Two-loop investigation of new physics effects on M_W from a doublet extension of the SM Higgs sector

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

DESY.

Introduction: M_W and the CDF result

M$_W$ as an Electroweak Precision Observable (EWPO)

➢ **Electroweak precision observables**, including
  - ➢ **W-boson mass M$_W$**
  - ➢ (Squared sine of) Effective leptonic weak mixing angle $\sin^2\theta_{eff}^{lep}$
  - ➢ Z-boson decay width Γ_Z
  - ➢ Muon anomalous magnetic moment (g-2)$_\mu$
    etc.
  are **measured** very precisely, and can also be **computed** to high level of accuracy in terms of **G$_F$, α(0), M$_Z$** (most precisely measured EW quantities) and m$_h$, m$_t$, α$_S$, Δα$_{had}$, Δα$_{lept}$, m$_b$, etc.
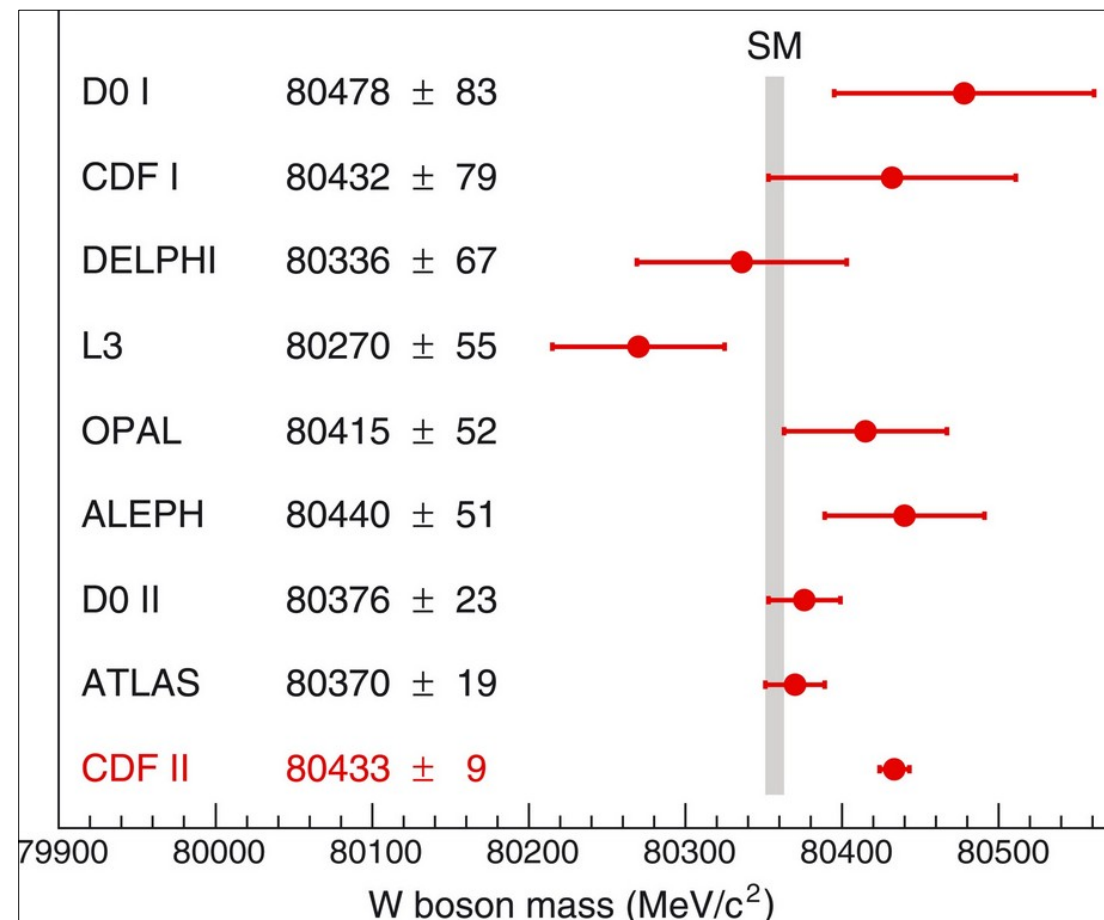
➢ Allow testing the SM as well as BSM models

➢ Before April, experimental *world average* was [PDG 2020]
  M$_W^{exp}$ = 80 379 ± 12 MeV

➢ *SM prediction* (full 1L+2L, partial 3L and 4L, see [Awramik, Czakon, Freitas, Weiglein '03])
  M$_W^{SM}$ = 80 353 ± 6 MeV (see e.g. discussion in [Bagnaschi, Chakraborti, Heinemeyer, Saha, Weiglein '22])
  → **already a small discrepancy!**

CDF measurement of M_W [Science **376**, 170 (2022)]

➢ April 7, 2022: CDF collaboration at Fermilab Tevatron released a measurement of M_W, using 8.8fb^{-1} of data taken between 2002 and 2011

$$M_W^{CDF} = 80\ 433.5 \pm 9.4 \text{ MeV}$$

➢ Most precise result from a *single experiment*, **~7σ away from SM prediction!**

➢ Possible issues remain to be discussed about the CDF measurement and its compatibility with previous results → central value could decrease and/or uncertainty could be augmented

➢ Even so, **inclusion of CDF II into world average will most certainly increase the** *already existing* **pull from the SM prediction**

➢ **Strong motivation to investigate BSM contributions to W-boson mass**

		SM
D0 I	80478 ± 83	
CDF I	80432 ± 79	
DELPHI	80336 ± 67	
L3	80270 ± 55	
OPAL	80415 ± 52	
ALEPH	80440 ± 51	
D0 II	80376 ± 23	
ATLAS	80370 ± 19	
CDF II	80433 ± 9	

W boson mass (MeV/c^2)

M$_W$ calculation in the SM and beyond

- Base for M$_W$ calculation is the decay of the muon
  - **Extract G$_F$ from muon lifetime τ$_μ$** by computing τ$_μ$ in the **Fermi theory**
  - Relate M$_W$, M$_Z$, α, G$_F$ by **computing muon decay in full theory**, and **matching to Fermi theory result**

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1 + \Delta r) \quad \Rightarrow \quad M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1 + \Delta r) \qquad \textit{(OS scheme)}$$

Δr ≡ Δr(M$_W$, M$_Z$, m$_h$, m$_t$, …) denotes corrections to muon decay (w/o finite QED effects)

- Previous relation used to determine M$_W$ as solution, via iterations, of

$$M_W^2 = M_Z^2\left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1 + \Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\,\right] \qquad \textit{(OS scheme)}$$

- Inclusion of known higher-order SM corrections *crucial* $\Delta r = \Delta r^{\mathrm{SM}} + \Delta r^{\mathrm{BSM}}$

- Δr^{SM} known to full 1L & 2L + leading 3L & 4L → see e.g. [Awramik, Czakon, Freitas, Weiglein '03]

Solving the M_W discrepancy at loop level

$$M_W^{CDF} = 80\,433.5 \pm 9.4 \text{ MeV}$$

Note 1: solutions at tree level are also possible, e.g. contribution from a triplet scalar (complete discussion e.g. in [Bagnaschi et al. 2204.05260] + see Prof. Ellis' talk)

Note 2: many models have been considered at loop level (large number of papers compute the S, T, U parameters at 1L and check if they can reproduce the preferred values obtained by a global fit including the CDF result, see e.g. [Strumia, 2204.04191])

Some models work, some don't

e.g. singlet extension, c.f. [Sakurai, Takahashi, Yin 2204.04770] which found that $\Delta M_W \leq 5$ MeV

$\rightarrow$ in what follows, we will consider whether the **2HDM** can accommodate a value M_W as high as the CDF result (future world-average will certainly be lower, hence needed BSM deviation will be smaller, and easier to reproduce) employing a **2L calculation of M_W**

The Two-Higgs-Doublet Model (2HDM)

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge 1

$$V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right)$$

$$+ \frac{1}{2}\Lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\Lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \Lambda_3 \left(\Phi_2^\dagger \Phi_2 \right) \left(\Phi_1^\dagger \Phi_1 \right) + \Lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right)$$

$$+ \left[\frac{1}{2}\Lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Lambda_6 \Phi_1^\dagger \Phi_1 + \Lambda_7 \Phi_2^\dagger \Phi_2 \right) \Phi_1^\dagger \Phi_2 + \text{h.c.} \right]. \qquad v_1^2 + v_2^2 = v^2 \simeq (246\ \text{GeV})^2$$

- CP-conserving 2HDM, with softly-broken Z_2 symmetry $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$ to avoid tree-level FCNCs → m_{12}^2 and Λ_5 real, $\Lambda_6 = \Lambda_7 = 0$

- **Mass eigenstates**:
  - h, H: CP-even Higgs bosons *(h → 125-GeV SM-like state)*
  - A: CP-odd Higgs boson
  - $H^\pm$: charged Higgs boson
  - α: CP-even Higgs mixing angle

- **BSM parameters**: 3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale M (defined by $M^2 \equiv m_{12}^2/c_\beta s_\beta$), angles α and β (defined by $\tan\beta = v_2/v_1$)

- We take the **alignment limit α=β-π/2** → all Higgs couplings are SM-like at tree level
  → compatible with current experimental data + no mixing of CP-even scalars!

Calculation of M$_W$ including 2L BSM effects: `THDM_EWPOS`

- Code written by Stefan Hessenberger, based on [Hessenberger, Hollik '16] and [Hessenberger '18]

- Computes Δρ and EWPOs in (aligned) 2HDM as well as IDM to full 1L + leading 2L BSM (+ higher SM)

- Specifically, the computed EWPOs are **M$_W$** and observables at Z pole, namely

  - **Z-boson width** $\Gamma_Z = \Sigma_f \Gamma(Z \to f\bar{f})$ with $\Gamma(Z \to f\bar{f}) = \dfrac{G_F M_Z^3}{6\sqrt{2}\pi} N_c^f \left[(g_V^f)^2 R_V^f + (g_A^f)^2 R_A^f \right]$

  - **Effective leptonic weak mixing angle** $\sin^2 \theta_{\text{eff}}^{\text{lep}} \equiv \dfrac{1}{4}\left(1 - \dfrac{g_V^{\text{lep}}}{g_A^{\text{lep}}} \right)$

    (assuming lepton universality)

  N_c^f: colour factor
  $g_{V,A}^f$: eff. vector/axial coup. of Z boson to fermion f
  $R_{V,A}^f$: radiation factors (final state QCD & QED corr.)

