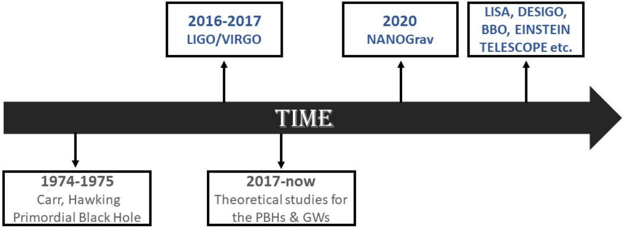
1. Introduction
2. An inflection point in the potential
3. Steep step-like potential
4. Model with a waterfall trajectory
5. Conclusions-Perspectives

Introduction

Introduction

Why do we study the production of PBHs and GWs ?

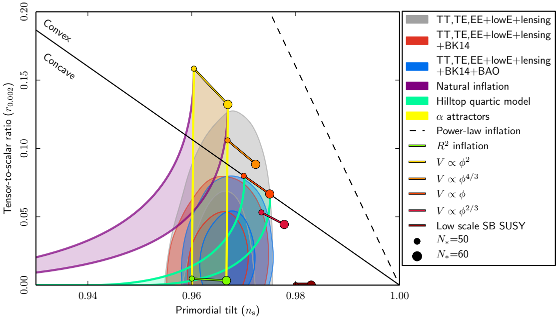
- The detection of Gravitational Waves (GWs) by a binary black hole merge by LIGO/VIRGO rekindles the old study of the Primordial Black Holes (PBHs).
- As a result there are numerous recent studies which show that the origin of PBHs can explain a fraction of Dark Matter (DM) in the Universe.



- The signal of the GWs is expected to be detected by future space-based GW interferometers such as LISA, BBO and DECIGO.
- Both the generation of PBHs and GWs can be explained in the framework of inflation. It is proposed that a significant amplification in the scalar power spectrum can explain both PBHs and GWs.

Constraints on the inflationary models

- The new theoretical models which have been proposed for explaining the generation of PBHs and GWs have to be in accordance with observable constraints on inflation released by Planck collaboration. [A&A 641 (2020)A10]



- Models based on Starobinsky-like potential give acceptable values for the spectral index n_s and tensor-to-scalar ratio r.
- Models which leads to Starobinsky-like effective scalar potential can be found through no-scale supergravity (SUGRA) theory.

How to obtain an enhancement in scalar power spectrum

We investigate three different mechanisms in order to obtain an amplification in scalar power spectrum:

1. An inflection point in the potential.
2. Steep step-like potential.
3. Models with a waterfall trajectory.

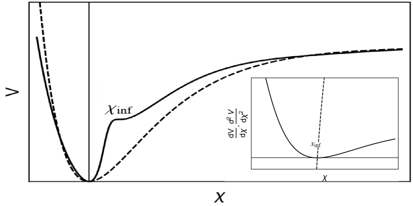
We match each mechanism with a model based on SUGRA theory in order to obtain an explicit model which respects the observable constraints:

1. No-Scale SUGRA Theory.
2. α-Attractor SUGRA.
3. Hybrid model with SUGRA corrections.

An inflection point in the potential

Inflection point in the scalar potential

Significant peaks in the scalar power spectrum, which can interpret the production of PBHs & GWs, can be produced by a near inflection point in the scalar potential.



This inflection point:

$$\frac{dV(\chi_{inf})}{d\chi_{inf}} \simeq 0, \qquad \frac{d^2V(\chi_{inf})}{d\chi_{inf}^2} = 0\,.$$

Pros:

- Simple mechanism for single field inflation & many recent works have adopted it.
- Provides significant results for abundances of PBHs and GWs.

Cons:

- Fine-tuning is required in order to achieve the proper peak in scalar power spectrum.

▶ We apply this mechanism to no-scale SUGRA.

Basic aspects of no-scale theory

The general Lagrangian in the context of SUGRA:

$$\mathcal{L} = K_i^{\bar{j}} \partial_\mu \varphi^i \partial^\mu \bar{\varphi}_{\bar{j}} - V(\varphi, \bar{\varphi}). \tag{1}$$

The F-term of the scalar potential:

$$V = e^K (D_\varphi W K^{\bar{\varphi}\varphi} D_{\bar{\varphi}} \bar{W} - 3|W|^2) \tag{2}$$

where K is the Kähler potential, W is the superpotential and D is the covariant derivative.

