

1 July 2022
SUSY conference
University of Ioannina

PBHs and GWs from an early matter era



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University of Athens



ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
Εθνικόν και Καποδιστριακόν
Πανεπιστήμιον Αθηνών

PBHs and GWs

A brief summary

- ★ *The network of operating and designed GW detectors*
- ☆ *PBH generation and the induced GWs*
- ★ *The predicted stochastic gravitational wave background (SGWB)*
- ☆ *Inflationary model building*
- ★ *The spectrum of GWs from an early matter domination era*


Detection of Gravitational Waves

PRL 116, 061102 (2016)

Selected for a Viewpoint in Physics

PHYSICAL REVIEW LETTERS

week ending
12 FEBRUARY 2016

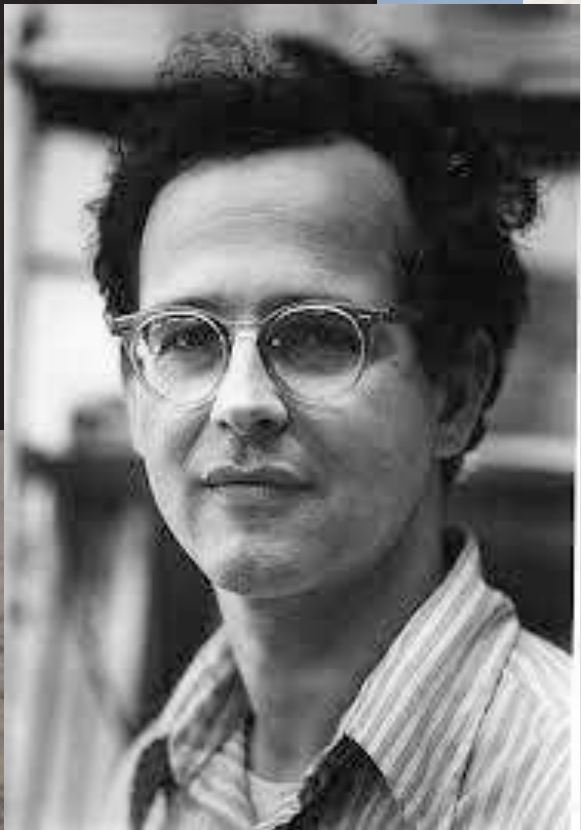
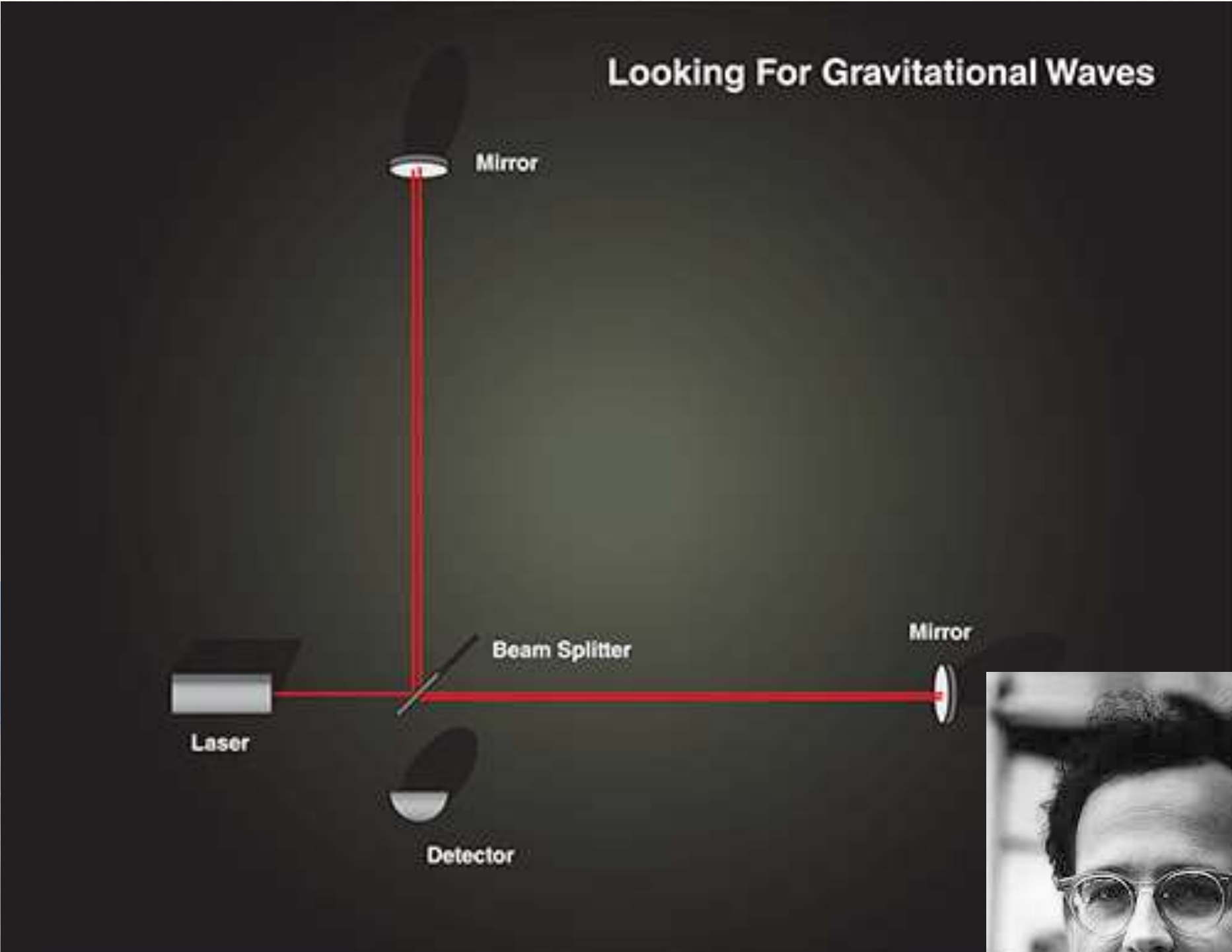
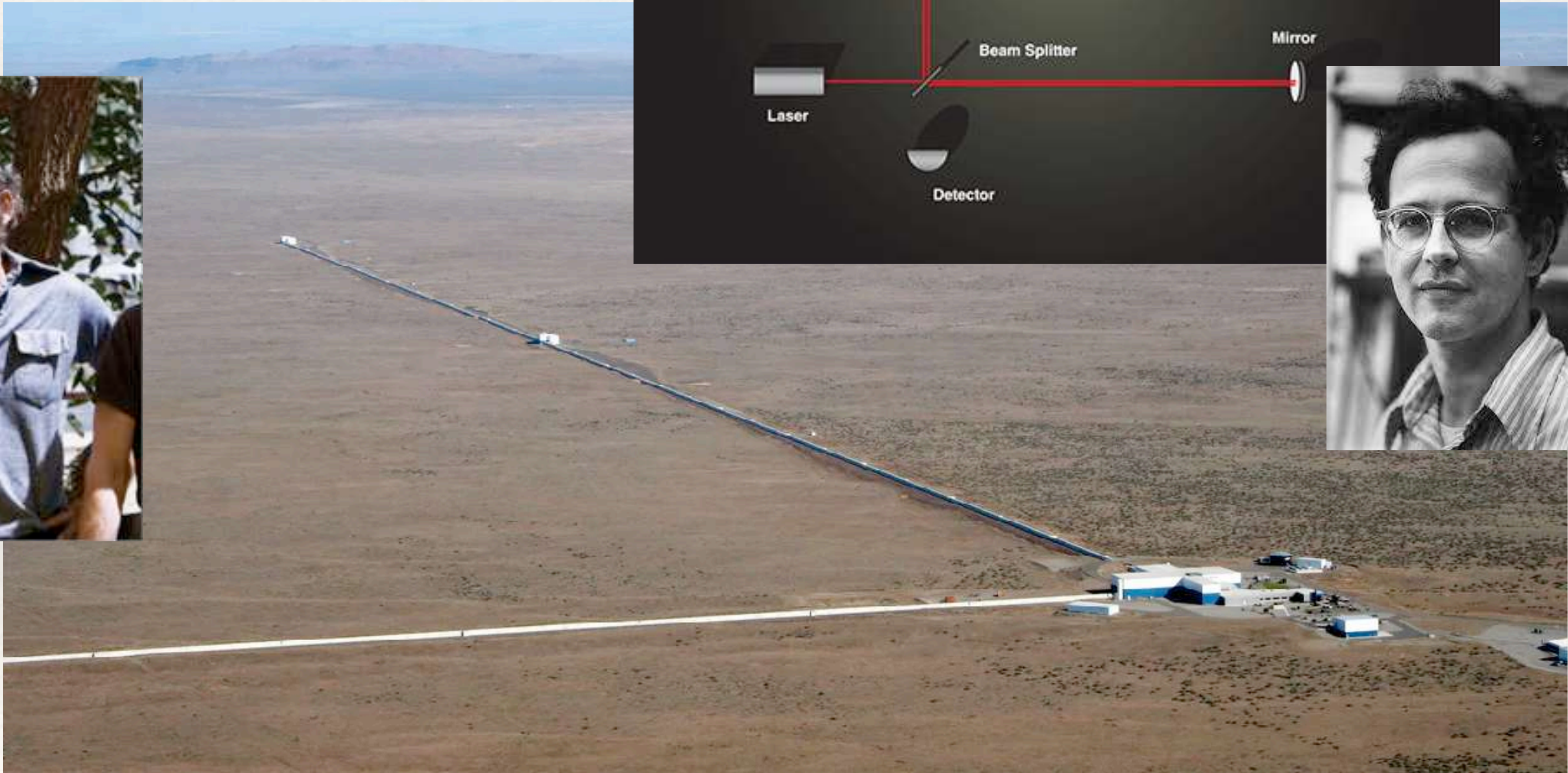


Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)



The detection of GWs from the Black Hole (BH) merger *(Talk of M. Agathos)*

- The spacetime evolution during the merging and of two BHs was understood and described in 2005
- The GWs inform us about the characteristics of the BHs

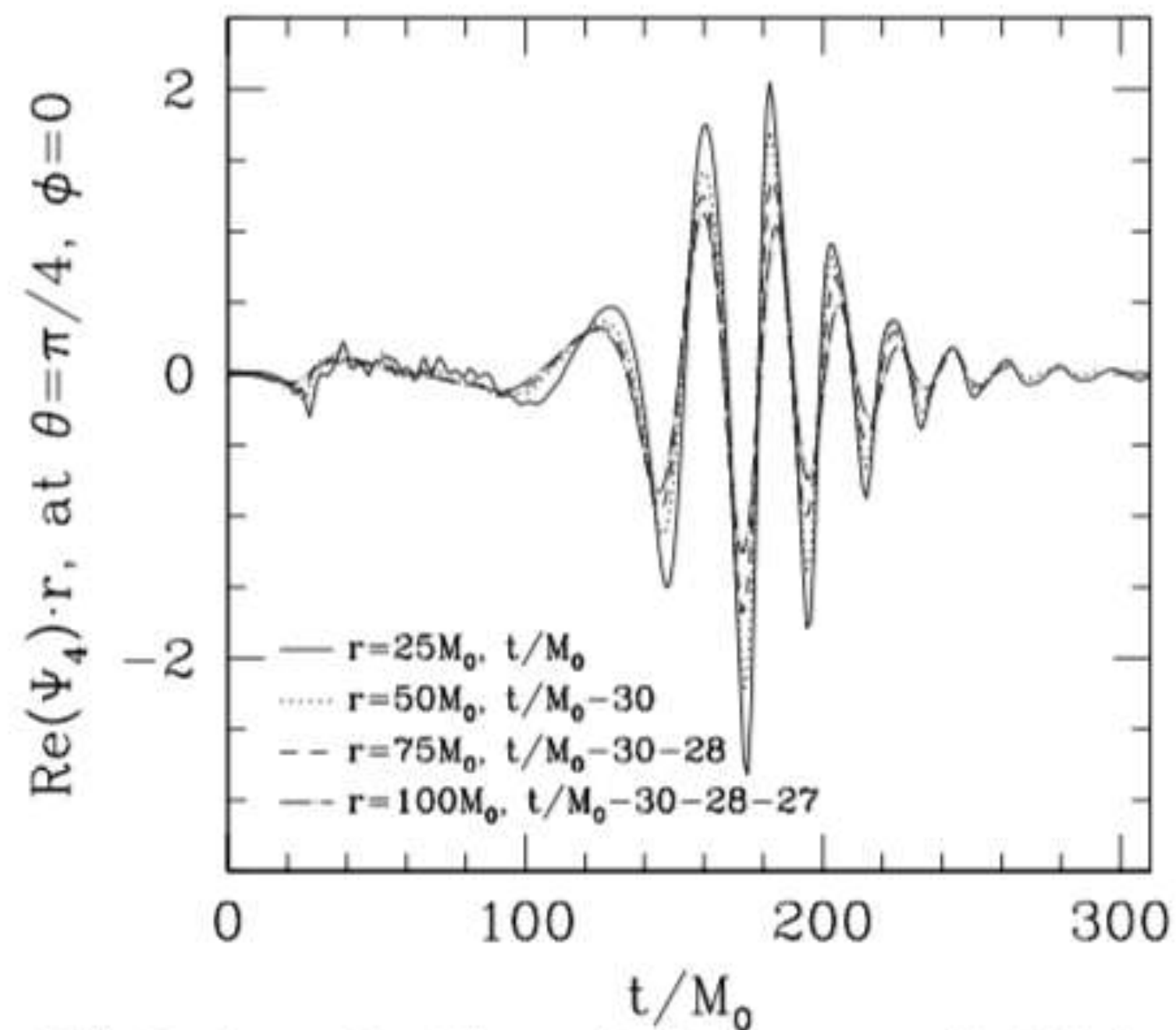
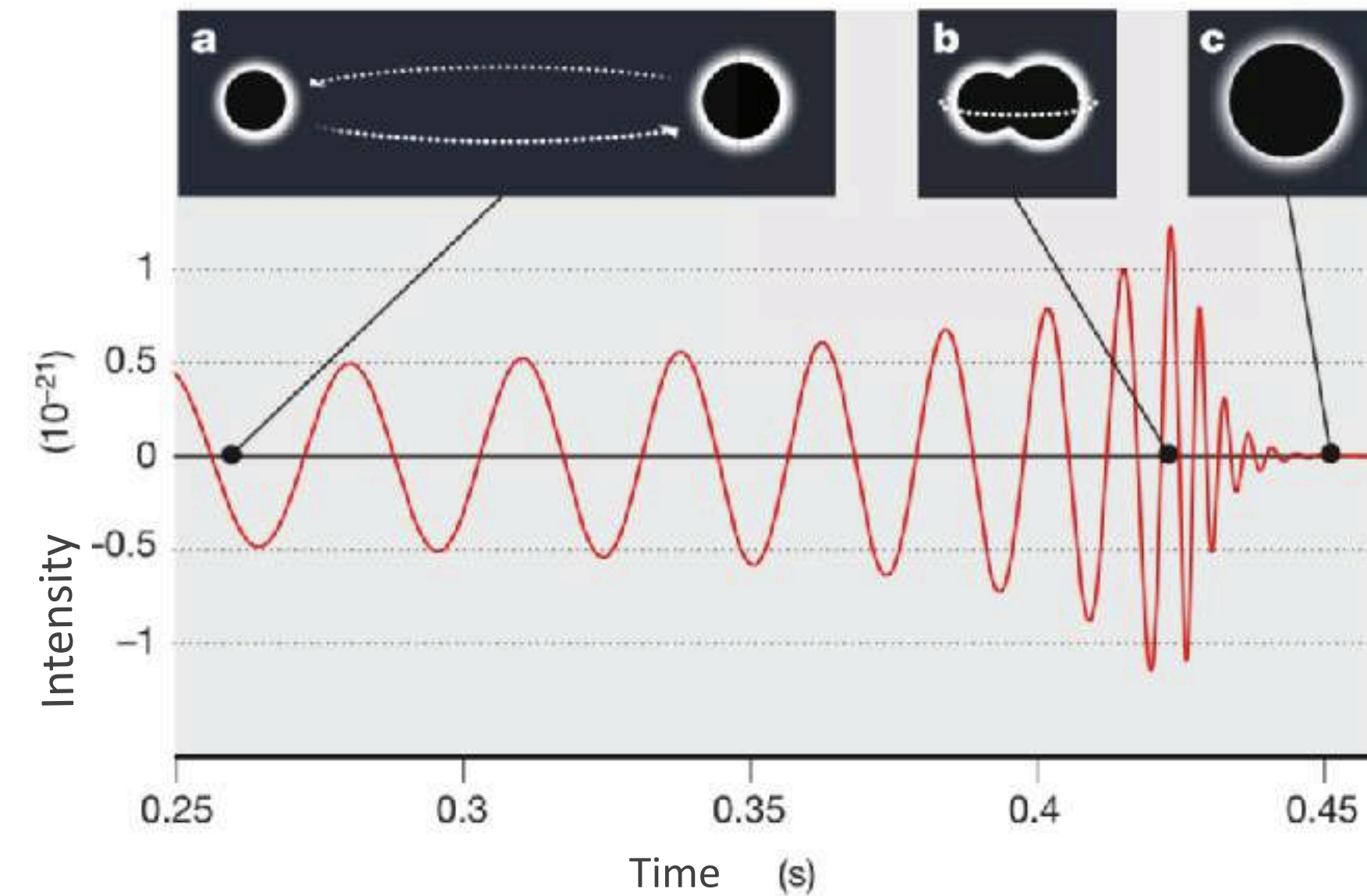
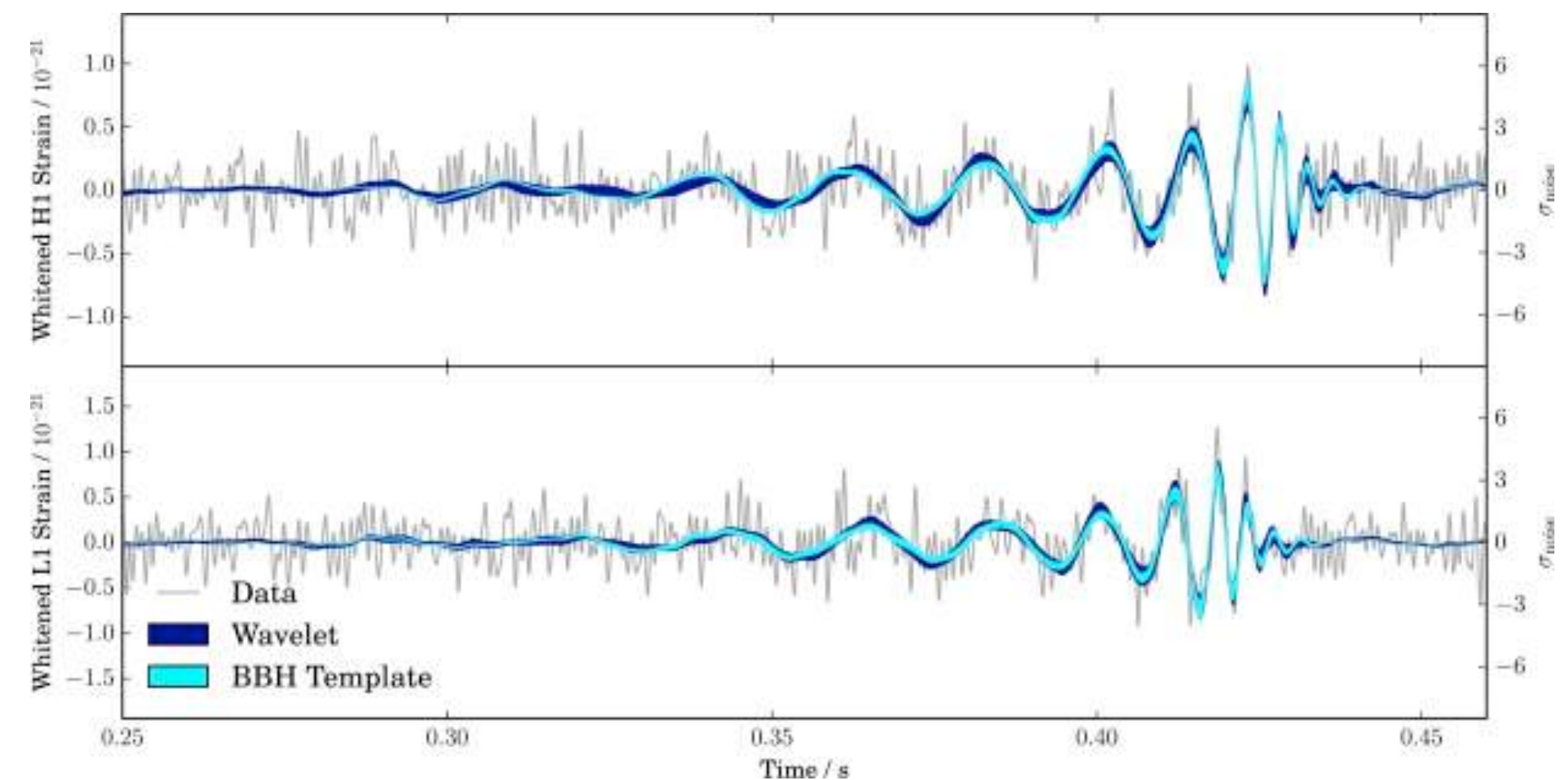


FIG. 3: A sample of the gravitational waves emitted during the merger, as estimated by the Newman-Penrose scalar Ψ_4

Frans Pretorius, Phys.Rev.Lett. (2005)



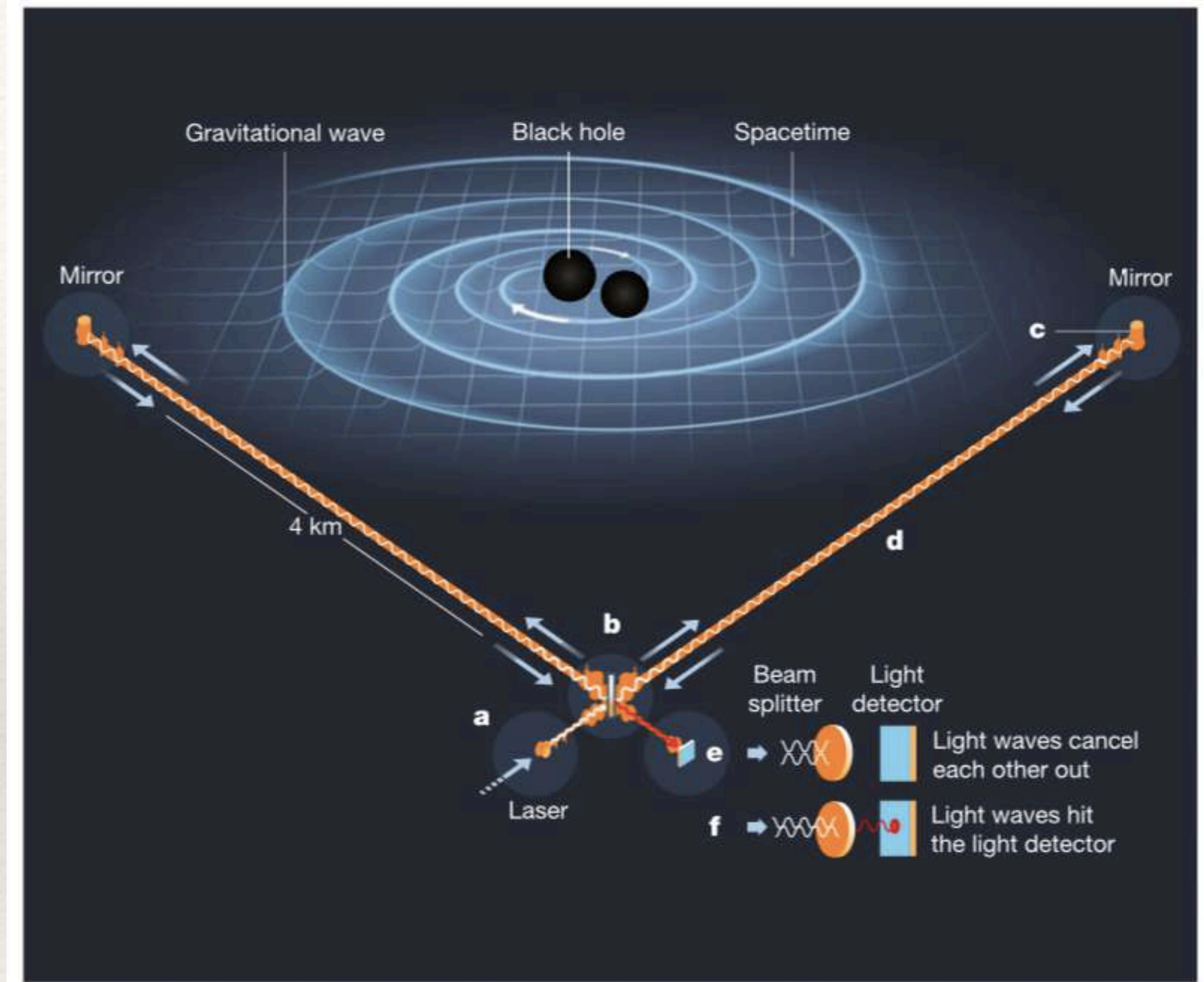
The theoretical prediction



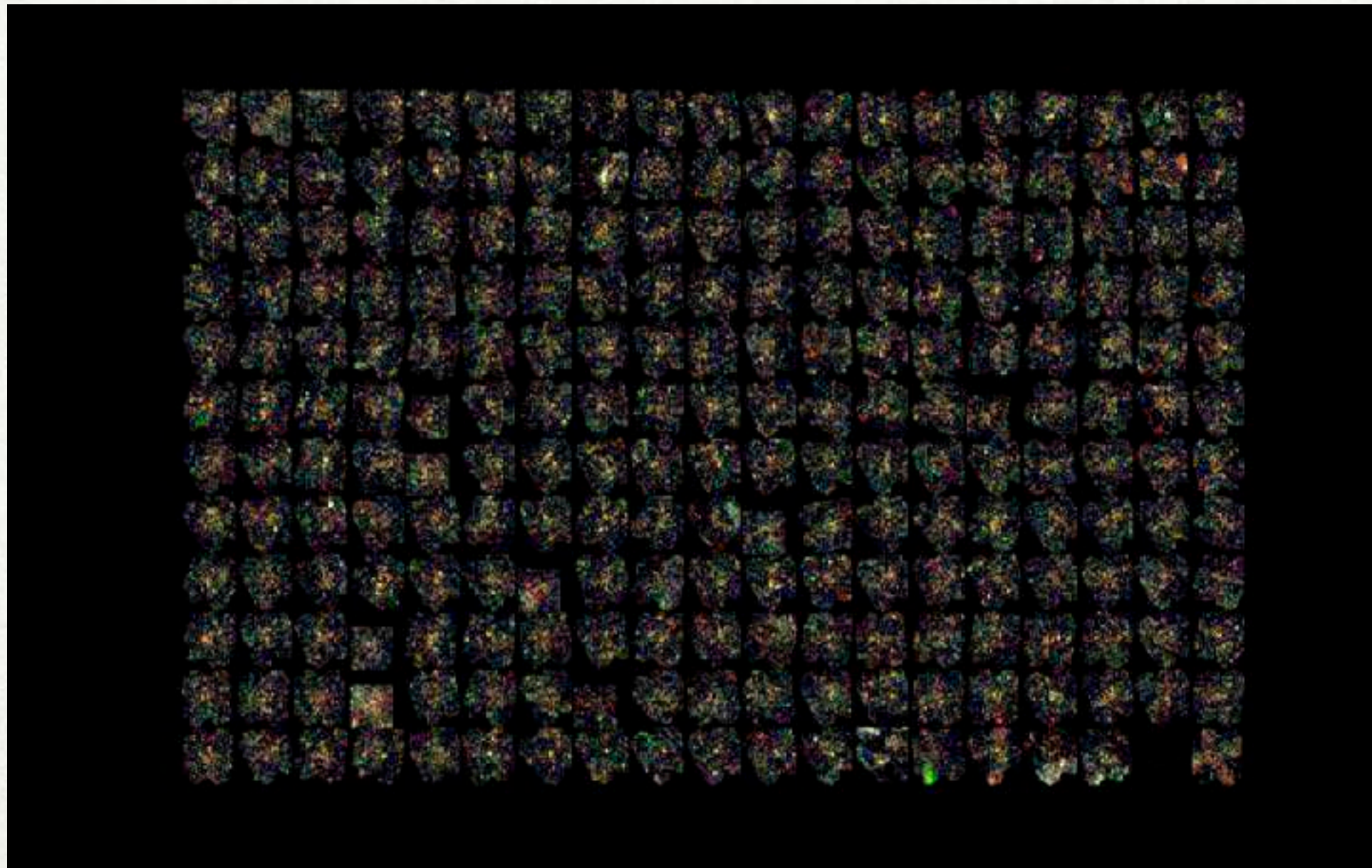
Observation

MESSAGE:

1. The universe is filled with GWs
2. BHs exist and they are abundant in the universe



STOCHASTIC GRAVITATIONAL WAVES



*“Electromagnetic NOISE” that indicates early concentrations of galaxies
(Planck-Herschel mission)*

- Apart from the distinct GW events the universe contains a stochastic GW background due to the superposition of a great number of distinct GW emitting systems together with GWs emitted in the early universe.
- The signal from the primordial universe, produced $10^{-30} - 10^{-40}$ s after the big bang is expected to be weak. It contains information about the early universe phase: its temperature, the primordial curvature spectrum, and BSM physics.

The **energy density** is (of a flat wave)

$$\rho_{GW} = \frac{\pi}{4} f^2 \langle h^2 \rangle$$

The amplitude is a stochastic variable and we estimate the statistical average per frequency

STOCHASTIC GRAVITATIONAL WAVES

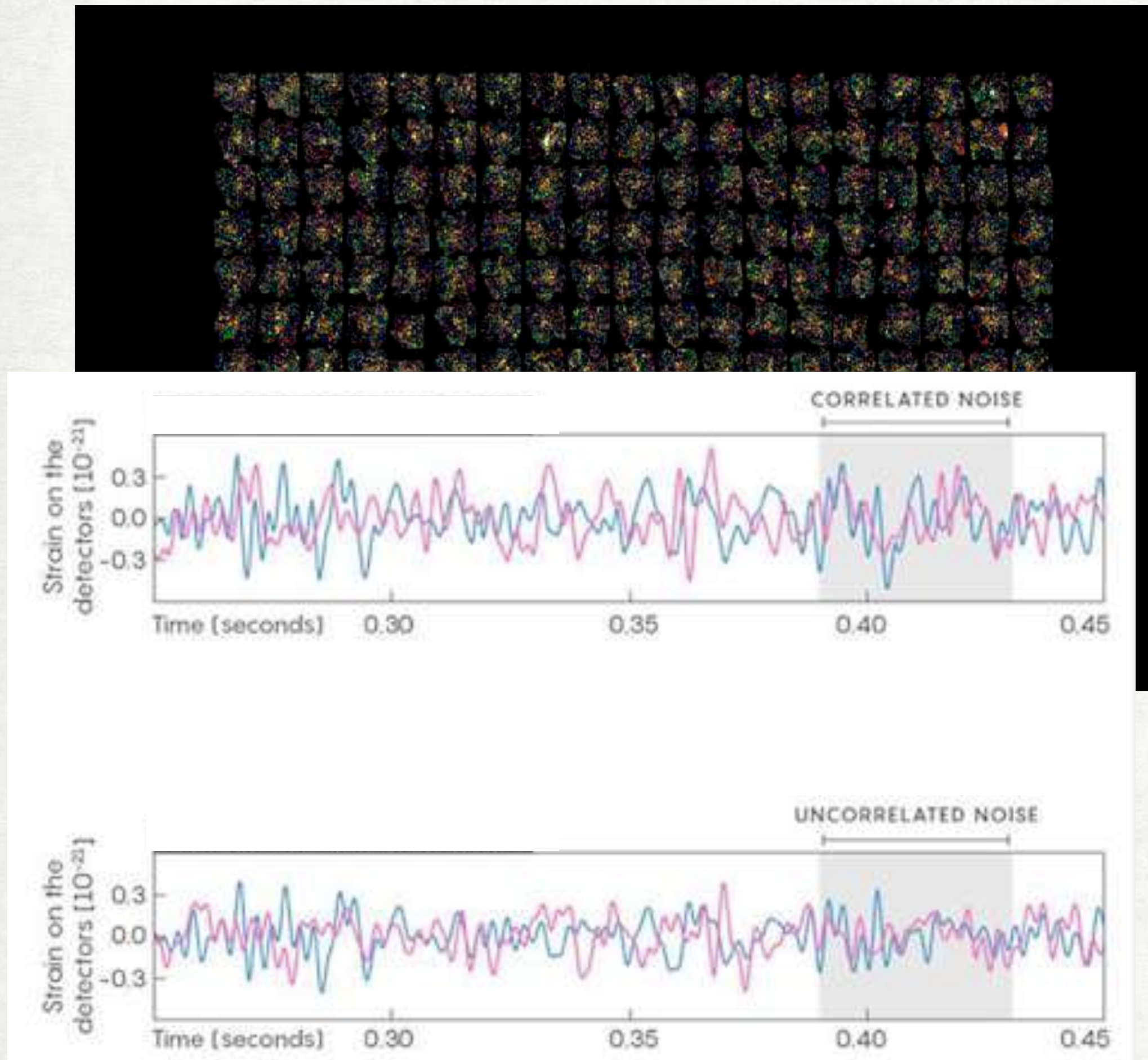
We measure the energy density of the SGW as a fraction of the critical energy density which is the **dimensionless energy density parameter** $\rho_c = 3H_0^2/8\pi G$,

$$\Omega_{GW}(f) = \frac{d\rho_{GW}}{d \ln f} \frac{1}{\rho_c} = \frac{10\pi^2 G}{3H_0^2} f^3 \langle h^2(f) \rangle$$

We know that $\Omega_{GW} < 10^{-5}$, from the BBN observables

The stochastic background is hard to be distinguished from the noise of an individual detector. We need

- 1) (A network of) sensitive detectors
- 2) Knowledge of the spectral energy density and of possible anisotropies.



Sources of Stochastic GWs

OF STELLAR AND COSMIC ORIGIN

B. S. Sathyaprakash &
Bernard F. Schutz
*Living Reviews in
Relativity* (2009)

- Black hole, neutron star, white dwarf binaries
- Stellar collapse
- Neutron star instabilities R-mode /magnetars

-
- First order phase transitions

Talk of M. Quiros Carcelen

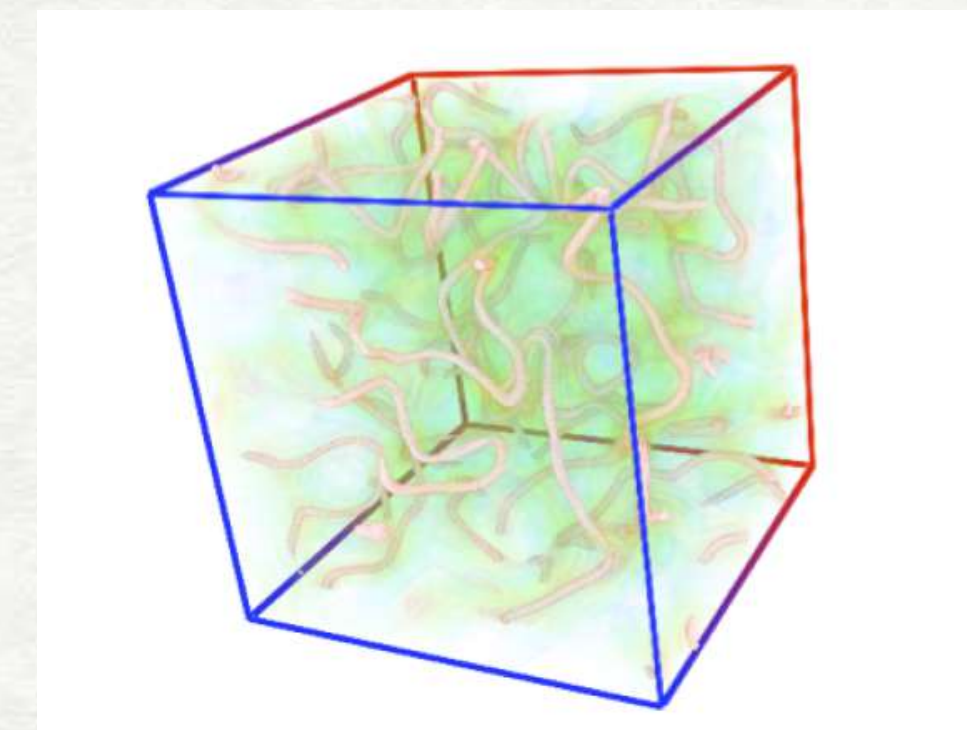
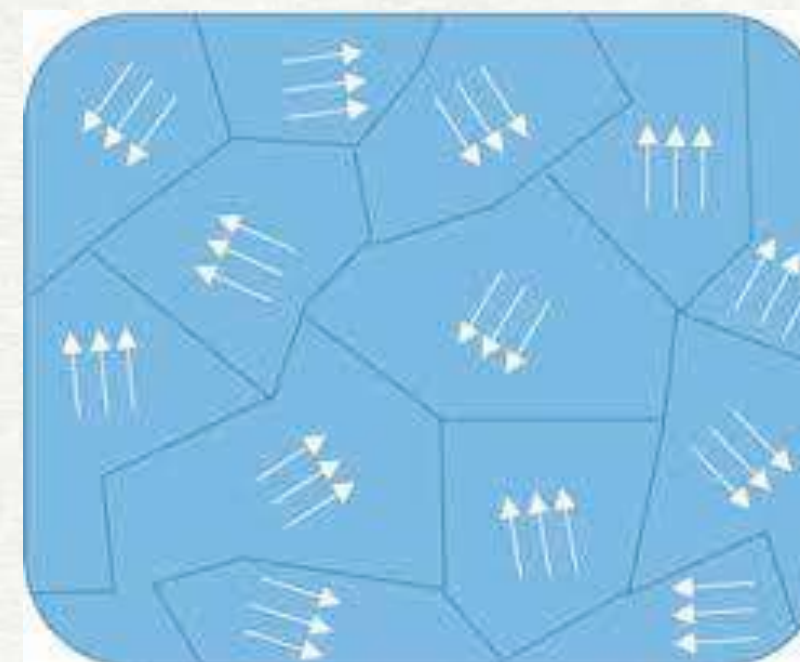
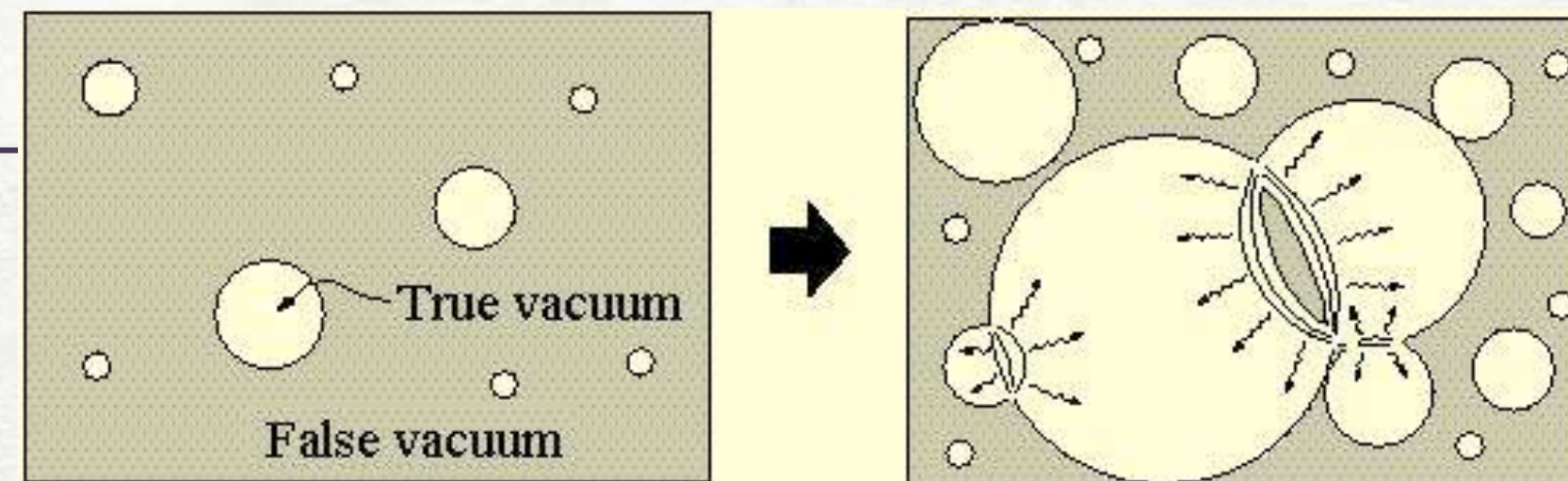
- Topological defects

- Inflation

Talk of K. Dimopoulos

- **INDUCED GWs**

- ...



Sources of Stochastic GWs

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- First order phase transitions
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- **INDUCED GWs**

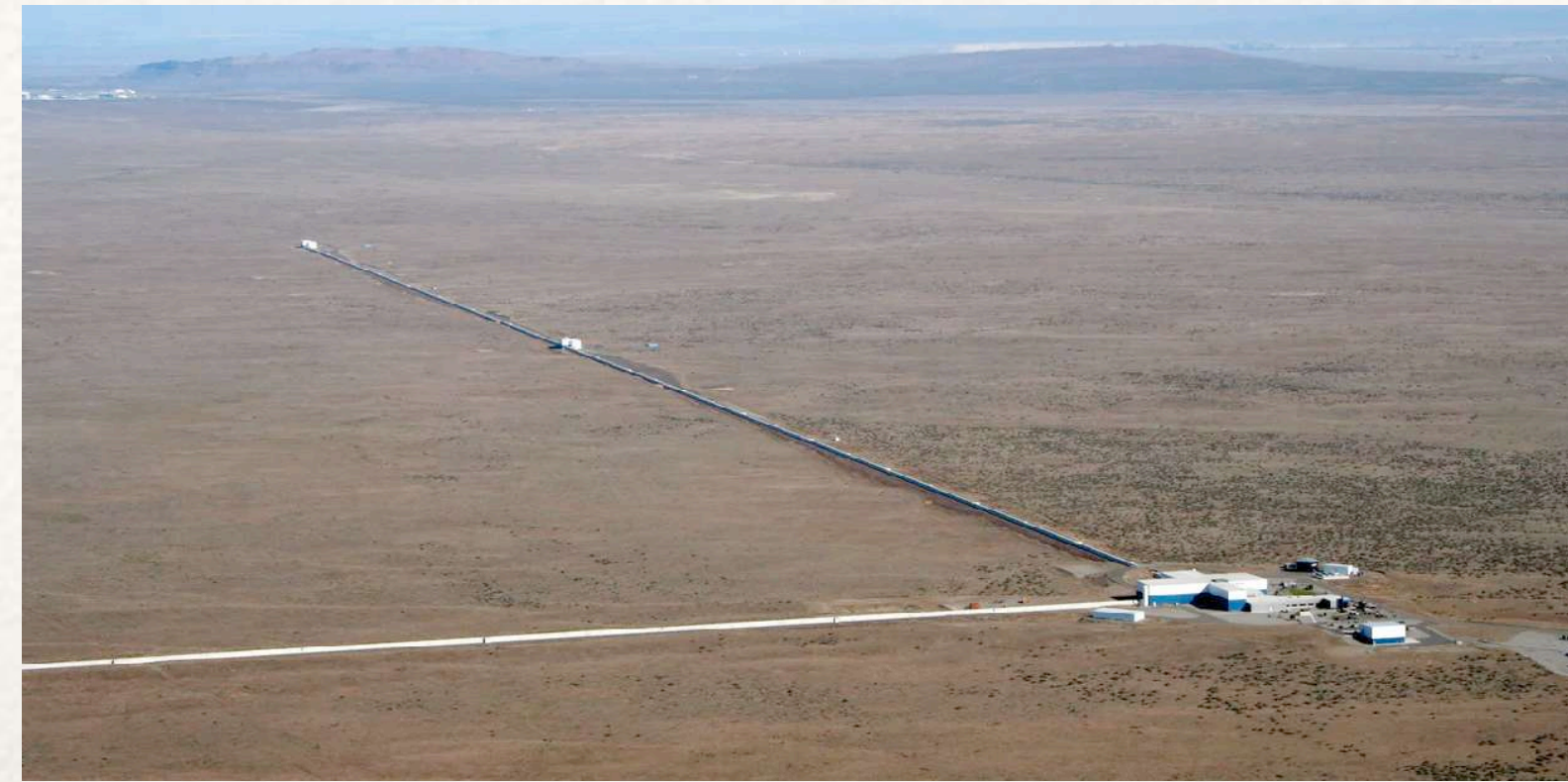
They exist independently the cosmological model for the primordial universe (inflation, etc) !!

GW detectors with sensitivity at the ~ 100 Hz band

(Talk of M. Agathos)



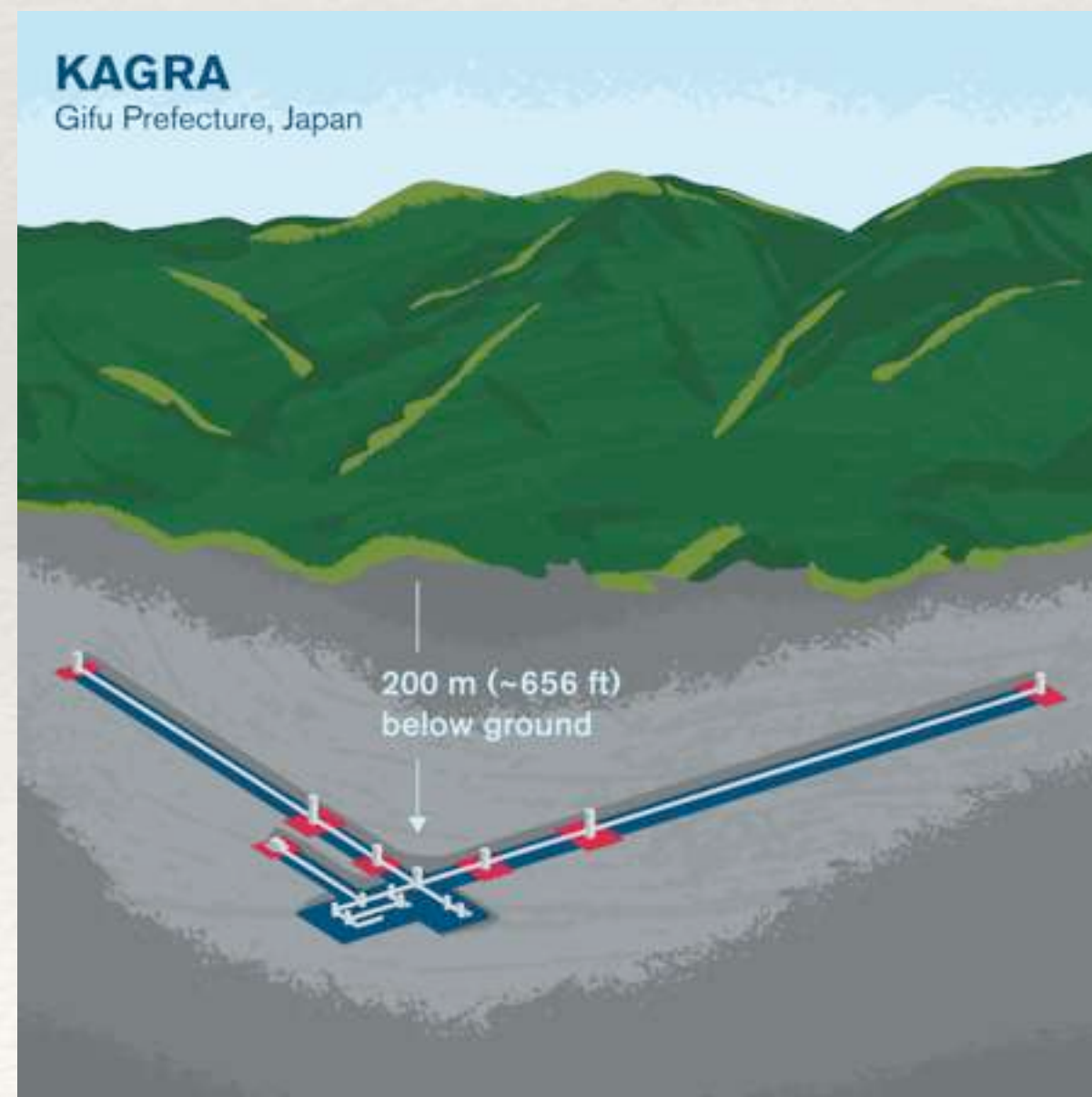
VIRGO



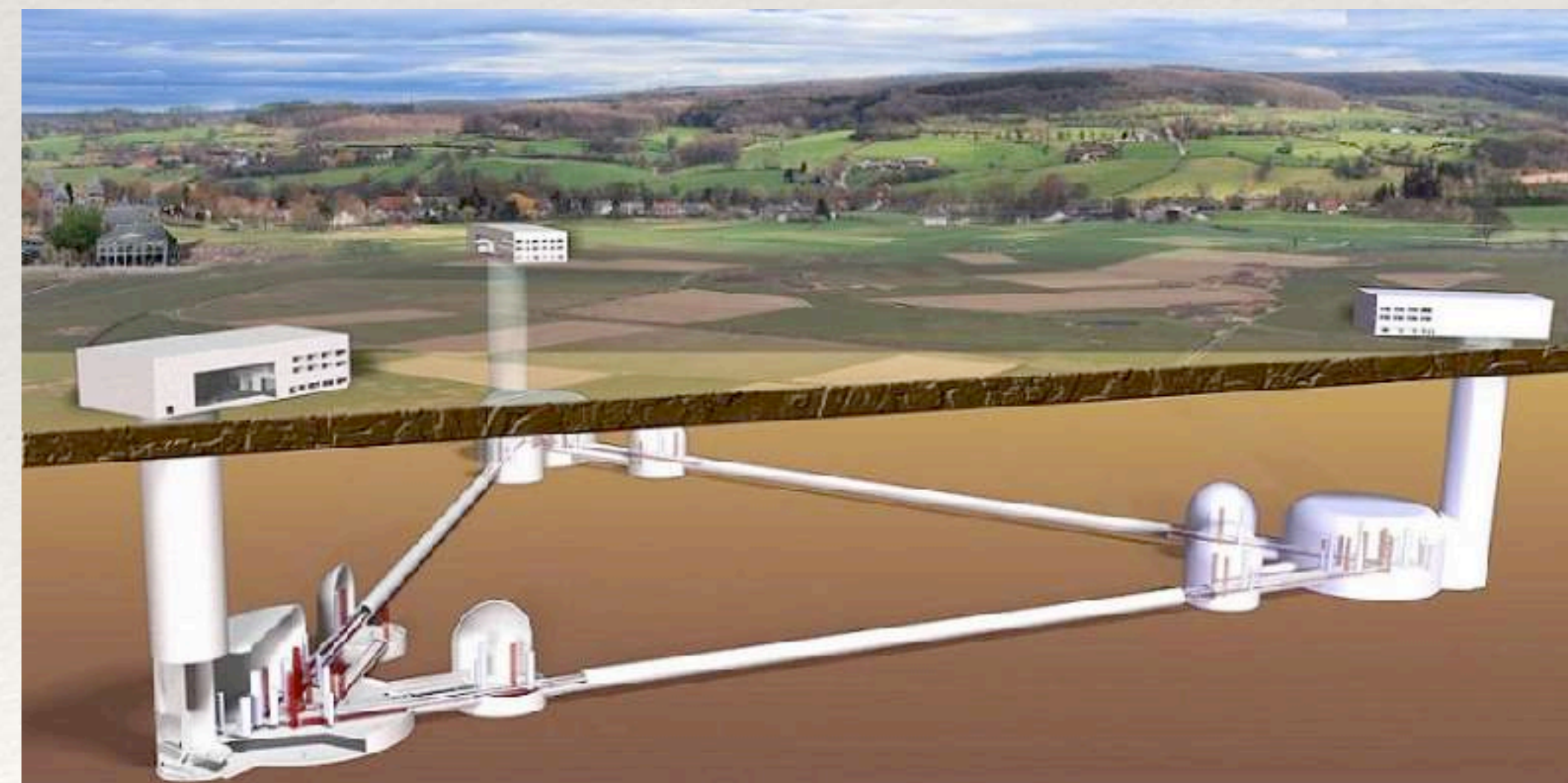
LIGO



LIGO

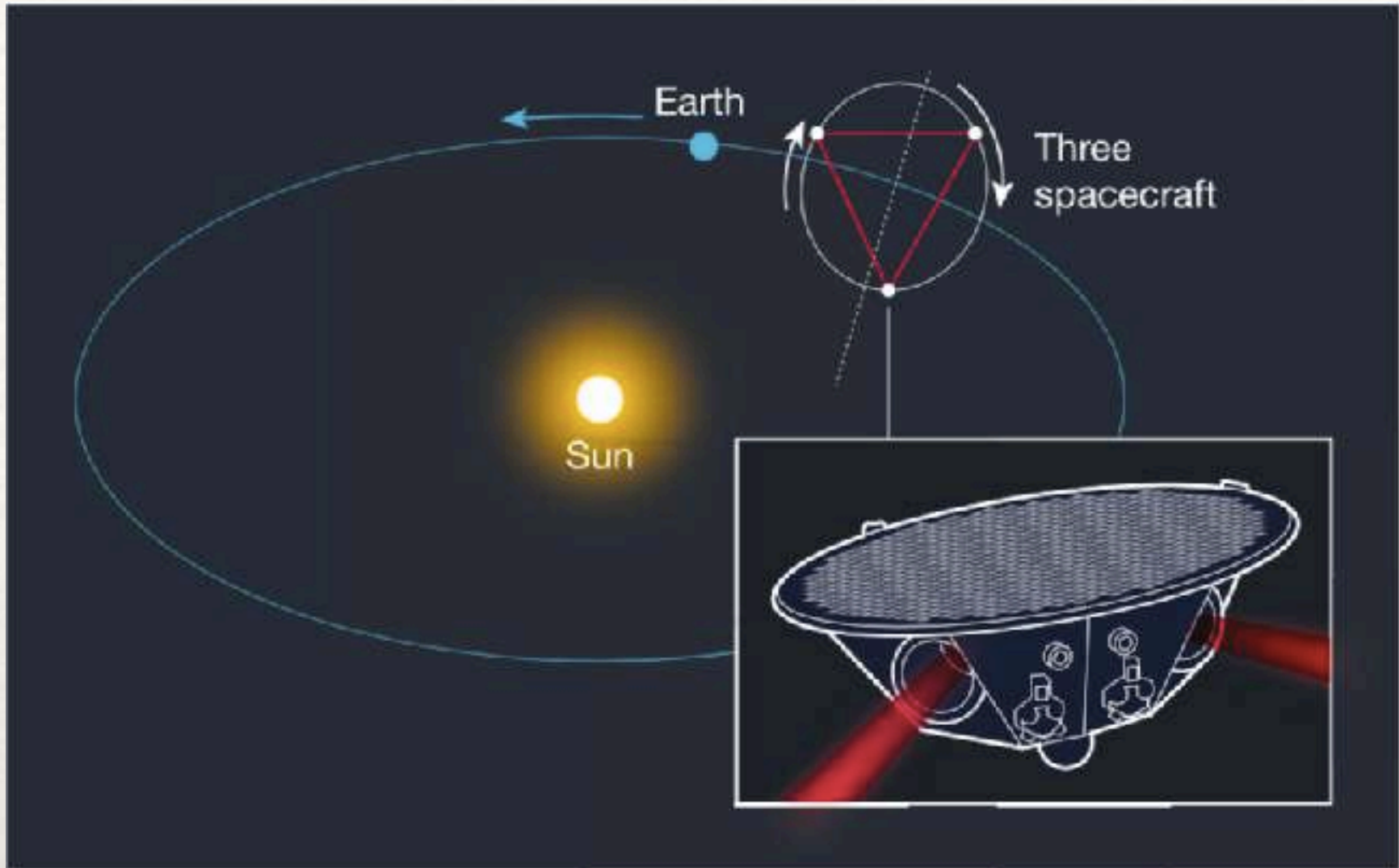


KARGA

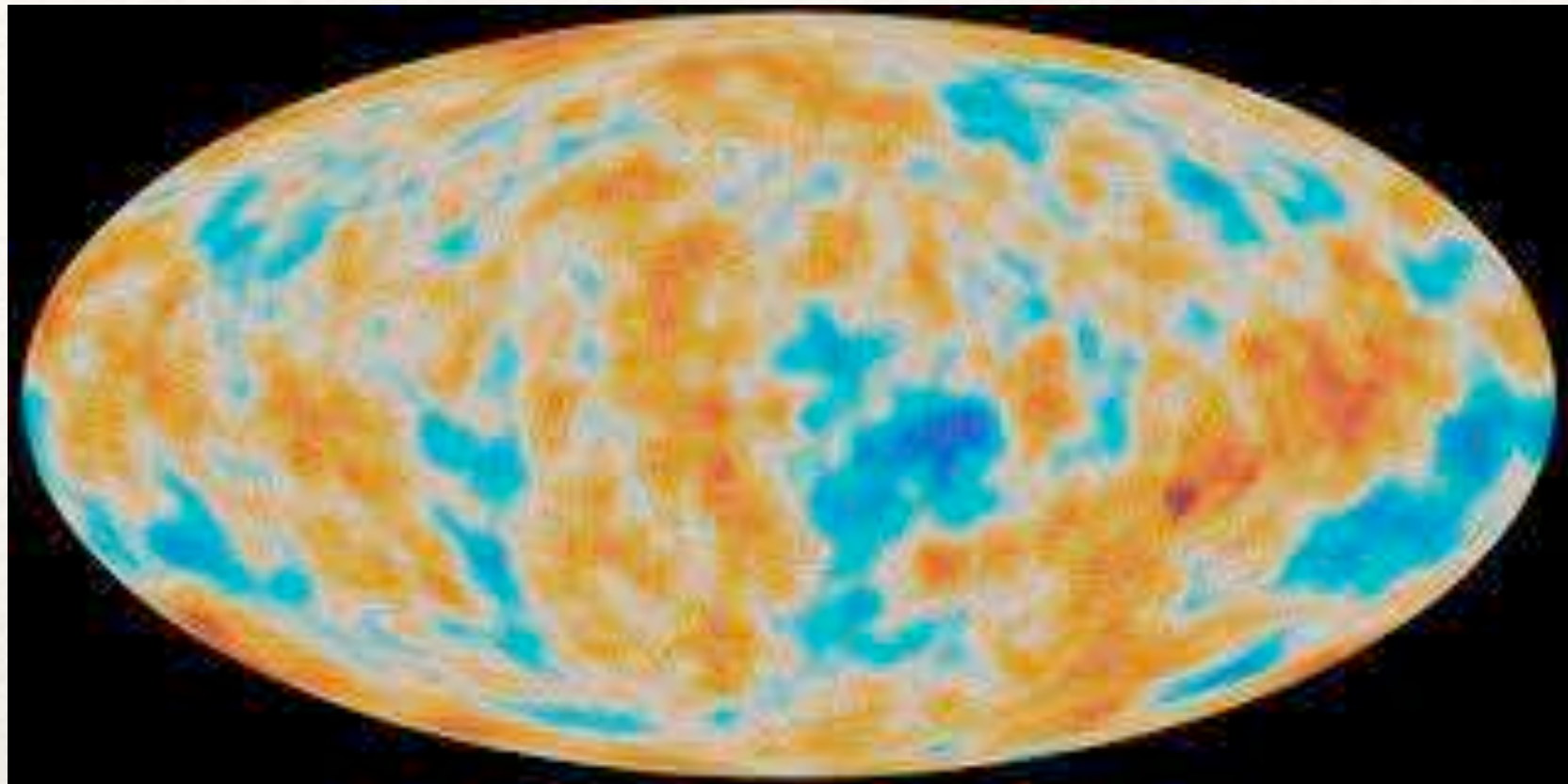


Einstein Telescope

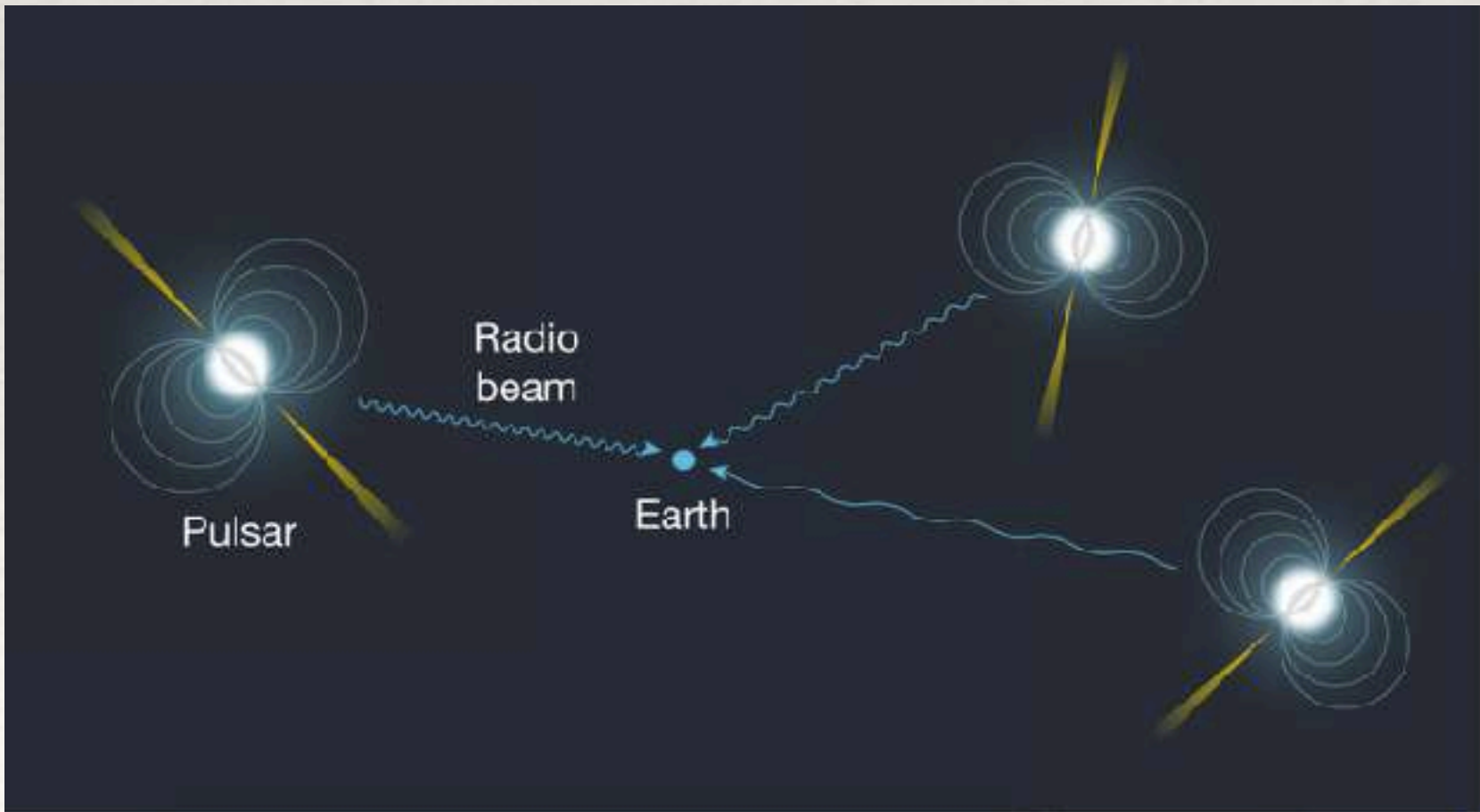
GW detectors with sensitivity at the mHz, nHz, <nHz bands