- Corrections to Δρ:

  - **1L**: SM-like top quark piece + BSM scalar piece

  - **2L**: (1L)^2 pieces + **genuine pieces**, i.e. *{top+SM scalars}, {top+BSM scalars}, {BSM scalars only}, {SM+BSM scalars}* – all computed in gaugeless limit

- 2L BSM corrections to Δr, Γ$_Z$, sin²θ$_{eff}^{lep}$ can always be split between a **reducible part** (i.e. (1L)^2 terms) and an **irreducible part**, which is **proportional to 2L BSM corrections to Δρ**

- Higher order SM corrections to Δr, Γ$_Z$, sin²θ$_{eff}^{lep}$ included via known parametrisations

  → see details in [Hessenberger '18]

A parameter scan to investigate EWPOs

[Bahl, JB, Weiglein `2204.05269`]

➢ *Here*: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types

➢ Constraints in our parameter scan (all except the last are checked with `ScannerS` [Mühlleitner et al. `2007.02985`])

experimental

- SM-like Higgs measurements with `HiggsSignals`
- Direct searches for BSM scalars with `HiggsBounds`
- b-physics constraints, using results from [Gfitter group `1803.01853`]

theoretical

- Vacuum stability
- Boundedness-from-below of the potential
- NLO perturbative unitarity, using results from [Grinstein et al. `1512.04567`], [Cacchio et al. `1609.01290`]

➢ For points passing these constraints, we compute M_W, $\sin^2\theta_{eff}^{lep}$ and Γ_Z using `THDM_EWPOS`

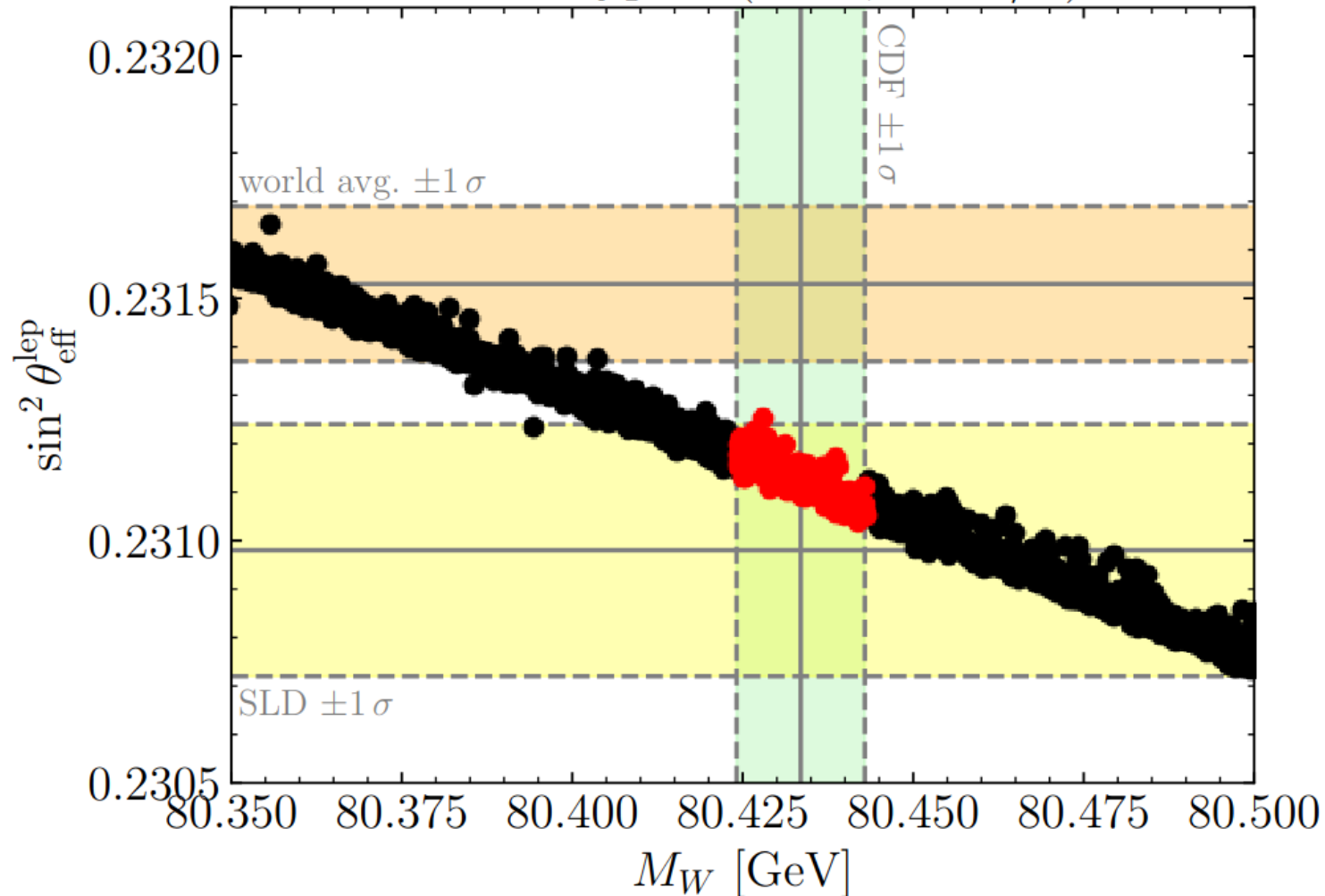- **red points** ≡ parameter points that reproduce CDF value for M_W within 1σ, i.e.

$$80\ 424\ \text{MeV} \leq (M_W^{(2)})^{2HDM} \leq 80\ 442\ \text{MeV}$$

- **black points** ≡ all other points

Results: M$_W$ vs sin²θ$_{eff}^{lep}$

- **2HDM can explain the discrepancy in M$_W$!**

- Light tension with world average for sin²θ$_{eff}^{lep}$ but good agreement with SLD result

- *World average*: using both LEP result (based on forward-backward asymmetry of bottom quarks) + SLD result (based on left-right asymmetry) which show a *3σ discrepancy* between each other

- **SLD**: most precise *single* measurement of sin²θ$_{eff}^{lep}$ and only depends on leptonic couplings

2HDM type-I $(\alpha = \beta - \pi/2)$
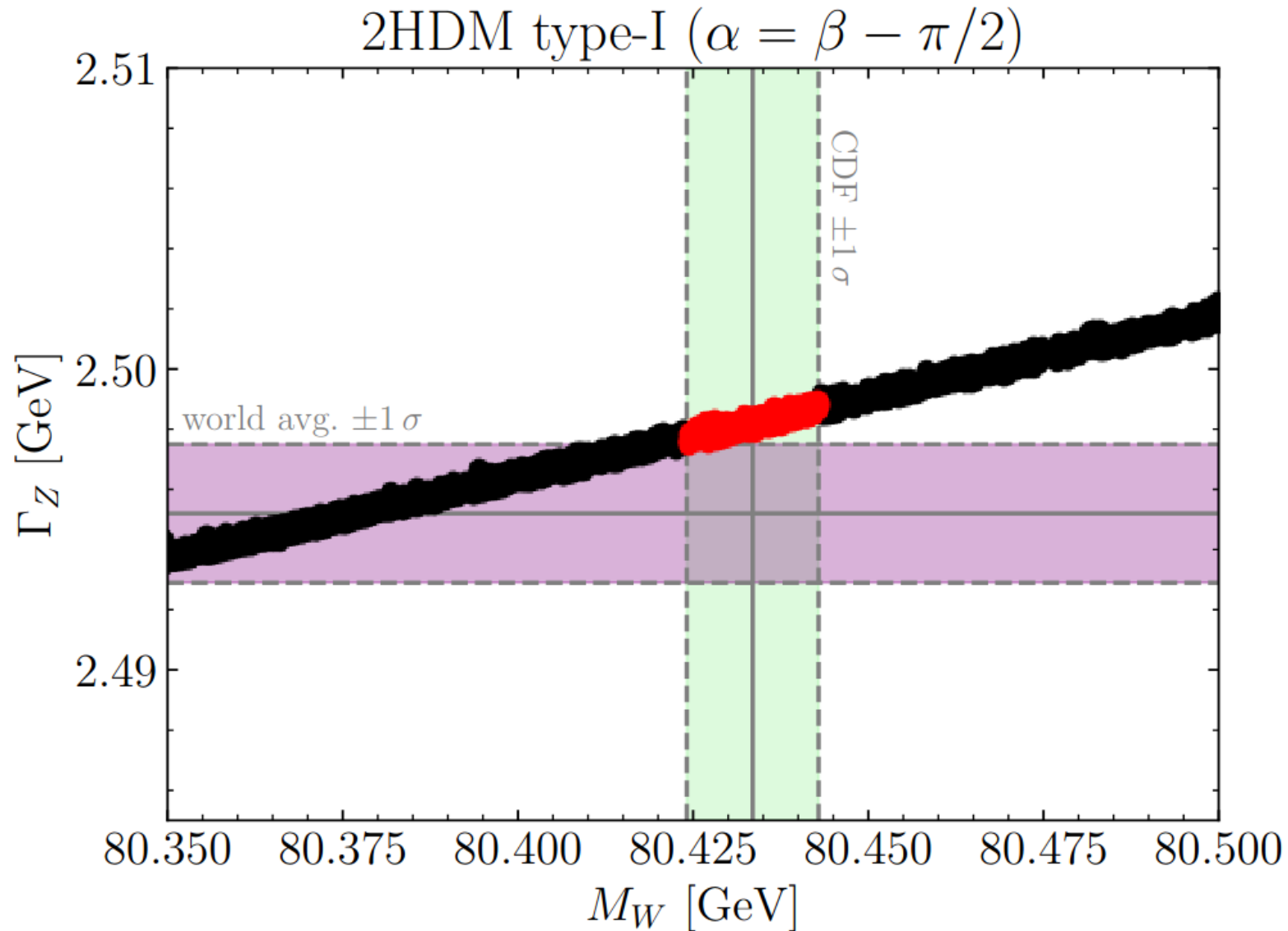
> ➤ Result for Γ_z compatible within $1 - 1.5\sigma$ of world average

[Bahl, JB, Weiglein 2204.05269]

2HDM type-I ($\alpha = \beta - \pi/2$)
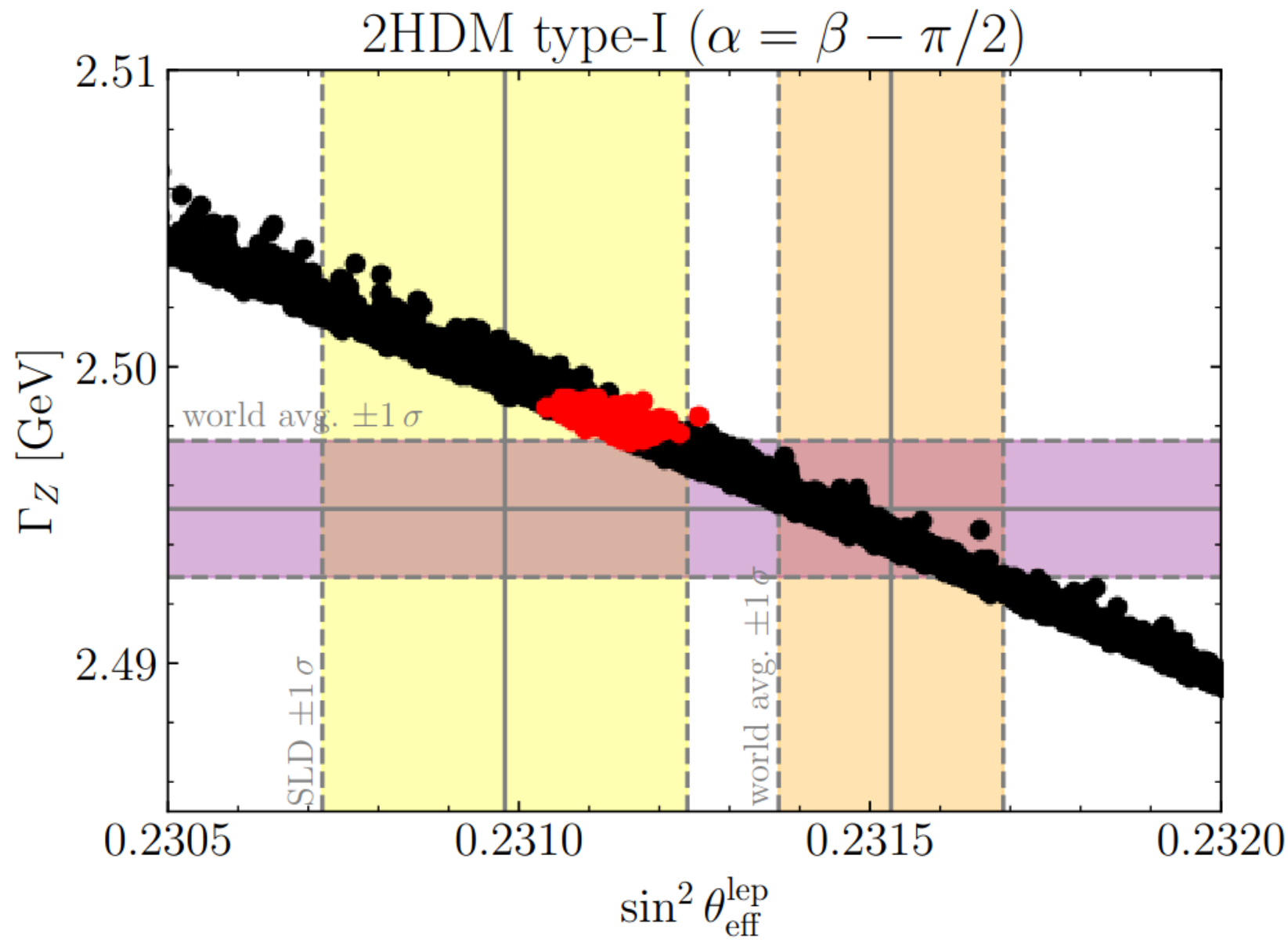
Results in the (M_H-$M_{H\pm}$, M_A-$M_{H\pm}$) plane