- The cosmological constant vanishes due to the identity:
  $$K^{\varphi\bar{\varphi}} K_\varphi K_{\bar{\varphi}} = 3.$$
  A flat potent can be found by the Kähler potential [Cremmer et. al. (1983)]:

$$K = -3 \ln(\varphi + \bar{\varphi}). \tag{3}$$

- Consider $\varphi = (y+1)/(y-1)$, we derive $K = -3 \ln \left(1 - \frac{|y|^2}{3}\right)$, which is invariant under the transformation of $y \to (\alpha y + \beta)/(\bar{\beta} y + \bar{\alpha}), \quad |\alpha|^2 - |\beta|^2 = 1.$ **SU(1,1)**

SU(1,1) group for a vanishing cosmological constant:

$$K=-3\ln(\varphi+\bar{\varphi}) \quad \text{or} \quad K=-3\ln(1-\tfrac{|y|^2}{3})$$

[Ellis, Kounnas, Nanopoulos (1984)]

SU(2,1)/SU(2)×U(1) group for finding Starobinsky-like scalar potential:

$$K=-3\ln(1-\tfrac{|y_1|^2}{3}-\tfrac{|y_2|^2}{3}) \quad \text{or} \quad K=-3\ln(T+\bar{T}-\tfrac{|\varphi|^2}{3})$$

$$W_{WZ}=\left(\tfrac{\hat{\mu}}{2}\left(y_1^2+\tfrac{y_1^2y_2}{\sqrt{3}}\right)-\lambda\tfrac{y_1^3}{3}\right) \quad \text{or} \quad W_C=m\left(-y_1y_2+\tfrac{y_2y_1^2}{l\sqrt{3}}\right)$$

where $(y_1,y_2)\rightarrow\left(\frac{2\varphi}{1+2T},\sqrt{3}\left(\frac{1-2T}{1+2T}\right)\right)$ and

$$W(T,\varphi)\rightarrow\bar{W}(y_1,y_2)=(1+y_2/\sqrt{3})^3W$$

[Ellis, Nanopoulos, Olive, Verner (2018)]

SU(2,1)/SU(2)×U(1) group for explaining the generation of PBHs & GWs:

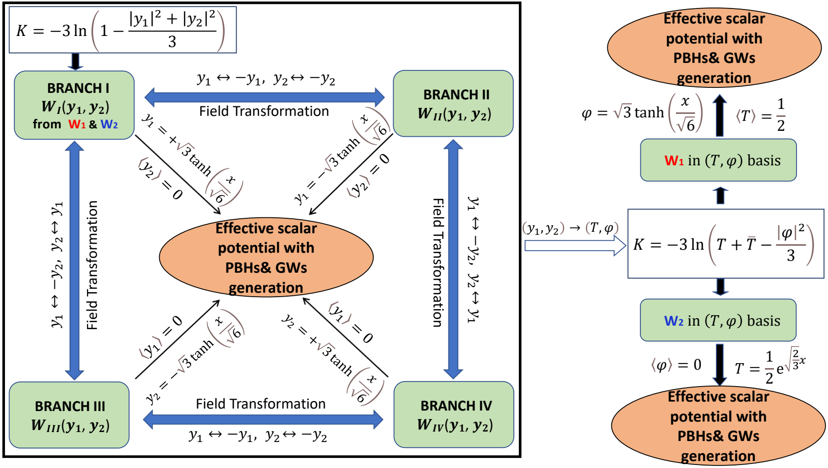
$$K=-3\ln(1-\tfrac{|y_1|^2}{3}-\tfrac{|y_2|^2}{3}) \quad \text{or} \quad K=-3\ln(T+\bar{T}-\tfrac{|\varphi|^2}{3})$$

$$W_1=\left(\tfrac{\hat{\mu}}{2}\left(y_1^2+\tfrac{y_1^2y_2}{\sqrt{3}}\right)-\lambda\tfrac{y_1^3}{3}\right)\left(1+g_1(y_1)\right) \quad \text{or} \quad W_2=m\left(-y_1y_2+\tfrac{y_2y_1^2}{l\sqrt{3}}\right)\left(1+g_2(y_1)\right)$$

Superpotentials

$$W_1 = \left(\frac{\hat{\mu}}{2}\left(y_1^2 + \frac{y_1^2 y_2}{\sqrt{3}}\right) - \lambda\frac{y_1^3}{3}\right)\left(1 + e^{-b_1 y_1^2}\left(c_1 y_1^2 + c_2 y_1^4\right)\right)\&$$

$$W_2 = m\left(-y_1 y_2 + \frac{y_2 y_1^2}{l\sqrt{3}}\right)\left(1 + c_3 e^{-b_2 y_1^2} y_1^2\right)$$