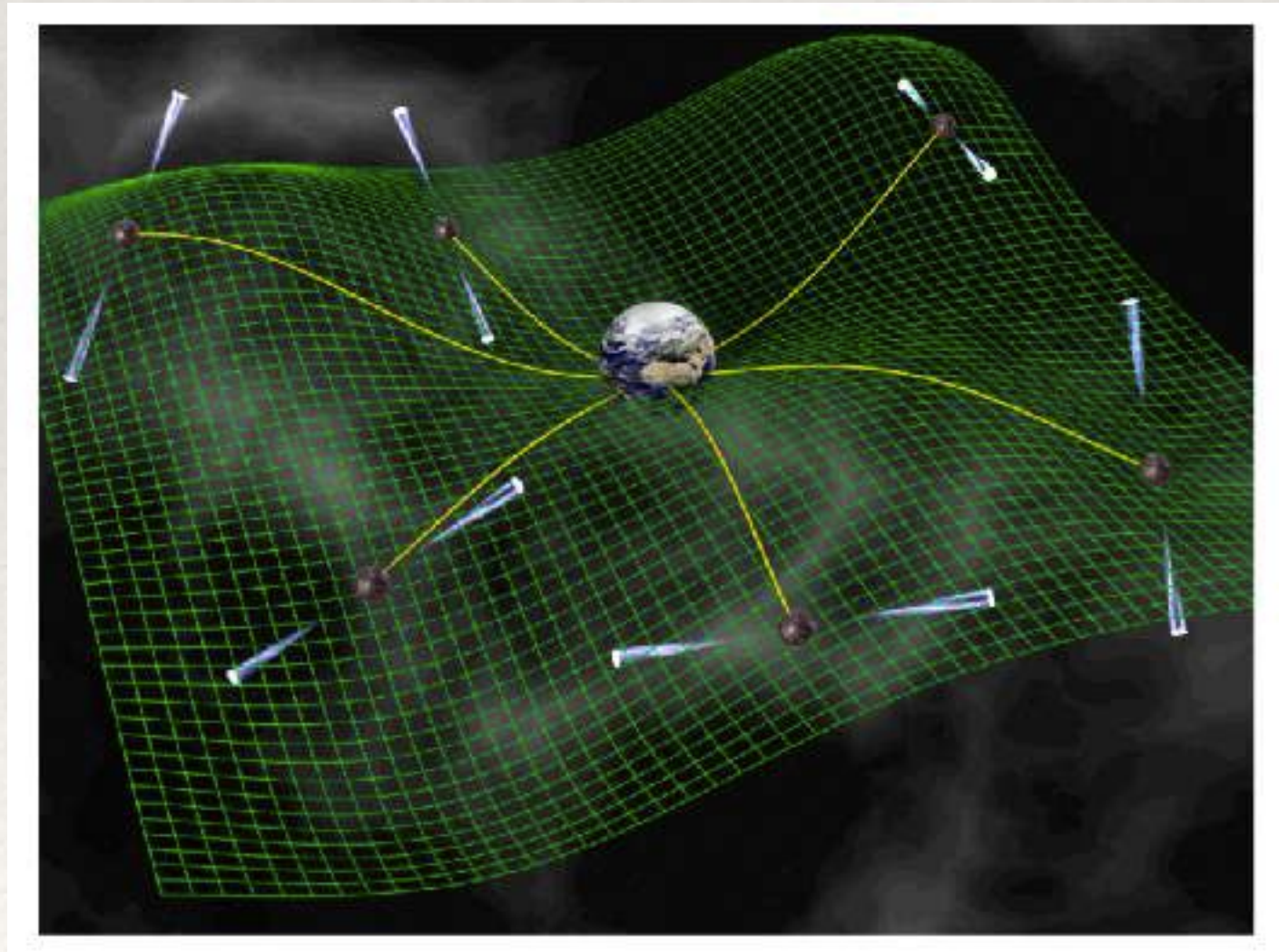
LISA



B-mode CMB



PTA experiments



PTA experiments

NETWORK OF GW DETECTORS

FUTURE

- Advanced-LIGO (aLIGO)
- Advanced Virgo (aVirgo)
- KARGA
- Einstein Telescope

Maximum sensitivity $\Omega_{\text{GW}} \sim 10^{-9}$ at the frequency band 10-100 Hz

- LISA

$\Omega_{\text{GW}} \sim 10^{-12} - 10^{-13}$ in the frequency band $f = 1 - 10 \text{ mHz}$

- IPTA

$\Omega_{\text{GW}} \sim 10^{-16}$ in the frequency band $f = 1 \text{ nHz}$

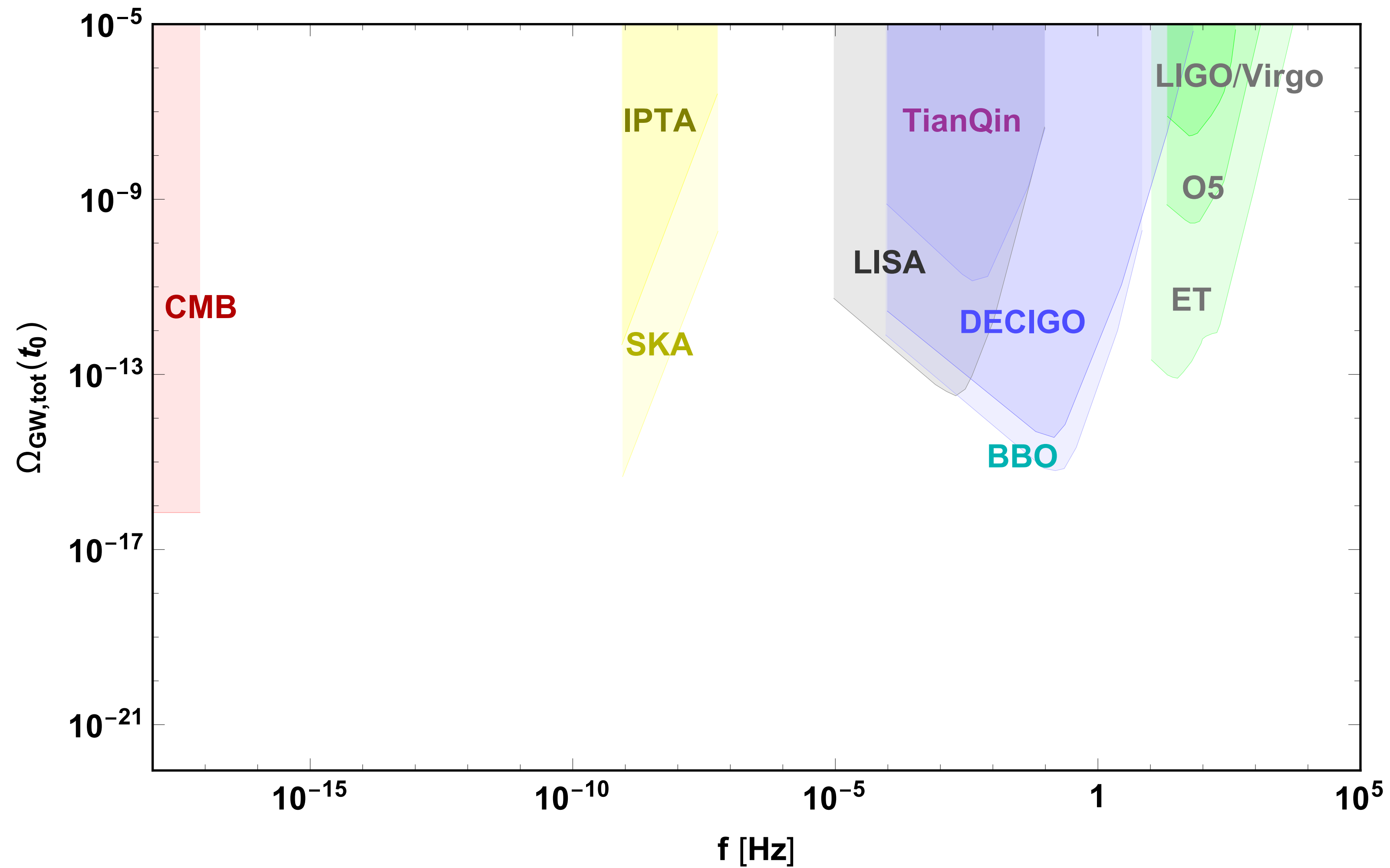
- DECIGO

$\Omega_{\text{GW}} \sim 10^{-16}$ in the frequency band $f = 0.1 - 1 \text{ Hz}$

- BBO

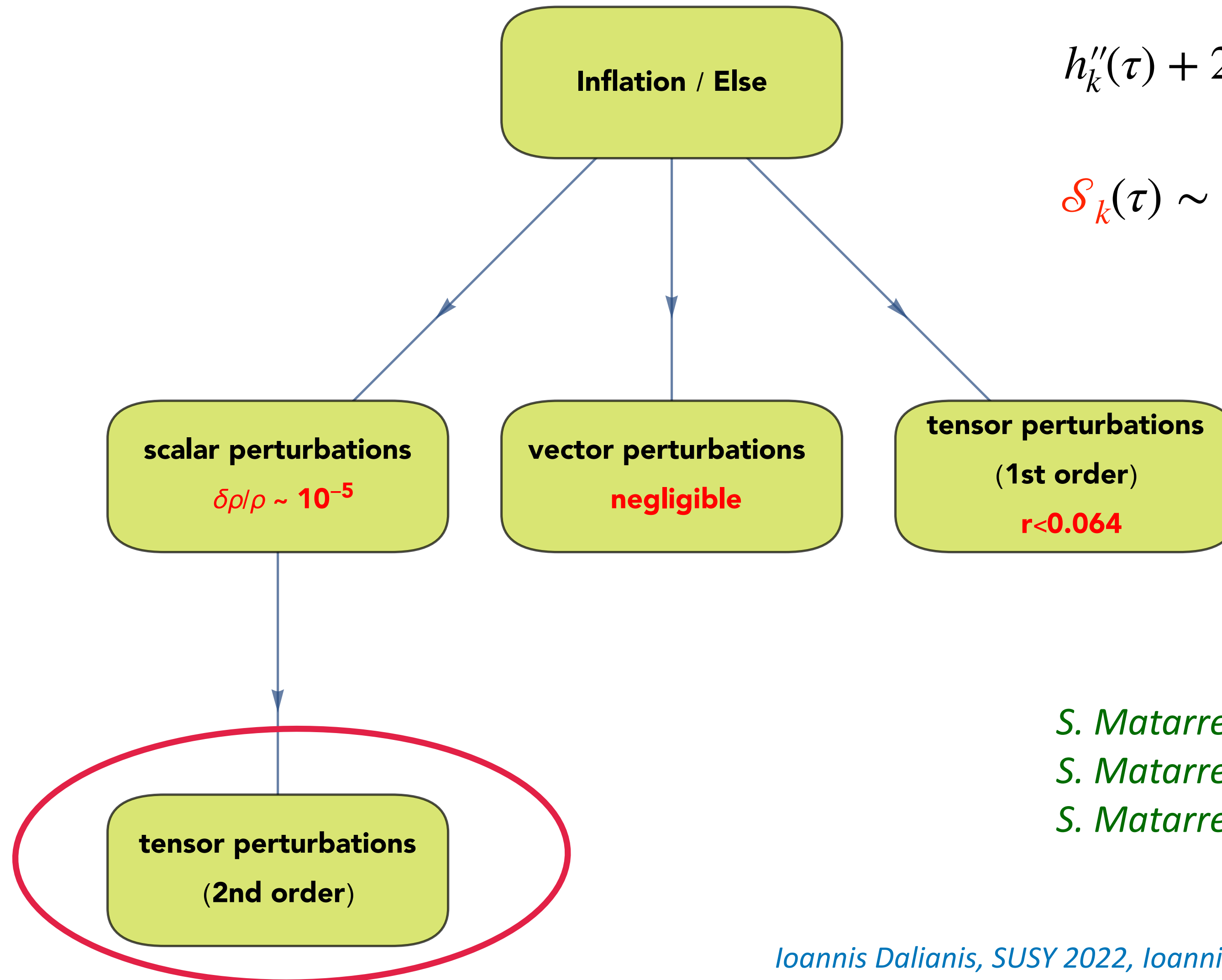
$\Omega_{\text{GW}} \sim 10^{-18}$ in the frequency band $f = 0.1 - 1 \text{ Hz}$

Constraints from GW experiments



Primordial cosmological perturbations & Induced GWs

$$ds^2 = a^2(\tau) \left[-(1 + 2\Phi)d\tau^2 + \left((1 - 2\Psi)\delta_{ij} + \frac{1}{2}h_{ij} \right) dx^i dx^j \right],$$



$$h_k''(\tau) + 2\mathcal{H}(\tau)h_k'(\tau) + k^2 h_k(\tau) = \mathcal{S}_k(\tau)$$

$$\mathcal{S}_k(\tau) \sim \int \frac{d^3 q}{(2\pi)^{3/2}} e^{ij}(\mathbf{k}) q_i q_j \left(\Phi_k \Phi_{k-q} + \frac{\Phi'_q \Phi_{k-q} + \dots}{3(1+w)/4} \right)$$

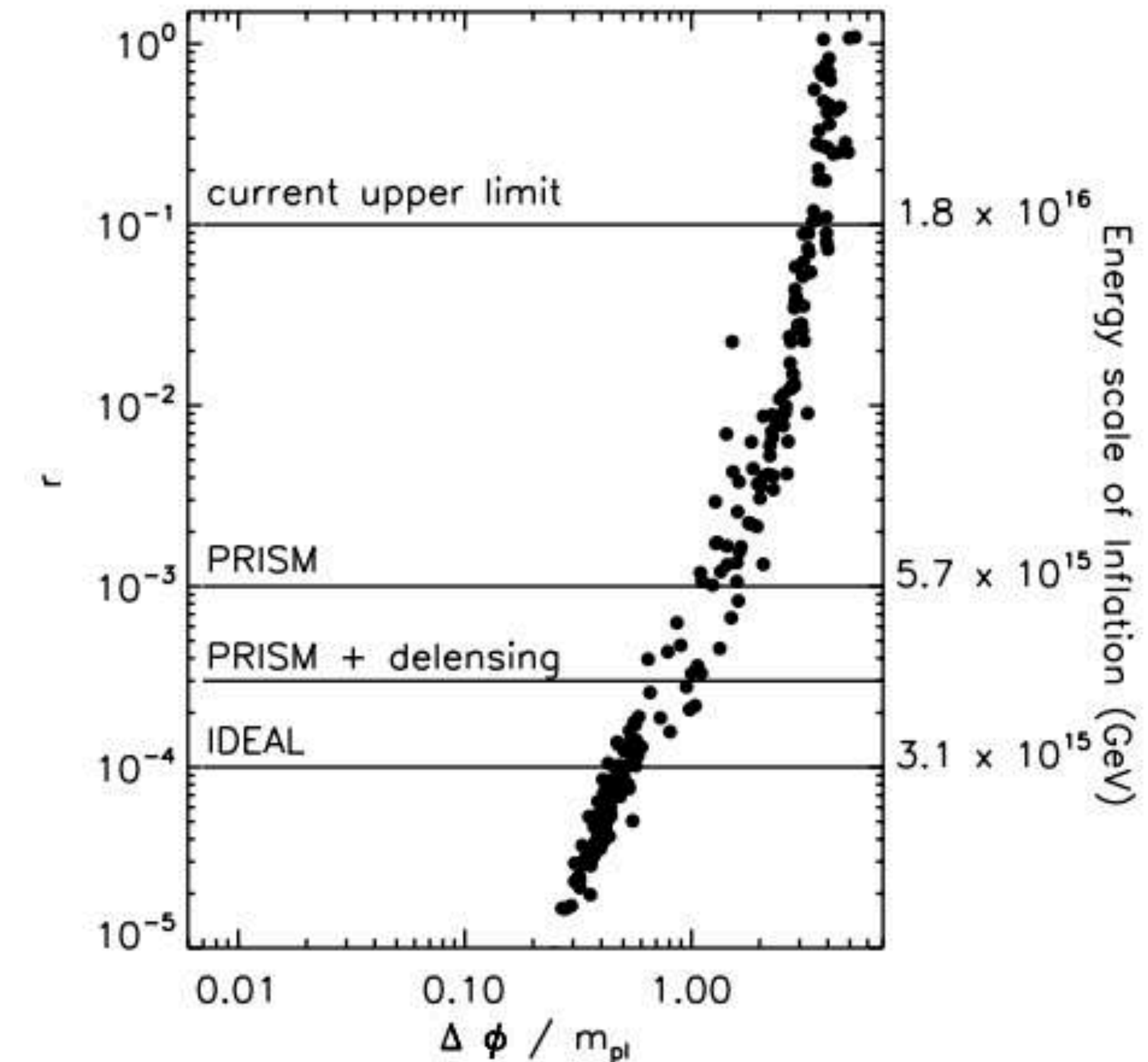
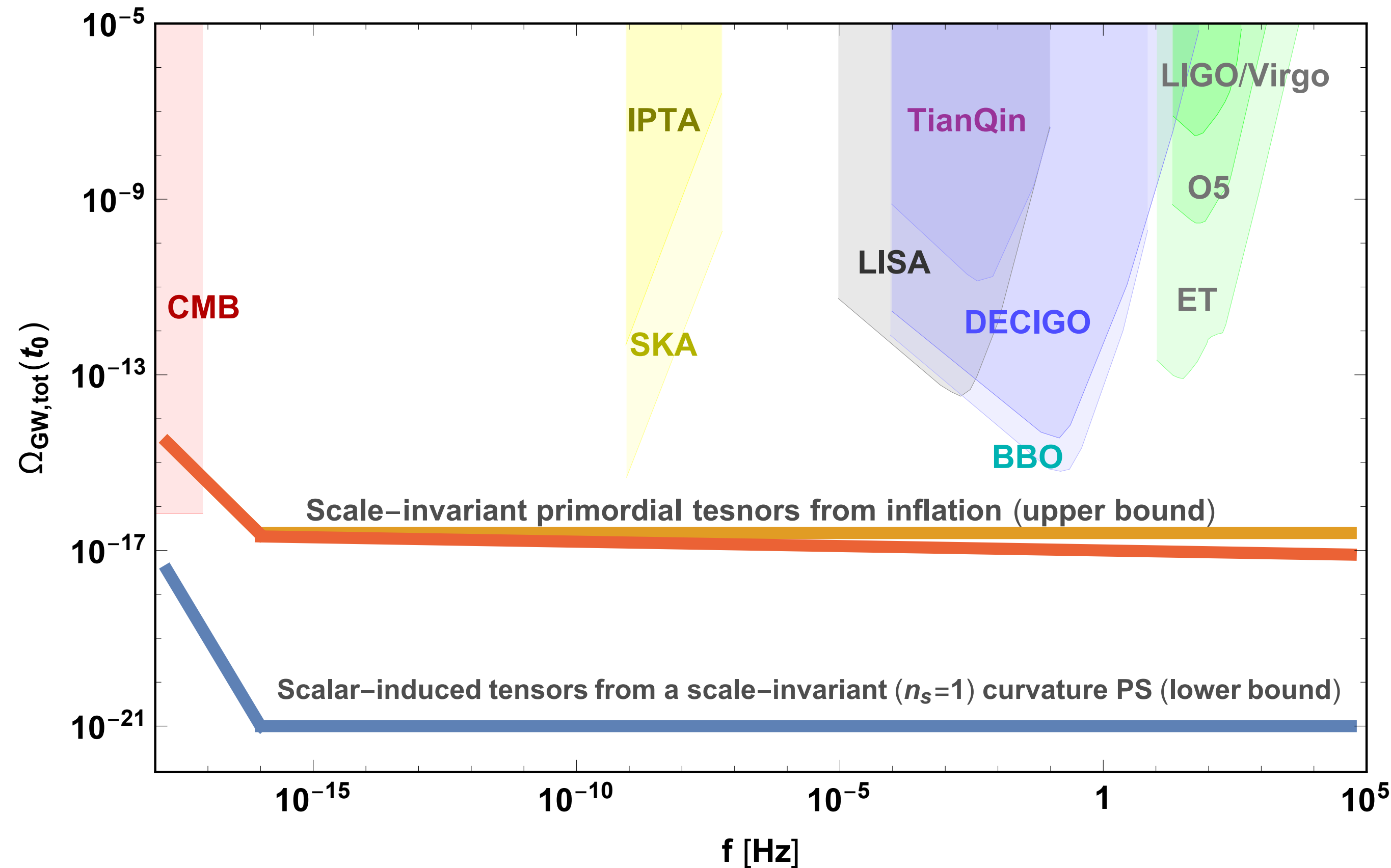
S. Matarrese, O. Pantano and D. Saez, Phys. Rev. D 47, 1311 (1993);
S. Matarrese, O. Pantano and D. Saez, Phys. Rev. Lett. 72, 320 (1994)
S. Matarrese, S. Mollerach and M. Bruni Phys. Rev. D 58 (1998)

Spectrum of primary and secondary (induced) Gravitational Waves

Example 1 (Schematic illustration)

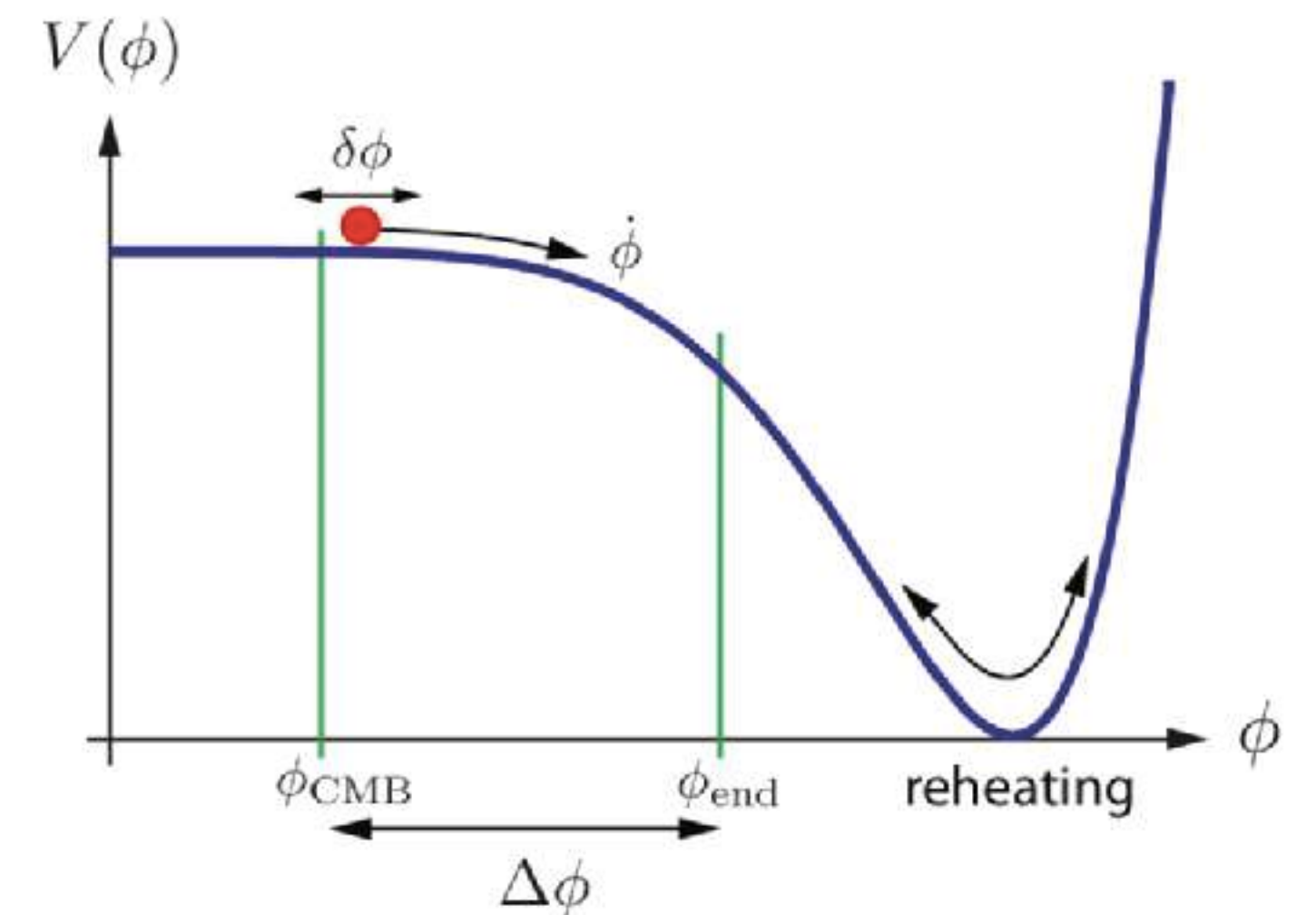
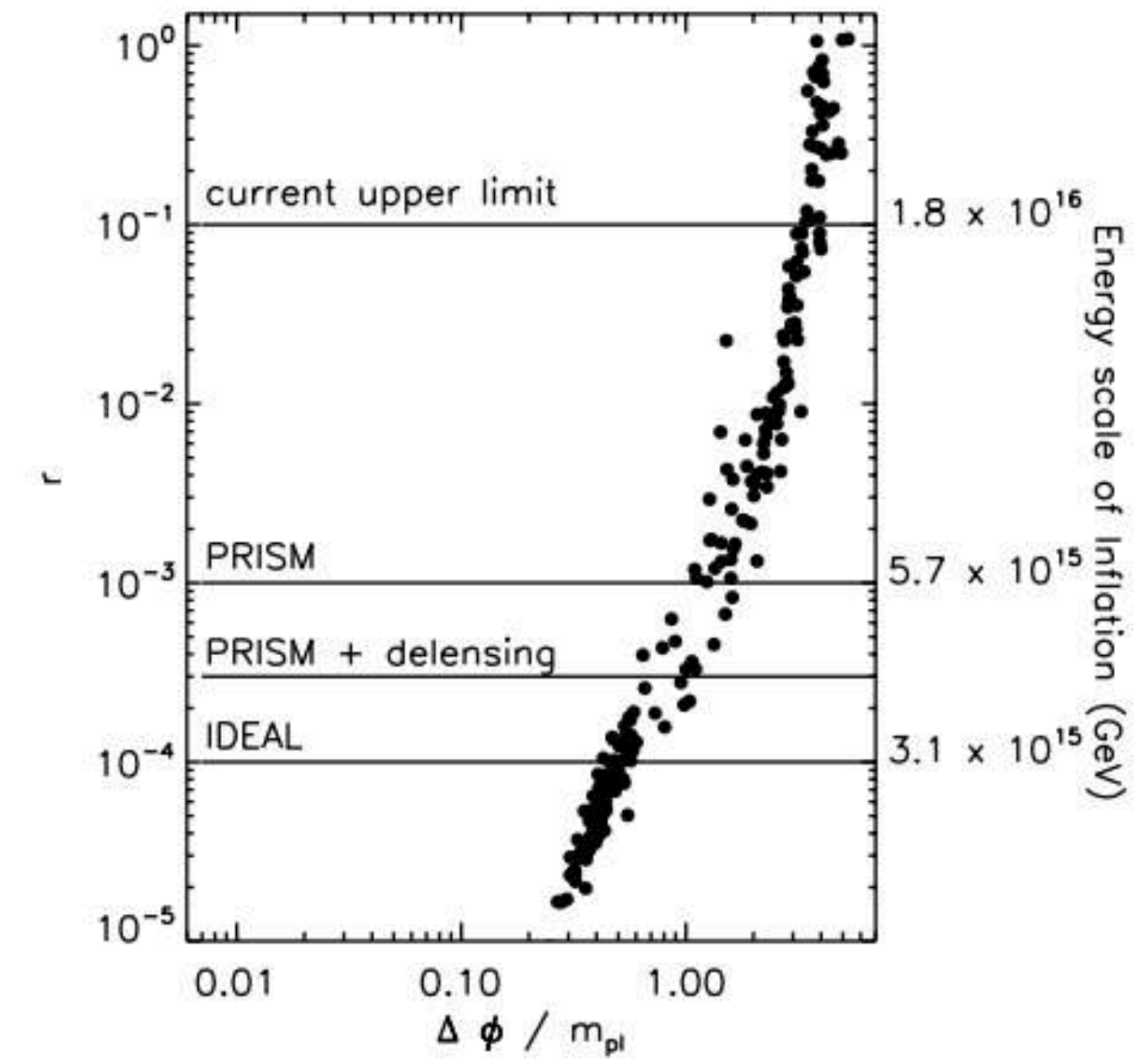
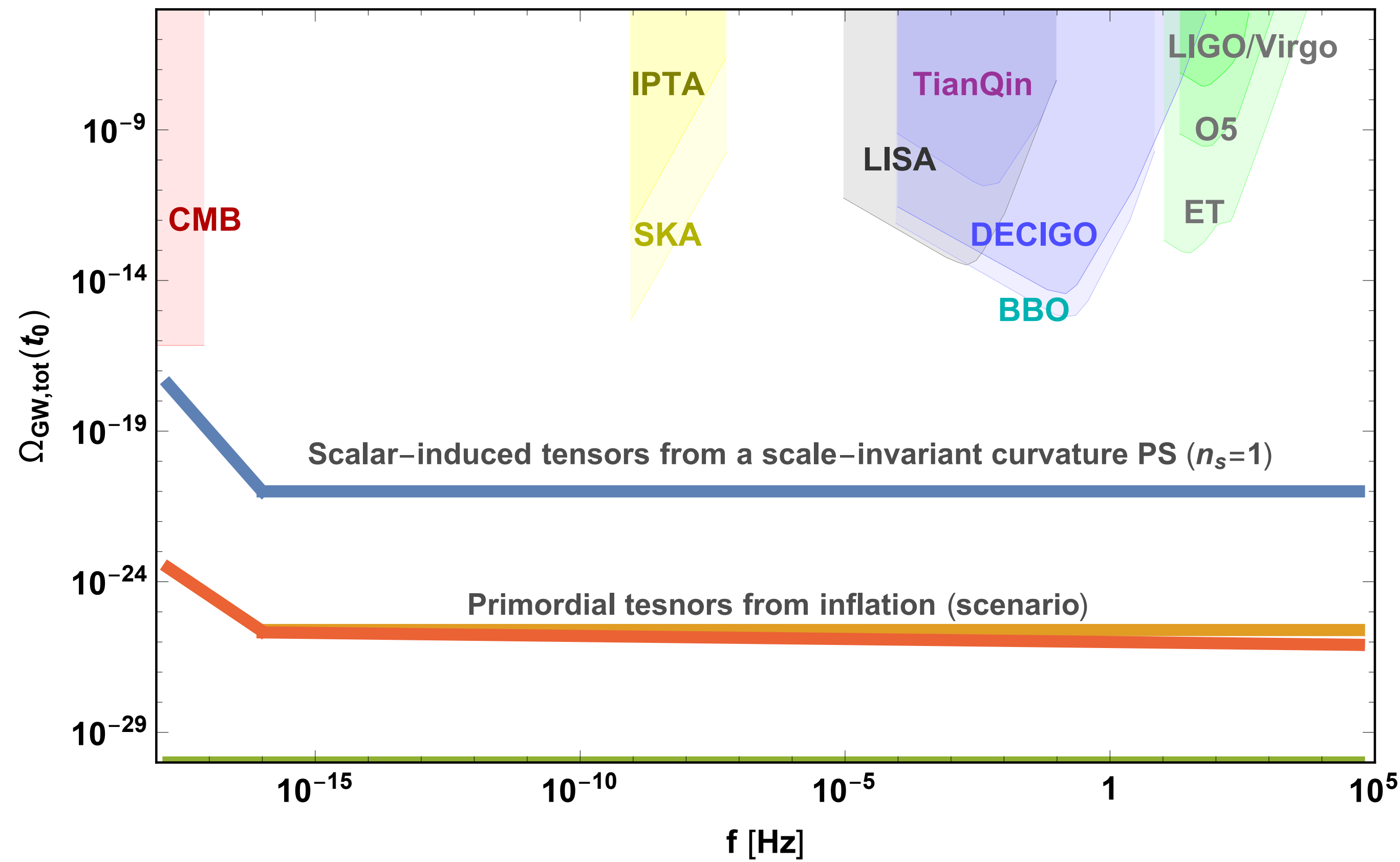
See also the talk of K. Dimopoulos at SUSY 2022

Prism white paper



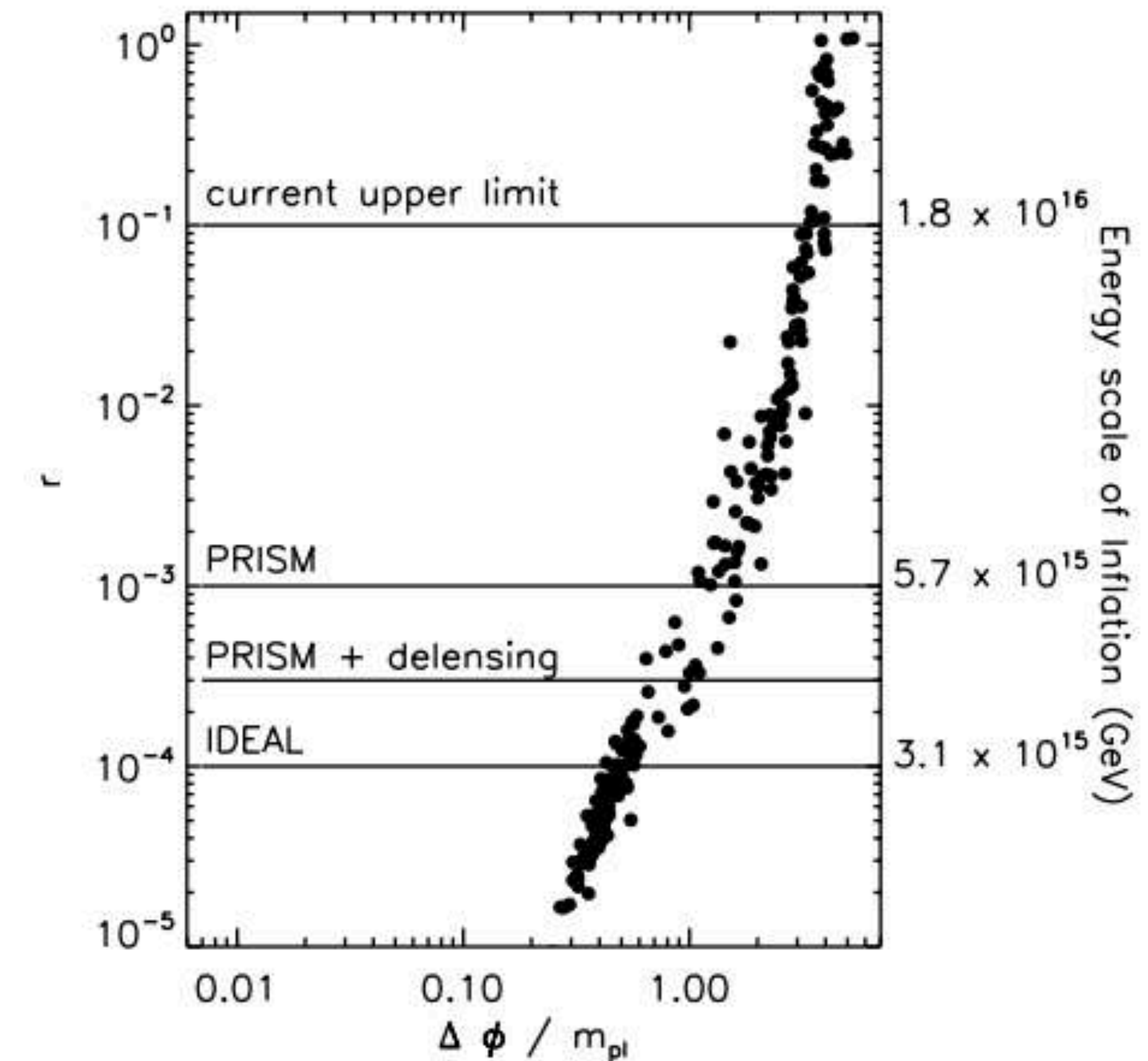
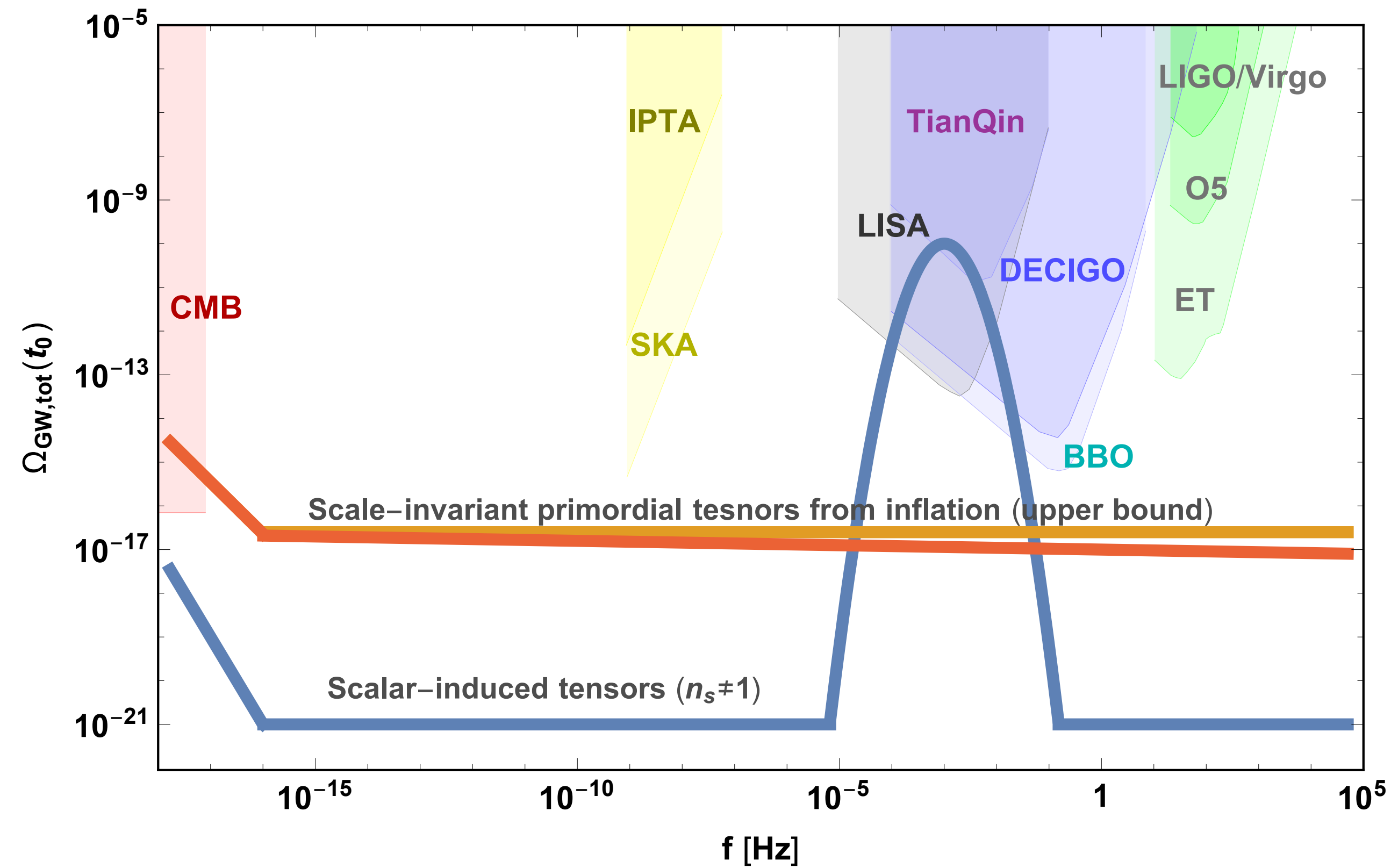
Spectrum of primary and secondary (induced) Gravitational Waves

Example 2 (Schematic illustration)

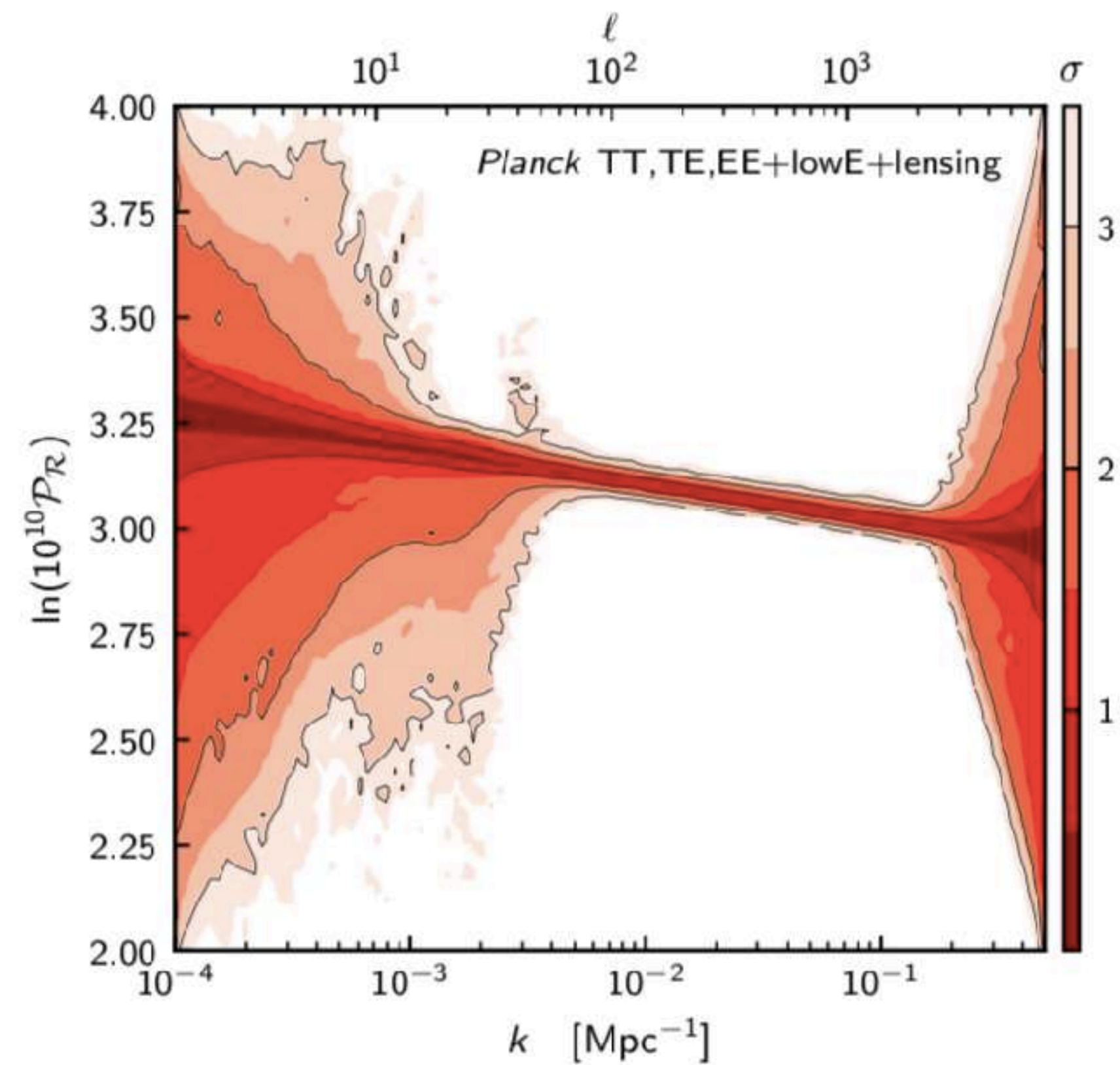


Spectrum of primary and secondary (induced) Gravitational Waves

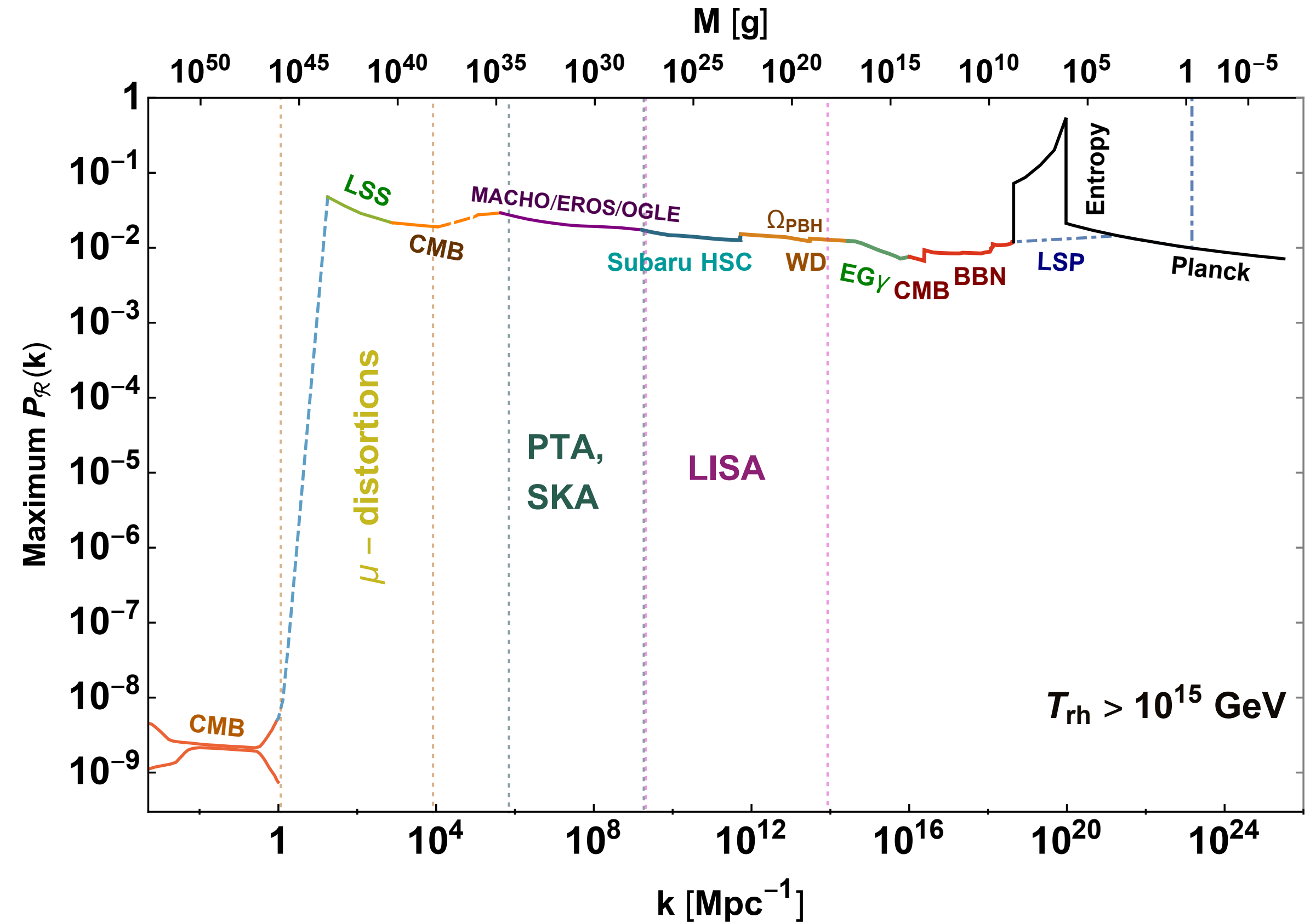
Example 3 (Schematic illustration)



What do we know about the spectrum of the primordial perturbations



Planck 2018

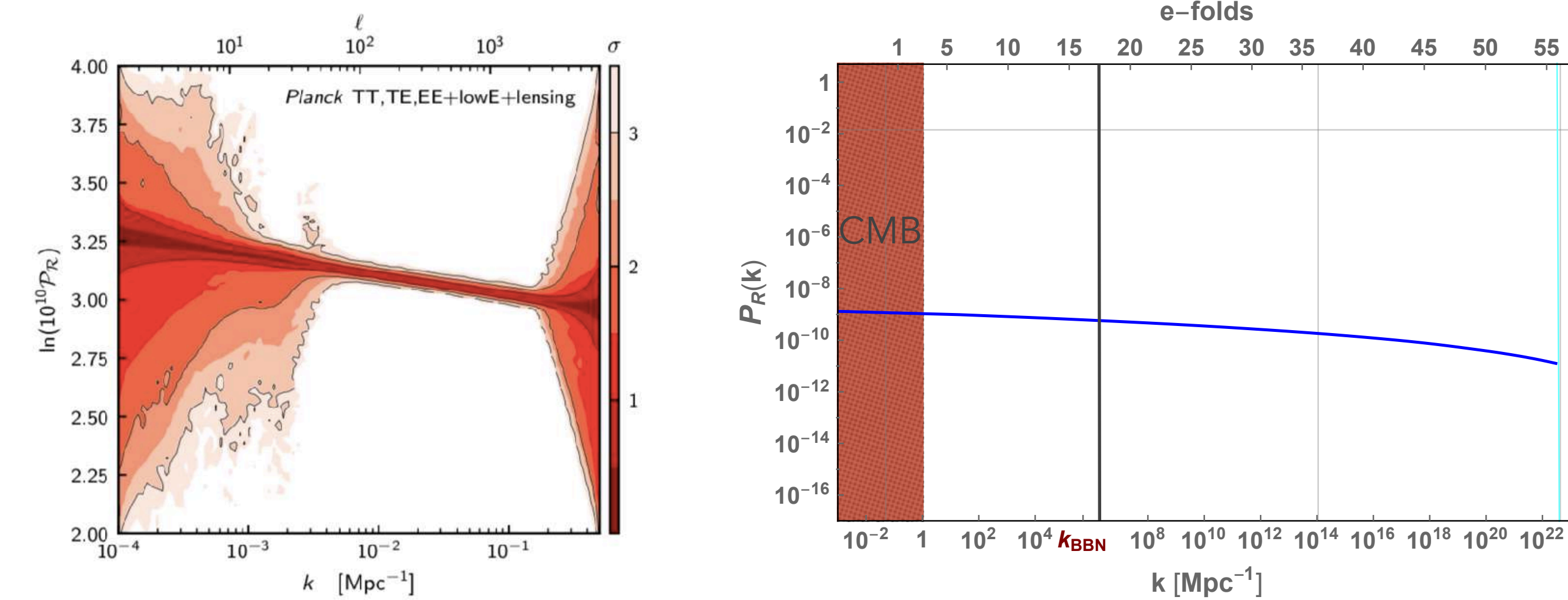


Dalianis, JCAP 08 (2019) 032

Inflation and the power spectrum of the curvature perturbations

Planck 2018

The PS of a “conventional” inflationary model:

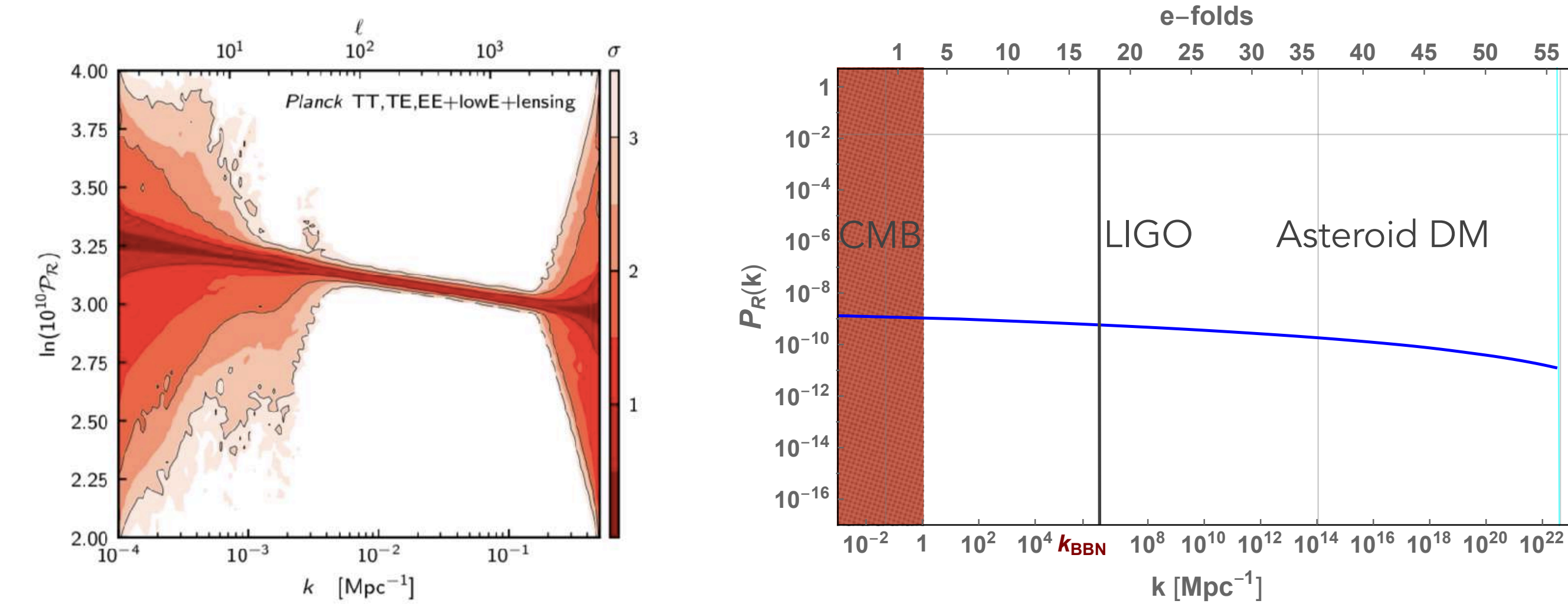


Q: How many PBHs in our observable universe?

Inflation and the power spectrum of the curvature perturbations

Planck 2018

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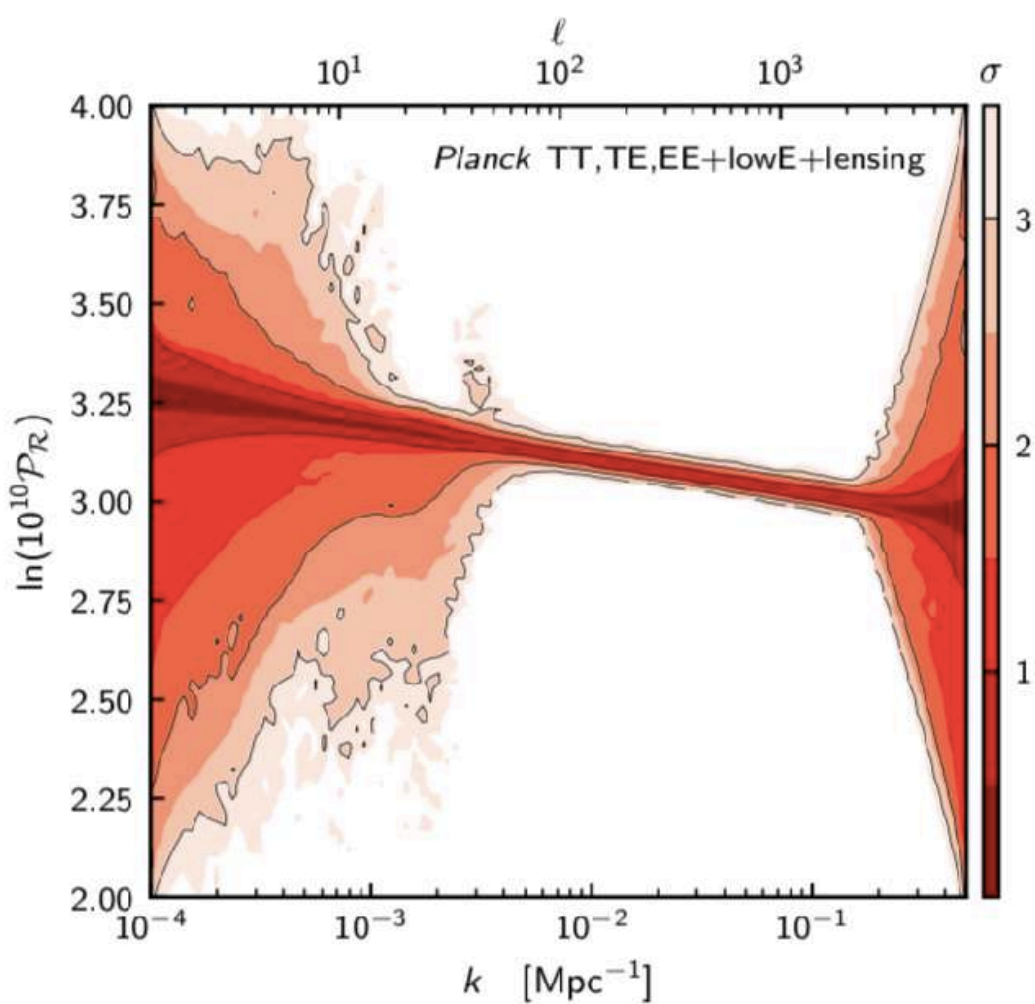


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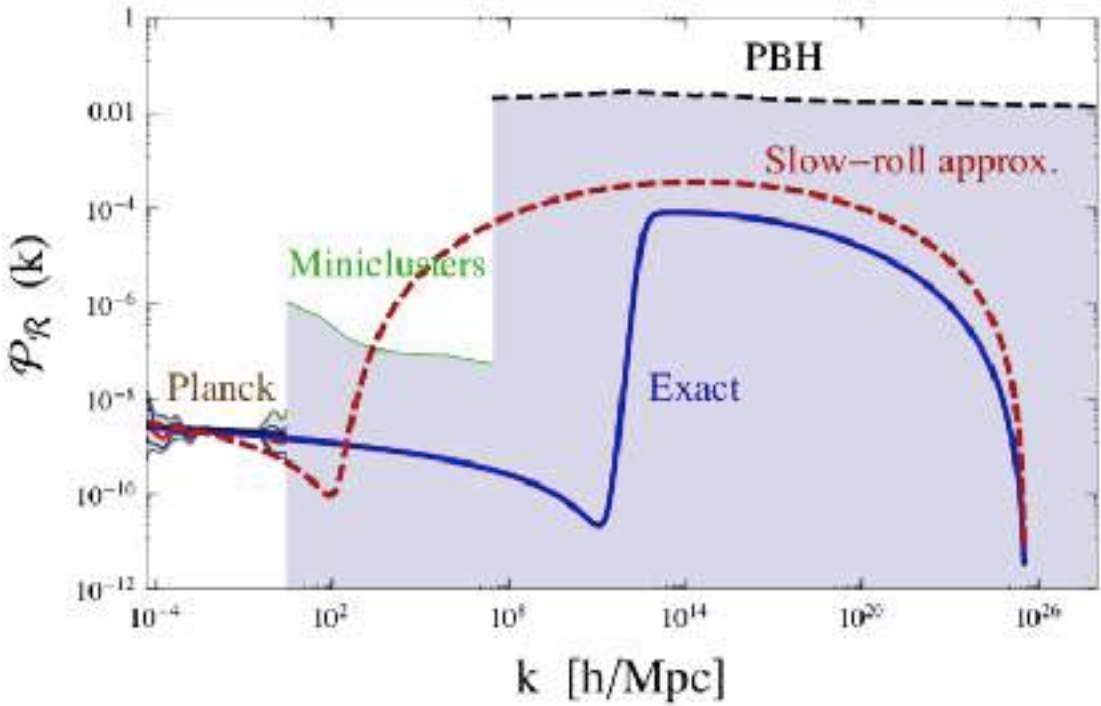
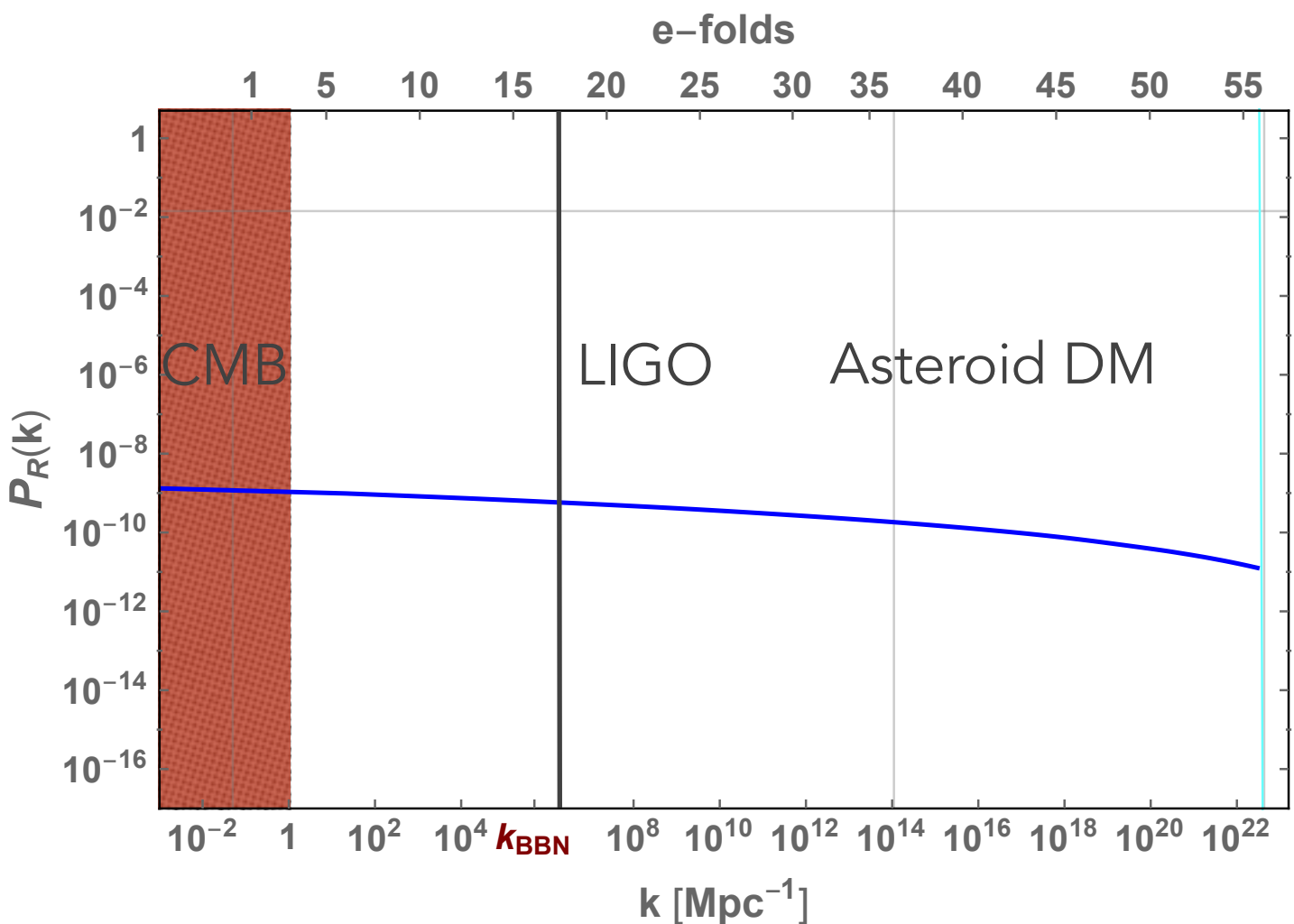
A: According to the PS there is the probability factor : $10^{-38,0000,000}$