[Bahl, JB, Weiglein 2204.05269]

2HDM type-I ($\alpha = \beta - \pi/2$)
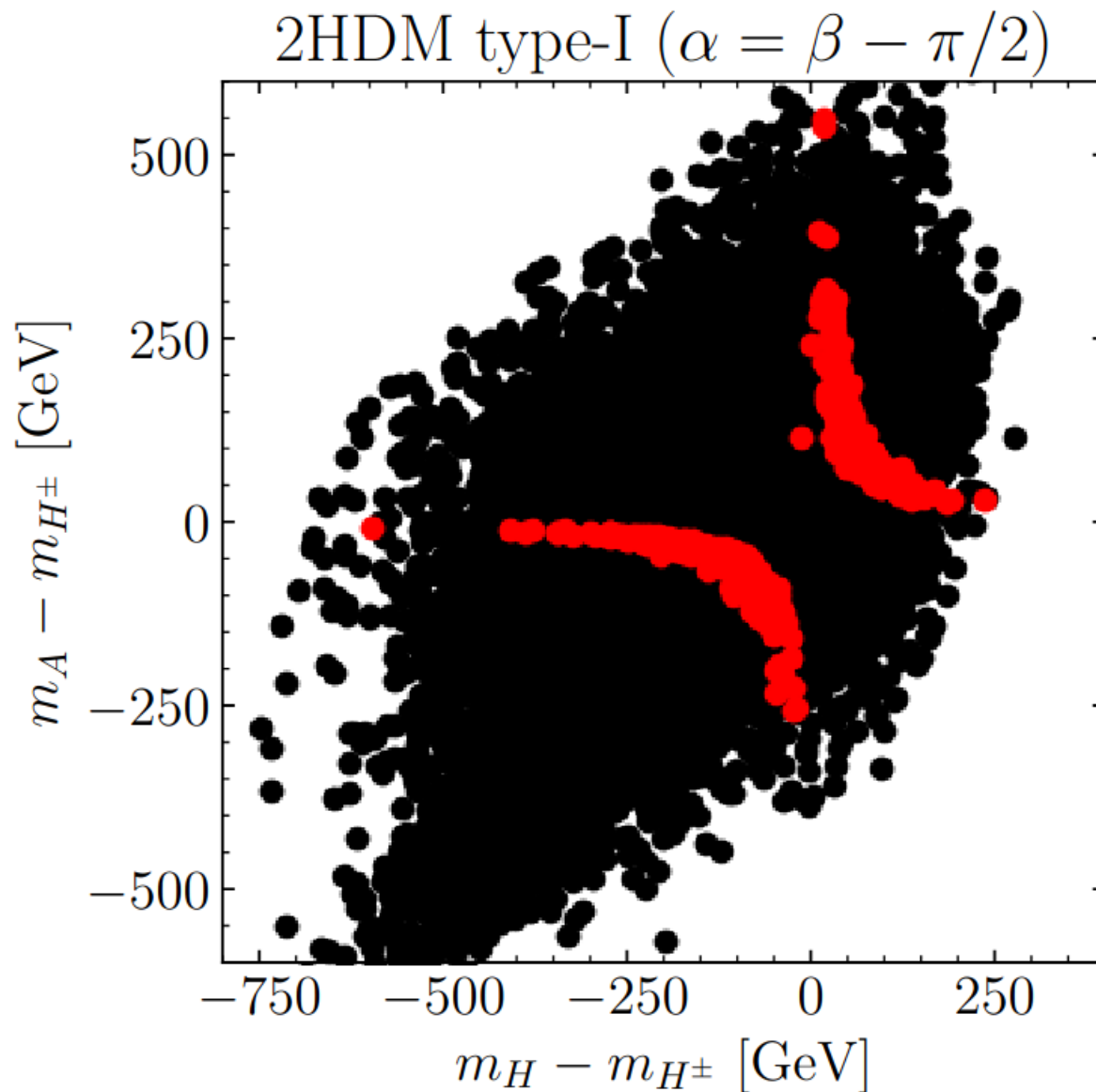
- **Mass hierarchy where m_H=m_A=$m_{H\pm}$ is no longer allowed because it cannot reproduce M_W!**

- The reason is that in this limit, the *custodial symmetry is restored in the 2HDM scalar sector*
  → scalar contributions to Δρ *vanish*
  → no way of getting a large enough contribution to M_W!

- Need **m_H-$m_{H\pm}$ < 0 and m_A-$m_{H\pm}$ < 0** or **m_H-$m_{H\pm}$ > 0 and m_A-$m_{H\pm}$ > 0** to have a **positive** contribution to Δρ

- Needed mass splitting of ≥ 50 GeV translates into an **upper bound on BSM scalar masses** of O(few TeV)

Impact of two-loop corrections to M_W

2HDM type-I $(\alpha = \beta - \pi/2)$

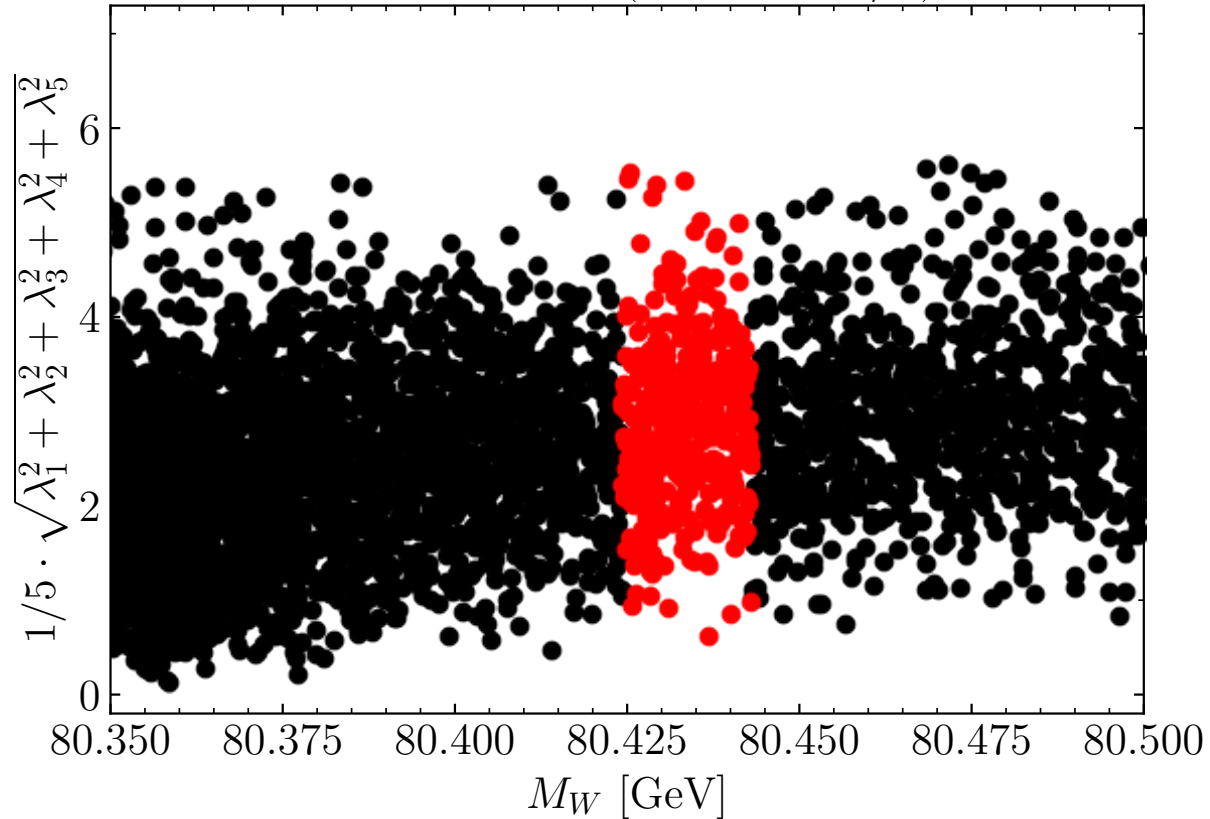- ➢ 2L corrections to M_W often significant, and can play an important role in reaching the values of M_W compatible with the CDF result

- ➢ **Shows the importance of including 2L BSM effects!**
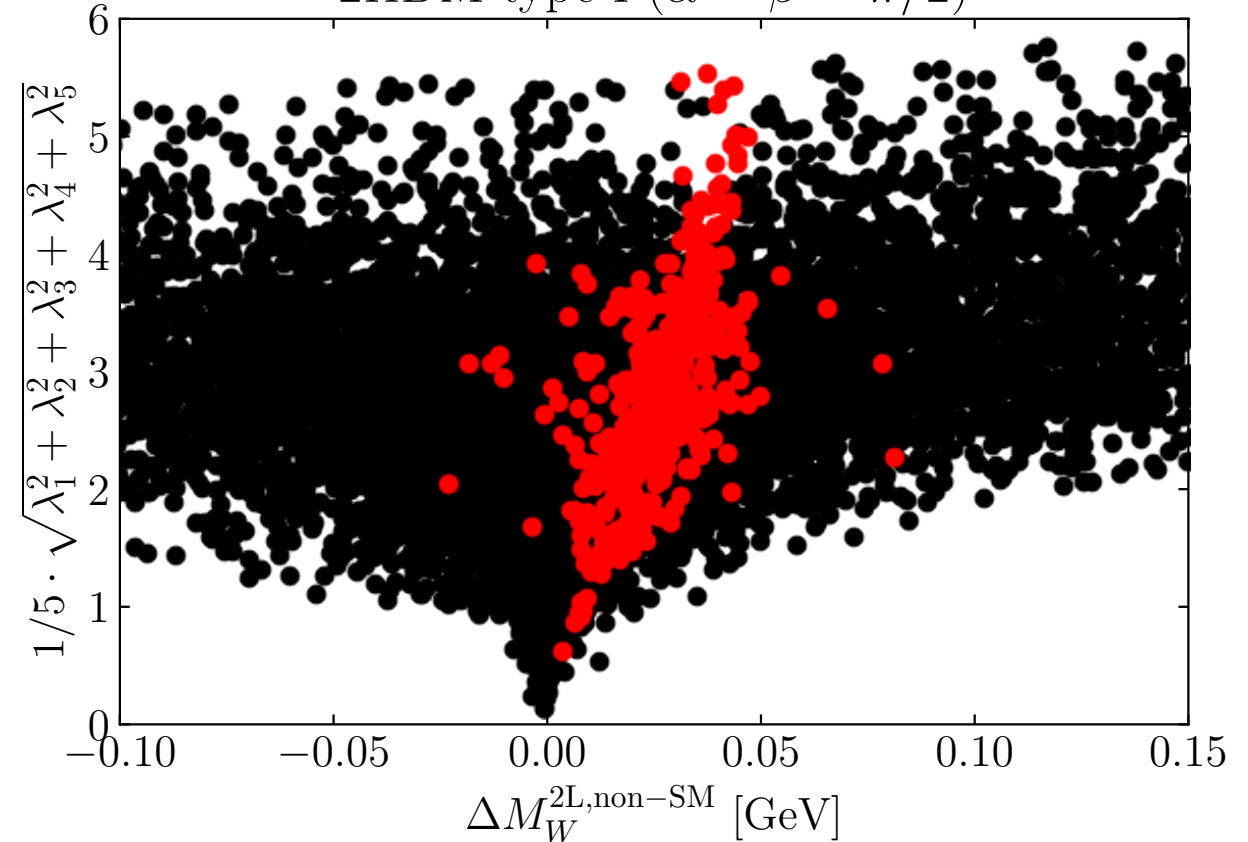
Typical couplings sizes

2HDM type-I $(\alpha = \beta - \pi/2)$

2HDM type-I $(\alpha = \beta - \pi/2)$

- ➢ Large scalar couplings are **not necessary** to reproduce the CDF value for M_W
  (with or without large 2L effects)

Summary

➢ M_W is one of the best measured EWPO, and comparison of theory prediction and experimental results allow stringent tests of SM as well as BSM theories

➢ Recent excitement related to **CDF result**, seemingly **7σ away from SM → strong motivation to consider BSM contributions to M_W**

➢ [Bahl, JB, Weiglein `2204.05269`] investigated situation in 2HDM, with calculation of M_W including leading 2L BSM (+ h.o. SM) effects using `THDM_EWPOS` → **2HDM can accommodate M_W discrepancy while keeping satisfactory agreement for $\sin^2\theta_{eff}^{lep}$ and Γ_Z**

➢ CDF result is not compatible with degenerate mass hierarchies in 2HDM → **upper bound** on BSM scalar masses

➢ **Impact of 2L (BSM) corrections to M_W can be significant** (not necessarily related to large couplings)

Thank you for your attention!