Scalar Power Spectrum

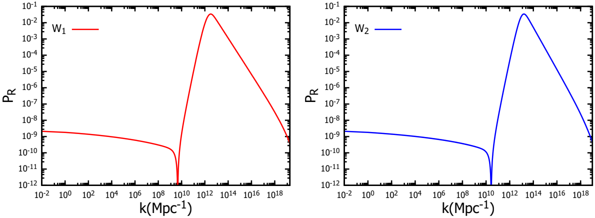
- The scalar power spectrum:

$$P_R = \frac{k^3}{2\pi^2}\left|R_k\right|^2, \tag{4}$$

where R_k is the comoving curvature perturbation:

$$R_k = \Psi + \frac{\delta\phi}{\phi'}.$$

- The power spectra for the cases W_1 & W_2: **[V. C. Spanos, IDS(2022)]**



We notice that the scalar power spectum has a significant enhancement due to inflection point.

Evaluating the production of PBHs

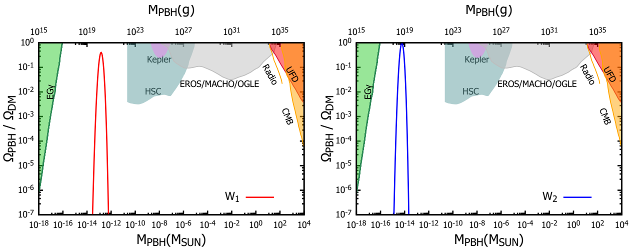
- We assume that PBHs are formed in the radiation dominated era.
- The present fraction of PBHs is given from:

$$\frac{\Omega_{PBH}}{\Omega_{DM}} = \frac{\beta(M_{PBH}(k))}{8\times 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g}{106.75}\right)^{-1/4}\left(\frac{M_{PBH}(k)}{10^{-18}grams}\right)^{-1/2}. \tag{5}$$

The mass is given as a function of k mode:

$$M_{PBH}(k) = 10^{18}\left(\frac{\gamma}{0.2}\right)\left(\frac{g}{106.75}\right)^{-1/6}\left(\frac{k}{7\times 10^{13}Mpc^{-1}}\right)^{-2}. \tag{6}$$

- The fractional abundance of PBHs for the cases W_1 & W_2:



Energy density of GWs

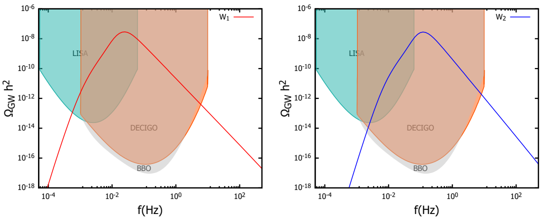
- The energy density of the GWs is given: [Espinosa, Racco, Riotto (2019)]

$$\Omega_{GW}(k) = \frac{\Omega_r}{36}\int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-1/3)(d^2-1/3)}{s^2+d^2}\right]^2 \times \\ P_R\left(k\frac{\sqrt{3}}{2}(s+d)\right) P_R\left(k\frac{\sqrt{3}}{2}(s+d)\right)(I_c^2+I_s^2) \qquad (7)$$

where the radiation density $\Omega_r \approx 5.4 \times 10^{-5}$. The functions I_c and I_s are given:

$$I_c = -36\pi\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1), \quad I_s = -36\frac{(s^2+d^2-2)^2}{(s^2-d^2)^2}\left[\frac{(s^2+d^2-2)}{(s^2-d^2)} log\left|\frac{d^2-1}{s^2-1}\right| + 2\right].$$

- The energy density of *GWs* for the cases W_1 & W_2: **[V. C. Spanos, IDS(2022)]**



Steep step-like potential

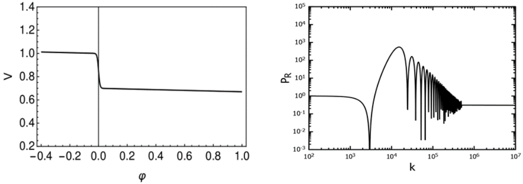
Step-like potential

The reinforcement in the scalar power spectrum can be achieved from the potential: **[K.Kefala, G.P. Kodaxis, N.Tetradis, IDS(2020)]**

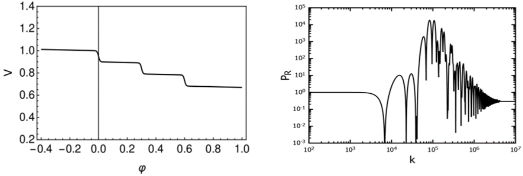
$$V(\varphi) = V_0 \left(1 + \frac{1}{2}\sum_i A_i \left(1 + \tanh\left[c_i\left(\varphi - \varphi_i\right)\right]\right)\right),$$

where A_i, c_i, φ_i and V_0 are the parameters.