Inflation and the power spectrum of the curvature perturbations

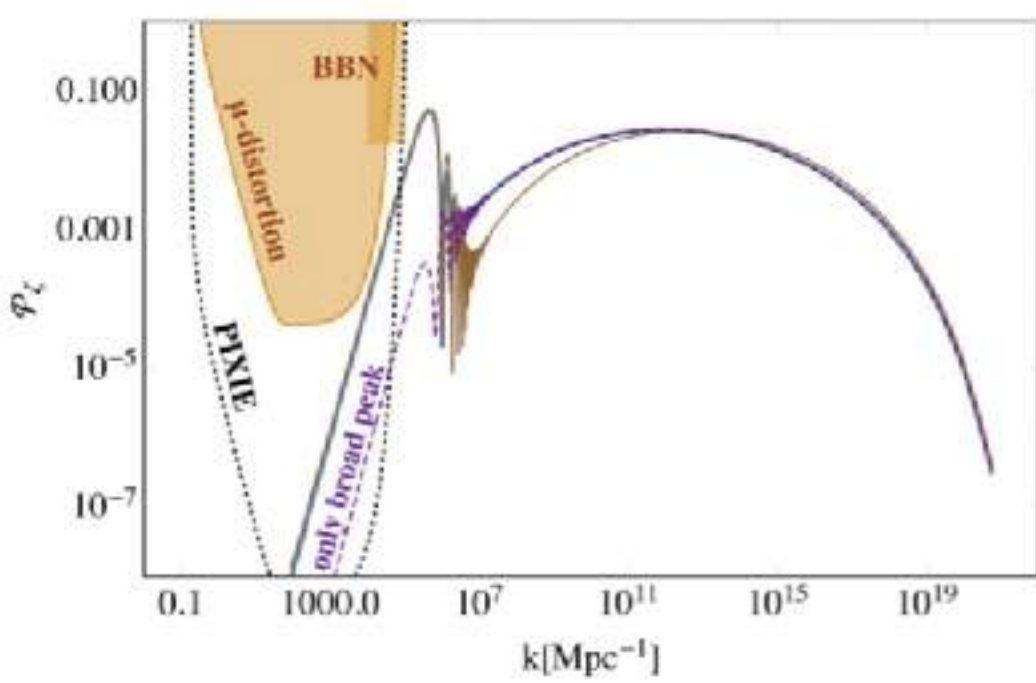
Planck 2018



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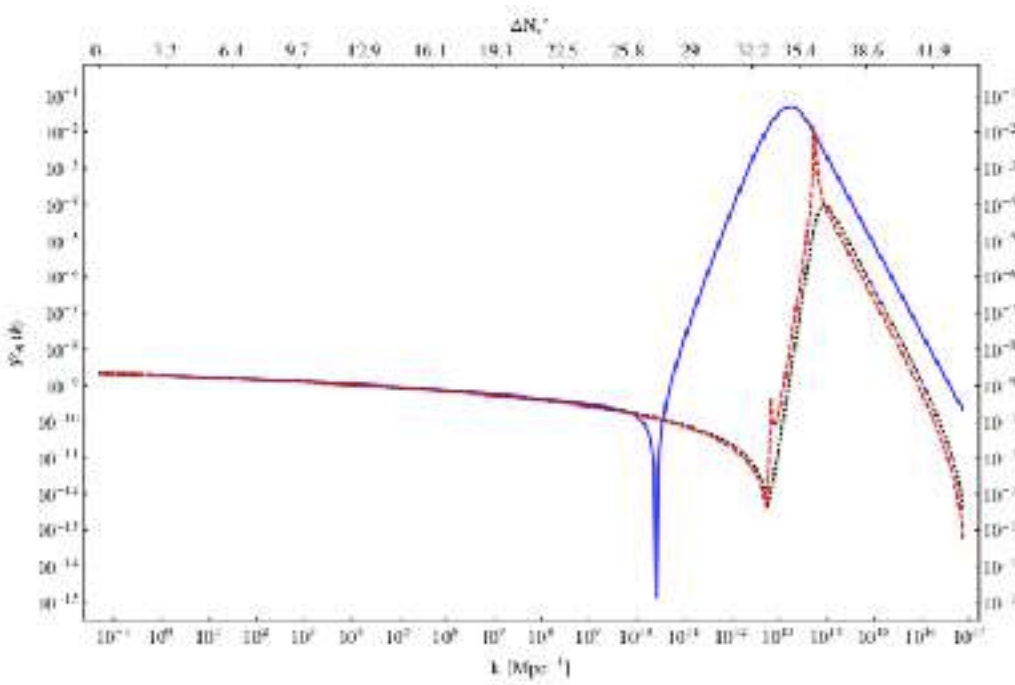
Bellido et.al



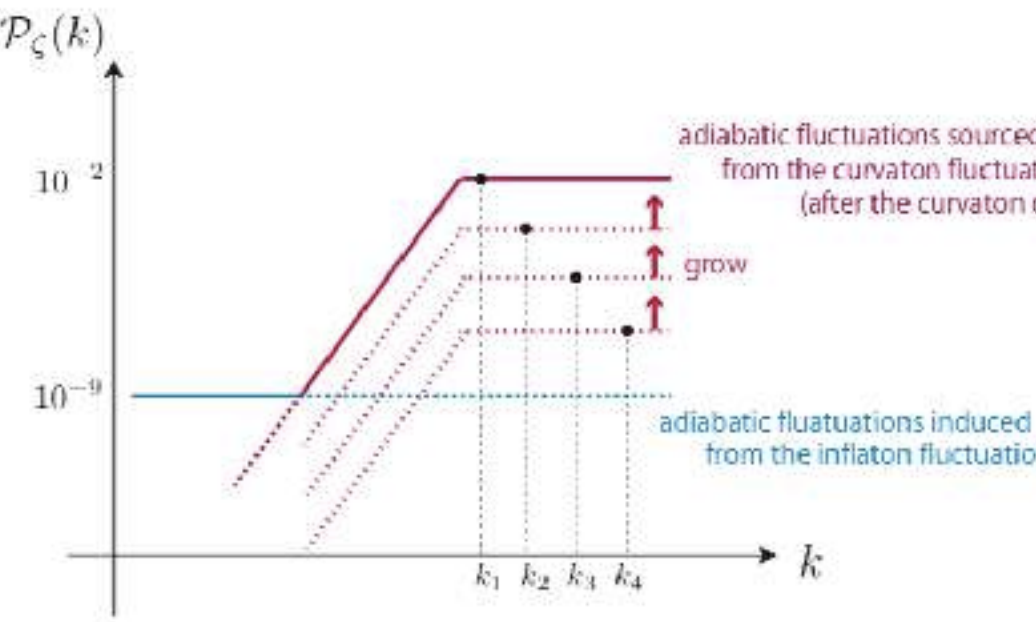
Yanagida et.al

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Ballesteros et.al

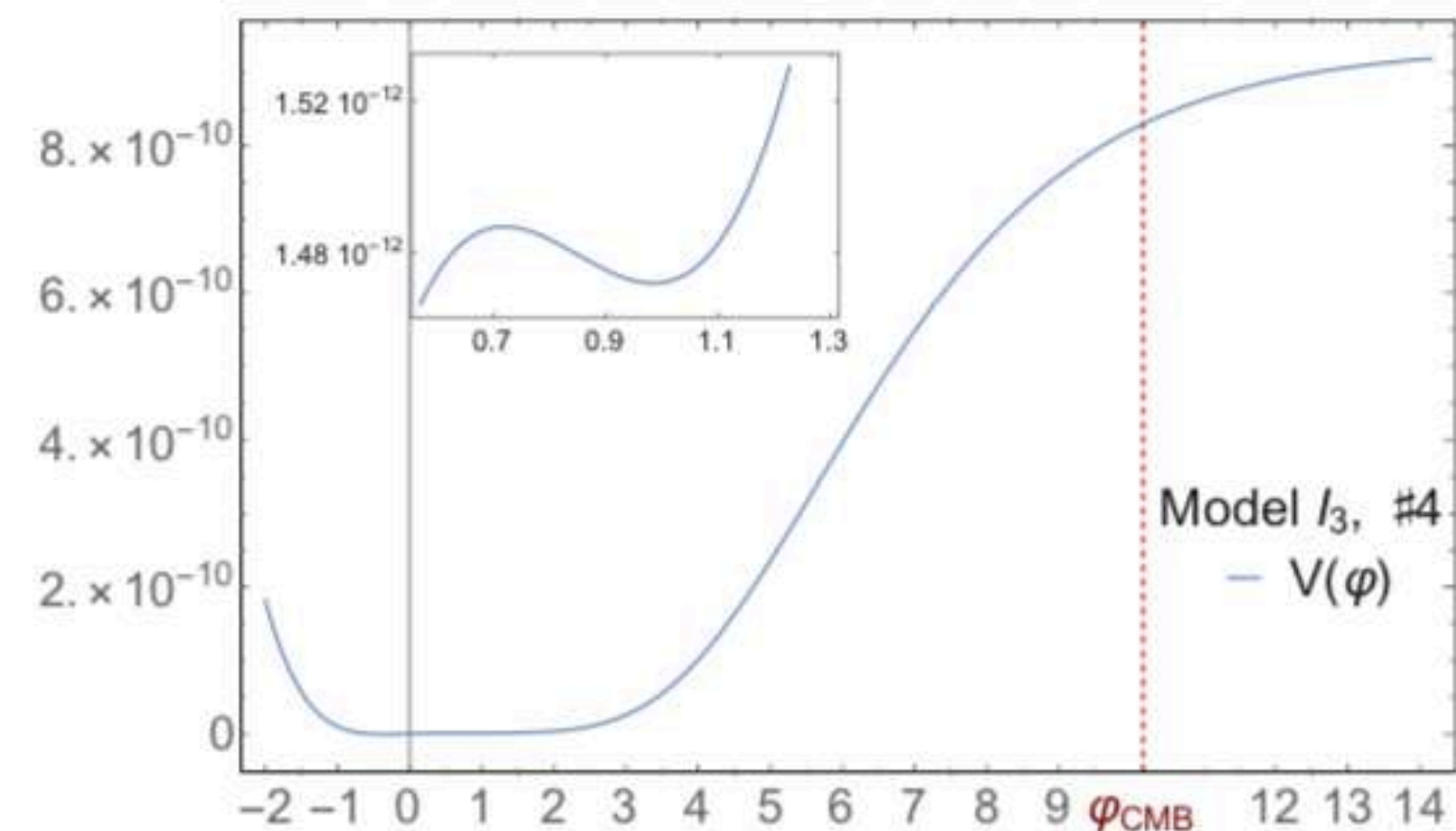
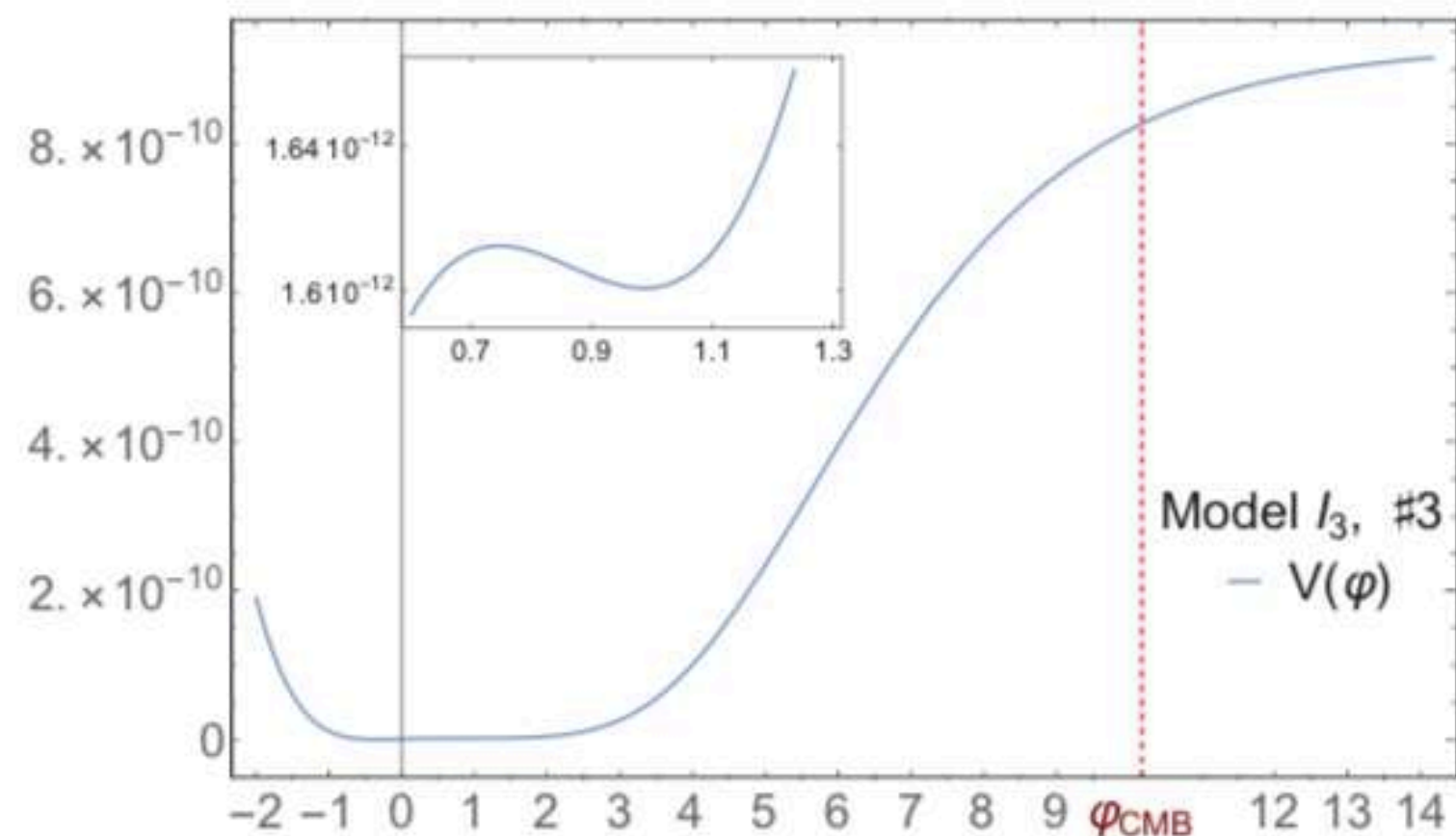
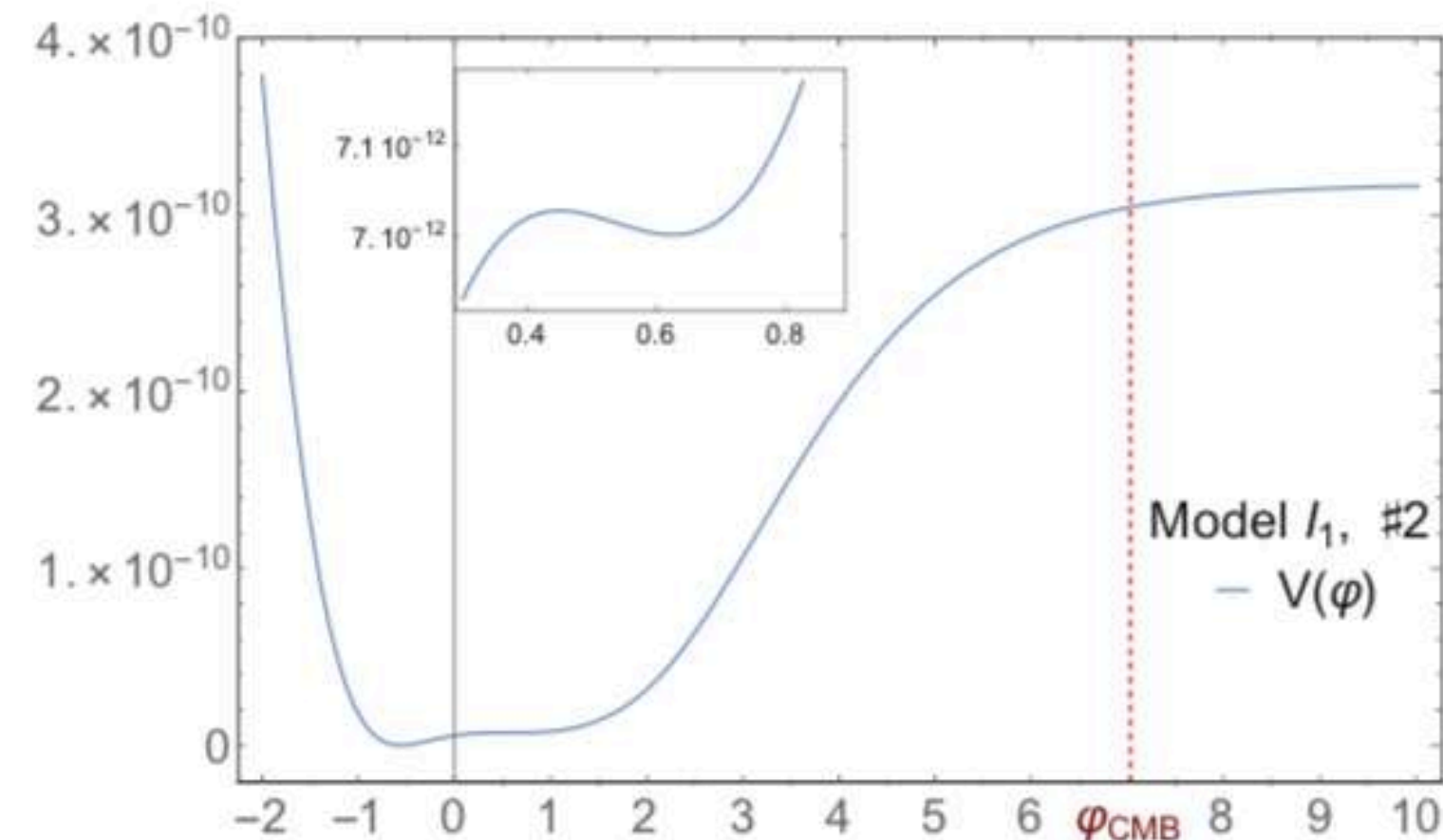
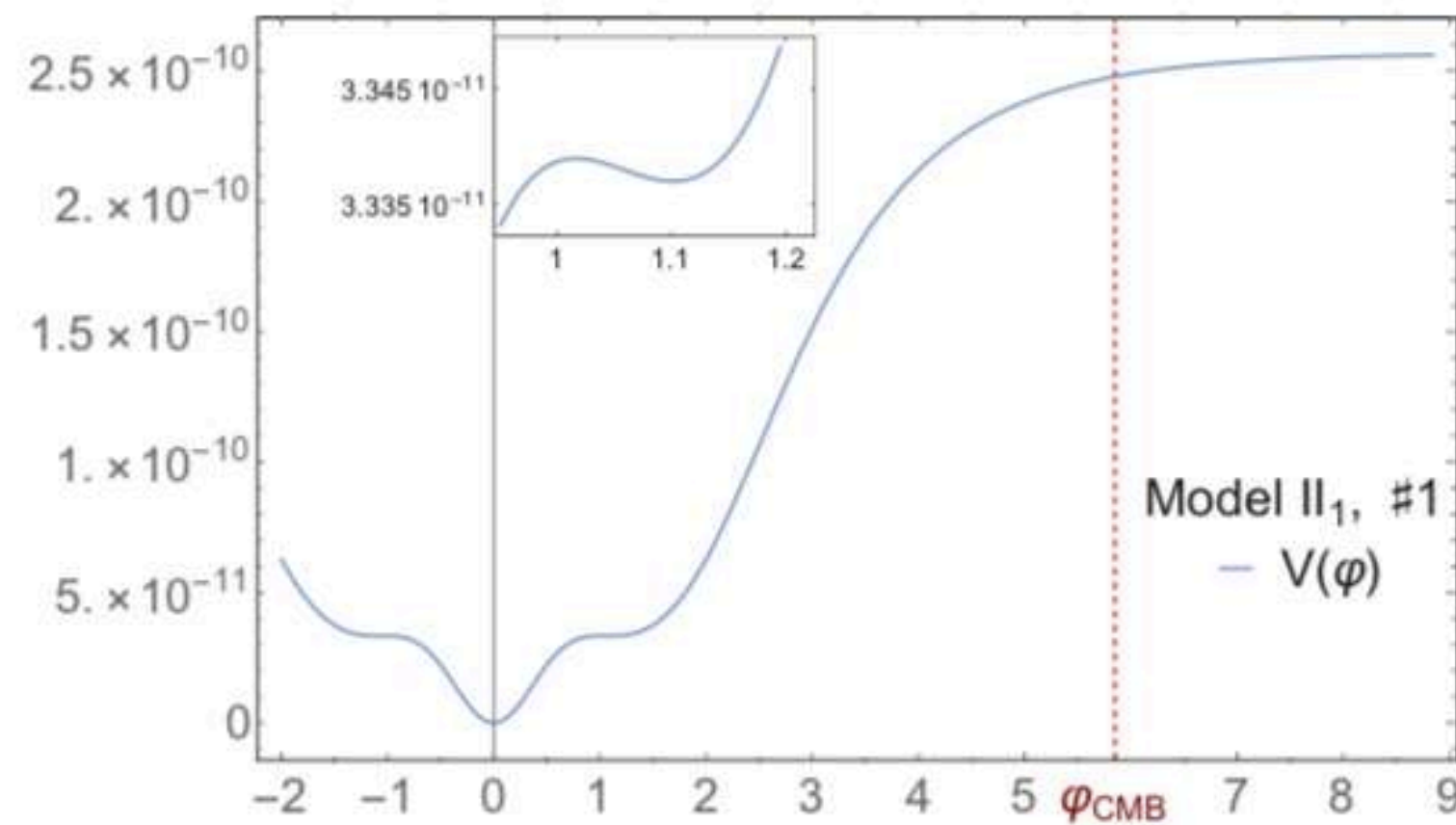


Sasaki et.al

Drees + Erfani (2011, 2012)
ID + Kehagias + Tringas (2018)
Nanopoulos, Spanos, Stamou (2020)]
Stamou (2021), Spanos, Stamou (2021)

$$V(\varphi) = V_0 \left[\tanh(\varphi/\sqrt{6}) + A \sin \left(\tanh(\varphi/\sqrt{6})/\theta \right) \right]^2$$

$$V(\varphi) = V_0 \left\{ c_0 + c_1 \tanh \left(\frac{\varphi}{\sqrt{6\alpha}} \right) + c_2 \tanh^2 \left(\frac{\varphi}{\sqrt{6\alpha}} \right) + c_3 \tanh^3 \left(\frac{\varphi}{\sqrt{6\alpha}} \right) \right\}^2$$



*“Inflection-point”
models*

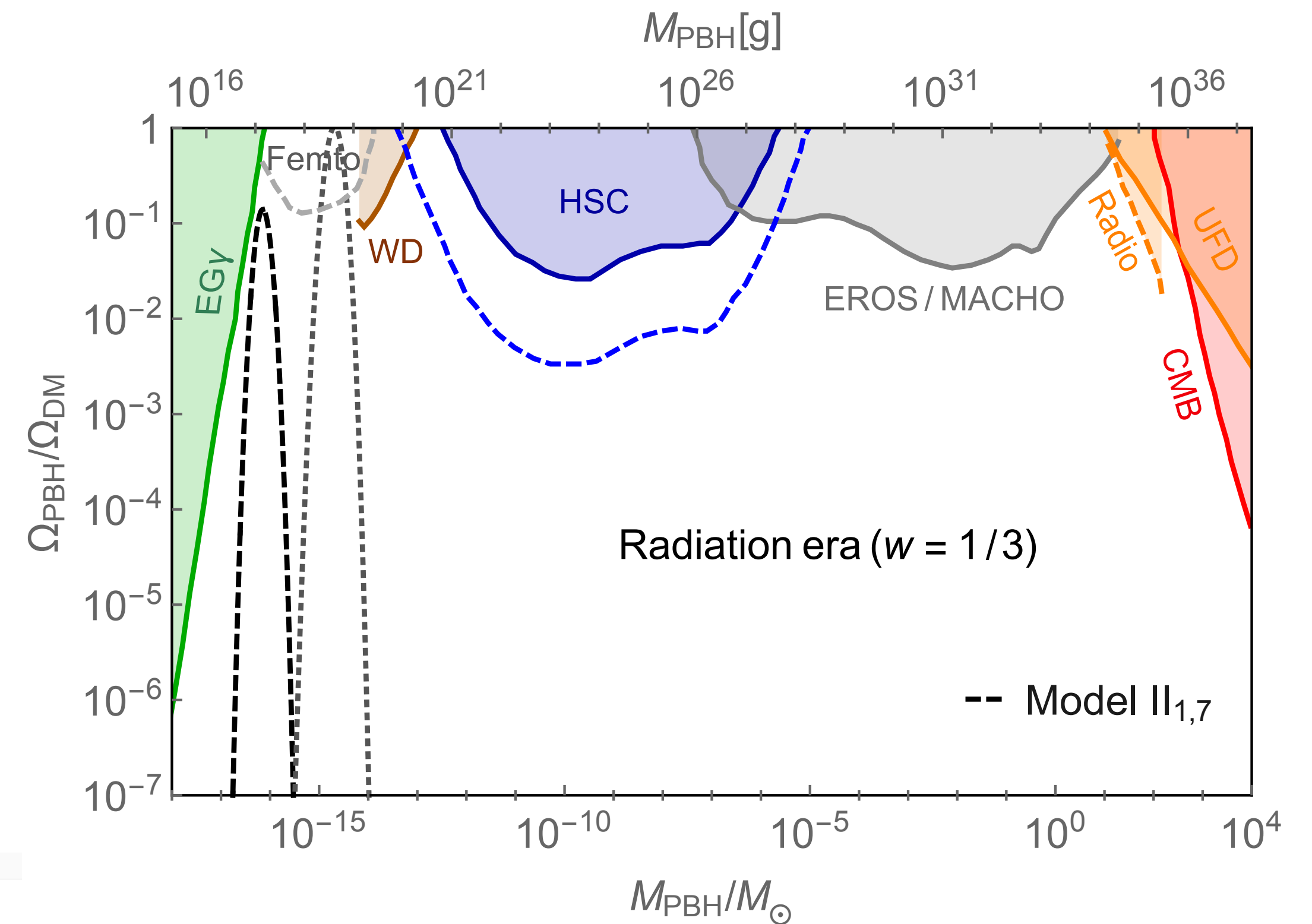
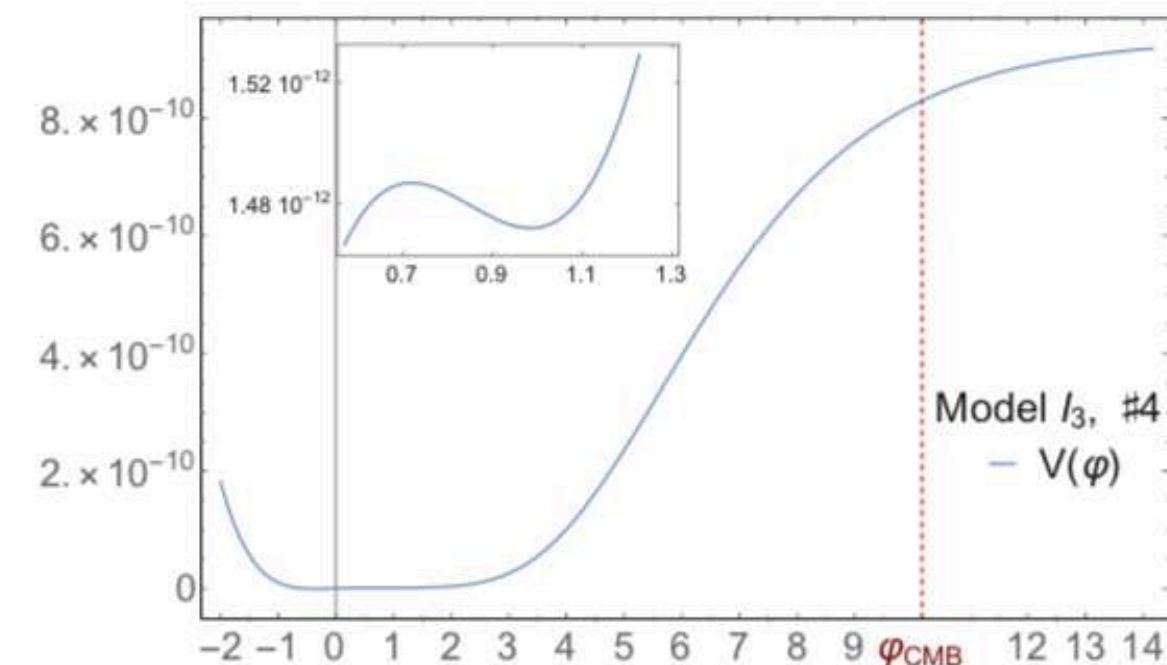
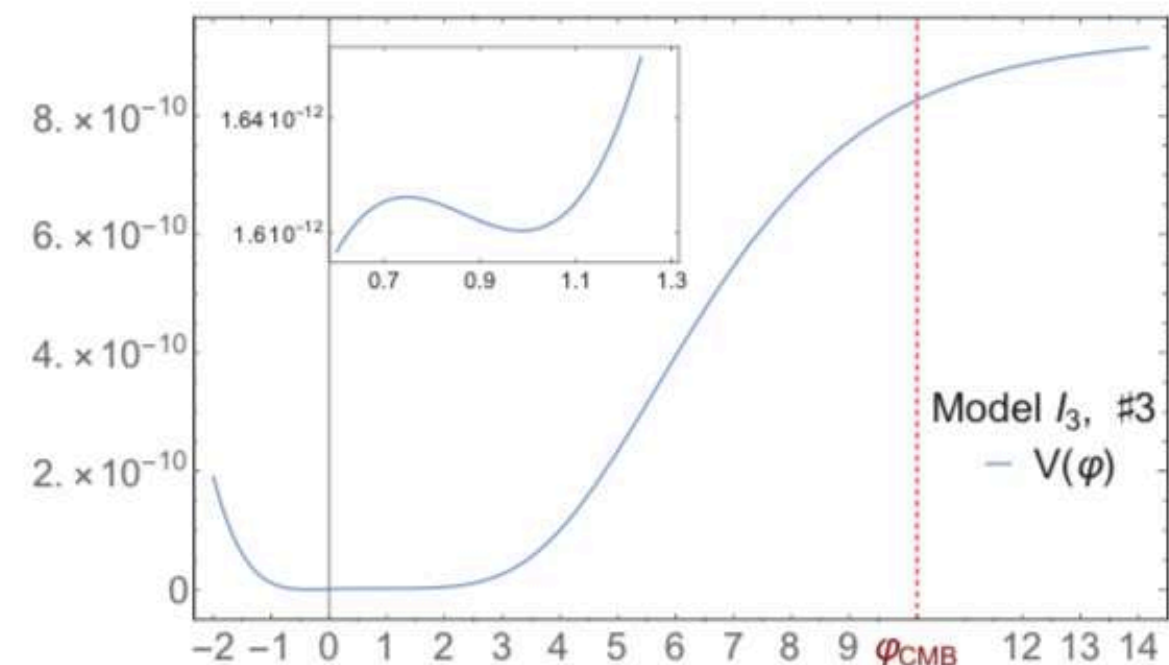
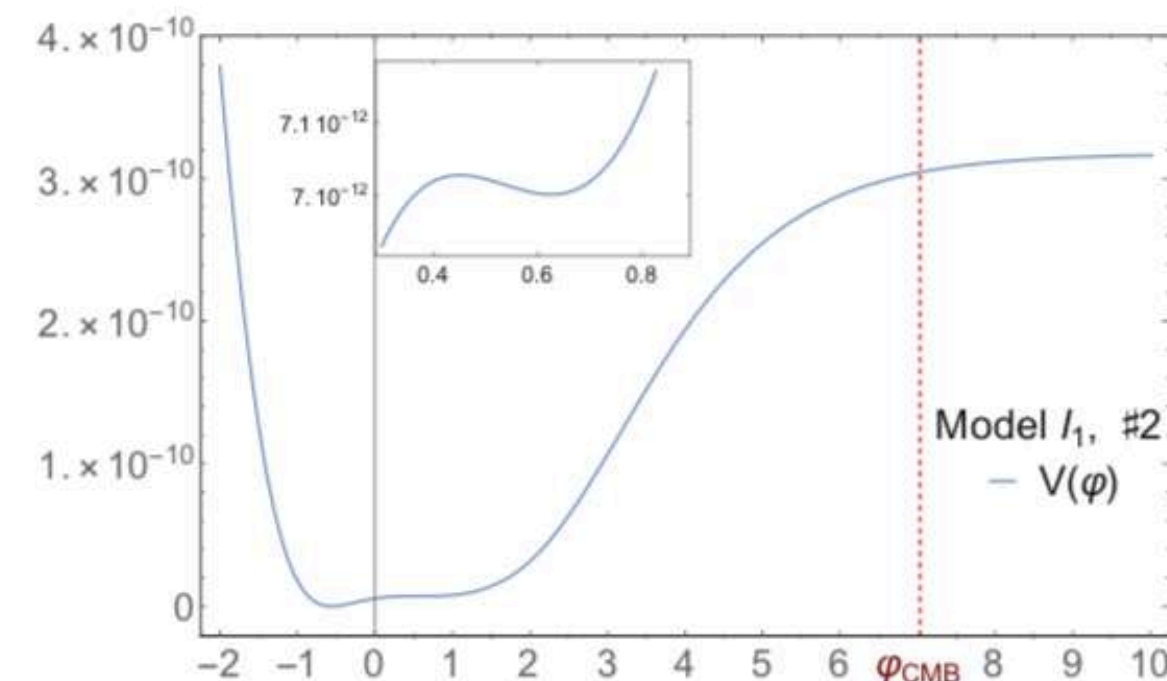
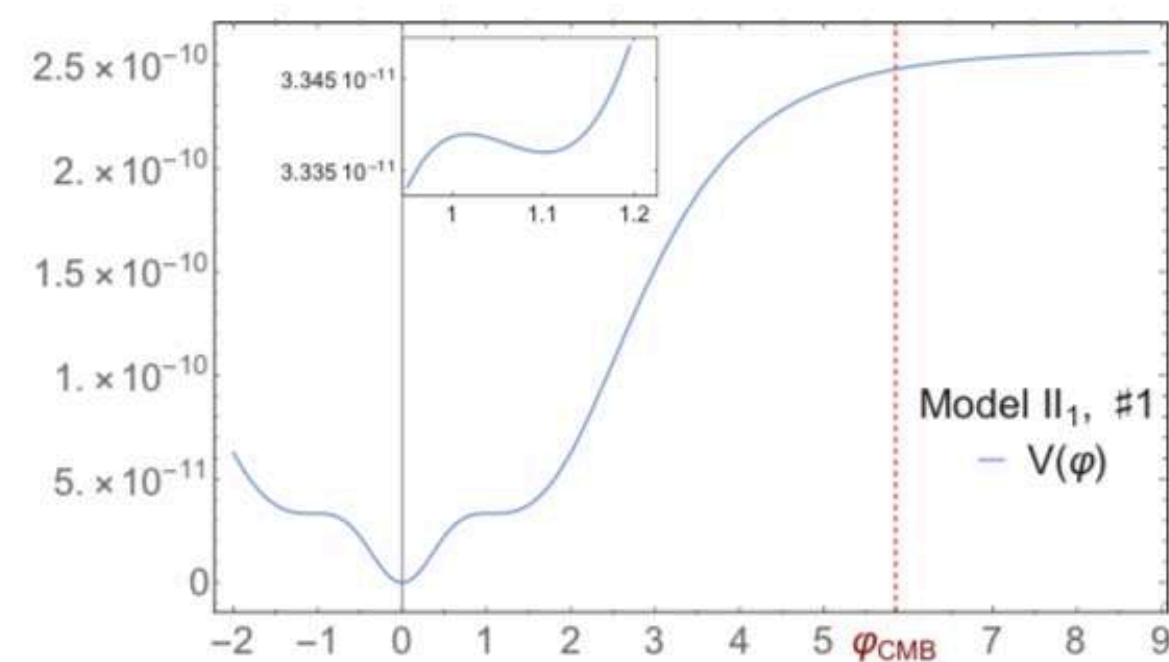
Superconformal α -Attractor inflationary models

ID + Kehagias + Tringas, JCAP (2019)

1

$$V(\varphi) = V_0 \left[\tanh(\varphi/\sqrt{6}) + A \sin \left(\tanh(\varphi/\sqrt{6})/\theta \right) \right]^2$$

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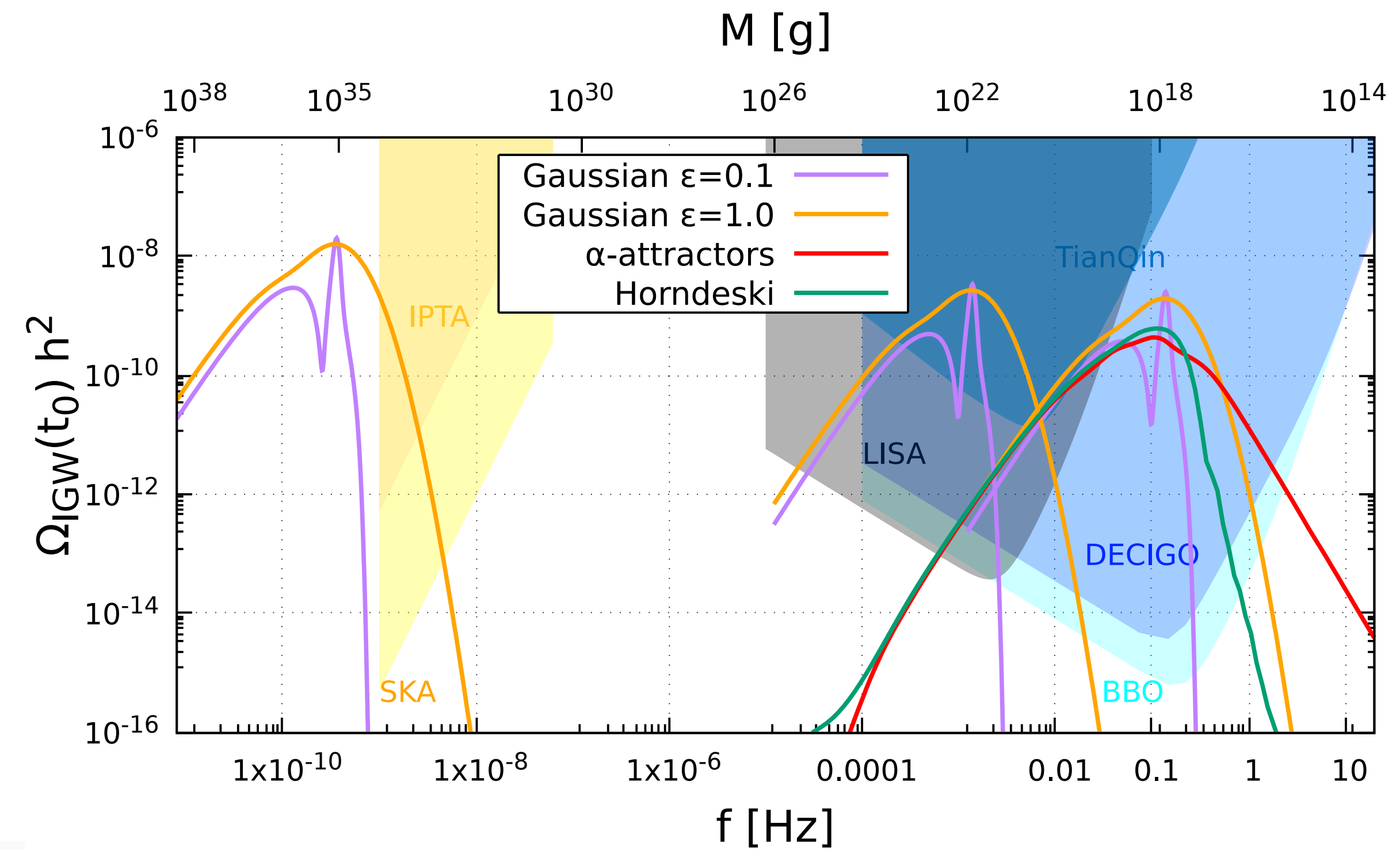
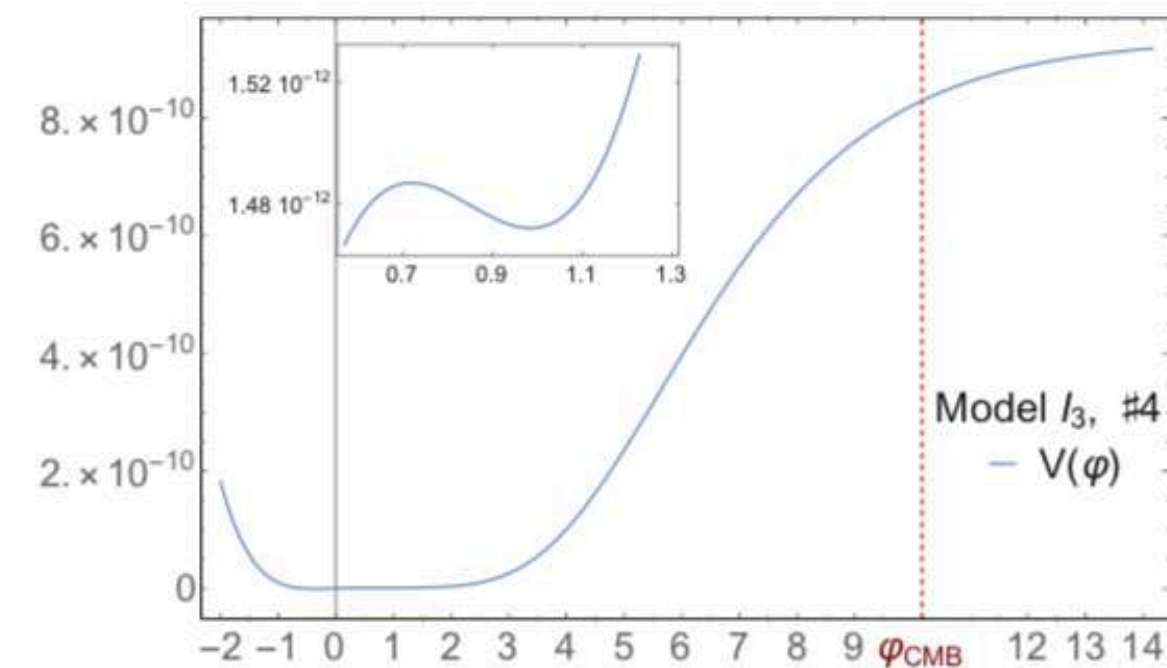
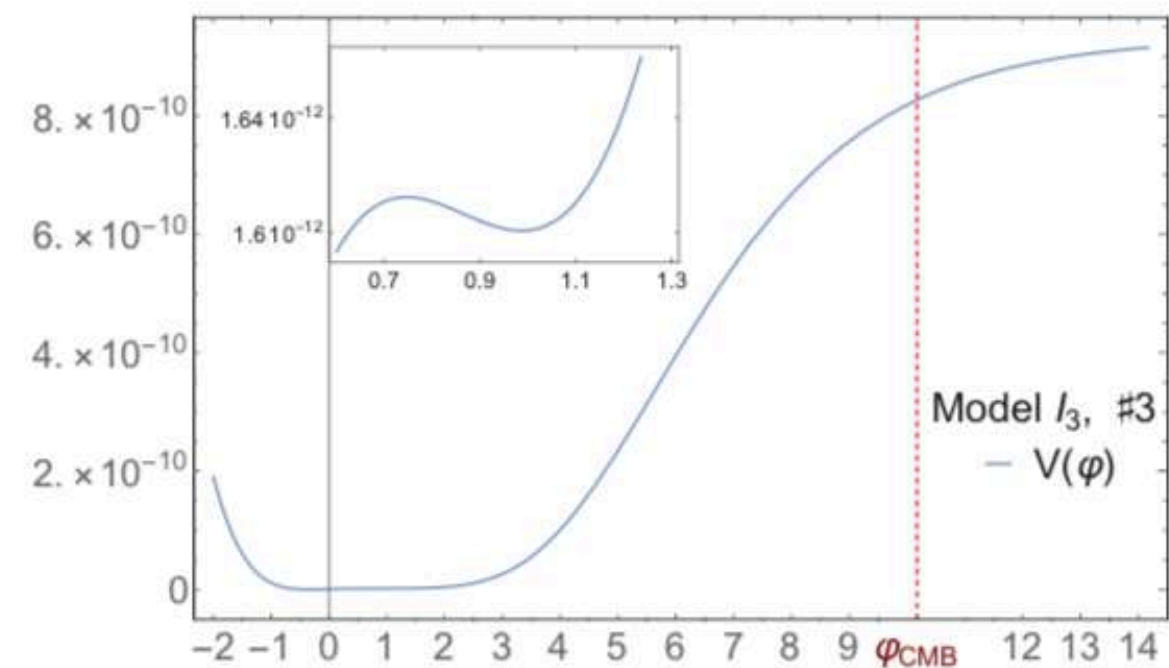
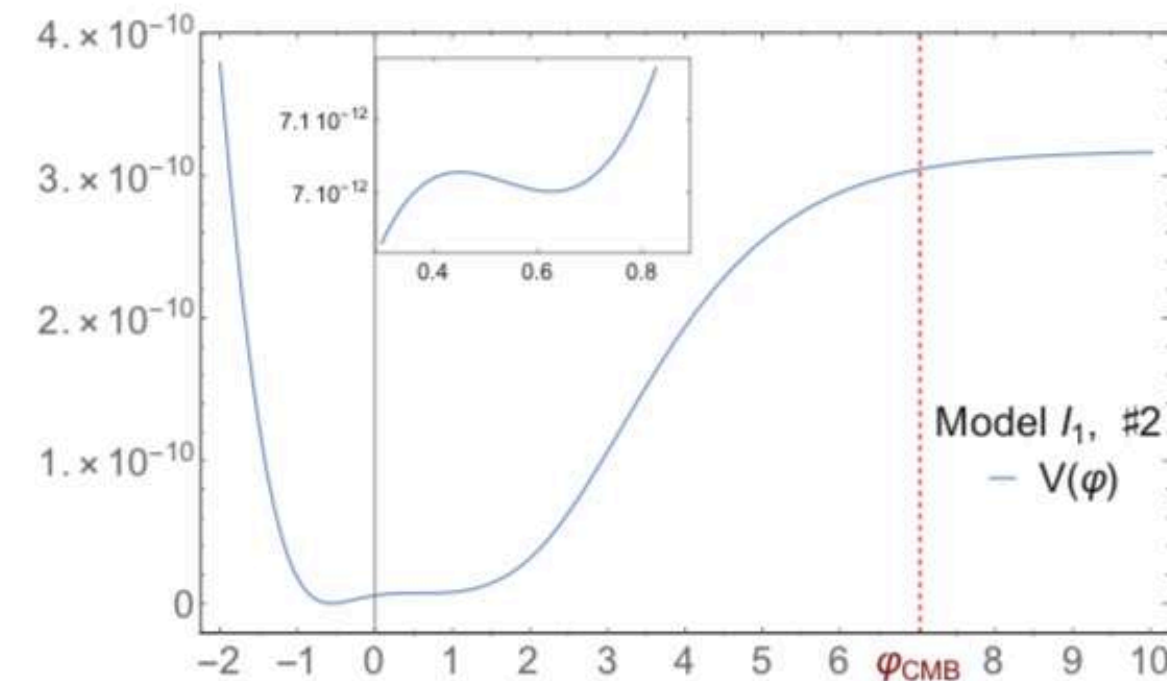
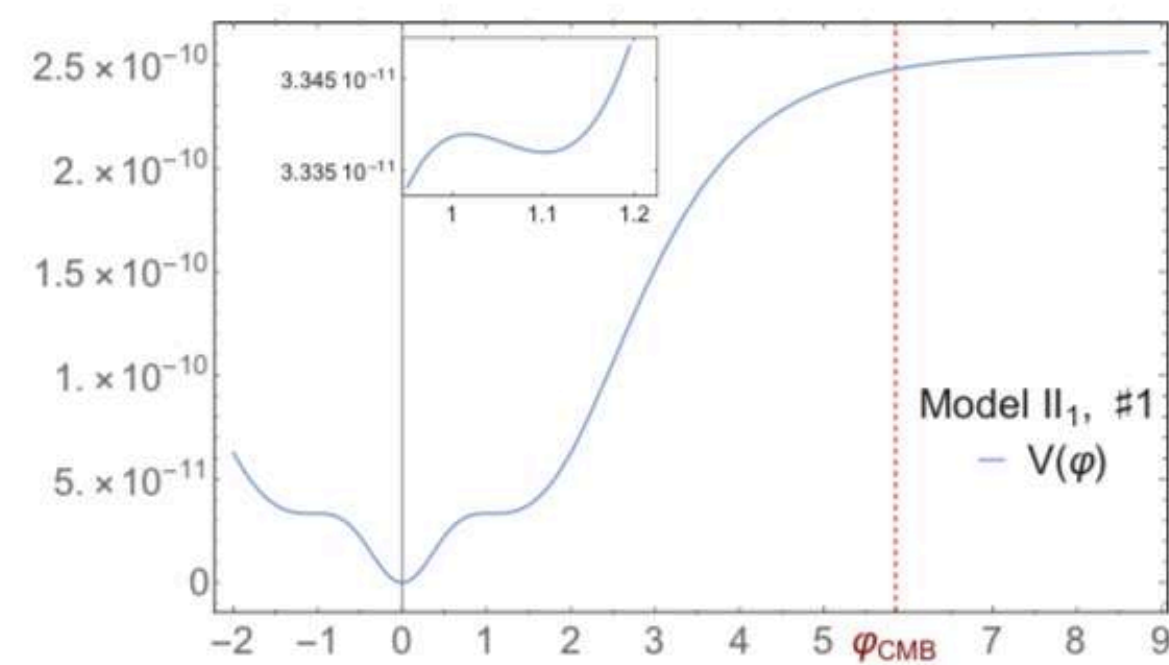
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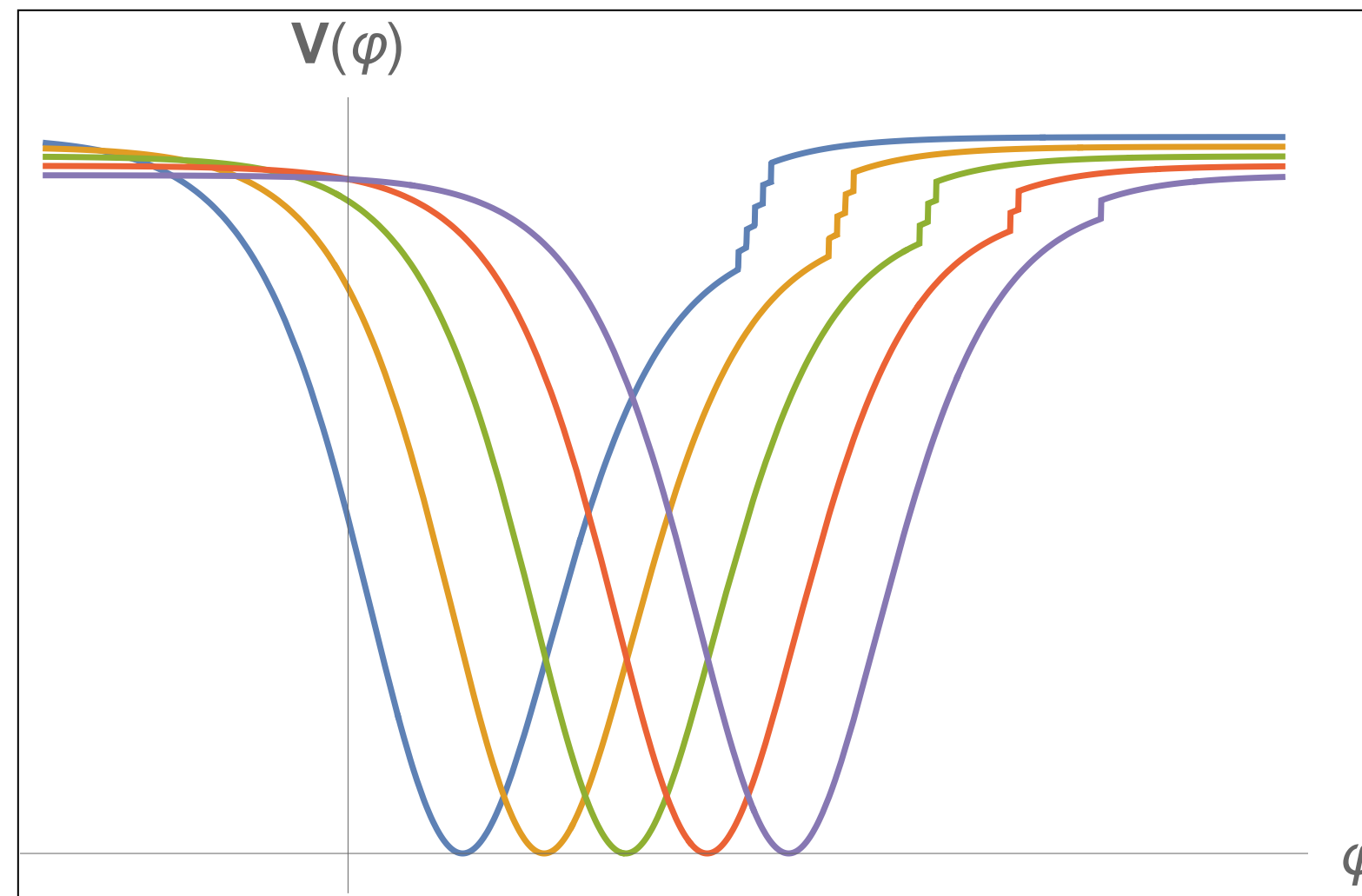


ID+ Kritos, Phys.Rev.D (2021)

Steps in the inflaton potential

Kefala + Kodaxis + Stamou + Tetradis *Phys.Rev.D* (2021)

ID + Kodaxis + Stamou + Tetradis + Tsigkas-Kouvelis, *Phys.Rev.D* (2021)



- We identify *the slow-roll parameter η* as the quantity that can trigger the rapid growth of perturbations.

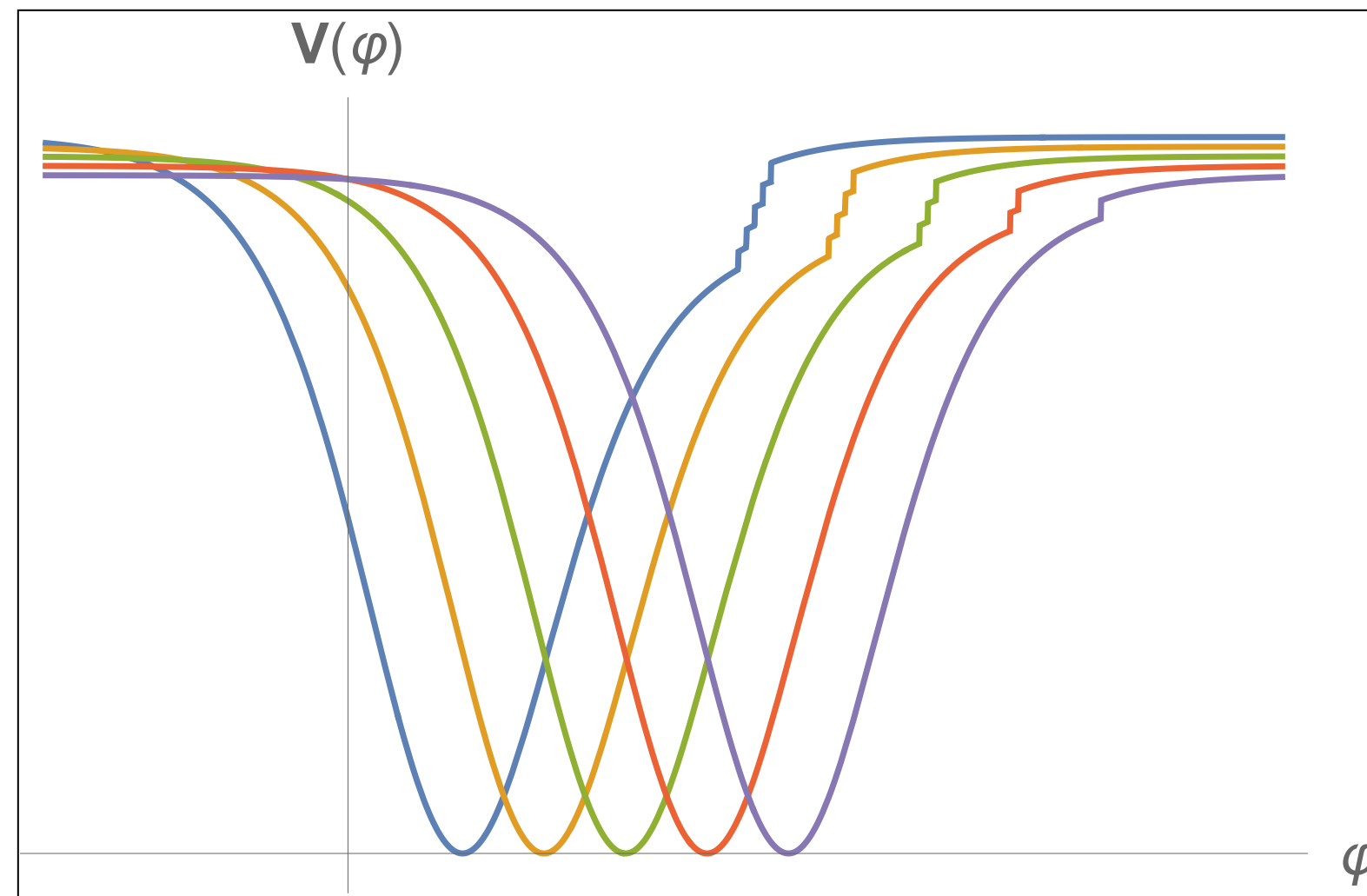
$$f(N) = 3 + \epsilon_H - 2\eta_H$$

- The typical underlying reason for the appearance of strong features in the evolution of η is the presence of points in the inflaton potential that **cannot support slow-roll**, such as sharp steps or inflection points.
- The acceleration of the background inflaton when a step in the potential is crossed makes η attain very **negative** values initially, followed by **positive** values when the inflaton settles in a slow-roll regime on a plateau following the step.
- The first part of the evolution is very short for sharp steps, while the second lasts longer with $\eta \approx 3$.
- During this second part, η **acts as negative friction**, leading to the rapid growth of the curvature perturbation.