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CDF measurement of M_W

➢ April 7, 2022: CDF collaboration at Fermilab Tevatron released a measurement of MW, using 8.8fb^{-1} of data taken between 2002 and 2011

$$M_W = 80\ 433.5 \pm 9.4 \text{ MeV}$$

➢ Most precise result from a *single experiment*

➢ **CDF value is ~7σ away from SM prediction!**

➢ Tevatron (claimed) advantages over LHC
  ➢ p$\bar{\text{p}}$ collisions rather than pp → processes involve mainly (anti)quark momentum distributions (PDFs), which are better known than that of gluons → lower uncertainty than processes at LHC
  ➢ Lower centre-of-mass energy
    → PDFs known more precisely at low √s
    → less QCD backgrounds
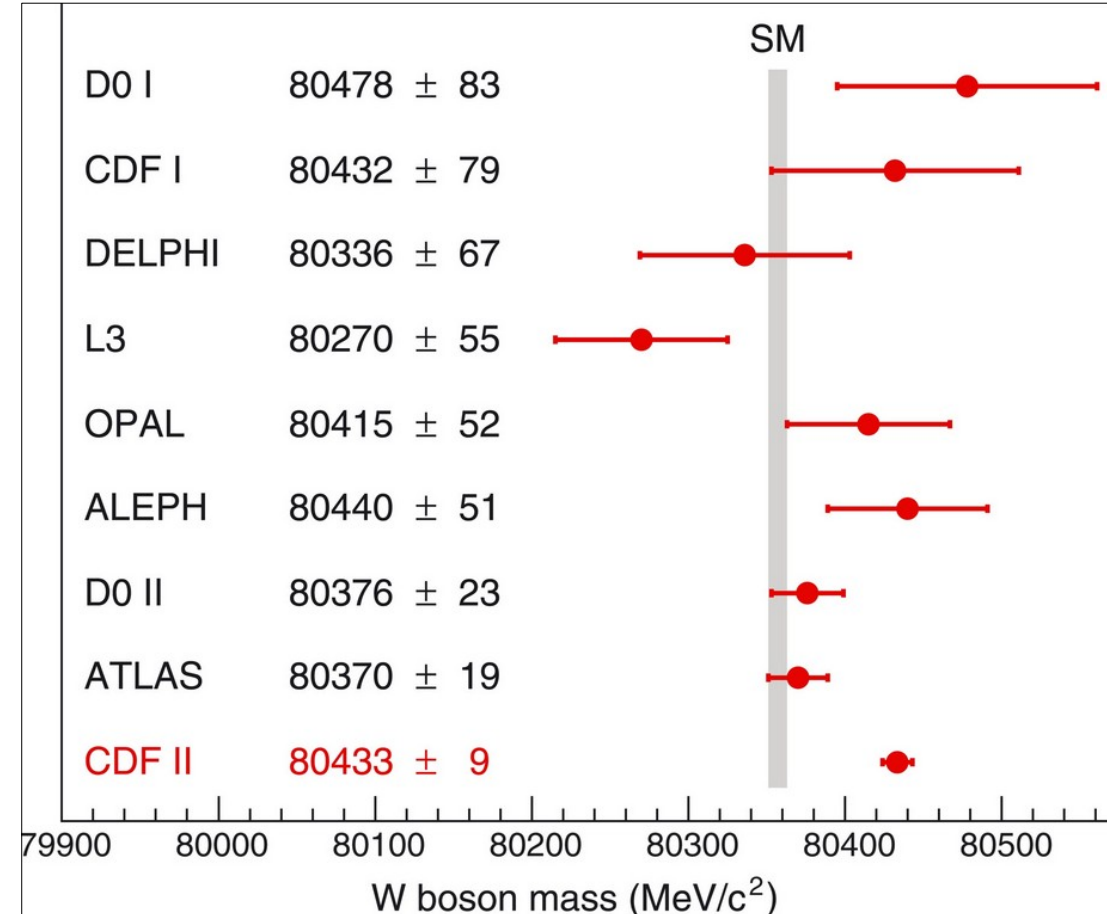
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SM

W boson mass (MeV/c^2)

79900 80000 80100 80200 80300 80400 80500

Some concerns raised about the CDF measurement of M_W

➢ Several points in the CDF analysis have drawn criticisms/skepticisms (see also colloquium by J. Ellis on 3/5)
  ➢ Measurement of lepton momenta
  ➢ Version of ResBos used to model p_T (v1 used rather than v2)
  ➢ Version of PDF and their uncertainties

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>> Version of ResBos used to model p_T (v1 used
   rather than v2) → not the bigest issue, see
   [Isaacson, Fu, Yuan 2205.02788]
>> Version of PDF and their uncertainties

ResBos2 and the CDF W Mass Measurement

Joshua Isaacson,[1, *] Yao Fu,[2] and C.-P. Yuan[3]

[1] Theoretical Physics Department, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA
[2] Department of Modern Physics, University of Science and Technology of China, Jinzhai Road 96, Hefei, Anhui, 230026, China
[3] Department of Physics and Astronomy, Michigan State University,
567 Wilson Road, East Lansing, MI 48824, USA

The recent CDF W mass measurement of 80,433 ± 9 MeV is the most precise direct measurement. However, this result deviates from the Standard Model predicted mass of 80,359.1 ± 5.2 MeV by 7σ. The CDF experiment used an older version of the ResBos code that was only accurate at NNLL+NLO, while the state-of-the-art ResBos2 code is able to make predictions at N^3LL+NNLO accuracy. We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV.

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  - ➤ Version of PDF and their uncertainties → see [Gao, Liu, Xie 2205.03942]

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Understanding PDF uncertainty on the W boson mass measurements in CT18 global analysis

Jun Gao,[1, 2, *] DianYu Liu,[1, 2, †] and Keping Xie[3, ‡]

δM_W in MeV	sta.	NNPDF3.1	CT18	MMHT14	NNPDF4.0	MSHT20	CTEQ6M
$\langle M_T\rangle$(LO)	–	$0^{+8.3}_{-8.3}$	$-1.0^{+8.3}_{-11.4}$	$-3.3^{+7.4}_{-4.2}$	$+7.8^{+5.1}_{-5.1}$	$-3.1^{+6.7}_{-5.7}$	$-7.3^{+8.4}_{-12.0}$
χ^2 fit (LO)	8.0	$0^{+7.6}_{-7.6}$	$-1.0^{+5.4}_{-8.6}$	$-3.3^{+6.1}_{-3.0}$	$+8.0^{+3.7}_{-3.7}$	$-3.0^{+5.0}_{-4.0}$	$-7.3^{+5.6}_{-9.3}$
$\langle M_T\rangle$(NLO)	–	$0^{+5.9}_{-5.9}$	$-4.2^{+8.8}_{-13.3}$	$-5.0^{+6.7}_{-5.3}$	$+6.9^{+6.2}_{-6.2}$	$-7.6^{+7.9}_{-6.7}$	$-14.0^{+9.0}_{-11.9}$
χ^2 fit (NLO)	8.0	$0^{+4.2}_{-4.2}$	$-4.3^{+5.4}_{-10.1}$	$-5.1^{+4.8}_{-3.4}$	$+7.1^{+4.5}_{-4.5}$	$-7.8^{+5.7}_{-4.5}$	$-14.6^{+5.8}_{-5.4}$
CDF	9.2	$0^{+3.9}_{-3.9}$	–	–	–	–	-3.3

TABLE I. Estimated shifts and PDF uncertainties at 68% C.L. on the extracted W boson mass for the CDF scenario for various PDF sets with respect to a common reference of using NNPDF3.1 NNLO central PDF. We show results using the simplified prescription, compared to those from a χ^2 fit as well as results reported in the CDF analysis. In the case of the χ^2 fit, we also show the expected experimental statistical error of the extracted W boson mass compared to the actual one in the CDF analysis.

Thus we suggest analyzing the experimental data using up-to-date PDFs could be highly desirable, especially considering tensions between different W boson mass measurements. We

Some concerns raised about the CDF measurement of M_W

- Several points in the CDF analysis have drawn criticisms/skepticisms (see also colloquium by J. Ellis on 3/5)
  - Measurement of lepton momenta
  - Version of ResBos used to model p_T (v1 used rather than v2) → not the bigest issue, see [Isaacson, Fu, Yuan 2205.02788]
  - Version of PDF and their uncertainties → see [Gao, Liu, Xie 2205.03942]

- CDF value is in tension with several of the earlier results (esp. LEP, ATLAS) → at least some of the experiments might have underestimated their uncertainties

- *Note*: pre-CDF II, measurements from LEP, Tevatron, and ATLAS were not yet combined, and investigation/evaluation of uncertainties was ongoing
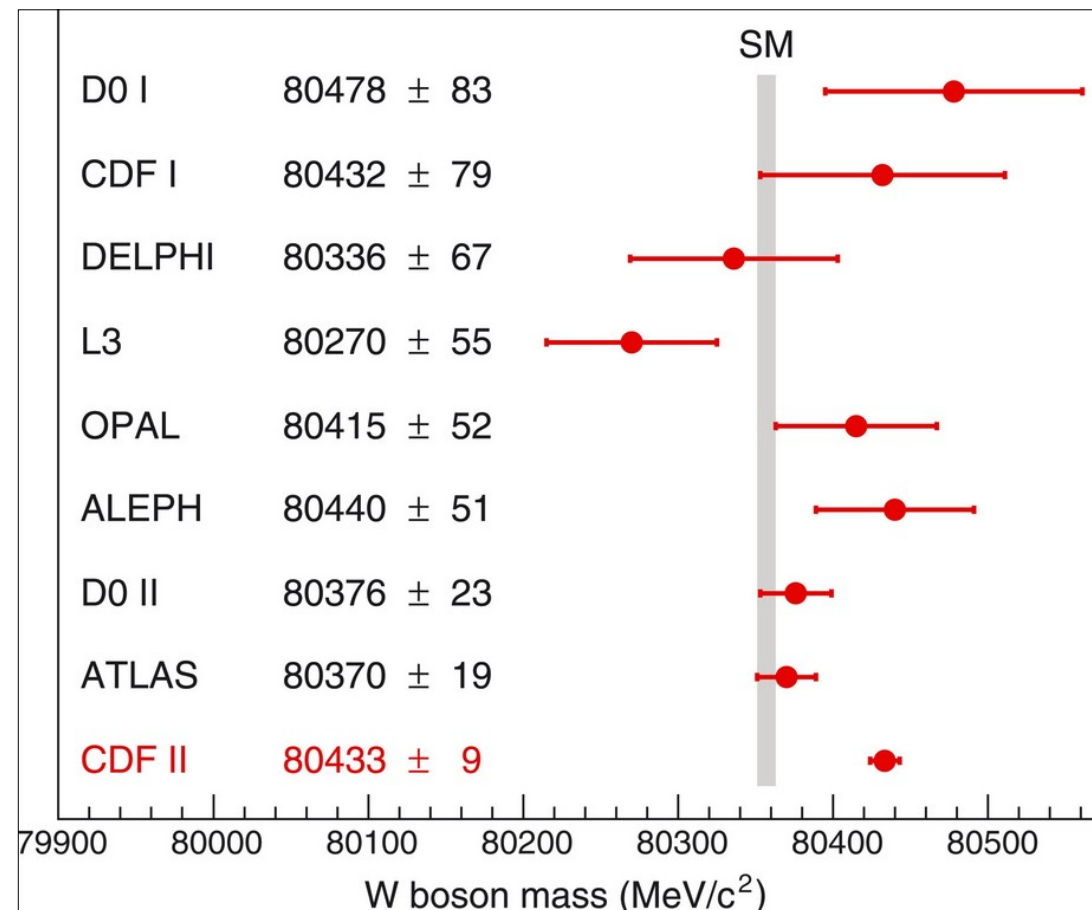
M_W calculation in the SM I