1 STEP



3 STEPS



An explicit model

• There are studies of production of PBHs from α attractors SUGRA [Dalianis, Kehagias, Tringas (2018)]

• These models are based on the scheme: [Kallosh, Linde, Roest (2014)]

$$K = -3\alpha \ln\left(1 - \frac{|S|^2 + |\Phi|^2}{3} - \frac{g|S|^4}{3 - |\Phi|^2}\right), \quad W = Sf\left(\frac{\Phi}{\sqrt{3}}\right)\left(3 - \Phi^2\right)^{(3\alpha-1)/2}.$$

If we consider the direction of inflation as:

$$\mathrm{Re}\Phi = \varphi, \quad \mathrm{Im}\Phi = S = 0$$

the Lagrangian takes the form:

$$\mathcal{L} = \sqrt{-g}\left[\tfrac{1}{2}R - \tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - F^2\left(\tanh \tfrac{\varphi}{\sqrt{6\alpha}}\right)\right].$$

Therefore the potential is given:

$$V(\varphi) = F^2(\tanh \tfrac{\varphi}{\sqrt{6}}).$$

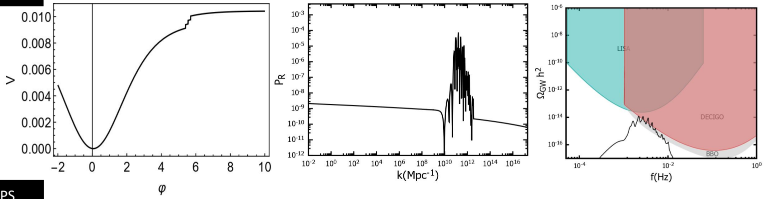
• In our study we consider: **[I. Dalianis, G. P. Kodaxis, N. Tetradis, A. Tsigkas-Kouvelis, IDS(2021)]**

$$F(x) = F_0\left(x + \sum_i^n c_i \tanh\left[d(x - x_i)\right]\right).$$

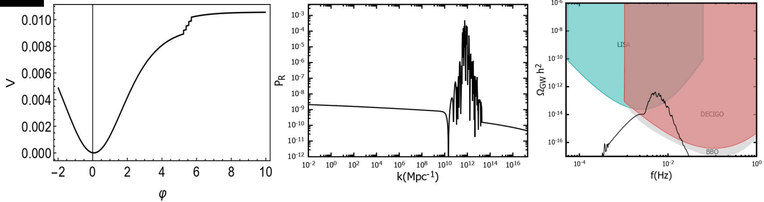
Production of GWs

We obtain oscillation feautures in the energy density of GWs.

3 STEPS



4 STEPS



Model with a waterfall trajectory

Basic aspect in hybrid models

The hybrid model is derived by the globally supersymmetric (SUSY) renormalizable superpotential: [Copeland, Liddle, Lyth, Stewart, Wands (1994)]

$$W = \kappa\, S(\bar{\Psi}_1 \Psi_2 - m^2)\,. \tag{8}$$

The F-term SUSY potential takes the form:

$$V_F^{\rm SUSY} = \kappa^2 \left[\left(\psi^2 - m^2\right)^2 + \phi^2 \psi^2 \right] \tag{9}$$

where we have assumed $|S| = \phi/\sqrt{2}$ and $|\Psi_1| = |\bar{\Psi}_2| = \psi$. The field ψ develops tachyonic solutions, if

$$\kappa^2(-2m^2 + \phi^2 + 6\psi^2) < 0\,.$$

Along the flat direction ($\psi = 0$) we have:

$$\phi^2 < \phi_c^2 = 2m^2 \equiv M^2\,,$$

where ϕ_c is the critical value of the field ϕ, after which this field ψ becomes tachyonic.

- ✘ Hybrid models predict spectral index $n_s = 1$.
- ✓ One loop corrections for $n_s < 1$. [Dvali, Shafi, Schaefer (1994)]
  We can use SUGRA corrections for obtaining an acceptable value for n_s.