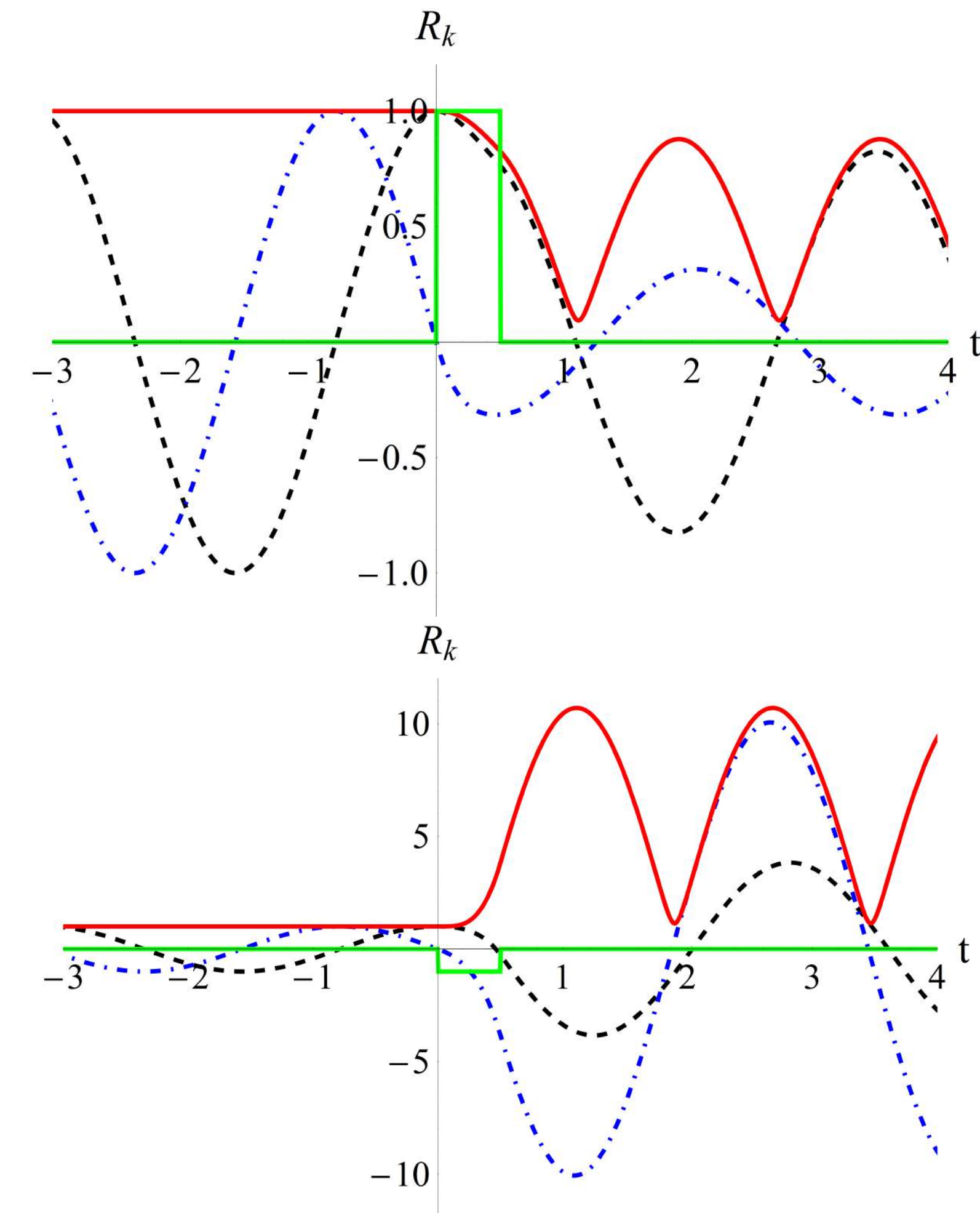
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Kefala + Kodaxis + Stamou + Tetradis *Phys.Rev.D* (2021)

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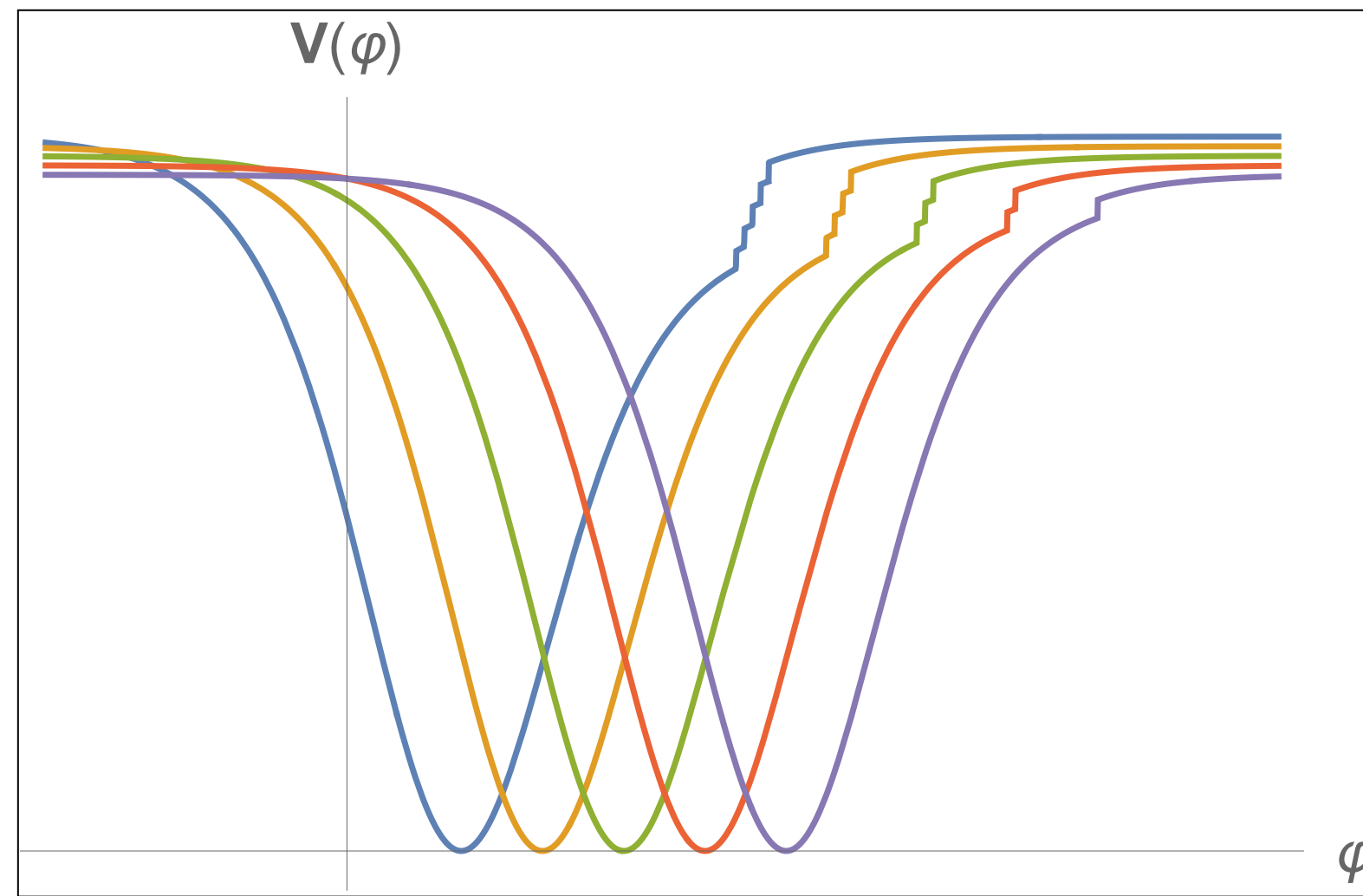
- A sharp drop in the potential of the inflaton field **detunes the relative phase of the real and imaginary parts of the curvature perturbation**, so that oscillations in the amplitude of the spectrum appear



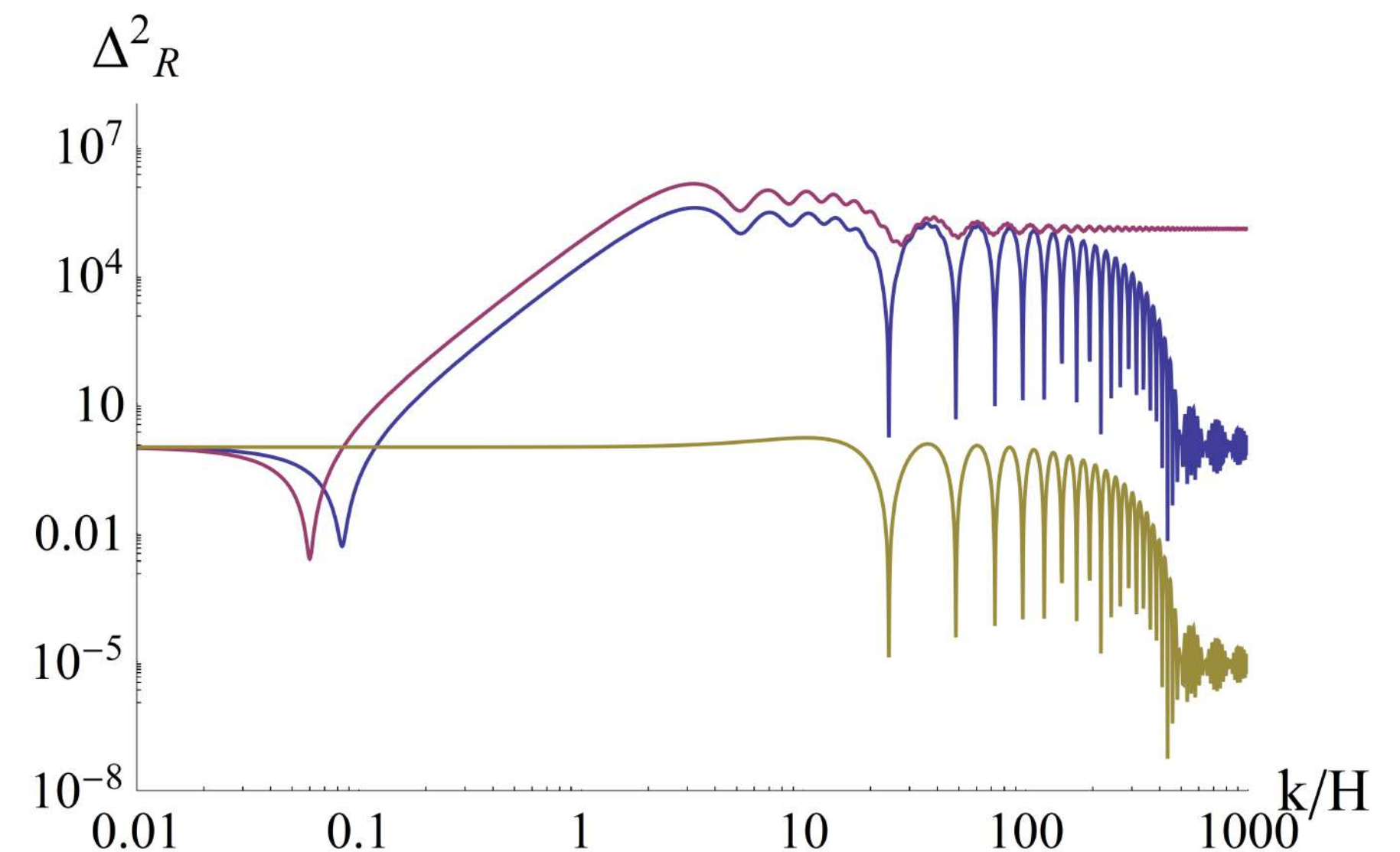
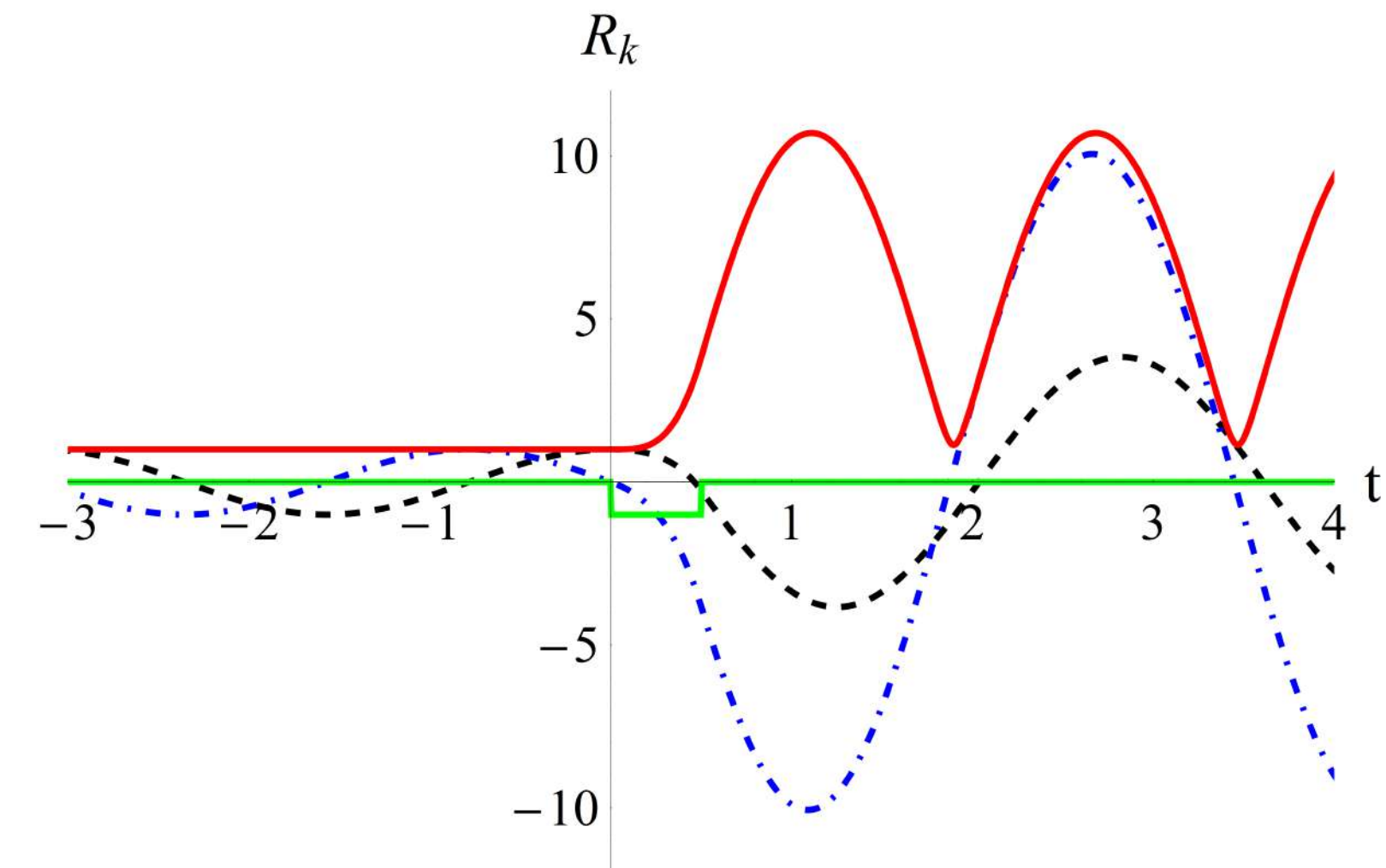
Steps in the inflaton potential

Kefala + Kodaxis + Stamou + Tetradis *Phys.Rev.D* (2021)

ID + Kodaxis + Stamou + Tetradis + Tsigkas-Kouvelis, *Phys.Rev.D* (2021)



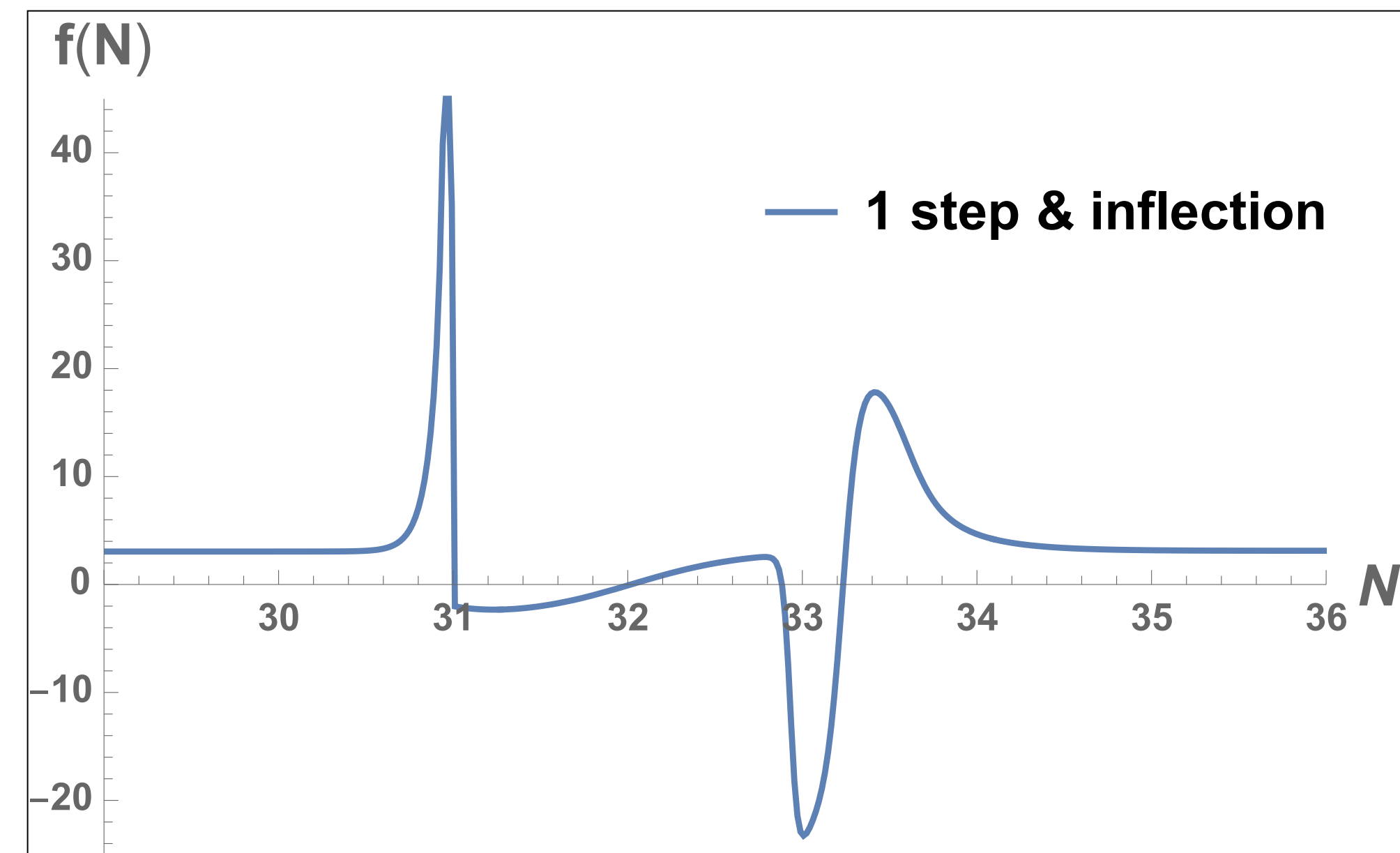
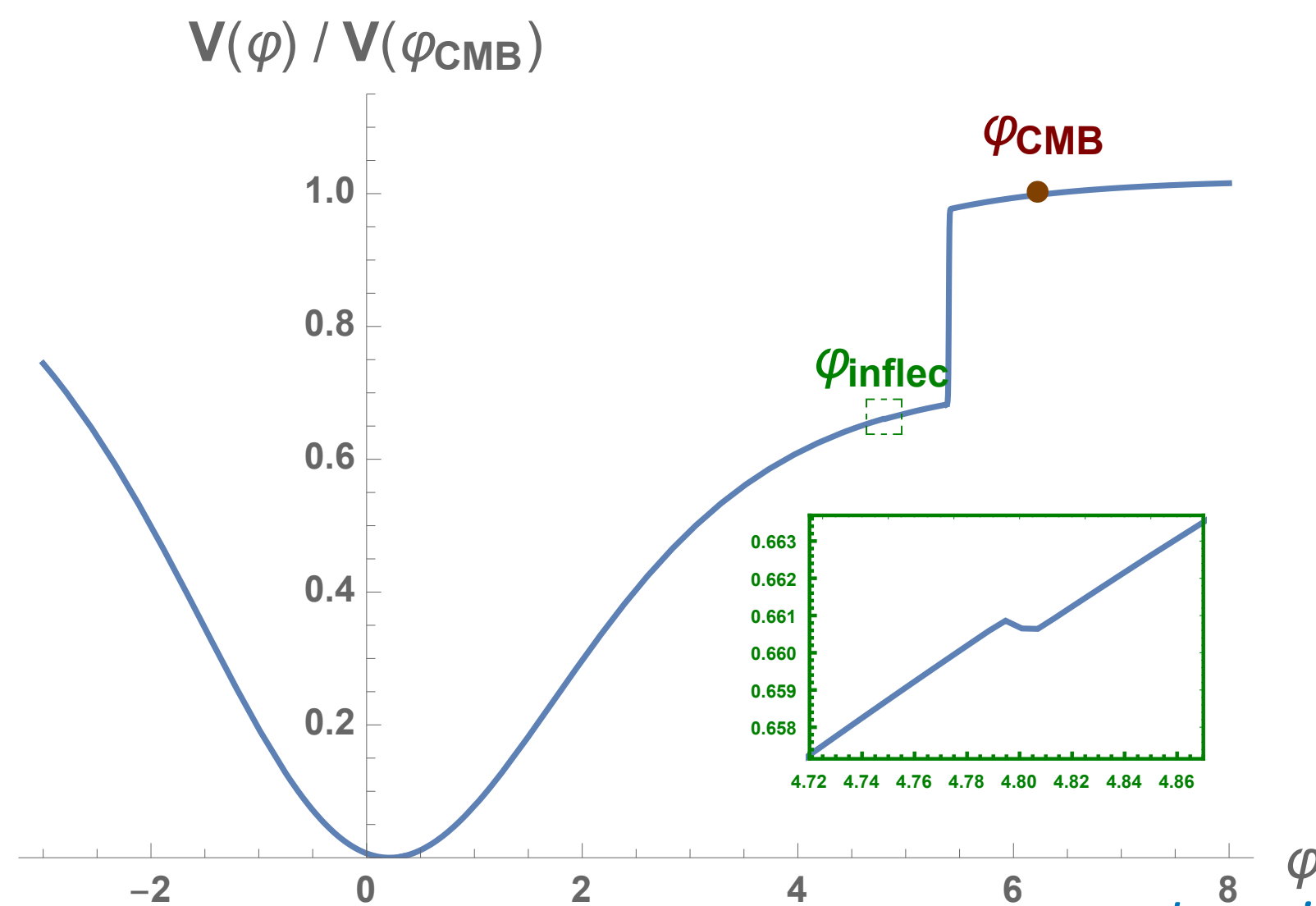
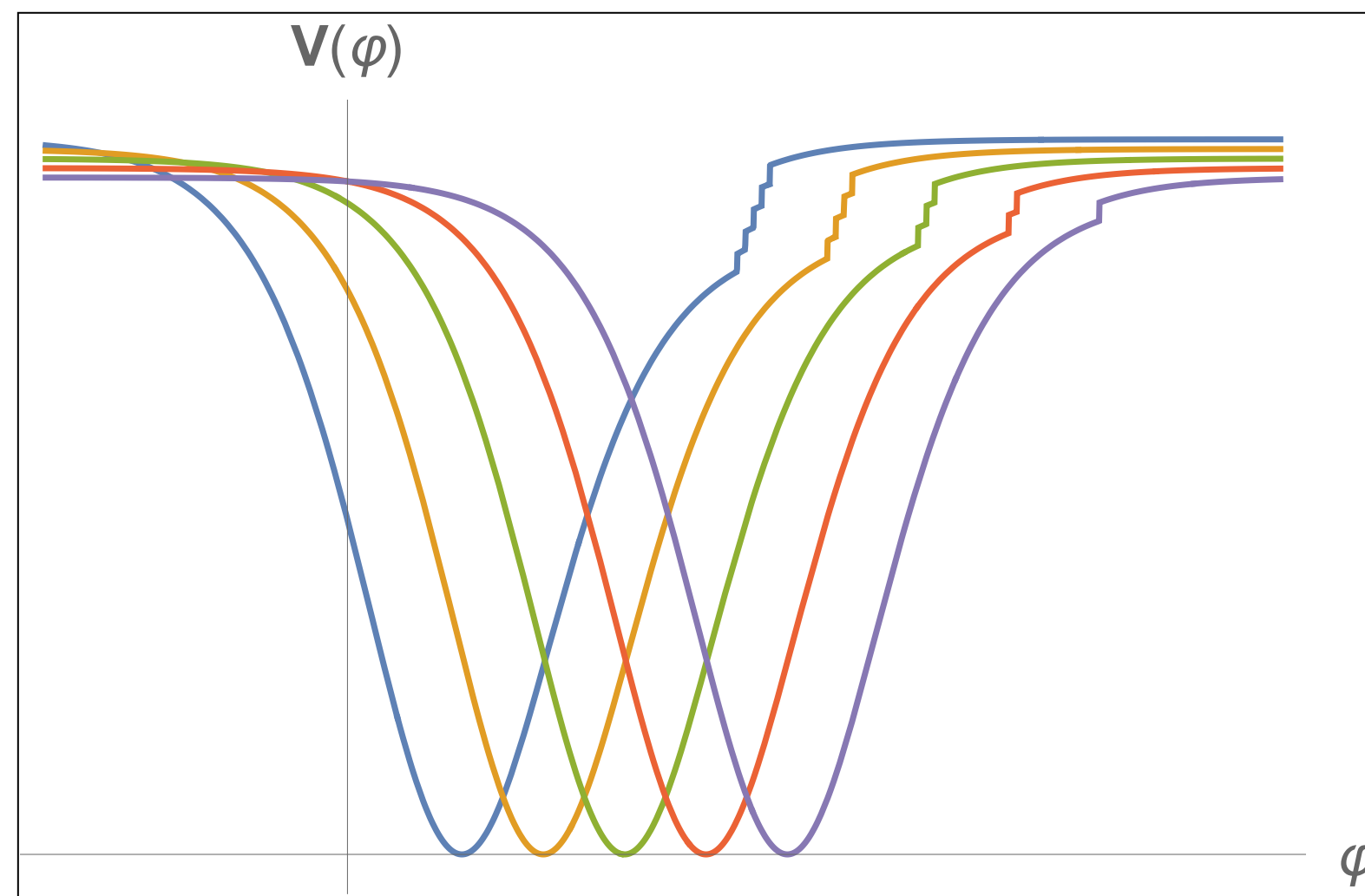
- A sharp drop in the potential of the inflaton field *detunes the relative phase of the real and imaginary parts of the curvature perturbation*, so that oscillations in the amplitude of the spectrum appear



Steps in the inflaton potential

Kefala + Kodaxis + Stamou + Tetradis *Phys.Rev.D* (2021)

ID + Kodaxis + Stamou + Tetradis + Tsigkas-Kouvelis, *Phys.Rev.D* (2021)

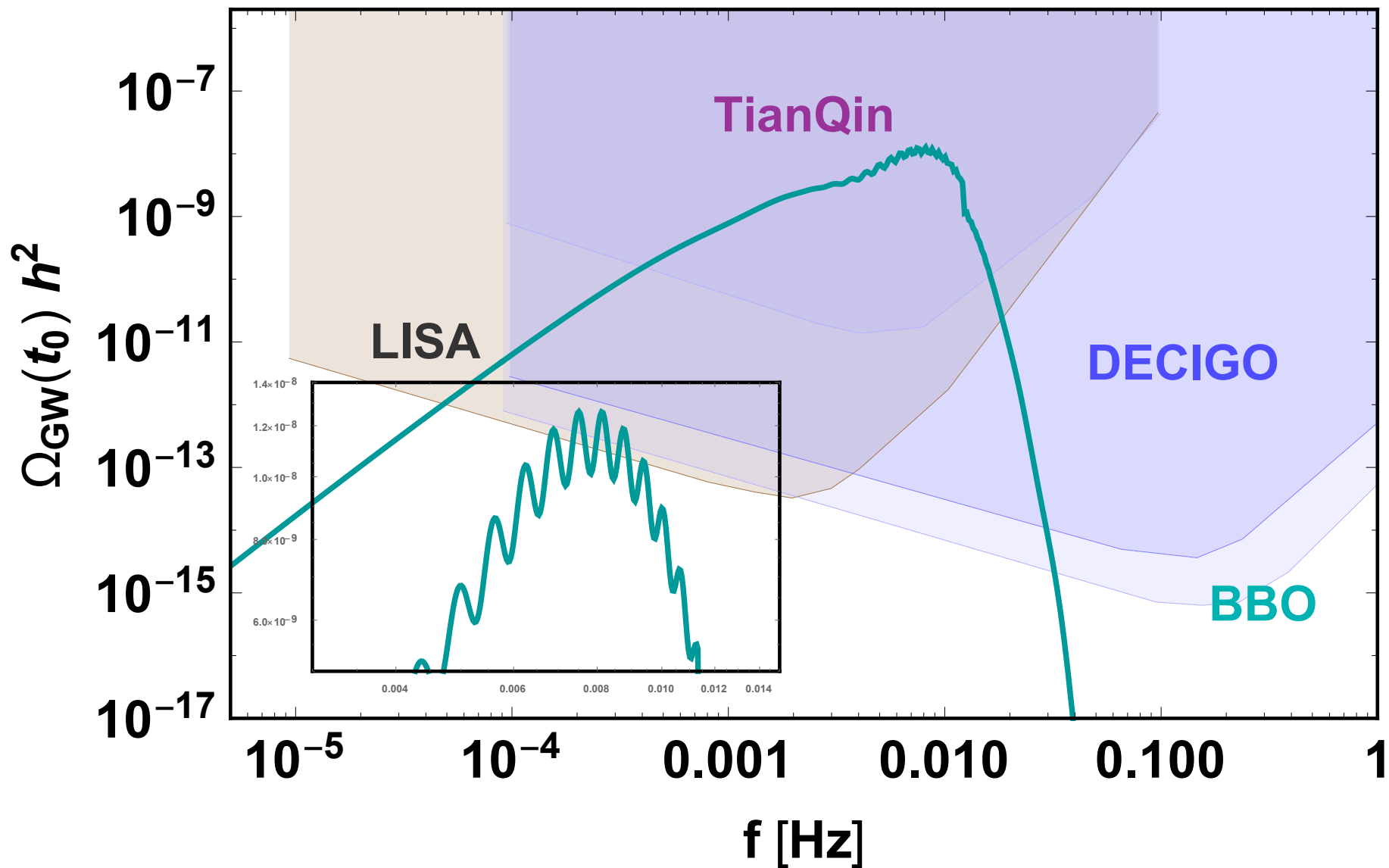
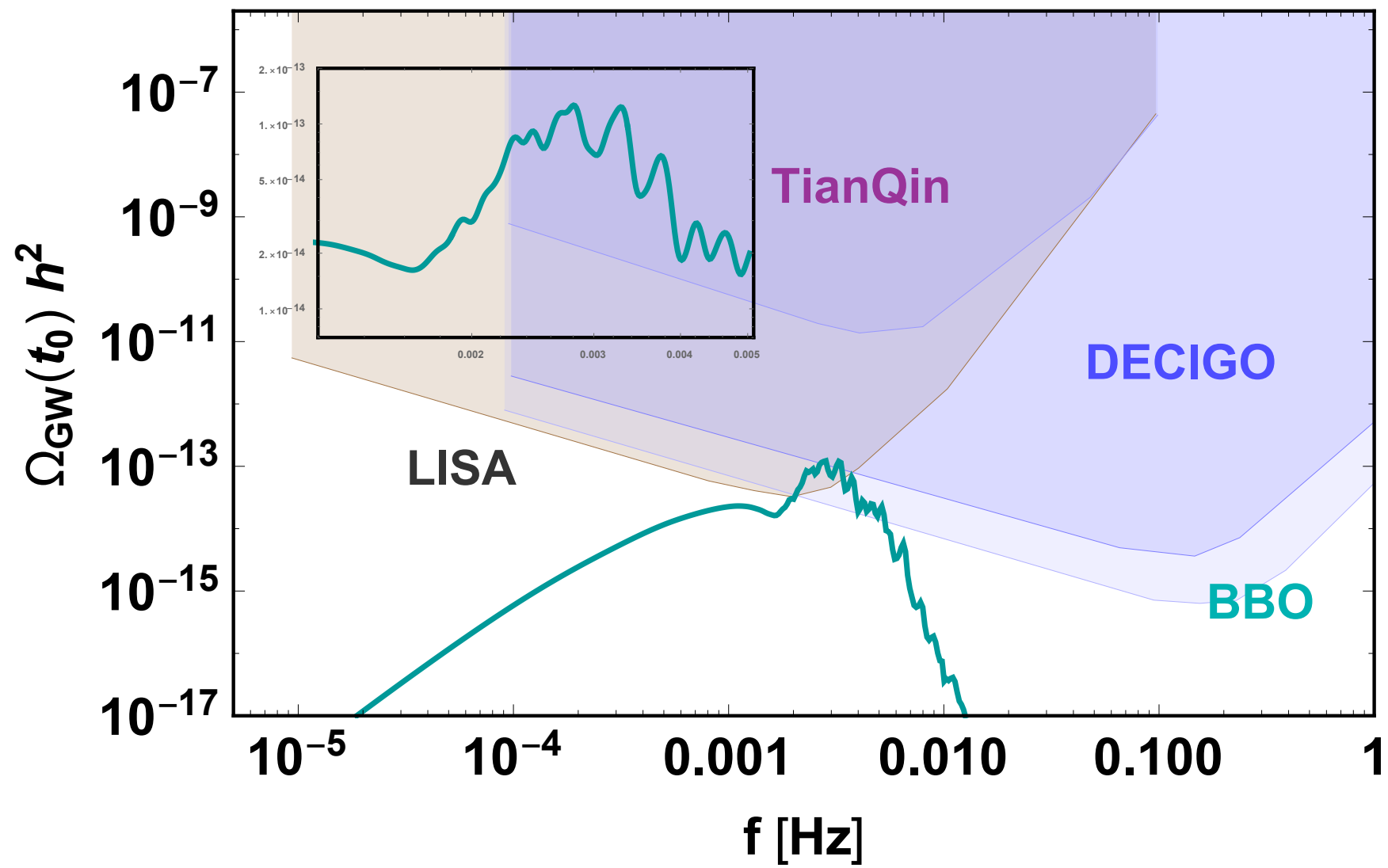
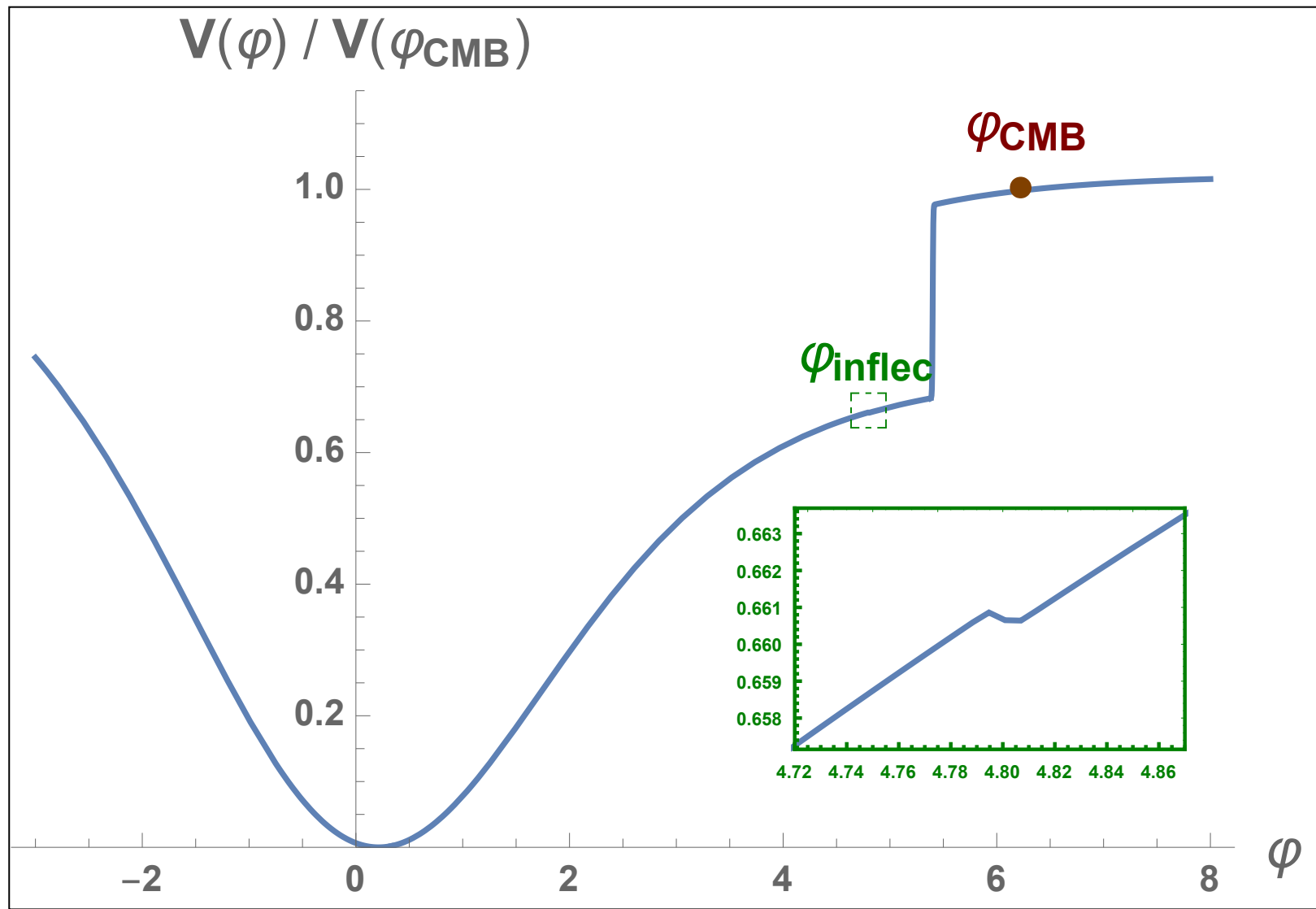
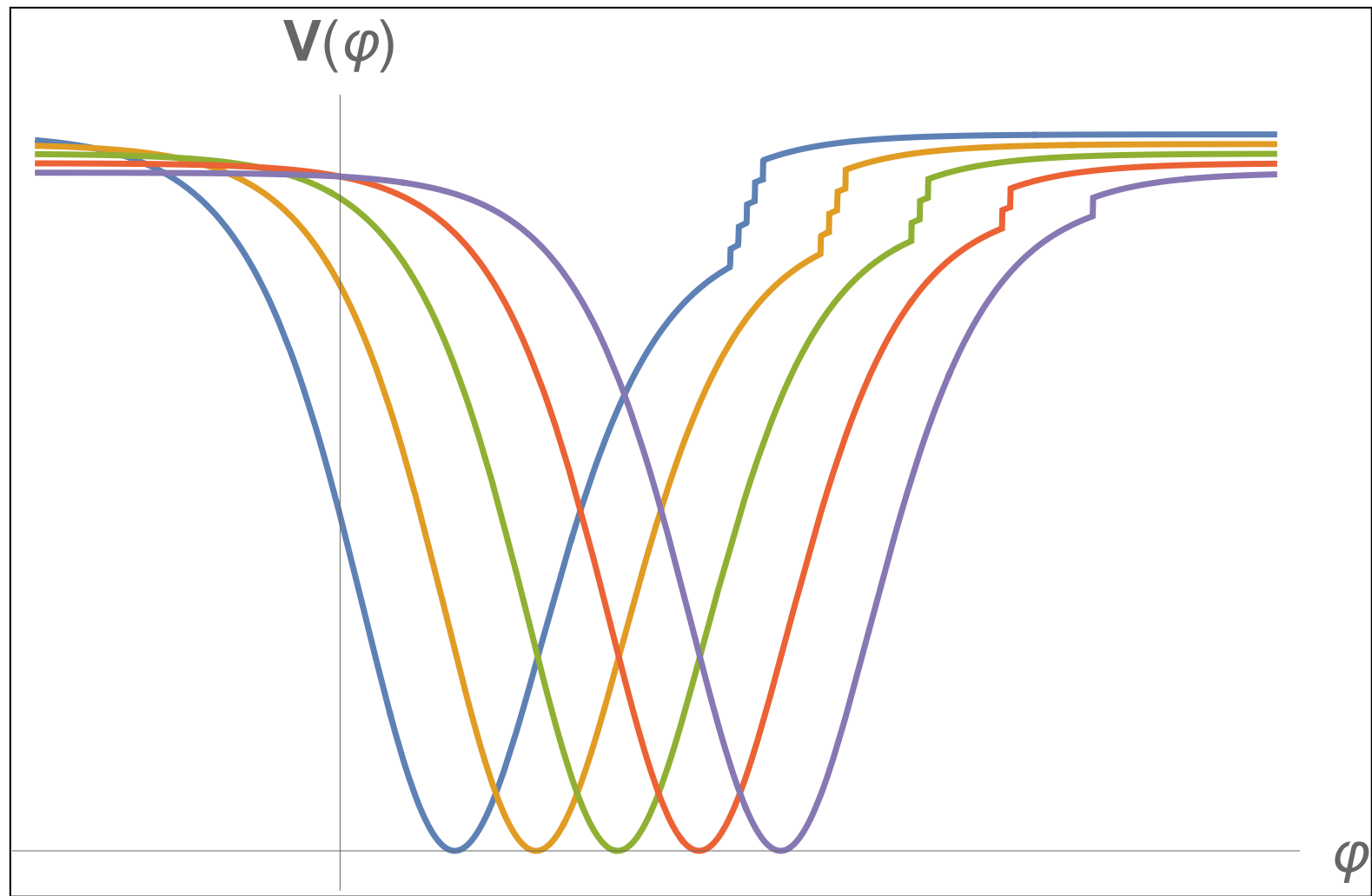


$$f(N) = 3 + \varepsilon_H - 2\eta_H$$

Steps in the inflaton potential

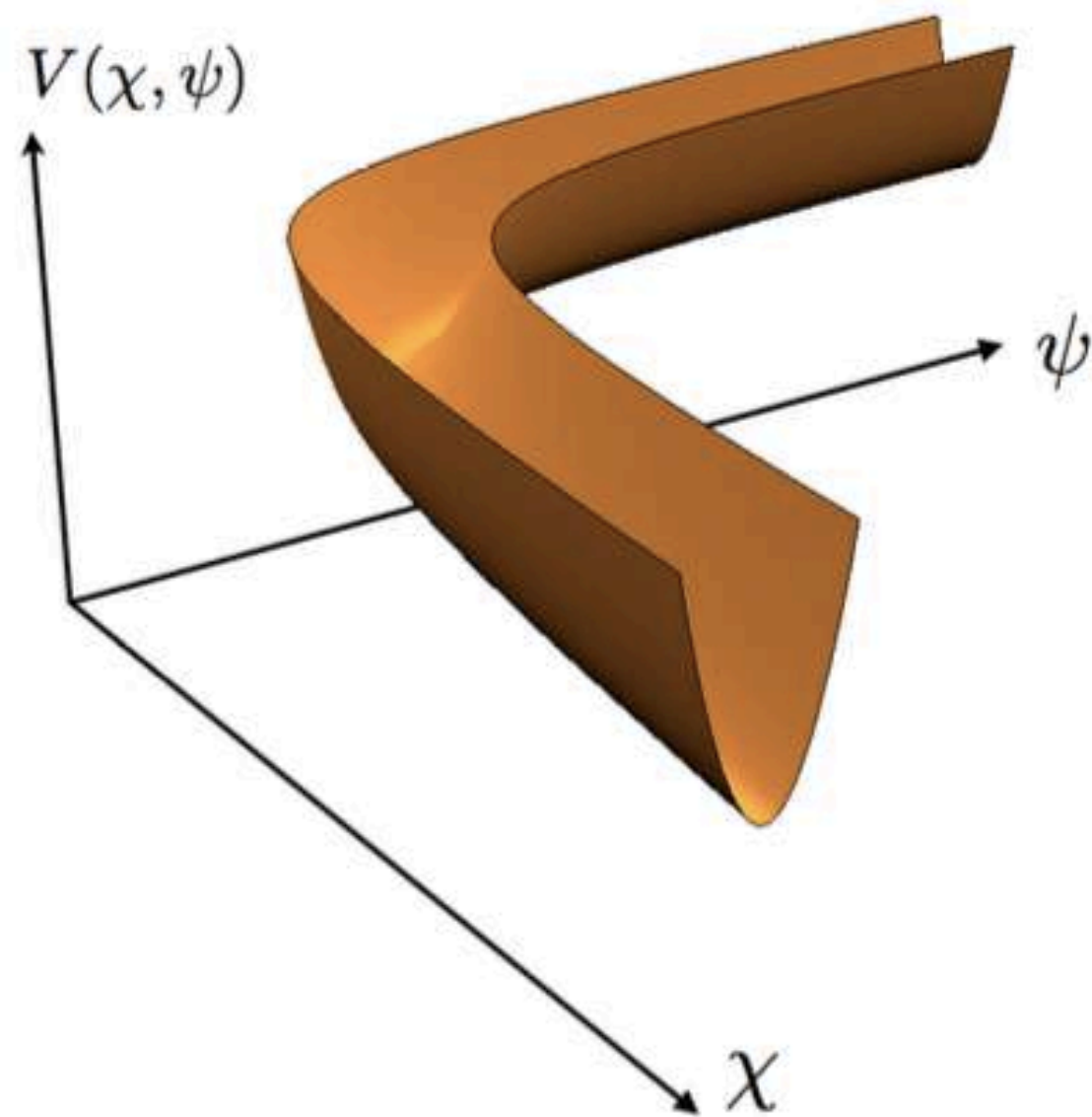
Kefala + Kodaxis + Stamou + Tetradis *Phys.Rev.D* (2021)

ID + Kodaxis + Stamou + Tetradis + Tsigkas-Kouvelis, *Phys.Rev.D* (2021)



Turns in the inflaton potential

Cespedes + Atal + Palma JCAP (2012)



$$\mathcal{R}_{k,NN} + 3\mathcal{R}_{k,N} + \frac{k^2}{H^2}e^{-2N}\mathcal{R}_k = -2(\eta_{\perp}\mathcal{S}_{k,N} + \eta_{\perp,N}\mathcal{S}_k + 3\eta_{\perp}\mathcal{S}_k)$$

$$\mathcal{S}_{k,NN} + 3\mathcal{S}_{k,N} + \frac{k^2}{H^2}e^{-2N}\mathcal{S}_k + \frac{M^2}{H^2}\mathcal{S}_k = 2\eta_{\perp}\mathcal{R}_{k,N}$$

- The inflationary phase is implemented by *more than one fields*.
- Multi-field inflation models can describe well the cosmological data of primordial perturbations, assuming that the latter make their multi-field dynamics *manifest at scales smaller than those directly observed in the CMB sky*.
- The phenomenology of interest in multi-field inflation models is related to the fact that the *curvature perturbations can evolve even on super-Hubble scales* because of the presence of isocurvature perturbations called also entropy perturbations or non-adiabatic pressure perturbations.

Starobinsky+Yokoyama, Sasaki+Stewart, Bellido+Wands (1995), ...

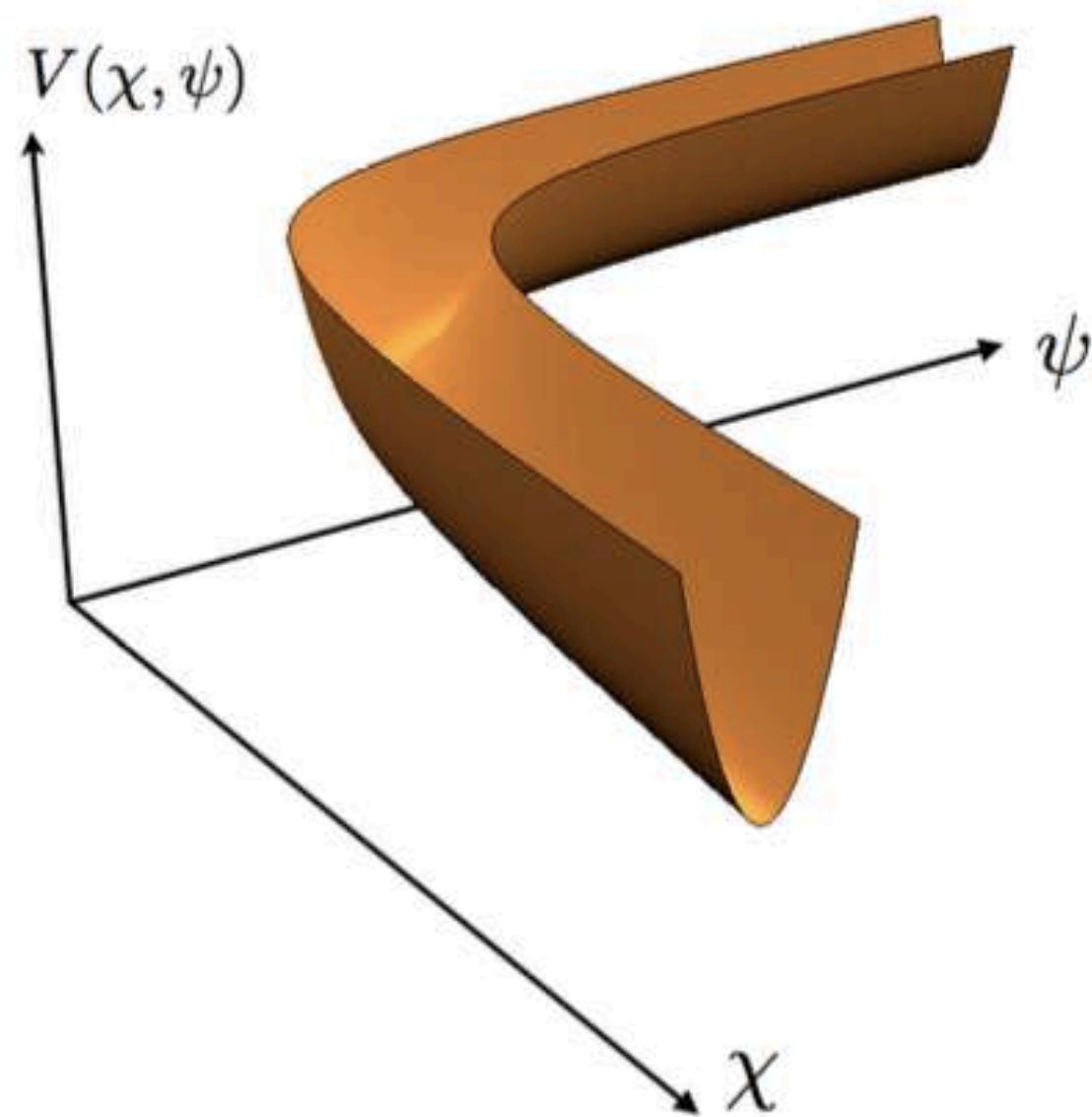
- In particular inflationary set-ups, the evolution of the curvature perturbations triggered by isocurvature modes can be *dramatic*, generating an observable GW signal and potentially a significant PBH abundance.

Palma+Sypsas+Zenteno (2020), Fumagulli+Renaux-Petel+Witkowski (2021), Braglia+Chen+Hazra (2021)

Turns in the inflaton potential

Boutivas+ ID + Kodaxis + Tetradis, (2022) to appear in JCAP

Cespedes + Atal + Palma JCAP (2012)



$$\mathcal{R}_{k,NN} + 3\mathcal{R}_{k,N} + \frac{k^2}{H^2}e^{-2N}\mathcal{R}_k = -2(\eta_{\perp}\mathcal{S}_{k,N} + \eta_{\perp,N}\mathcal{S}_k + 3\eta_{\perp}\mathcal{S}_k)$$

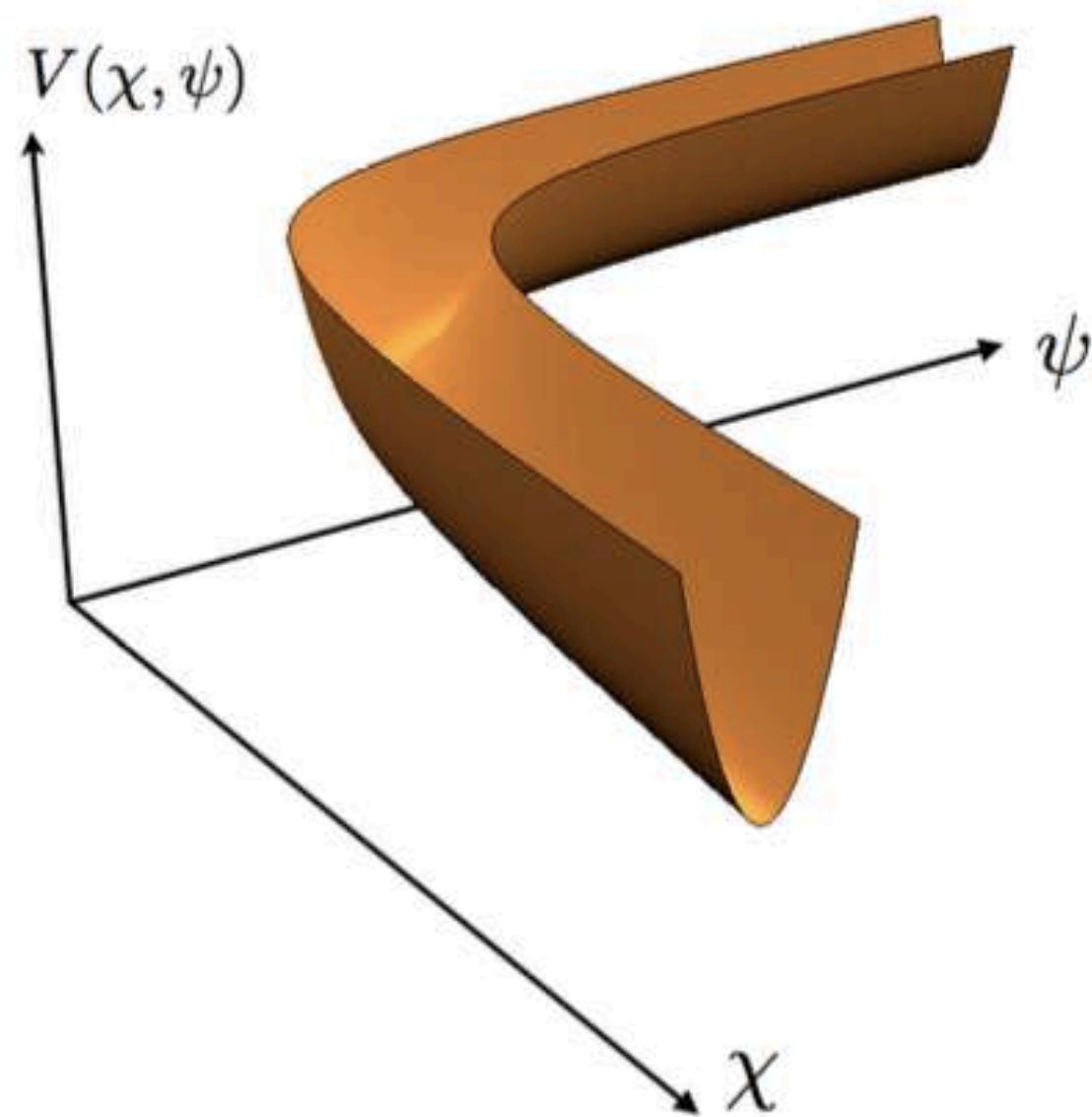
$$\mathcal{S}_{k,NN} + 3\mathcal{S}_{k,N} + \frac{k^2}{H^2}e^{-2N}\mathcal{S}_k + \frac{M^2}{H^2}\mathcal{S}_k = 2\eta_{\perp}\mathcal{R}_{k,N}$$

- We studied the resonant effect that may occur when there are several instances *with strong deviations from approximate scale invariance* during the inflationary evolution.
 - We did not focus on a particular model, but looked instead for *generic properties* of the equations of motion for the perturbations which would lead to their enhancement.
 - We identified *the slow-roll parameter η* as the quantity that can trigger the rapid growth of perturbations.
 - η can be projected onto the directions parallel and perpendicular to the trajectory of the background fields.
 - η_{\perp} grows large during sharp turns in field space.
- ★ We assume that $M \gtrsim H$, so that the isocurvature perturbations are suppressed apart from short periods during which the parameter η_{\perp} becomes large.

Turns in the inflaton potential

Boutivas+ ID + Kodaxis + Tetradis, (2022) to appear in JCAP

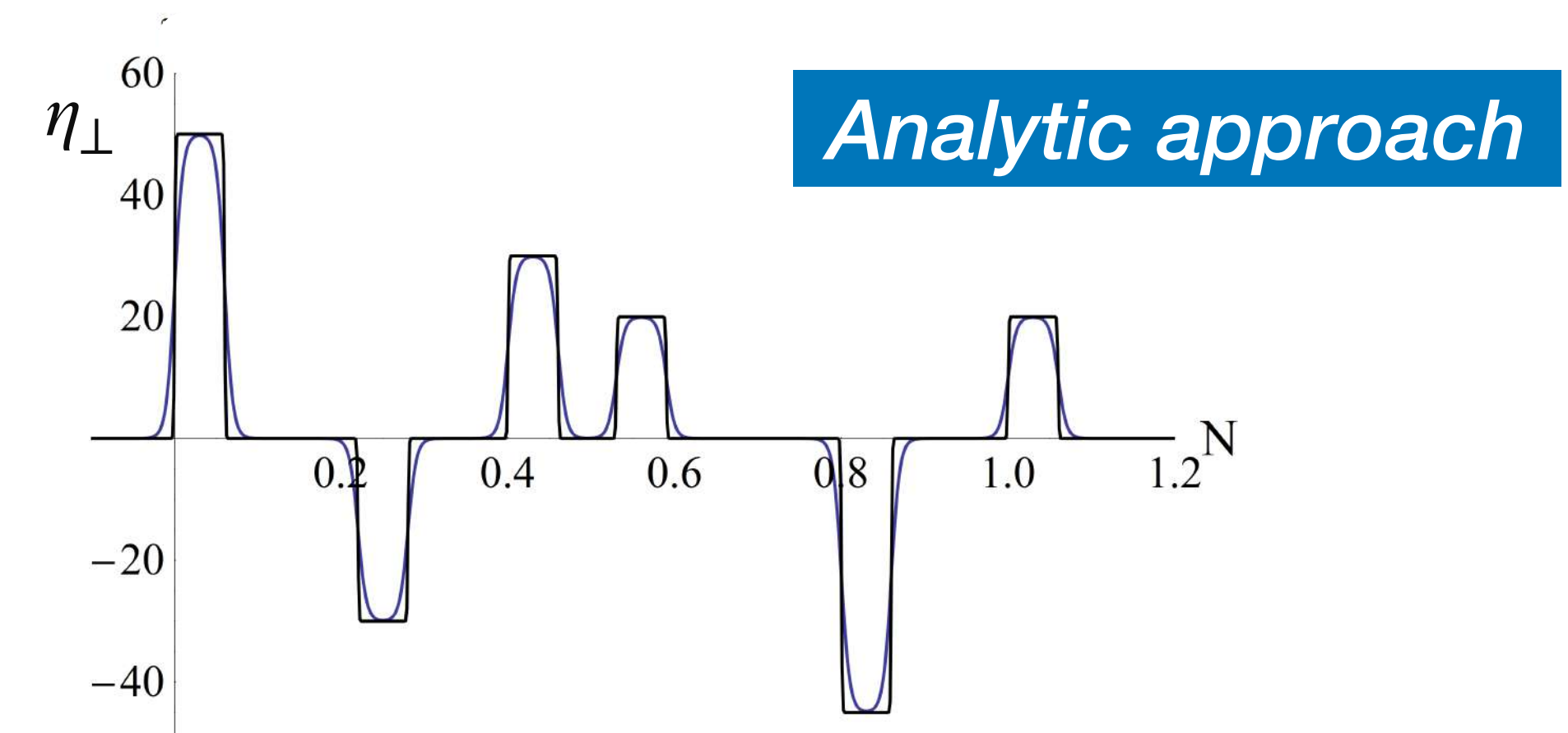
Cespedes + Atal + Palma JCAP (2012)



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$$\mathcal{R}_{k,NN} + 3\mathcal{R}_{k,N} + \frac{k^2}{H^2}e^{-2N}\mathcal{R}_k = -2(\eta_{\perp}\mathcal{S}_{k,N} + \eta_{\perp,N}\mathcal{S}_k + 3\eta_{\perp}\mathcal{S}_k)$$

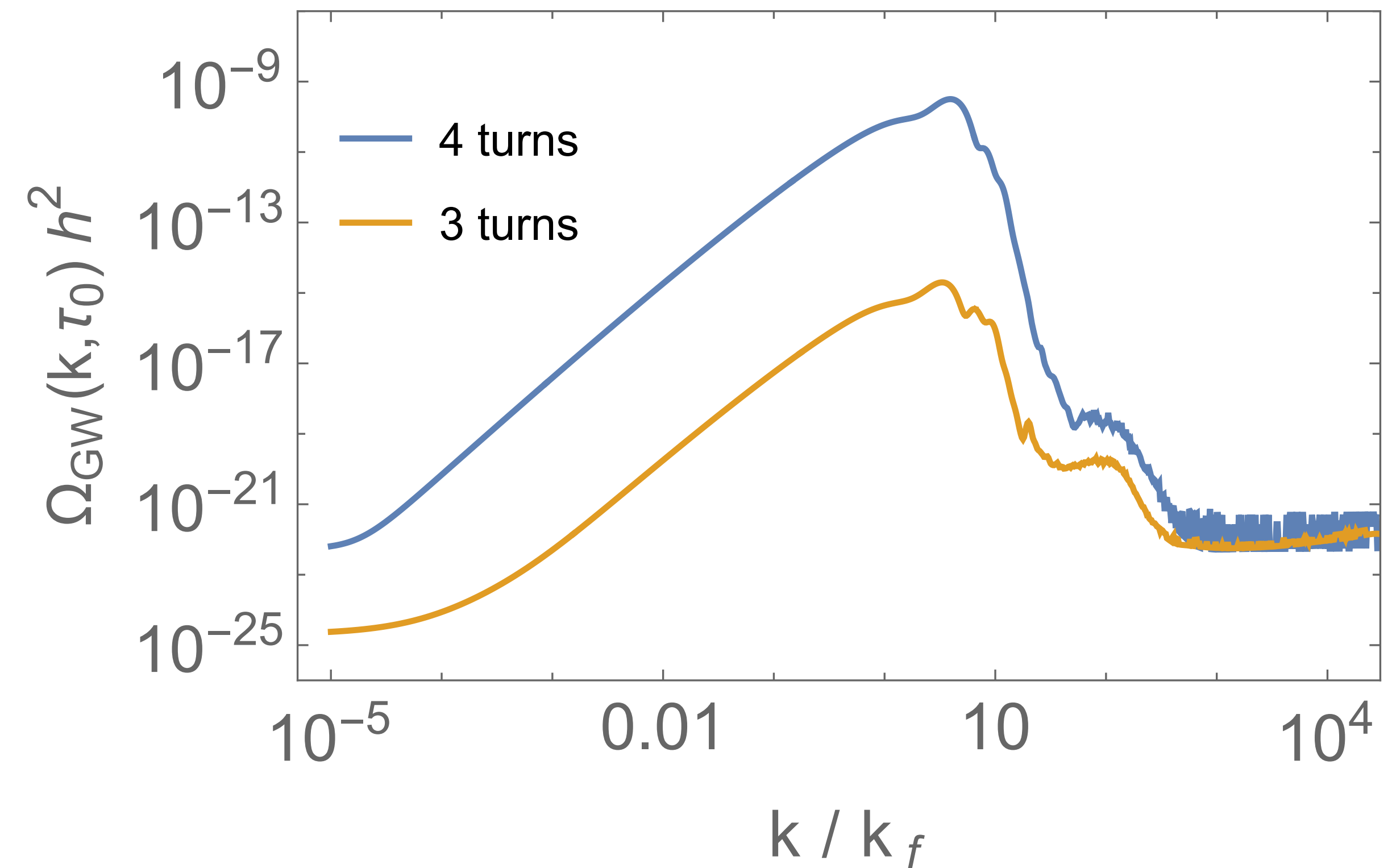
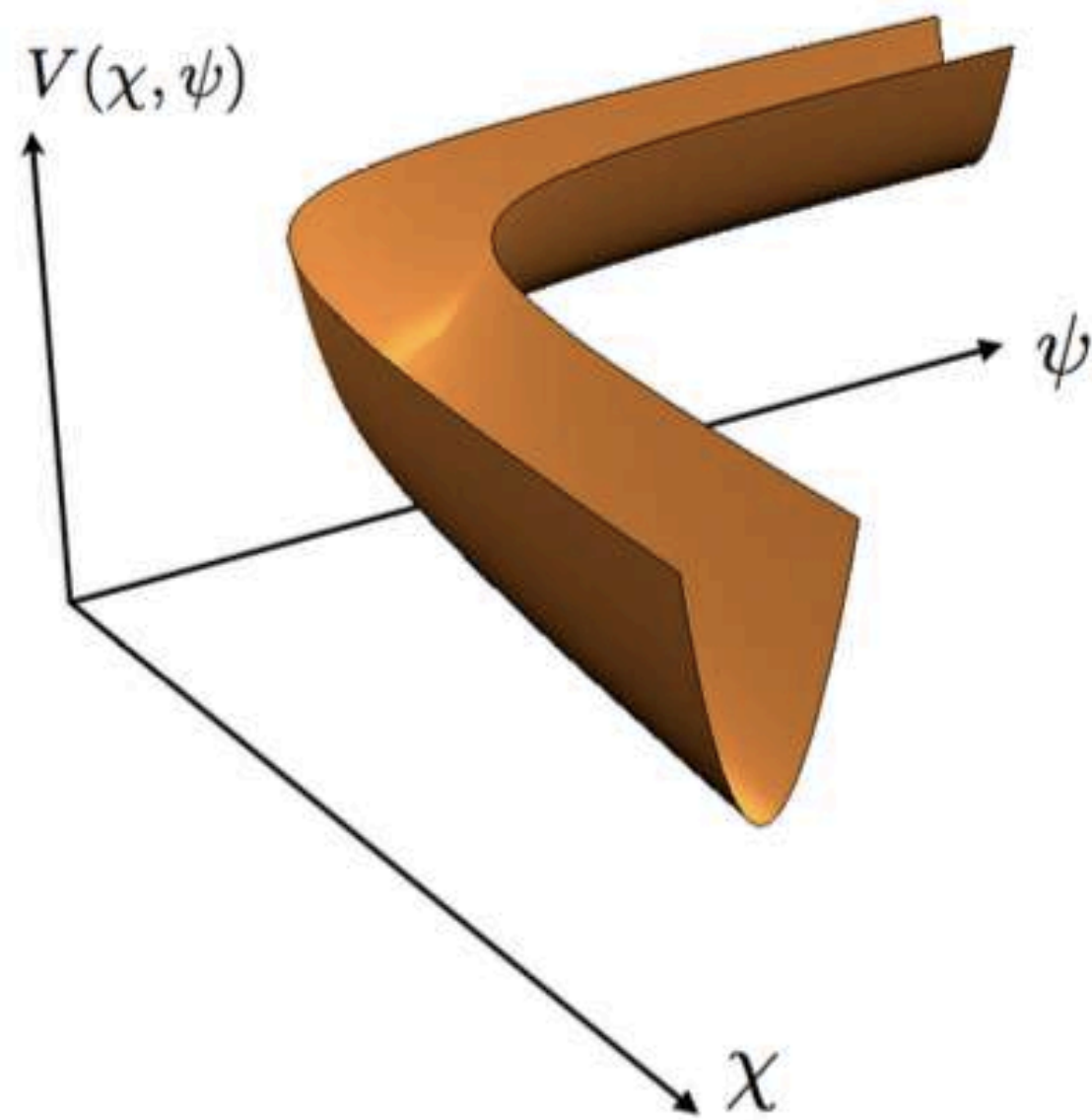
$$\mathcal{S}_{k,NN} + 3\mathcal{S}_{k,N} + \frac{k^2}{H^2}e^{-2N}\mathcal{S}_k + \frac{M^2}{H^2}\mathcal{S}_k = 2\eta_{\perp}\mathcal{R}_{k,N}$$



Turns in the inflaton potential

Boutivas+ ID + Kodaxis + Tetradis, (2022) to appear in JCAP

The spectrum of the induced GWs



$$\mathcal{R}_{k,NN} + 3\mathcal{R}_{k,N} + \frac{k^2}{H^2} e^{-2N} \mathcal{R}_k = -2(\eta_{\perp} \mathcal{S}_{k,N} + \eta_{\perp,N} \mathcal{S}_k + 3\eta_{\perp} \mathcal{S}_k)$$

$$\mathcal{S}_{k,NN} + 3\mathcal{S}_{k,N} + \frac{k^2}{H^2} e^{-2N} \mathcal{S}_k + \frac{M^2}{H^2} \mathcal{S}_k = 2\eta_{\perp} \mathcal{R}_{k,N}$$

What if the deviation from scale invariance happens at the very last stage of inflation?