- Base for M_W calculation is the decay of the muon
  - **Extract G_F from muon lifetime τ_μ** by computing τ_μ in the **Fermi theory**

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(m_e^2/m_\mu^2)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

with $F(x) \equiv 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$

Tree-level W propagator contributions (not in Fermi th. but numerically tiny)

QED corrections (known to 1L+2L)

- Relate M_W, M_Z, α, G_F by **computing muon decay in SM**, and **matching to Fermi theory result**

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 M_W^2 s_W^2}(1 + \Delta r) \quad \Rightarrow \quad M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1 + \Delta r) \qquad \textit{OS scheme}$$

$\Delta r \equiv \Delta r(M_W, M_Z, m_h, m_t, \ldots)$ denotes corrections to muon decay (w/o finite QED effects)

- Previous relation used to determine M_W as solution, via iterations, of

$$M_W^2 = M_Z^2\left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1 + \Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right] \qquad \textit{OS scheme}$$

M_W calculation in the SM II

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1 + \Delta r) \quad \Rightarrow \quad M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1 + \Delta r) \qquad\qquad M_W^2 = M_Z^2\left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1 + \Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

➢ At one loop

$$\Delta r^{(1)} = 2\delta^{(1)}Z_e + \frac{\Sigma_{WW}^{(1)}(p^2 = 0) - \delta^{(1)}M_W^2}{M_W^2} - \frac{\delta^{(1)}s_W^2}{s_W^2} + \{\text{vertex} + \text{box corrections}\}$$

Σ_{WW}: transverse part of the W-boson self-energy, $\delta^{(1)}$X: 1L counterterm to quantity X

➢ One can show that

$$\delta^{(1)}Z_e \simeq \frac{1}{2}\Delta\alpha + \cdots \quad \text{and} \quad \frac{\delta^{(1)}s_W^2}{s_W^2} \simeq \frac{c_W^2}{s_W^2}\Delta\rho^{(1)}$$

$$\text{with} \quad \Delta\alpha = \frac{\partial}{\partial p^2}\Sigma_{\gamma\gamma}\big|_{p^2 = 0} - \frac{\text{Re}\Sigma_{\gamma\gamma}(p^2 = M_Z^2)}{M_Z^2}$$

➢ Leading terms can be rewritten as [Sirlin '80]

$$\Delta r^\alpha = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho^{(1)} + \Delta r_{\text{remainder}}(m_h)$$

with Δα: contribution from light fermion loops to photon vacuum polarisation
Δρ: corrections to the ρ parameter

$$\rho \equiv \frac{G_{\text{NC}}}{G_{\text{CC}}} \quad \Rightarrow \quad \rho^{(0)} = \frac{M_W^2}{c_W^2 M_Z^2} = 1 \text{ and } \Delta\rho^{(1)} = \frac{\Sigma_{ZZ}^{(1)}(p^2 = 0)}{M_Z^2} - \frac{\Sigma_{WW}^{(1)}(p^2 = 0)}{M_W^2}$$

M_W calculation in the SM III

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1 + \Delta r) \quad \Rightarrow \quad M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1 + \Delta r) \qquad M_W^2 = M_Z^2\left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1 + \Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

➢ At higher orders

$$\Delta r = \Delta r^\alpha + \Delta r^{\alpha\alpha_s} + \Delta r^{\alpha\alpha_s^2} + \Delta r^{\alpha\alpha_s^3 m_t} + \Delta r_{\text{ferm}}^{\alpha^2} + \Delta r_{\text{bos}}^{\alpha^2} + \Delta r^{G_F^2 \alpha_s m_t^4} + \Delta r^{G_F^3 m_t^6}$$

QCD (2L+3L+approx.4L)   *EW (2L)*   *leading 3L corr. to Δρ*

➢ [Awramik, Czakon, Freitas, Weiglein '03] gives a parametrisation as

$$M_W = M_W^0 - c_1\,\text{dH} - c_2\,\text{dH}^2 + c_3\,\text{dH}^4 + c_4(\text{dh} - 1) - c_5\,\text{d}\alpha + c_6\,\text{dt} - c_7\,\text{dt}^2$$
$$- c_8\,\text{dH}\,\text{dt} + c_9\,\text{dh}\,\text{dt} - c_{10}\,\text{d}\alpha_s + c_{11}\,\text{dZ},$$

with

$$\text{dH} = \ln\left(\frac{M_H}{100\text{ GeV}}\right), \quad \text{dh} = \left(\frac{M_H}{100\text{ GeV}}\right)^2, \quad \text{dt} = \left(\frac{m_t}{174.3\text{ GeV}}\right)^2 - 1,$$

$$\text{dZ} = \frac{M_Z}{91.1875\text{ GeV}} - 1, \quad \text{d}\alpha = \frac{\Delta\alpha}{0.05907} - 1, \quad \text{d}\alpha_s = \frac{\alpha_s(M_Z)}{0.119} - 1,$$

$$M_W^0 = 80.3779\text{ GeV}, \quad c_1 = 0.05263\text{ GeV}, \quad c_2 = 0.010239\text{ GeV},$$
$$c_3 = 0.000954\text{ GeV}, \quad c_4 = -0.000054\text{ GeV}, \quad c_5 = 1.077\text{ GeV},$$
$$c_6 = 0.5252\text{ GeV}, \quad c_7 = 0.0700\text{ GeV}, \quad c_8 = 0.004102\text{ GeV},$$
$$c_9 = 0.000111\text{ GeV}, \quad c_{10} = 0.0774\text{ GeV}, \quad c_{11} = 115.0\text{ GeV},$$

➢ *Note*: Δr also serves to extract the Higgs VEV from G_F

$$v^2 = \frac{1}{\sqrt{2}G_F}(1 + \Delta r)$$

M$_W$ calculation beyond the SM

- Idea of the calculation remains the same, but full theory calculation (that is matched with the Fermi theory one) is now done in the **BSM model**

- In BSM models, M$_W$ ($\leftrightarrow$ muon decay) can receive contributions both at **tree level** and at **loop level**. Considering a model with both sources (and turning to $\overline{\text{MS}}$ for simplicity just here), one can write at 1L [Athron et al. 1710.03760, 2204.05285]

$$M_W^2\big|^{\overline{\text{MS}}} = (M_W^{\text{SM}}|^{\overline{\text{MS}}})^2\left\{1 + \frac{s_W^2}{c_W^2 - s_W^2}\left[\frac{c_W^2}{s_W^2}(\Delta\rho_{\text{tree}} + \Delta\rho_{\text{loop}}^{\text{BSM}}) - \Delta r_{\text{remainder}}^{\text{BSM}} - \Delta\alpha^{\text{BSM}}\right]\right\}$$

- In the following, we will only discuss models with **$\rho^{(0)}$=1**, and we stay in **OS scheme**

- Some 2L corrections to Δρ known in BSM models
  - O(αα$_s$) SUSY corrections in [Djouadi et al. '96, '98]
  - O(α$_t^2$,α$_t$α$_b$,α$_b^2$) in MSSM in [Heinemeyer, Weiglein '02], [Hastier, Heinemeyer, Stöckinger, Weiglein '05]
  - BSM scalar + top quark corrections in (aligned) 2HDM and IDM [Hessenberger, Hollik '16]
- Inclusion of known higher-order SM corrections crucial $\quad \Delta r = \Delta r^{\text{SM}} + \Delta r^{\text{BSM}}$

- Calculations of M$_W$ with Δr to full BSM 1L + partial BSM 2L (from resummation and Δρ) + SM up to 4L
  - MSSM [Heinemeyer, Hollik, Weiglein, Zeune '13]
  - NMSSM [Stål, Weiglein, Zeune '15]
  - MRSSM [Diessner, Weiglein '19]
  - **2HDM & IDM** [Hessenberger '18] (TUM thesis and code `THDM_EWPOS`)

Fixed vs running width

➢ OS renormalisation conditions: W- (and Z-) boson mass defined as **real part of the complex pole of the propagator** → gauge invariant definition

➢ Expanding propagator around complex pole → Breit-Wigner shape with a **fixed width**

➢ W- (and Z-) boson mass measured experimentally corresponds (usually) to a definition of the mass with a Breit-Wigner shape with **running width**

➢ Comparison of theory and experiment requires a conversion:

$$M_W^{\text{run. width}} = M_W^{\text{fix. width}} + \frac{\Gamma_W^2}{2M_W^{\text{run. width}}}$$

where for the W decay width one uses a result parametrised in terms of G_F and including 1L QCD corrections

$$\Gamma_W = \frac{3G_F(M_W^{\text{run. width}})^3}{2\sqrt{2}\pi}\left(1 + \frac{2\alpha_s}{3\pi}\right)$$

➢ Resulting shift of **~27 MeV**

Custodial symmetry in the scalar sector of the 2HDM I

- ➤ In SM (and at 0L) the Higgs potential is *invariant under global transformations* of $SU(2)_L \times SU(2)_R$
- ➤ After EWSB, this invariance group is *broken by the Higgs VEV down to* **$SU(2)_{L+R}$**
  - → **custodial symmetry**, which ensures **$\rho^{(0)}=1$**
  - → quark sector breaks the custodial symmetry → **$\Delta\rho_{tb}^{SM} \neq 0$**
- ➤ What about the 2HDM? → let's follow the the discussion in [Hessenberger '18]
- ➤ Using the Higgs basis Φ_{SM}, Φ_{NS} one can first rewrite the scalar potential as

$$V = V_{\mathrm{I}} + V_{\mathrm{II}} + V_{\mathrm{III}} + V_{\mathrm{IV}};$$

$$V_{\mathrm{I}} = \frac{m_{h^0}^2}{2v^2}\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{SM}}\right)^2 - \frac{1}{2}m_{h^0}^2\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{SM}}\right),$$

$$V_{\mathrm{II}} = \left[\frac{1}{2v^2}\left(m_{h^0}^2 + \frac{4}{t_{2\beta}^2}\left(m_{H^0}^2 - \frac{m_{12}^2}{s_\beta c_\beta}\right)\right) + \frac{\Lambda_6\left(2c_{2\beta}-1\right)}{4c_\beta s_\beta^3} - \frac{\Lambda_7\left(2c_{2\beta}+1\right)}{4c_\beta^3 s_\beta}\right]\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right)^2$$
$$+ \left(\frac{m_{12}^2}{c_\beta s_\beta} - \frac{m_{h^0}^2}{2}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right),$$

$$V_{\mathrm{III}} = \left(\frac{m_{A^0}^2}{v^2} - \frac{2m_{H^\pm}^2}{v^2} + \frac{m_{H^0}^2}{v^2}\right)\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{NS}}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{SM}}\right)$$
$$+ \left(\frac{m_{H^0}^2}{2v^2} - \frac{m_{A^0}^2}{2v^2}\right)\left(\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{SM}}\right)^2 + \left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{NS}}\right)^2\right)$$
$$+ \left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right)\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{SM}}\right)\left(\frac{2m_{H^\pm}^2}{v^2} + \frac{m_{h^0}^2}{v^2} - \frac{2m_{12}^2}{v^2 c_\beta s_\beta}\right),$$

$$V_{\mathrm{IV}} = \left(\frac{2}{v^2 t_{2\beta}}\left(m_{H^0}^2 - \frac{m_{12}^2}{c_\beta s_\beta}\right) - \frac{\Lambda_7}{2c_\beta^2} + \frac{\Lambda_6}{2s_\beta^2}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{SM}} + \Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{NS}}\right).$$

with

$$\Phi_{\mathrm{SM}} = \begin{pmatrix} \phi_{\mathrm{SM}}^+ \\ \phi_{\mathrm{SM}}^0 \end{pmatrix} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG) \end{pmatrix}$$

$$\Phi_{\mathrm{NS}} = \begin{pmatrix} \phi_{\mathrm{NS}}^+ \\ \phi_{\mathrm{NS}}^0 \end{pmatrix} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(-H+iA) \end{pmatrix}$$