SUGRA corrections

• SUGRA correction in the context of hybrid inflation had been studied. [Lazarides, Tetradis (1998)]
• We assume SUGRA corrections in order to obtain acceptable n_s. We consider the following Kähler potential: **[V. C. Spanos, IDS(2021)]**

$$K = S\bar{S} + b_1(S+\bar{S}) + b_2(S+\bar{S})^2 + \frac{1}{2}\Psi_1\bar{\Psi}_1 + \frac{1}{2}\Psi_2\bar{\Psi}_2 \,.$$

The general F-term of scalar potential is

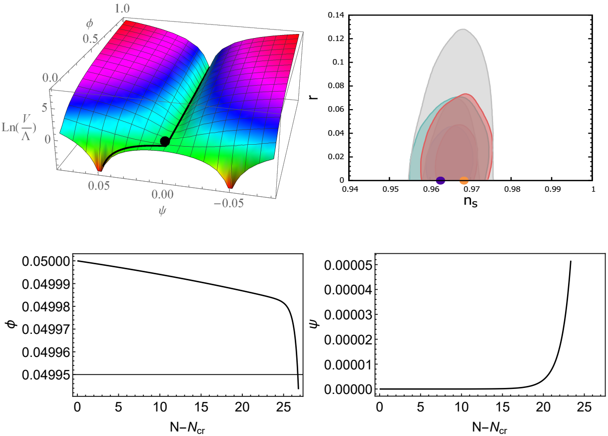
$$V_F^{\mathrm{SUGRA}} = e^{K/{M_P}^2}\left[(K^{-1})^i_{\bar{j}}\left(W^{\bar{j}} + \frac{WK^{\bar{j}}}{{M_P}^2}\right)\left(\bar{W}_i + \frac{\bar{W}K_i}{{M_P}^2}\right) - \frac{3|W|^2}{{M_P}^2}\right] ,$$

and it takes the form:

$$V_F^{\mathrm{SUGRA}} = \frac{\kappa^2(M^4 - 4M^2\psi^2 + 4\psi^2\phi^2 + 4\psi^4)}{4+8b_2} + \frac{\mathcal{A}_1}{M_P^2} + \frac{\mathcal{A}_2}{M_P^4} + \mathcal{O}\left(\frac{1}{M_P^6}\right) ,$$

where $S = \frac{\phi}{\sqrt{2+4b_2}}$ and $|\Psi_1| = |\bar{\Psi}_2| = \psi$.

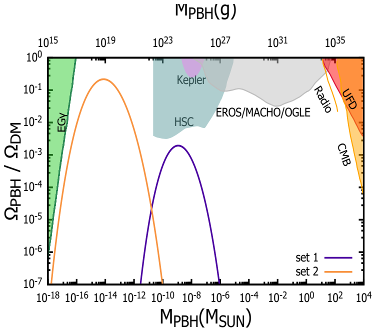
• The inflaton field ϕ slowly rolls through the valley until it reaches the critical point. After that the waterfall field ψ acquires tachyonic solutions.
•The potential of our proposed model, the prediction of observable constraints and the evolution of the fields: (• $b_1 = 3.51 \times 10^{-4} M_P$, $b_2 = -3.5$, $M = 0.05 M_P$ and
• $b_1 = 8.92 \times 10^{-4} M_P$, $b_2 = -5.0$, $M = 0.1 M_P$).

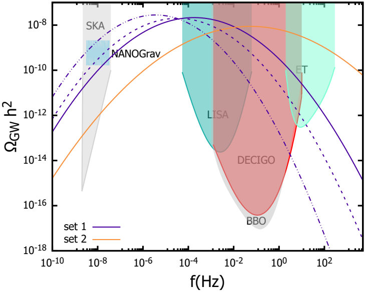


• The waterfall trajectory in the framework of hybrid inflation in order to explain the production of PBHs have been previous been studied. [Clesse & Bellido (2015)]

Generation of PBHs & GWs

For the previous sets we evaluate the amount of PBHs and GWs.





- The set 1 satisfies the NANOGrav signal and it can explain the 1% of the Dark Matter in the Universe. This is due to restriction era of HSC etc.
- The set 2 can explain both the whole DM though hybrid model and LISA, DECIGO etc.

Conclusions-Perspectives

Conclusions-Perspectives

In this presentation:

- We present three different mechanisms in order to obtain an enhancement in scalar power spectrum
    1. An inflection point in potential
    2. Steep step-like potential
    3. Waterfall trajectory in the context of hybrid inflation
- All models proposed give us acceptable values for the observable constraints of inflation.
- We evaluate the power spectrum in our models and we find significant peaks.
- We evaluate the abundances of PBHs and GWs by using the scalar power spectra.

Perspectives
A study in order to explain the generation of PBHs as well as the GWs without fine-tuning .

Thank you!