Runaway inflationary models and mini PBHs

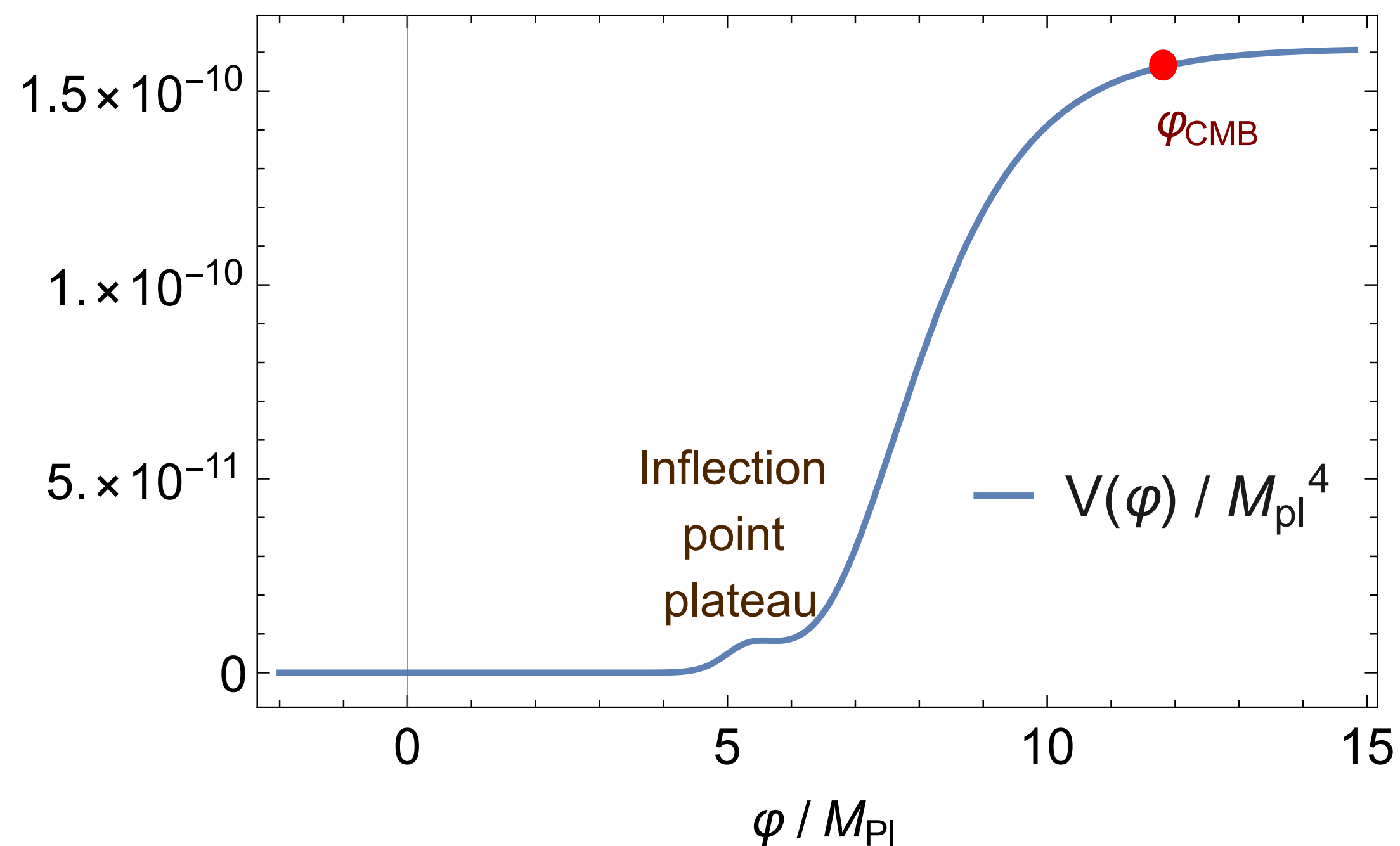
ID+ Tringas, *Phys.Rev.D* (2019)

ID+ Kodaxis, *Galaxies* (2022)

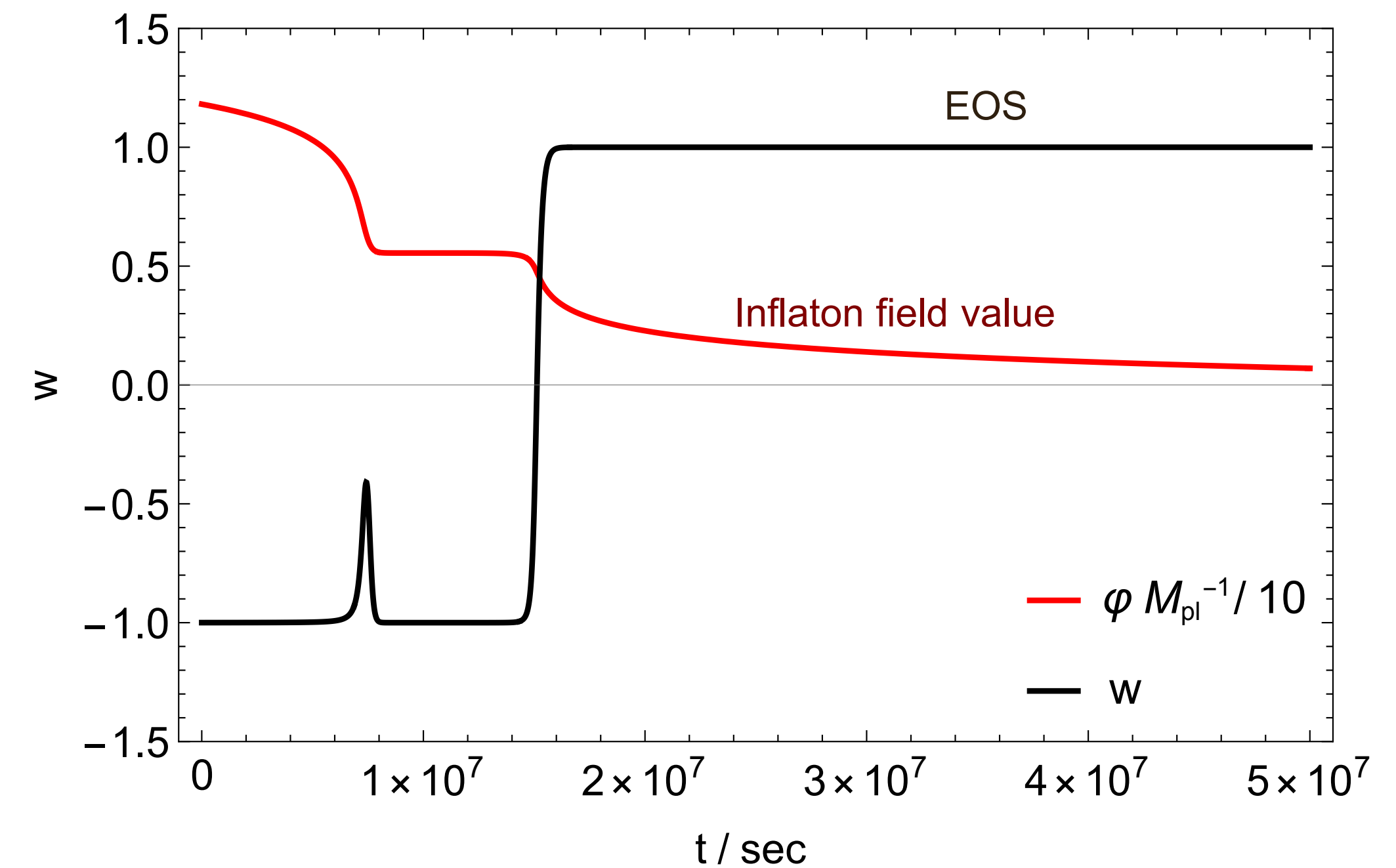
4

and mini black holes that evaporate promptly

$$V(\varphi) = f_0^2 \left[c_0 + c_1 e^{\lambda_1 \tanh \varphi / \sqrt{6}} + c_2 e^{\lambda_2 \left(\tanh(\varphi / \sqrt{6}) - \tanh(\varphi_P / \sqrt{6}) \right)} \right]^2$$



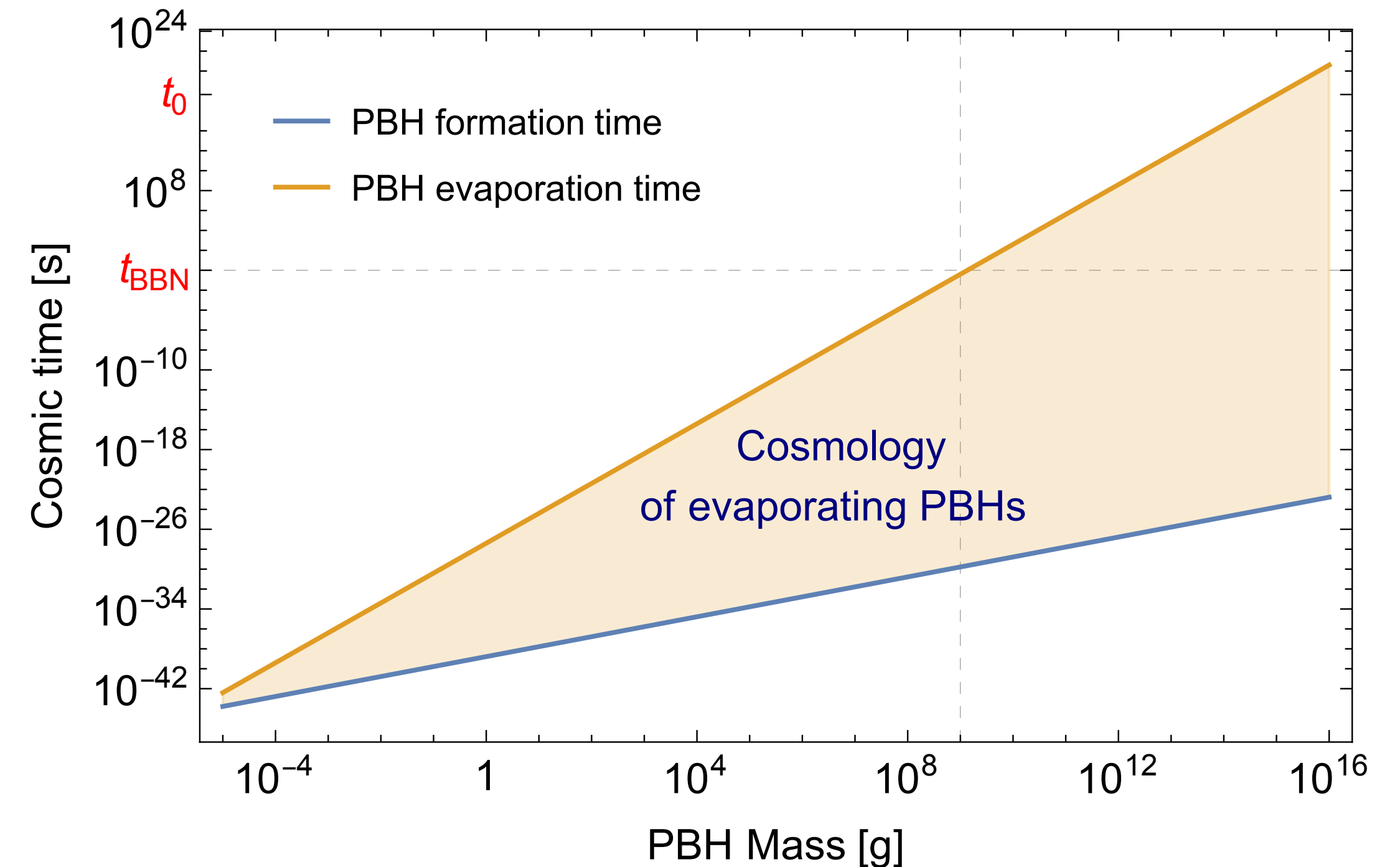
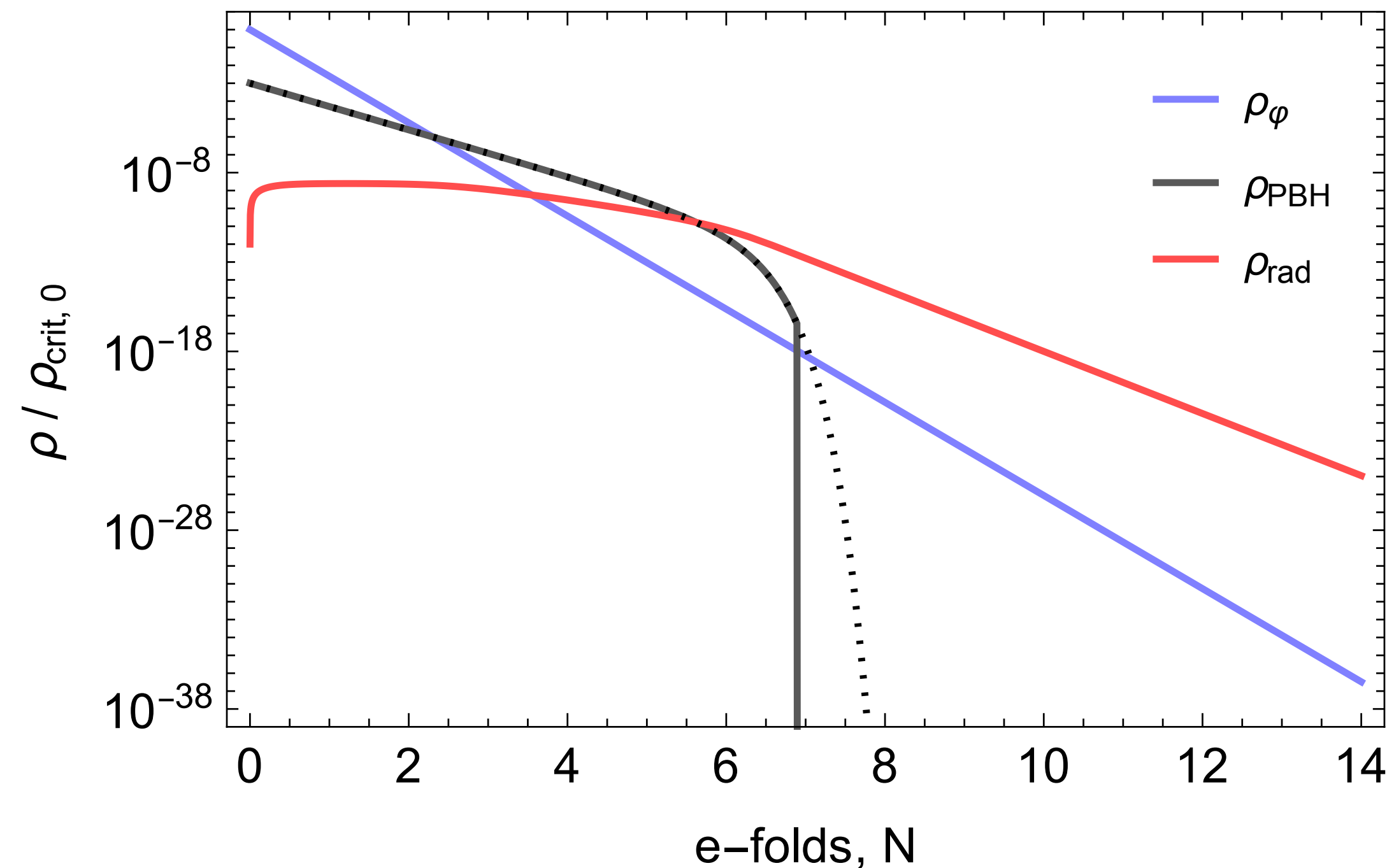
- Mini PBHs evaporate promptly and **reheat** the early universe. *Barrow+Copeland+Kolb+Liddle (1991)*
- PBH evaporation **remnants present in galaxies** ?
- A non-zero residual potential energy plays the role of the **dark energy**? *Dimopoulos + Owen (2017)*
Dimopoulos + Donaldson + Owen (2018)



and mini black holes that evaporate promptly

$$V(\varphi) = f_0^2 \left[c_0 + c_1 e^{\lambda_1 \tanh \varphi / \sqrt{6}} + c_2 e^{\lambda_2 \left(\tanh(\varphi / \sqrt{6}) - \tanh(\varphi_P / \sqrt{6}) \right)} \right]^2$$

- Mini PBHs evaporate promptly and **reheat** the early universe.
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Dimopoulos + Donaldson + Owen (2018)

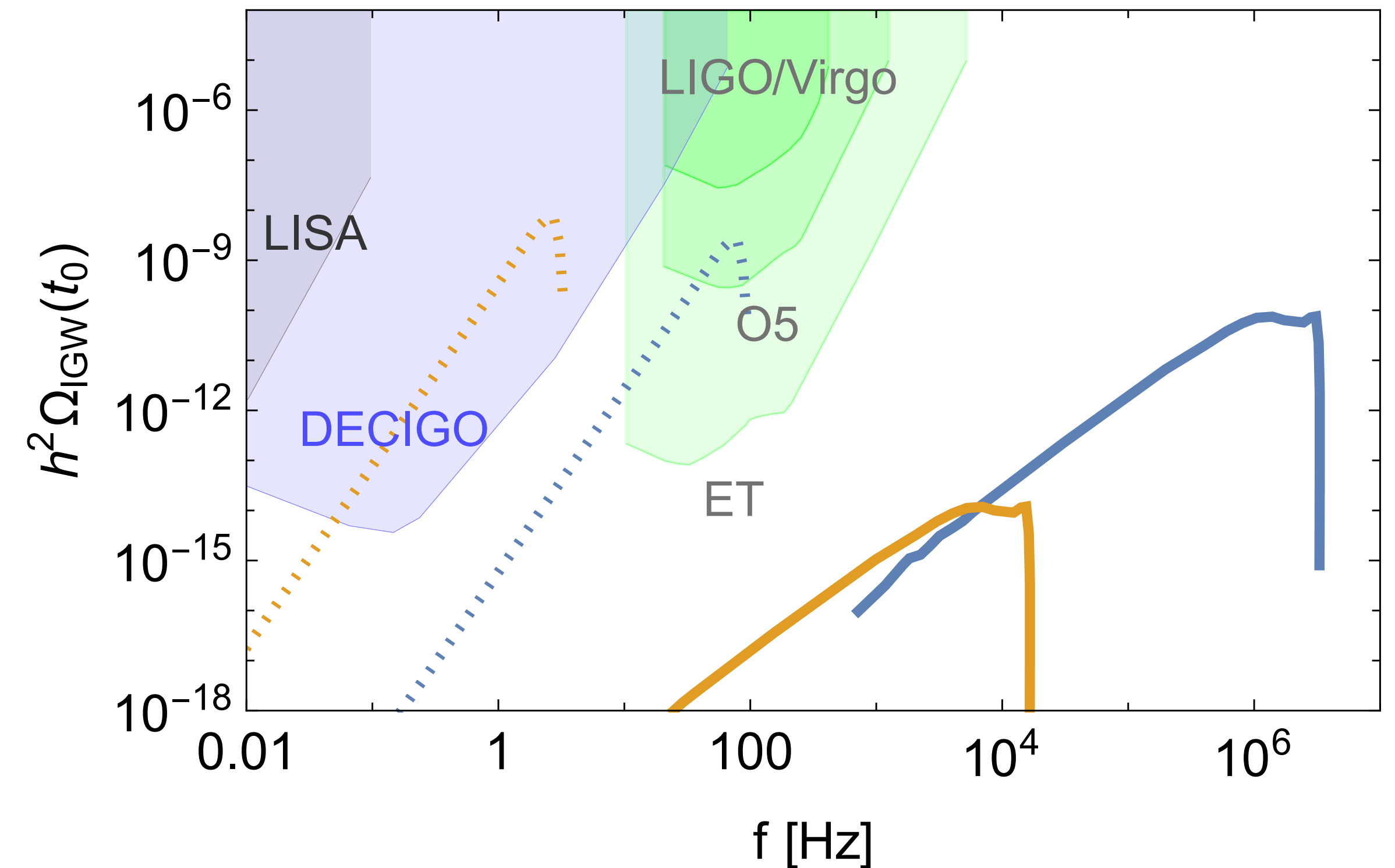
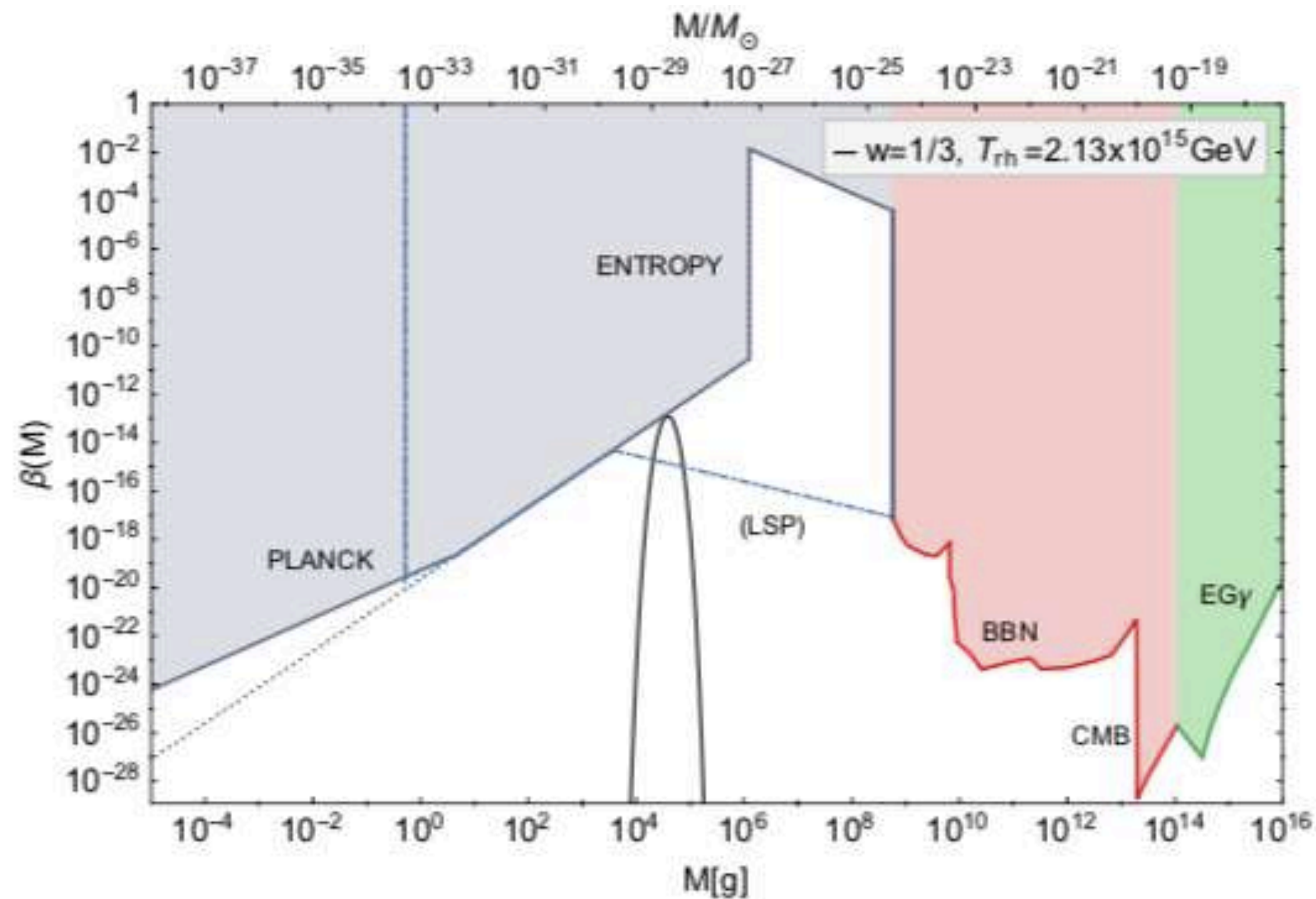


Runaway inflationary models and PBHs

and mini black holes that evaporate promptly

ID + Tringas, *Phys.Rev.D* (2019)

ID + Kodaxis, *Galaxies* (2022)



- A transit PBH domination phase is necessary due to GW constraints from CMB and BBN.
- A distinct prediction of the scenario is a **compound GW signal** that might be probed by current and future experiments.

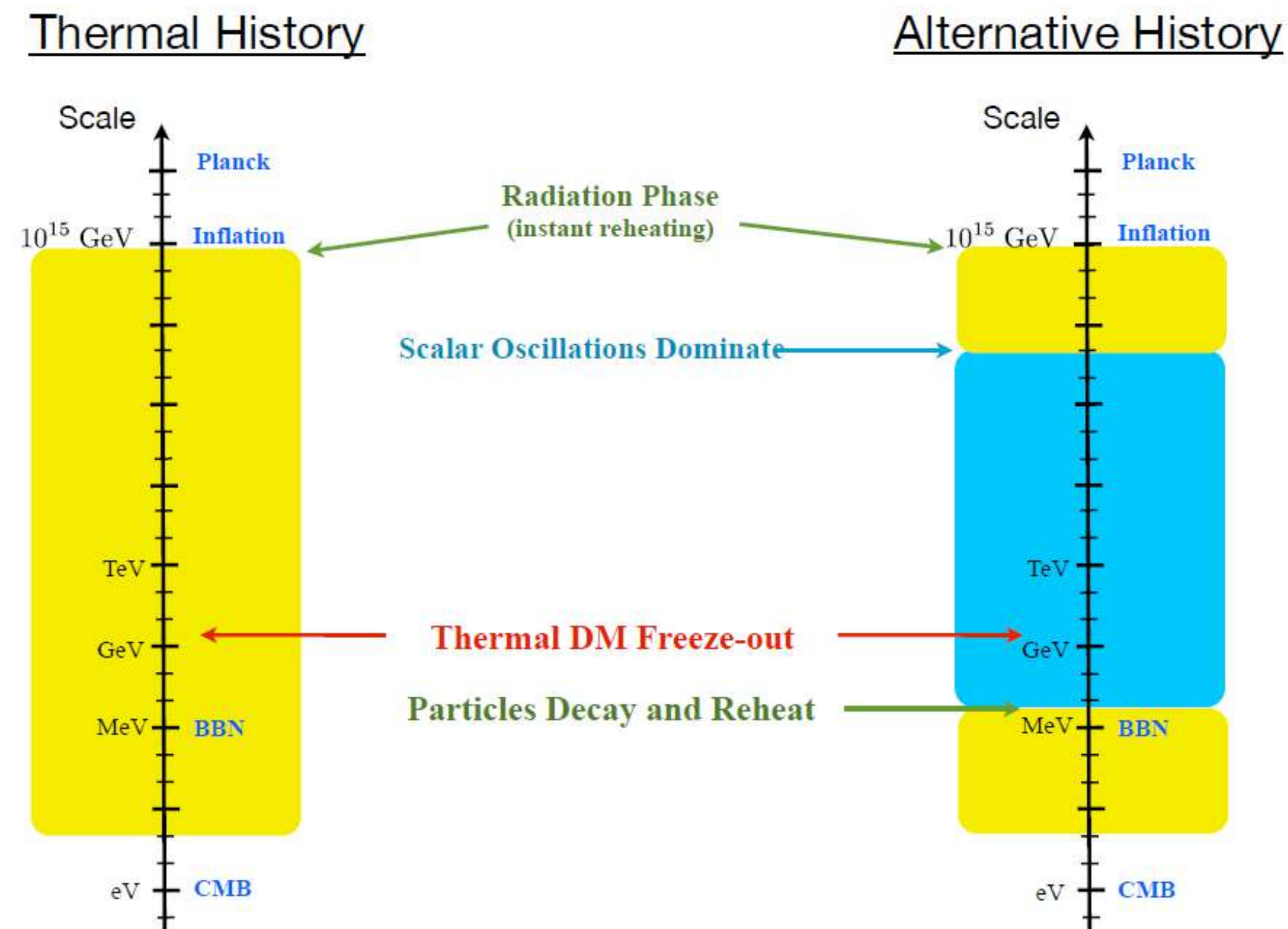
Papanikolaou+Vennin+Langlois,

Domenech+Lin+Sasaki (2021)

An early matter domination era (or late entropy production)?

Transit phases and induced GWs

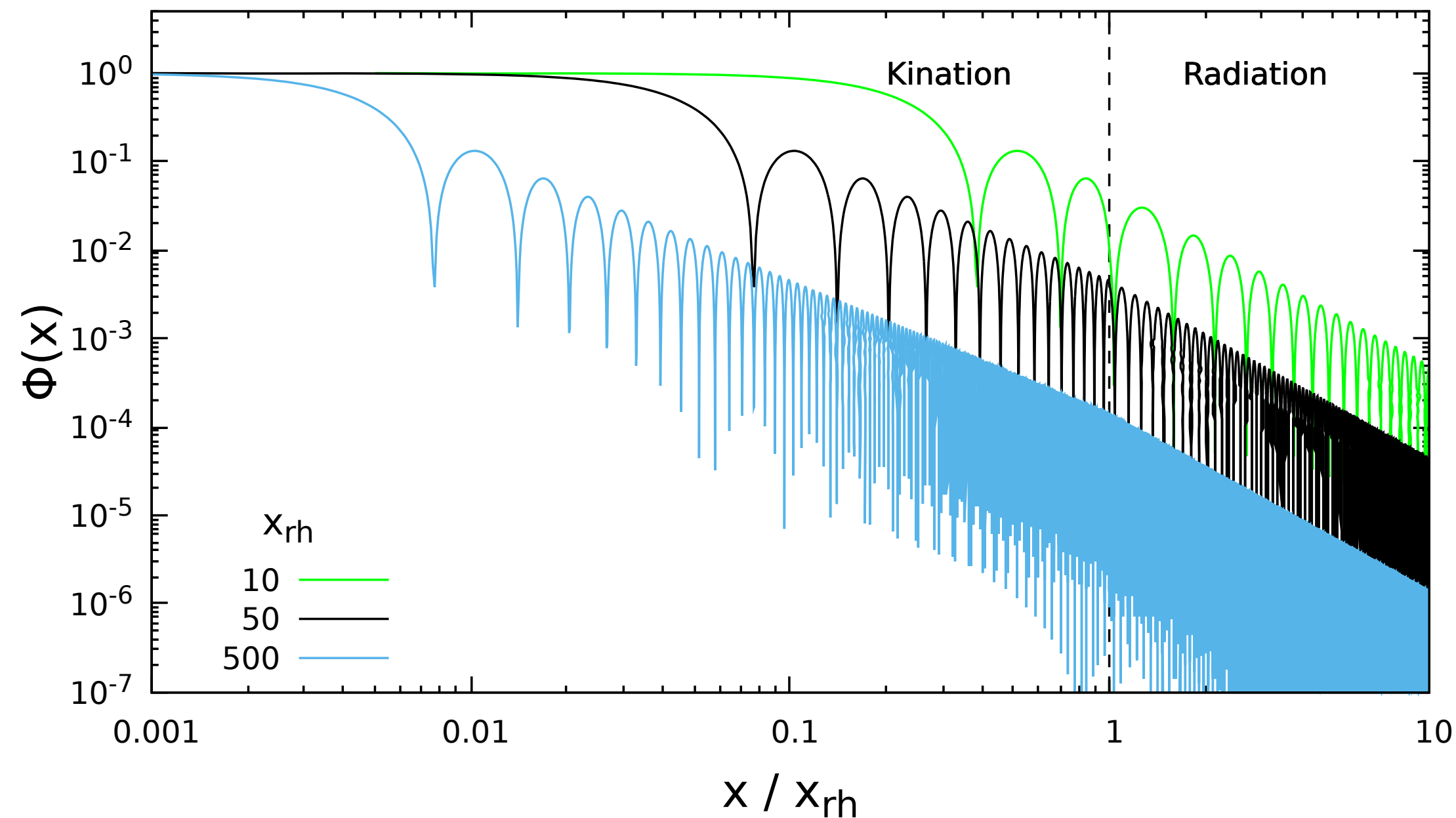
- *The secondary GWs and the Reheating Temperature or Late Entropy Production* (Several SUSY 2022 talks)



Transit phases and induced GWs

- The GW detection and the Reheating Temperature

ID + Kritos, Phys.Rev.D (2021)



$$\Phi''_k + \frac{6(1+w)}{1+3w} \frac{1}{\tau} \Phi'_k + w k^2 \Phi_k = 0$$

Kohri+Terada (2018)

Inomata+Kohri+Nakama+Terada (2019)

$$h''_k(\tau) + 2\mathcal{H}(\tau)h'_k(\eta) + k^2 h_k(\tau) = \mathcal{S}_k(\tau)$$

$$\mathcal{S}_k(\eta) \sim \int \frac{d^3 q}{(2\pi)^{3/2}} e^{ij(k)} q_i q_j \left(\Phi_k \Phi_{k-q} + \frac{\Phi'_q \Phi_{k-q} + \dots}{3(1+w)/4} \right)$$

For $w = 0 \rightarrow \Phi''_k + \frac{6}{\tau} \Phi'_k = 0 \Rightarrow \Phi = \text{σταθερό}$

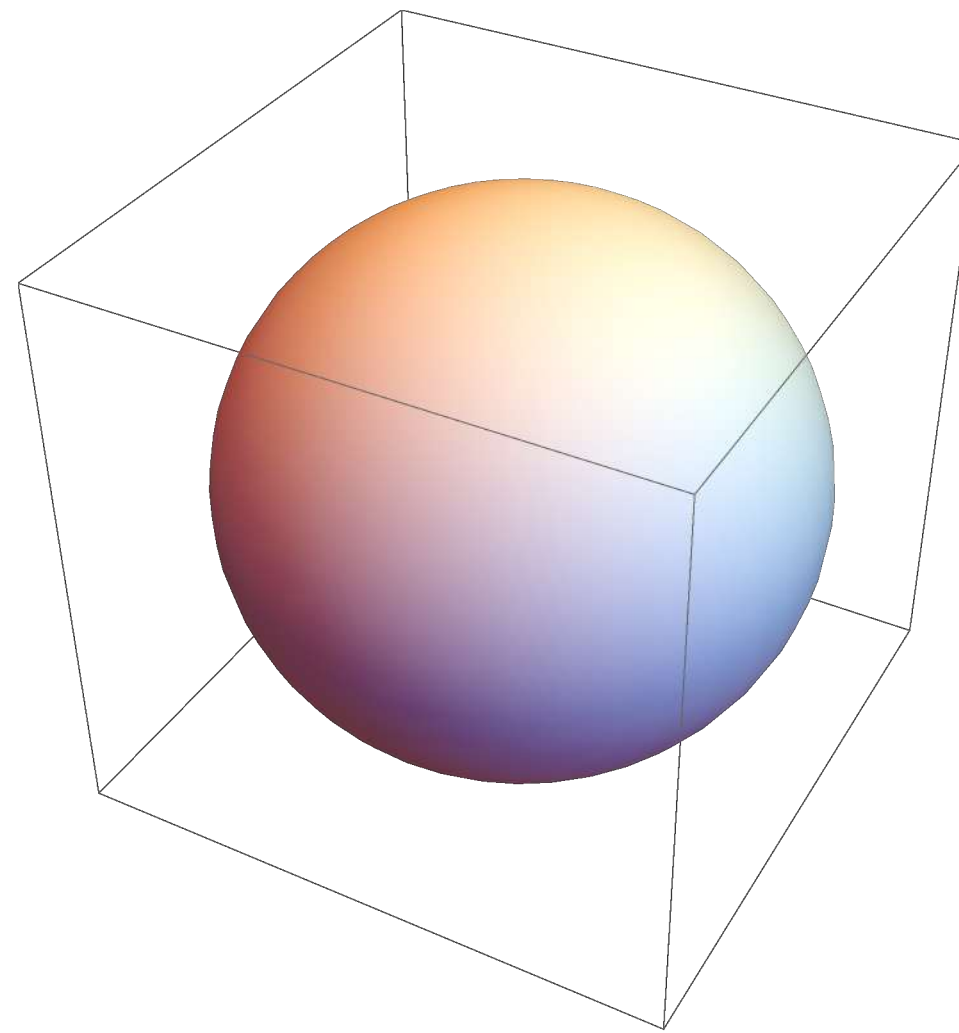
$$\delta\rho_m/\rho \sim 1 \Rightarrow k_{NL}(\tau) \sim \sqrt{3} \frac{\mathcal{H}(\tau)}{\mathcal{P}_\Phi^{1/4}}$$

There is threshold scale for the wavenumber that the perturbation analysis stops being valid

Y. B. Zeldovich (1970)
ID + Kouvaris JCAP 07 (2021)

- Zel'dovich have shown that there can be an approximation valid **well inside the nonlinear regime** (although eventually breaks down too).
- The Zel'dovich approximation describes the **nonlinear evolution of pressureless density perturbations** which deviate from the sphere.
- As the system evolves an initial deviation from the spherical shape becomes more pronounced as times goes on leading to flattened structures (**Zel'dovich pancakes**)
- This increasingly asymmetric evolution of the collapsing perturbation creates substantial quadrupole moment which in turn **creates GWs**.

$$\frac{dE_e}{dt} = \frac{G}{5c^5} \sum_{ij} \ddot{Q}_{ij} \ddot{Q}_{ji}(t)$$



Evolution and collapse of overdensities in eMD

$$r_1(t) = \frac{3}{2} t_q^{1/3} t^{2/3} \left(1 - \frac{1}{2} \left(\frac{t}{t_{\max}} \right)^{2/3} \right)$$

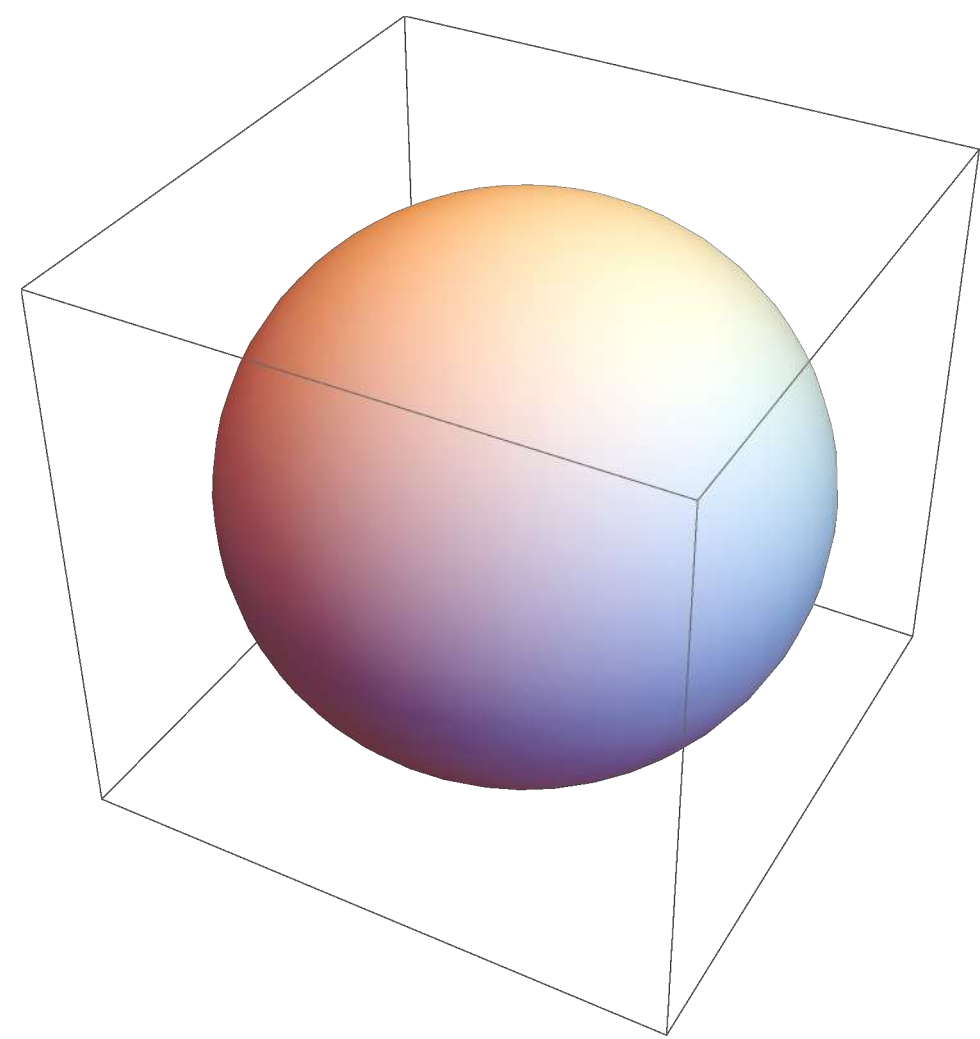
$$r_2(t) = \frac{3}{2} t_q^{1/3} t^{2/3} \left(1 - \frac{\beta}{2\alpha} \left(\frac{t}{t_{\max}} \right)^{2/3} \right)$$

$$r_3(t) = \frac{3}{2} t_q^{1/3} t^{2/3} \left(1 - \frac{\gamma}{2\alpha} \left(\frac{t}{t_{\max}} \right)^{2/3} \right)$$

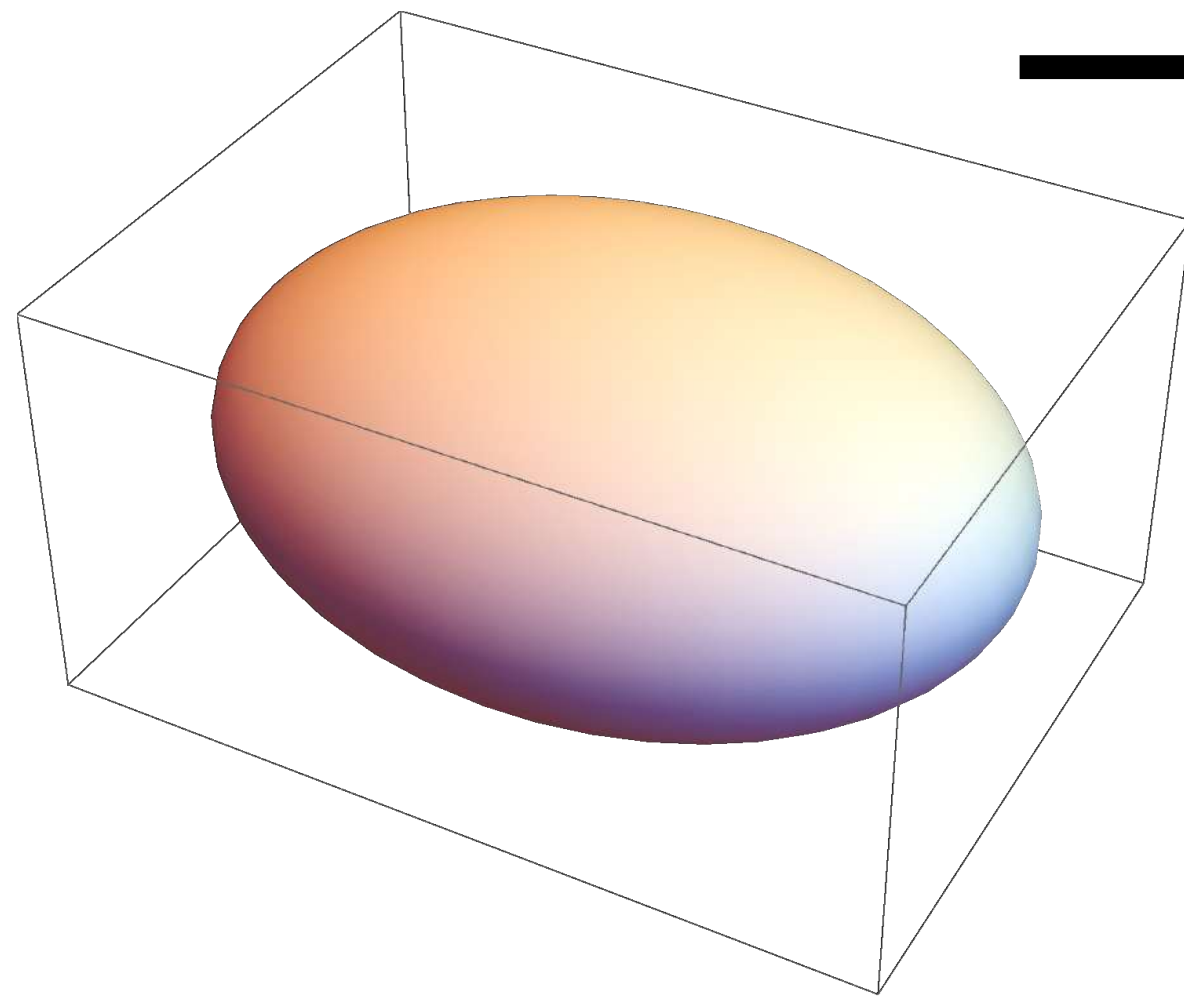
$$I_{ij} = \frac{1}{5} M \begin{pmatrix} r_2^2 + r_3^2 & 0 & 0 \\ 0 & r_1^2 + r_3^2 & 0 \\ 0 & 0 & r_1^2 + r_2^2 \end{pmatrix}$$

$$Q_{ij} = -I_{ij}(t) + \frac{1}{3} \delta_{ij} \text{Tr } I(t)$$

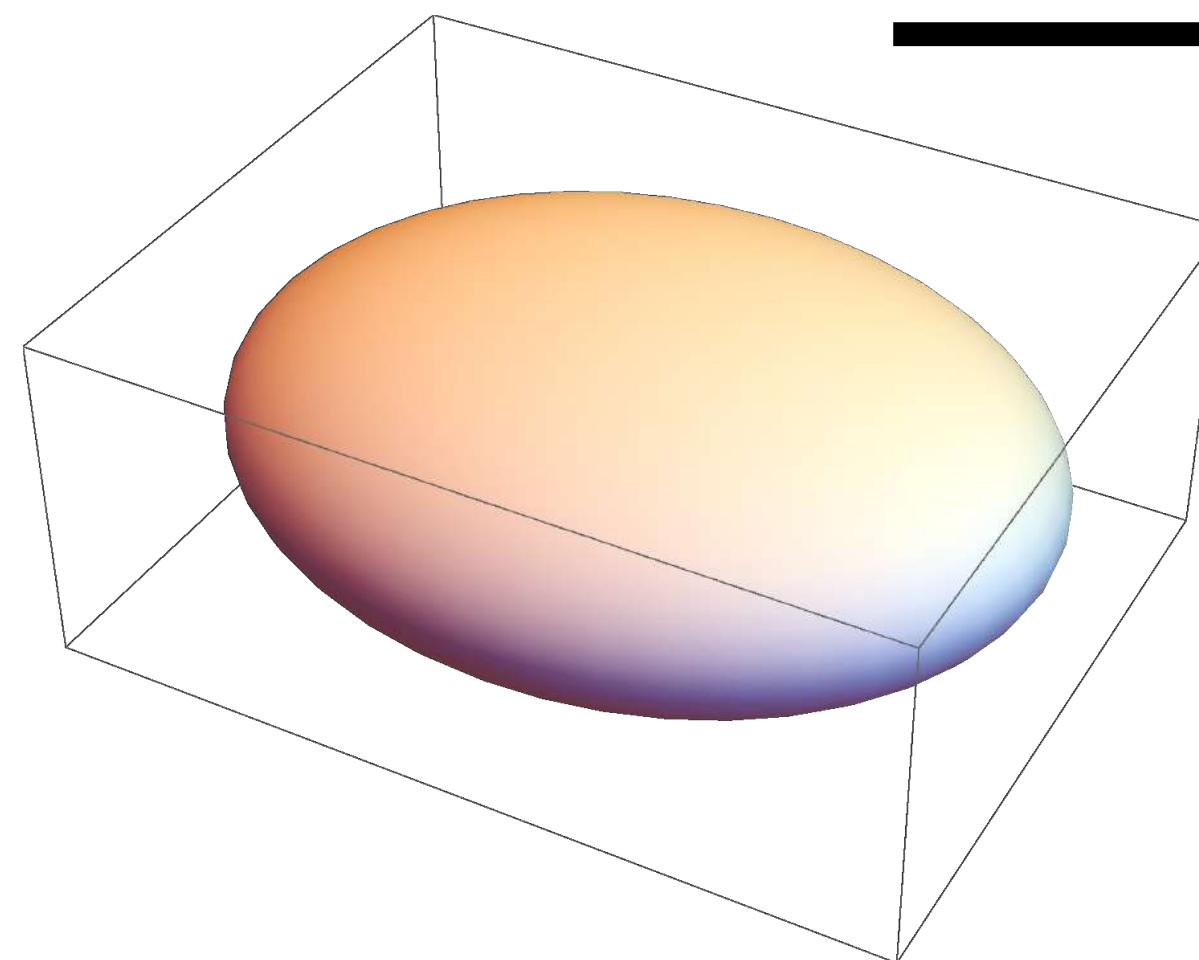
Evolution and collapse of overdensities in eMD



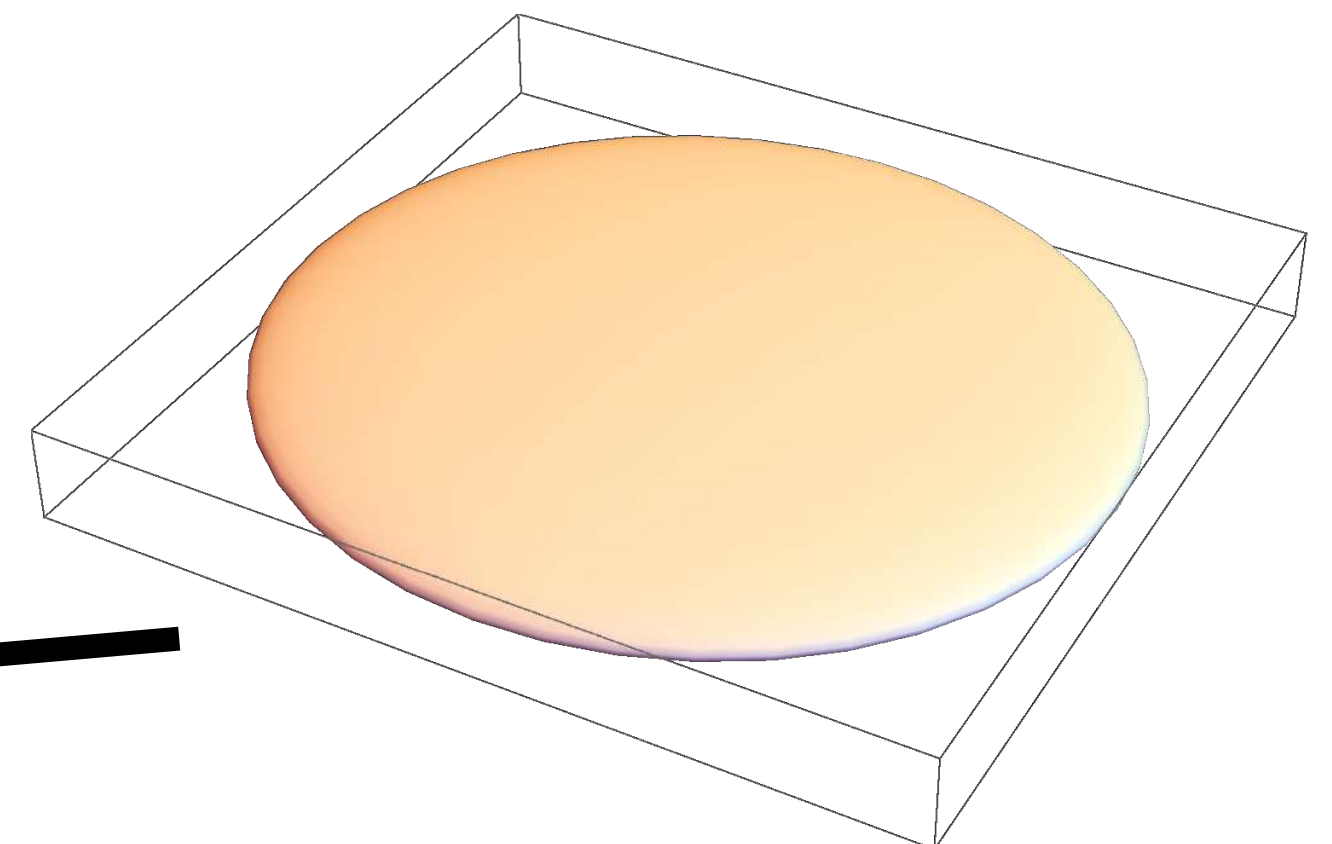
Time of entry



Time of maximum expansion



Time of minimum density



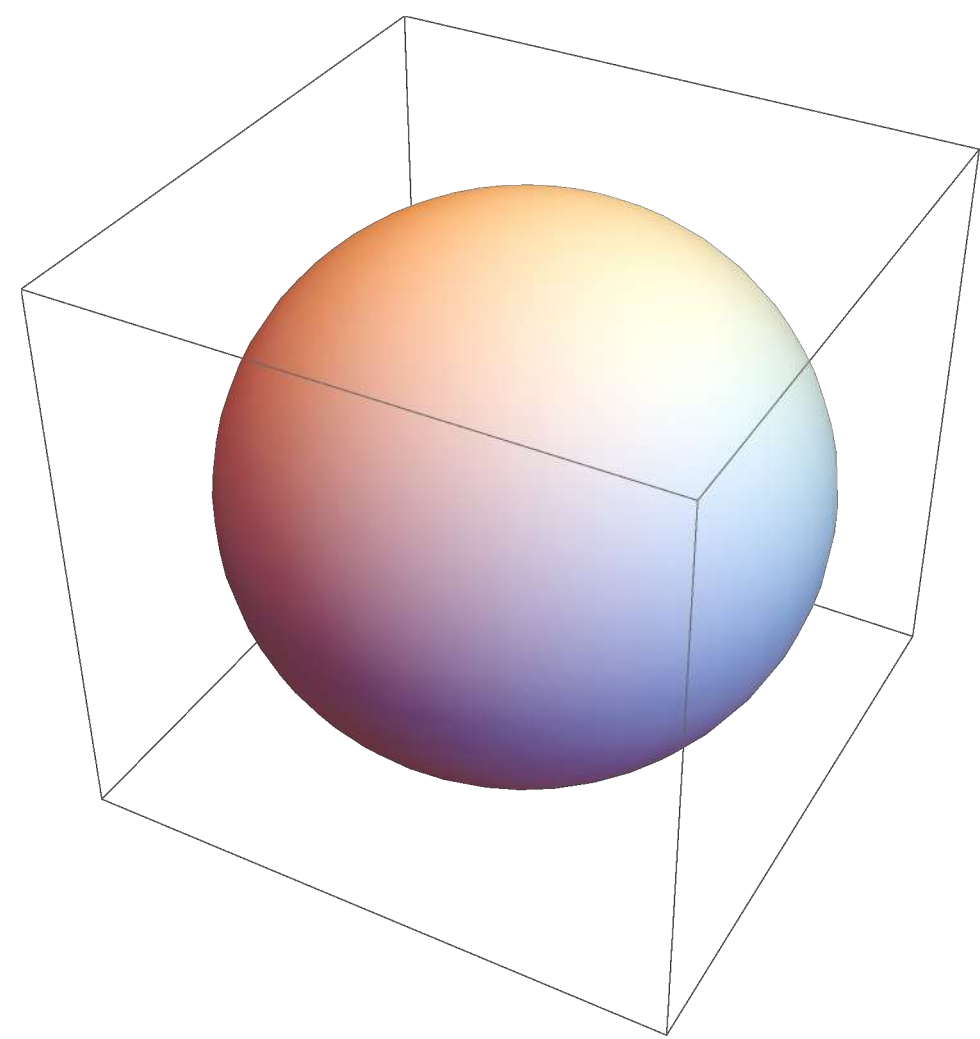
Time of pancake collapse

Y. B. Zeldovich (1970)

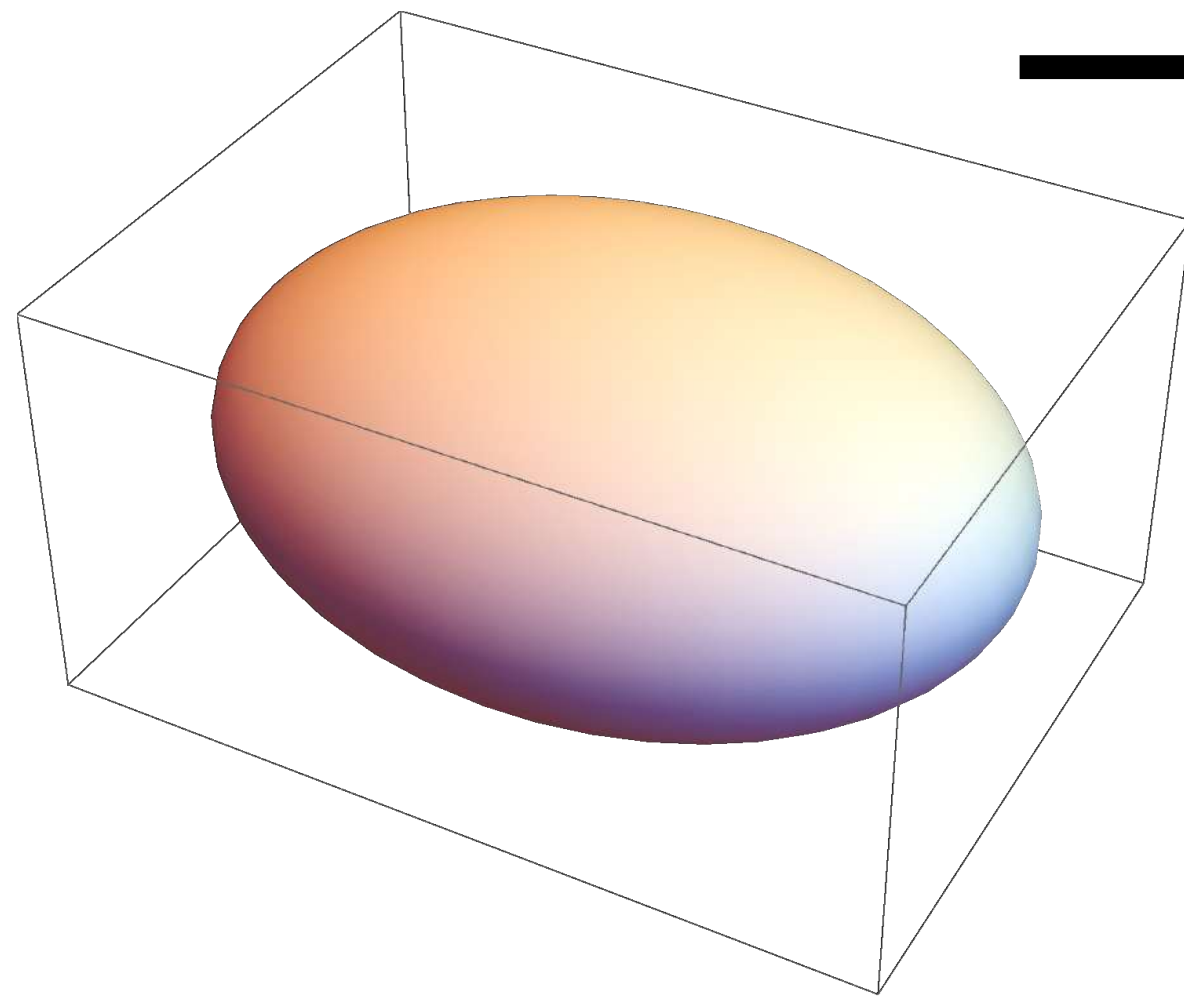
Harada + Yoo + Kohri + Nakao + Jhingan (2016)

ID + Kouvaris JCAP 07 (2021)

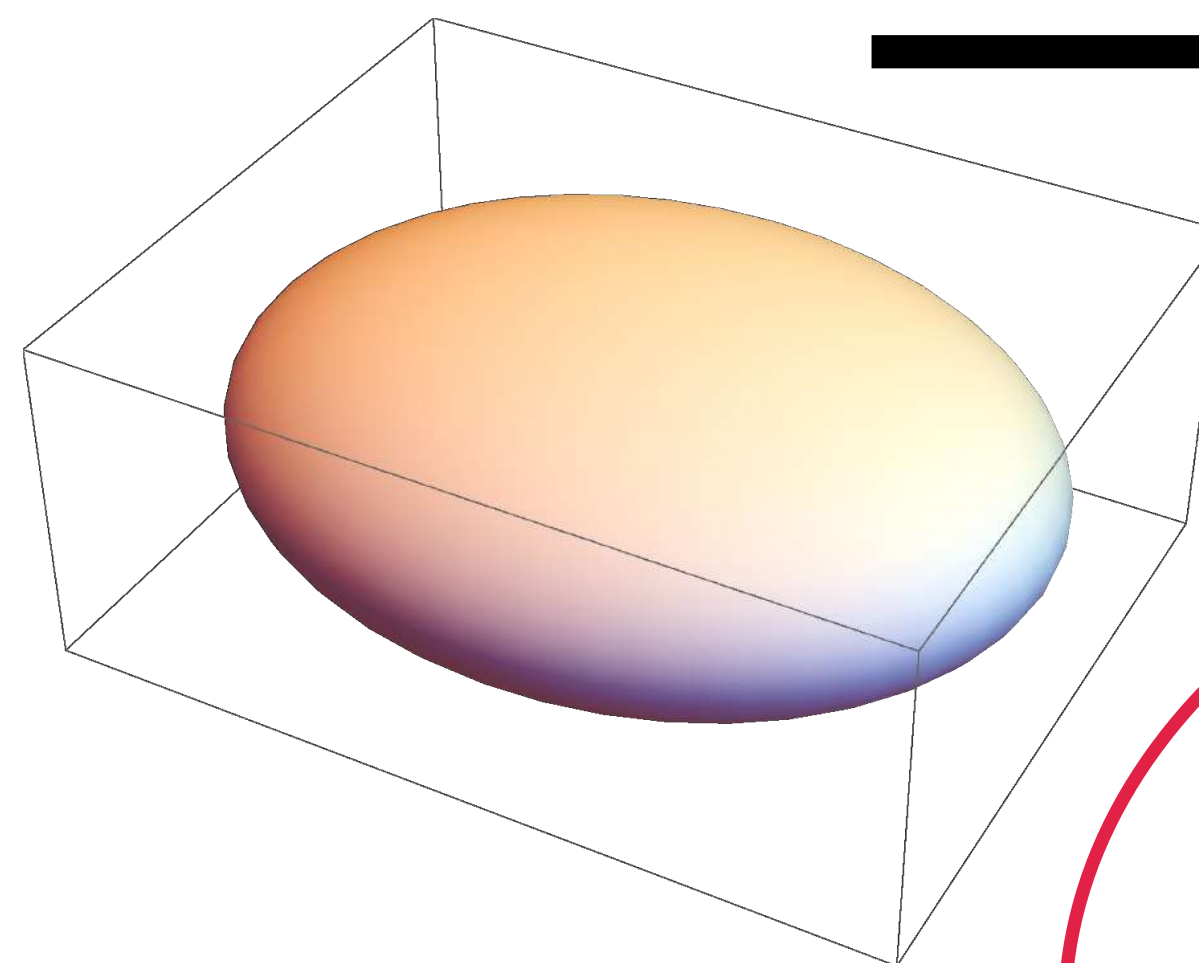
Evolution and collapse of overdensities in eMD



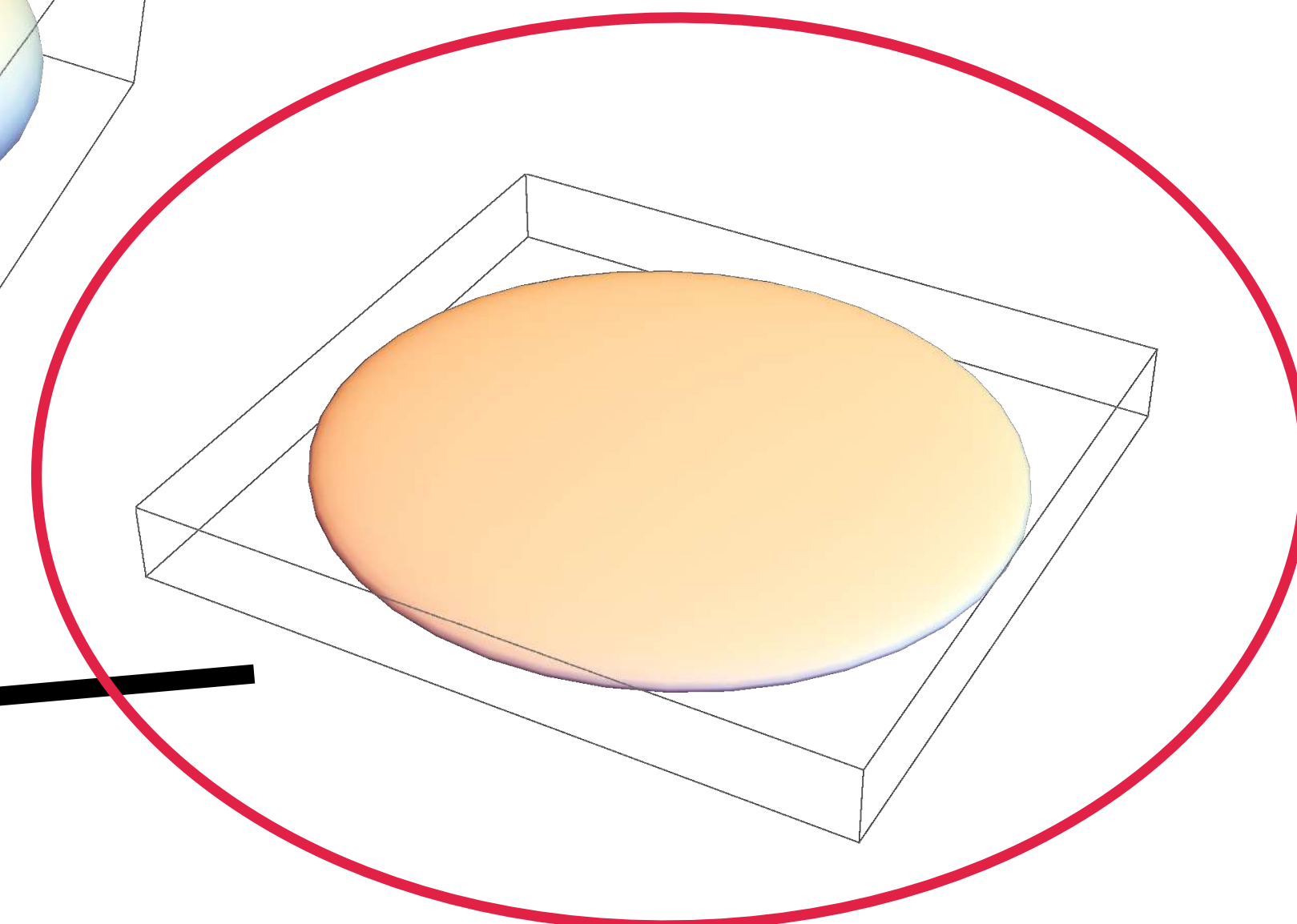
Time of entry



Time of maximum expansion



Time of minimum density



Time of pancake collapse.
Hoop-conjecture: BH forms

Y. B. Zeldovich (1970)

Harada + Yoo + Kohri + Nakao + Jhingan (2016)