Custodial symmetry in the scalar sector of the 2HDM II

➢ Then one can construct bidoublets transforming under SU(2)$_L$xSU(2)$_R$ as

$$\mathcal{M}_{\mathrm{SM,NS}} = \left(i\sigma_2 \Phi^*_{\mathrm{SM,NS}} \middle| \Phi_{\mathrm{SM,NS}} \right) = \begin{pmatrix} \phi^{0\,*}_{\mathrm{SM,NS}} & \phi^+_{\mathrm{SM,NS}} \\ -\phi^-_{\mathrm{SM,NS}} & \phi^0_{\mathrm{SM,NS}} \end{pmatrix}$$

$$\mathcal{M}_{\mathrm{SM}} \to L\mathcal{M}_{\mathrm{SM}}R^\dagger \quad \text{and} \quad \mathcal{M}_{\mathrm{NS}} \to L\mathcal{M}_{\mathrm{NS}}R'^{\,\dagger} \text{ with } L \in SU(2)_L,\ R,R' \in SU(2)_R$$

➢ 2 custodial symmetric invariant quantities $\mathrm{tr}\left(\mathcal{M}^\dagger_X \mathcal{M}_X\right) = 2\Phi^\dagger_X \Phi_X \text{ with } X = \mathrm{SM} \text{ or } \mathrm{NS}$
   → V_I and V_{II} respect custodial invariance!

➢ V_{III} and V_{IV} involve the non-invariant combinations $\Phi^\dagger_{\mathrm{NS}}\Phi_{\mathrm{SM}} \pm \Phi^\dagger_{\mathrm{SM}}\Phi_{\mathrm{NS}}$
   → break custodial symmetry
   → enter scalar corrections to Δρ at 1L and 2L respectively → potential contributions to Δρ and hence Δr and M$_W$!

➢ Φ$_{SM}$ and Φ$_{NS}$ have same hypercharge Y=1 → R and R' are related as R=X^{-1}R'X and due to CP invariance, X=Id or X=-iσ$_3$
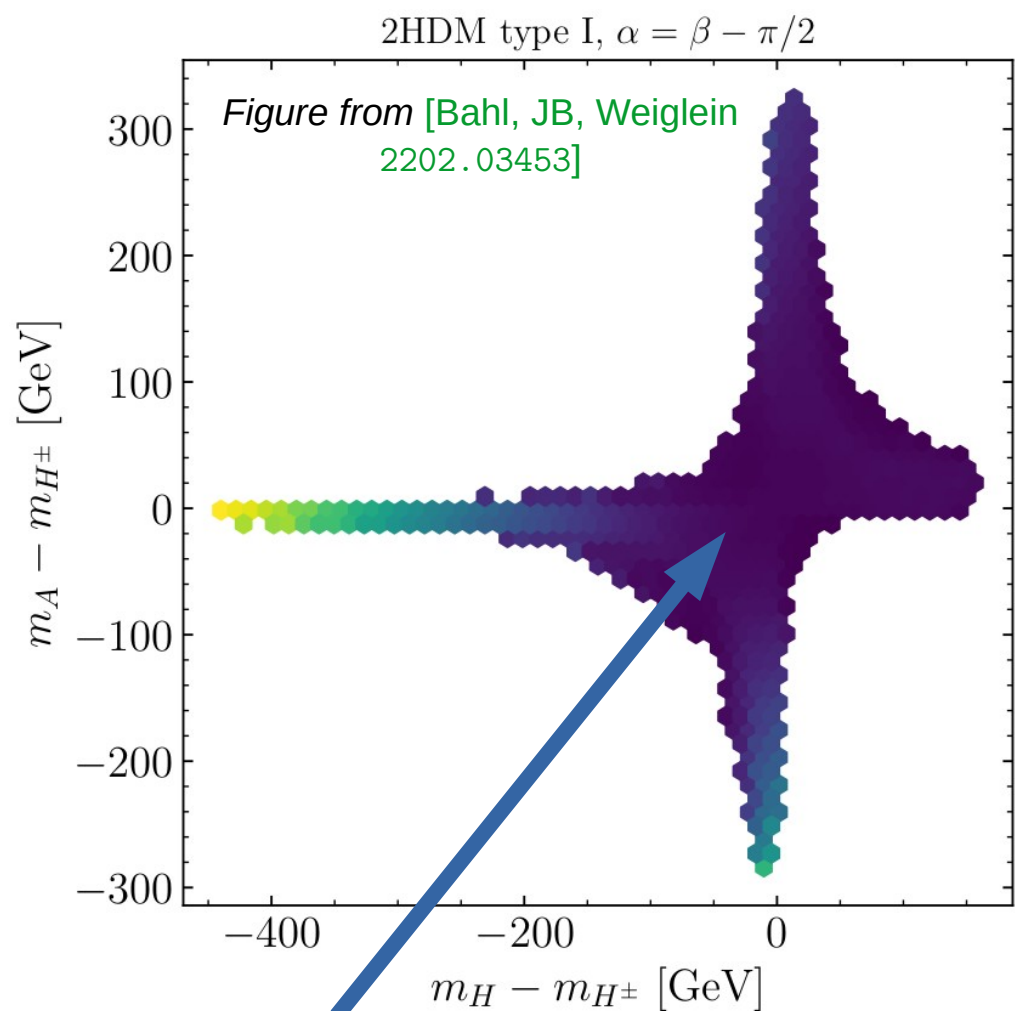   ➢ X=Id → $\mathrm{tr}\left(\mathcal{M}^\dagger_{\mathrm{SM}}\mathcal{M}_{\mathrm{NS}}X\right) = \Phi^\dagger_{\mathrm{NS}}\Phi_{\mathrm{SM}} + \Phi^\dagger_{\mathrm{SM}}\Phi_{\mathrm{NS}}$ is invariant → **V$_{IV}$ invariant and V$_{III}$ invariant if m$_A$=m$_{H\pm}$**

   ➢ X=-iσ$_3$ → $\mathrm{tr}\left(\mathcal{M}^\dagger_{\mathrm{SM}}\mathcal{M}_{\mathrm{NS}}X\right) = -i\Phi^\dagger_{\mathrm{NS}}\Phi_{\mathrm{SM}} + i\Phi^\dagger_{\mathrm{SM}}\Phi_{\mathrm{NS}}$ is invariant → **V$_{III}$ invariant if m$_H$=m$_{H\pm}$**
   while **V$_{IV}$ must vanish → imposes either m$_H$^{2}=m$_{12}$^{2}/(s$_\beta$c$_\beta$)=M^2 or t$_\beta$=1**
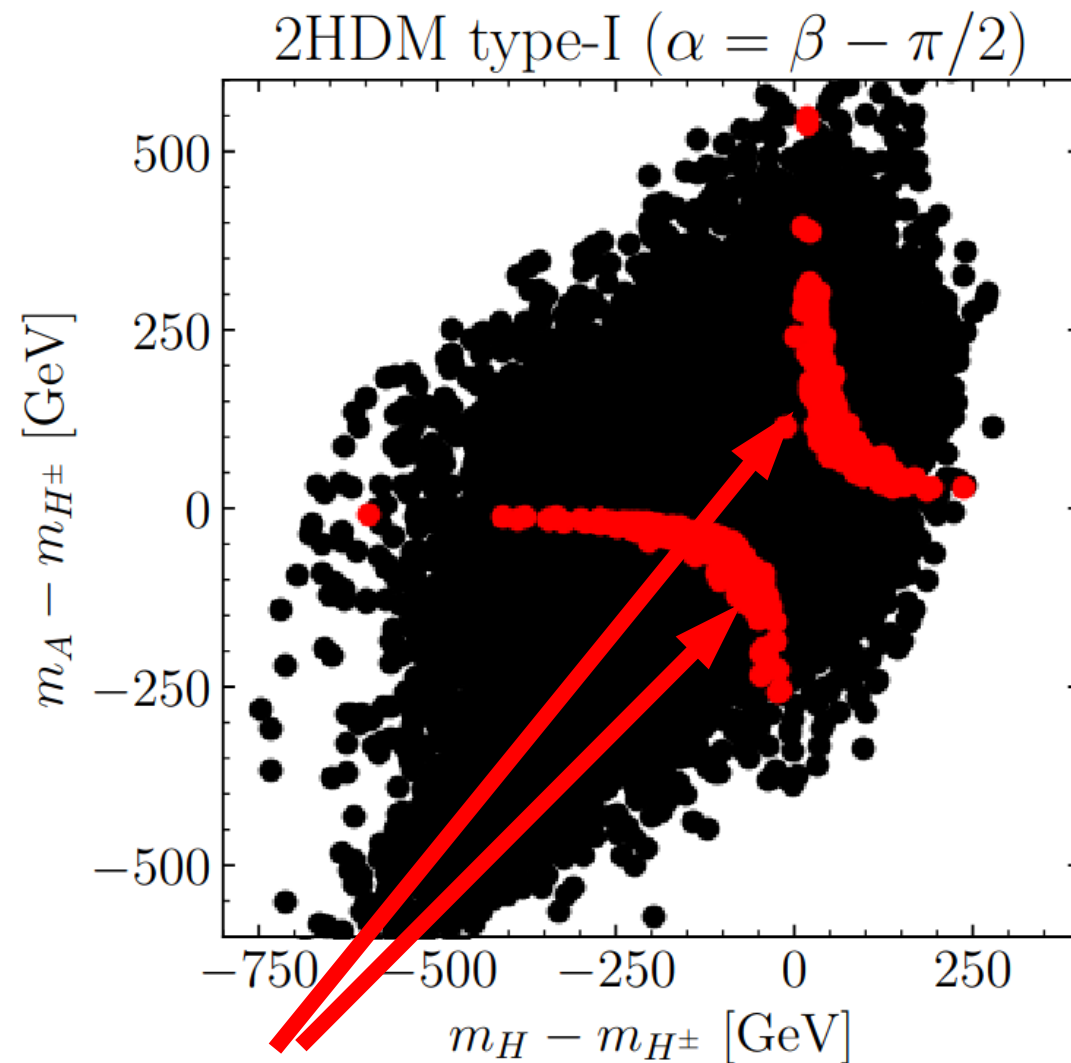
➢ E.g. at 1L, explicitly
$$\Delta\rho^{(1)}_{\mathrm{non\text{-}SM}} = \frac{\alpha}{16\pi^2 s_W^2 M_W^2}\left\{ \frac{m_A^2 m_H^2}{m_A^2 - m_H^2}\ln\frac{m_A^2}{m_H^2} - \frac{m_A^2 m_{H\pm}^2}{m_A^2 - m_{H\pm}^2}\ln\frac{m_A^2}{m_{H\pm}^2} - \frac{m_H^2 m_{H\pm}^2}{m_H^2 - m_{H\pm}^2}\ln\frac{m_H^2}{m_{H\pm}^2} + m_{H\pm}^2 \right\} \xrightarrow{m_{H\pm} \to m_H \text{ or } m_A} 0$$