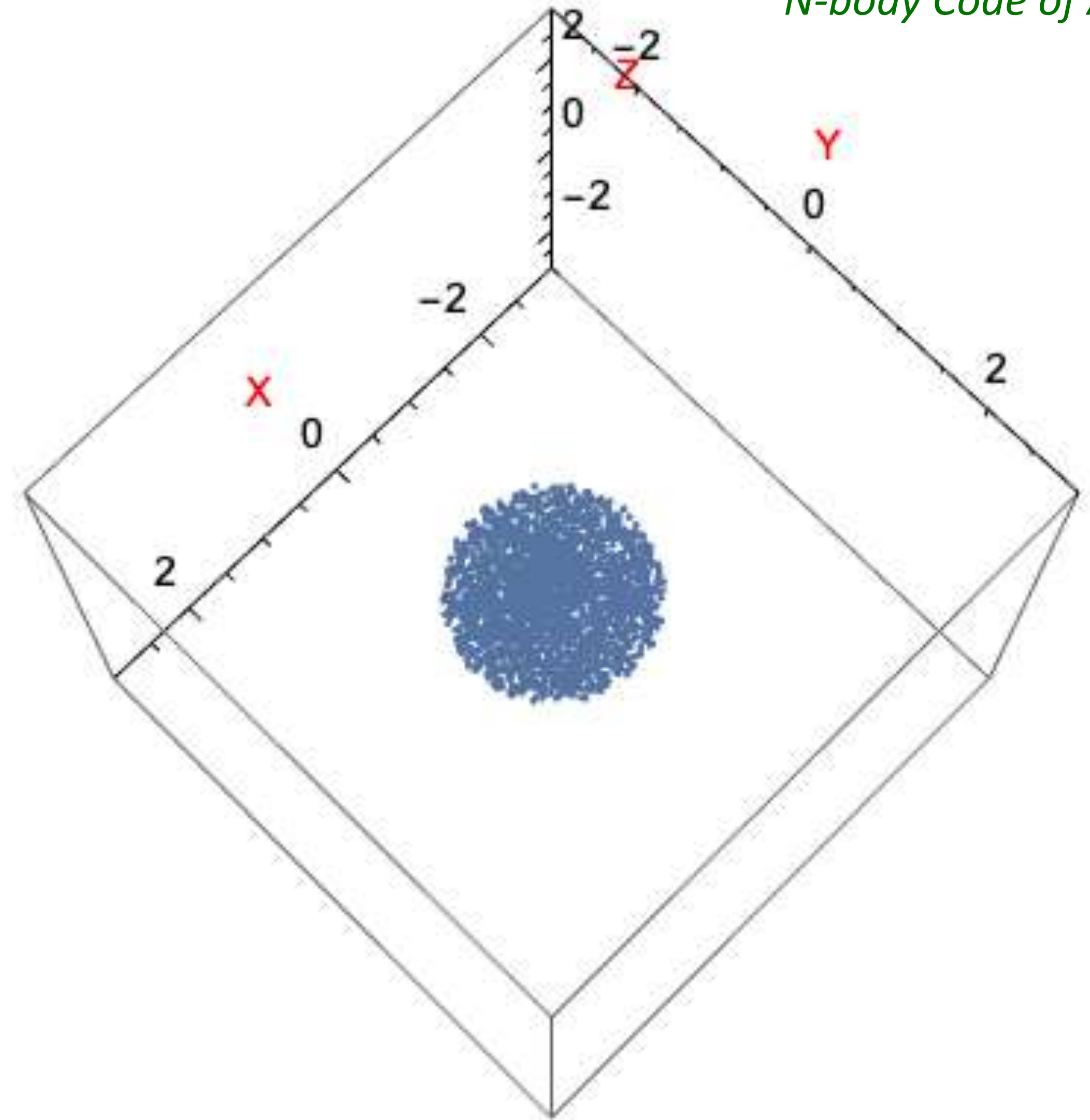
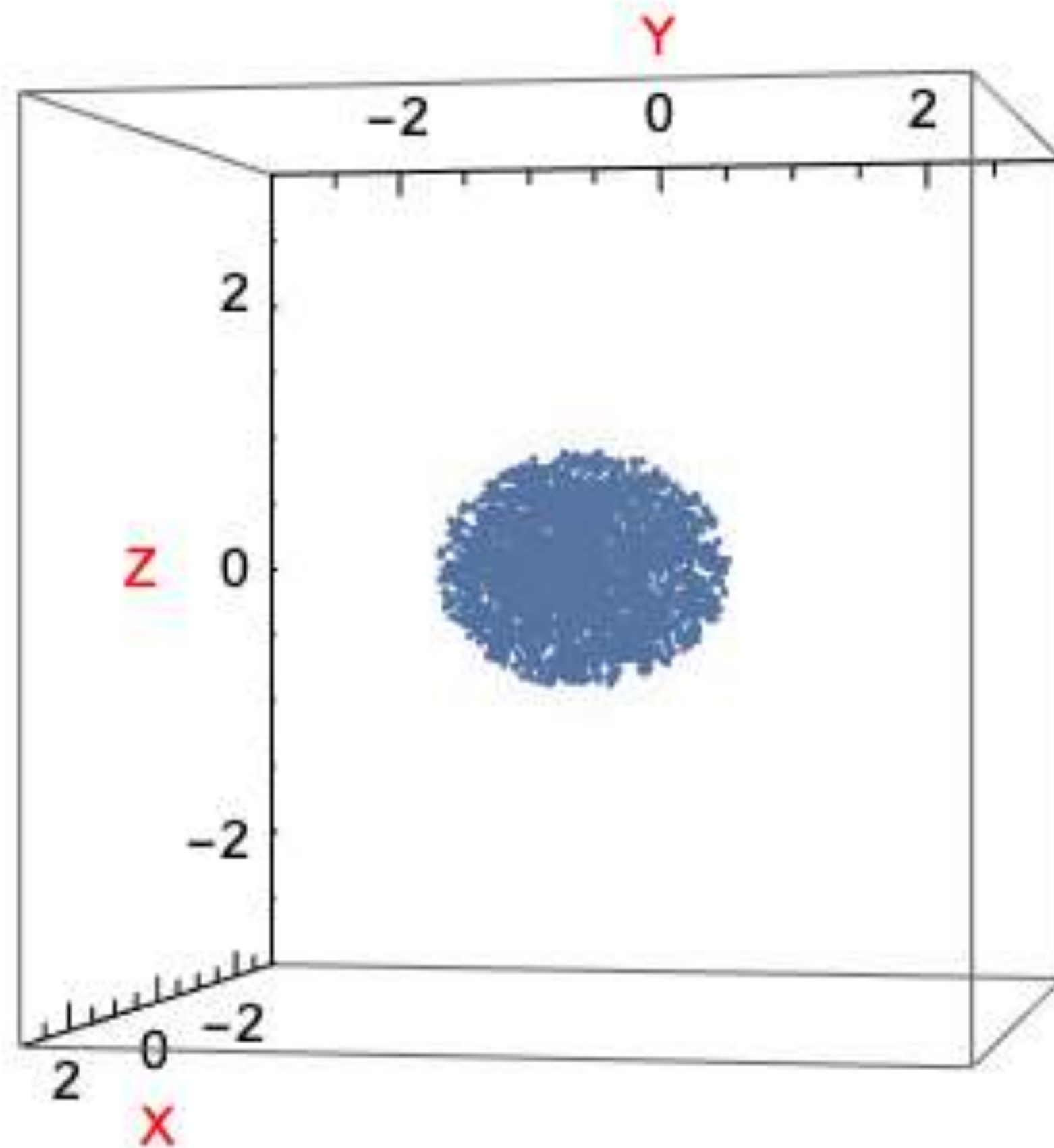
ID + Kouvaris JCAP 07 (2021)

Numerical Simulations

(Work in progress)

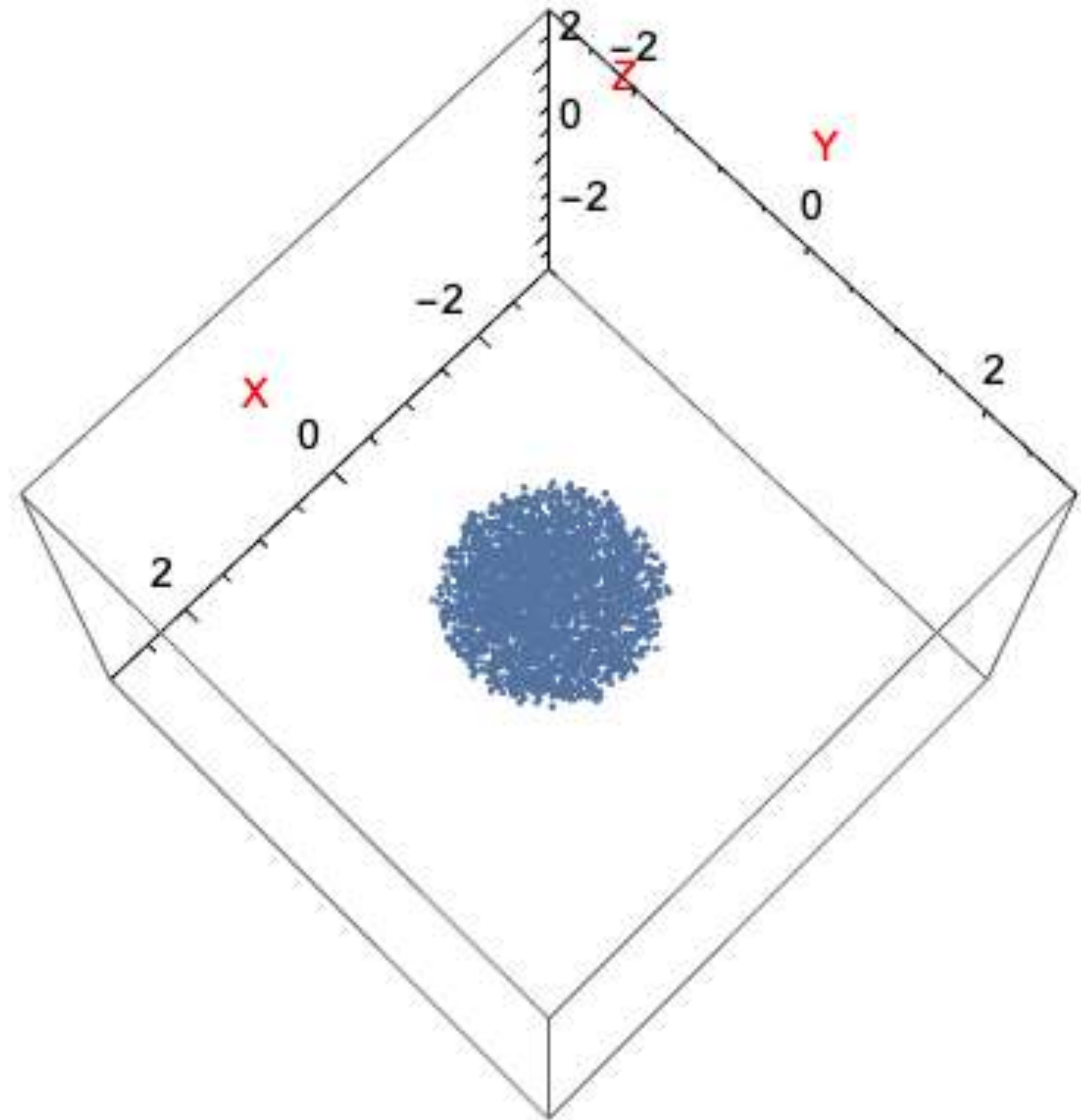
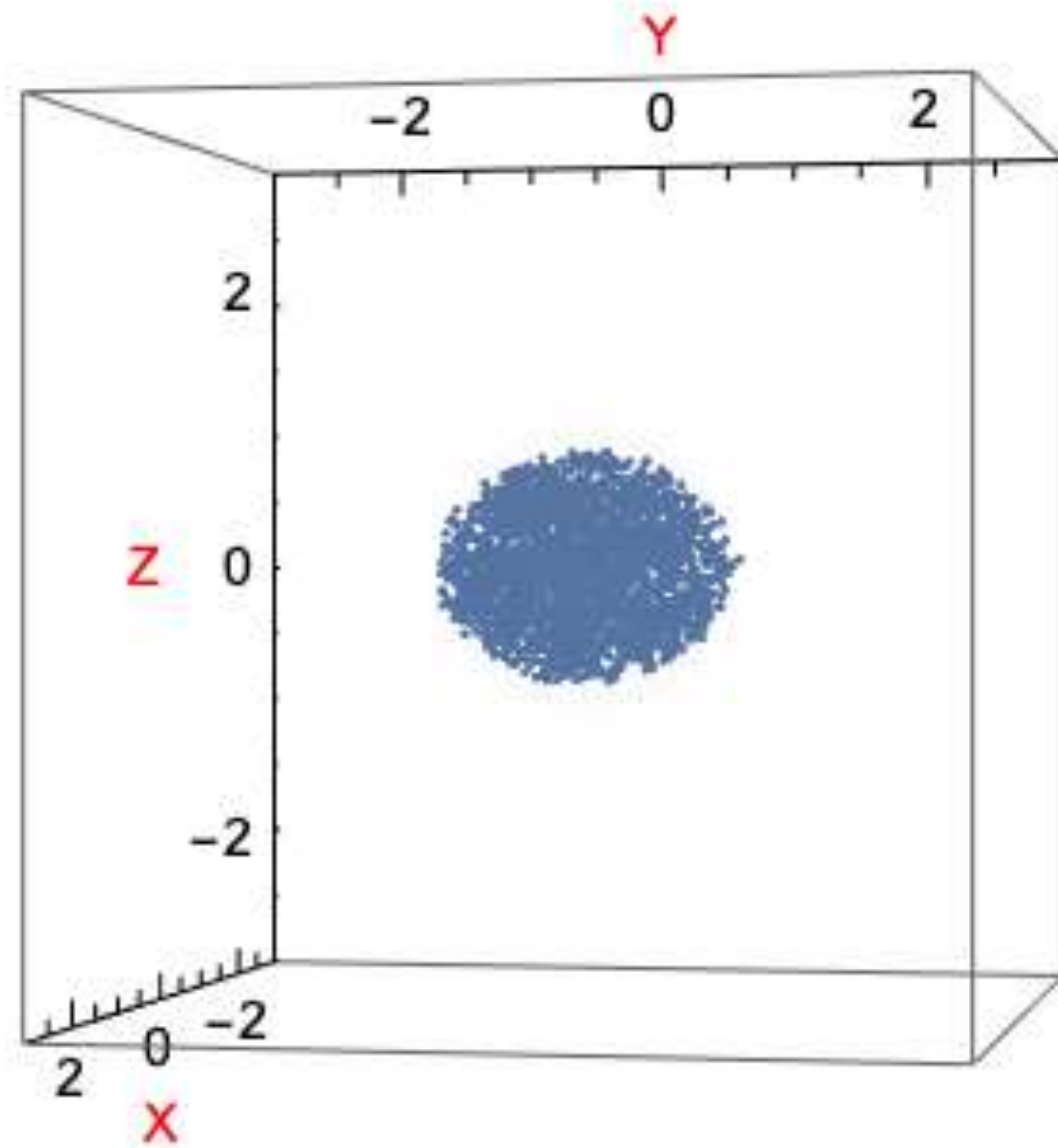
N-body Code of Aarseth

$j=1, Q=0.5$

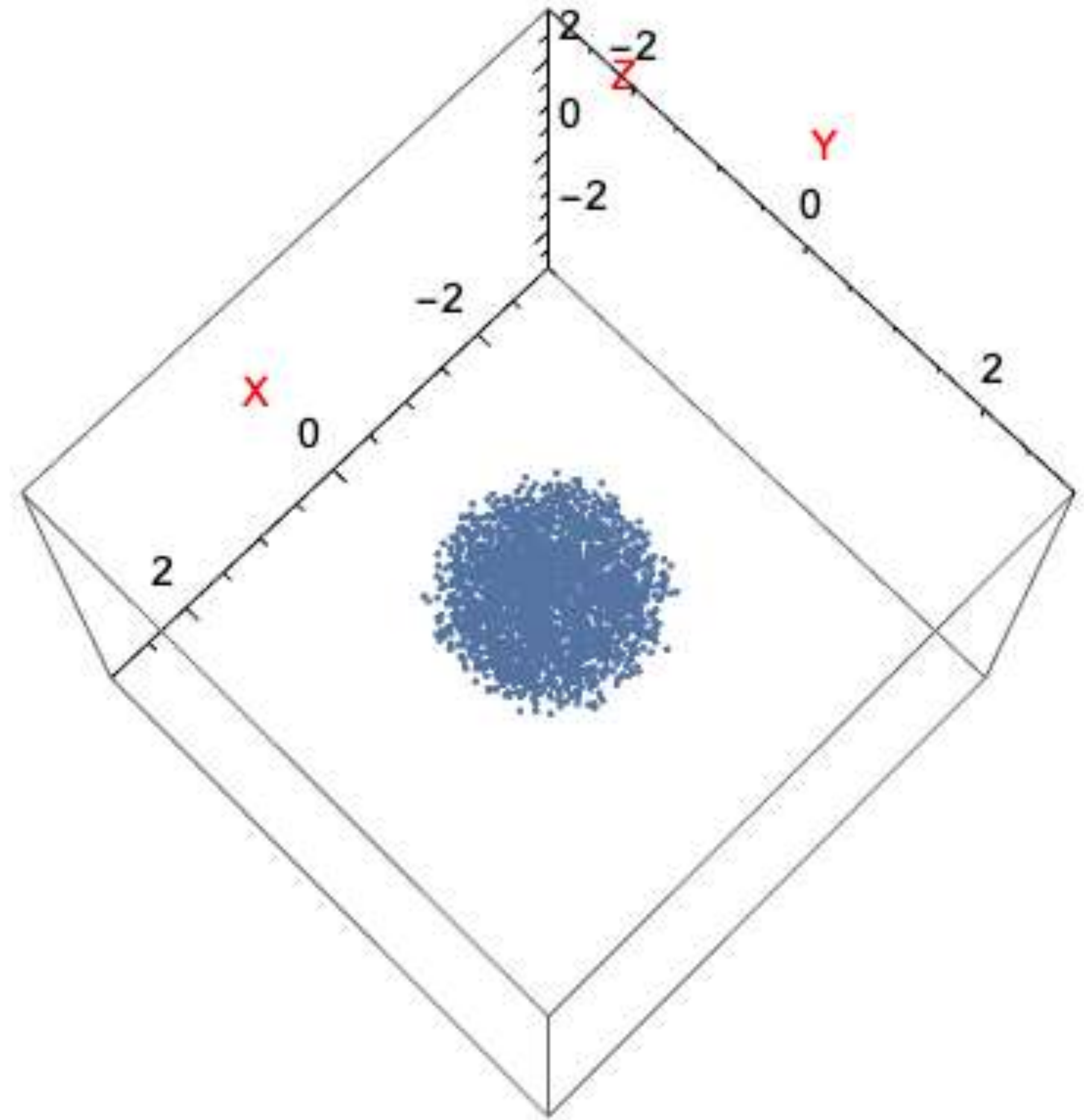
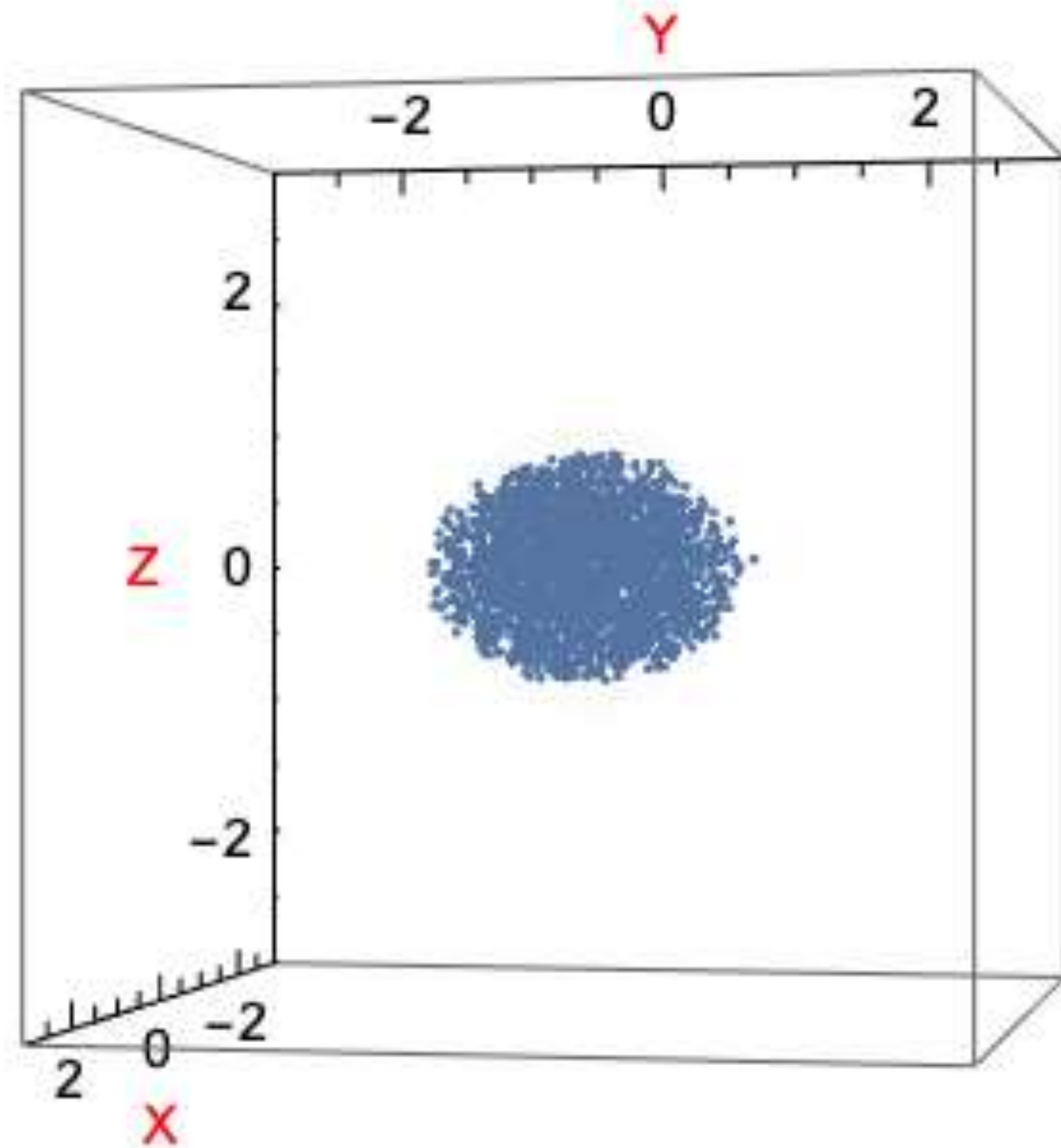


I thank M. Chira for her guidance in N-body numerical simulation

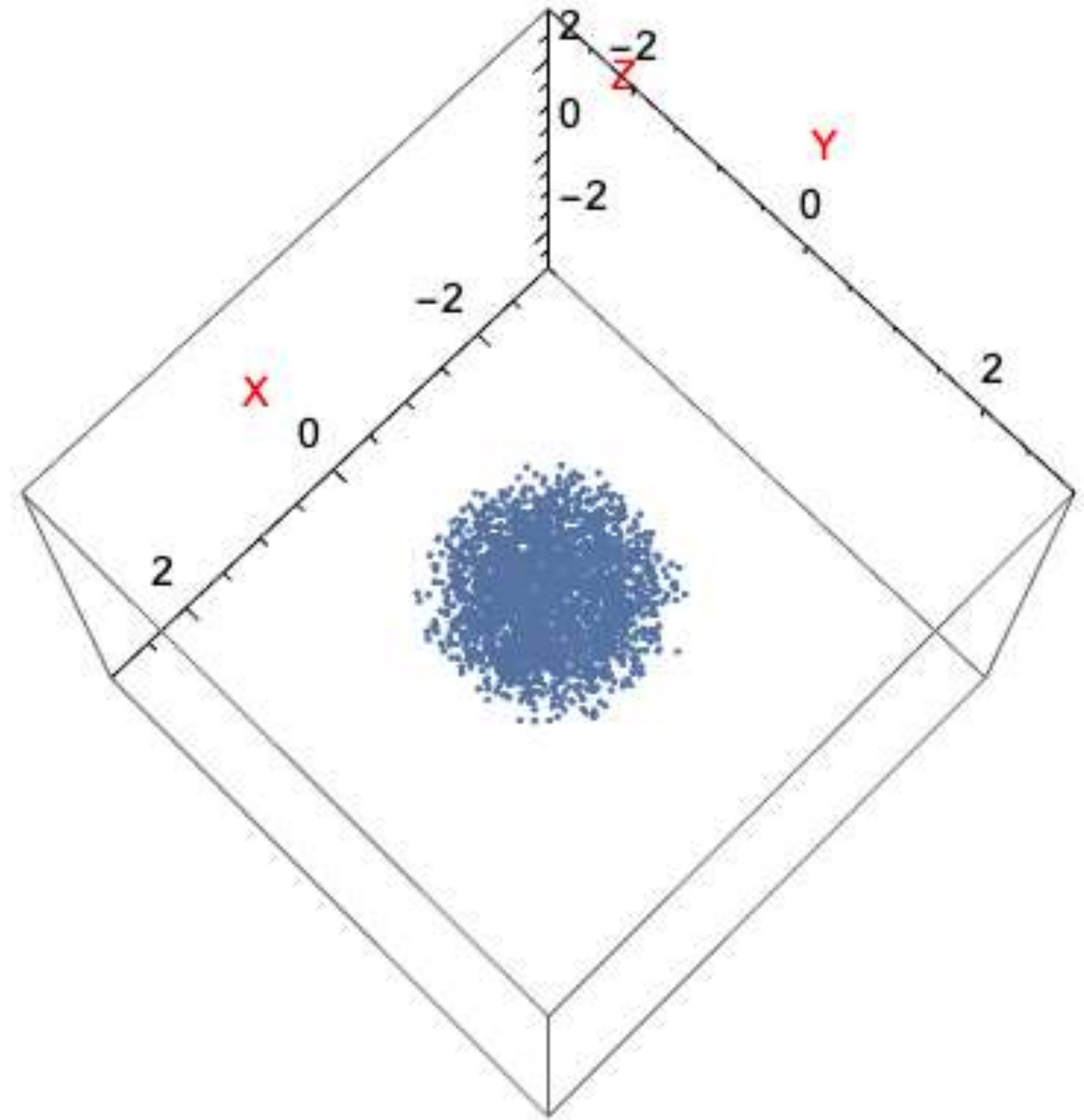
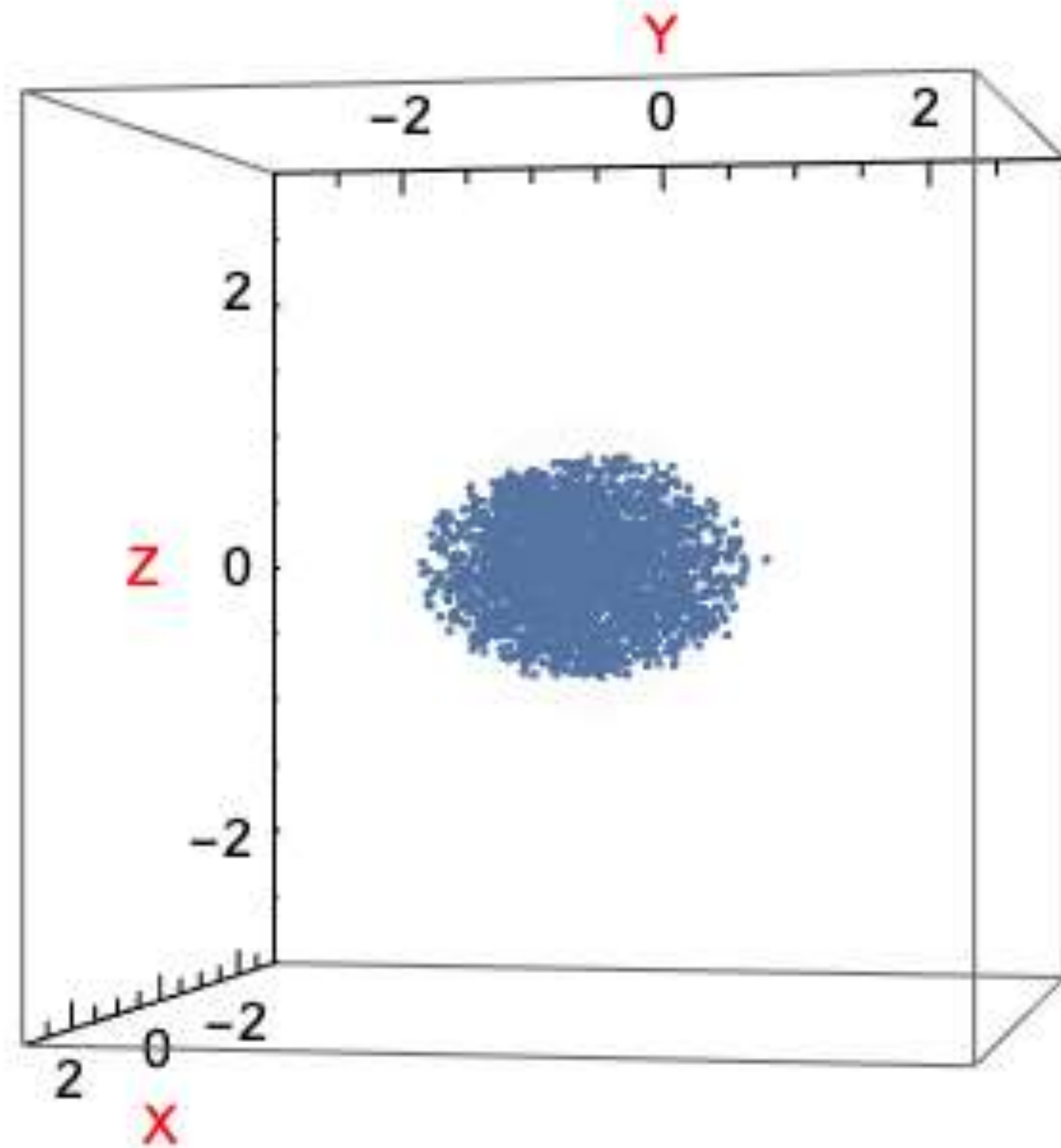
$j=5, Q=0.5$



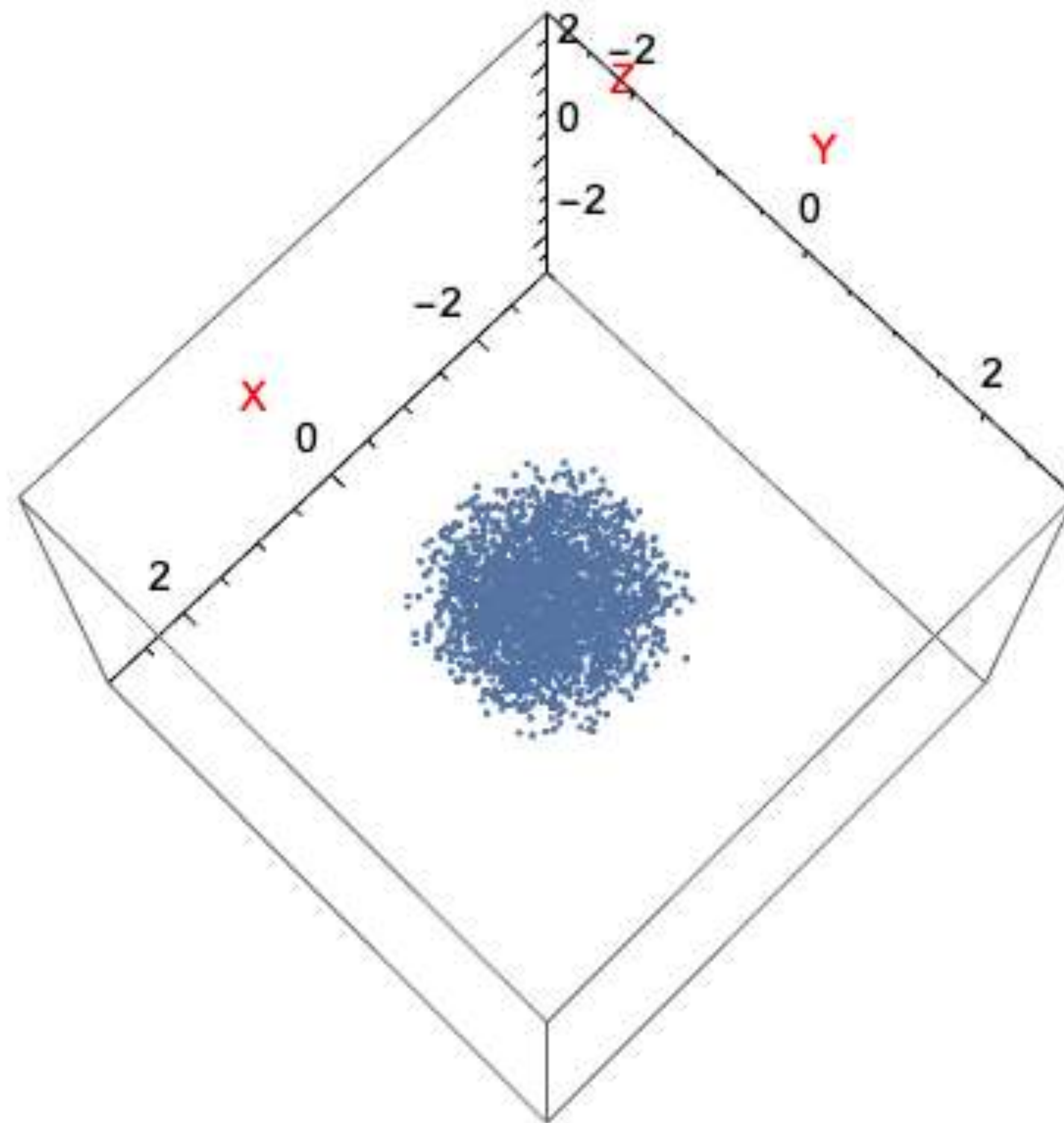
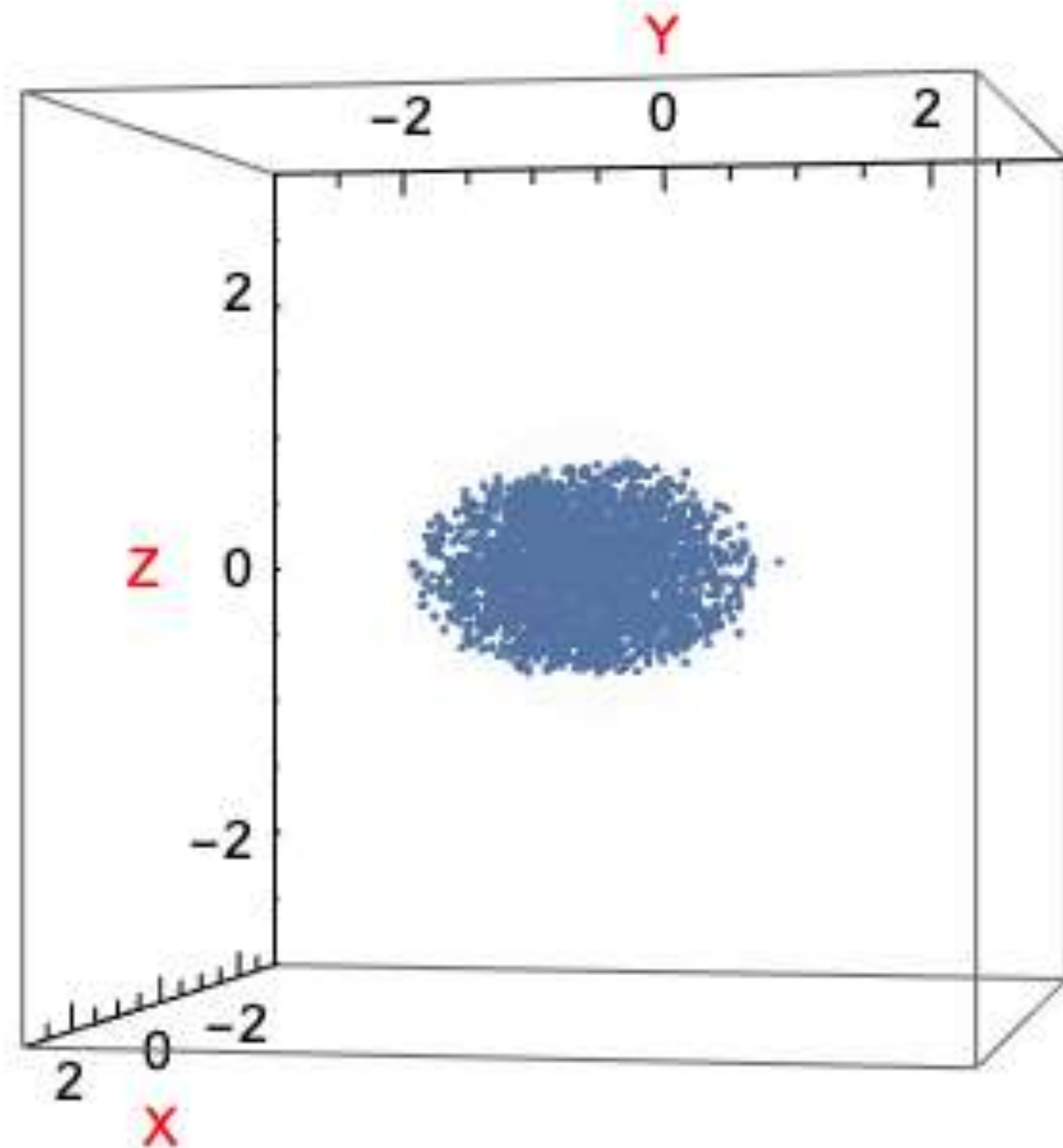
$j=10, Q=0.5$



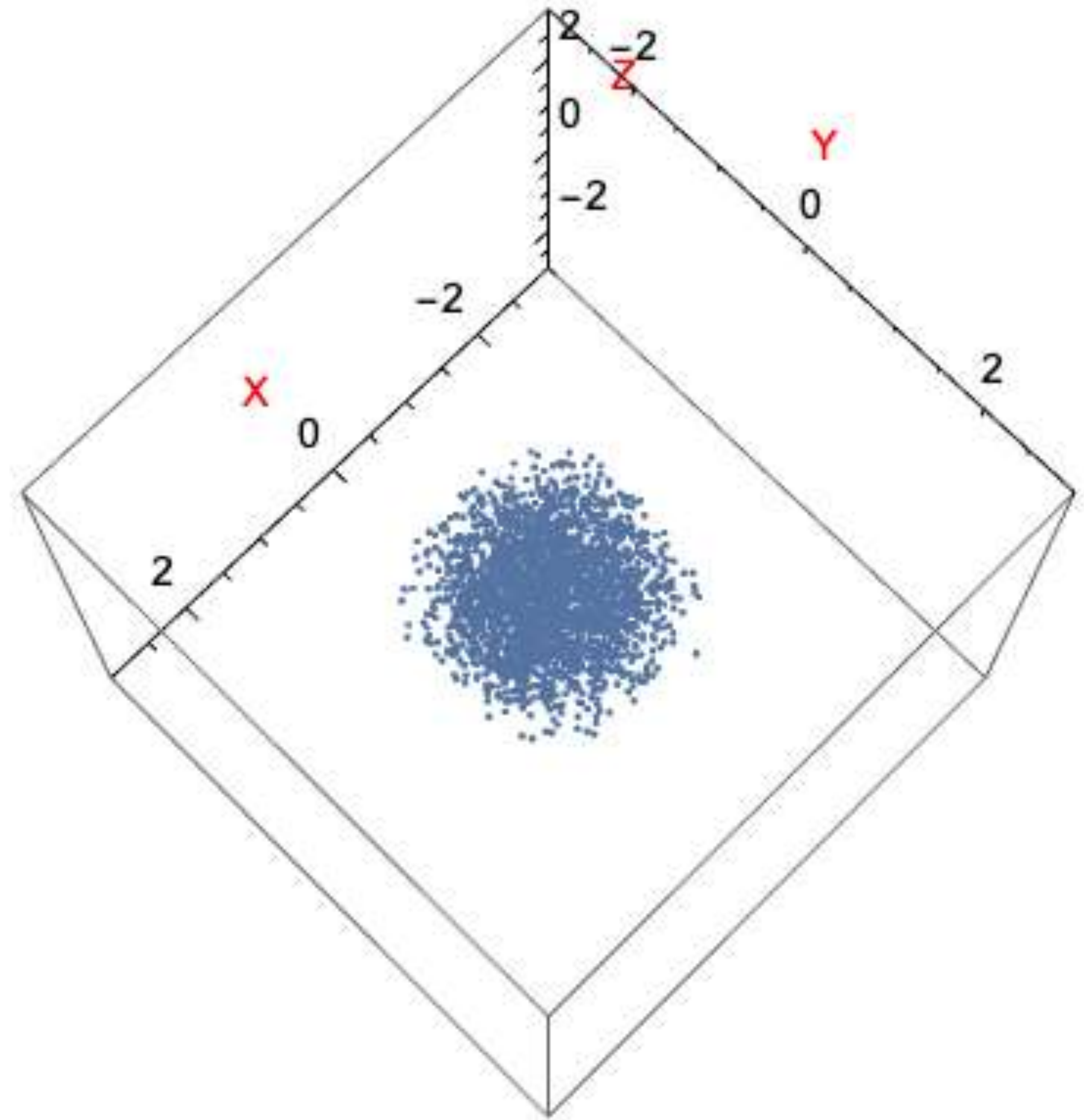
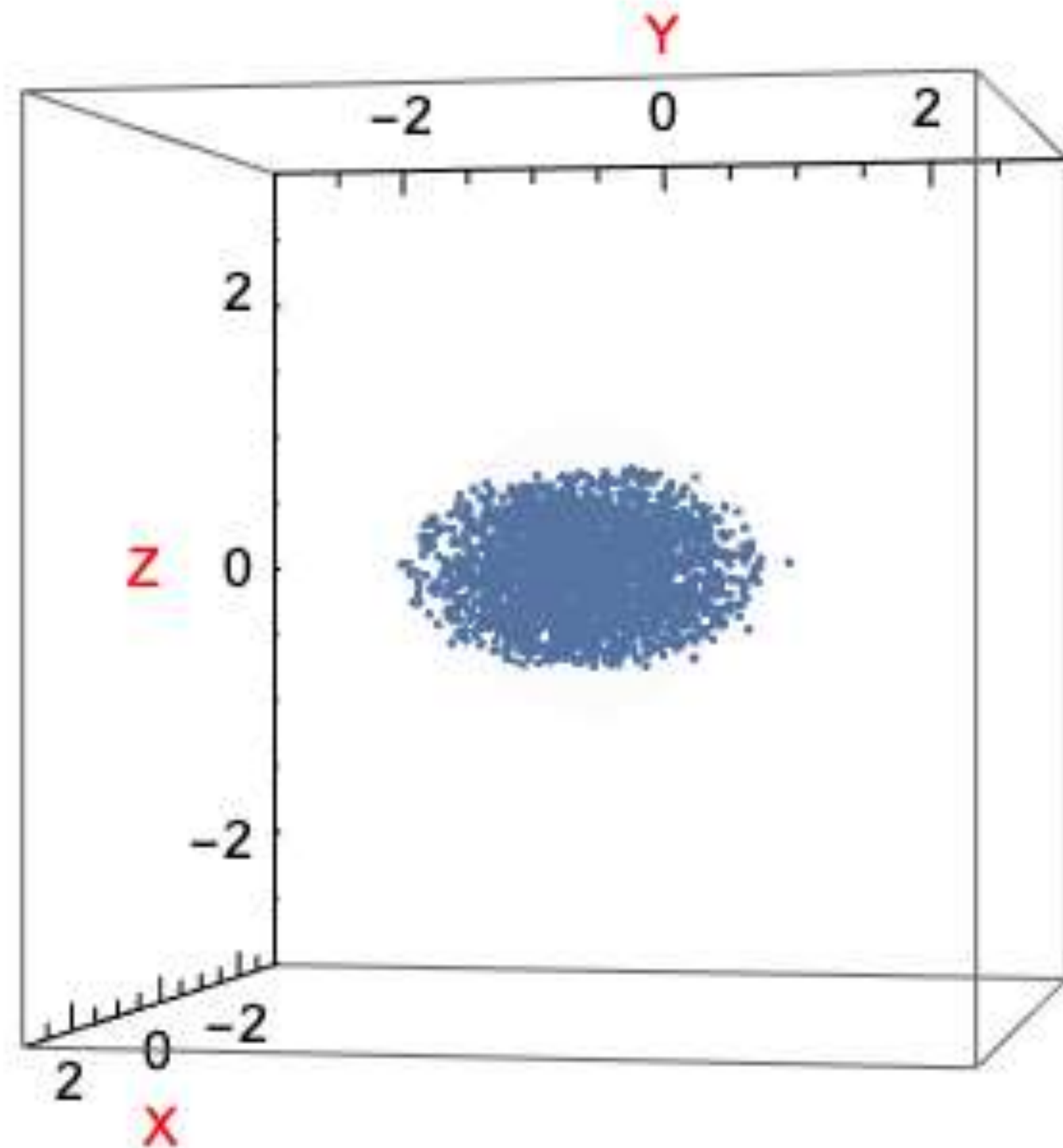
$j=15, Q=0.5$



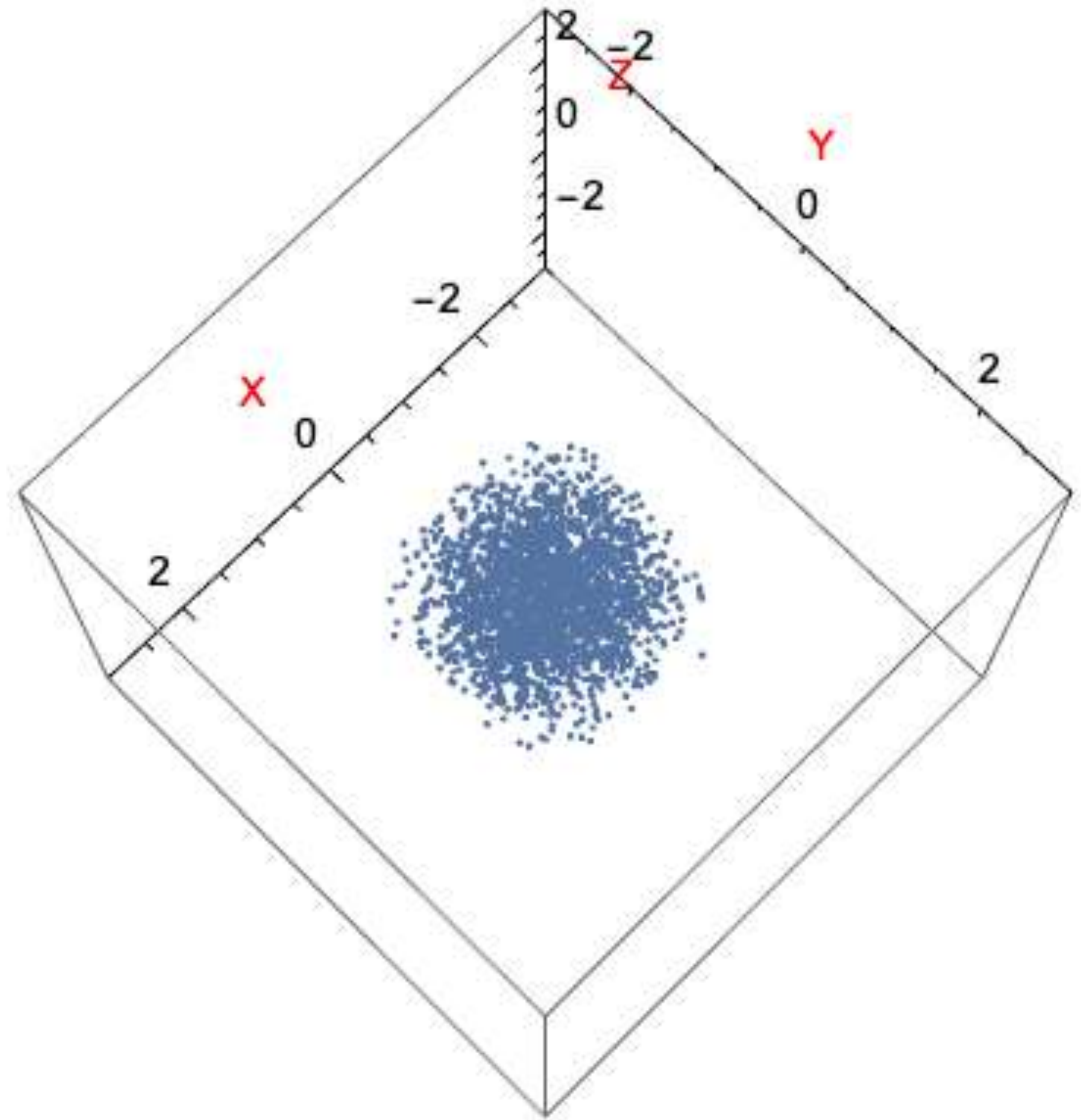
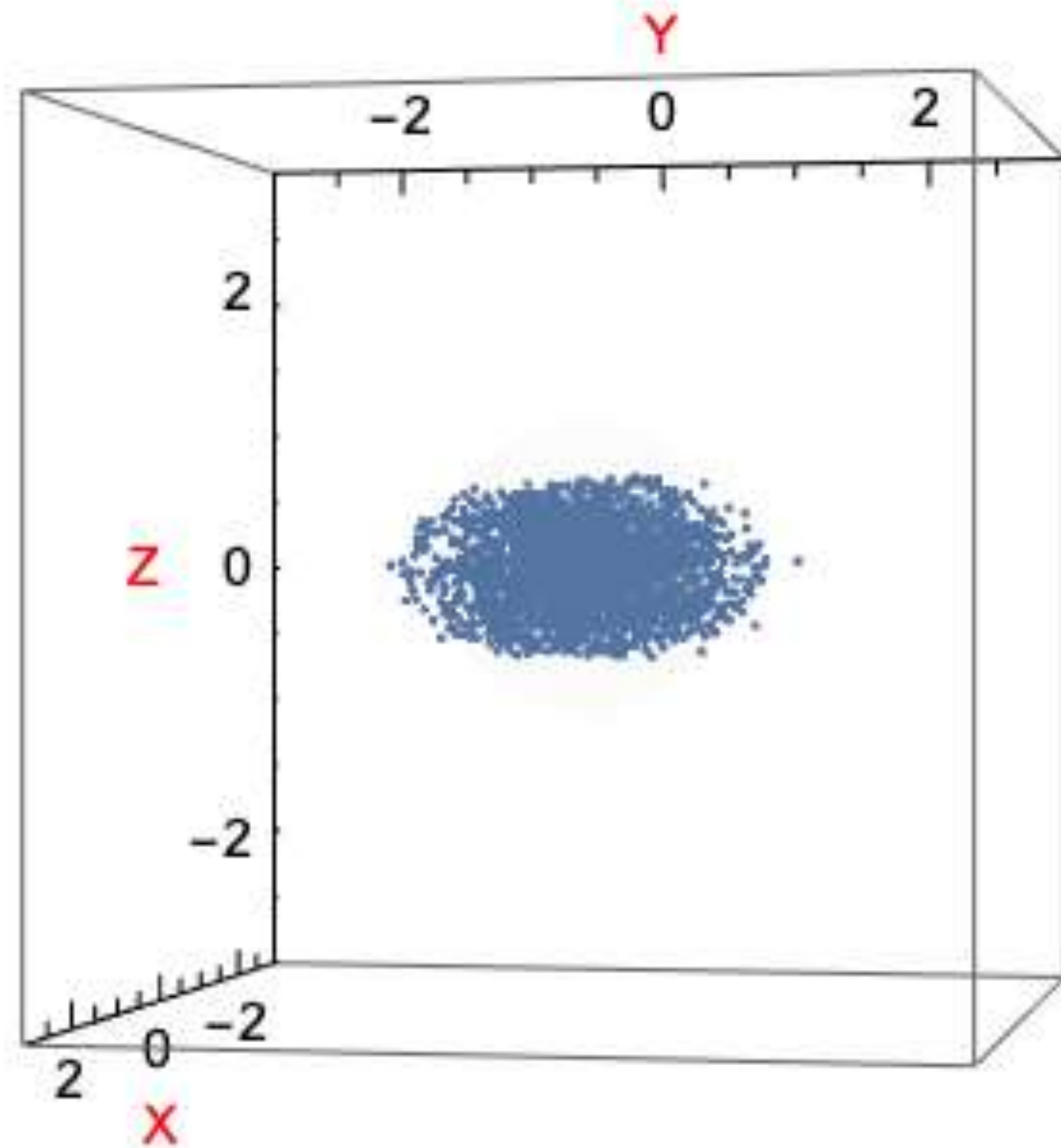
$j=20, Q = 0.51$



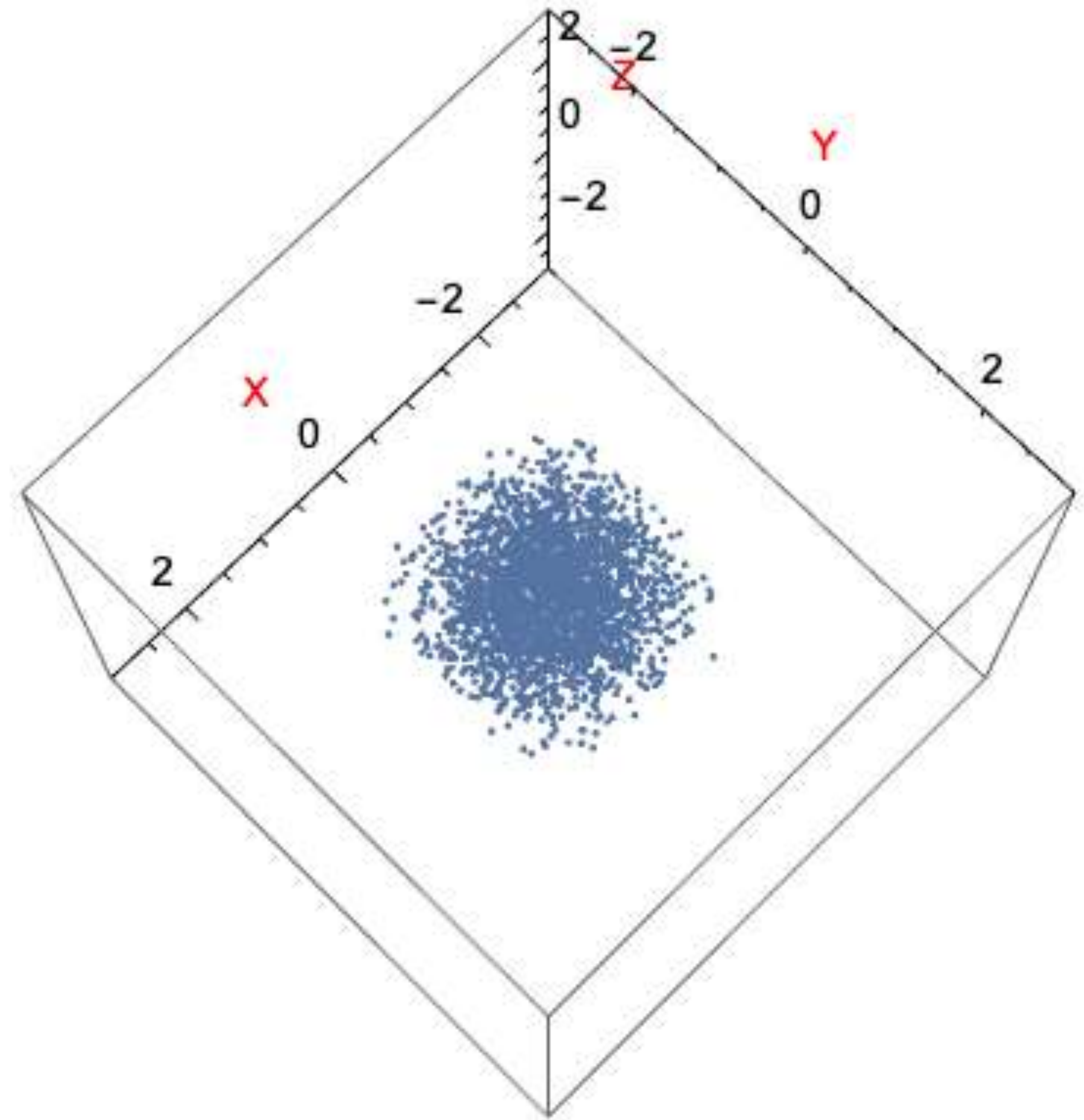
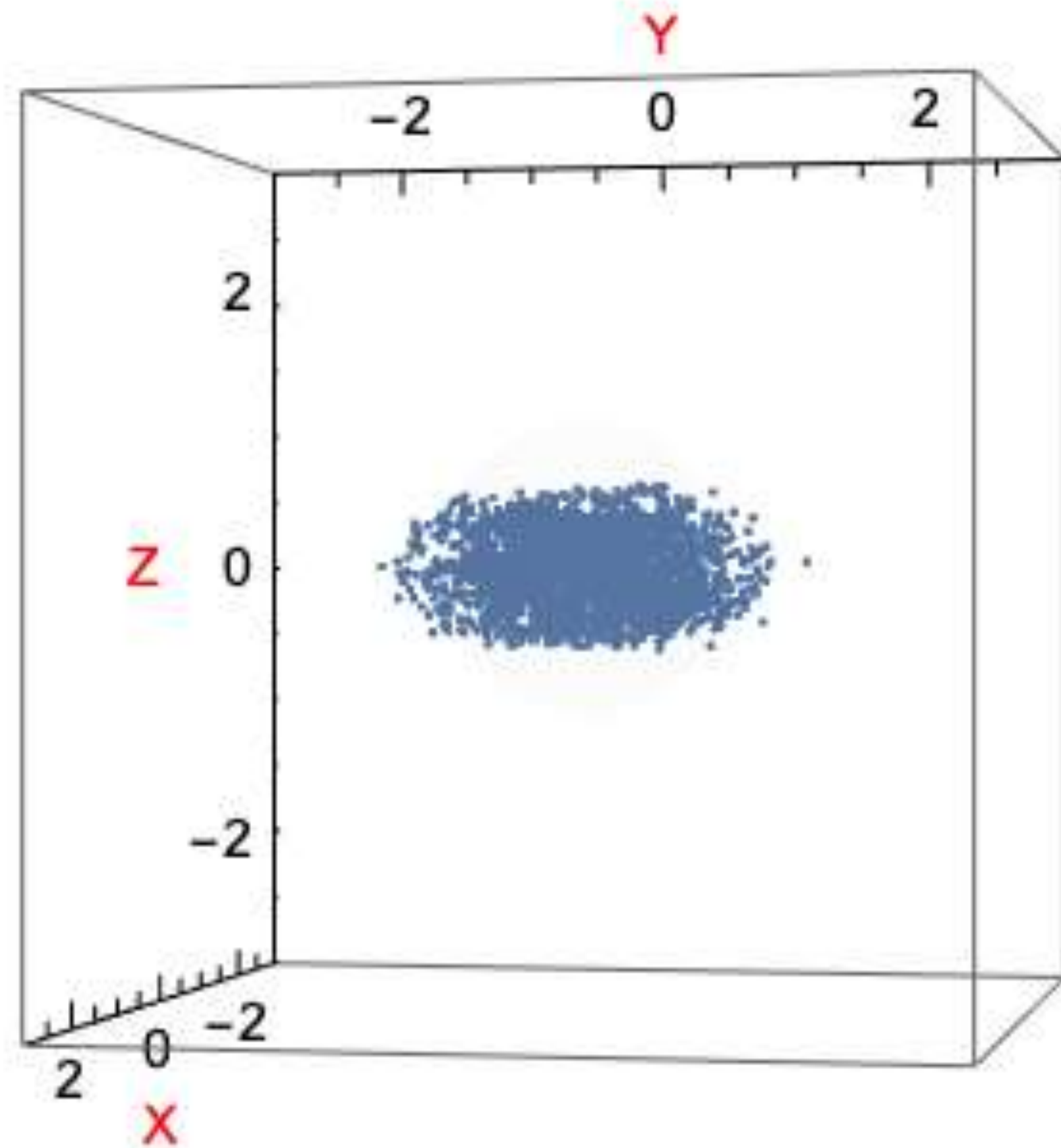
$j=25, Q = 0.51$



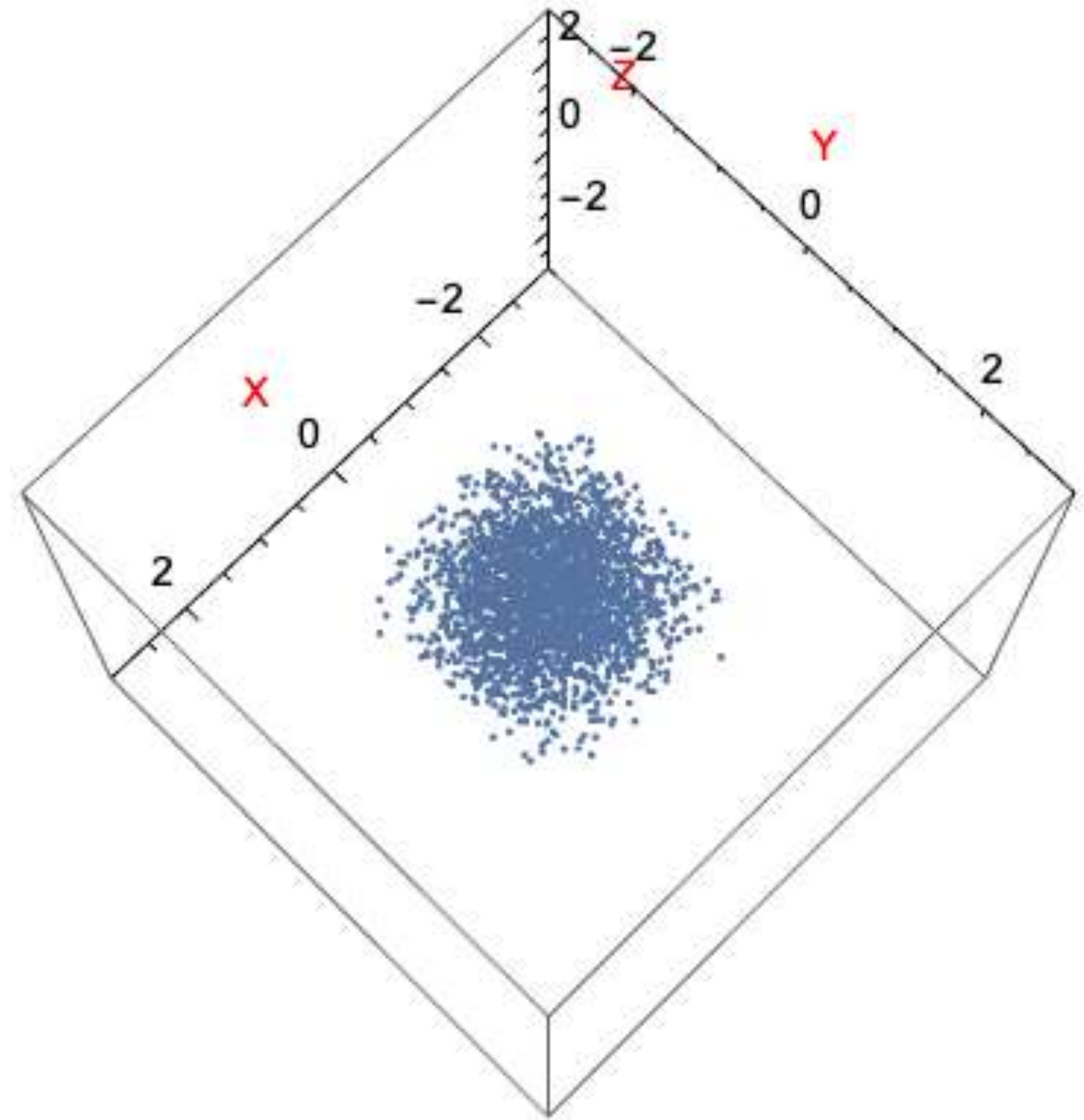
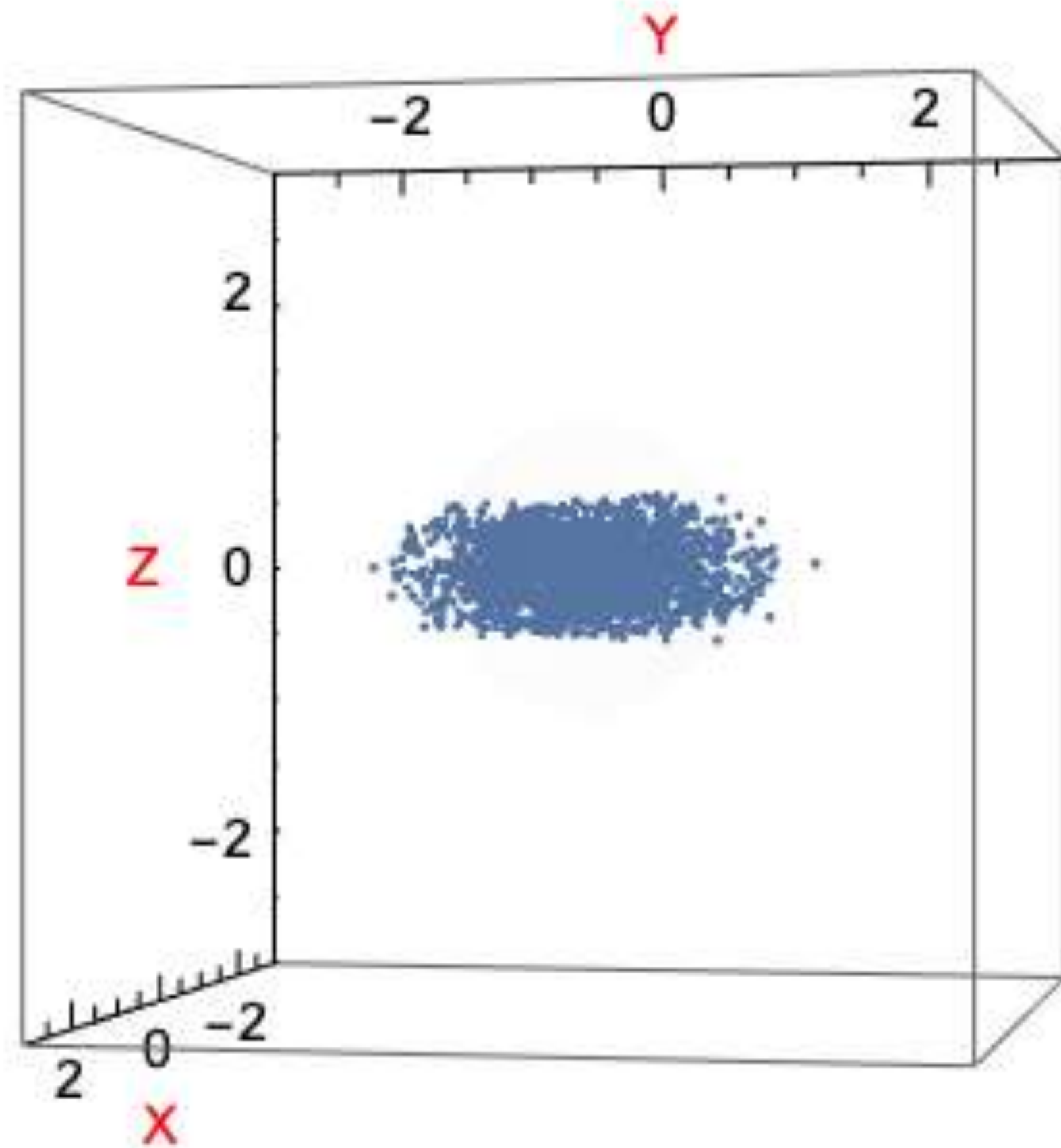
$j=30, Q = 0.52$



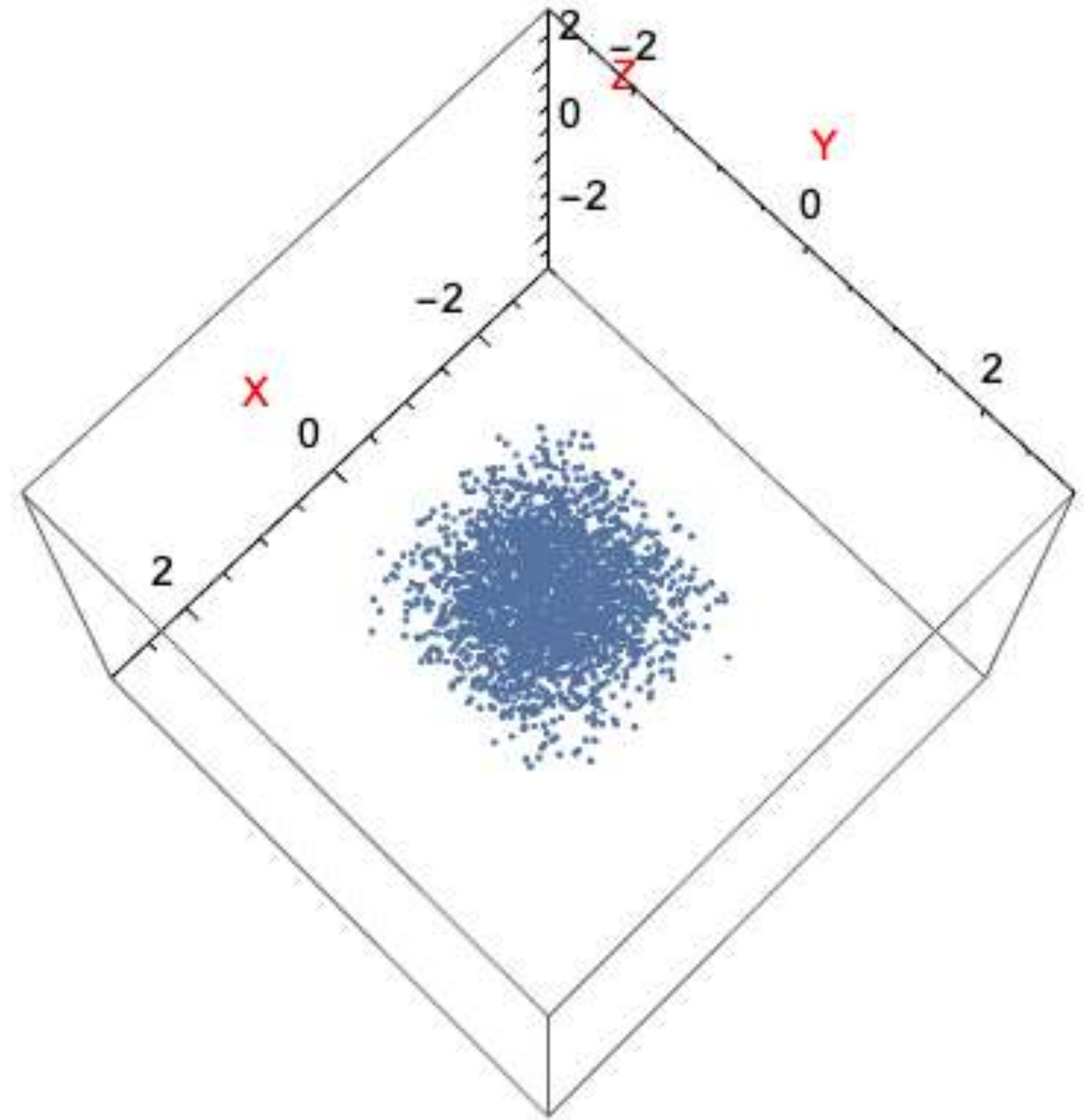
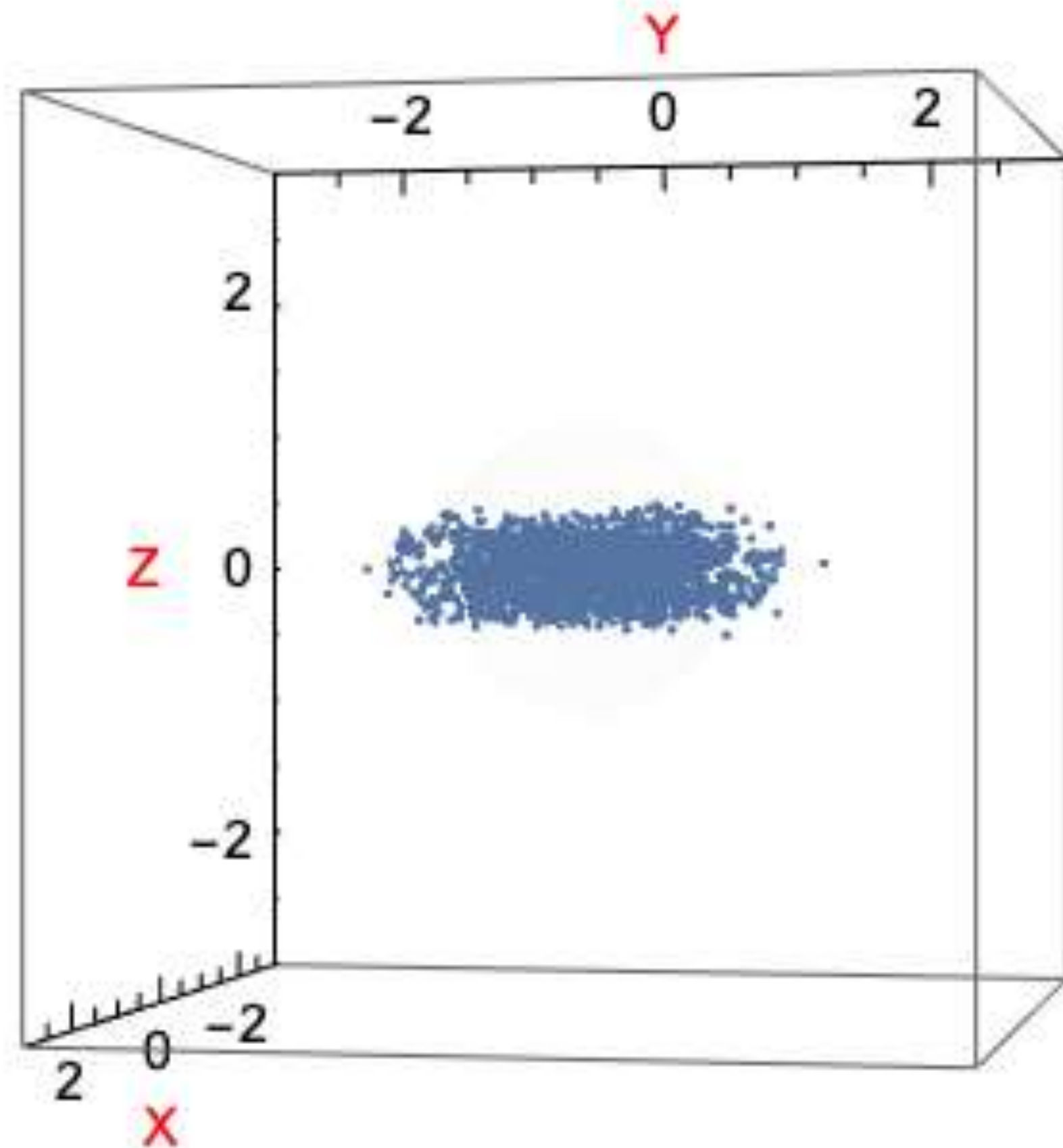
$j=35, Q = 0.53$



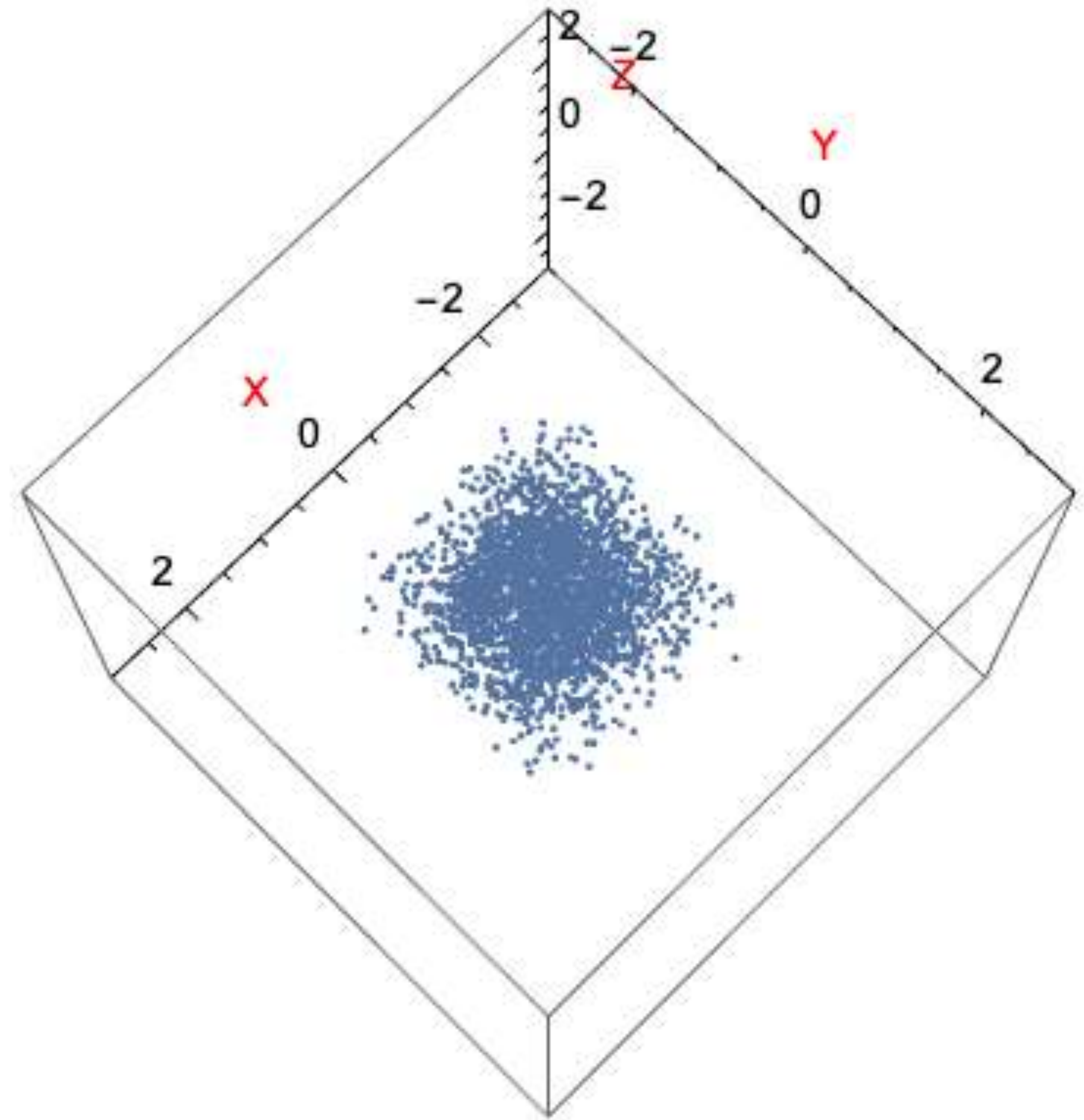
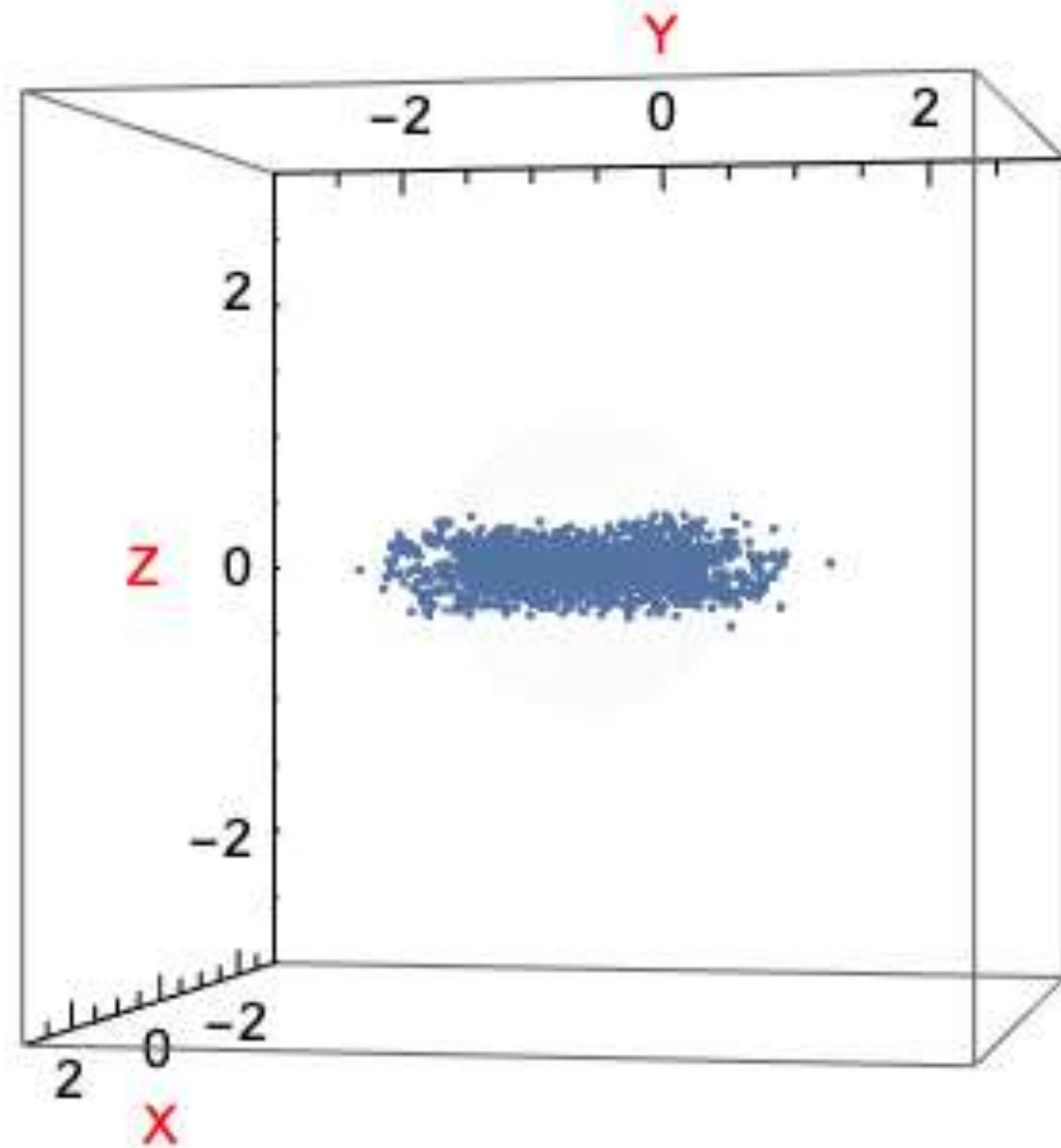
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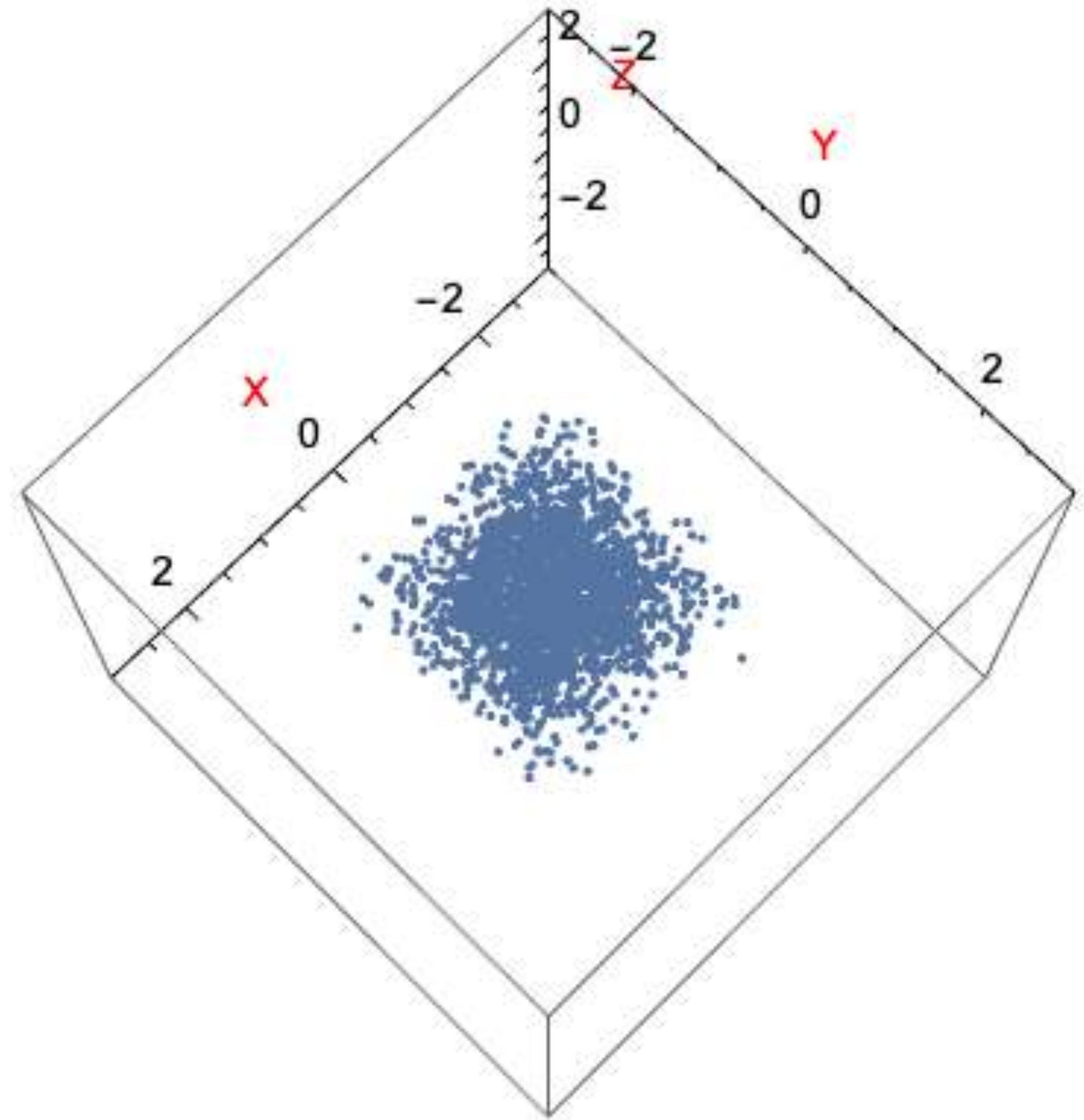
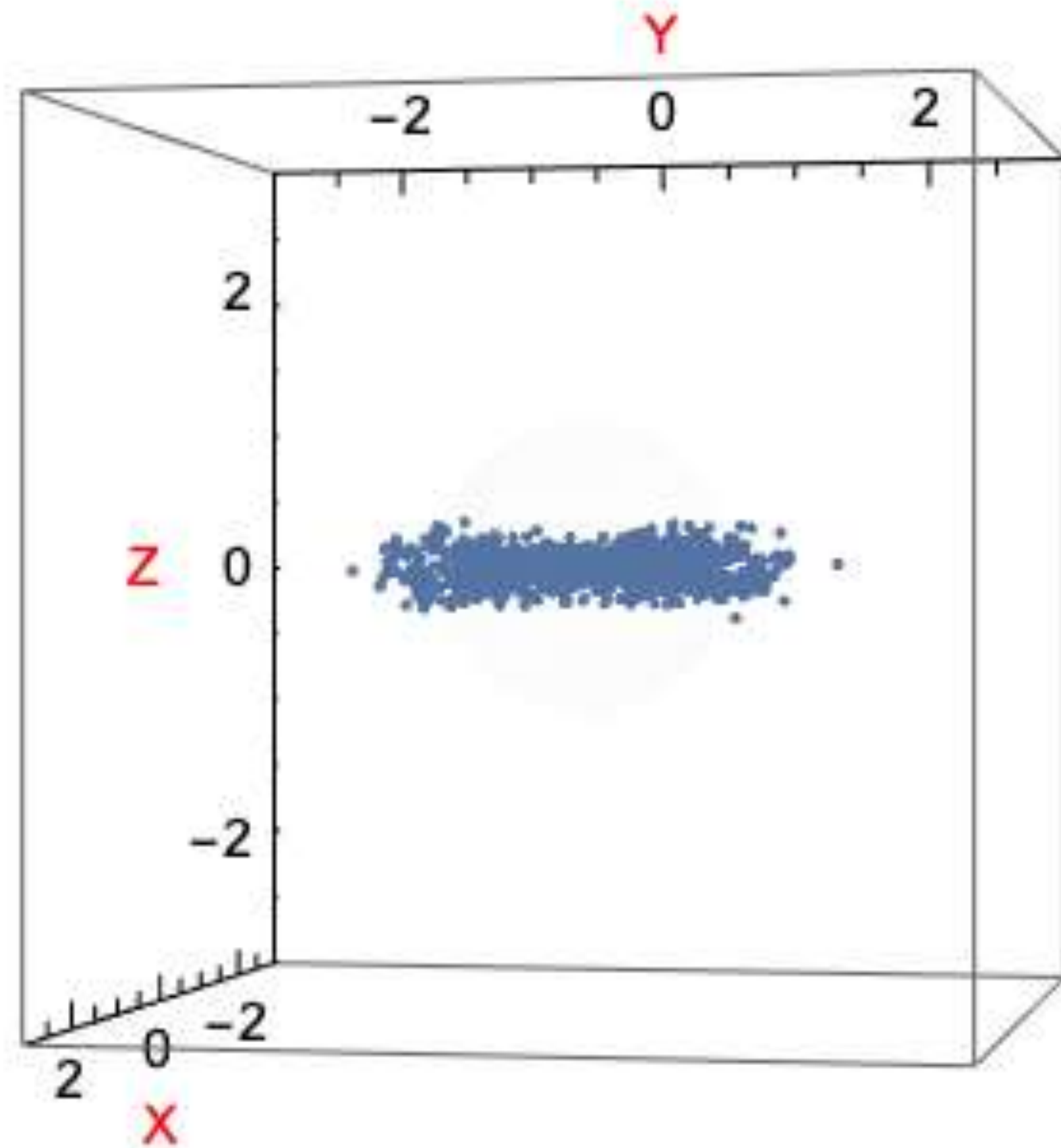
$j=45, Q = 0.56$



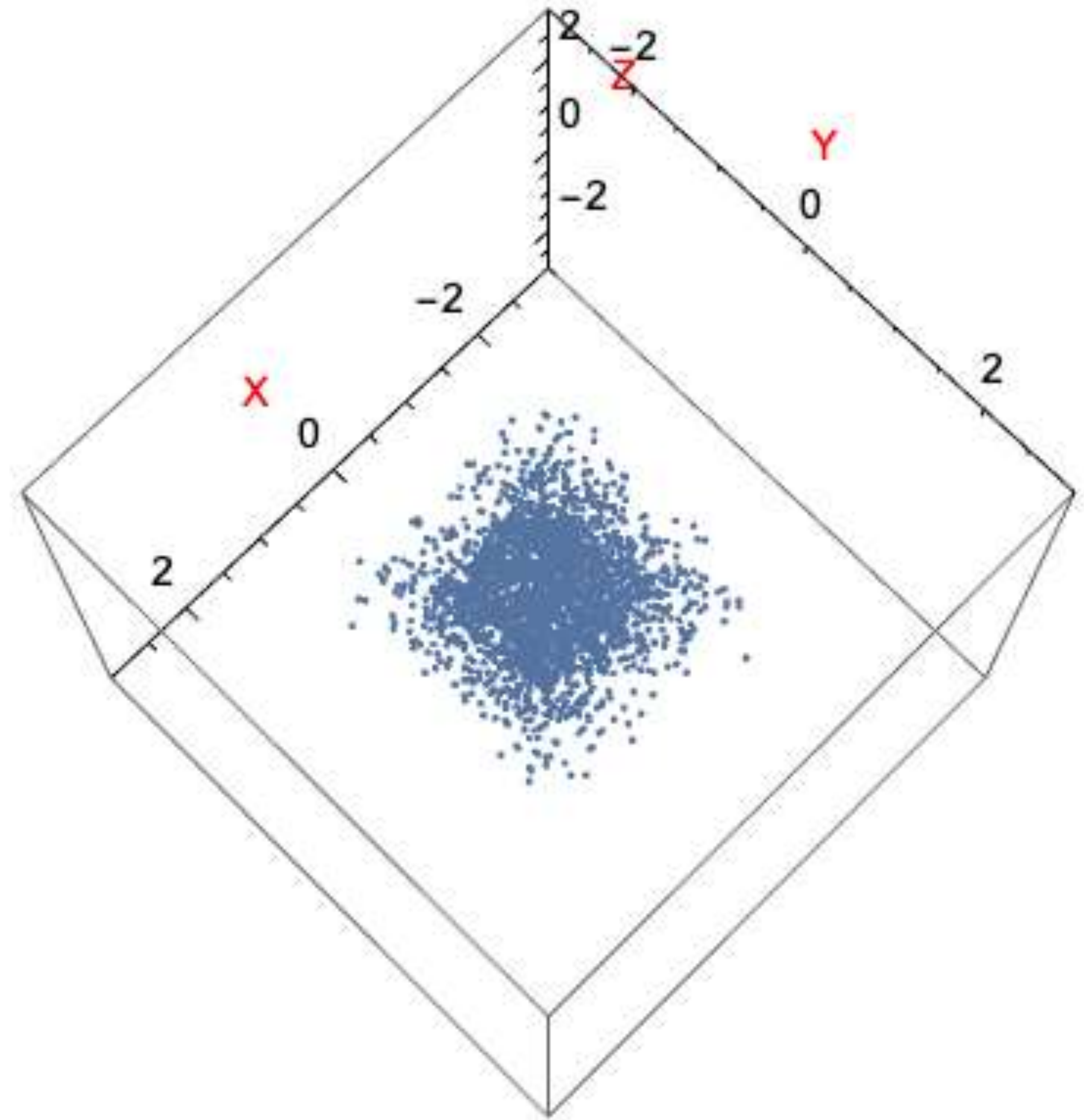
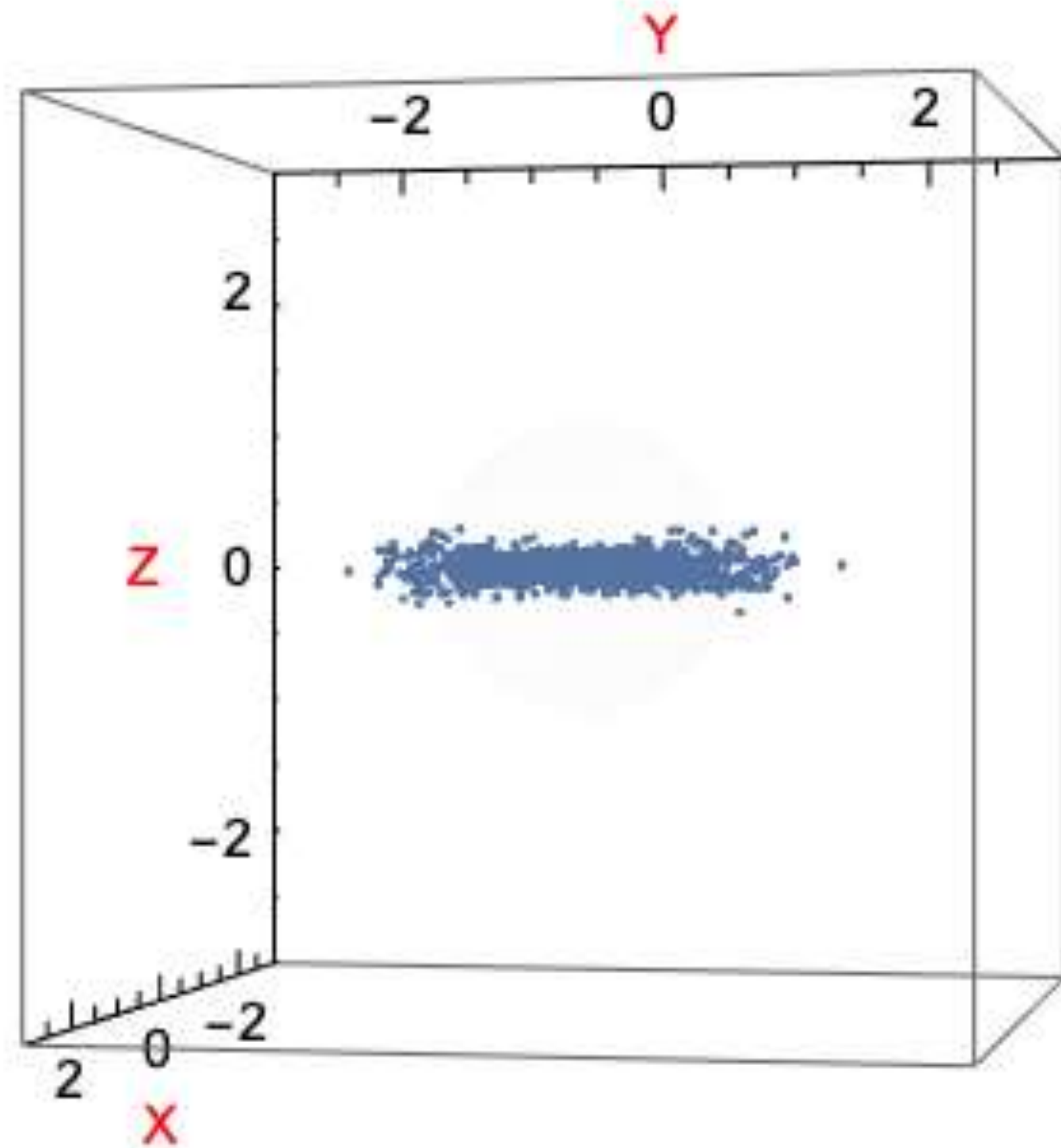
$j=50, Q = 0.58$



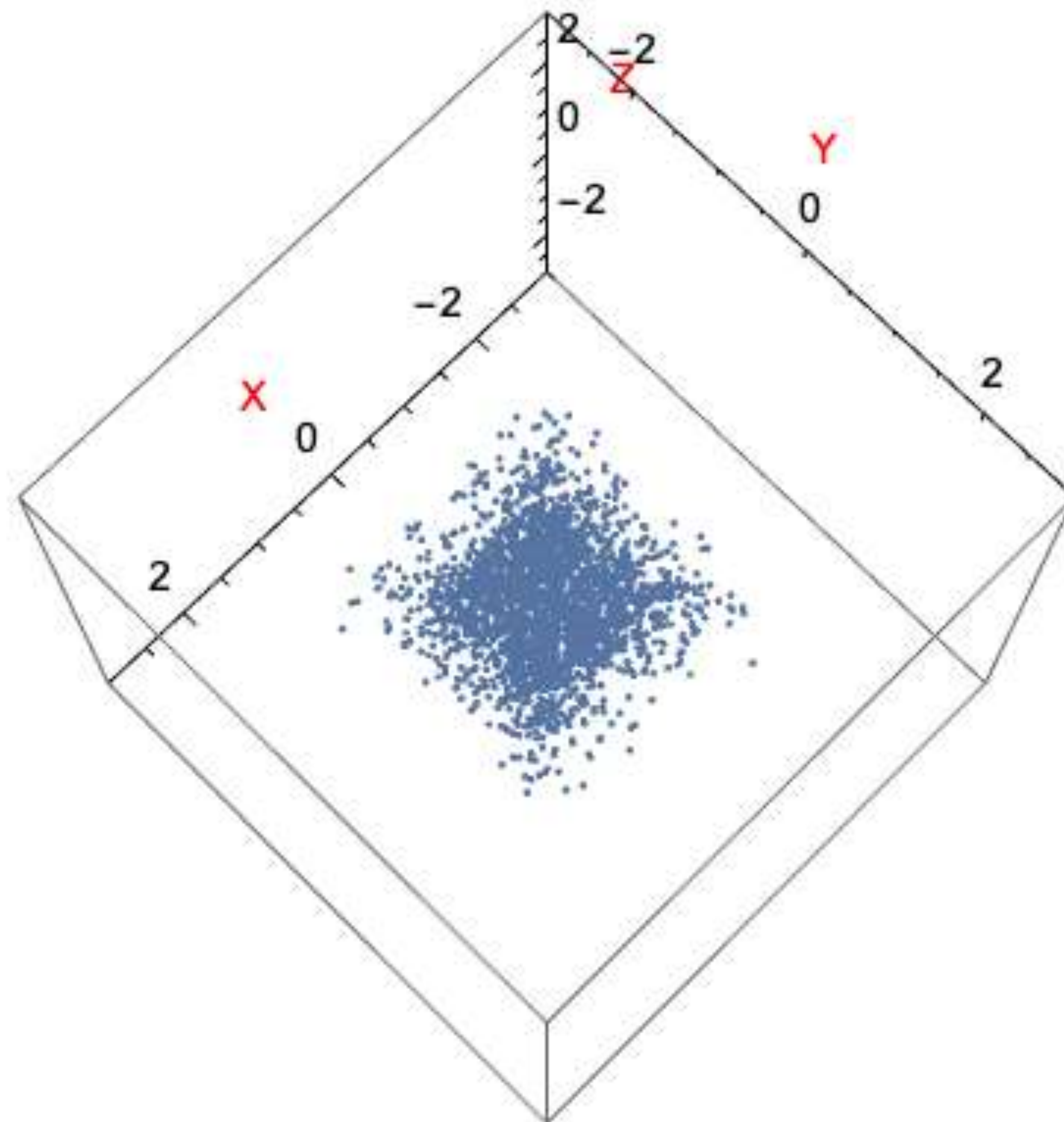
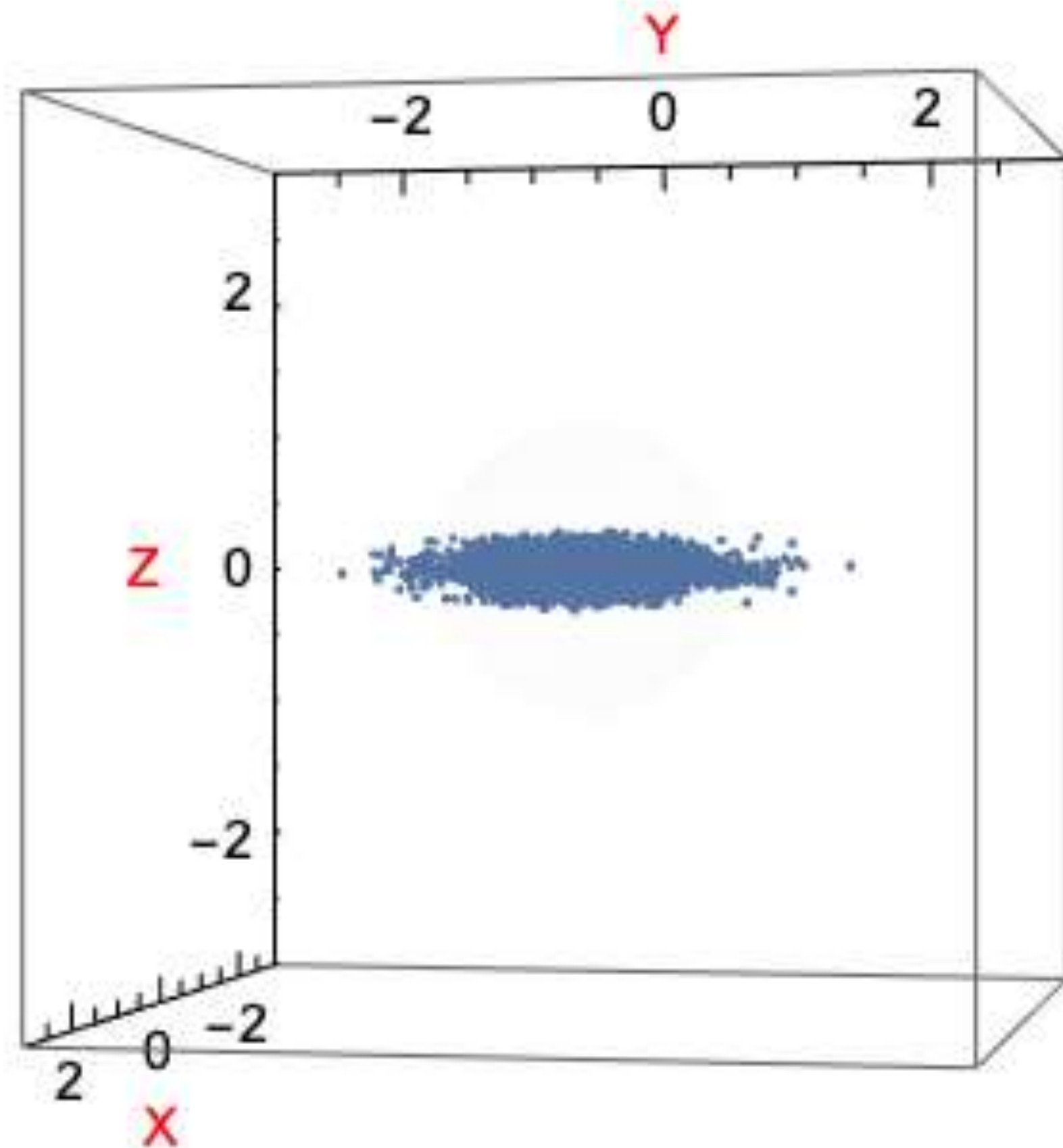
$j=55, Q = 0.59$



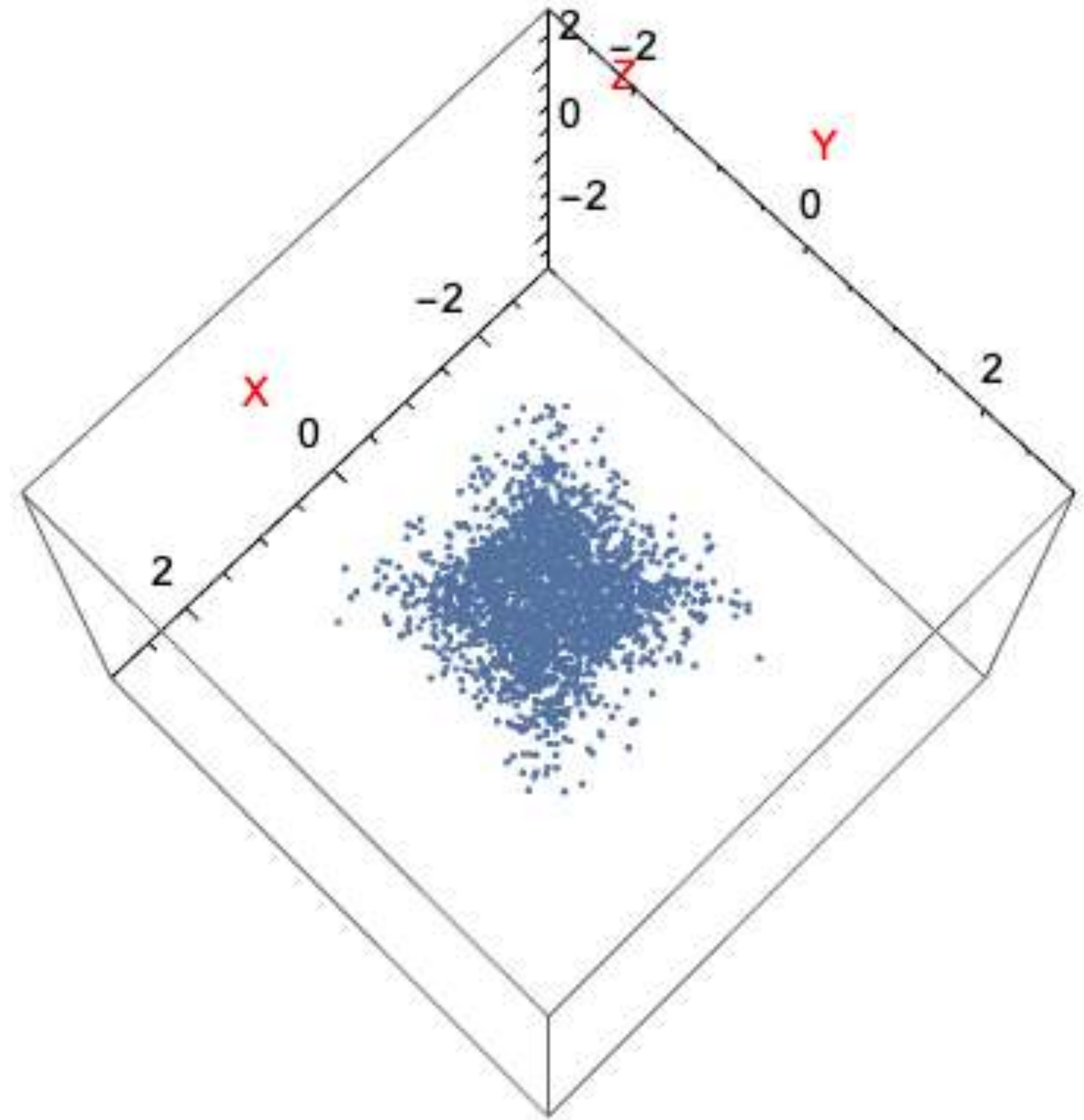
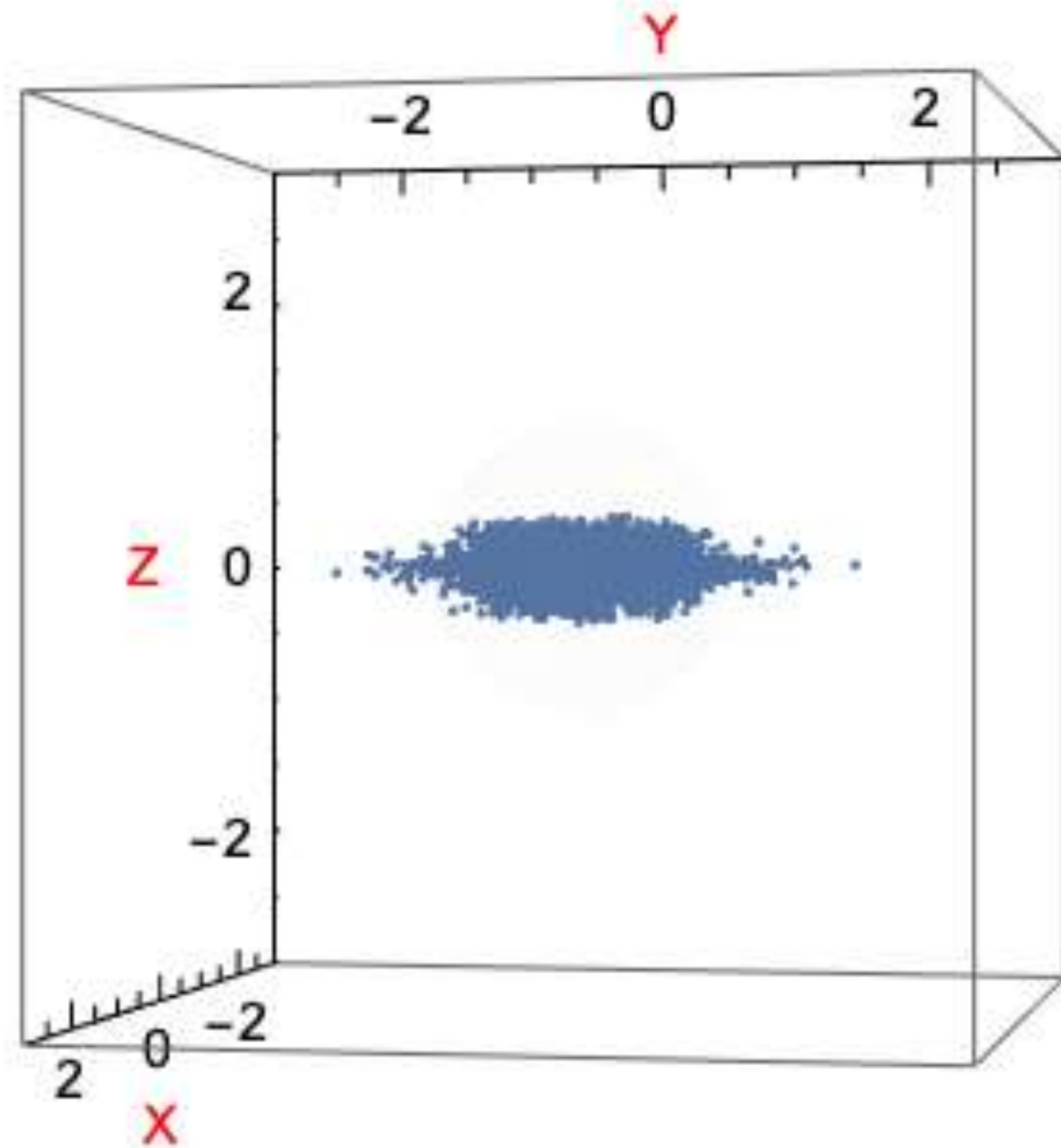
$j=59, Q = 0.59$



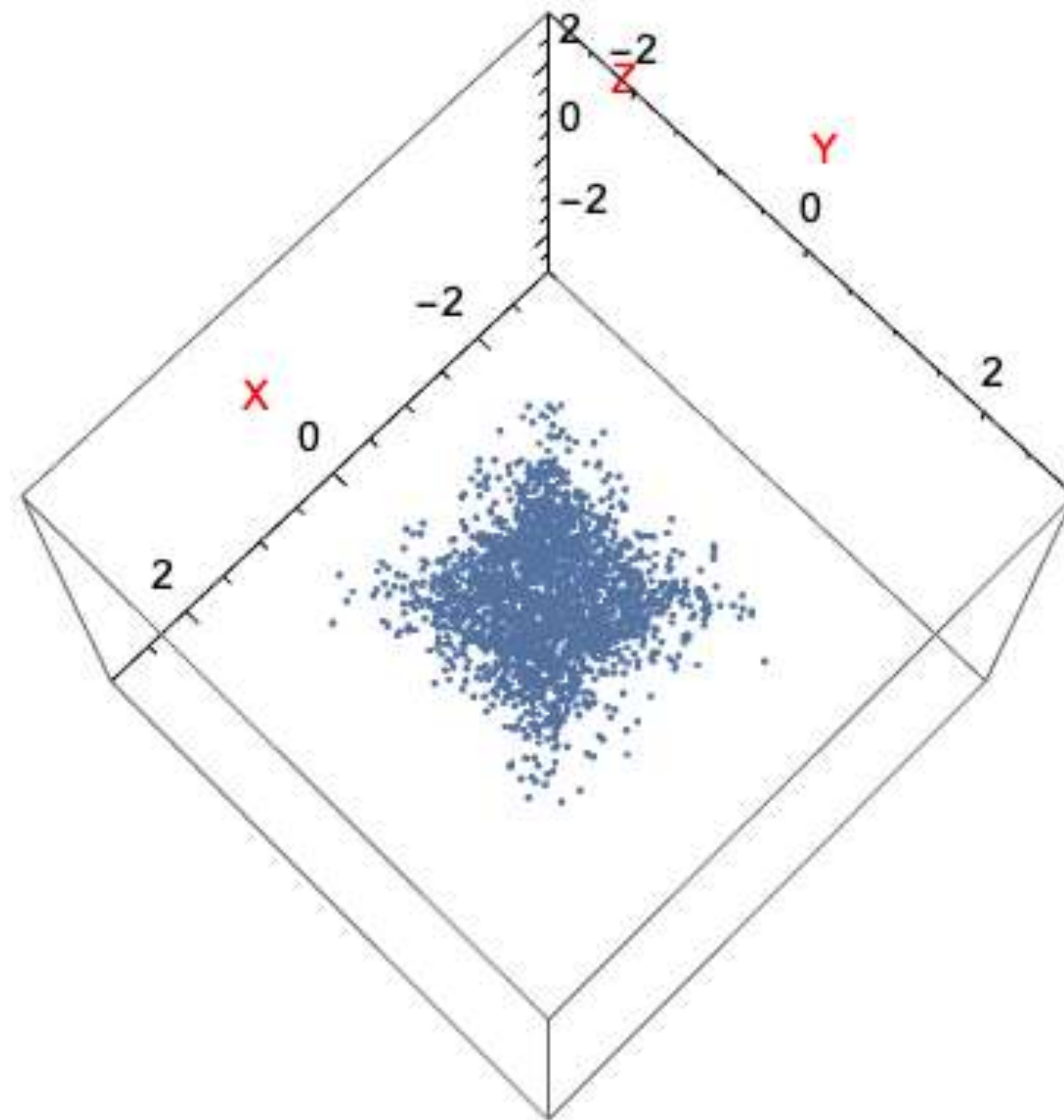
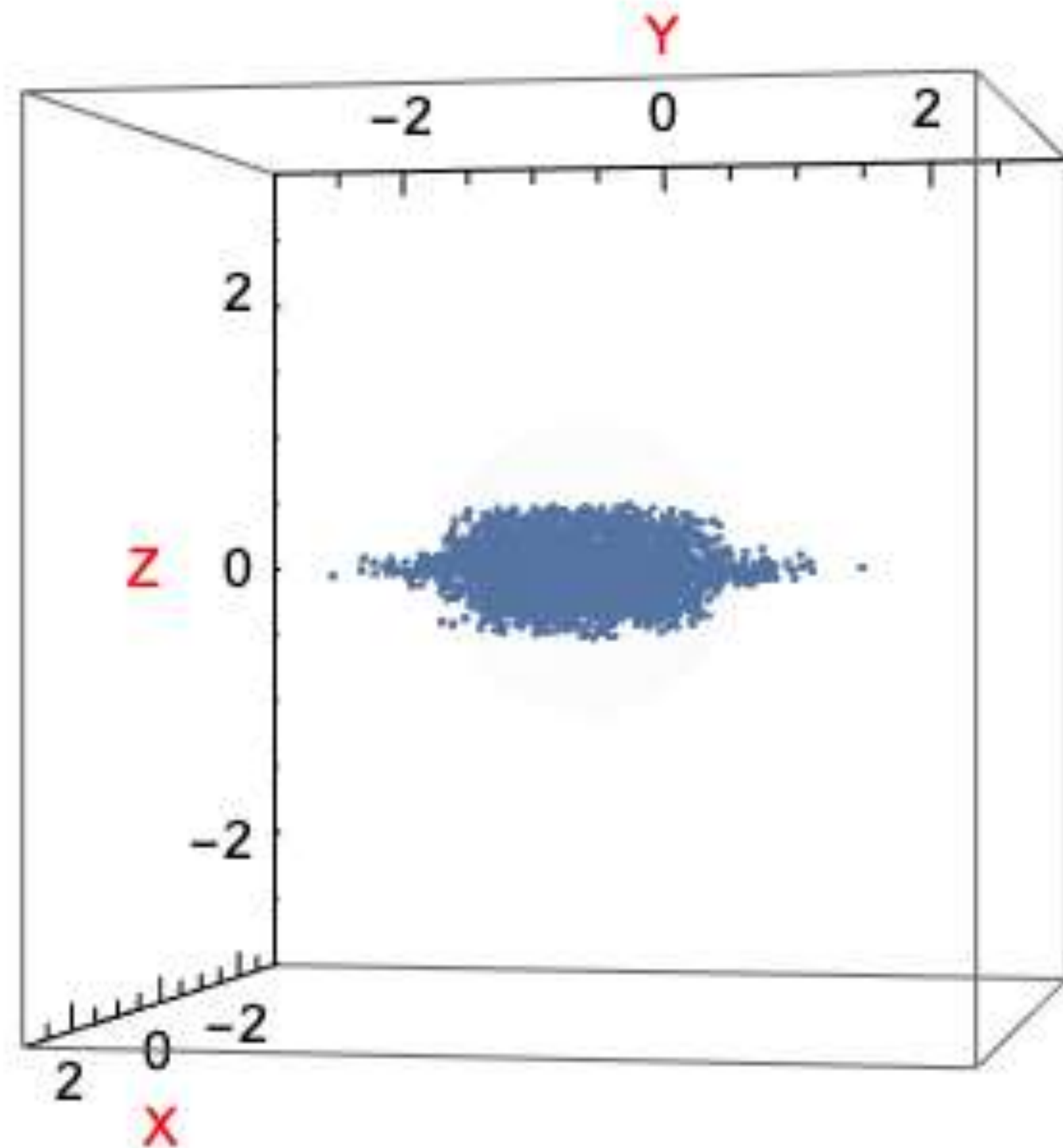
$j=65, Q = 0.56$



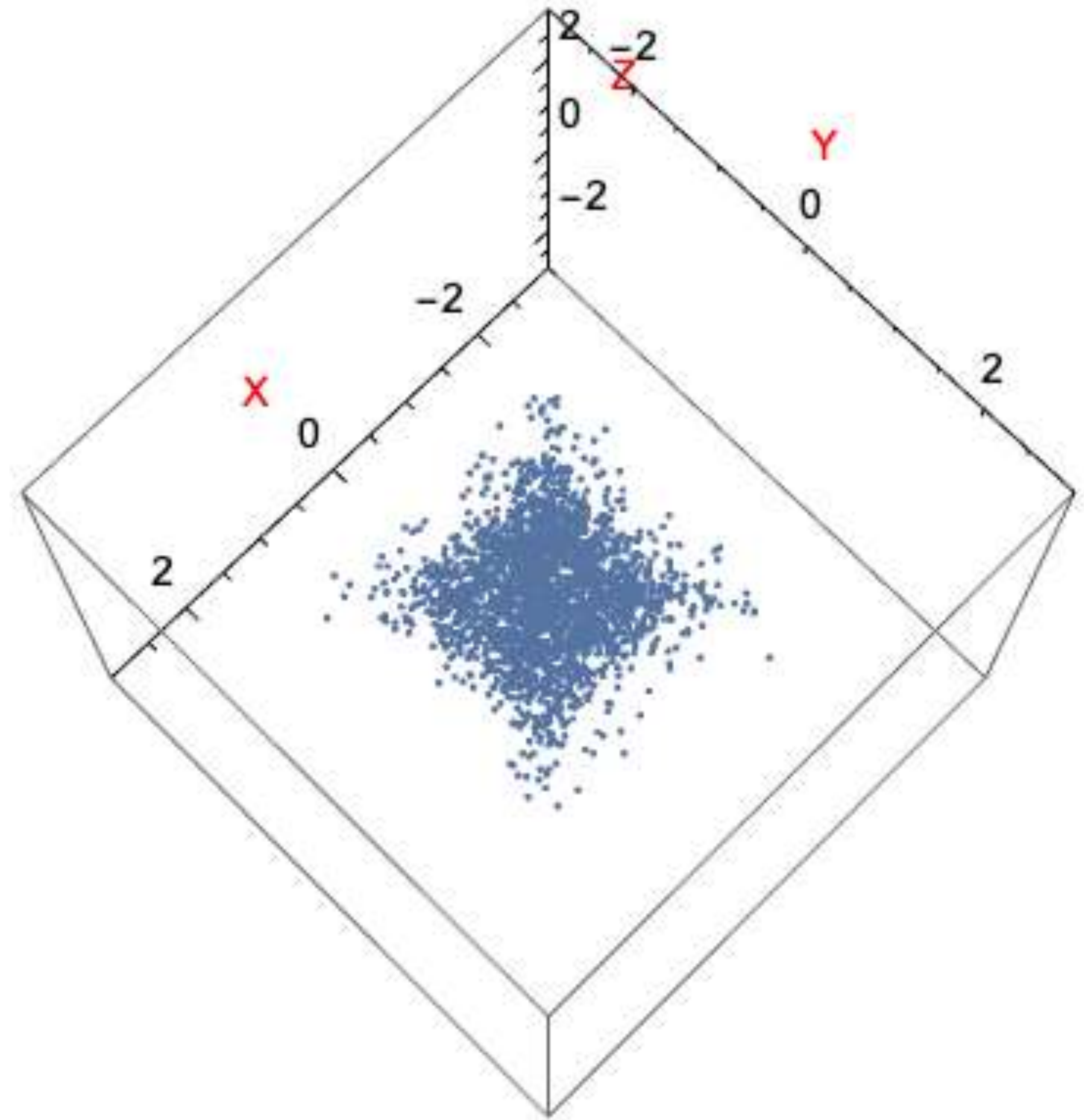
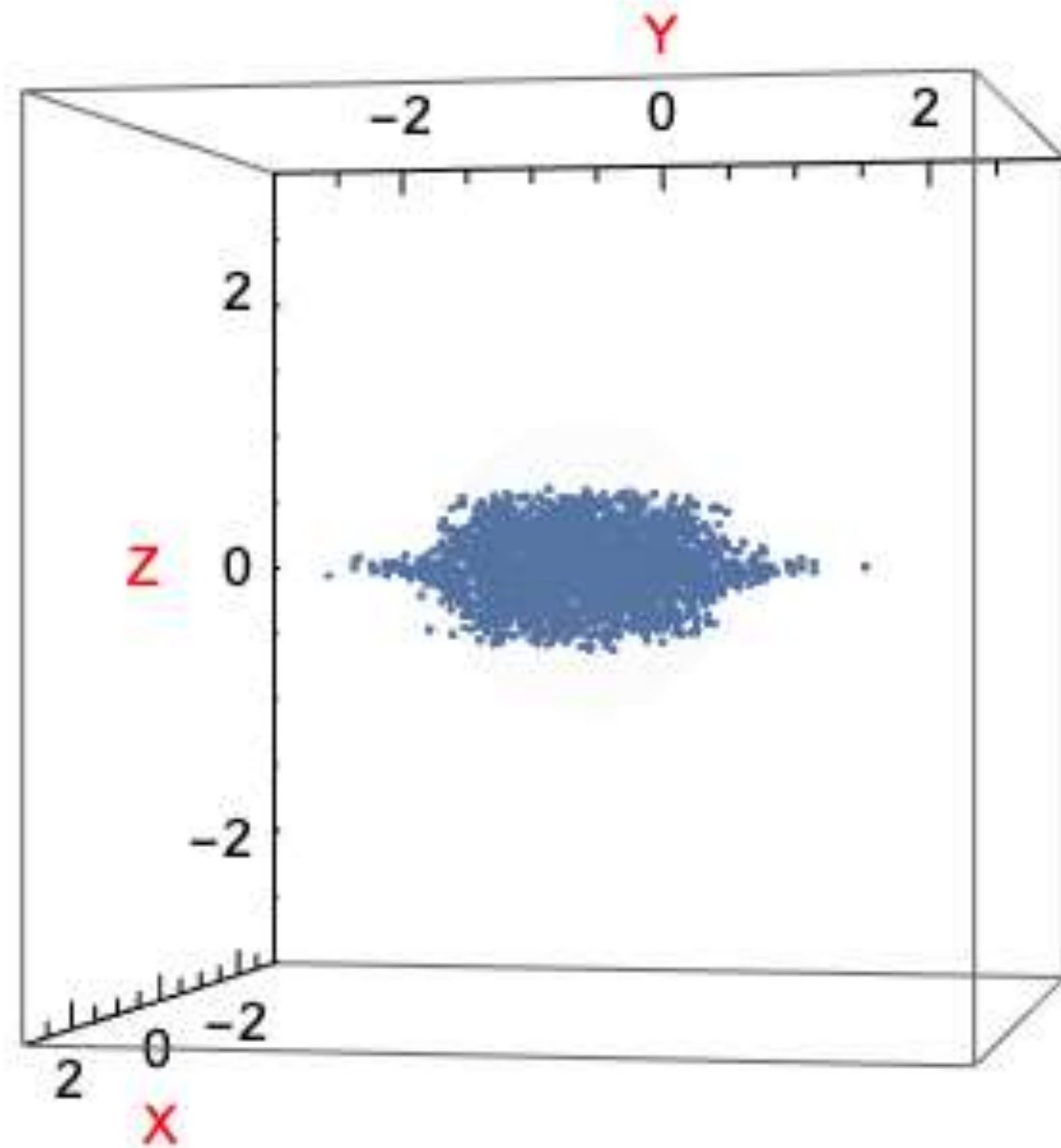
$j=70, Q = 0.54$