Results in the (M_H-$M_{H^\pm}$, M_A-$M_{H^\pm}$) plane II
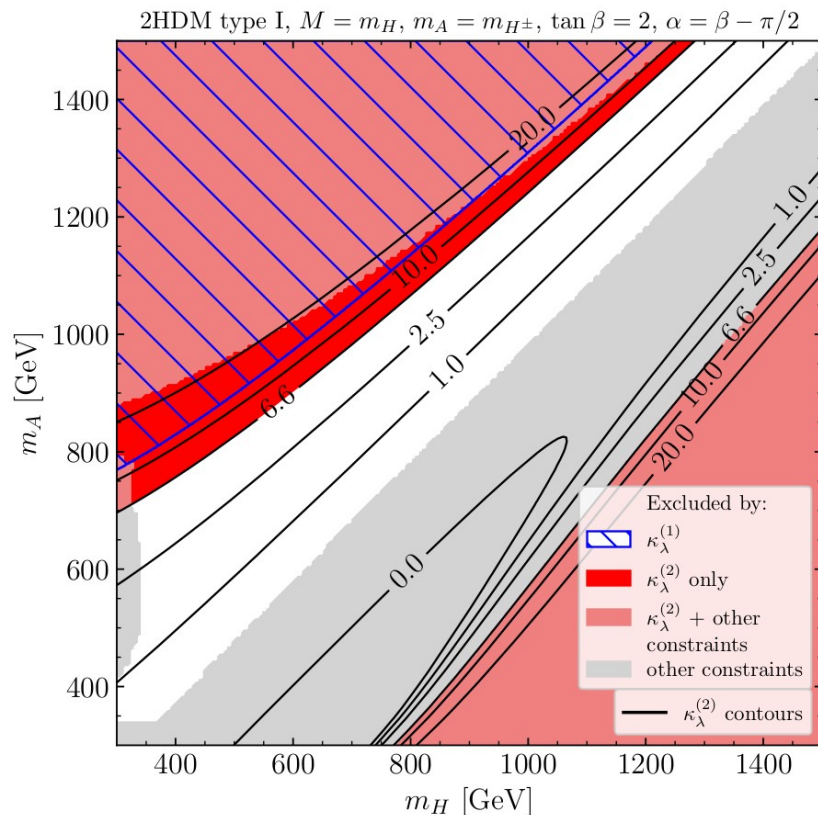
[Bahl, JB, Weiglein 2204.05269]

2HDM type I, $\alpha = \beta - \pi/2$

Figure from [Bahl, JB, Weiglein 2202.03453]

$m_A - m_{H^\pm}$ [GeV]

$m_H - m_{H^\pm}$ [GeV]

Reproducing the world average value for M_W (w/o CDF)

2HDM type-I ($\alpha = \beta - \pi/2$)

$m_A - m_{H^\pm}$ [GeV]

$m_H - m_{H^\pm}$ [GeV]

Reproducing the CDF result for M_W

Correlation between M_W and κ_λ

2HDM type I, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

Excluded by:
- $\kappa_\lambda^{(1)}$
- $\kappa_\lambda^{(2)}$ only
- $\kappa_\lambda^{(2)}$ + other constraints
- other constraints
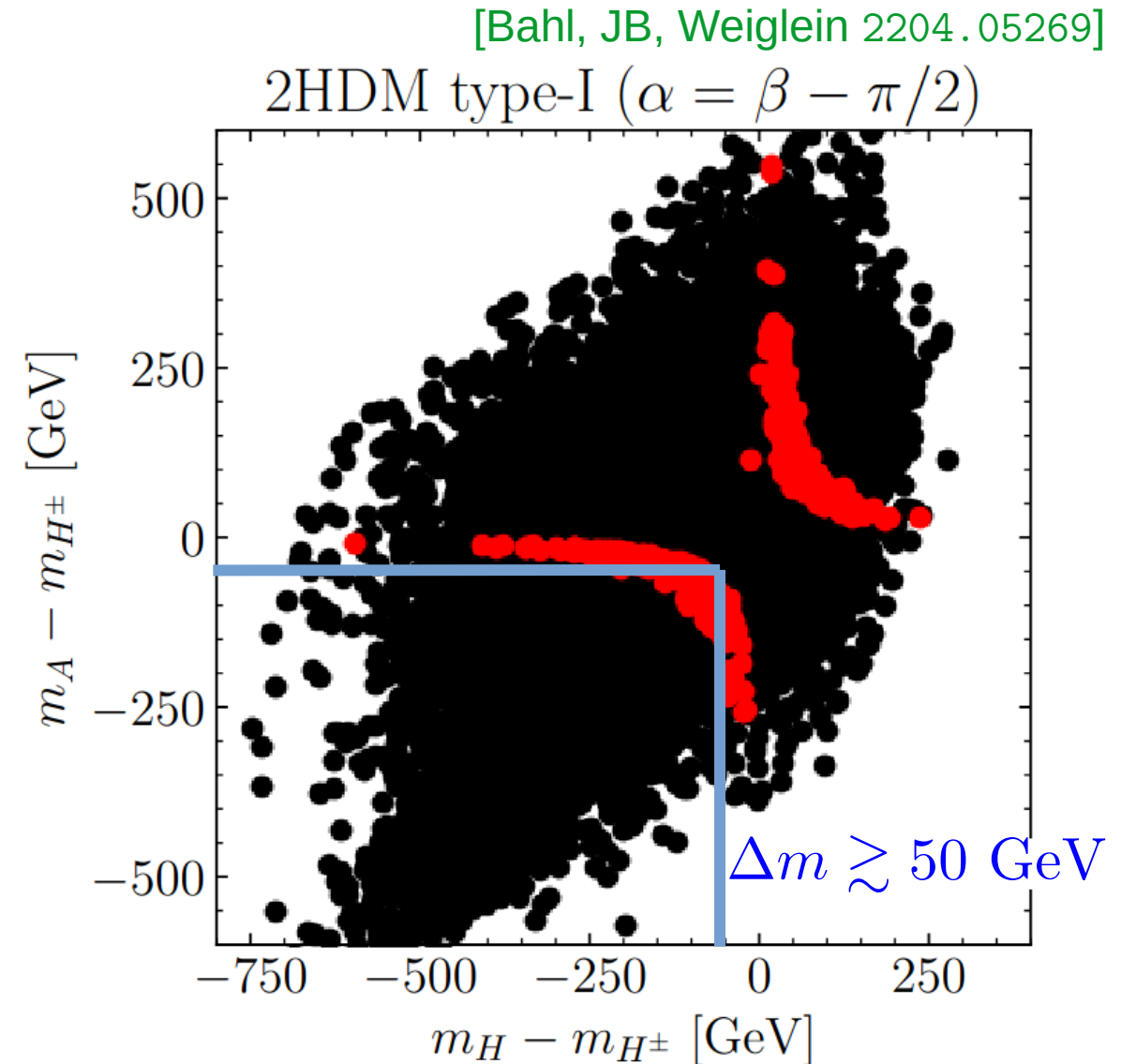- $\kappa_\lambda^{(2)}$ contours
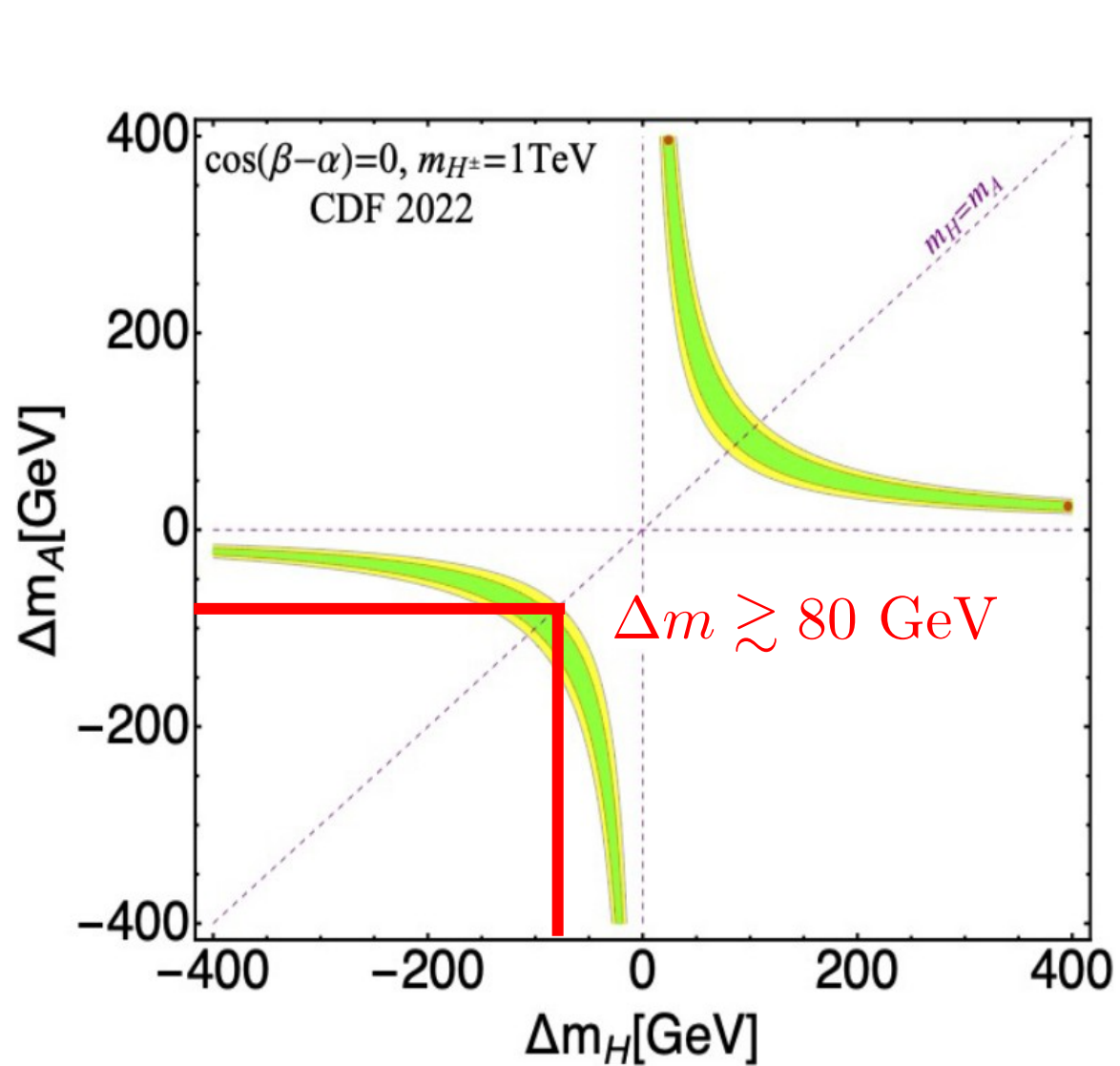

2HDM type-I $(\alpha = \beta - \pi/2)$

[Bahl, JB, Weiglein 2202.03453]
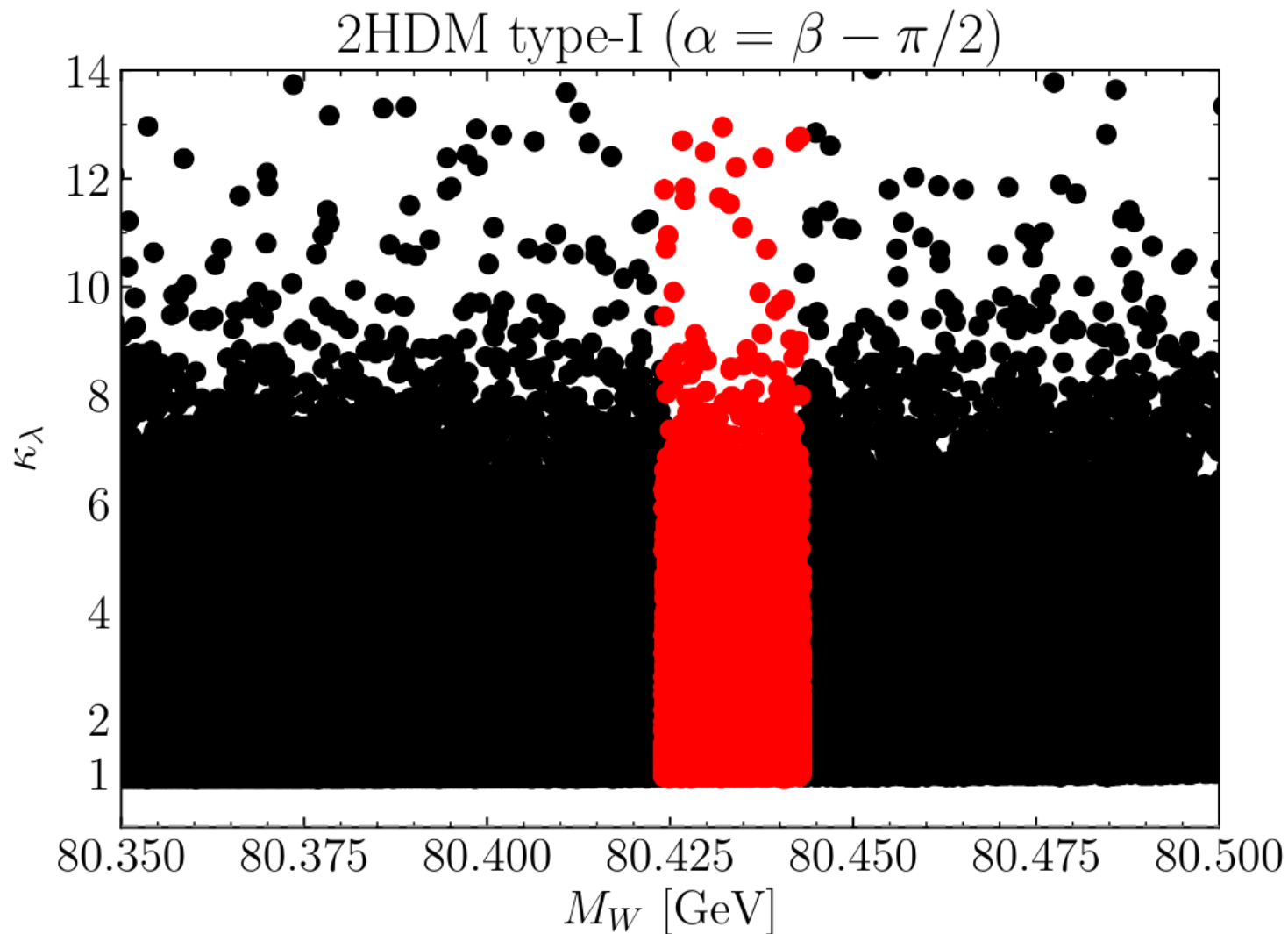
> No apparent correlation between M_W and κ_λ

> Only few points excluded by -1.0 < κ_λ < 6.6 [ATLAS-CONF-2021-052]

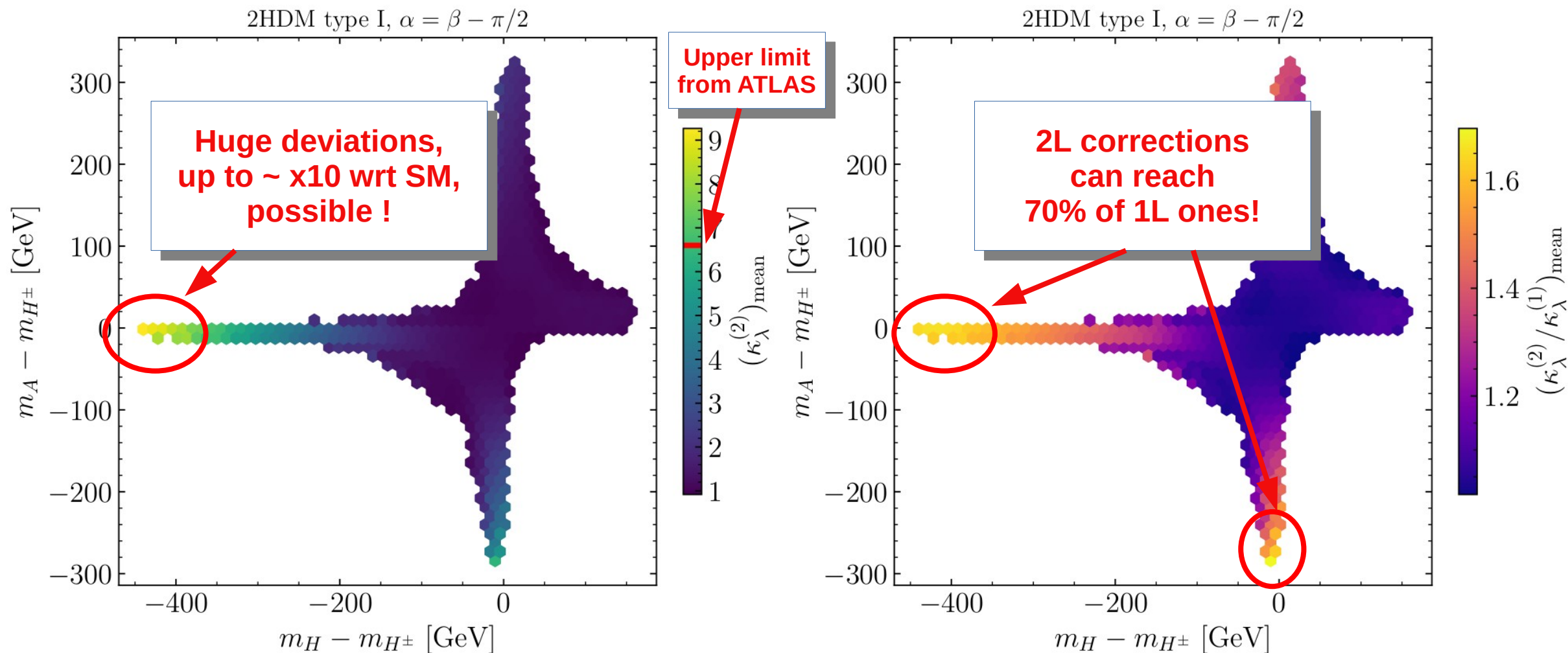# Impact of two-loop corrections to M$_W$ II

2HDM type-I ($\alpha = \beta - \pi/2$)

$\cos(\beta-\alpha)=0$, $m_{H^\pm}=1\,\mathrm{TeV}$
CDF 2022

$m_H = m_A$

$\Delta m \gtrsim 80\ \mathrm{GeV}$

$\Delta m \gtrsim 50\ \mathrm{GeV}$

Plot from [Lu, Wu, Wu, Zhu 2204.03796] using 1L S, T, U

Parameter scan results

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $\{m_H-m_{H^\pm}, m_A-m_{H^\pm}\}$ plane

- ➢ 2L corrections can become **significant** (up to ~70% of 1L)
- ➢ **Huge enhancements** (by a factor ~10) of λ_{hhh} possible for $m_A \sim m_{H^\pm}$ and $m_H \sim M$
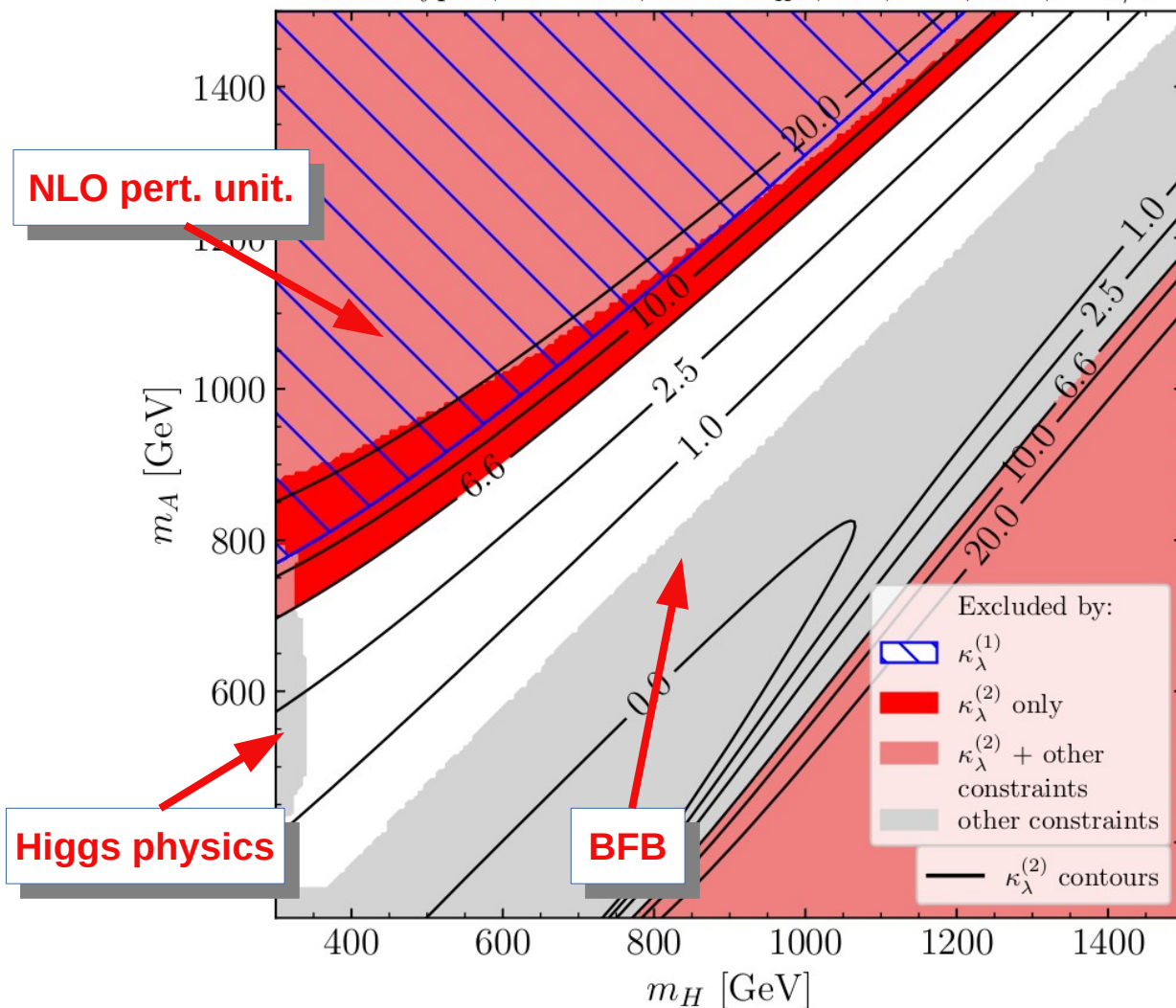
A benchmark plane in the aligned 2HDM

[Bahl, JB, Weiglein 2202.03453]

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$

➢ ***Grey area:*** area excluded by other constraints, in particular Higgs physics, boundedness-from-below (BFB), perturbative unitarity

➢ ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.6$ [in region where $\kappa_\lambda^{(2)} < -1.0$ the calculation isn't reliable]

➢ ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.6$**. Would otherwise not be excluded!

➢ ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.6$ → impact of including 2L corrections is significant!