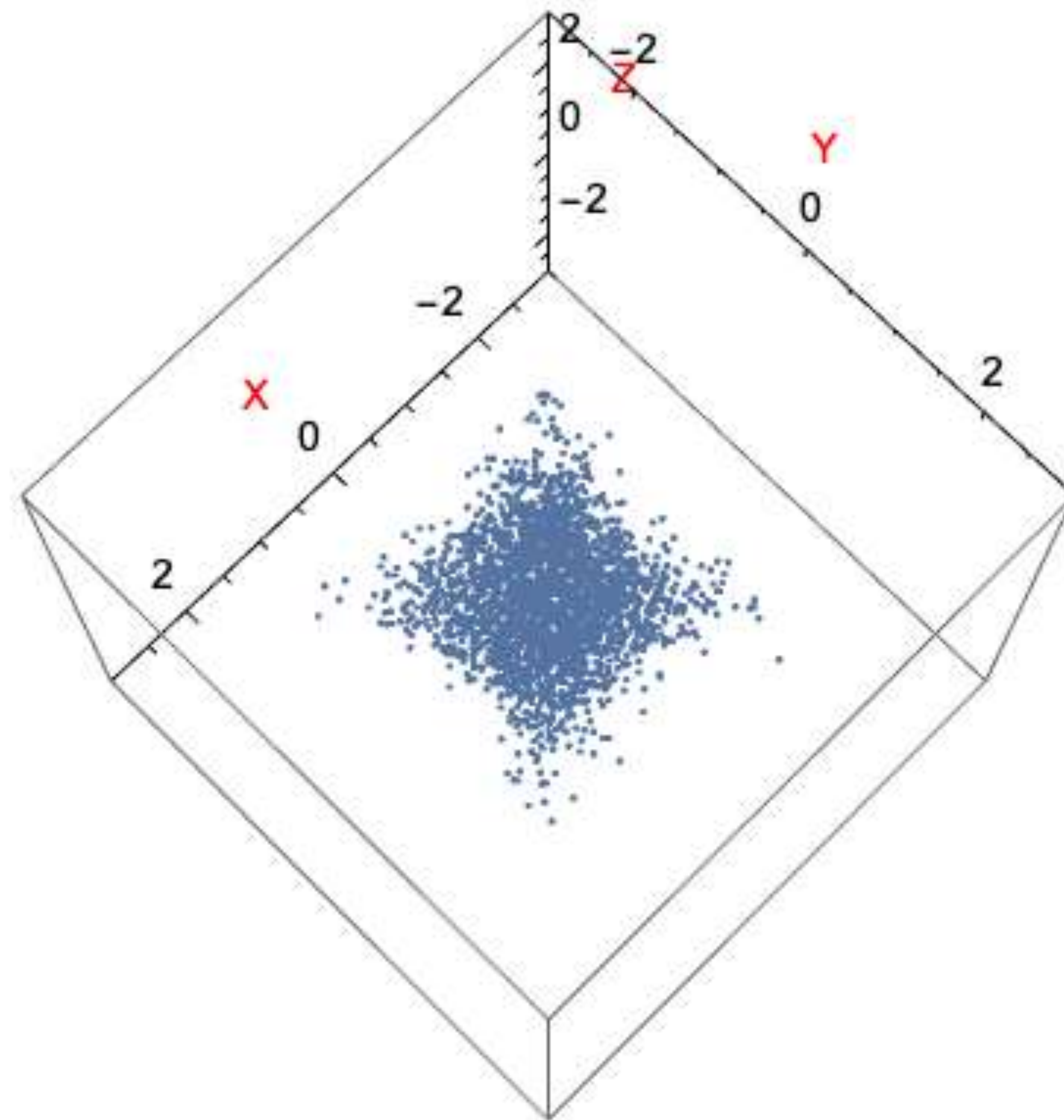
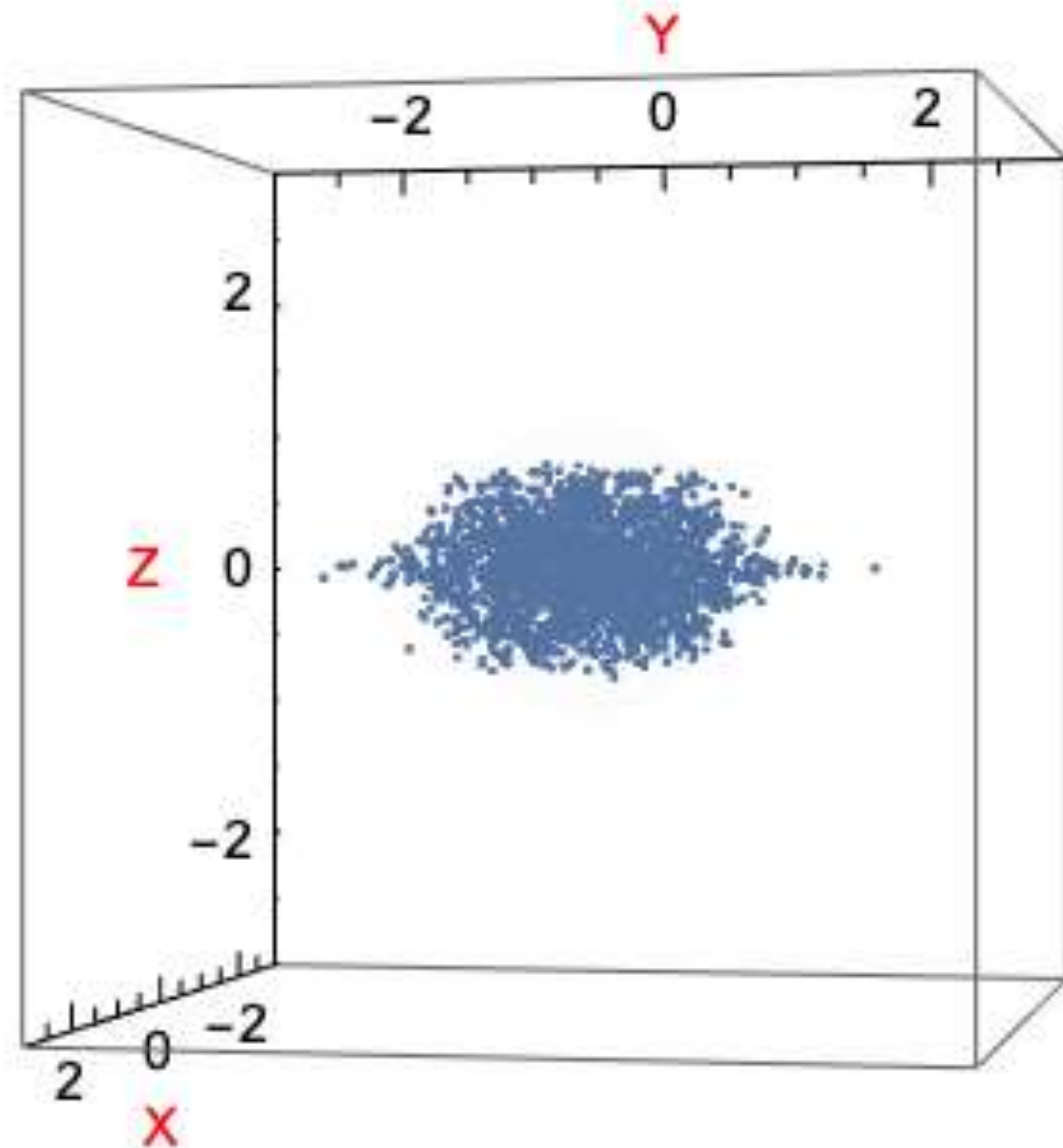
$j=75, Q = 0.52$



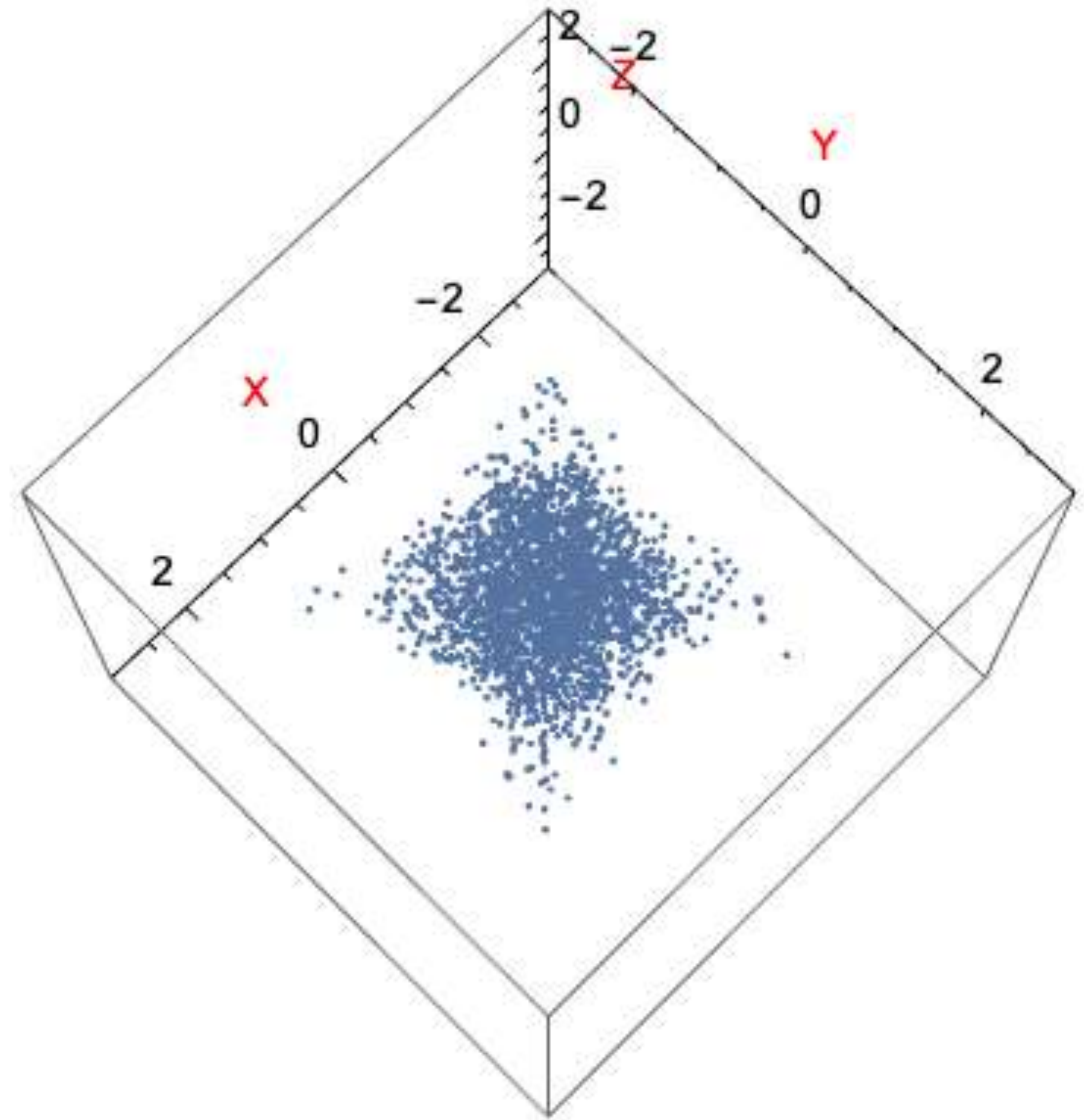
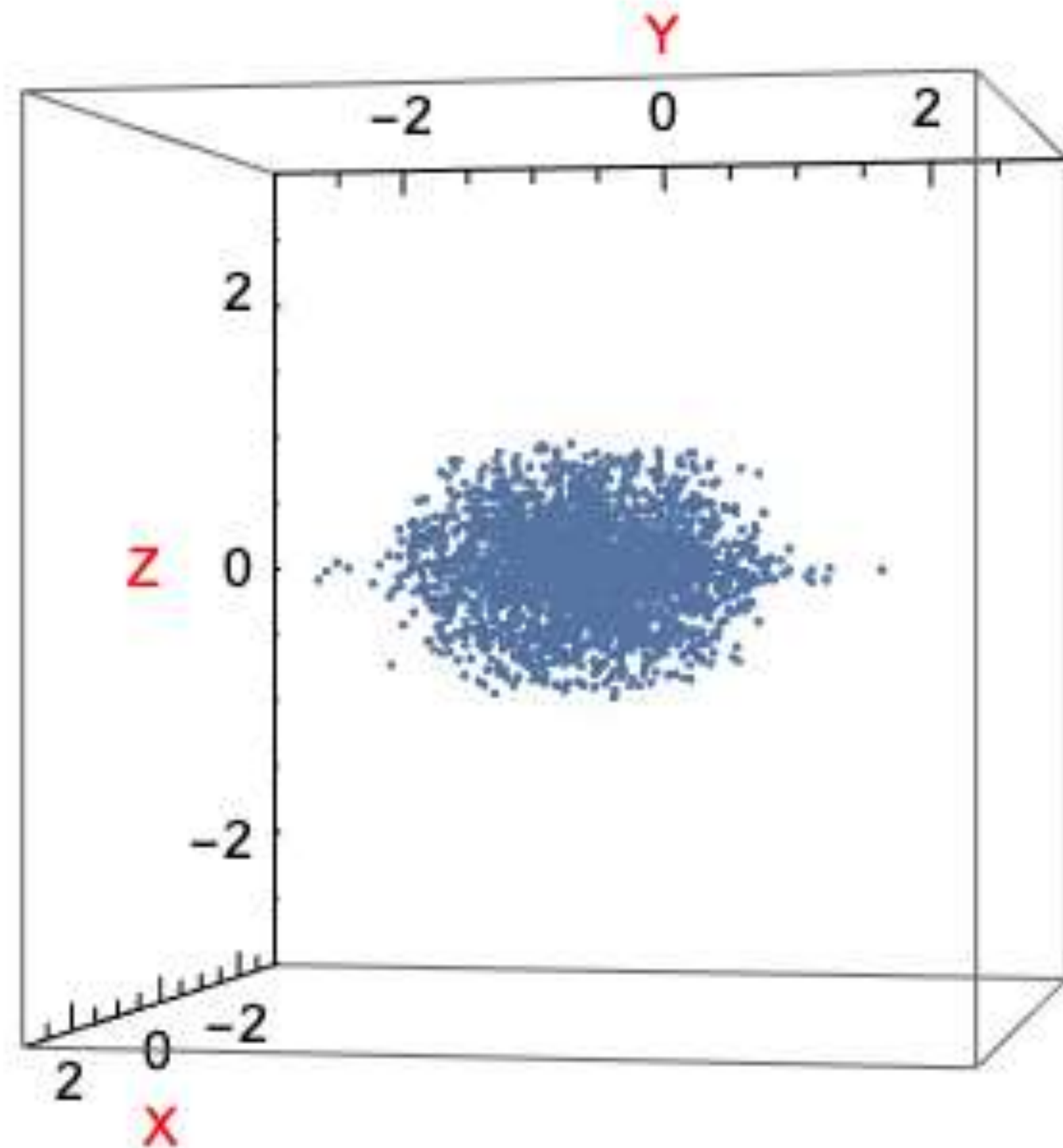
$j=80, Q = 0.51$



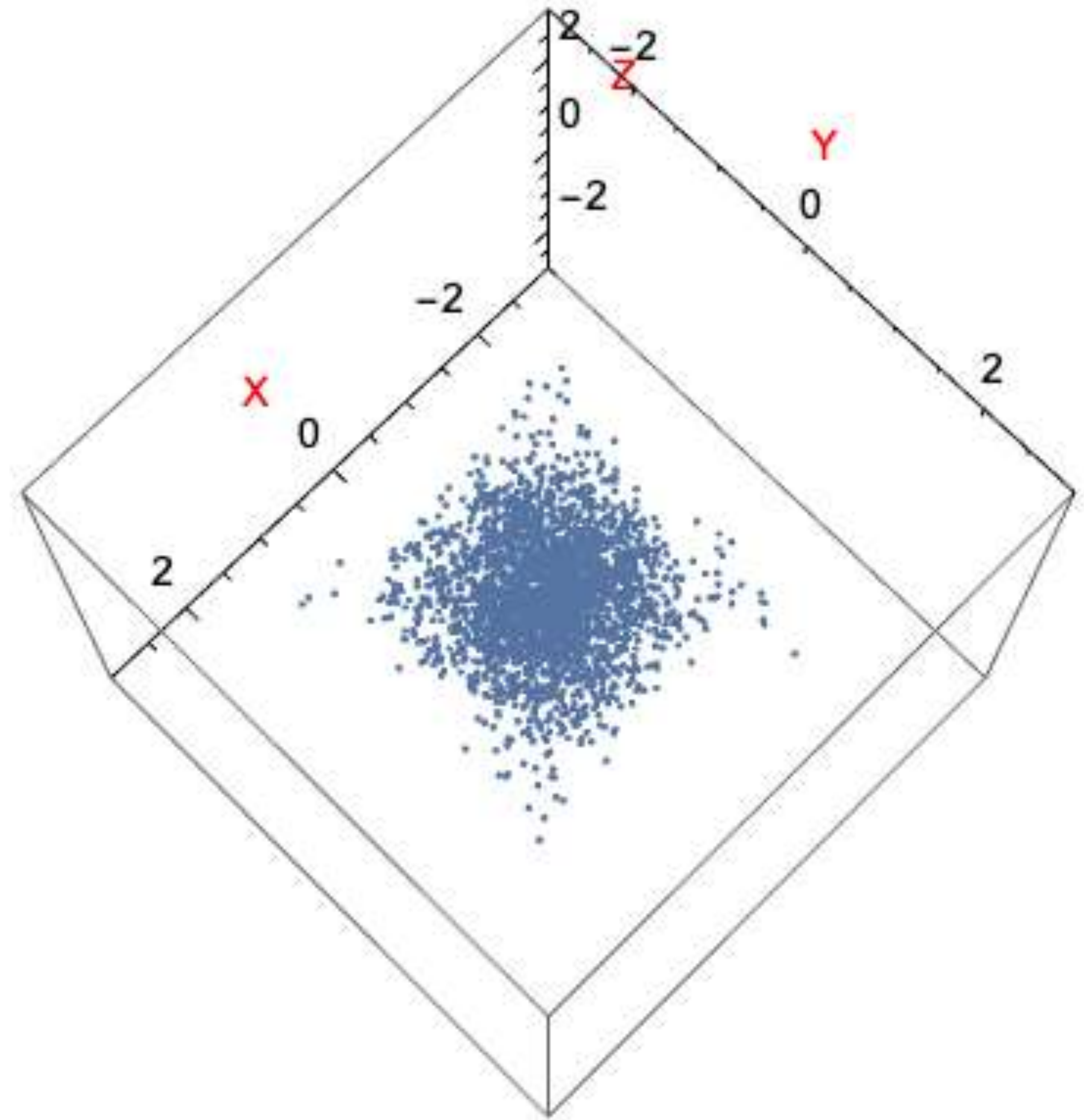
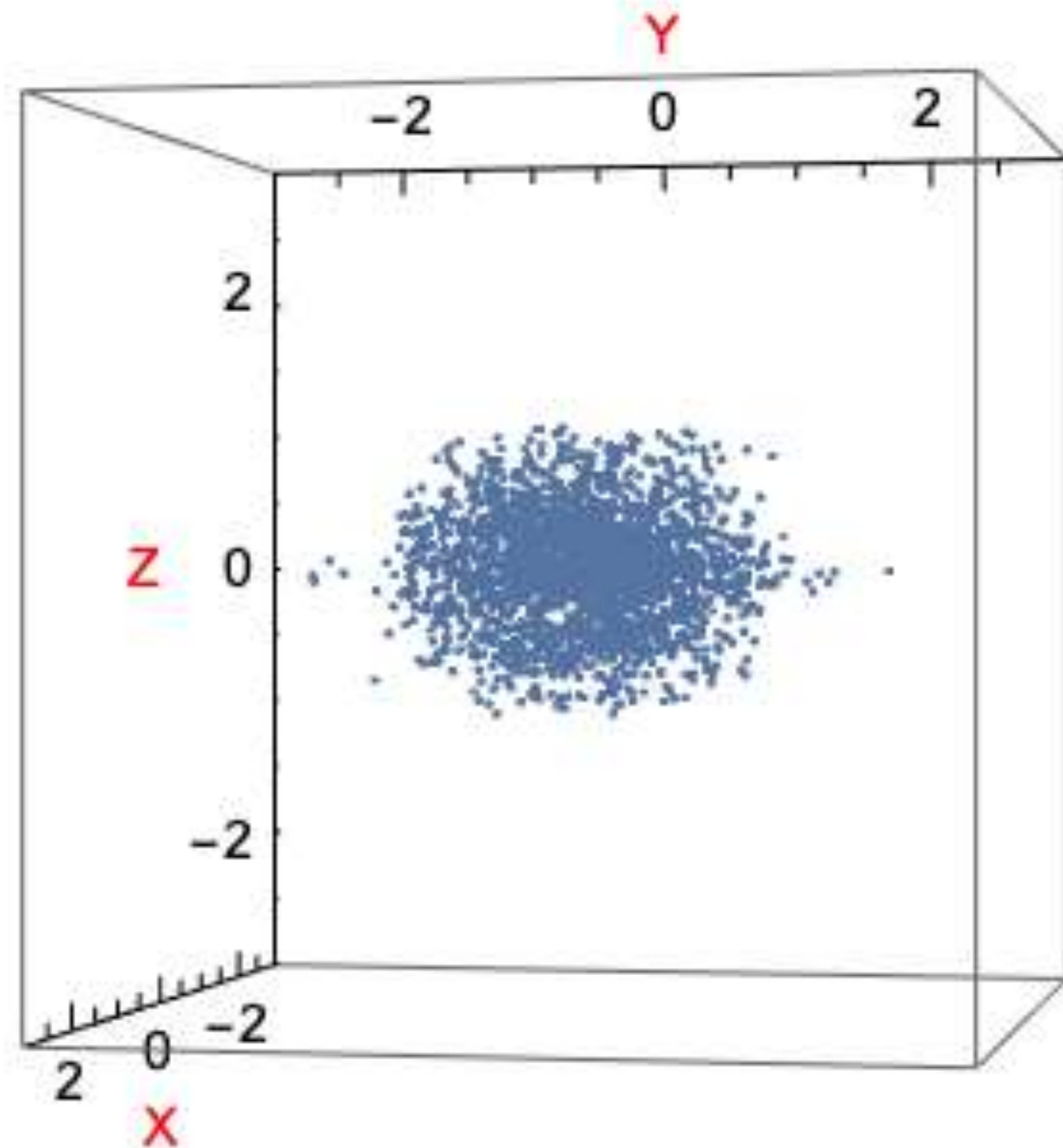
$j=90, Q = 0.48$



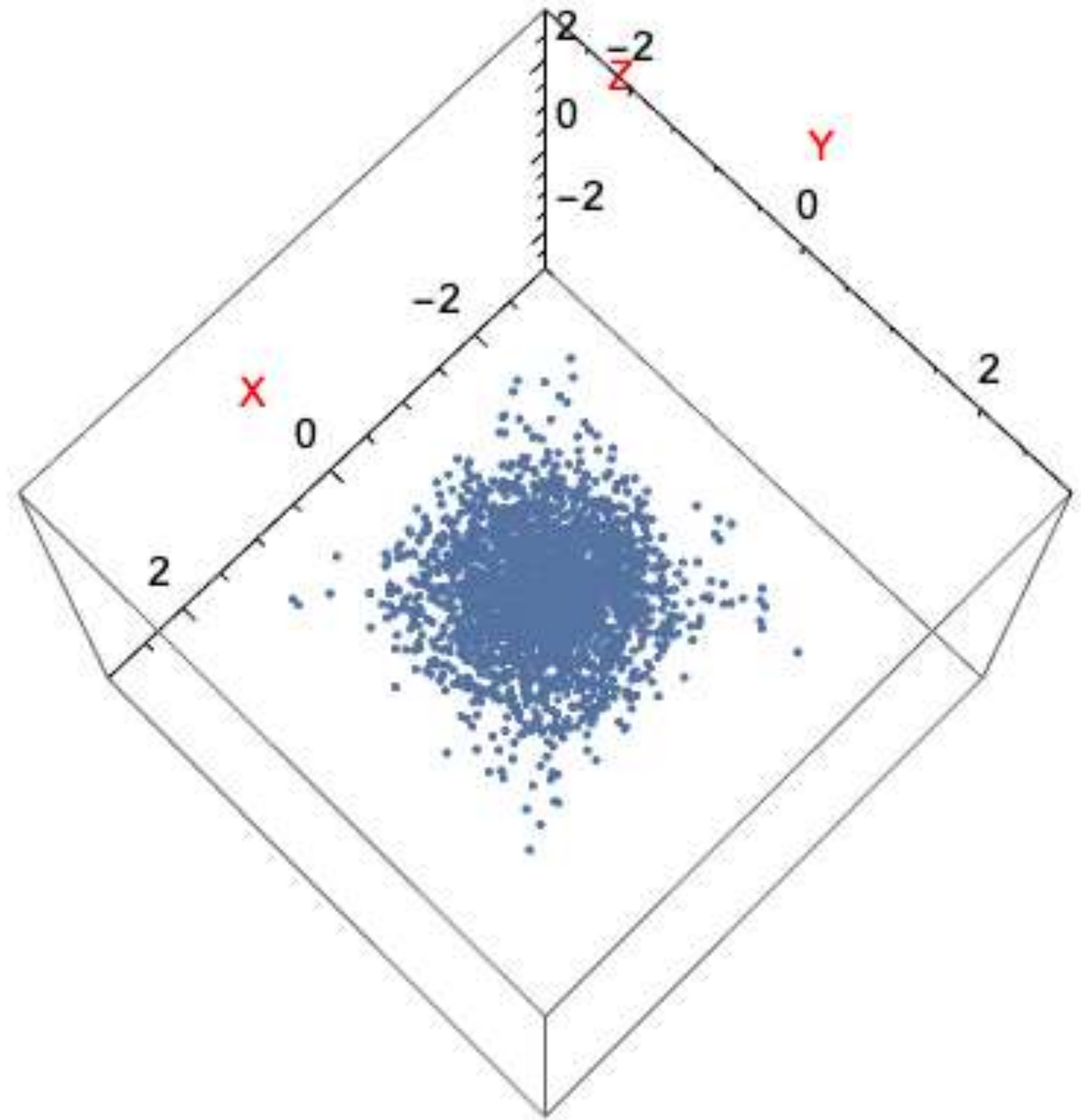
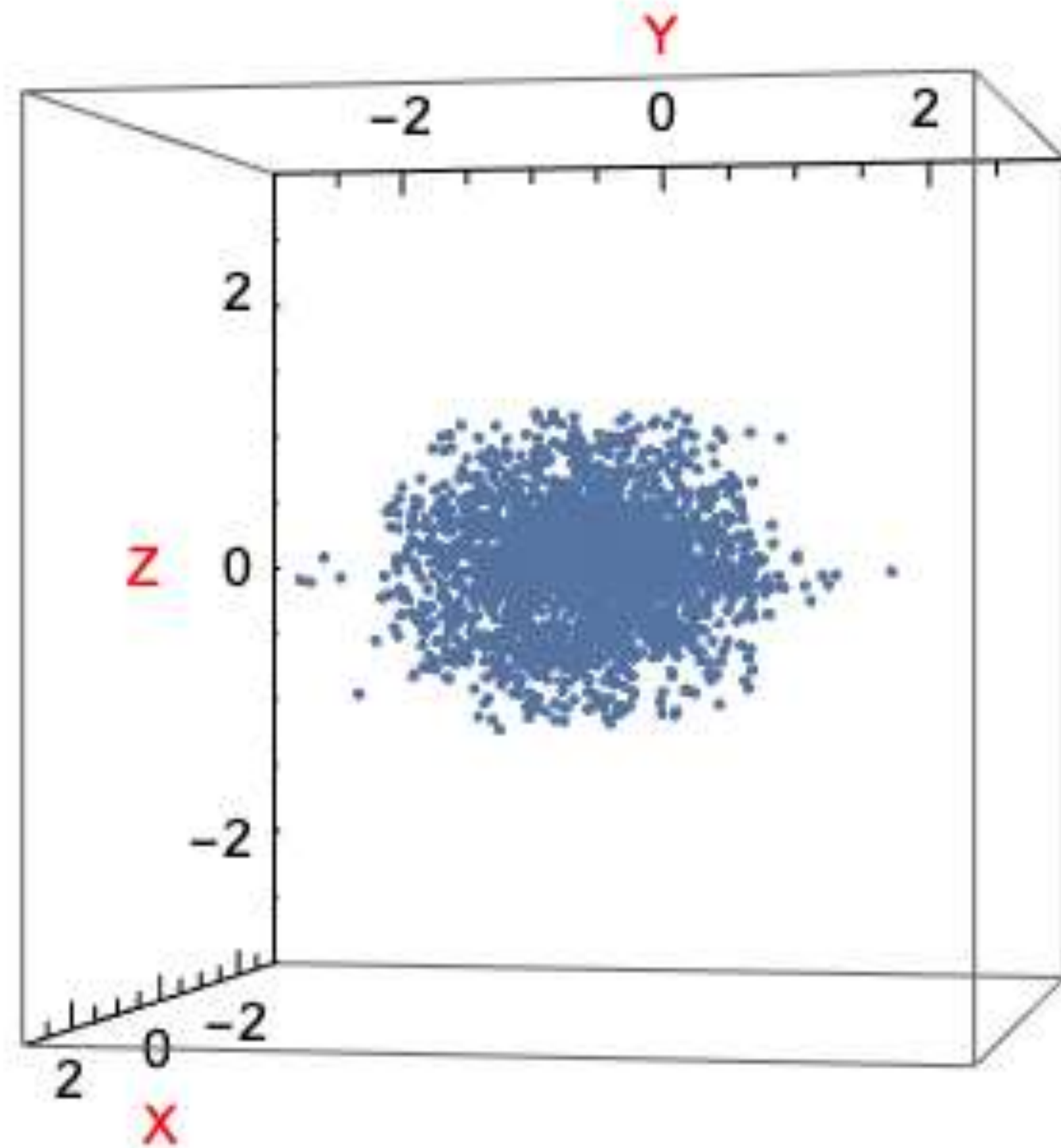
$j=100, Q = 0.47$



$j=110, Q = 0.46$



$j=120, Q = 0.45$



- ◆ The power emitted in the form of GWs from each Hubble patch that encloses a perturbation with a quadrupole tensor Q_{ij} is

$$\frac{dE_e}{dt} = \frac{G}{5c^5} \sum_{ij} \ddot{Q}_{ij}(t) \ddot{Q}_{ji}(t)$$

- ◆ We estimate the size of deformation (α , β , γ) from the spherical size in each direction utilising the Doreshkevich probability density

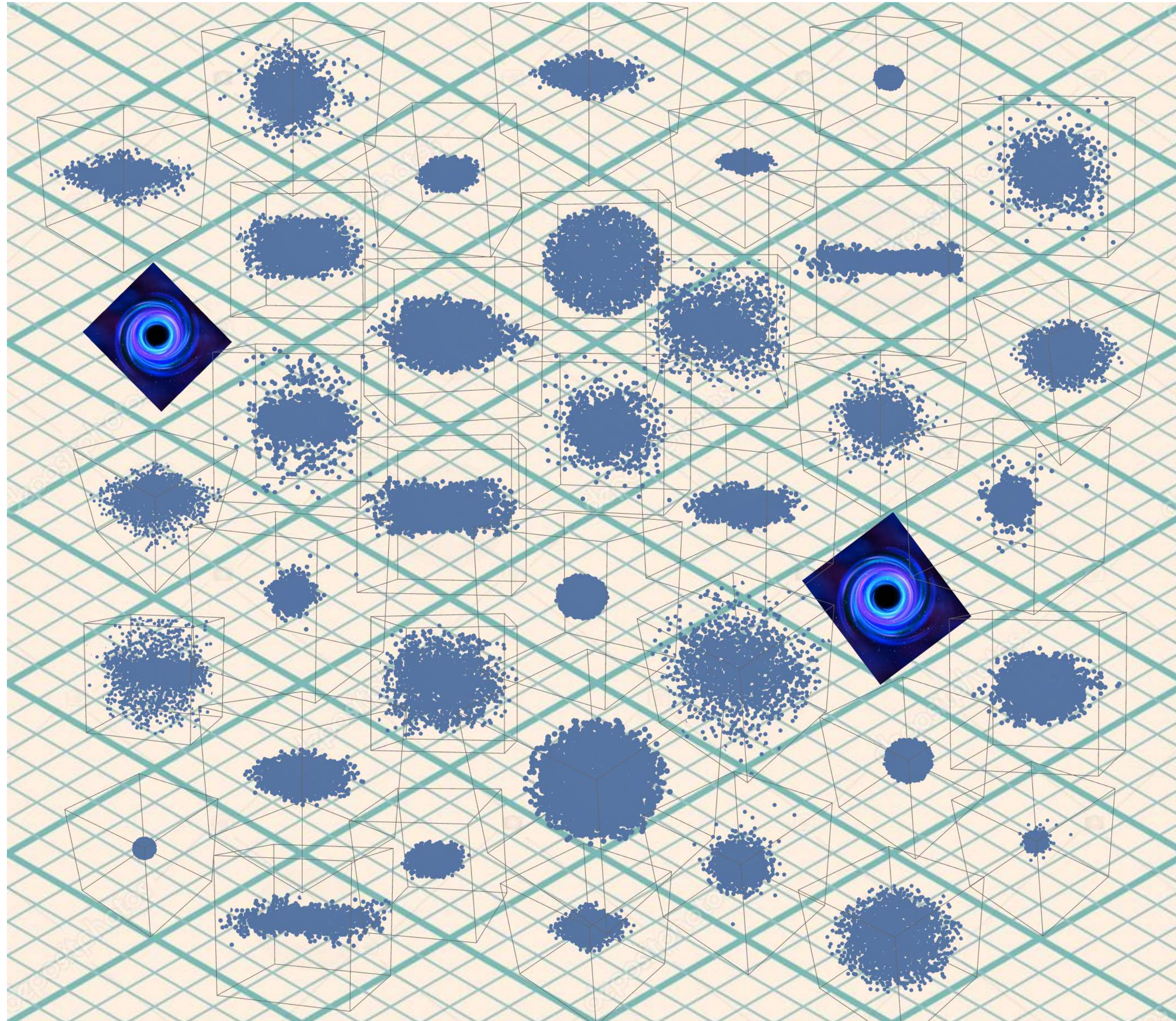
$$\mathcal{F}_D(\alpha, \beta, \gamma, \sigma_3) d\alpha d\beta d\gamma = -\frac{27}{8\sqrt{5}\pi\sigma_3^6} \times \exp \left[-\frac{3}{5\sigma_3^2} \left((\alpha^2 + \beta^2 + \gamma^2) - \frac{1}{2}(\alpha\beta + \beta\gamma + \gamma\alpha) \right) \right] \times (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha) d\alpha d\beta d\gamma$$

A.G. Doroshkevich, 1970

- ◆ The differential energy density per observed logarithmic frequency interval and deformation configuration is

$$\frac{d\rho_{GW}(t_0, f_0)}{d\alpha d\beta d\gamma d \ln f_0} = \sum_N \frac{1}{1+z_N} \frac{4\pi G}{5c^5} \sum_{ij} |\tilde{Q}_{ij}^N(2\pi f_0(1+z_N))|^2 (2\pi f_0(1+z_N))^7 \Theta(t_{rh} - t_{col}(\alpha, \beta, \gamma)) \left(\frac{4\pi}{3} q^3 \right)^{-1} \mathcal{F}_D(\alpha, \beta, \gamma, \sigma_3)$$

Schematic illustration of different Hubble patches after the horizon entry of inhomogeneities



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- ◆ The spectral energy density parameter is

$$\Omega_{GW}(t_0, f_0) = \frac{1}{\rho_{crit}(t_0)} \iiint_{\mathcal{S}} d\alpha d\beta d\gamma \sum_N \frac{1}{1+z_N} \frac{4\pi G}{5c^5} \sum_{ij} |\tilde{Q}_{ij}^N(2\pi f_0(1+z_N))|^2 (2\pi f_0(1+z_N))^7 \Theta(t_{rh} - t_{col}(\alpha, \beta, \gamma, \sigma)) \left(\frac{4\pi}{3}q^3\right)^{-1} \mathcal{F}_D(\alpha, \beta, \gamma, \sigma)$$

- ◆ **Our analytic result: The GWs emitted from time of maximum expansion until the time of pancake collapse:**

$$\Omega_{GW}(t_0, f_0) = \frac{1}{\rho_{crit}} \left(\frac{4\pi}{3}q^3\right)^{-1} \frac{4\pi G}{5c^5} \frac{2\pi f_0}{54\pi^2} M^2 \sigma^2 \iiint_{\mathcal{S}} d\alpha d\beta d\gamma \left(\frac{2\alpha}{\alpha + \beta + \gamma}\right)^2 \times$$

$$\left[1 + \left(\frac{\beta}{\alpha}\right)^4 + \left(\frac{\gamma}{\alpha}\right)^4 - \left(\frac{\gamma}{\alpha}\right)^2 - \left(\frac{\beta}{\alpha}\right)^2 \left(1 + \left(\frac{\gamma}{\alpha}\right)^2\right) \right] \times$$

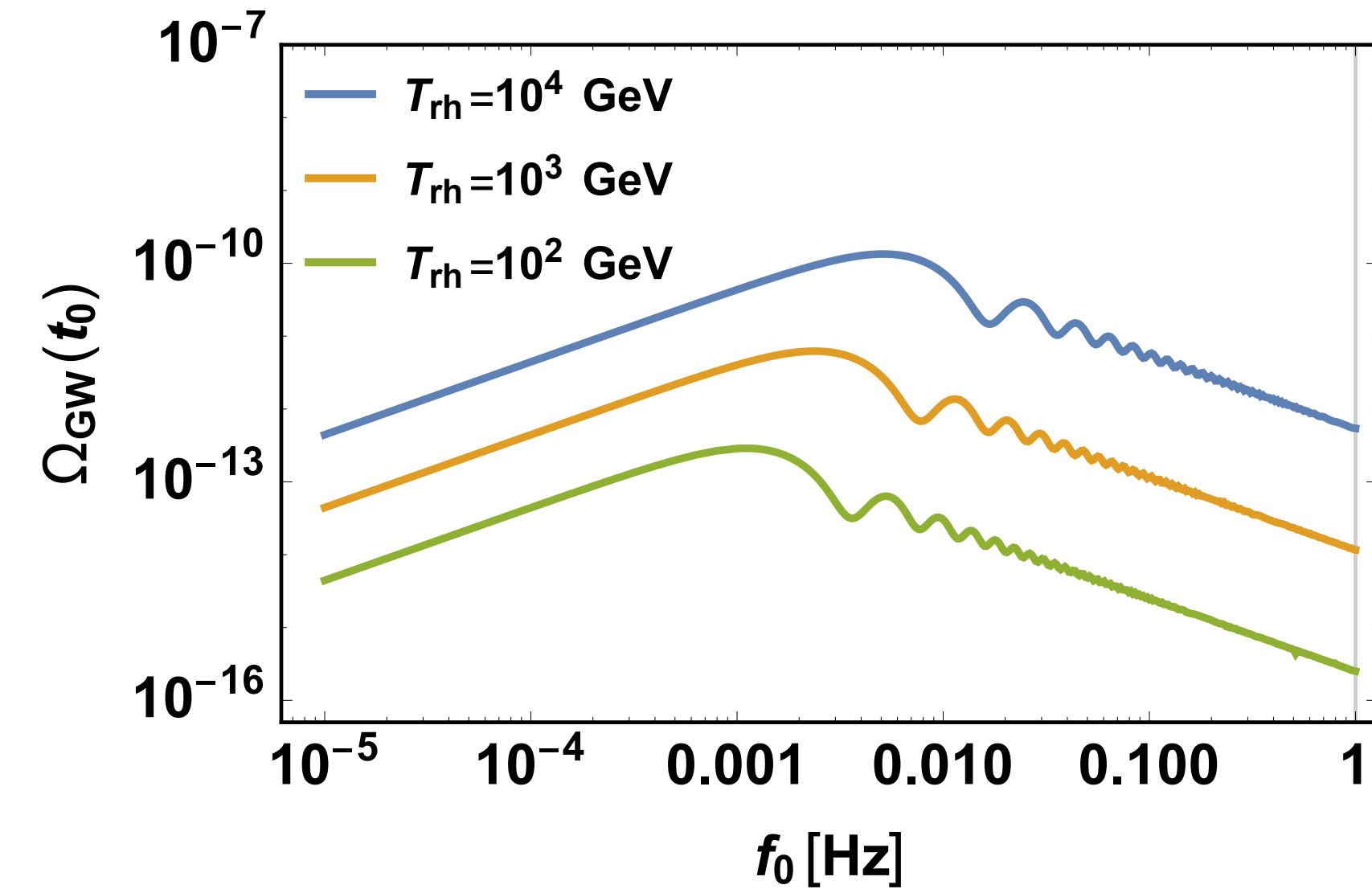
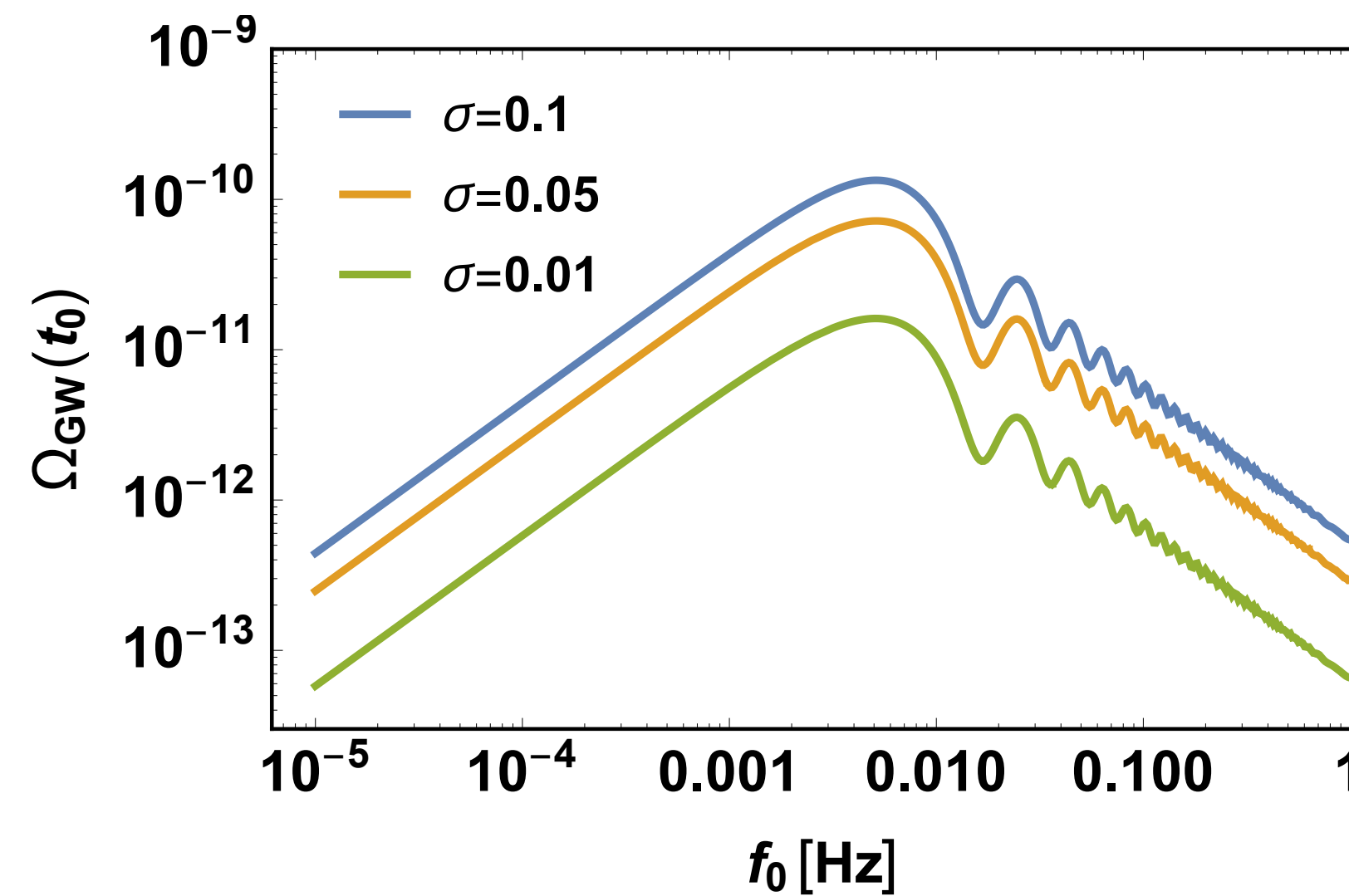
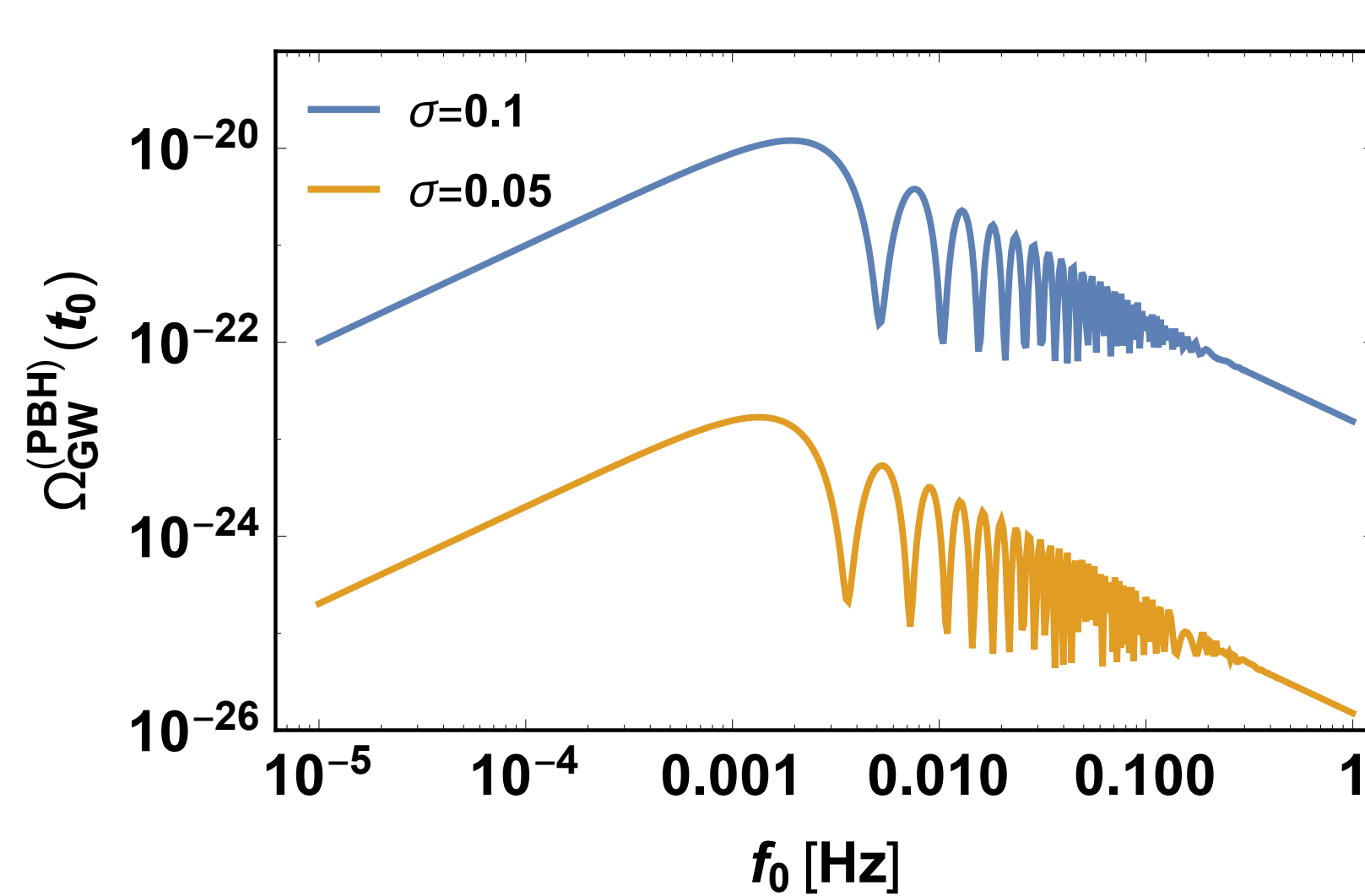
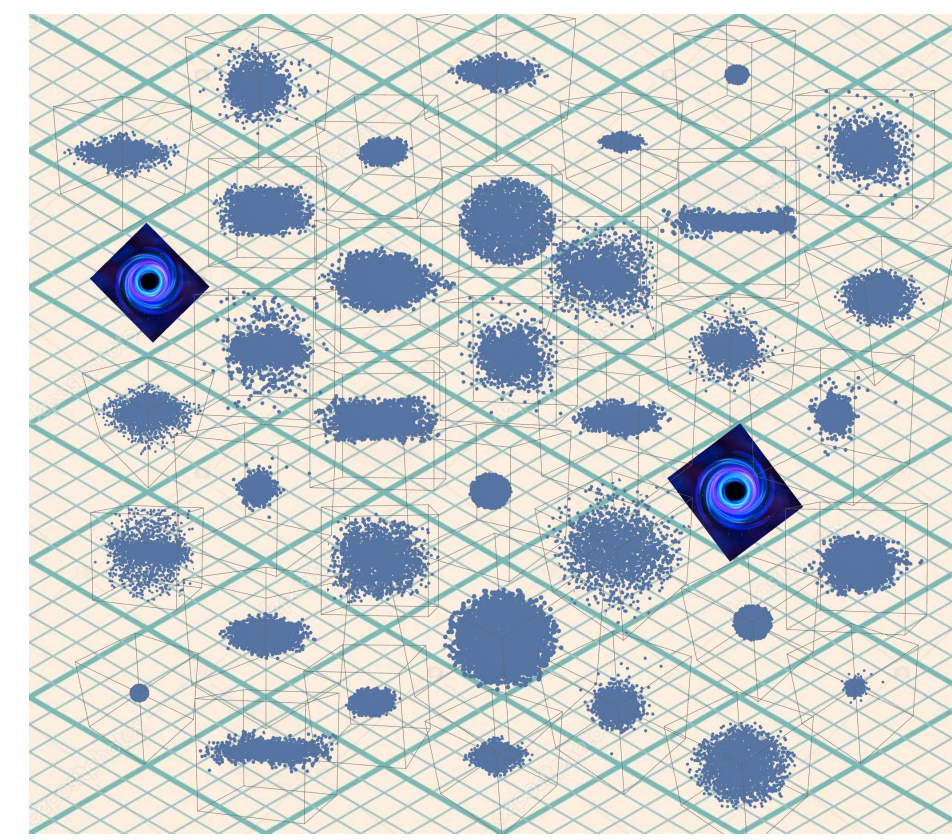
$$\left| \text{Ei} \left[\frac{1}{3}, it_{max}(\alpha, \beta, \gamma, \sigma) 2\pi f_0(1+z_{col}(\alpha, \beta, \gamma, \sigma)) \right] - 2\text{Ei} \left[\frac{1}{3}, it_{col}(\alpha, \beta, \gamma, \sigma) 2\pi f_0(1+z_{col}(\alpha, \beta, \gamma, \sigma)) \right] \right|^2 \times$$

$$\Theta(t_{rh} - t_{col}(\alpha, \beta, \gamma, \sigma)) \mathcal{F}_D(\alpha, \beta, \gamma, \sigma_3)$$

$$\begin{aligned} 0 < \alpha < \infty \\ \frac{\alpha}{2}(1-2\sigma) < \beta < \alpha \\ -\beta - \alpha(1-2\sigma) < \gamma < \beta \end{aligned}$$

The spectrum of GW produced during eMD (halo formation)

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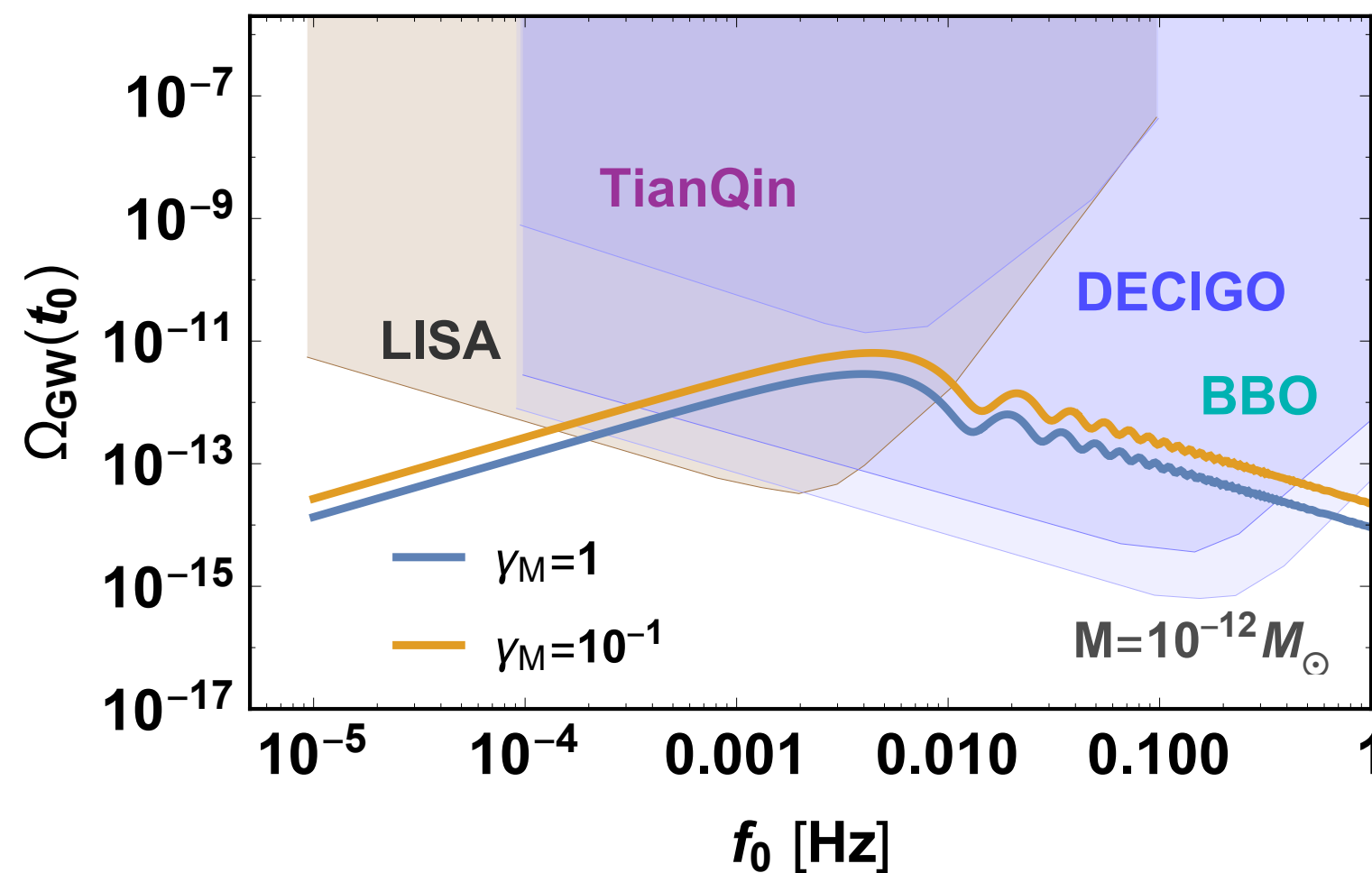


- 1) The GW signal from the patches that collapsed and formed PBHs
- 2) The GW signal for different amplitudes of the density variance (or enhancements of the $\mathcal{P}_{\mathcal{R}}(k)$)
- 3) The GW signal for different reheating temperatures

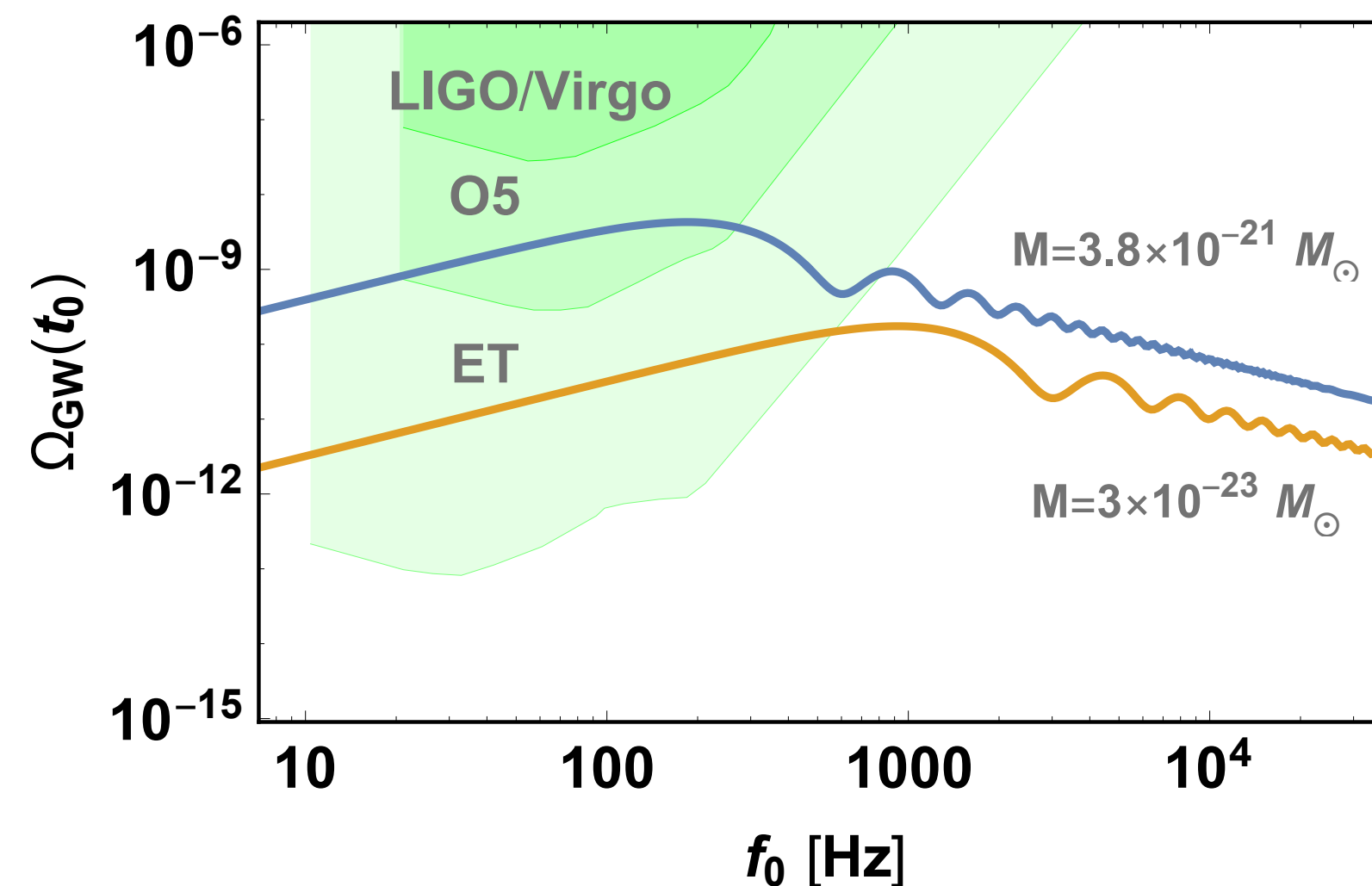
The spectrum of GW produced during eMD (halo formation)

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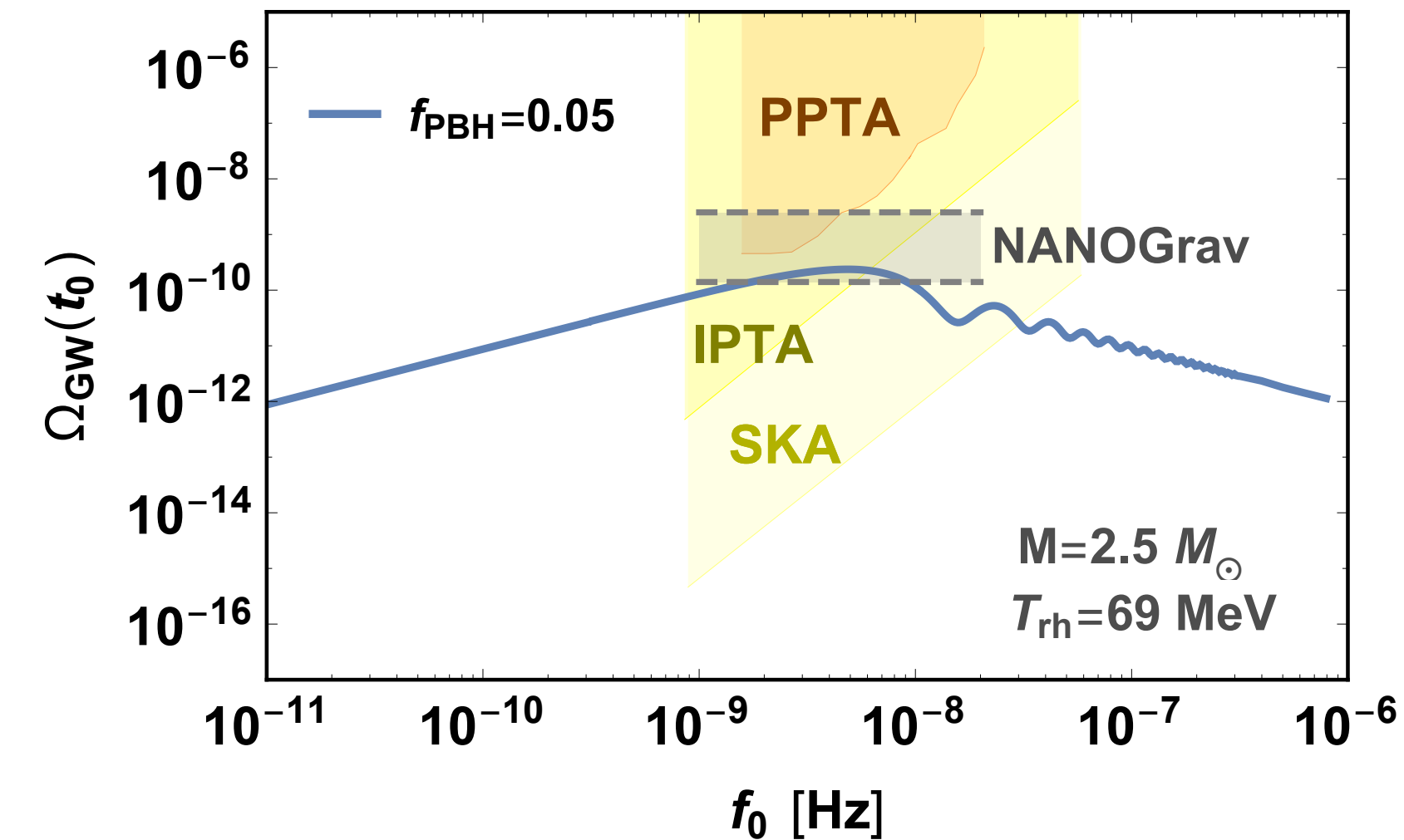
Z. Arzoumanian et al. [NANOGrav], “The NANOGrav 12.5 yr Data Set: Search for an Isotropic Stochastic Gravitational-wave Background,”
Astrophys. J. Lett. 905 (2020) no.2, L34 [astro-ph.HE/2009.04496].



The maximum value of the GW signal on the frequency band of LISA-BBO for PBH dark matter.
The reheating temperature is
 $T_{rh} = 6 - 5 \times 10^3 \text{ GeV}$



The GW for PBHs that evaporate in the very early universe. The reheating temperature is
 $T_{rh} = 2 \times 10^9 \text{ GeV}$.
Such a signal is compatible with BBN for
 $10^{-4} \lesssim \gamma \lesssim 2 \times 10^{-2}$



The analysis 12.5 years PTA data from the North American NanonHertz Observatory for GW, that observes in the frequency band around 5.5 nHz.
The signal amplitude is estimated To $\pi\lambda\tau\sigma$
 $3 \times 10^{-10} - 2 \times 10^{-9}$
GWs associated with PBHS produced during eMD with 5% fractional abundance can be an explanation

Summary

- *The early universe cosmic era that corresponds to temperatures $T \gtrsim 5\text{MeV}$ is dark (no direct observables)*
- *An early matter domination era (eMD) is often postulated within BSM scenarios, and in particular SUSY scenarios*
 - *An enhanced spectrum of curvature perturbation at small scales can be produced within inflationary theory and points to a more complex inflationary paradigm that features*
 - ▶ *inflection point,*
 - ▶ *step-like transition in the inflaton potential energy,*
 - ▶ *sharp turns in the inflationary trajectory*
- *If PBHs are a fraction of the Dark Matter in the universe a SGWB, associated with their production, exists*
- *PBH production during eMD gives a distinct GW signal and we have been working out to determine its detail spectrum*

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 - ▶ *PBH reheating*
- *If PBHs are a fraction of the Dark Matter in the universe a SGWB, associated with their production, exists*
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Thank you!